Theoritical Neuroscience: Dayan and Abbott

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1 Plasticity and Learning

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1.1 Introduction

- Activity dependent synaptic plasticity is considered to be the basic learning phenomenon.
- Hebb rule says that "if firing of A often leads to firing of B, then their synapse should be strengthened". Can be generalized to inhibition too.
- High-frequency stimulation induced synaptic potentiation, and low-frequency stimulation result in synaptic depression.
- If changes >10mins. Called Long term potentiation(LTP) and long term depression(LTD).
- Training procedures can be of 3 types.
  - *Supervised*: where input-output relationships are learned. Biologically plausible models have one network that acts as a teacher for another network.
  - *Un-Supervised*: where network response is solely based on intrinsic connections and dynamics of inputs.
  - *Reinforcement Learning*: where evaluative feedback is provided in the form of a reward or a punishment.
- Non-hebbian learning include when all the inputs to a neuron are simultaneously strengthened, or when the plasticity depends only on pre or post firings.
- Stability
  - Hebbian modification leads to uncontrollable growth. There should be some contraint.
  - Synaptic Saturation: Activity should be between 0 and a maximum value ω_{max}
  - Synaptic competition: If all neurons would be maxed out, there would be no competition.

1.2 Synaptic plasticity Rules

In our models, we use a continuous varaiable instead of a spike train.
u is the pre-synaptic firing rates, v is the post-synaptic firing rate
For simplification: they can be negative too. Also non-dimensional

Linear version of firing-rate model(unsupervised)

$$\tau_r \frac{dv}{dt} = -v + \mathbf{w}.\mathbf{u} = -v + \sum_{b=1}^{N_u} w_b u_b \tag{1}$$

where, w_b is the synaptic weight between b and postsynaptic neuron.

1. The Basic Hebb Rule
   - $\tau_w \frac{d\mathbf{w}}{dt} = v\mathbf{u}$ The basic rule. Simultaneous post and pre firings lead to increase in synaptic strength. Generally Slow so can be averaged for good approximation
   - $\tau_w \frac{d\mathbf{w}}{dt} = \langle v\mathbf{u} \rangle$ Averaged Hebb rule
   - $\tau_w \frac{d\mathbf{w}}{dt} = \mathbf{Q.w}$ Correlation based rule. Where Q is the input correlation matrix $\mathbf{Q} = \langle \mathbf{uu} \rangle$
   - Basic Hebb's rule is unstable. Does not have saturation constraint, and even if we add a constraint, it fails to induce competition.
2. The Covariance Rule
   - Hebbs rule only describes LTP, a threshold can be set which determines the switch from LTP, LDP.
   - $\tau_w \frac{d\mathbf{w}}{dt} = (v - \theta_v)\mathbf{u} = v(\mathbf{u} - \theta_u)$
   - We can set the threshold to their corresponding activity averages, and then take a average and use $v = \mathbf{w.u}$
   - $\tau_w \frac{d\mathbf{w}}{dt} = \mathbf{C.w}$ where C is the input covariance matrix, $C = \langle (\mathbf{u} - \langle \mathbf{u} \rangle)(\mathbf{u} - \langle \mathbf{u} \rangle) \rangle$
   - Unstable, because covariance rules are non-competitive.
3. BCM rule
   - The previous rule can produce activity without both pre and post.
   - $\tau_w \frac{d\mathbf{w}}{dt} = v\mathbf{u}(v - \theta_v)$. This is unstable if θ_v is fixed, but it grows rapidly than v.
   - $\tau_\theta \frac{d\theta_v}{dt} = v^2 - \theta_v$
   - BCM rule stabilises Hebbian plsticity by means of a sliding threshold.
4. Synaptic Normalizations
   - Global normalization is a more direct way to impose competetiveness and stability
   - Subtractive Normalization
     - Constrain the sum of weight matrix. So if one increases, the other decreases.
     - $\tau_w \frac{d\mathbf{w}}{dt} = v\mathbf{u} - \frac{v(textbf(n.u)n}{N_u}$
     - Where $\sum w_b = \mathbf{n.w}$ is a constant. N_u is a n-dimensional vector with all elements 1.
     - Saturation can be included by setting any element that is saturated to 0 in w_b.
   - Multiplicative Normalization aka Oja's Rule
     - $\tau_w \frac{d\mathbf{w}}{dt} = v\mathbf{u} - \alpha v^2 \mathbf{w}$
     - $|\mathbf{w}|^2$ will relax over time to $1/\alpha$
     - Induces competition because the length of the weight vector is constant
5. Timing Based Rules
   - Synaptic Plasticity occurs only if difference between pre and post synaptic spike times fall within $\pm$ 50ms
   - Firing rates can be used approximately instead of spiking models with a temporal difference τ
   - A function $H(\tau)$ determines the rate of synaptic modification.
   - In case of Hebbian rule, the rate of change of weights can be given as $\tau_w \frac{d\mathbf{w}}{dt} = \int_0^\infty d\tau (H(\tau)v(t)\mathbf{u}(t-\tau) + H(-\tau)v(t-\tau)\mathbf{u}(t))$

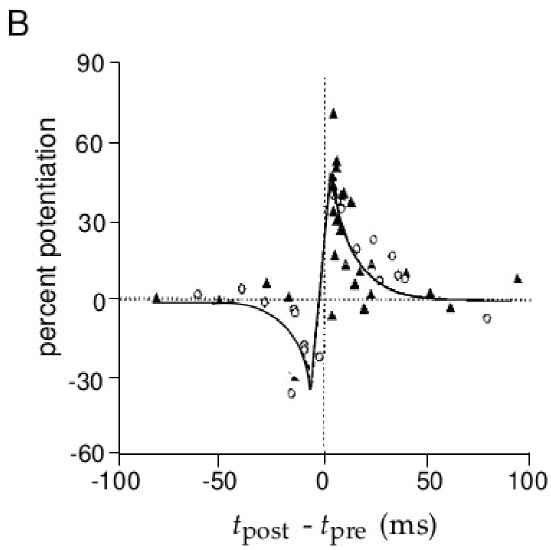

Figure 1: The filled symbols correspond to extracellular stimulation of the postsynaptic neuron, and the open symbols, to intracellular stimulation. The H function in is proportional to the solid curve.

Figure 2: