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Institut national de la recherche agronomique

DROSOPHILA SUZUKII'S (SWD) OVERVIEW

MATHEMATICAL MODELING

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OUTLINE

THE INVASIVE SPECIES

- Drosophila suzukii is considered an r-strategist species
- Drosophila suzukii exhibits a lek mating system
- High rate of ovipostion.

ECONOMIC IMPACT OF DROSOPHILA SUZUKII

- **The economic impact of Drosophila suzukii: perceived costs and revenue losses of Swiss cherry, plum and grape growers (2020)**
- the study estimates the overall economic impact of Drosophila suzukii on the Swiss fruit industry to be around CHF 13 million (approx. USD 14.3 million) per year 13 millions euro
- **Recent Trends in the Economic Impact of Drosophila suzukii**
- In the United States, estimated losses due to D. suzukii infestation ranged from USD 511 million to USD 2.6 billion in 2010-2016. In Europe, the economic impact of D. suzukii varied depending on the crop and region. For example, in Italy, losses in cherry production ranged from 13% to 80%. In Asia, D. suzukii has been a significant pest of raspberry and blueberry crops, causing up to 100% yield losses in some cases.

THE FEMALE'S CHARACTERISTIC

- high reproductive rate and a short generation time,
- females mated on average twice with an average of 16 days between each mating. (Debatable)
- females more resistant to cold than males
- A winter with several days of intense cold causes high mortality in the population especially.
- spring, females are always more numerous than males
- summer: the proportion of males and females is then balanced
- The sex ratio of newly emerged adults is on average 1

THE MALE'S CHARACTERISTIC

- high reproductive rate and a short generation time,
- autumn: finally reversed males more than females
- males more resistant to heat than females
- In some cases, males may exhibit aggressive behavior towards each other as they compete for access to females.
- autumn: finally reversed males more than females
- have a high re-mating rate and can mate multiple times in a day. In laboratory studies, males have been observed to mate up to 20 times in a 24-hour period.

MATHEMATICAL MODEL

With and Without the release of sterile species

- without the release we have

$$
\begin{cases}
\frac{dL}{dt} = \beta\left(1-\frac{L}{K}\right)F - (v_L+\mu_L)L\\
\frac{dM}{dt} = v_L m L - \mu_M M\\
\frac{dV}{dt} = v_L(1-m)L - \mu_F V - v_F \min\left(\frac{M}{V},1\right)V\\
\frac{dF}{dt} = v_F \min\left(\frac{M}{V},1\right)V - (\mu_F+\gamma)F
\end{cases}
$$

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$$

-

$$
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MATHEMATICAL MODEL

With and Without the release of sterile species

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$$\begin{cases} \frac{dL}{dt} = \beta\left(1-\frac{L}{K}\right)F-(\nu_L+\mu_L)L \\ \frac{dM}{dt} = \nu_L mL-\mu_M M \\ \frac{dV}{dt} = \nu_L(1-m)L-\mu_F V-\nu_F\min\left(\frac{M}{V},1\right)V \\ \frac{dF}{dt} = \nu_F\min\left(\frac{M}{V},1\right)V-(\mu_F+\gamma)F \end{cases}$$

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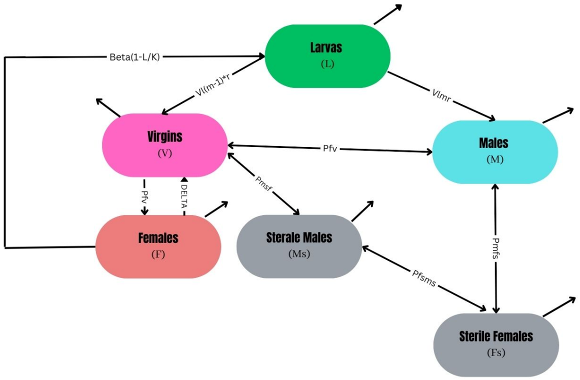
MATHEMATICAL MODEL

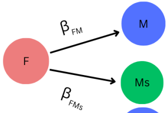
With and Without the release of sterile species



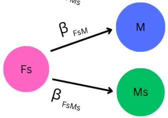
MATHEMATICAL MODEL

With and Without the release of sterile species

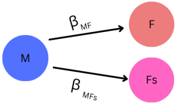
» with the release of both sexes



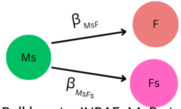
$$P_{FM} = \frac{\beta_{FM}M\beta_{MF}F}{\beta_{FM}M\beta_{MF}F + \beta_{FM_s}M_s\beta_{M_sF}F + \beta_{F_sM}M\beta_{MF_s}F_s + \beta_{F_sM_s}M_s\beta_{M_sF_s}F_s}$$



$$P_{F_sM} = \frac{\beta_{F_sM}M\beta_{MF_s}F_s}{\beta_{FM}M\beta_{MF}F + \beta_{FM_s}M_s\beta_{M_sF}F + \beta_{F_sM}M\beta_{MF_s}F_s + \beta_{F_sM_s}M_s\beta_{M_sF_s}F_s}$$



$$P_{FM_s} = \frac{\beta_{FM_s}M_s\beta_{M_sF}F}{\beta_{FM}M\beta_{MF}F + \beta_{FM_s}M_s\beta_{M_sF}F + \beta_{F_sM}M\beta_{MF_s}F_s + \beta_{F_sM_s}M_s\beta_{M_sF_s}F_s}$$



$$P_{F_sM_s} = \frac{\beta_{F_sM_s}M_s\beta_{M_sF_s}F_s}{\beta_{FM}M\beta_{MF}F + \beta_{FM_s}M_s\beta_{M_sF}F + \beta_{F_sM}M\beta_{MF_s}F_s + \beta_{F_sM_s}M_s\beta_{M_sF_s}F_s}$$

MATHEMATICAL MODEL

With and Without the release of sterile species

- $$\begin{cases} \dot{L} = \beta\left(1 - \frac{L}{K}\right)F - (\mu_L + \nu_L)L \\ \dot{V} = \nu_L(1-m)L + \delta F + \gamma V_s - (\mu_V + \nu_v(P_{VM} + P_{VM_s}))V \\ \dot{V}_s = \nu_v P_{VM_s} V - (\mu_V + \gamma)V_s \\ \dot{F} = \nu_v P_{VM} V - (\mu_F + \delta)F \\ \dot{M} = \nu_L m L - \mu_M M \\ \dot{M}_s = \phi_1(t) - \mu_{Ms} M_s \\ \dot{F}_s = \phi_2(t) - \mu_{F_s} F_s \end{cases}$$

- $$P_{VM} = \frac{\beta_{VM} M \beta_{MV} V}{\beta_{VM} M \beta_{MV} V + \beta_{VM_s} M_s \beta_{M_s V} V + \beta_{F_s M} M \beta_{MF_s} F_s + \beta_{F_s M_s} M_s \beta_{M_s F_s} F_s}$$

- $$M_s(t) = e^{-\mu_{M_s} t}\left(M_s(0) + \int_0^t e^{\mu_{M_s} s}\phi_1(s)ds\right).$$

- $$F_s(t) = e^{-\mu_{F_s} t}\left(F_s(0) + \int_0^t e^{\mu_{F_s} s}\phi_2(s)ds\right).$$

MATHEMATICAL MODEL

With and Without the release of sterile species

- **we didn't take into consideration the fact of the competitiveness of males, sperms, residual fertility, polygamy immigration and emigration.**
- multiple mating is not considered here because of the type of equation we are working with

CRITICAL POINTS

Without the release of sterile species

- without the release of sterile insects the problem is

$$\begin{cases} \dot{L} = \beta\left(1 - \frac{L}{K}\right)F - (\mu_L + v_L)\,L \\ \dot{V} = v_L(1-m)L + \delta F - \mu_V V - v_v min(\frac{\gamma M}{V})V \\ \dot{F} = v_v min(\frac{\gamma M}{V})V - (\mu_F + \delta)F \\ \dot{M} = v_L m L - \mu_M M \end{cases}$$

- we can have two critical points at this stage the first one is $E^{\#} = (0,0,0,0)$ and the second is $T^{\#} = (L^*, V^*, F^*, M^*)$

  $L^* = K - \frac{K(\mu_L+v_L)}{\beta(m-1)\mu_L}(\delta - \frac{(\mu_V+v_v)(\mu_F+\delta)}{v_v})$

  $F^* = \frac{K\beta(m-1)v_L v_v - K(v_L+\mu_L)(\delta v_v - (\mu_v+v_v)(\delta+\mu_F))}{\beta(\delta v_v - (v_v+\mu_V)(\delta+\mu_F))}$

  $V^* = \frac{(\delta+\mu_F)}{v_v}F^*$

  $M^* = \frac{v_L m}{\mu_M}L^*$

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$T^{\#} = (L^*, V^*, F^*, M^*)$
$L^* = K - \frac{K(\mu_L+v_L)}{\beta(m-1)\mu_L}(\delta - \frac{(\mu_V+v_v)(\mu_F+\delta)}{v_v})$
$F^* = \frac{K\beta(m-1)v_L v_v - K(v_L+\mu_L)(\delta v_v - (\mu_v+v_v)(\delta+\mu_F))}{\beta(\delta v_v - (v_v+\mu_V)(\delta+\mu_F))}$
$V^* = \frac{(\delta+\mu_F)}{v_v}F^*$
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MATHEMATICAL MODEL

Realeasing only males without preferences

$$\begin{cases} \dot{L} = \beta\left(1 - \frac{L}{K}\right) F - (\mu_L + v_L) L \\ \dot{V} = v_L(1-m)L + \delta F + \gamma V_s - (\mu_F + v_v)V \\ \dot{V}_s = v_v P_{VM_s} V - (\mu_F + \gamma)V_s \\ \dot{F} = v_v P_{VM} V - (\mu_F + \delta)F \\ \dot{M} = v_L m L - \mu_M M \\ \dot{M}_s = \phi_1(t) - \mu_{Ms} M_s \end{cases}$$

- critical points

MATHEMATICAL MODEL

Realeasing only males without preferences

- critical points

$$[\eta_0 + \frac{\delta m v_L}{\mu_M\left(\mu_F+\delta\right)} - \frac{(\mu_F + v_v)v_L m}{\mu_M v_v}]L^* - \frac{\eta_0}{K}L^{*2} = [\frac{\mu_F + v_v}{v_v} - \frac{\gamma}{\mu_F+\gamma}]M_s$$

$$F^* = \frac{\mu_L + v_L}{\beta(1-\frac{L}{K})}L^*$$

$$M^* = \frac{m v_L}{\mu_M}L^*$$

$$V_s^* = \frac{M_s(\mu_F+\delta)}{M(\mu_F+\gamma)}\frac{\mu_L + v_L}{\beta(1-\frac{L}{K})}L^*$$

$$V^* = \frac{(\mu_F+\delta)}{v_v}(1+\frac{M_s}{M})\frac{\mu_L + v_L}{\beta(1-\frac{L}{K})}L^*$$

MATHEMATICAL MODEL

Realeasing only males without preferences

• critical points

with

$$\eta_0 = \frac{v_L^2(1-m)m\beta}{\mu_M\left(\mu_F+\delta\right)\left(\mu_L+v_L\right)}$$

MATHEMATICAL MODEL

Other sophisticated models, A Physiologically Based Approach

$$\begin{cases} \frac{\partial}{\partial t}N(t,x) + \frac{\partial}{\partial x}[G(t,x)N(t,x)] = -M(t,x)N(t,x) \\ N(t,0) = \int_0^{x_m} \beta\left(t,x'\right) N\left(t,x'\right) dx' \\ N(0,x) = n^0(x) \end{cases}$$

OUTLINE

GOOD LUCK!

- I will add this part later
- If you have some pieces of advice or suggestions,

BIBLIOGRAPHY

In [1] a detailed description of the use of LaTeX is given.

[1] Santosh Revadi et al. "Sexual Behavior of Drosophila suzukii". In: *(Insects)* (2015).


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