In the name of God

HW3 – Programming Part

1st MATLAB Assignment

DSP – Dr. Babaei-Zadeh

Student: Mehrsa Pourya

Student ID: 95101247

Sharif University of Technology

Electrical Engineering department

Winter 2019

PROJECT I: TIME-DOMAIN RESPONSE OF DIFFERENCE EQUATIONS

EXERCISE 1.1:

$$y[n] + 0.9y[n-2] = 0.3x[n] + 0.6x[n-1] + 0.3x[n-2].$$

a) I defined a, b vectors corresponded to above differential equation in MATLAB, the result is:

```
a =

    1.0000         0    0.9000


b =

    0.3000    0.6000    0.3000
```

b) Here we calculate impulse response of above system analytically, x[n] = $[n]$:

We use Z transform:

$$Y(z) + 0.9z^{-2}\,Y(z) = 0.3\,X(z) + 0.6z^{-1}X(z) + 0.3z^{-2}\,X(z)$$

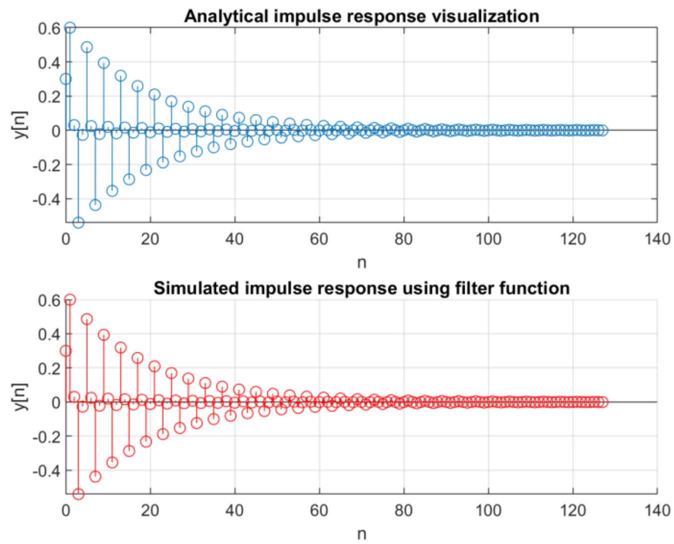
$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.3 + 0.6z^{-1} + 0.3z^{-2}}{1 + 0.9z^{-2}} = \frac{1}{3} + \frac{0.6\,z^{-1} - \frac{1}{30}}{1 + 0.9z^{-2}} = 0.3 + \frac{\frac{-0.6\frac{1}{\sqrt{0.9}j} - \frac{1}{30}}{2}}{1 + \sqrt{0.9}jz^{-1}} + \frac{\frac{0.6\frac{1}{\sqrt{0.9}j} - \frac{1}{30}}{2}}{1 - \sqrt{0.9}jz^{-1}}$$

$$h[n] = 0.3\delta[n] + \left(\frac{0.6\frac{1}{\sqrt{0.9}j} - \frac{1}{30}}{2}\right)\left(\sqrt{0.9}j\right)^{n} u[n] + \left(\frac{-0.6\frac{1}{\sqrt{0.9}j} - \frac{1}{30}}{2}\right)0.3\left(-\sqrt{0.9}j\right)^{n} u[n]$$

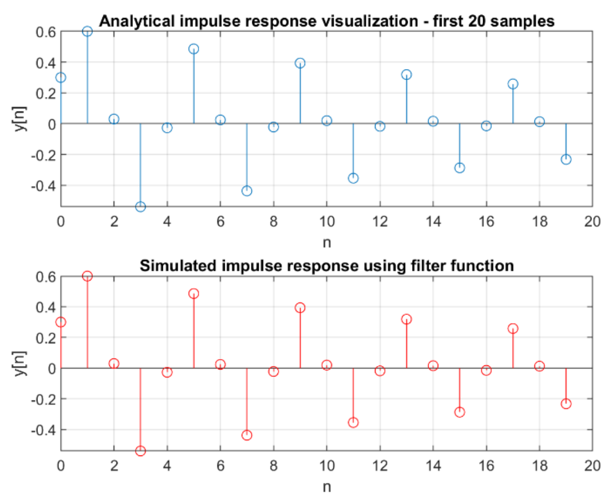
$$h[n] = \begin{cases} 0.3\delta[n] + 0.3\left(\sqrt{0.9}j\right)^{n-1}u[n] + 0.3\left(-\sqrt{0.9}j\right)^{n-1}u[n] & n\ odd \\ 0.3\delta[n] - \frac{1}{60}\left(\sqrt{0.9}j\right)^{n}u[n] - \frac{1}{60}\left(-\sqrt{0.9}j\right)^{n}u[n] & n\ even \end{cases}$$

$$h[n] = \begin{cases} 0 & n = 0 \\ 0.3 & n = 0 \\ -\frac{1}{30}\sqrt{0.9}^{n} & n = 4k\,, n > 0 \\ 0.6\sqrt{0.9}^{n-1} & n = 4k+1, n > 0 \\ +\frac{1}{30}\sqrt{0.9}^{n} & n = 4k+2, n > 0 \\ -0.6\sqrt{0.9}^{n-1} & n = 4k+3, n > 0 \end{cases}$$

c) Here we calculate y[n] using MATLAB (filter function) , then we plot both practical and analytical (part b) results for y[n] using stem function. We see that analytical and simulation results are the same.



For a better visualization, we plot only first 20 points then:

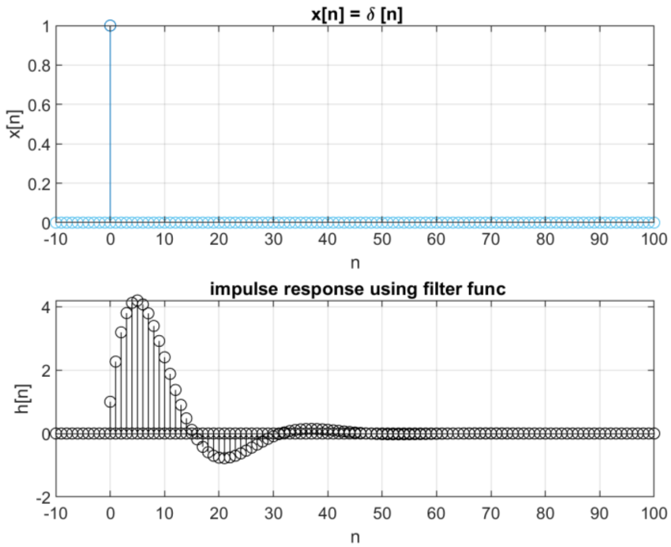


EXERCISE 1.2:

a) Here we simulate impulse response of given equation, to do that we define corresponded a, b vectors, generate impulse function and then calculate h[n] using filter function :

$$y[n] - 1.8\cos\left(\frac{\pi}{16}\right)\, y[n-1] + 0.81\, y[n-2] = x[n] + \frac{1}{2}x[n-1]$$

The result is:



b) Analytical answer:

$$H(z)=\frac{Y(z)}{X(z)}=\frac{1+0.5z^{-1}}{1-1.8\cos\left(\frac{\pi}{16}\right)z^{-1}+0.81z^{-2}}=\frac{A_1}{1-\dfrac{1.8cos\left(\frac{\pi}{16}\right)+\sqrt{\left(1.8cos\left(\frac{\pi}{16}\right)\right)^2-4*0.81}}{2}z^{-1}}+\frac{A_2}{1-\dfrac{1.8cos\left(\frac{\pi}{16}\right)-\sqrt{\left(1.8cos\left(\frac{\pi}{16}\right)\right)^2-4*0.81}}{2}z^{-1}}$$

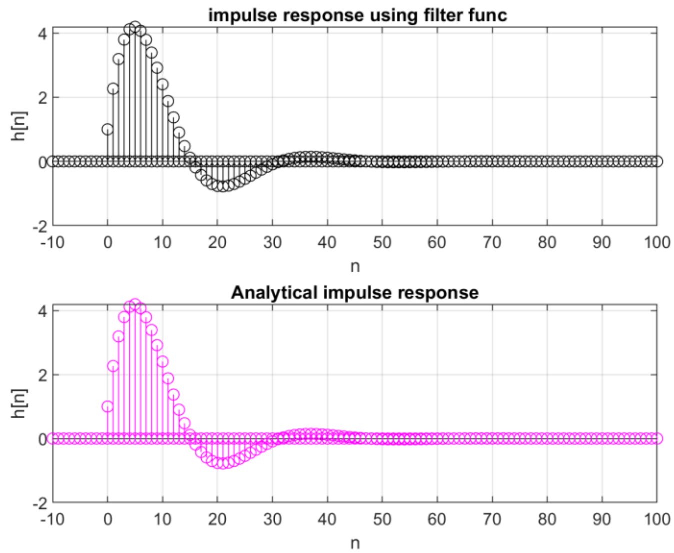
$$H(z)=\frac{A_1}{1-0.9\left(cos\left(\frac{\pi}{16}\right)+jsin\left(\frac{\pi}{16}\right)\right)z^{-1}}+\frac{A_2}{1-0.9\left(cos\left(\frac{\pi}{16}\right)-jsin\left(\frac{\pi}{16}\right)\right)z^{-1}}$$

$$H(z)=\frac{A_1}{1-0.9e^{j\left(\frac{\pi}{16}\right)}z^{-1}}+\frac{A_2}{1-0.9e^{-j\left(\frac{\pi}{16}\right)}z^{-1}}$$

$$H(z)=\frac{\dfrac{1+\frac{5}{9}e^{-j\frac{\pi}{16}}}{1-e^{-\frac{j\pi}{8}}}}{1-0.9e^{j\left(\frac{\pi}{16}\right)}z^{-1}}+\frac{\dfrac{1+\frac{5}{9}e^{\frac{j\pi}{16}}}{1-e^{\frac{j\pi}{8}}}}{1-0.9e^{-j\left(\frac{\pi}{16}\right)}z^{-1}}$$

$$h[n]=\frac{1+\frac{5}{9}e^{-j\frac{\pi}{16}}}{1-e^{-\frac{j\pi}{8}}}\left(0.9e^{\frac{j\pi}{16}}\right)^n u[n]+\frac{1+\frac{5}{9}e^{j\frac{\pi}{16}}}{1-e^{\frac{j\pi}{8}}}\left(0.9e^{-\frac{j\pi}{16}}\right)^n u[n]$$

In order to confirm our analytical answer we plot both simulation analytical results to see whether they are the same or not, as we see below they are the same so our results are correct.



EXERCISE 1.3 : Natural Frequencies

a) For system given in previous exercise we determine natural frequencies.

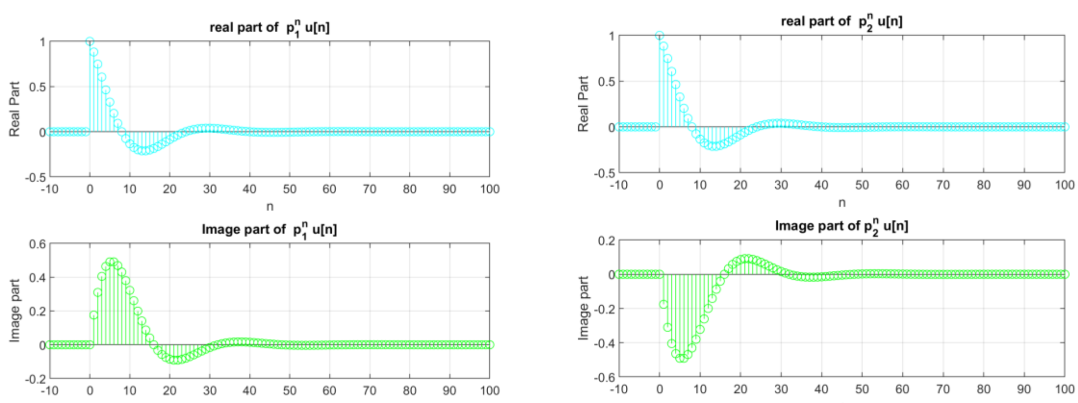
$$y[n] - 1.8\cos\left(\frac{\pi}{16}\right)\, y[n-1] + 0.81\, y[n-2] = x[n] + \frac{1}{2}x[n-1]$$

We calculate roots using MATLAB root function , roots are:

```
roots =

   0.8827 + 0.1756i
   0.8827 - 0.1756i
```

Then we plot Real and imaginary parts of $p_k^n\, u[n]$, *here* $k = 1,2$ *and* p_1 *and* p_2 are roots derived above.



b)

$$y[n] = x[n] + \frac{1}{2}x[n-1] + 1.8\cos\left(\frac{\pi}{16}\right)y[n-1] - 0.8y[n-2]$$

$$x[n] = \delta[n] \rightarrow \ h[0] = 1\ , due\ to\ causality\ y[n<0] = 0$$

$$h[1] = 0.5 + 1.8\cos\left(\frac{\pi}{16}\right)$$

$$\boldsymbol{h[n] = \left(\alpha p_1^n + \beta p_2^n\right)\ u[n]}$$

So we can write: $$h[0] = \alpha + \beta = 1$$

$$h[1] = \alpha p_1 + \beta p_2 = 0.5 + 1.8\cos\left(\frac{\pi}{16}\right)$$

We solve this using MATLAB \ operator and the result is:

```
alpha =

   0.5000 - 3.9375i


beta =

   0.5000 + 3.9375i
```

Now we generate h[n] as below using derived values for $\alpha, \beta, p_1\ , p_2\ above.$

$$\boldsymbol{h[n] = \left(\alpha p_1^n + \beta p_2^n\right)\ u[n]}$$

We stem the result and Compare it with simulated result of exercise 1.2, we see that they are the same, this confirms our answers.(The analytical answer(1.2 b) is also the same with calculated h[n] in this part)