03 - Symbols, No Numbers Lean: First Steps

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Symbolic Statements

- Let's see Lean prove purely symbolic statements.
- Because these don't mention specific numbers, they can be stronger, more general, statements.

Task

• Given the following facts about $x, y, z, c \in \mathbb{R}$.

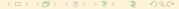
$$z = y^2$$

$$y = x + c$$

• Our task is to prove

$$z = (x+c)^2$$

• If true, this statement holds for any values of $x, c \in \mathbb{R}$.



Maths

• The task is not difficult. Even so, let's write out a structured proof.

$$z = y^2$$
 given fact (1)

$$y = x + c$$
 given fact (2)

$$z = y^2$$
 using fact (1)
= $(x + c)^2$ substitution using fact (2)

Maths

- We start by listing the two given facts, $z = y^2$ and y = x + c.
- We want to prove something about z. The first fact gives us $z = y^2$.
- We'd like to express y in terms of x and c, if possible.
- The second fact tells us y = x + c. We can use it to substitute for the y in y^2 . This gives us $z = (x + c)^2$, our objective.

Maths

• This proof has no numbers, it is entirely symbolic.

Code

```
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import Mathlib.Tactic

example {x y z c: R} (h1 : z = y^2) (h2: y = x + c) : z = (x + c)^2 := by

calc

z = y^2 := by rw [h1]

_ = (x + c)^2 := by rw [h2]
```

Code

- Notice how similar this code is to that from the previous chapter.
- We can see in the proof header how we
 - define the variables x , y , z and c as real numbers
 - list the two hyoptheses
 - specify the overall proof objective $z = (x + c)^2$.

Code

- The main body of the proof is also similar.
- We first establish that $z = y^2$, justifying it by the first hypothesis h1.
- Then we say this is the same as $(x + c)^2$, justifying it with second hypothesis h2.
- That completes the proof, which is one line shorter than the previous one because we don't need to do a numerical calculation.

Easy Exercise

• Write a Lean program to prove that z = x given z = y and y = x, where $x, y, z \in \mathbb{R}$.