# 10 - "And" Goal

Lean: First Steps

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October 4, 2024

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#### "And" Goal

- We've seen disjunctive and conjunctive hypotheses.
- We've also seen disjunctive proof goals.
- Here we look at a conjunctive proof goal.

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## Task

• For integer x, given that x = -1, show that

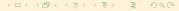
$$(x^2 = 1) \wedge (x^3 = -1)$$



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## Maths

- $P \wedge Q$  true means at **both** P and Q must be true.
- So we have to **prove** both  $x^2 = 1$  and  $x^3 = -1$ .



## Maths

$$x=-1$$
 given fact (1)  $(x^2=1) \wedge (x^3=-1)$  proof objective

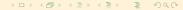
$$x^2 = (-1)^2$$
 using fact (1)  
= 1 using arithmetic (2)

$$x^3 = (-1)^3$$
 using fact (1)  
= -1 using arithmetic (3)

$$(x^2 = 1) \land (x^3 = -1)$$
 using results (2, 3)

#### Code

```
-- 10 - Conjunctive "and" Goal
import Mathlib.Tactic
example \{x : \mathbb{Z}\} \{x : x = -1\} \{x : x^2 = 1 \land x^3 = -1\}
  constructor
  calc
      x^2 = (-1)^2 := by rw [h]
      _ = 1 := by norm_num
  calc
      x^3 = (-1)^3 := by rw [h]
      _{-} = -1 := by norm_num
```



#### Code

- The objective here is a conjunction, uses the symbol  $\wedge$  to denote "logical and".
- constructor splits the conjunctive goal into two separate goals.
- Infoview will confirm constructor replaces the single goal x^2 = 1
   \( x^3 = 1 \) with two new goals, \( x^2 = 1 \) and \( x^3 = -1 \).
- The rest of the proof has two calc structures, one after the other, to show  $x^2 = 1$  and  $x^3 = -1$ .
  - Focussing dots are used to make clear there are two sub-proofs.

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#### Infoview

Placing the cursor before constructor shows the original proof goal.

```
x : \mathbb{Z}

h : x = -1

\vdash x ^ 2 = 1 \land x ^ 3 = -1
```

• The cursor after constructor shows the two replacement goals.

```
x : \mathbb{Z}

h : x = -1

\vdash x ^ 2 = 1

x : \mathbb{Z}

h : x = -1

\vdash x ^ 3 = -1
```

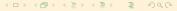
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# Easy Exercise

• For integer x, given that x = -1, write a Lean program to show that

$$(x^3 = -1) \wedge (x^4 = 1) \wedge (x^5 = -1)$$

- A longer conjunction could be  $P \wedge Q \wedge R \wedge S$ .
  - constructor splits this into two goals, P and  $Q \wedge R \wedge S$ .
- The proof will require two uses of constructor.



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