09 - "Or" Goal

Lean: First Steps

Tariq Rashid

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"Or" Goal

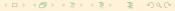
- We've seen disjunctive and conjunctive hypotheses.
- Here we look at a disjunctive proof goal.

Task

• For integer x = -1, show that

$$(x^2 = 1) \lor (x^3 = 1)$$

• $x^3 = 1$??



Maths

- Clearly $(-1)^3 = 1$ is not true.
- $P \vee Q$ being true means at least one of P and Q is true.
- If Q is false, but P is true, then the disjunction $P \vee Q$ is still true.
- If we can prove $x^2 = 1$, then we have proven the disjunction $(x^2 = 1) \lor (x^3 = 1).$



Maths

Step-by-step proof

$$x = -1$$
$$(x^2 = 1) \lor (x^3 = 1)$$

$$x^2 = 1$$

$$x^2 = (-1)^2$$
$$= 1$$

$$(x = -1) \implies (x^2 = 1) \lor (x^3 = 1)$$

given fact
$$(1)$$

Choice

Consider proof objective

$$(x^2 = 1) \lor (x^3 = -1)$$

- Both statements $x^2 = 1$ and $x^3 = -1$ can be proven to be true.
- We can choose which one of the two statements we want to prove. One is sufficient.



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Code

```
-- 09 - Disjunctive "or" Goal

import Mathlib.Tactic

example {x : Z} (h : x = -1) : x^2 = 1 \lor x^3 = 1 := by

left

calc

x^2 = (-1)^2 := by rw [h]

_ = 1 := by norm_num
```

Code

- The objective is a disjunction, and uses the symbol ∨ to denote "logical or".
- We state our intention to prove only the "left" part of the disjunction using left.
- Infoview will confirm left replaces the goal $x^2 = 1 \lor x^3 = 1$ with $x^2 = 1$.
- Rest of the proof uses the familiar calc to show $x^2 = 1$.

Infoview

Placing the cursor before left shows the original proof goal.

```
x : \mathbb{Z}

h : x = -1

\vdash x ^ 2 = 1 \lor x ^ 3 = 1
```

 Placing the cursor on the next line after left confirms the proof goal has been replaced by a smaller, but sufficient, statement.

```
x : \mathbb{Z}
h : x = -1
\vdash x ^ 2 = 1
```

Manipulating the proof goal is quite normal.

Easy Exercise

• For integer x, given that x = -1, write a Lean program to show

$$(x = 1) \lor (x^2 = 1) \lor (x^3 = 1)$$

- A longer disjunction could be $P \lor Q \lor R \lor S$.
 - left selects the left-most statement P as the new goal.
 - right selects the remainder $Q \lor R \lor S$ as the new goal.
- You'll need both right and left to write the proof.

