

# 01 - First Proof

Lean: First Steps

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# Task

- Let's start with a very simple task. Imagine we're given the following fact.

$$a = 4$$

- Our task is to prove that

$$a > 1$$

- Here,  $a$ , 1 and 4 are all natural numbers.

- Seems obvious that if  $a = 4$  then  $a > 1$ .
- Challenge is thinking about easy tasks in a structured way.
- So how can we justify  $a > 1$ , given  $a = 4$ ?
  - Question - how can we justify  $4 > 1$  ?
  - Answer - the natural numbers  $\mathbb{N}$  are ordered by “greater than”  $>$ .

- Let's write down these thoughts in a structured way.

$$a = 4$$

given fact

$$> 1$$

by the ordering of natural numbers



- We'll need this kind of structured step-by-step thinking to write proofs in Lean.

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```
-- 01 - First Proof

import Mathlib.Tactic

example {a : ℕ} (h1: a = 4) : a > 1 := by
  calc
    a = 4 := by rw [h1]
    _ > 1 := by norm_num
```

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- The proof starts on the line beginning with `example`. This line states the **theorem** we want to prove.
- `{a : ℕ}` tells Lean the variable `a` is a natural number.
- `(h1 : a = 4)` is the given fact, or **hypothesis**.
  - It's given a label `h1` so we can refer to it later.
- `a > 1` is the statement we want to prove.
- Finally the `:=` by signals we're about to prove that statement.

The last three lines of code prove the theorem.

- We state  $a = 4$ , and **justify** it by referring to the given fact, previously labelled `h1`.
- We complete the proof by saying this is  $> 1$ , and **justify** it by the ordering of natural numbers.

# Tactics

- We justified  $4 > 1$  with `norm_num`. How did this happen?
- The **Peano axioms** describe the natural numbers. On top of these axioms, we can define addition, then the “greater than”  $>$  relation.
- We did not go that deep in our maths proof!
- For a large body of fundamental results, we don’t need to prove them every time we use them. The same idea applies to Lean proofs.
- Many of those fundamental results have been written in Lean and packaged up as **tactics**, ready for us to use.
- `norm_num` is a tactic that includes knowledge about the order of natural numbers.



# Easy Exercise

- The Lean program above proves  $a > 1$  given  $a = 4$ .
- Change it to prove  $a < 10$  given  $a = 4$ .
- Tip: the `norm_num` tactic understands “less than”  $<$ , as well as “greater than”  $>$ .