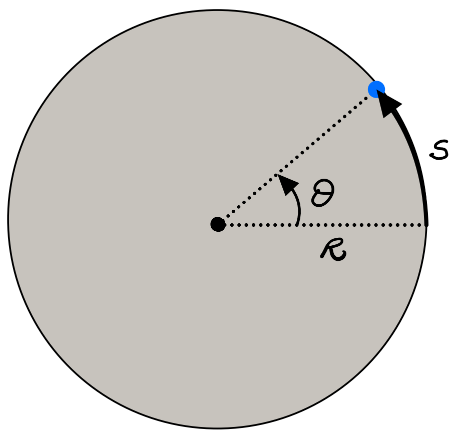
(SIDDON)

9. Rotation

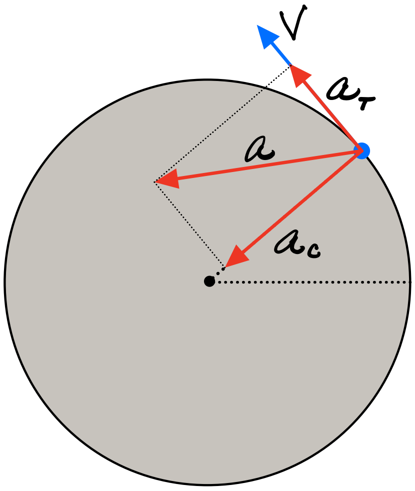
A. Kinematics



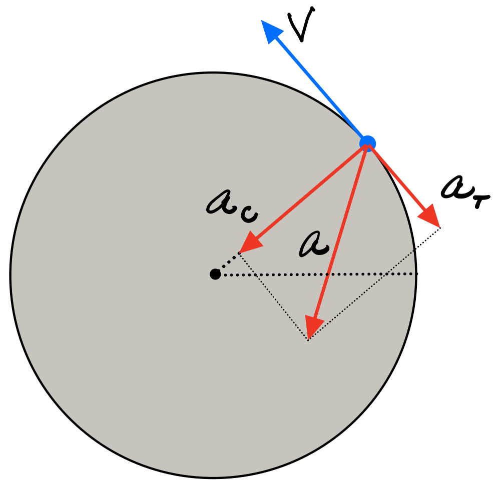
Definition of θ in radians

$\boxed{\theta \equiv s/R}$

Variables:	Angular	Linear
position	θ	$s = \theta R$
velocity	$\omega = \frac{d\theta}{dt}$	$v = \frac{ds}{dt} = \omega R$
tangential acceleration	$\alpha = \frac{d\omega}{dt}$	$a_T = \frac{dv}{dt} = \alpha R$
radial (centripetal) acceleration		$a_c = \frac{v^2}{R} = \omega^2 R$

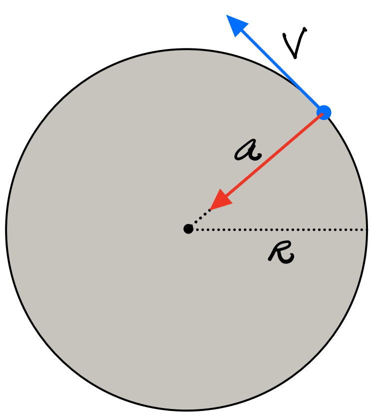


speeding up



slowing down

c) $\alpha = \phi$ Uniform Circular Motion



$V = \omega R$

$a = V^2/R = \omega^2 R$

$V = 2\pi R / T$

T: period of rotation

d) α = constant (1D Kinematics)

a) $\frac{d\omega}{dt} = \alpha \rightarrow \omega - \omega_0 = \int_0^t \alpha\, dt = \alpha t$

$$\rightarrow \boxed{\omega = \omega_0 + \alpha t}$$

b) $\frac{d\theta}{dt} = \omega \rightarrow \theta - \theta_0 = \int_0^t \omega\, dt = \omega_0 t + \frac{1}{2}\alpha t^2$

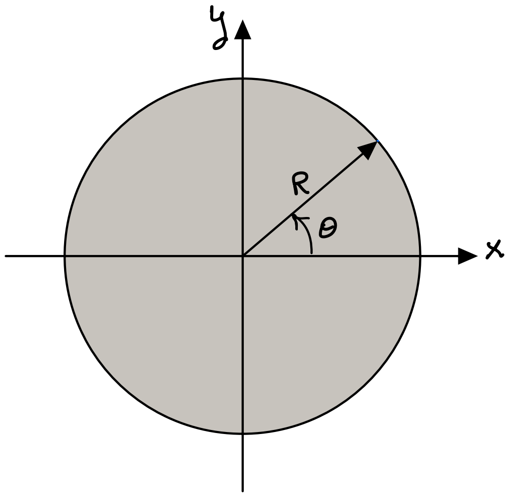
$$\rightarrow \boxed{\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2}$$

c) Combine (a) & (b) to eliminate t!

1. $\omega^2 = \omega_0^2 + 2\alpha\omega_0 t + \alpha^2 t^2$ (square eqn a)
2. $\omega^2 = \omega_0^2 + 2\alpha\left(\omega_0 t + \frac{1}{2}\alpha t^2\right)$ (insert eqn b)

$$\rightarrow \boxed{\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)}$$

Circular Motion (derivation)



$$\boxed{\omega = d\theta/dt}$$

$$\boxed{\alpha = d\omega/dt}$$

1. $\vec{R} = R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}$

2. $\vec{V} = -R\omega\sin\theta\,\hat{i} + R\omega\cos\theta\,\hat{j} \longrightarrow \boxed{V = R\omega}$

   $\vec{R}\cdot\vec{V} = \phi \longrightarrow \boxed{\vec{V} \text{ is tangent}}$

3. $\vec{a} = -\omega^2(R\cos\theta\,\hat{i} + R\sin\theta\,\hat{j}) + \frac{\alpha}{\omega}(-R\omega\sin\theta\,\hat{i} + R\omega\sin\theta\,\hat{j})$

$$= -\omega^2\vec{R} + \frac{\alpha}{\omega}\vec{V} = -\frac{V^2}{R}\hat{R} + \alpha R\,\hat{V} = \vec{a}_c + \vec{a}_T$$

$$\longrightarrow \boxed{\begin{array}{l} a_c = \frac{V^2}{R} \text{ towards center} \\ a_T = \alpha R \text{ tangent} \end{array}}$$

32. • A 12-m-radius Ferris wheel rotates once each 27 s. (*a*) What is its angular speed (in radians per second)? (*b*) What is the linear speed of a passenger? (*c*) What is the acceleration of a passenger?

a) $\omega = \frac{2\pi}{27} = \boxed{0.233 \text{ rad/s}}$

b) $v = \omega R = \boxed{2.79 \text{ m/s}}$

c) $a_c = \frac{v^2}{R} = \boxed{0.65 \text{ m/s}^2}$

34. • What is the angular speed of Earth in radians per second as it rotates about its axis?

$$\omega = \frac{2\pi}{24 \cdot 3600} = \boxed{7.27 \times 10^{-5} \text{ rad/s}}$$

36. • A bicycle has 0.750-m-diameter wheels. The bicyclist accelerates from rest with constant acceleration to 24.0 km/h in 14.0 s. What is the angular acceleration of the wheels?

1. $v = \cancel{v_0}^{0} + at \longrightarrow a = \boxed{0.476 \text{ m/s}^2}$

2. $\alpha = a/R = \boxed{1.27 \text{ rad/s}^2}$

29. • A wheel released from rest is rotating with constant angular acceleration of 2.6 rad/s^2. At 6.0 s after the release: (*a*) What is its angular speed? (*b*) Through what angle has the wheel turned? (*c*) How many revolutions has it completed? (*d*) What is the linear speed and what is the magnitude of the linear acceleration of a point 0.30 m from the axis of rotation? SSM

a) $\omega = \cancel{\omega_0}^{\phi} + \alpha t = \boxed{15.6 \text{ rad/s}}$

b) $\theta = \cancel{\theta_0}^{\phi} + \cancel{\omega_0}^{\phi} t + \frac{1}{2}\alpha t^2 = \boxed{46.8 \text{ rad}}$

c) $rev = \theta / 2\pi = \boxed{7.45 \text{ rev}}$

d) $v = \omega R = \boxed{4.7 \text{ m/s}}$

$a_T = \alpha R = \boxed{0.78 \text{ m/s}^2}$

12. A centrifuge spinning at 16 rad/s is brought to rest at a constant angular acceleration of magnitude 8.0 rad/s^2. Through how many radians does the centrifuge spin as it winds down?

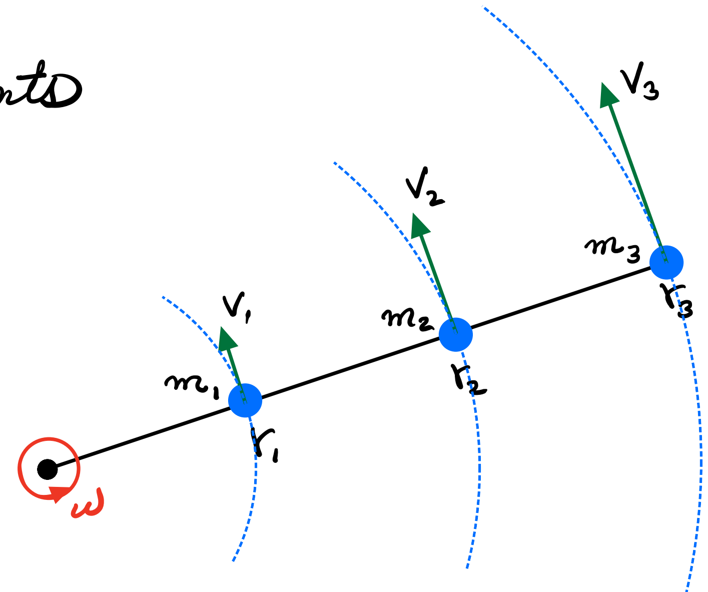
A. 4.0 rad
B. 32 rad
C. 2.0 rad
D. 8.0 rad
E. 16 rad

Given: $\omega_0 = 16$ rad/s   Find: $\theta - \theta_0$
$\omega = \phi$
$\alpha = -8$ rad/s^2 (slowing down)

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0) \longrightarrow \theta - \theta_0 = \boxed{16 \text{ rad}}$

B. Rotational Kinetic Energy

1. Points



1) $K = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 + \frac{1}{2} m_3 V_3^2$ Kinetic Energy

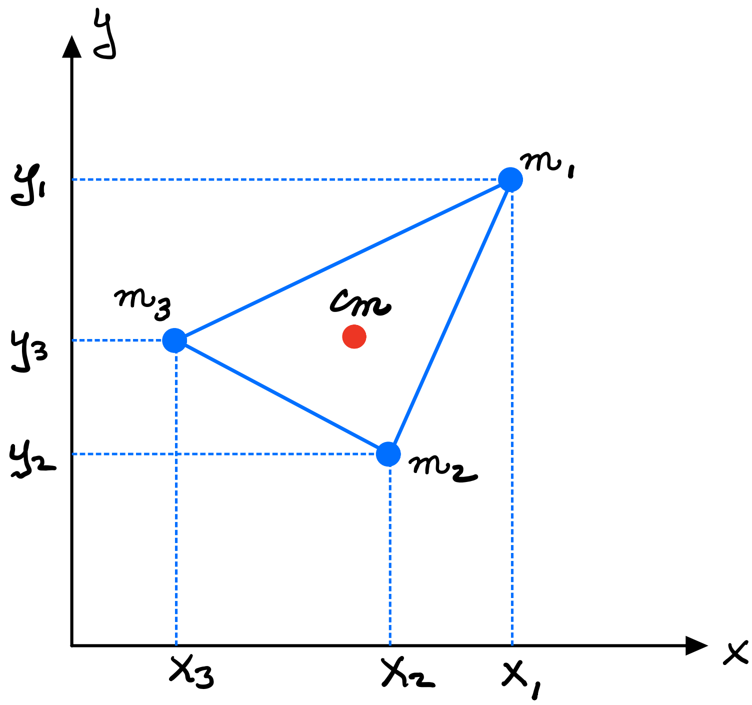
2) $V = \omega r$

3) $K = \frac{1}{2} \underbrace{(m_1 r_1^2 + m_2 r_2^2 + m r_3^2)}_{I \text{ (moment of inertia)}} \omega^2$

4) $\boxed{K = \frac{1}{2} I \omega^2}$

5) Moment of Inertia $\boxed{I = m r^2}$ (point)

Potential Energy



1. $$U = m_1 g y_1 + m_2 g y_2 + m_3 g y_3 + \cdots$$
$$= (m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots)\, g$$

2. $$y_{cm} = \frac{(m_1 y_1 + m_2 y_2 + m_3 y_3 + \cdots)}{(m_1 + m_2 + m_3 + \cdots)}$$ m

3. $$\boxed{U = m g y_{cm}}$$

34. These three small spheres (they can be approximated as point particles) are fastened to a massless rod that spins around the pivot. The speed of the sphere furthest out is 4.2 m/s. The kinetic energy of rotation of this system is

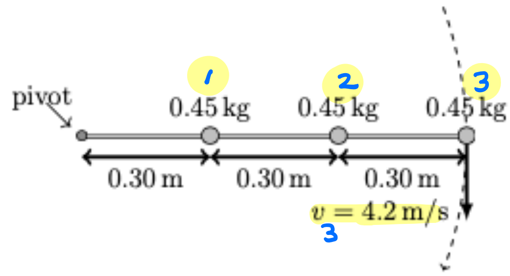
A. 8.0 J

B. 4.0 J

C. 12 J

D. 24 J

E. 6.2 J



1. $I = m_1 R_1^2 + m_2 R_2^2 + m_3 R_3^2$

$= (0.45)(0.3)^2 + (0.45)(0.6)^2 + (0.45)(0.9)^2$

$= 0.567 \text{ kg m}^2$

2. $V_3 = \omega R_3 \longrightarrow \omega = \frac{4.2}{0.9} = 4.667 \text{ rad/s}$

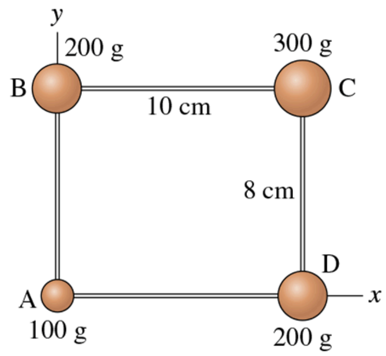
3. $K = \frac{1}{2} I \omega^2 = \boxed{6.18 \text{ J}}$

13. || The four masses shown in FIGURE EX12.13 are connected by massless, rigid rods.

a. Find the coordinates of the center of mass.

b. Find the moment of inertia about an axis that passes through mass A and is perpendicular to the page.

a)

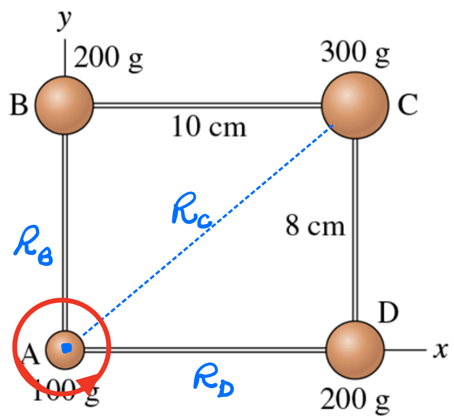


	m	x	y
A	0.1	0.00	0.00
B	0.2	0.00	0.08
C	0.3	0.10	0.08
D	0.2	0.10	0.00

$$x_{cm} = \frac{m_A x_A + m_B x_B + m_c x_c + m_D x_D}{m_A + m_B + m_c + m_D} = \boxed{6.25\text{cm}}$$

$$y_{cm} = \frac{m_A y_A + m_B y_B + m_c y_c + m_D y_D}{m_A + m_B + m_c + m_D} = \boxed{5.0\text{cm}}$$

b)



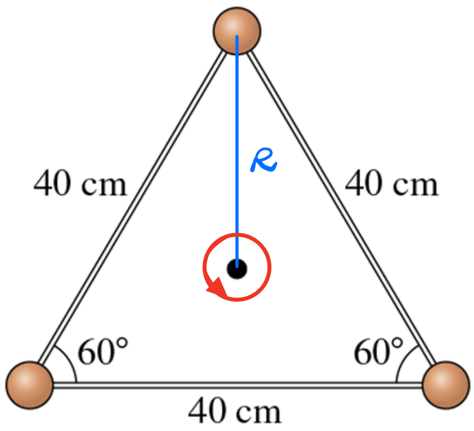
	m	R
A	0.1	0.00
B	0.2	0.08
C	0.3	0.128
D	0.2	0.10

$$I = m_A R_A^2 + m_B R_B^2 + m_c R_c^2 + m_D R_D^2 = \boxed{0.0082\ kg \cdot m^2}$$

11. || The three 200 g masses in FIGURE EX12.11 are connected by massless, rigid rods.

a. What is the triangle's moment of inertia about the axis through the center?

b. What is the triangle's kinetic energy if it rotates about the axis at 5.0 rev/s?



Given:

$m = 0.2\ kg$

$\omega = 31.42\ rad/s$

$R = 0.4/\sqrt{3} = 0.231\ m$

Find: I, K

a) $I = 3mR^2 = \boxed{0.032\ kg \cdot m^2}$

b) $K = \frac{1}{2} I \omega^2 = \boxed{15.79\ J}$

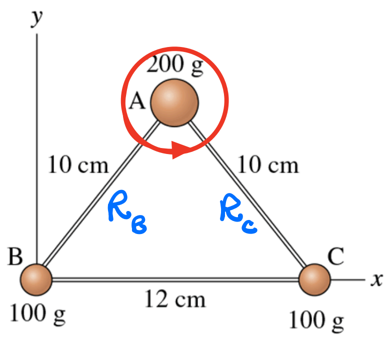
15. | The three masses shown in FIGURE EX12.15 are connected by massless, rigid rods.

a. Find the coordinates of the center of mass.

b. Find the moment of inertia about an axis that passes through mass A and is perpendicular to the page.

c. Find the moment of inertia about an axis that passes through masses B and C.

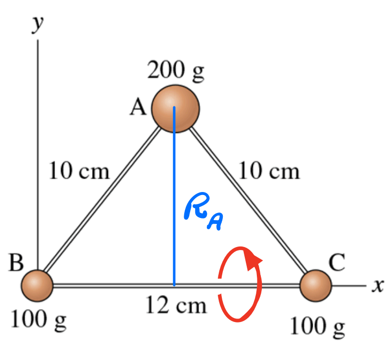
b)



	m	R
A	0.2	0.00
B	0.1	0.10
C	0.1	0.10

$$I = m_A R_A^2 + m_B R_B^2 + m_C R_C^2 = \boxed{2\times 10^{-3}\ kg\cdot m^2}$$

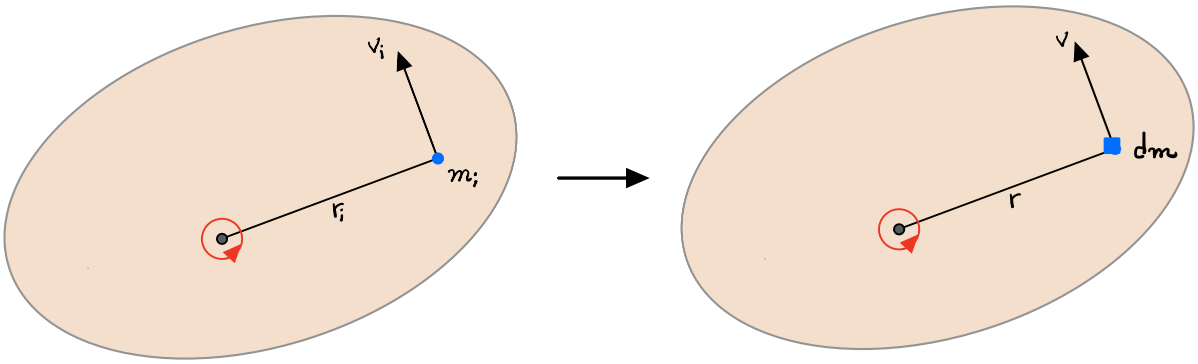
c)



	m	R
A	0.2	0.08
B	0.1	0.00
C	0.1	0.00

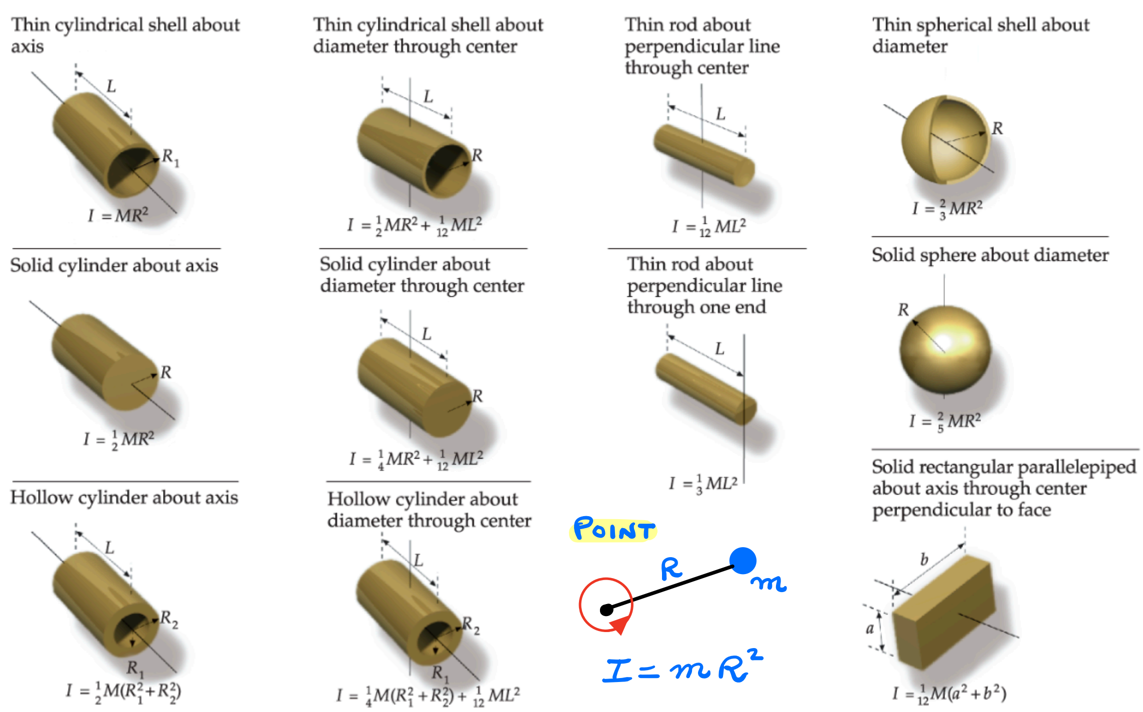
$$I = m_A R_A^2 + m_B R_B^2 + m_C R_C^2 = \boxed{1.28\times 10^{-3}\ kg\cdot m^2}$$

2. Continuous Object



$$\boxed{I = \sum_i m_i r_i^2 \longrightarrow \int_{\text{OBJECT}} r^2\,dm}$$

Table 9-1 Moments of Inertia of Uniform Bodies of Various Shapes

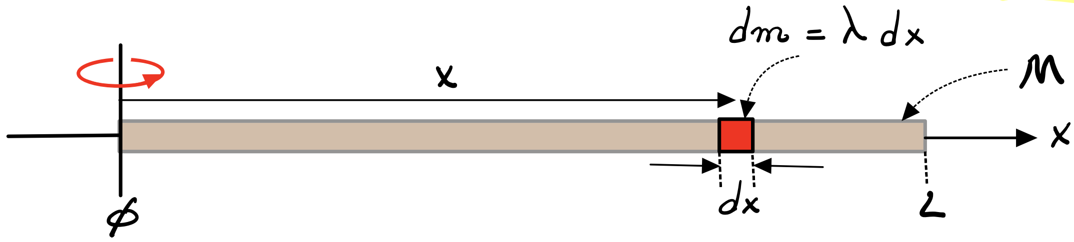


Tipler & Mosca, *Physics for Scientists and Engineers*, 6e © 2008 W.H. Freeman and Company

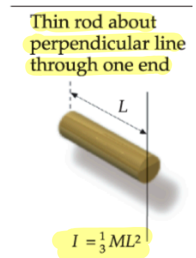
*A disk is a cylinder whose length L is negligible. By setting $L = 0$, the above formulas for cylinders hold for disks.

Ex: Rod (rotate about end)

$\lambda = M/L$

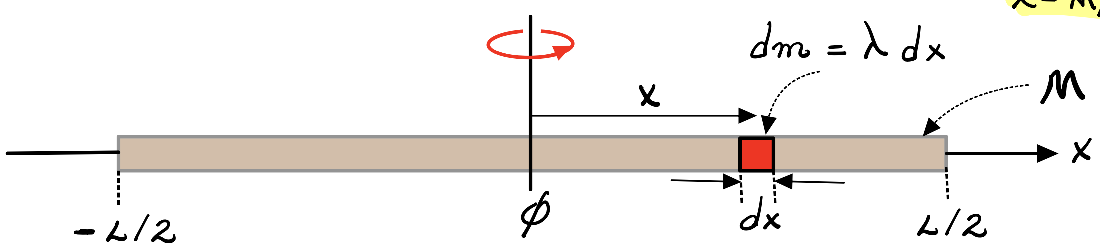


$$I = \int_{\text{OBJECT } ②} x^2\, dm^{①} = \lambda \int_0^L x^2\, dx = \boxed{\frac{1}{3} M L^2}$$

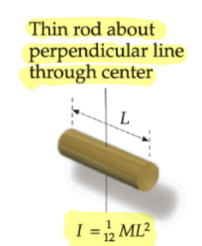


Ex: Rod (rotate about cm)

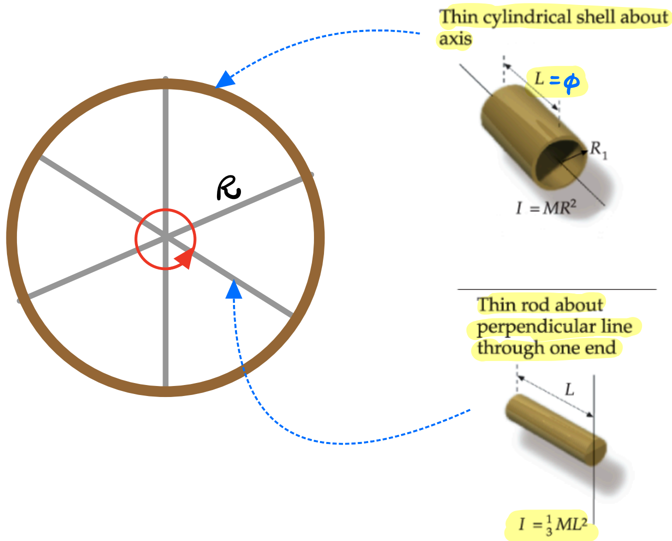
$\lambda = M/L$



$$I = \int_{\text{OBJECT } ②} x^2\, dm^{①} = \lambda \int_{-L/2}^{L/2} x^2\, dx = \boxed{\frac{1}{12} M L^2}$$



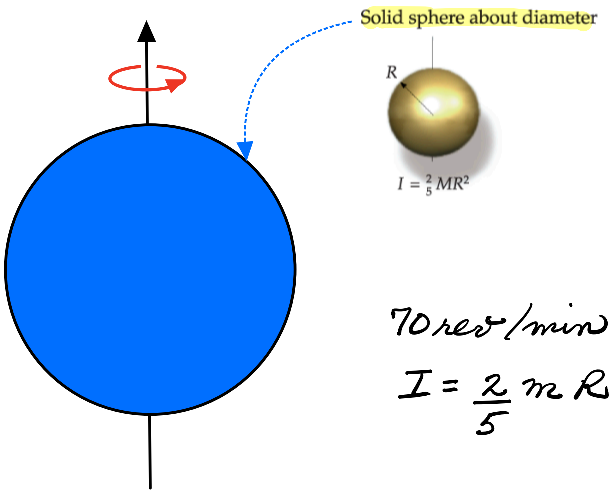
45. •• A 1.00-m-diameter wagon wheel consists of a thin rim having a mass of 8.00 kg and 6 spokes, each with a mass of 1.20 kg. Determine the moment of inertia of the wagon wheel about its axis.



1. $I_{WHEEL} = m_{WHEEL} R^2 \qquad I_{SPOKE} = \frac{1}{3} m_{SPOKE} R^2$

2. $I = I_{WHEEL} + 6\, I_{SPOKE} = \boxed{2.6 \text{ kg} \cdot \text{m}^2}$

65. • A 1.4-kg 15-cm-diameter solid sphere is rotating about its diameter at 70 rev/min. (*a*) What is its kinetic energy? (*b*) If an additional 5.0 mJ of energy are added to the kinetic energy, what is the new angular speed of the sphere? SSM



70 rev/min = 7.33 rad/s

$I = \frac{2}{5} m R^2$ (sphere)

a) $K = \frac{1}{2} I \omega^2 = \boxed{0.085 \text{ J}}$

b) $K = 0.090 \text{ J} \longrightarrow \omega = \boxed{7.56 \text{ rad/s } (72.2 \text{ rev/min})}$

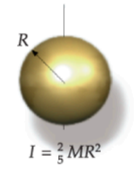
66. •• Calculate the kinetic energy of Earth due to its spinning about its axis, and compare your answer with the kinetic energy of the orbital motion of Earth's center of mass about the Sun. Assume Earth to be a homogeneous sphere of mass 6.0×10^{24} kg and radius 6.4×10^{6} m. The radius of Earth's orbit is 1.5×10^{11} m.

1. axis: $I = \frac{2}{5} mR^2 = 9.83 \times 10^{37}\ kg \cdot m^2$

$\omega = \frac{2\pi\ rad}{24 \cdot 3600\ s} = 7.27 \times 10^{-5}\ rad/s$

$K = \frac{1}{2} I \omega^2 = \boxed{2.60 \times 10^{29}\ J}$

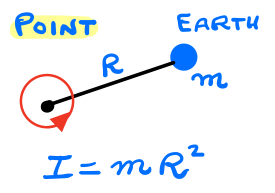
Solid sphere about diameter



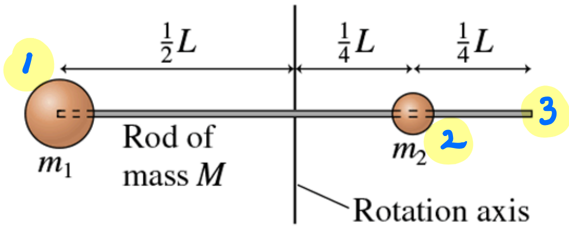
2. sun: $I = mR^2 = 1.35 \times 10^{47}\ kg \cdot m^2$

$\omega = \frac{2\pi\ rad}{365 \cdot 24 \cdot 3600\ s} = 1.99 \times 10^{-7}\ rad/s$

$K = \frac{1}{2} I \omega^2 = \boxed{2.68 \times 10^{33}\ J}$



52. || Determine the moment of inertia about the axis of the object shown in FIGURE P12.52.

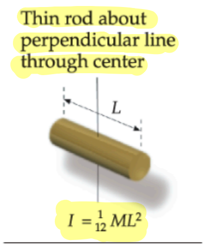


$$\boxed{I = I_1 + I_2 + I_3 + \ldots}$$

1. $I_1 = m_1 R_1^2 = m_1 \left(\frac{L}{2}\right)^2 = \frac{1}{4} m_1 L^2$

$I_2 = m_2 R_2^2 = m_2 \left(\frac{L}{4}\right)^2 = \frac{1}{16} m_2 L^2$

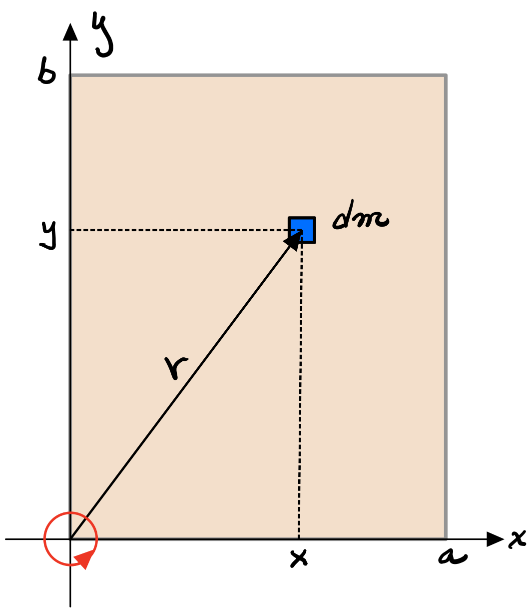
$I_3 = \frac{1}{12} M L^2$



2. $I = I_1 + I_2 + I_3 = \boxed{\left(\frac{1}{4} m_1 + \frac{1}{16} m_2 + \frac{1}{12} M\right) L^2}$

47. •• A uniform rectangular plate has mass m and edges of lengths a and b. (a) Show by integration that the moment of inertia of the plate about an axis that is perpendicular to the plate and passes through one corner is $\frac{1}{3}m\left(a^2+b^2\right)$. ($b$) What is the moment of inertia about an axis that is perpendicular to the plate and passes through its center of mass?

a)



1. $I = \int r^2 \, dm$ ③ ① ②

2. ① $r^2 = (x^2 + y^2)$

② $dm = \left(\frac{M}{ab}\right) dx\,dy$

$$I = \frac{M}{ab} \int_0^b \int_0^a (x^2+y^2)\,dx\,dy = \boxed{\frac{1}{3} M (a^2+b^2)}$$

③

Definite integral

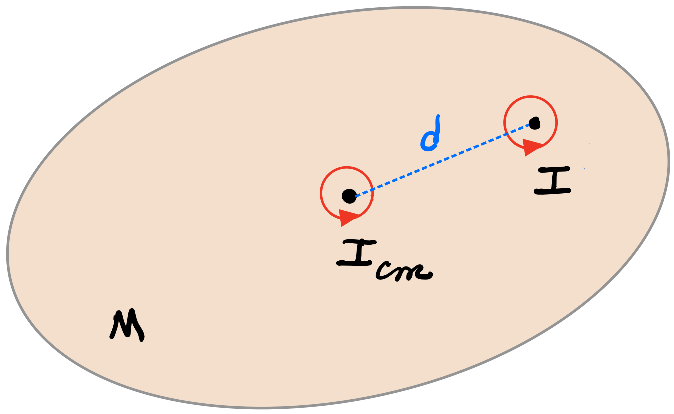
$$\int_0^b \int_0^a (x^2+y^2)\,dx\,dy = \frac{1}{3} a\,b\,(a^2+b^2)$$

b) $$I = \frac{M}{ab} \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} (x^2+y^2)\,dx\,dy = \boxed{\frac{1}{12} M (a^2+b^2)}$$

Definite integral

$$\int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} (x^2+y^2)\,dx\,dy = \frac{1}{12} a\,b\,(a^2+b^2)$$

3. Parallel Axis Theorem

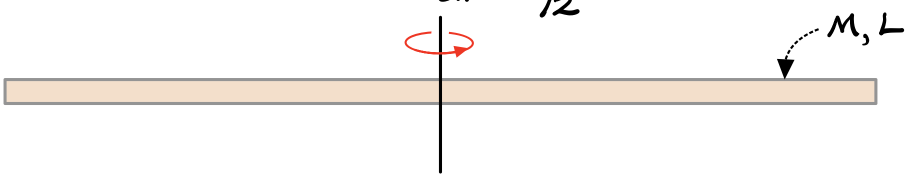


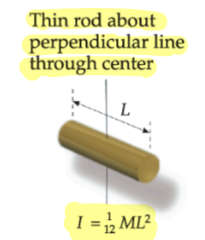
$$\boxed{I = I_{cm} + Md^2}$$

Example:

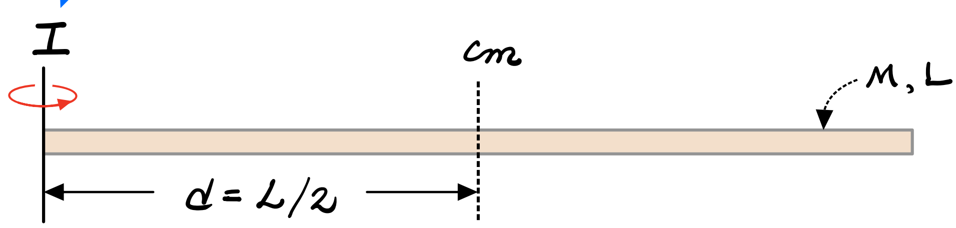
given this

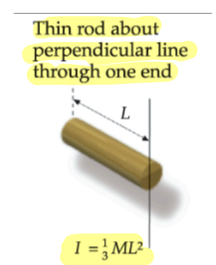
$$I_{cm} = \frac{1}{12}ML^2$$





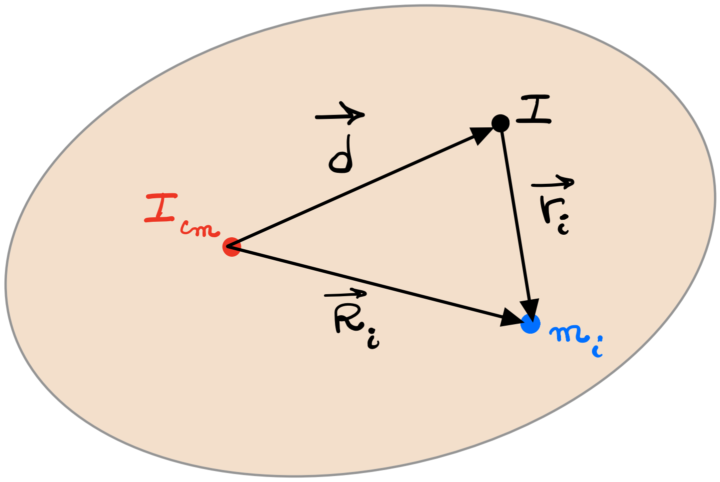
find this





$$\longrightarrow \quad I = \underbrace{\frac{1}{12}ML^2}_{I_{cm}} + \underbrace{M\left(\frac{L}{2}\right)^2}_{Md^2} = \frac{1}{3}ML^2$$

Derivation: $I = I_{cm} + Md^2$



1. $\vec{r}_i = \vec{R}_i - \vec{d}$

2. $r_i^2 = R_i^2 + d^2 - 2\vec{R}_i \cdot \vec{d}$

3. $\underbrace{\sum m_i r_i^2}_{I} = \underbrace{\sum m_i R_i^2}_{I_{cm}} + \underbrace{(\sum m_i) d^2}_{Md^2} - 2 \underbrace{(\sum m_i \vec{R}_i)}_{\phi} \cdot \vec{d}$

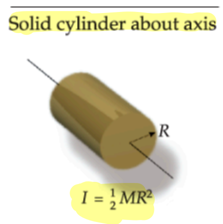
4. $\sum m_i \vec{R}_i$ is the position of cm with respect to the cm, which is ϕ.

$\rightarrow$ $\boxed{I = I_{cm} + Md^2}$

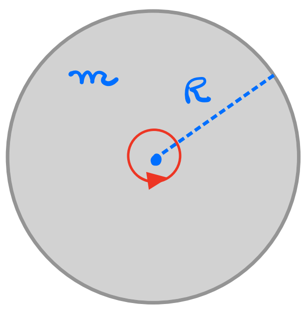
17. | | A 12-cm-diameter DVD has a mass of 21 g. What is the DVD's moment of inertia for rotation about a perpendicular axis (a) through its center and (b) through the edge of the disk?

Given: $R = 0.06\,m$

$m = 0.021\,kg$



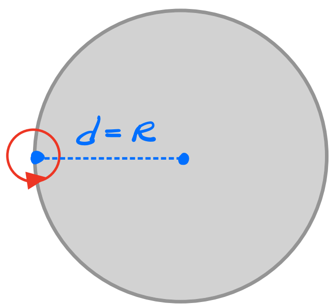
a)



given this

$$I_{cm} = \frac{1}{2} m R^2 = \boxed{3.78 \times 10^{-5}\ kg \cdot m^2}$$

b)



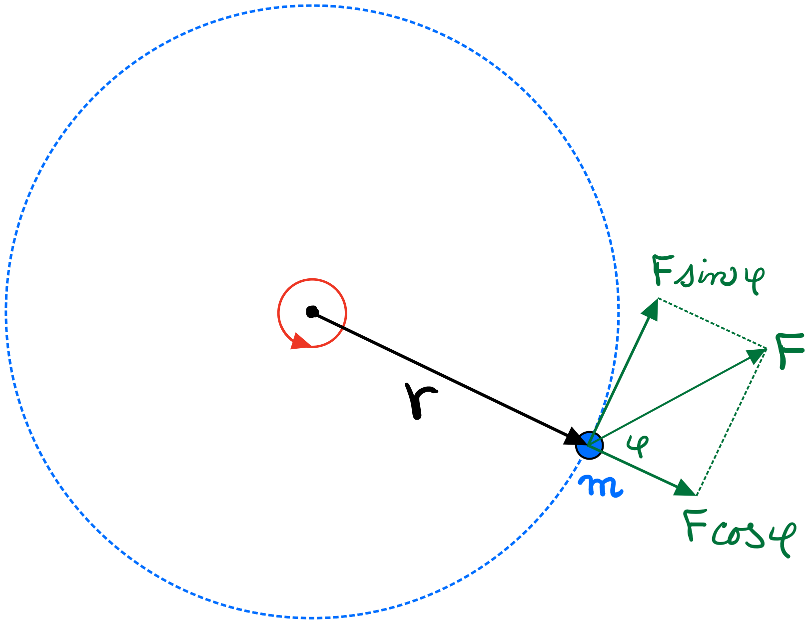
find this

$$I = I_{cm} + m d^2$$

$$= \frac{1}{2} m R^2 + m R^2 = \frac{3}{2} m R^2$$

$$= \boxed{1.134 \times 10^{-4}\ kg \cdot m^2}$$

D. Newton's 2nd Law for rotation ($\tau = I\alpha$)



1. $F \sin\varphi = m a_T$

2. $\underbrace{r F \sin\varphi}_{\tau} = \underbrace{(m r^2)}_{I} \underbrace{\left(\frac{a_T}{r}\right)}_{\alpha} \longrightarrow \boxed{\tau = I\alpha}$

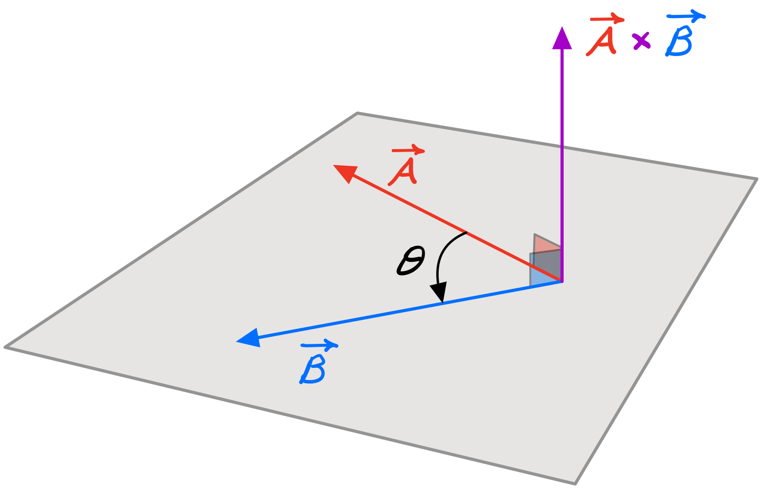
3. $\boxed{\tau = r F \sin\varphi}$ torque (units: $N \cdot m$)

   $\boxed{\tau > 0 \text{ ccw} \quad \tau < 0 \text{ cw}}$ sign convention

4. $\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$ vector definition

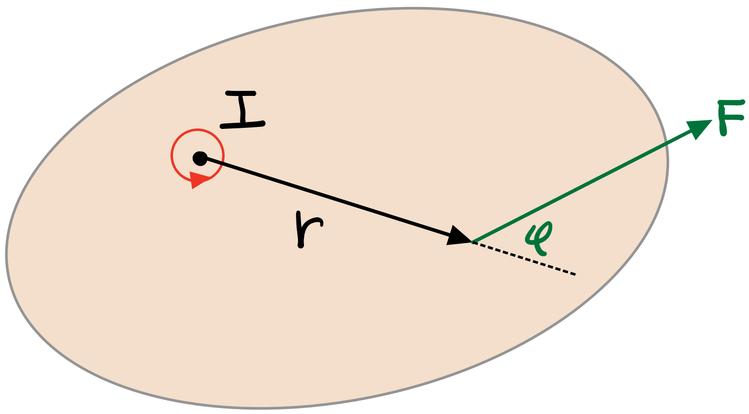
magnitude: $|\vec{A} \times \vec{B}| = AB\sin\theta$

direction: Right-Hand-Rule



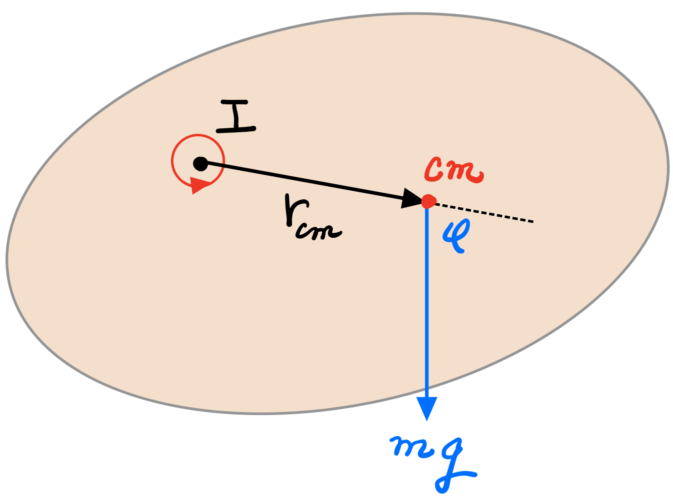
1. Solid Object (τ from F)

$\boxed{\tau = r F \sin\varphi}$ $\boxed{\tau = I\alpha}$

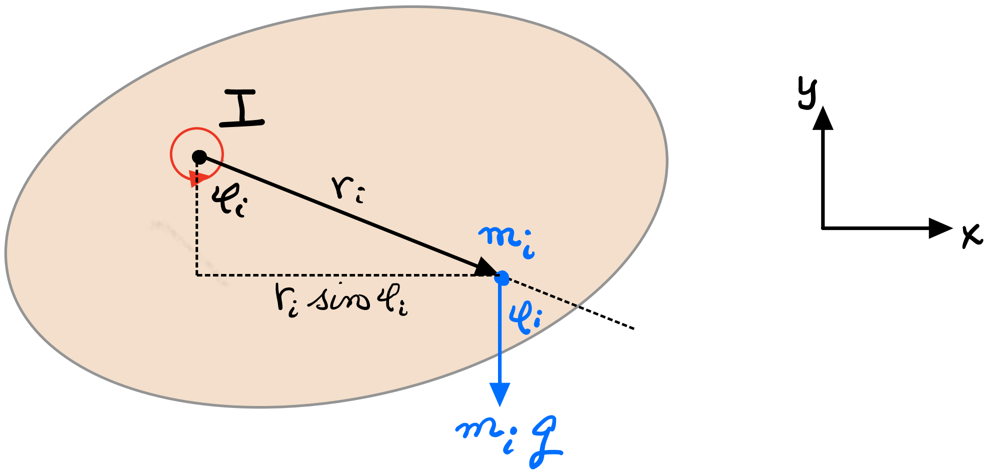


2. Solid Object (τ from gravity)

F_{mg} acts at the center of mass

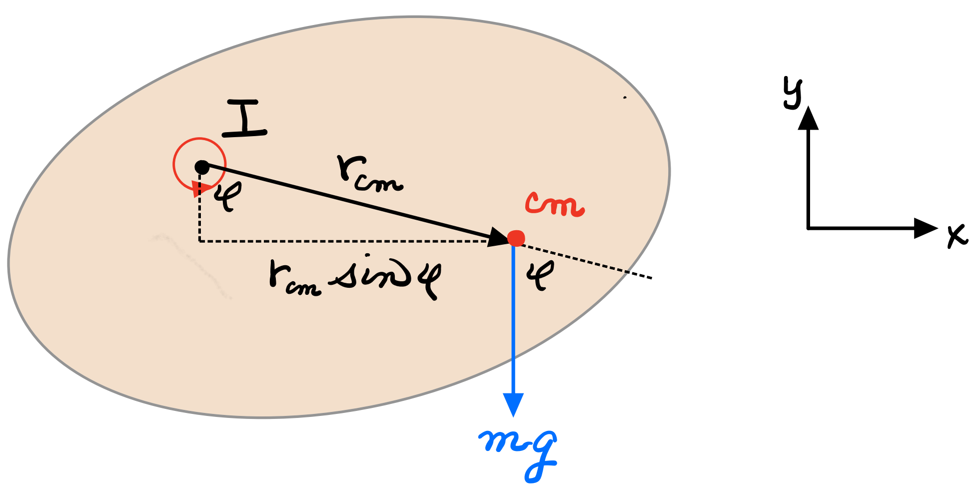


Derivation: Torque from gravity

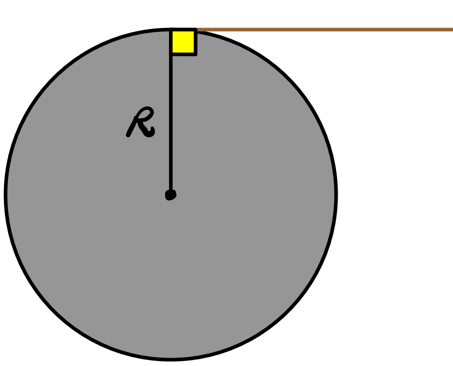


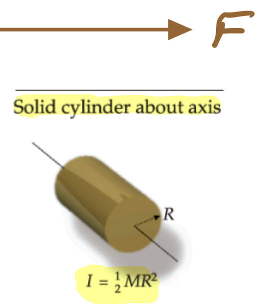
1. $$\tau = \sum_i r_i m_i g \sin\varphi_i$$
$$= \underbrace{\left(\sum_i m_i x_i\right)}_{m x_{cm}} g = m g x_{cm}$$

2. $$\boxed{\tau = m g r_{cm} \sin\varphi}$$



59. • A 2.5-kg 11-cm-radius cylinder, initially at rest, is free to rotate about the axis of the cylinder. A rope of negligible mass is wrapped around it and pulled with a force of 17 N. Assuming that the rope does not slip, find (*a*) the torque exerted on the cylinder by the rope, (*b*) the angular acceleration of the cylinder, and (*c*) the angular speed of the cylinder after 0.50 s. SSM





$m = 2.5\ kg$

$R = 0.11\ m$

$F = 17\ N$

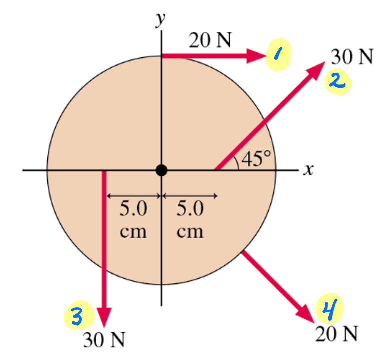
$\varphi = 90°$

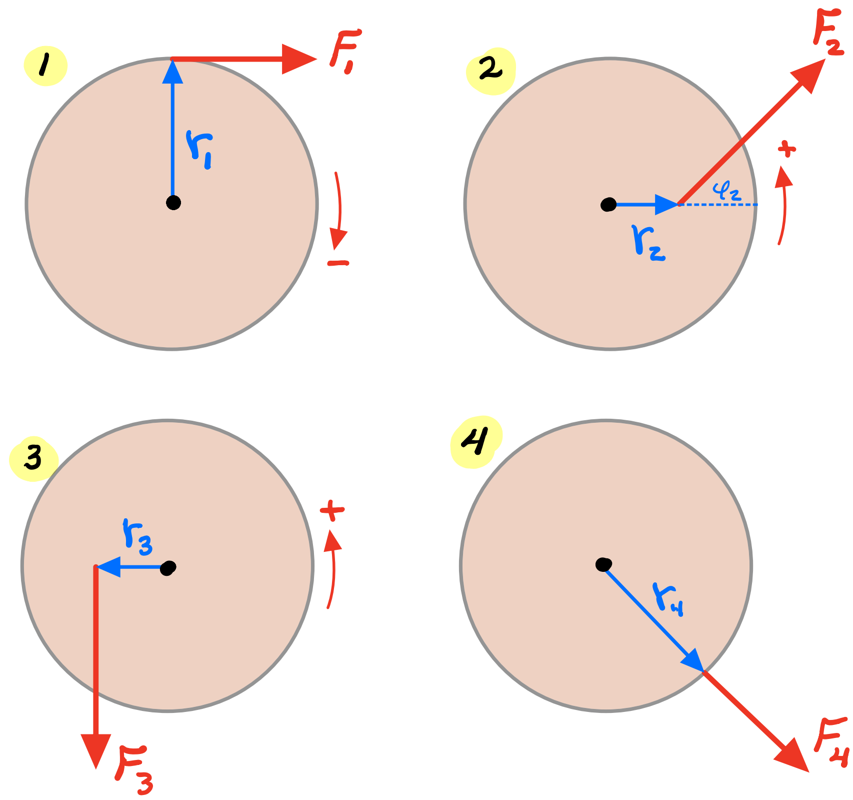
a) $\tau = RF\sin\varphi = \boxed{1.87\ N \cdot m}$

b) $\tau = I\alpha \quad I = \frac{1}{2}mR^2 \longrightarrow \alpha = \boxed{123.6\ rad/s^2}$

c) $\omega = \cancel{\omega_0}^{\,0} + \alpha t = \boxed{61.8\ rad/s}$

21. ||| The 20-cm-diameter disk in FIGURE EX12.21 can rotate on an axle through its center. What is the net torque about the axle?

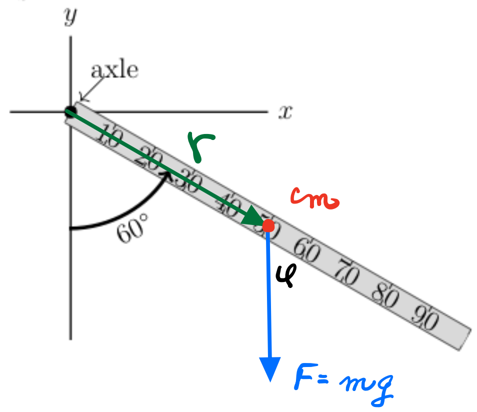




	r	F	φ	τ
1	0.10	20	90°	−2.00
2	0.05	30	45°	+1.06
3	0.05	30	90°	+1.50
4	0.10	20	∅°	∅

$$\tau_{NET} = \boxed{+0.56\ \text{N}\cdot\text{m}}$$

36. A 0.45 kg meter stick is fixed to an axle at its end. At the instant shown, the magnitude of the gravitational torque on the meter stick about the axle is



A. 2.2 N·m

B. 3.8 N·m

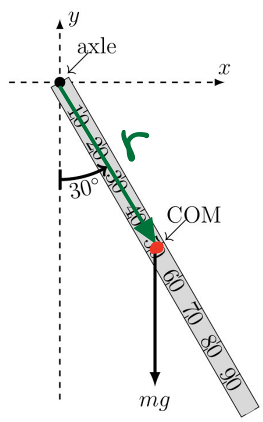
C. 1.9 N·m

D. 1.1 N·m

E. 4.4 N·m

$\tau = rF\sin\varphi = \boxed{1.91\ \text{N}\cdot\text{m}}$

68. A 0.45 kg meter stick is mounted to an axle at its end. At the instant shown what is the torque exerted by gravity on the meter stick about the axle?



A. 2.2 N·m clockwise

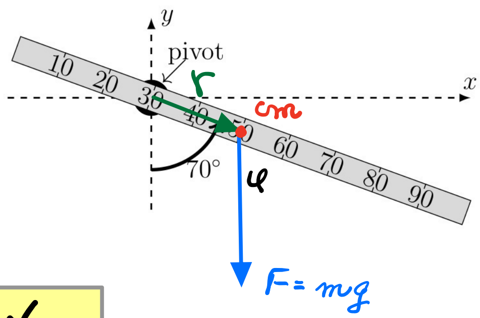
B. 1.1 N·m counterclockwise

C. 2.2 N·m counterclockwise

D. 1.1 N·m clockwise

$\tau = rF\sin\varphi = \boxed{1.1\ \text{N}\cdot\text{m}\quad \text{cw}}$

69. This 0.65 kg uniform aluminum meter stick is mounted to a pivot at the 30 cm mark. At 70° away from vertical, what is the torque exerted by gravity on this meter stick about the pivot?



A. 1.2 N·m

B. 4.5 N·m

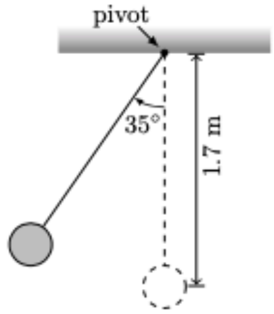
C. 3.7 N·m

D. 3.5 N·m

E. 4.2 N·m

$\tau = rF\sin\varphi = \boxed{1.2\ \text{N}\cdot\text{m}}$

35. A 0.4 kg ball swings from the end of a string of length 1.7 m. When the string makes a 35° angle with the vertical, the magnitude of the torque on the ball about the pivot is



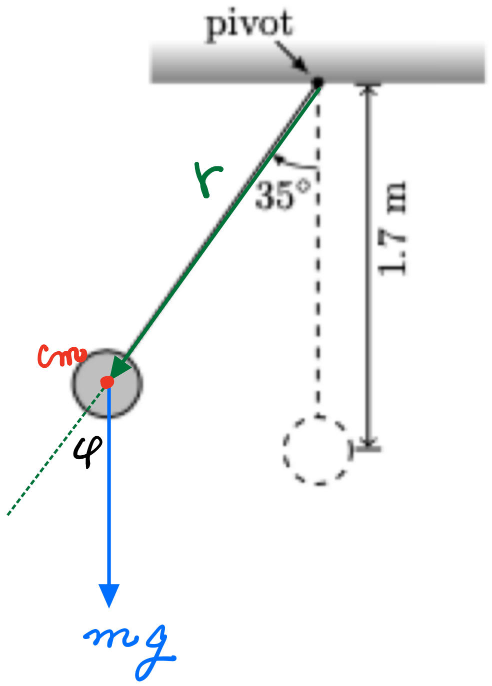
A. 2.3 N·m

B. 0

C. 3.8 N·m

D. 3.2 N·m

E. 5.5 N·m



Given:

$m = 0.4\,kg$

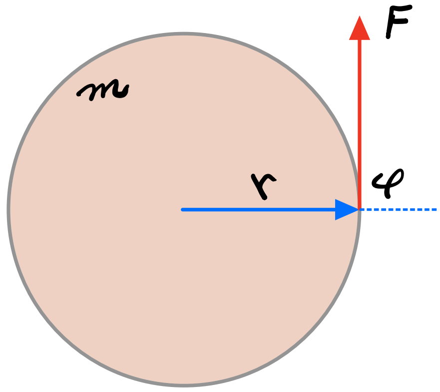
$r = 1.7\,m$

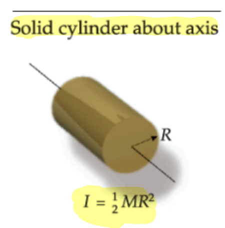
$\varphi = 35°$

$F = mg$

$$\tau = rF\sin\varphi = \boxed{3.82\,N\cdot m}$$

28. | | A 4.0 kg, 36-cm-diameter metal disk, initially at rest, can rotate on an axle along its axis. A steady 5.0 N tangential force is applied to the edge of the disk. What is the disk's angular velocity, in rpm, 4.0 s later?





Given: $m = 4\,kg$

$r = 0.18\,m$

$F = 5\,N$

$\varphi = 90°$

$t = 4\,s$

Find: ω

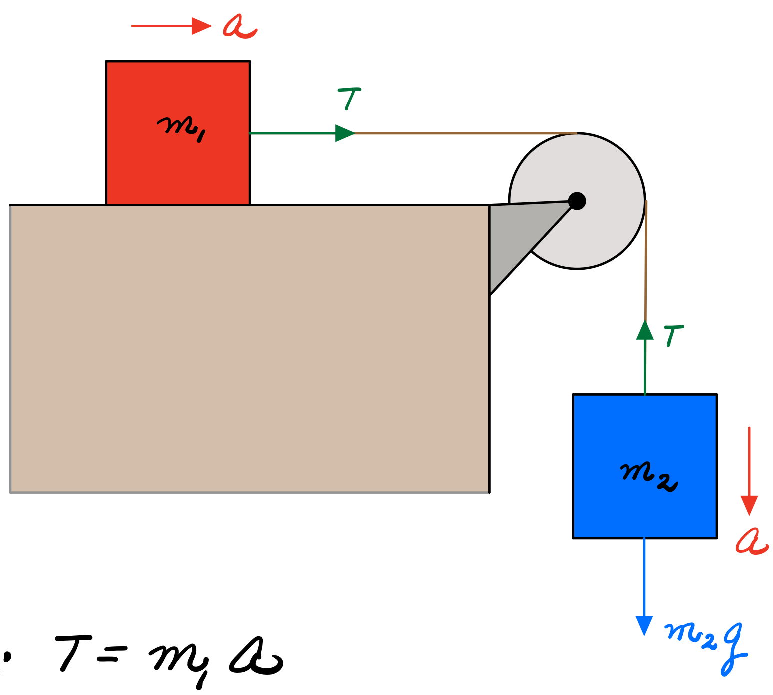
1. $\tau = rF\sin\varphi \qquad I = \frac{1}{2}mr^2$

$$\tau = I\alpha \longrightarrow \alpha = 13.89\ rad/s^2$$

2. $\omega = \cancel{\omega_0}^{\,0} + \alpha t = 55.6\ rad/s \qquad \boxed{530.5\ rpm}$

Pulleys:

Ex 1. Pulley is massless (find a, T)



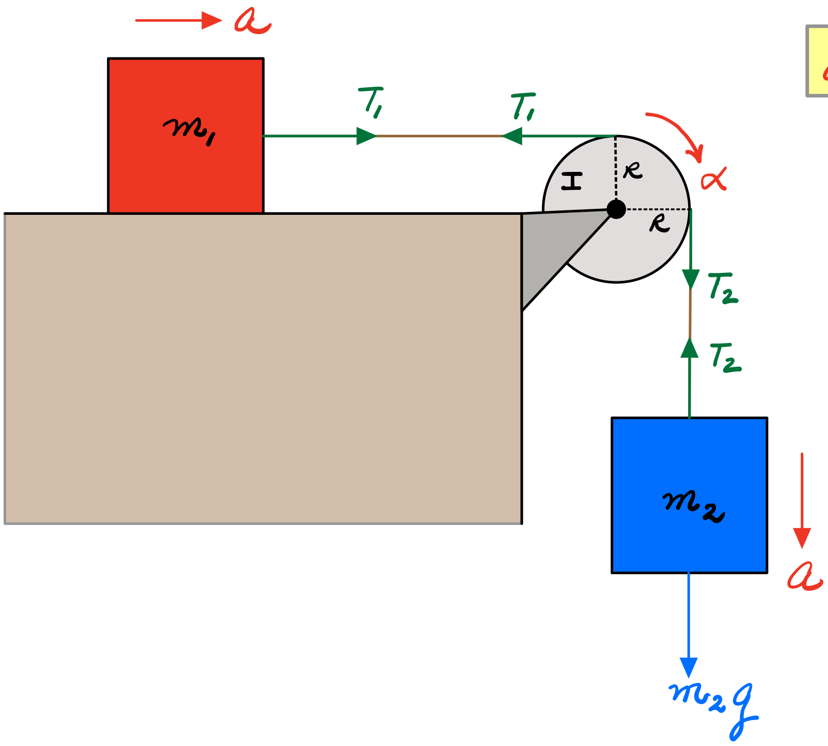
1. m_1: $T = m_1 a$

m_2: $m_2 g - T = m_2 a$

2. $m_2 g = (m_1 + m_2) a$

$\rightarrow$ $\boxed{a = \left(\frac{m_2}{m_1 + m_2}\right) g}$ $\boxed{T = m_1 a}$

Ex 2. Pulley has I (find a, T_1, T_2)



$$\boxed{a = \alpha R}$$

1. $F_1 = m_1 a \longrightarrow T_1 = m_1 a \longleftarrow$

$F_2 = m_2 a \longrightarrow m_2 g - T_2 = m_2 a \longleftarrow$

a/R

2. $\tau = I\alpha \longrightarrow T_2 R - T_1 R = I\alpha$

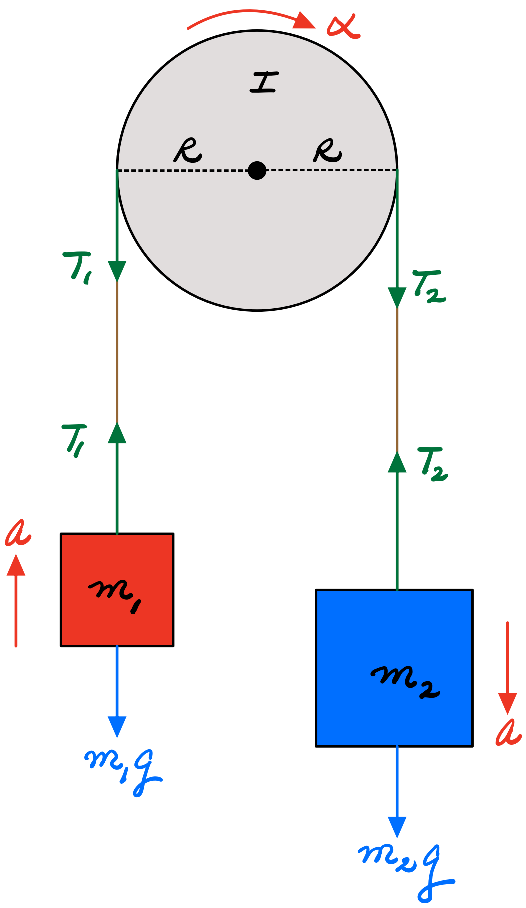
$\longrightarrow T_2 - T_1 = \dfrac{Ia}{R^2} \longleftarrow$

3. $\cancel{T_1} + (m_2 g - \cancel{T_2}) + (\cancel{T_2} - \cancel{T_1}) = (m_1 + m_2 + I/R^2)a$

$$\boxed{a = \left(\frac{m_2}{m_1 + m_2 + I/R^2}\right) g \qquad T_1 = m_1 a \qquad T_2 = m_2(g - a)}$$

Ex 3. Pulley has I (find a, T_1, T_2)

$$\boxed{a = \alpha R}$$



1. m_1: $T_1 - m_1 g = m_1 a$

   m_2: $m_2 g - T_2 = m_2 a$

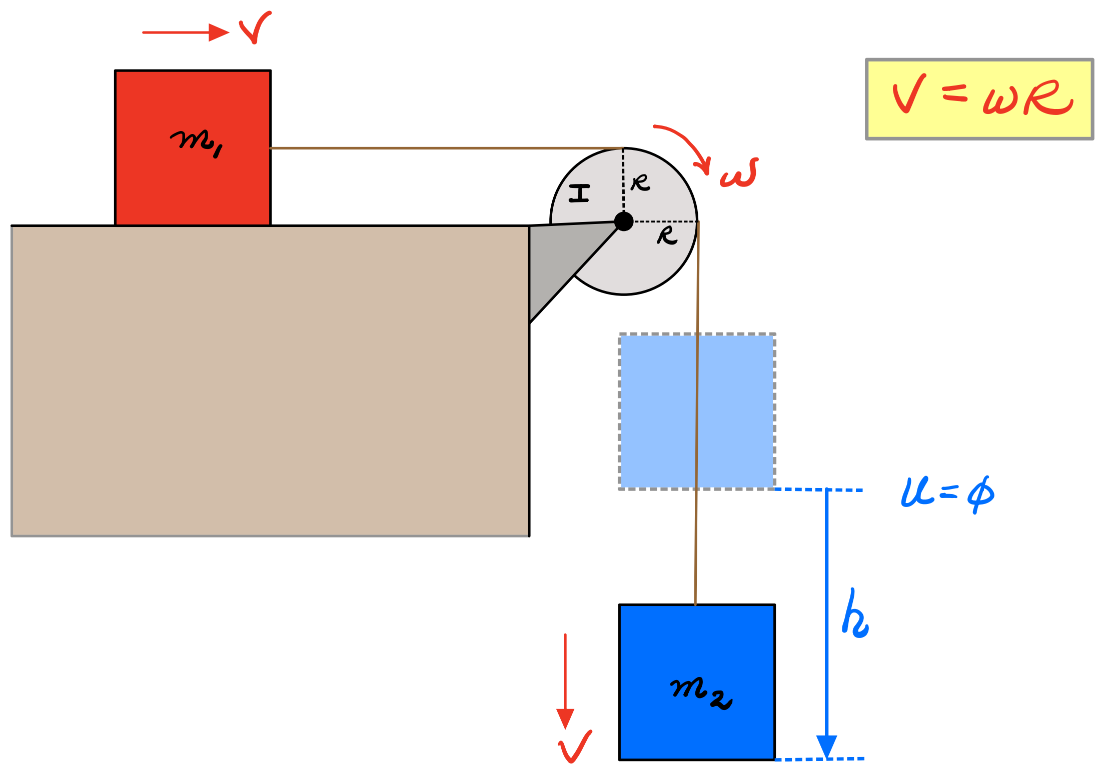
2. I: $T_2 R - T_1 R = I \alpha$ ← a/R

   $\longrightarrow T_2 - T_1 = \dfrac{Ia}{R^2}$

3. $$\boxed{a = \left(\frac{m_2 - m_1}{m_1 + m_2 + I/R^2}\right) g}$$

$$\boxed{\begin{aligned} T_1 &= m_1(g+a) \\ T_2 &= m_2(g-a) \end{aligned}}$$

Ex 4. Pulley has I (find v after falls h)



1. $K_F + U_F = \cancel{K_I}^{\phi} + \cancel{U_I}^{\phi}$

2. $\frac{1}{2} m_1 V^2 + \frac{1}{2} m_2 V^2 + \frac{1}{2} I \omega^2 - m_2 g h = \phi$

$\omega = V/R$

3. $\boxed{V = \left(\frac{2 m_2 g h}{m_1 + m_2 + I/R^2}\right)^{1/2}}$

71. •• The system shown in Figure 9-55 consists of a 4.0-kg block resting on a frictionless horizontal ledge. This block is attached to a string that passes over a pulley, and the other end of the string is attached to a hanging 2.0-kg block. The pulley is a uniform disk of radius 8.0 cm and mass 0.60 kg. Find the acceleration of each block and the tension in the string. SSM

pulley:

$m_p = 0.6\ kg$

$R = 0.08\ m$

$I = \frac{1}{2} m_p R^2$

See Ex 2:

$$\boxed{a = \left(\frac{m_2}{m_1 + m_2 + I/R^2}\right) g \qquad T_1 = m_1 a \qquad T_2 = m_2 (g - a)}$$

$a = \boxed{3.11\ m/s^2}$ $\quad T_1 = \boxed{12.44\ N}$ $\quad T_2 = \boxed{13.38\ N}$

72. •• For the system in Problem 71, the 2.0-kg block is released from rest. (*a*) Find the speed of the block after it falls a distance of 2.5 m. (*b*) What is the angular speed of the pulley at this instant?

pulley:

$m_p = 0.6\ kg$
$R = 0.08\ m$
PROB. 71

$I = \frac{1}{2} m_p R^2$

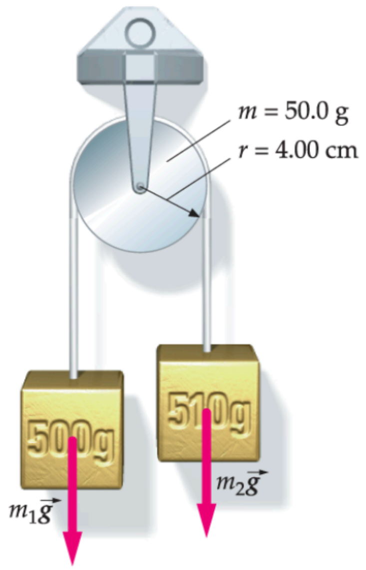
$h = 2.5\ m$

See Ex 4: $\boxed{v = \left(\dfrac{2 m_2 g h}{m_1 + m_2 + I/R^2}\right)^{1/2}}$

a) $v = \boxed{3.94\ m/s}$

b) $\omega = v/R = \boxed{49.3\ rad/s}$

78. •• Two objects, of masses $m_1 = 500$ g and $m_2 = 510$ g, are connected by a string of negligible mass that passes over a pulley with frictionless bearings (Figure 9-59). The pulley is a uniform 50.0-g disk with a radius of 4.00 cm. The string does not slip on the pulley. (*a*) Find the accelerations of the objects. (*b*) What is the tension in the string between the 500-g block and the pulley? What is the tension in the string between the 510-g block and the pulley? By how much do these tensions differ? (*c*) What would your answers be if you neglected the mass of the pulley?



pulley:

$m = 0.05\,kg$

$R = 0.04\,m$

$I = \frac{1}{2} m R^2$

See Ex 3:

$$\boxed{a = \left(\frac{m_2 - m_1}{m_1 + m_2 + I/R^2}\right) g}$$

$$\boxed{\begin{aligned} T_1 &= m_1(g+a) \\ T_2 &= m_2(g-a) \end{aligned}}$$

$a = \boxed{0.0947\,m/s^2}$ $T_1 = \boxed{4.95\,N}$ $T_2 = \boxed{4.95\,N}$

67. The pulley has rotational inertia $I = 0.0030$ kg m^2 and radius $r = 0.050$ m. The 0.60 kg block descends as the cord unwinds without slipping. The magnitude of the block's acceleration is

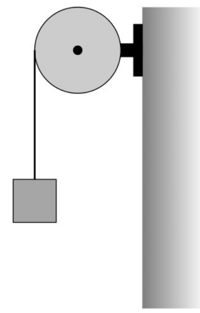
A. 8.9 m/s^2

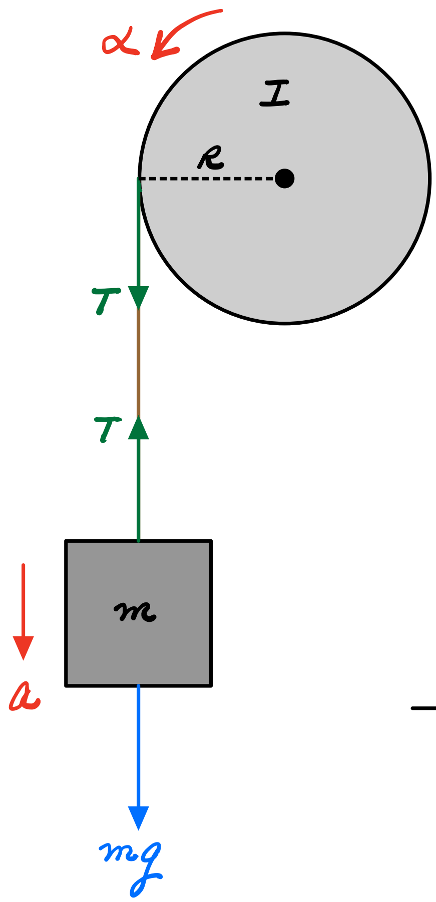
B. 3.3 m/s^2

C. 4.5 m/s^2

D. 9.8 m/s^2

E. 6.5 m/s^2





1. $F = ma$ $\qquad mg - T = ma$

a/R

2. $\tau = I\alpha$ $\qquad TR = I\alpha$

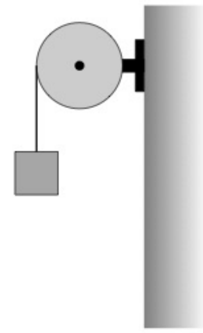
$$T = Ia/R^2$$

3. $mg = ma + I\dfrac{a}{R^2}$

$$\rightarrow a = \left(\frac{m}{m + I/R^2}\right) g = \boxed{3.27\,m/s^2}$$

$$T = m(g - a) = \boxed{3.92\,N}$$

70. Released from rest, the 0.30 kg block descends as the frictionless pulley unwinds the string. The pulley's rotational inertia is 0.0040 kg · m^2, and it's radius is 0.060 m. The block's speed after descending 1.0 m is



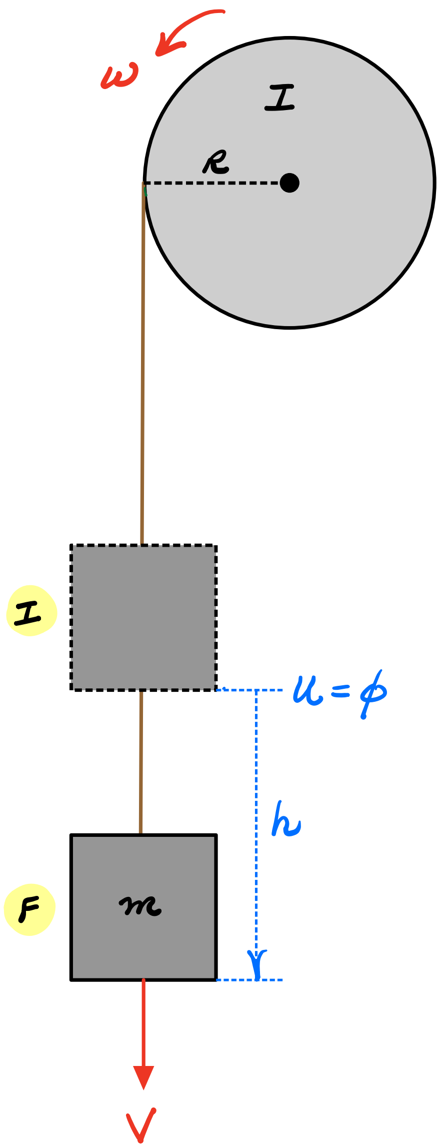
A. 4.5 m/s

B. 1.6 m/s

C. 3.1 m/s

D. 4.9 m/s

E. 2.0 m/s



1. $K_F + U_F = \cancel{K_I}^{\phi} + \cancel{U_I}^{\phi}$

$$\rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 - mgh = \phi$$

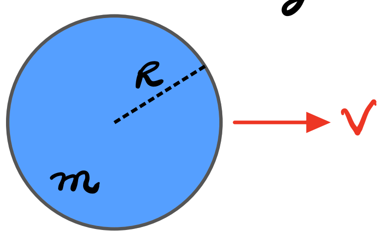
V/R

2. $\frac{1}{2}mv^2 + \frac{1}{2} I \frac{V^2}{R^2} - mgh = \phi$

$$\rightarrow V = \left(\frac{2mgh}{m + I/R^2}\right)^{1/2} = \boxed{2.04\ m/s}$$

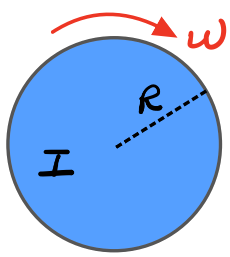
D. Rolling Without Slipping

1. Translation only



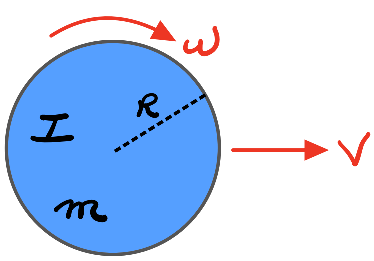
$$K = \frac{1}{2} m v^2$$

2. Rotation only



$$K = \frac{1}{2} I \omega^2$$

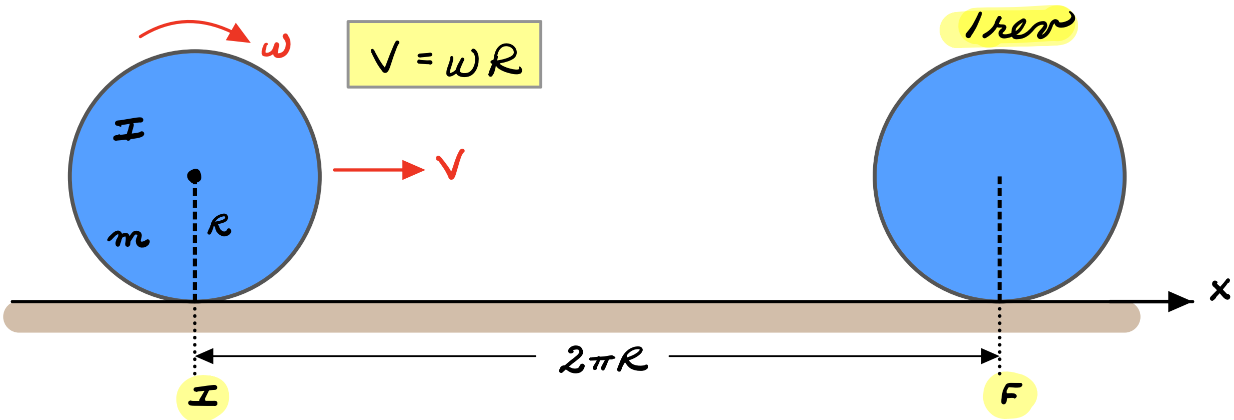
3. Translation and Rotation



$$K = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

ω and v independent

4. Rolling without Slipping



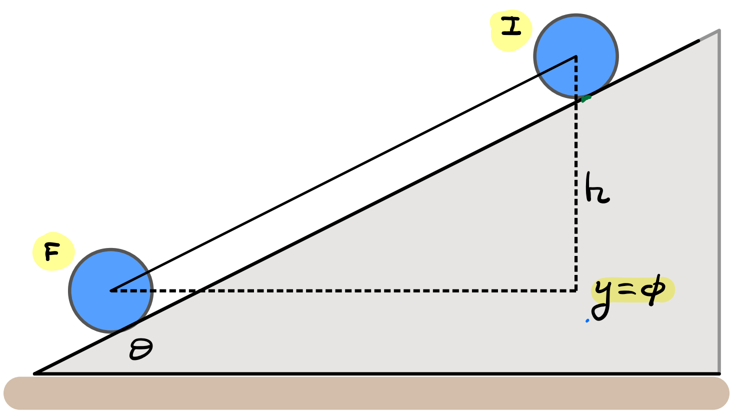
1. $K = \frac{1}{2} m V^2 + \frac{1}{2} I \omega^2$

2. $\omega = \frac{2\pi}{T}$ rad/s $\qquad V = \frac{2\pi R}{T}$ m/s

$\longrightarrow \boxed{V = \omega R}$

36. A solid sphere, a solid cylinder, and a thin hoop are simultaneously released from rest from the top of a ramp and they all roll without slipping. They all have the same radius but different masses. Which one reaches the bottom first?

$$I_{\text{solid sphere}} = \frac{2}{5}MR^2, \qquad I_{\text{solid cylinder}} = \frac{1}{2}MR^2, \qquad I_{\text{hoop}} = MR^2$$



$I = \beta m R^2$

$\beta = 2/5$ sphere

$\beta = 1/2$ cylinder

$\beta = 1$ hoop

$\boxed{v = R\omega}$

1. $K_F + \cancel{U_F}^{\,\phi} = \cancel{K_I}^{\,\phi} + U_I$

2. $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$

$\longrightarrow \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}(\beta \cancel{m} \cancel{R^2})\left(\frac{v}{\cancel{R}}\right)^2 = \cancel{m}gh$

$$\boxed{v = \left(\frac{2gh}{1+\beta}\right)^{1/2}}$$

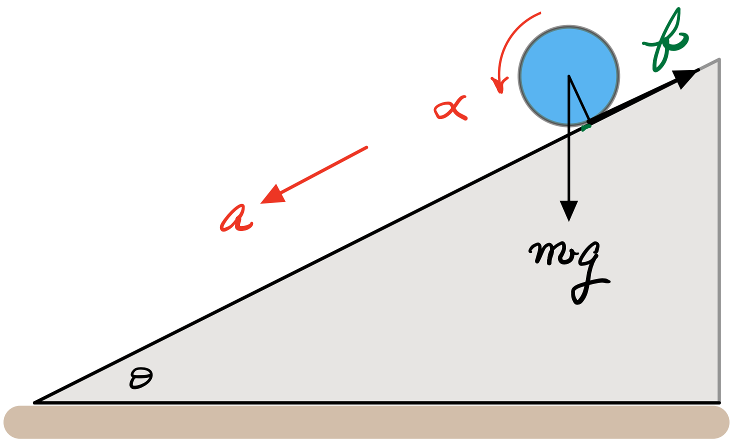
1. $v = 0.845(2gh)^{1/2}$ sphere

2. $v = 0.817(2gh)^{1/2}$ cylinder

3. $v = 0.707(2gh)^{1/2}$ hoop

36. A solid sphere, a solid cylinder, and a thin hoop are simultaneously released from rest from the top of a ramp and they all roll without slipping. They all have the same radius but different masses. Which one reaches the bottom first?

$$\boxed{I_{\text{solid sphere}} = \frac{2}{5}MR^2,} \qquad I_{\text{solid cylinder}} = \frac{1}{2}MR^2, \qquad I_{\text{hoop}} = MR^2$$



$I = \beta m R^2$

$\beta = 2/5$ sphere

$\beta = 1/2$ cylinder

$\beta = 1$ hoop

$$\boxed{a = R\alpha}$$

1. $F = ma$: $\quad mg\sin\theta - f = ma$

2. $\tau = I\alpha$: $\quad fR = I\alpha \longrightarrow f\cancel{R} = \beta m R^{\cancel{2}}\left(\frac{a}{\cancel{R}}\right)$

3. $\cancel{m}g\sin\theta - \beta\cancel{m}a = \cancel{m}a$

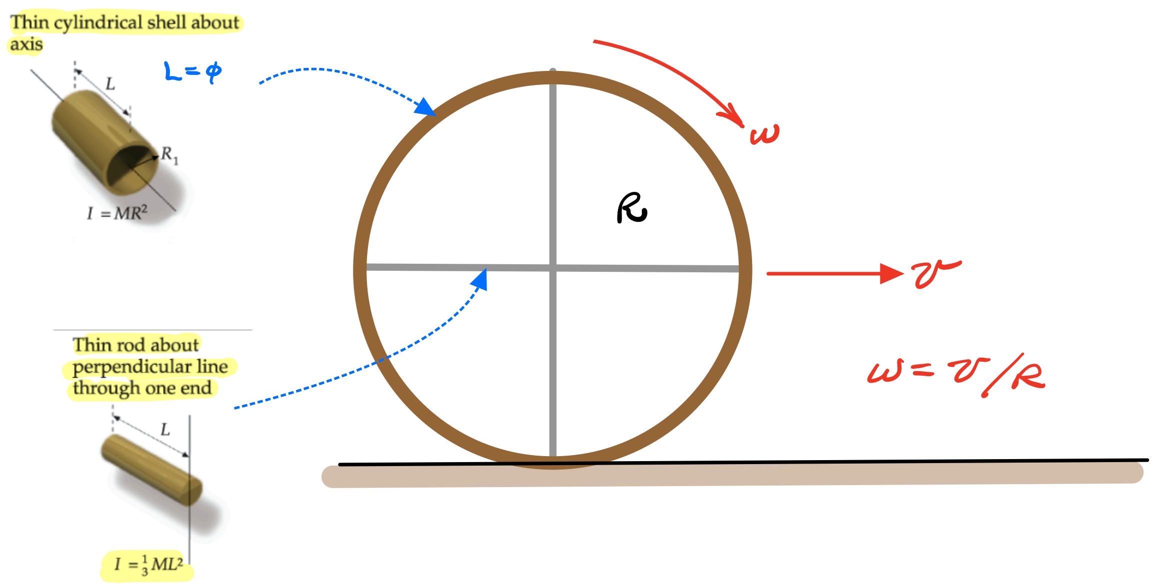
$$\boxed{a = \frac{g\sin\theta}{1+\beta}}$$

1. $a = \frac{5}{7}g\sin\theta$ sphere

2. $a = \frac{2}{3}g\sin\theta$ cylinder

3. $a = \frac{1}{2}g\sin\theta$ hoop

95. •• A wheel has a thin 3.0-kg rim and four spokes, each of mass 1.2 kg. Find the kinetic energy of the wheel when it is rolling at 6.0 m/s on a horizontal surface.



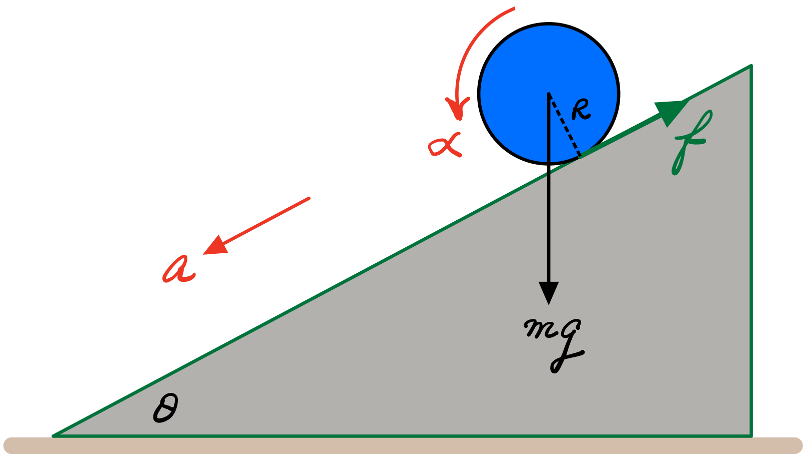
1. $I_{WHEEL} = m_{WHEEL} R^2$ $\qquad I_{SPOKE} = \frac{1}{3} m_{SPOKE} R^2$

2. $I = I_{WHEEL} + 4 I_{SPOKE} = 4.6 R^2$

3. $K = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

$$= \frac{1}{2}(7.8) v^2 + \frac{1}{2}(4.6 \cancel{R^2})\left(\frac{v}{\cancel{R}}\right)^2 = \boxed{223.2 \text{ J}}$$

88. •• A uniform solid sphere rolls down an incline without slipping. If the linear acceleration of the center of mass of the sphere is $0.20g$, then what is the angle the incline makes with the horizontal?



$I = \frac{2}{5} m R^2$

$\alpha = a/R$

$a = 0.2g$

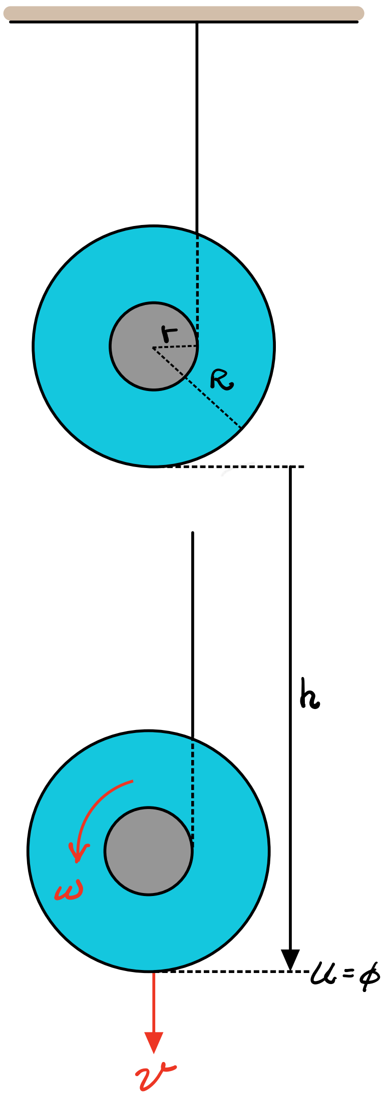
1. Rotation: $fR = I\alpha = \frac{2}{5} m R^2 \left(\frac{a}{R}\right) \longrightarrow f = \frac{2}{5} ma$

2. Translation: $mg\sin\theta - f = ma$

$\longrightarrow a = \frac{5}{7} g \sin\theta \longrightarrow \theta = \boxed{16.26^\circ}$

85. •• In 1993 a giant 400-kg yo-yo with a radius of 1.5 m was dropped from a crane at height of 57 m. One end of the string was tied to the top of the crane, so the yo-yo unwound as it descended. Assuming that the axle of the yo-yo had a radius of 0.10 m, estimate its linear speed at the end of the fall.

SSM



Given: $m = 400\,kg$ $R = 1.5\,m$

$r = 0.1\,m$ $h = 57\,m$

$I = \frac{1}{2} m R^2$ $\omega = v/r$

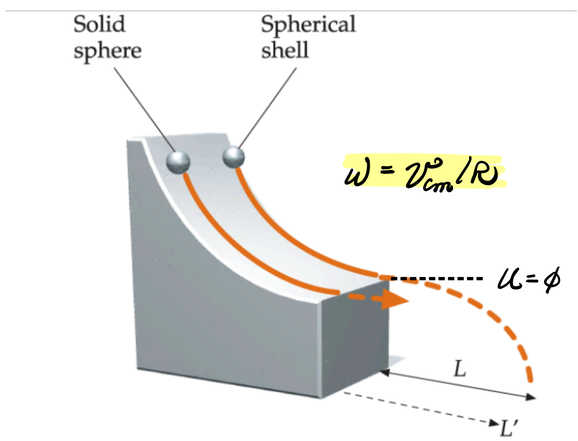
1. $K_F + \cancel{U_F}^{\phi} = \cancel{K_I}^{\phi} + U_I$

$$\frac{1}{2} m v^2 + \underbrace{\frac{1}{2} I \omega^2}_{\frac{1}{2}\left(\frac{1}{2} m R^2\right)\left(\frac{v}{r}\right)^2} = m g h$$

2. $\frac{1}{2} v^2 + \frac{1}{4} \frac{R^2}{r^2} v^2 = g h$

$\longrightarrow v = \boxed{3.14\ m/s}$

92. •• Released from rest at the same height, a thin spherical shell and solid sphere of the same mass m and radius R roll without slipping down an incline through the same vertical drop H (Figure 9-64). Each is moving horizontally as it leaves the ramp. The spherical shell hits the ground a horizontal distance L from the end of the ramp and the solid sphere hits the ground a distance L' from the end of the ramp. Find the ratio L'/L.



$$I = \beta m R^2$$

solid $\beta = 2/5$

shell $\beta = 2/3$

1. $K_F + \cancel{U_F}^{\,\phi} = \cancel{K_I}^{\,\phi} + U_I$

$$\frac{1}{2} m v_{cm}^2 + \underbrace{\frac{1}{2} I \omega^2}_{\frac{1}{2}(\beta m R^2)\left(\frac{v_{cm}}{R}\right)} = mgh \quad \longrightarrow \quad v_{cm} = \left(\frac{2gh}{1+\beta}\right)^{1/2}$$

2. $\left(x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2\right)$ $\qquad L = v_{SHELL}\, t \qquad L' = v_{SOLID}\, t$

$$\longrightarrow \frac{L'}{L} = \frac{v_{SOLID}}{v_{SHELL}} = \left(\frac{1+\beta_{SHELL}}{1+\beta_{SOLID}}\right)^{1/2} = \boxed{1.09}$$