Communication System Lab 2

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(在做完所有實驗後，10/19寫結報時不小心把jupyter notebook刪掉了.....
有些code和實際機器跑的結果有所出入，是因為code是重新寫的)

1. Superdense coding

(a) In Fig.1, a quantum circuit for transmitting a 2-bit message “10” via the superdense coding protocol is demonstrated. The circuit is separated by barriers into four parts: Bell’s state generator, encoding circuit ($X^{b_1} \cdot Z^{b_2}$ for a two-bit message “b1b2”, in this case it reduces to a single X), post-rotation for Bell’s measurement, and finally the measurements in computational basis. Ideally, the measurement outcome “c_0c_1” should be same as Alice’s message “b_1b_2”. On `qasm_simulator`, the information can be reliably reconstruct by Bob with zero error rate. On real device, the noise and decoherence effects would decrease the performance of the circuit. The circuit shown in Fig.1 was executed on `ibmq_rome` and the ratio of successful reconstruction was 97.66% as shown in Fig.2.

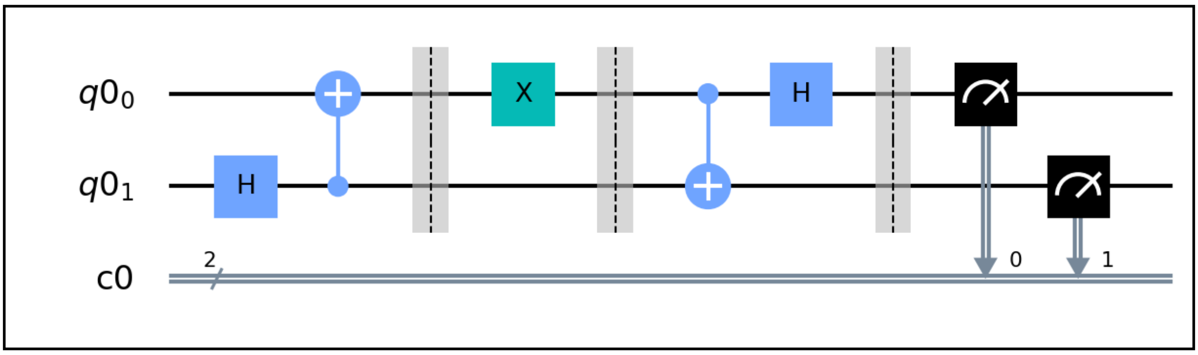

Fig.1
Quantum circuit for superdense coding

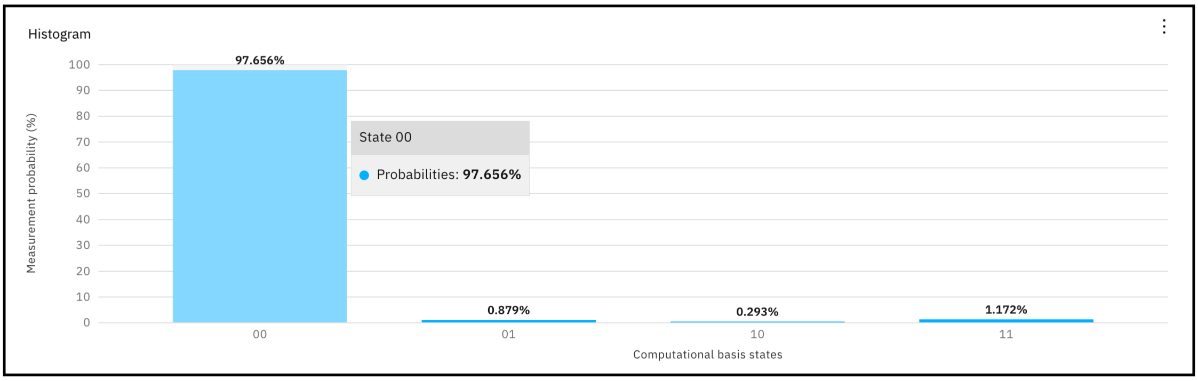

Fig.2
Result of circuit in Fig.1 executed on `ibmq_rome`

2. Teleportation

(a)

The circuit shown in Fig.3 presents a quantum teleportation between qubit #0 and #2. I used a random u3-gate for generating arbitrary single-qubit states on qubit #0, and after teleporting it to qubit #2, an inverse of the original u3-gate was used to check if the state was transmitted correctly.

*Note that in qiskit, $U_3(\theta, \phi, \lambda)^{-1} = U_3(\theta, \phi, \lambda)^{\dagger} = U_3(-\theta, -\lambda, -\phi)$.

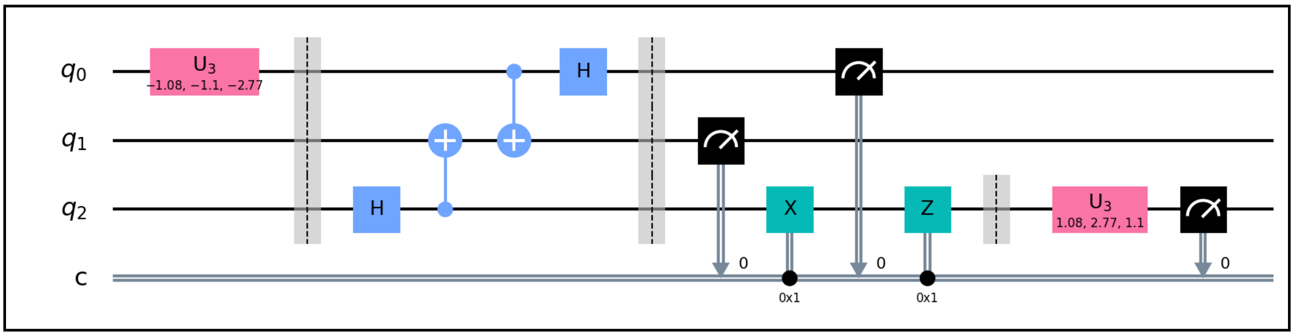

Fig.3

Quantum circuit for quantum teleportation

On simulator, the performance is perfect—in each shots the state was transmitted from qubit #0 to #2 correctly, and thus after the inverse of U, all measurement outputs are 0. Due to some (unknown to me, but I guess it's about classical-if operations) restrictions on IBMQ's devices, the circuit in Fig.3 CANNOT run successfully on them. Alternatively, I use the noisy simulator in `Aer` to obtain a qualitative estimation about the result on real device. The ratio of measuring 0 is about 90.8%.

(b) Entanglement swap

For our demonstration of entanglement swapping, a four-qubit quantum circuit is used. Qubit #0 and #3 belongs to Alice and Bob, respectively. Qubit #1 and #2 belong to Charlie, who is able to create entanglement between one of his own qubits and one of Alice's or Bob's qubit, i.e. Bell's state on (#0, #1) or (#2, #3). The goal of entanglement swap is to achieve an entanglement between Alice and Bob (#0 & #3) under this circumstance.

The circuit in Fig.4 composes of five parts: first, Charlie generates two pair of Bell's states sharing with Alice and Bob. Then, Charlie performs some local operations and local measurements. From theoretical analysis, the state of qubit #0 and #3 will collapse to one of the four Bell's states. In the third part, Charlie announces his measurement results and Bob should accordingly perform some local correction (I, X, Z, or XZ) to his state. In this stage, Alice's and Bob's qubits should now be in $|00\rangle + |11\rangle$ state. The fourth and the fifth part perform a Bell measurement on their qubits, to check if their qubits are entangled as we expected.

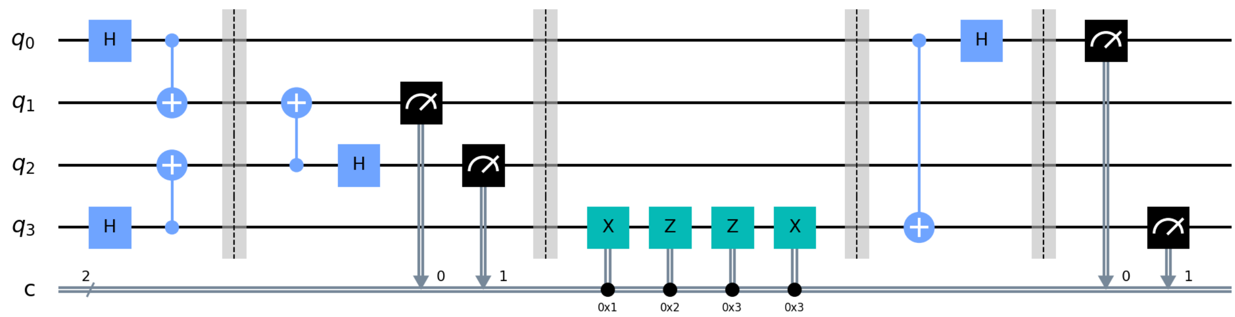

Fig.4
Quantum circuit for entanglement swap

In simulation the final measurement outcome is all “00”. Using the noisy simulator, only 80.3% of the outcome read “00”. The error rate is significantly higher than that in teleportation circuit is mainly due to the depth and the number of CNOT gates we used. In particular, our circuit design requires a 4-qubit-ring structure in qubit connectivity, which is not a common topology in current quantum devices. Implementing this circuit on real devices needs extra swap gates, and it would significantly increase the error rate of the circuit.

3. BB84 protocol

(a) The probability of that Eve correctly guesses Alice’s basis is 50%. Furthermore, even if Even used a wrong basis, Bob still has 50% probability of measuring correct outcome. Thus, Eve has a probability of 75% to guess Alice’s bit correctly. In the code marked with #3a, 500 bits are transmitted from Alice to Bob under eavesdropping. The probability of Eve’s getting correct bit in the experiment is 0.7856.

(b) The code for this part is marked “#3b” . Using Breidbart basis, the experimental probability of Eve’s correct guess has decreased to around 0.517.