

10 - “And” Goal

Lean: First Steps

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“And” Goal

- We’ve seen disjunctive and conjunctive **hypotheses**.
- We’ve also seen disjunctive proof **goals**.
- Here we look at a conjunctive proof **goal**.

Task

- For integer x , given that $x = -1$, show that

$$(x^2 = 1) \wedge (x^3 = -1)$$

- $P \wedge Q$ true means at **both** P and Q must be true.
- So we have to **prove** both $x^2 = 1$ and $x^3 = -1$.

$$\begin{array}{ll} x = -1 & \text{given fact} \\ (x^2 = 1) \wedge (x^3 = -1) & \text{proof objective} \end{array} \quad (1)$$

$$\begin{array}{ll} x^2 = (-1)^2 & \text{using fact (1)} \\ = 1 & \text{using arithmetic} \end{array} \quad (2)$$

$$\begin{array}{ll} x^3 = (-1)^3 & \text{using fact (1)} \\ = -1 & \text{using arithmetic} \end{array} \quad (3)$$

$$(x^2 = 1) \wedge (x^3 = -1) \quad \text{using results (2, 3)} \quad \square$$

```
-- 10 - Conjunctive "and" Goal
```

```
import Mathlib.Tactic
```

```
example {x :  $\mathbb{Z}$ } (h : x = -1) : x^2 = 1  $\wedge$  x^3 = -1 := by
  constructor
  · calc
    x^2 = (-1)^2 := by rw [h]
    _ = 1 := by norm_num
  · calc
    x^3 = (-1)^3 := by rw [h]
    _ = -1 := by norm_num
```

- The objective here is a conjunction, uses the symbol \wedge to denote “logical and”.
- `constructor` splits the conjunctive goal into two separate goals.
- Infview will confirm `constructor` replaces the **single** goal $x^2 = 1 \wedge x^3 = 1$ with **two** new goals, $x^2 = 1$ and $x^3 = -1$.
- The rest of the proof has two **calc** structures, one after the other, to show $x^2 = 1$ and $x^3 = -1$.
 - Focussing dots are used to make clear there are two sub-proofs.

- Placing the cursor before `constructor` shows the original proof goal.

```
x : ℤ
h : x = -1
⊢ x ^ 2 = 1 ∧ x ^ 3 = -1
```

- The cursor after `constructor` shows the two replacement goals.

```
x : ℤ
h : x = -1
⊢ x ^ 2 = 1
```

```
x : ℤ
h : x = -1
⊢ x ^ 3 = -1
```

Easy Exercise

- For integer x , given that $x = -1$, write a Lean program to show that

$$(x^3 = -1) \wedge (x^4 = 1) \wedge (x^5 = -1)$$

- A longer conjunction could be $P \wedge Q \wedge R \wedge S$.
 - `constructor` splits this into two goals, P and $Q \wedge R \wedge S$.
- The proof will require two uses of `constructor`.