00 - Proofs & Proof Assistants
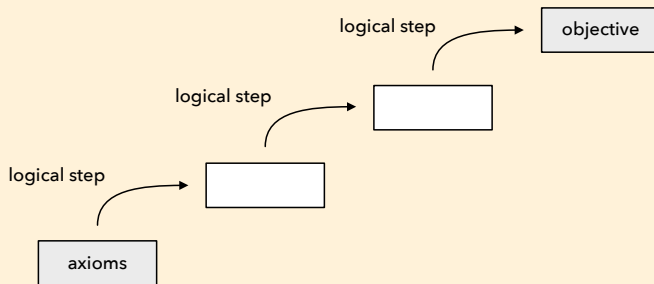## Lean: First Steps

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Proof?

- Thousands of years of discovering mathematical truths - eg

  - Inifinitude of primes

  - Irrationality of $\sqrt{2}$

  - Four colours to colour a geographic map

- Truth is not determined by weight of experimental evidence.

- Truth is determined by **proof** - reasoned argument, based on a foundation of **axioms** & framework of **logic**.

Proof

Axioms & Logic

- Axiom?

  - Peano Axioms for natural numbers - eg if n is a natural number, then so is its successor $n + +$.

  - Zermelo-Fraenkel axioms for set theory - eg two sets are the same if they contain the same elements.

- Logic?

  - eg $A \implies B$ is not the same as $B \implies A$.

Proof Assistant

- Some software helps automate the creative process of constructing proofs.

- A proof assistant is different - it checks proofs **we** write.

  - A powerful capability because human expertise is rare, and we humans make errors.

Lean 4

- Lean is a programming language.

- With mathlib, used as a proof assistant.

  - Microsoft Visual Studio Code with Lean extension is popular

  - Web version is useful - only requires a browser:
    `https://live.lean-lang.org`