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محدثه علیرضایی طهرانی

تمرین ۱

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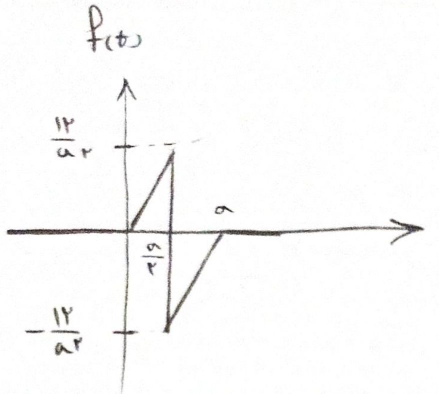

$$t \rightarrow f(t) = \begin{cases} 0 & t<0 \\ \frac{24}{a^3}t & 0\le t<\frac{a}{2} \\ +\frac{24}{a^3}t-\frac{24}{a^2} & \frac{a}{2}\le t<a \\ 0 & t\ge a \end{cases}$$

$$\rightarrow y = A_1 x + b \rightarrow \quad \frac{12}{a^2} = A_1\frac{a}{2} + \cancel{b} \qquad A_1 = \frac{2\times 12}{a^3} = \frac{24}{a^3}$$

$$0 = 0 + b_1 \rightarrow b_1 = 0$$

$$\rightarrow y = A_2 x + b_2 \rightarrow \quad -\frac{12}{a^2} = A_2\frac{a}{2} + b_2 \qquad -\frac{12}{a^2} = A_2\Big(\underbrace{\frac{a}{2}-a}_{-\frac{a}{2}}\Big)$$

$$0 = A_2 a + b_2 \rightarrow b_2 = -A_2 a$$

$$\rightarrow +\frac{24}{a^3} = A_2 \qquad b_2 = -\frac{24}{a^2} \rightarrow f(t) = +\frac{24}{a^3}t - \frac{24}{a^2}$$

$$\mathcal{L} \rightarrow \int_0^\infty f(t)e^{-st}\,dt \rightarrow ① = 0 \qquad ④ = 0$$

$$② = \int_0^{\frac{a}{2}} \frac{24}{a^3} t e^{-st}\,dt \rightarrow \frac{24}{a^3}\int_0^{\frac{a}{2}} te^{-st}\,dt = \Big[-ste^{-st} - s^2e^{-st}\Big]\Big|_0^{\frac{a}{2}}$$

$$\rightarrow -\frac{24s}{a^3}\times e^{-st}[t-s] = -\frac{24}{a^3}\times se^{-\frac{as}{2}}\Big(\frac{a}{2}-s\Big) = -\frac{12}{a^2}se^{-\frac{as}{2}} +$$

$$+\frac{24}{a^3}s^2e^{-\frac{a}{2}s}\Big(-\Big(-\frac{24}{a^3}s\times 1(0-s)\Big)\Big) + \frac{24}{a^3}s^2 = F(s)_2$$

$$③ = \int_{\frac{a}{2}}^{a}\Big(-\frac{24}{a^3}t + \frac{24}{a^2}\Big)e^{-st}\,dt = \int_{\frac{a}{2}}^{a} +\frac{24}{a^3}te^{-st}\,dt + \int_{\frac{a}{2}}^{a} -\frac{24}{a^2}e^{-st}\,dt$$

انتگرال بالا با حدود متفاوت ← مشابه انتگرال بالا با کران متفاوت و a در مخرج و ضریب منفی

$$\rightarrow +\frac{24}{a^3}\Big(\Big[-s\times ae^{-sa} - s^2e^{-sa}\Big] - \Big[-\frac{sa}{2}e^{-\frac{a}{2}s} - s^2e^{-\frac{a}{2}s}\Big]\Big)$$

$$-\frac{24}{a^2}\Big(\Big[-s\times a\times e^{-as} - s^2e^{-as}\Big] - \Big[-s\frac{a}{2}e^{-\frac{a}{2}s} - s^2e^{-\frac{a}{2}s}\Big]\Big)$$

$$= \frac{24(1-a)}{a^3}\Big[\Big(-se^{-as}(a+s)\Big) - \Big(-se^{-\frac{a}{2}s}\Big(\frac{a}{2}+s\Big)\Big)\Big] = F(s)_3$$

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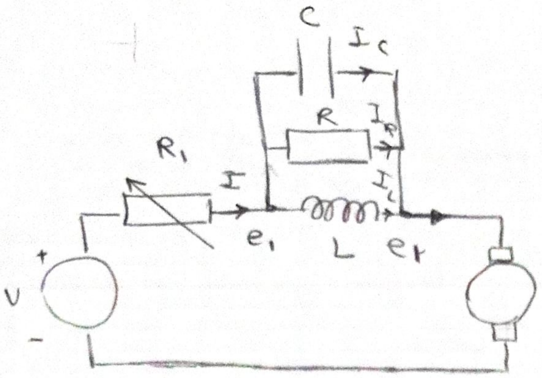
الف) مدلسازی



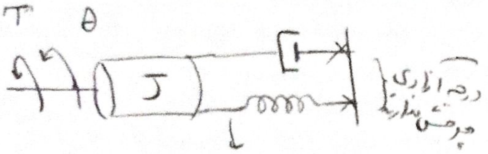
$\omega = \frac{d\theta}{dt}$ $\quad V_C = \frac{1}{SC} I_C \quad V_L = LIs_L \quad V_R = RI_R$

→ $V_C = V_L = V_R$

$V = k_v \omega \quad f_b = bV \quad f_k = kx \quad V_{emf} = k_v \omega(t)$

→ $b\frac{dx}{dt}$

$I = I_L + I_C + I_R$



$V(t) = R_1 i(t) + L\frac{di_L(t)}{dt} + V_{emf}$

هر دو دیسک هم با θ می‌چرخند → $V(t) = R_1 i(t) + L\frac{di_L(t)}{dt} + k_v\omega(t)$

→ T_ω گشتاور چرخشی $- T_{\text{دمپر}} - T_{\text{ورودی}} = \frac{d^2\theta}{dt^2}J$ یعنی $\frac{d\omega}{dt}J$

→ $\frac{1}{k}\frac{dT}{dt} - B\cdot\omega - T_d = \frac{d\omega}{dt}J$ → $k_m i(t) = B\omega + J\frac{d\omega}{dt} + k\theta(t)$

$\mathcal{L}$ → $V(s) = R_1 I(s) + LsI_L + k_v\omega(s)$

$k_m I(s) = B\omega(s) + Js\omega(s) + k\theta(s)$

$RI_R = \frac{1}{SC}I_C = sLI_L \quad I_R = \frac{Ls}{R}I_L \quad I_C = s^2CLI_L$

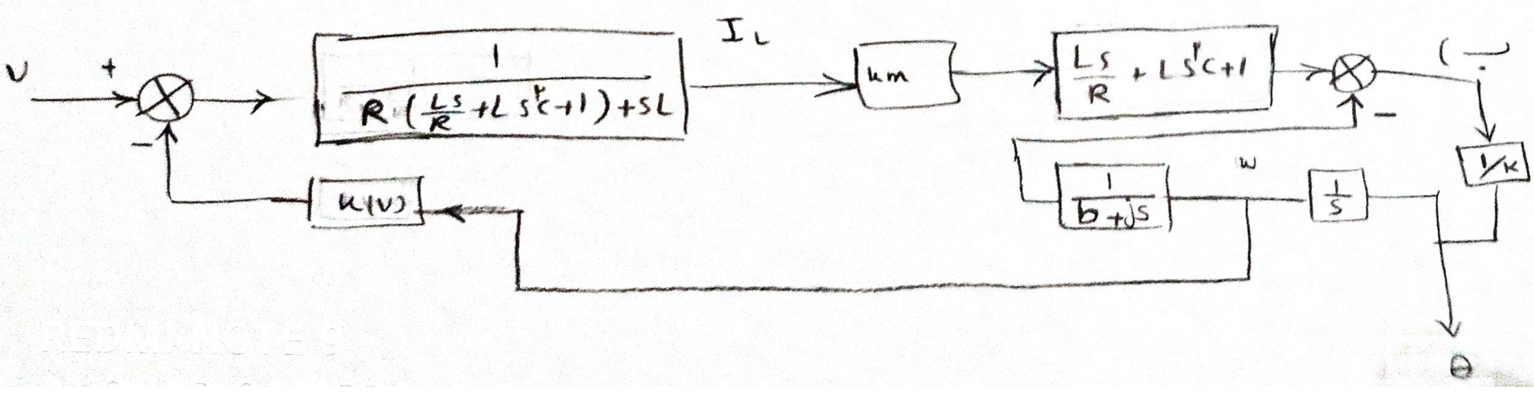
$I = \frac{Ls}{R}I_L + s^2CLI_L + I_L = I = \left(\frac{Ls}{R} + Ls^2C + 1\right)I_L$

جایگذاری در روابط بالا → $I_L R_1\left(\frac{Ls}{R} + Ls^2C + 1\right) + LsI_L + k_v\omega(s) = V(s)$

$k_m I_L\left(\frac{Ls}{R} + Ls^2C + 1\right) = b\omega(s) + Js\omega(s) + k\theta(s)$

→ $\frac{I_L}{V(s) - k_v\omega(s)} = \frac{1}{R_1\left(\frac{Ls}{R} + Ls^2C + 1\right) + sL} = (V(s) - k_v\omega(s)) \times \frac{1}{R_m} = I_L$

ب)



ج)

$V_{in} = u(t)$ $\quad G(s) = \frac{\theta}{R_1}$

$\hookrightarrow V_{in}(s) = \frac{1}{s}$

$$\theta = \frac{k_m I_L\left(\frac{Ls}{R} + Ls^2c + 1\right) - b\,\omega(s) + js\,\omega(s)}{k}$$

$$R_1 = \frac{\cancel{V_s}\frac{1}{s} - LsI_L - k_v\,\omega(s)}{I_L\left(\frac{Ls}{R} + Ls^2c + 1\right)}$$

$G(s) = \frac{\theta}{R_1}$

$$G(s) = \frac{\left[k_m I_L\left(\frac{Ls}{R} + Ls^2c + 1\right) + \omega(s)\left(js - b\right)\right]\left[I_L\left(\frac{Ls}{R} + Ls^2c + 1\right)\right]}{k\left[\frac{1}{s} - LsI_L - k_v\,\omega(s)\right]}$$

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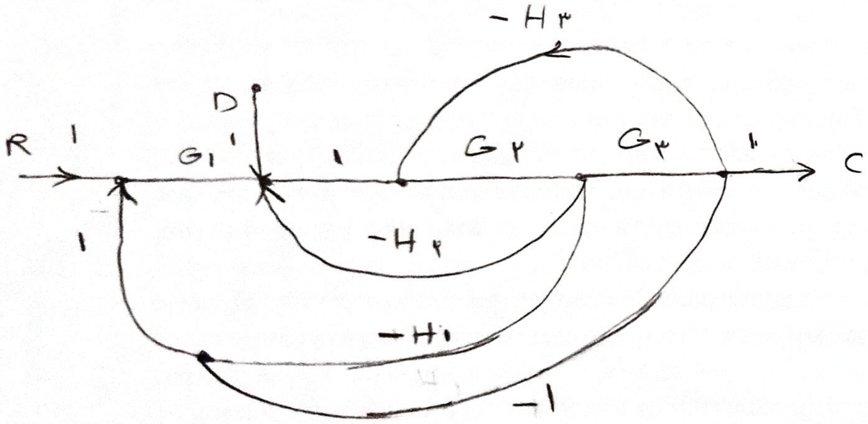
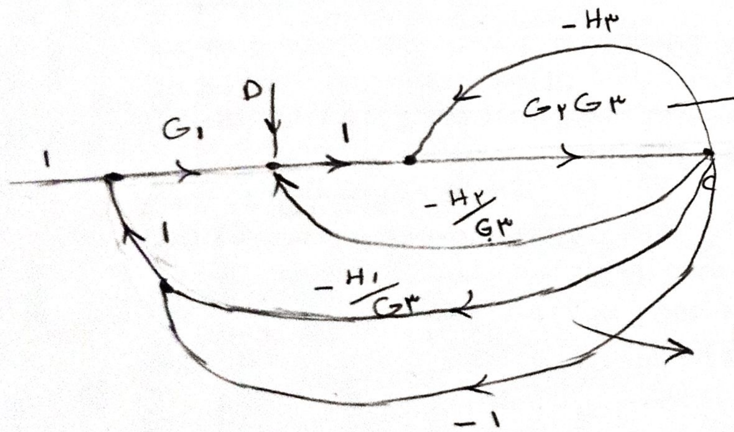
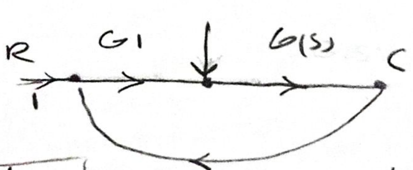




① $\frac{G_2G_3}{1 + H_3G_2G_3}$

② $-1-\frac{H_1}{G_3}$



③
$$\frac{\frac{G_2G_3}{1+H_3G_2G_3}}{1 + \frac{G_2\cancel{G_3}}{1+H_3G_2G_3} \times \frac{H_2}{\cancel{G_3}}} = \frac{\frac{G_2G_3}{1+H_3G_2G_3}}{1 + \frac{G_2H_2}{1+H_3G_2G_3}} = G(s)$$

$$Y(s) = \sum_{k=1}^{N} \frac{M_k \Delta_k}{\Delta}$$

$$L_1 = G_1 A\left(-1 - \frac{H_1}{G_3}\right)$$

$$M_1 = \frac{\dfrac{G_1 G_2 G_3}{1 + H_3 G_2 G_3}}{1 + \dfrac{G_2 H_2}{1 + H_3 G_2 G_3}}$$

$$\Delta = 1 - L_1 = 1 + G_1 A\left(1 + \frac{H_1}{G_3}\right)$$

$$\Delta_1 = 1$$

$$\rightarrow \frac{\dfrac{\dfrac{G_1 G_2 G_3}{1 + H_3 G_2 G_3}}{1 + \dfrac{G_2 H_2}{1 + H_3 G_2 G_3}}}{1 + \left[\dfrac{\dfrac{G_1 G_2 G_3}{1 + H_3 G_2 G_3}}{1 + \dfrac{G_2 H_2}{1 + H_3 G_2 G_3}}\left(1 + \dfrac{H_1}{G_3}\right)\right]} = \frac{\dfrac{G_1 G_2 G_3}{1 + H_3 G_2 G_3 + G_2 H_2}}{1 + \dfrac{G_1 G_2 G_3 + G_1 G_2 H_1}{1 + H_3 G_2 G_3 + G_2 H_2}}$$

$$= \frac{G_1 G_2 G_3}{1 + H_3 G_2 G_3 + G_2 H_2 + G_1 G_2 G_3 + G_1 G_2 H_1}$$