

Problem-1: Perceptron

Initial decision boundary & Samples

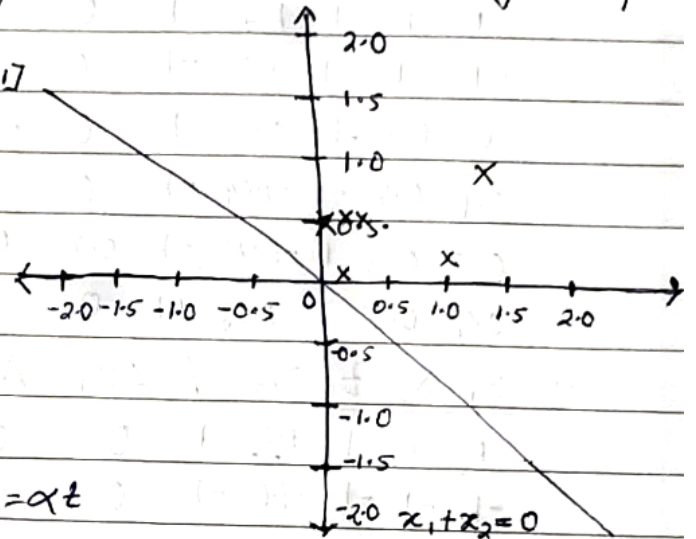
Assume weight vector of initial decision boundary $w^T x = 0$ as $w = [1, 1]$

$$\Rightarrow x_1 + x_2 = 0 \quad b = 0$$

$$Y_{in} = w^T x_i + b = w_1 x_1 + w_2 x_2 + b$$

Assume learning rate as 1.

$$y = \begin{cases} 1 & \text{if } Y_{in} > 0 \\ 0 & \text{if } Y_{in} = 0 \\ -1 & \text{if } Y_{in} < 0 \end{cases}$$



$$\Delta w_1 = \alpha t x_1, \quad \Delta w_2 = \alpha t x_2, \quad \Delta b = \alpha t$$

I)

x_1	x_2	class(t)	Y_{in}	Y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	1	2	1	0	0	0	1	1	0
-1	-1	-1	-2	-1	0	0	0	1	1	0
0	0.5	-1	0.5	1	0	-0.5	-1	1	0.5	-1
0.1	0.5	-1	-0.65	-1	0	0	0	1	0.5	-1
0.2	0.2	+1	-0.7	-1	0.2	0.2	1	1.2	0.7	0
0.9	0.5	+1	1.43	1	0	0	0	1.2	0.7	0

II)

x_1	x_2	t	Y_{in}	Y	Δw_1	Δw_2	Δb	w_1	w_2	b
+1	1	1	1.9	1	0	0	0	1.2	0.7	0
-1	-1	-1	-1.9	-1	0	0	0	1.2	0.7	0
0	0.5	-1	0.35	1	0	-0.5	-1	1.2	0.2	-1
0.1	0.5	-1	-0.78	-1	0	0	0	1.2	0.2	-1
0.2	0.2	1	-0.72	-1	0.2	0.2	1	1.4	0.4	0
0.9	0.5	1	1.46	1	0	0	0	1.4	0.4	0

III)

x_1	x_2	t	Y_{in}	Y	Δw_1	Δw_2	Δb	w_1	w_2	b
1	1	1	1.8	1	0	0	0	1.4	0.4	0
-1	-1	-1	-1.8	-1	0	0	0	1.4	0.4	0
0	0.5	-1	0.2	1	0	-0.5	-1	1.4	-0.1	-1
0.1	0.5	-1	-0.81	-1	0	0	0	1.4	-0.1	-1
0.2	0.2	1	-0.74	-1	0.2	0.2	1	1.6	0.1	0
0.9	0.5	1	1.44	1	0	0	0	1.6	0.1	0

iv) $x_1, x_2, t, y_{in}, \gamma, \Delta w_1, \Delta w_2, \Delta b, w_1, w_2, b$

1	1	1	1.7	1	0	0	0	1.6	0.1	0
-1	-1	-1	-1.7	-1	0	0	0	1.6	0.1	0
0	0.5	-1	0.05	1	0	-0.5	-1	1.6	-0.4	-1
0.1	0.5	-1	-1.04	-1	0	0	0	1.6	-0.4	-1
0.2	0.2	1	-0.76	-1	0.2	0.2	1	1.8	-0.2	0
0.9	0.5	1	1.52	+1	0	0	0	1.8	-0.2	0

v) $x_1, x_2, t, y_{in}, \gamma, \Delta w_1, \Delta w_2, \Delta b, w_1, w_2, b$

1	1	1	1.6	1	0	0	0	1.8	-0.2	0
-1	-1	-1	-1.6	-1	0	0	0	1.8	-0.2	0
0	0.5	-1	-0.1	-1	0	0	0	1.8	-0.2	0
0.1	0.5	-1	0.08	1	-0.1	-0.5	-1	1.7	-0.7	-1
0.2	0.2	1	-0.8	-1	0.2	0.2	1	1.9	-0.5	0
0.9	0.5	1	1.46	+1	0	0	0	1.9	-0.5	0

vi) $x_1, x_2, t, y_{in}, \gamma, \Delta w_1, \Delta w_2, \Delta b, w_1, w_2, b$

1	1	1	1.4	1	0	0	0	1.9	-0.5	0
-1	-1	-1	-1.4	-1	0	0	0	1.9	-0.5	0
0	0.5	-1	-0.25	-1	0	0	0	1.9	-0.5	0
0.1	0.5	-1	-0.06	-1	0	0	0	1.9	-0.5	0
0.2	0.2	1	0.28	+1	0	0	0	1.9	-0.5	0
0.9	0.5	1	1.46	+1	0	0	0	1.9	-0.5	0

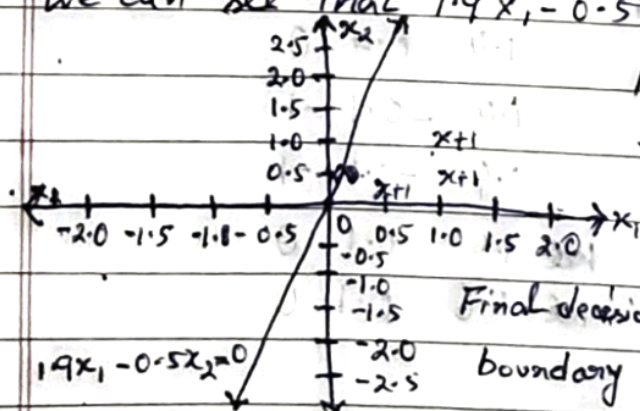
The perceptron learning algorithm covered in 6 steps!

The final weight vector of the decision boundary is $w = [1.9, -0.5]$

$$1.9x_1 + (-0.5)x_2 = 0 \Rightarrow 1.9x_1 - 0.5x_2 = 0$$

Let's plot the final decision boundary.

We can see that $1.9x_1 - 0.5x_2 = 0$ line separates the two classes



Neural network corresponding to the perceptron

