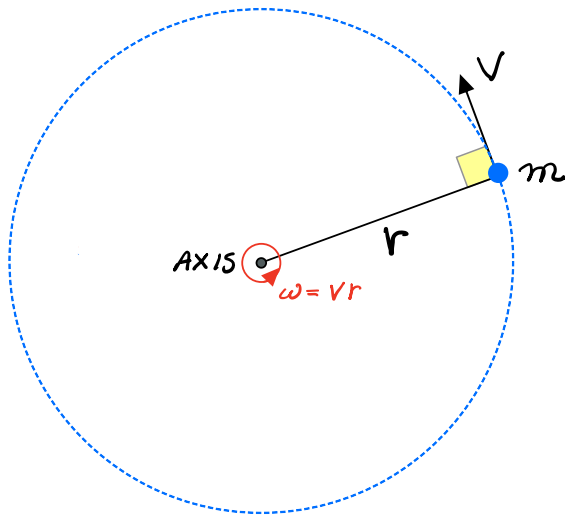


(SIDDON)

10. Angular Momentum (L)

1. Rotating Points



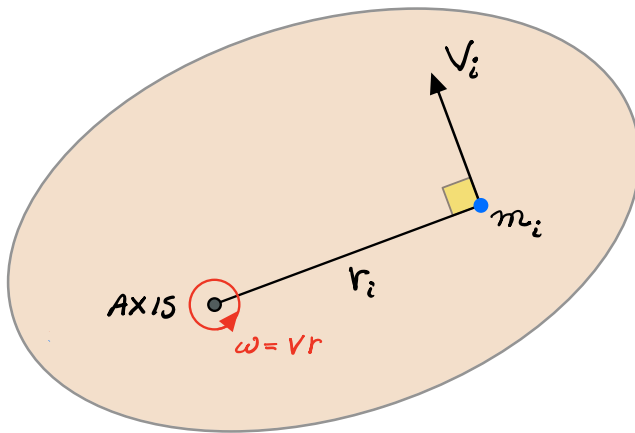
1. $\phi = m v$

2. $\underbrace{r}_{L} \underbrace{\phi}_{I} = m r^2 \underbrace{\left(\frac{v}{r}\right)}_{\omega}$

3. $L = I \omega$

$L > 0$ CCW $L < 0$ CW

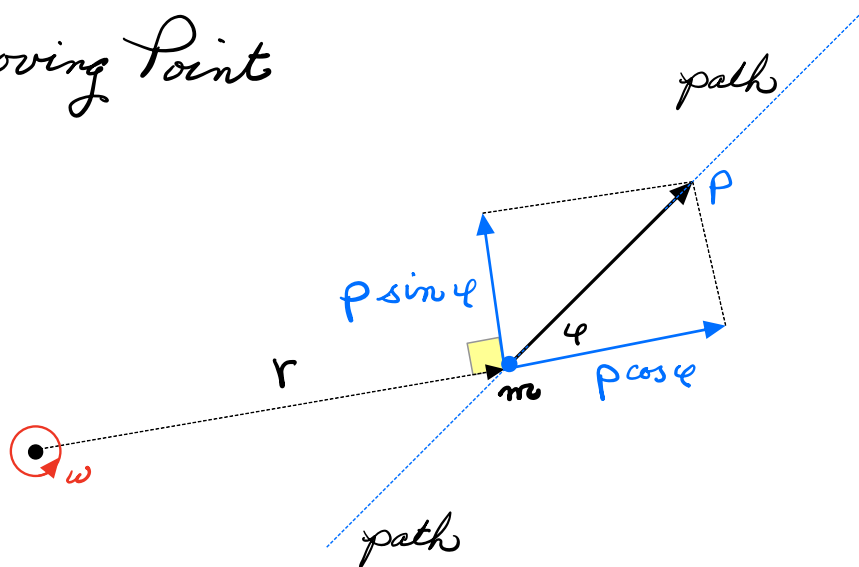
2. Rotating Object



$$L = \sum L_i = \underbrace{(\sum m_i r_i^2)}_I \omega$$

$\rightarrow L = I \omega$

3. Moving Point

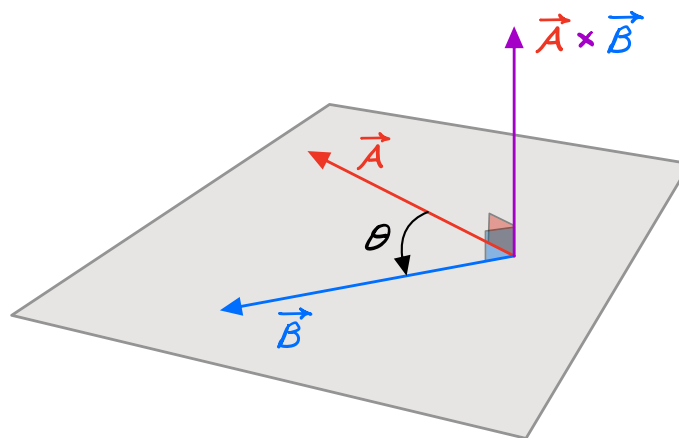


1. $L = r p \sin \varphi$

2. $\vec{L} = \vec{r} \times \vec{p}$ (vector definition)

magnitude: $|\vec{A} \times \vec{B}| = AB \sin \theta$

direction: Right-Hand-Rule

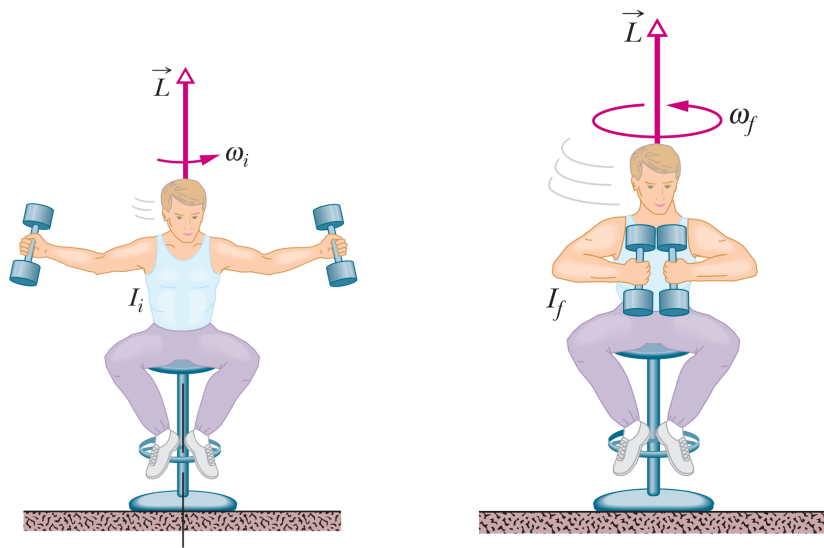


4. Conservation of Angular Momentum

1. $\frac{dL}{dt} = I\alpha = \tau_{NET}$

2. If $\tau_{NET} = 0 \rightarrow L = I\omega = \text{const}$

3. $I_F \omega_F = I_I \omega_I$



31. A skater rotates with her arms crossed at an angular speed of 1.30 revolutions per second. She extends her arms, and her angular speed drops to 0.8 rev/sec. The factor by which her moment of inertia has changed is closest to

- A. $I_f/I_i = 0.38$
- B. $I_f/I_i = 0.62$
- C. $I_f/I_i = 1.6$
- D. $I_f/I_i = 2.6$
- E. $I_f/I_i = 3.6$



$$1. \quad L_F = L_I \longrightarrow I_F \omega_F = I_I \omega_I$$

$$2. \quad \frac{I_F}{I_I} = \frac{\omega_I}{\omega_F} = \frac{1.3}{0.8} = 0.625$$

37. Due to violent internal processes, a spinning star collapses and its rotational inertia I shrinks to $1/3$ of its initial value ($I_f = \frac{1}{3}I_0$). How does the final angular speed after the collapse ω_f compare to the angular speed before the collapse ω_0 ?

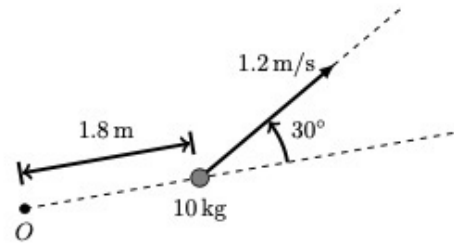
- A. $\omega_f = 9\omega_0$
- B. $\omega_f = \sqrt{3}\omega_0$
- C. $\omega_f = 3\omega_0$
- D. $\omega_f = \omega_0$
- E. the outcome depends on the details of the internal physics of the star.

$$1. \quad L_F = L_I \longrightarrow I_F \omega_F = I_I \omega_I$$

$$2. \quad \omega_F = \left(\frac{I_I}{I_F} \right) \omega_I = 3\omega_0$$

37. This 10 kg particle is 1.8 m from the origin O , moving at 1.2 m/s in the direction shown. Relative to O , the magnitude of the particle's angular momentum is

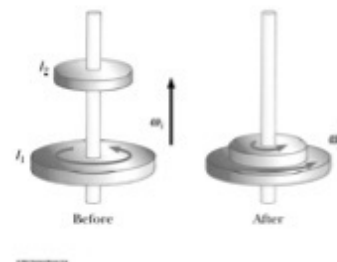
- A. 11 kg·m²/s
- B. 41 kg·m²/s
- C. 24 kg·m²/s
- D. 22 kg·m²/s
- E. 32 kg·m²/s



$$L = r p \sin \phi$$

$$= (1.8)(10)(1.2) \sin 30^\circ = 10.8 \text{ kg} \cdot \text{m}^2/\text{s}$$

30. A disk with moment of inertia I_1 rotates about a vertical, frictionless axle with angular speed ω_i . A second disk, with moment of inertia $I_2 = 1/2 I_1$ and not rotating, drops onto the first disk. Because of friction between the disks, they eventually reach a common angular speed ω_f . This final angular speed is closest to:

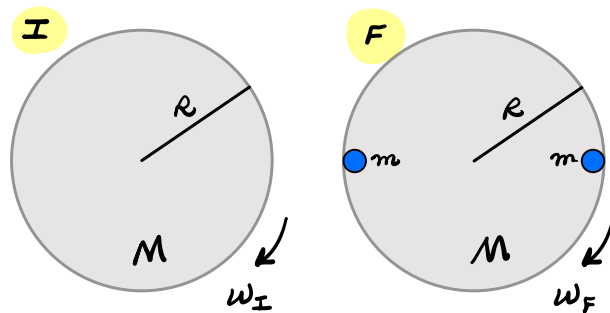


- A. $\omega_f = 1/2 \omega_i$
- B. $\omega_f = 2/3 \omega_i$
- C. $\omega_f = \omega_i$
- D. $\omega_f = 3/2 \omega_i$
- E. $\omega_f = 2 \omega_i$

$$1. \quad L_F = L_i \longrightarrow I_F \omega_F = I_i \omega_i$$

$$2. \quad \omega_F = \left(\frac{I_i}{I_F} \right) \omega_i = \left(\frac{I_1}{I_1 + I_2} \right) \omega_i = \frac{2}{3} \omega_i$$

45. || A 2.0 kg, 20-cm-diameter turntable rotates at 100 rpm on frictionless bearings. Two 500 g blocks fall from above, hit the turntable simultaneously at opposite ends of a diameter, and stick. What is the turntable's angular velocity, in rpm, just after this event?



Given: $M = 2 \text{ kg}$
 $m = 0.5 \text{ kg}$
 $R = 0.1 \text{ m}$
 $\omega_I = 10.47 \text{ rad/s}$

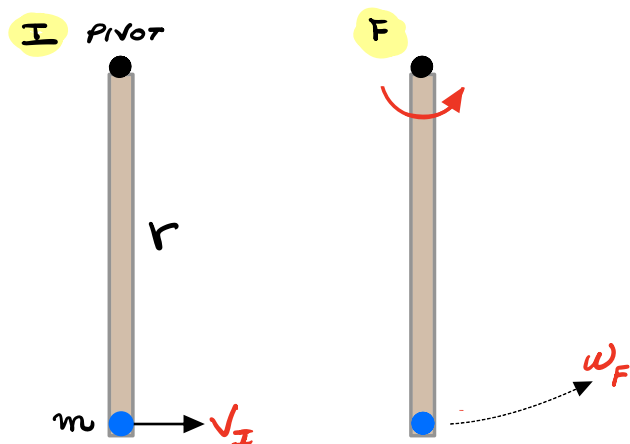
1. $I_I = \frac{1}{2}MR^2 = 0.01 \text{ kg} \cdot \text{m}^2$

$I_F = \frac{1}{2}MR^2 + mR^2 + mR^2 = 0.02 \text{ kg} \cdot \text{m}^2$

2. $I_I \omega_I = I_F \omega_F \rightarrow \omega_F = 5.24 \text{ rad/s}$

50 rpm

78. || A 10 g bullet traveling at 400 m/s strikes a 10 kg, 1.0-m-wide door at the edge opposite the hinge. The bullet embeds itself in the door, causing the door to swing open. What is the angular velocity of the door just after impact?



Given:

$$\begin{aligned} m &= 0.01 \text{ kg} \\ v_I &= 400 \text{ m/s} \\ M &= 10 \text{ kg} \\ r &= 1 \text{ m} \end{aligned}$$

$$1. \quad L_I = r p \sin \phi = r m v \sin \phi = r m v_I \quad \text{where } \phi = 90^\circ$$

$$L_F = I \omega = \left(\underbrace{\frac{1}{3} M r^2}_{\text{door}} + \underbrace{m r^2}_{\text{bullet (rotating particle)}} \right) \omega_F$$

$$2. \quad L_F = L_I \longrightarrow \omega_F = \frac{r m v_I}{\frac{1}{3} M r^2 + m r^2} = \boxed{1.2 \text{ rad/s}}$$

