10 - "And" Goal

Lean: First Steps

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October 27, 2024

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"And" Goal

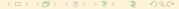
- We've seen disjunctive and conjunctive hypotheses.
- We've also seen disjunctive proof goals.
- Here we look at a conjunctive proof goal.

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Task

• For integer x, given that x = -1, show that

$$(x^2 = 1) \wedge (x^3 = -1)$$



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Maths

- $P \wedge Q$ true means at **both** P and Q must be true.
- So we have to **prove** both $x^2 = 1$ and $x^3 = -1$.



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Maths

$$x=-1$$
 given fact (1) $(x^2=1) \wedge (x^3=-1)$ proof objective

$$x^2 = (-1)^2$$
 using fact (1)
= 1 using arithmetic (2)

$$x^3 = (-1)^3$$
 using fact (1)
= -1 using arithmetic (3)

$$(x^2 = 1) \land (x^3 = -1)$$
 using results (2, 3)

Code

```
-- 10 - Conjunctive "and" Goal
import Mathlib.Tactic
example \{x : \mathbb{Z}\} \{x : x = -1\} \{x : x^2 = 1 \land x^3 = -1\}
  constructor
  calc
      x^2 = (-1)^2 := by rw [h]
      _ = 1 := by norm_num
  calc
      x^3 = (-1)^3 := by rw [h]
      _{-} = -1 := by norm_num
```



Code

- The objective here is a conjunction, uses the symbol ∧ to denote "logical and".
- constructor splits the conjunctive goal into two separate goals.
- Infoview will confirm constructor replaces the single goal $x^2 = 1 \land x^3 = 1$ with two new goals, $x^2 = 1$ and $x^3 = -1$.
- The rest of the proof has two calc structures, one after the other, to show $x^2 = 1$ and $x^3 = -1$.
 - Focussing dots are used to make clear there are two sub-proofs.



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Infoview

• Placing the cursor before constructor shows the original proof goal.

```
x : \mathbb{Z}

h : x = -1

\vdash x \land 2 = 1 \land x \land 3 = -1
```

• The cursor after constructor shows the two replacement goals.

```
x : \mathbb{Z}
h : x = -1
\vdash x ^ 2 = 1
x : \mathbb{Z}
h : x = -1
\vdash x ^ 3 = -1
```

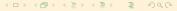
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Easy Exercise

• For integer x, given that x = -1, write a Lean program to show that

$$(x^3 = -1) \wedge (x^4 = 1) \wedge (x^5 = -1)$$

- A longer conjunction could be P ∧ Q ∧ R ∧ S.
 - constructor splits this into two goals, P and $Q \wedge R \wedge S$.
- The proof will require two uses of constructor.



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