01 - First Proof <u>Lea</u>n: First Steps

Tariq Rashid

August 25, 2024

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Task

• Let's start with a very simple task. Imagine we're given the following fact.

$$a = 4$$

Our task is to prove that

• Here, a, 1 and 4 are all natural numbers.

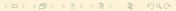


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Maths

- Seems obvious that if a = 4 then a > 1.
- Challenge is thinking about easy tasks in a structured way.
- So how can we justify a > 1, given a = 4?
 - Question how can we justify 4 > 1?
 - Answer the natural numbers $\mathbb N$ are ordered by "greater than" >.



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Maths

Let's write down these thoughts in a structured way.

$$a=4$$
 given fact

> 1 by the ordering of natural numbers

• We'll need this kind of structured step-by-step thinking to write proofs in Lean.



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Code

```
-- 01 - First Proof
import Mathlib.Tactic
example \{a : \mathbb{N}\}\ (h1: a = 4) : a > 1 := by
  calc
    a = 4 := by rw [h1]
    _ > 1 := by norm_num
```



Tarig Rashid

Code

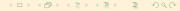
- The proof starts on the line beginning with example. This line states the **theorem** we want to prove.
- {a : N} tells Lean the variable a is a natural number.
- (h1 : a = 4) is the given fact, or hypothesis.
 - It's given a label h1 so we can refer to it later.
- a > 1 is the statement we want to prove.
- Finally the := by signals we're about to prove that statement.

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Code

The last three lines of code prove the theorem.

- We state a = 4, and justify it by referring to the given fact, previously labelled h1.
- We complete the proof by saying this is > 1, and justify it by the ordering of natural numbers.



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Tactics

- We justified 4 > 1 with norm_num. How did this happen?
- The Peano axioms describe the natural numbers. On top of these axioms, we can define addition, then the "greater than" > relation.
- We did not go that deep in our maths proof!
- For a large body of fundamental results, we don't need to prove them every time we use them. The same idea applies to Lean proofs.
- Many of those fundamental results have been written in Lean and packaged up as tactics, ready for us to use.
- norm_num is a tactic that includes knowledge about the order of natural numbers.

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Easy Exercise

- The Lean program above proves a > 1 given a = 4.
- Change it to prove a < 10 given a = 4.
- Tip: the norm_num tactic understands "less than" <, as well as "greater than" >.

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