EE 233 Circuit Theory
Lab 1: RC Circuits

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1 Introduction

This lab is designed to teach students methods for characterizing circuit systems, and more specifically, an RC circuit system. This lab will also familiarize students with the test bench instruments used in this class by having them use the equipment to analyze some fundamental response trends of step and sinusoidal input functions for an RC circuit.

A circuit system can be pictured as a box with inputs and outputs, and the characteristics of this system can be represented by its input and output signals, e.g. voltage and current. A signal contains three parameters: magnitude, frequency, and phase. Any change of these parameters in the input signal will affect the output signal.

The RC circuit has many interesting characteristics while staying one of the most basic circuit systems. This lab is going to allow students to observe these characteristics and teach them how to analyze the output signals with changes in input magnitude or frequency.

This lab is split into a prelab exercise and hardware implementation. Submit one prelab report and one lab report per group, with the members' names are clearly written on the front page. There is no template for the prelab report, and the lab report template is available on Canvas. These reports must be in pdf format. There are multiple apps, including CamScanner, for Apple and Android phones that turn photos into pdf's.

2 Precautions

None of the devices used in this set of experiments are particularly static sensitive; nevertheless, you should pay close attention to the circuit connections and the polarity of the power supplies, function generator, and oscilloscope inputs.

3 Prelab Exercises

3.1 The RC Response to a DC Input

3.1.1 Charging RC Circuit

The differential equation for $v_{\mathrm{out}}(t)$ is the most fundamental equation describing the RC circuit, and it can be solved if the input signal $v_{\mathrm{in}}(t)$ and an initial condition are given.

Prelab #1:
Derive the differential equation for $v_{\mathrm{out}}(t)$ in Figure 3.1.1, in terms of $v_{\mathrm{in}}(t)$, R, and C.

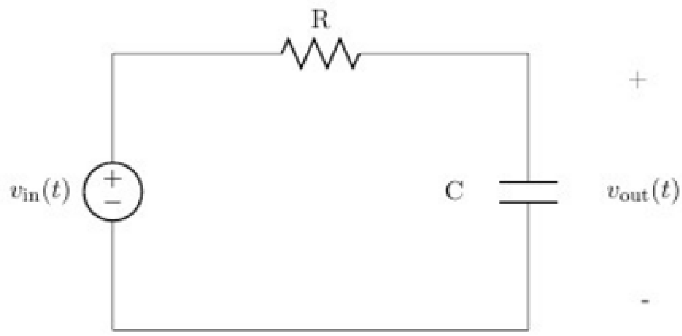

Figure 3.1.1: Single-stage RC circuit.

Now suppose the input signal $v_{\mathrm{in}}(t)$ has been zero for a long time, and then is changed to V_o, a positive constant, at time $t = 0$. The input signal is then a step function, which means:

$$v_{\mathrm{in}}(t) = V_o\, u(t) = \begin{cases} 0, & t < 0 \\ V_o, & t \geq 0 \end{cases}$$

The initial condition for $v_{\mathrm{out}}(t)$ is needed to solve the differential equation. The output voltage should be zero when $t < 0$, since there is no input until $t = 0$. Thus, the initial condition for $v_{\mathrm{out}}(t)$ is $v_{\mathrm{out}}(0) = 0$.

Prelab #2:
Derive $v_{\mathrm{out}}(t)$ for $t \geq 0$ in terms of V_o, R, and C.

Download **Lab1_Prelab.m** and **lab1plot.m** from the Canvas webpage, making sure they are in the same folder on your computer. Suppose $V_o = 5\mathrm{V}$, $R = 10\mathrm{k}\Omega$, and $C = 0.01\mu\mathrm{F}$.

Prelab #3:
Using the given Matlab scripts, plot $v_{\mathrm{in}}(t)$ and $v_{\mathrm{out}}(t)$ on the same set of axes.

To do this, open **Lab1_Prelab.m** using Matlab (there is no need to open the other file) and read the developer comments about how to use the **lab1plot** function. Run the script, select "Change Folder" if the warning appears, and the plot for Prelab #3 should appear. You are not expected to know how to use Matlab in this course, so feel free to ask the TA for assistance if you have difficulty using the script.

3.1.2 Discharging RC Circuit

You have now analyzed the RC circuit's step response, and you also have a general idea of what this response looks like by plotting it with the input voltage. Now suppose the input signal has been V_o, a positive constant, for a long time before being changed to zero at $t = 0$, which means

$$v_{\text{in}}(t) = V_o\, u(-t) = \begin{cases} V_o, & t < 0 \\ 0, & t \geq 0 \end{cases}$$

> **Prelab #4:**
> Find the initial condition for $v_{out}(t)$, then find $v_{out}(t)$ for $t \geq 0$ in terms of V_o, R, and C.

> **Prelab #5:**
> Using the given Matlab scripts, plot $v_{\text{in}}(t)$ and $v_{\text{out}}(t)$ on the same set of axes.

3.1.3 Square Wave Input

If the input signal is turned on and off periodically then it becomes a square wave. Suppose the period of this square wave is T, and its duty cycle (the ratio of how long the square wave is on vs. how long it's off) is 50%. If half of the period, $T/2 \gg RC$ then the output voltage goes to its limit before the input changes.

Example: If $T = 10RC$, the ratio $\frac{V_{out}(\mathrm{T}/2) - V_{out}(0)}{V_0} = \frac{V_0 \exp(-5)}{V_0} = 0.67\% < 1\%$. So the change of output voltage is almost equal to the change of the input voltage, and it means the output voltage is close to its limit.

Refer to Reference 5.1 to answer Prelab #6.

> **Prelab #6:**
> Find the time constant, rise time, fall time, and both delay times for the RC circuit in terms of R and C. Then replace R and C with their numeric values from Section 3.1.1. How long does the period T need to be for the output to be within 1% of its final value?

When deriving the expressions, notice that these timing parameters are independent of the input voltage.

> **Prelab #7:**
> Suppose the period of the turning switch is $T = 4$ms. Using the given Matlab scripts, plot the input and output voltages $v_{\text{in}}(t)$ and $v_{\text{out}}(t)$ over two periods.

3.1.4 Multiple-stage RC Circuits

Refer to Reference 5.2 Elmore Delay Estimation to answer Prelab #8.

> **Prelab #8:**
> Calculate the delay time for the circuit in Figure 3.1.1, Figure 3.1.2, and Figure 3.1.3 (located on the next page), in terms of R and C. Assume $R = R_1 = R_2 = R_3$ and $C = C_1 = C_2 = C_3$.

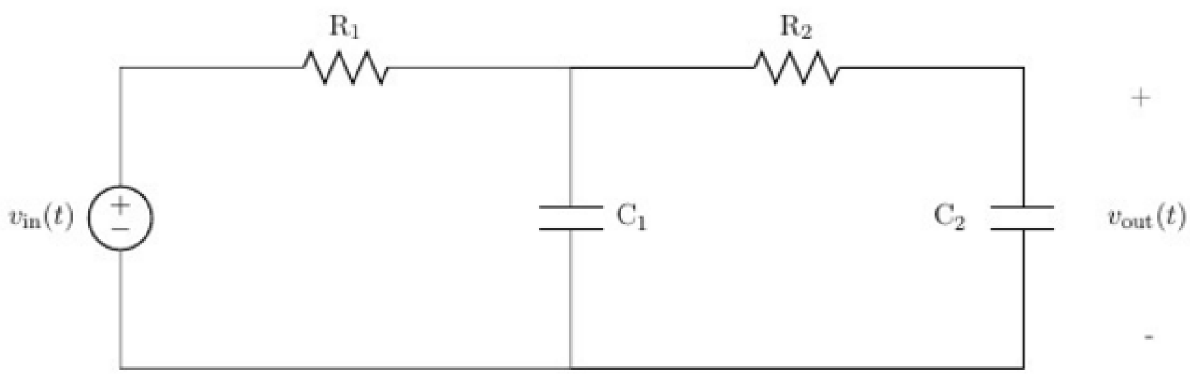

Figure 3.1.2: Two-stage RC circuit.

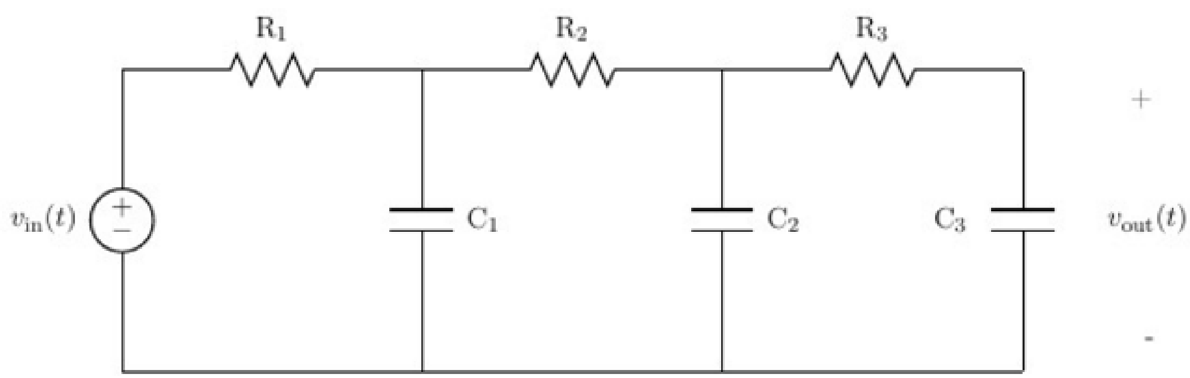

Figure 3.1.3: Three-stage RC circuit.

3.2 The RC Response to a Sinusoidal Input

3.2.1 Time-domain RC Response

While the input square wave changes the magnitude of the signal, exploration of the RC response to an AC signal can show more interesting characteristics of the RC circuit. Looking back on Figure 3.1.1, the single-stage RC circuit, suppose we are using a sinusoidal wave as an input signal, $v_{\text{in}}(t) = V_o \cos(\omega t)$, where ω is the angular frequency of the signal.

Prelab #9:
Derive the new differential equation for $v_{\text{out}}(t)$ in Figure 3.1.1, in terms of V_o, ω, R, and C.

This differential equation is the fundamental equation describing the RC circuit system. The solution for the steady-state output voltage is

$$v_{\text{out}}(t) = \frac{V_o}{1 + R^2 C^2 \omega^2}[\cos(\omega t) + RC\omega \sin(\omega t)]$$

This solution shows that $v_{\text{out}}(t)$ is a function of the signal's frequency f and time t. The relationship between angular frequency ω and signal frequency f is $\omega = 2\pi f$.

Suppose $V_o = 1\text{V}$ (notice it's different), $f = 1\text{kHz}$, $R = 10\text{k}\Omega$, and $C = 0.01\mu\text{F}$.

Prelab #10:
Using the given Matlab scripts, plot $v_{\text{out}}(t)$ from 0 to 5ms. Is the output signal a sinusoidal function? If so, what is the period T and the magnitude $|v_{\text{out}}(t)|$?

3.2.2 Frequency-domain RC Response

Now consider the solution for $v_{\text{out}}(t)$ with the signal's frequency f being the independent variable. The output voltage is a sinusoidal wave with the same frequency as the input voltage, and its magnitude is given by

$$|V_{\text{out}}(f)| = \frac{V_{\text{o}}}{\sqrt{1 + 4\pi^2 R^2 C^2 f^2}}$$

Suppose $V_o = 1\text{V}$, $R = 10\text{k}\Omega$, and $C = 0.01\mu\text{F}$.

Prelab #11:
Using the given Matlab scripts, plot the magnitude of the output voltage, $|V_{\text{out}}(f)|$, versus frequency. Comment on the output signal's characteristics at very low frequencies, e.g. 10Hz, and at very high frequencies, e.g. 1MHz. Read Reference 5.3 for more information.

Notice that the frequency-domain plot's *x*-axis is logarithmic, that is, each division is 10 times greater than the previous. This frequency-domain plot will become very important in subsequent labs, where you will use it to design filters for your audio mixer.

Now consider another RC system in Figure 3.2.1, in which the output voltage is over the resistor, rather than the capacitor.

The output voltage is now the input signal minus the voltage over the capacitor, and its magnitude is given by

$$|V_{\text{out}}(f)| = \frac{2\pi V_{\text{o}} R C f}{\sqrt{1 + 4\pi^2 R^2 C^2 f^2}}$$

Suppose $V_o = 1\text{V}$, $R = 10\text{k}\Omega$, and $C = 0.01\mu\text{F}$.

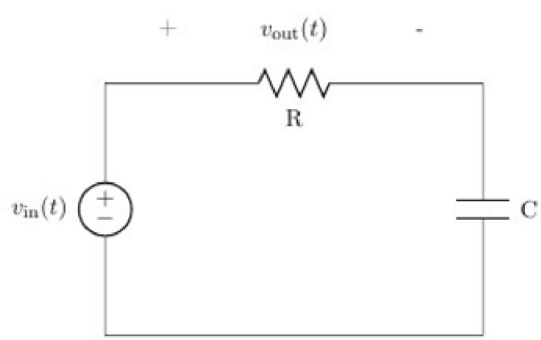

Figure 3.2.1: An RC circuit with the output over the resistor.

Prelab #12:
Using the given Matlab scripts, plot the magnitude of the output voltage, $|V_{\text{out}}(f)|$, versus frequency. Comment on the output signal's characteristics at very low frequencies, e.g. 1Hz, and at very high frequencies, e.g. 1MHz. In what way(s) does this output behave differently than the one over the capacitor? Explain.

4 Experimental Procedure and Data Analysis

4.1 The RC Response to a DC Input

4.1.1 Square Wave Input Analysis

Build the circuit in Figure 4.1.1 and set the function generator to provide a square wave input as follows:

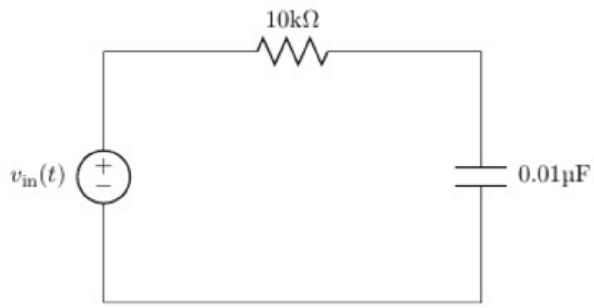

Figure 4.1.1: RC circuit for lab experiment.

a) The period $T \geq 4$ms (to ensure that $T \gg RC$). This value of T guarantees that the output signal has sufficient time to reach a final value before the next input transition. **Record your value of *T*.**

b) The minimum voltage is 0V and maximum voltage is 5V. Note that you may need to manually set the offset to achieve this waveform. Use the oscilloscope to display this waveform on Channel 1 to verify that the amplitude is correct. We use these amplitudes since it they are common in computer systems (false = 0V, true = 5V).

Use Channel 2 of the oscilloscope to display the output voltage over the capacitor. Adjust the time base to display 3 complete cycles of the signals. Capture the output from the scope display with both the waveforms and the measured values. **Turn this oscilloscope waveform in as part of your lab report.**

> **Analysis #1:**
> Does the oscilloscope display the same waveform that you plotted in Prelab #7? Explain any similarities or differences.

Using the oscilloscope's **Cursor** menu, record the period T of the input signal, as well as the maximum and minimum values of the output signal. Then measure the time value of the 10% point of V_{out}, the time value of the 90% point of V_{out}, and the time value of the 50% point of V_{out}.

Note: Instructions for using the lab equipment are found in **Lab Equipment.pdf**, on the Canvas webpage.

> **Analysis #2:**
> Calculate the rise time, fall time, and delay time of the RC circuit using the values from the oscilloscope, then compare them with the theoretical values from your Prelab using percent error. Explain the likely sources of errors leading to any differences.

Percent error is defined as:

$$\text{PE} = \frac{|\text{actual value} - \text{theoretical value}|}{\text{theoretical value}} \times 100\%$$

Now clear all the oscilloscope measurements. Use the measurement capability of the oscilloscope to measure the rise time of $v_{\text{out}}(t)$, the fall time of $v_{\text{out}}(t)$, and the two delay times t_{PHL} and t_{PLH}.

Analysis #3:
Compare the measured data with the theoretical values, as well as the measurements in Analysis #2, using percent error. Explain the likely sources of errors leading to any differences.

4.1.2 Time Constant Measurement

The time constant $\tau = RC$ is one of the most important characteristics of RC circuit, and its value can be extracted from measured data.

To measure the time constant τ, use the oscilloscope's **Cursor** menu to measure the voltage and time values at 10 points on the v_{out} waveform during one interval when v_{out} either rises or falls with time (pick one interval only). Note that the time values should be referred to time $t = 0$ at the point where the input signal rises from 0V to 5V or falls from 5V to 0V. **Record the 10 measurements.**

Analysis #4:
Plot the 10 measurements in Excel, then make the x-axis (time) logarithmic. The experimental time constant τ is the inverse of the slope of this now-linear graph. Compare this τ with theoretical value $\tau = RC$ using percent error. Explain the likely sources of errors leading to any differences.

Explanation: Consider the ratio of $|v_{out} - v_{in}|$ and high voltage V_0. It is

$$\text{Ratio}(t) = \frac{|v_{out}(t) - v_{in}|}{|V_0|} = e^{-\frac{t}{\tau}}$$

and it can be calculated by measured data. So the function $\ln(\text{Ratio}(t))$ is linear according to time, and the slope is $-\frac{1}{\tau}$. Read Reference 5.4 for more information.

Now build two-stage and three-stage RC circuits and measure time constant $\tau_{\text{two-stage}}$ and $\tau_{\text{three-stage}}$ using the same methods as the single stage circuit analysis. **Record all your measurements.**

Analysis #5:
Compare the measured values of the time constant with the theoretical values using percent error. Are they the same values? Explain the likely sources of errors leading to any differences.

4.2 The RC Response to a Sinusoidal Input

Rebuild the circuit in Figure 4.1.1 and set the function generator to provide a sinusoidal input with:

a) An amplitude of 1V, which means $V_{\text{pk-pk}} = 2\text{V}$

b) A frequency of 1kHz.

Connect Channel 1 to the input voltage and Channel 2 to the voltage over the capacitor as the output. Display the input and output voltages simultaneously on the oscilloscope in 3 complete cycles. Capture the output from the scope display with both the waveforms and the measured values. **Turn this oscilloscope waveform in as part of your lab report.**

Now measure the RC response to sinusoidal signals with various frequencies. Keep the input amplitude at 1V, but sweep the frequency from the starting input frequency of 10Hz, varying it using a 1-2-5 sequence up to 1MHz (i.e. set input frequency to 10Hz, 20Hz, 50Hz, 100Hz, 200Hz … up to 1MHz). **Record the amplitudes of the output signals.**

Analysis #6:
Using Microsoft Excel, plot the amplitude of the output voltage in terms of frequency. Make sure the frequency is plotted on a log scale. Compare it to what was plotted in Prelab #11. Explain the likely sources of errors leading to any differences.

Once done, switch the locations of the resistor and capacitor and change the output to be the voltage over the resistor. Set the function generator to provide a sinusoidal wave input with 1V amplitude. As before, sweep the frequency starting from 10Hz using the 1-2-5 sequence up to 1MHz. **Record the amplitudes of the output signals.**

Analysis #7:
Using Microsoft Excel, plot the amplitude of the output voltage in terms of frequency. Make sure the frequency is plotted on a log scale. Compare it to what was plotted in Prelab #12. Explain the likely sources of errors leading to any differences.

5 Reference Material

5.1 RC Step Response and Timing Parameters

The step response of a simple RC circuit, illustrated in Figure 5.1.1, is an exponential signal with time constant $\tau = RC$. Besides this timing parameter, four other timing parameters are important in describing how fast or how slow an RC circuit responds to a step input. These timing parameters are marked in Figure 5.1.1, as three voltage levels:

a) The 10%-point is the point at which the output voltage is 10% of the maximum output voltage.

b) The 50%-point is the point at which the output voltage is 50% of the maximum output voltage.

c) The 90%-point is the point at which the output voltage is 90% of the maximum output voltage.

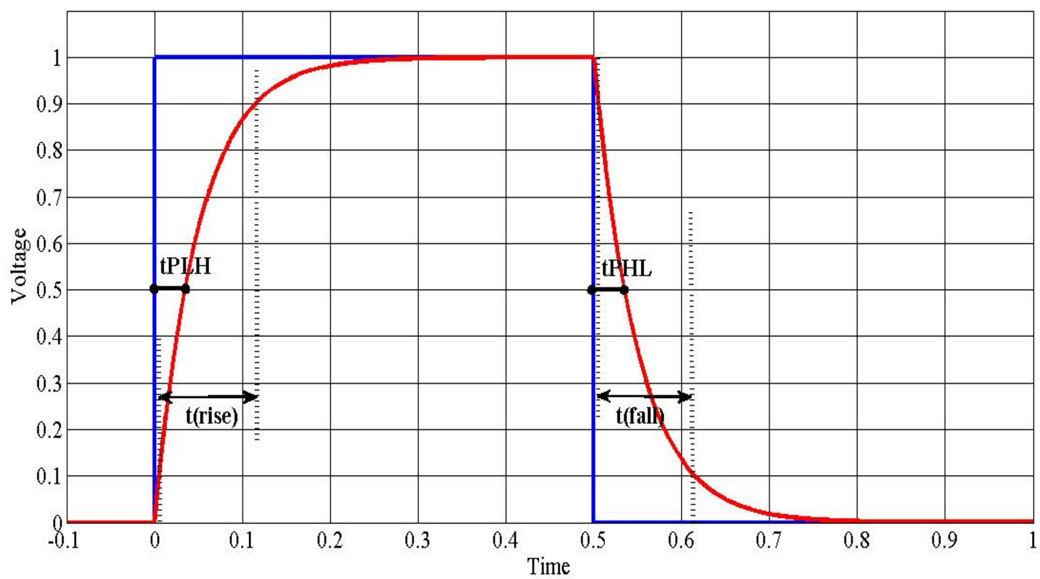

Figure 5.1.1: Timing parameters of signal waveforms.

The three timing parameters are defined as follows:

a) **Rise time:** the time interval between the 10%-point and the 90%-point of the waveform when the signal makes the transition from low voltage (L) to high voltage (H). Notation: t_r.

b) **Fall time:** the time interval between the 90%-point and the 10%-point of the waveform when the signal makes the transition from high voltage (H) to low voltage (L). Notation: t_f.

c) **Delay time** (or propagation delay time)**:** the time interval between the 50%-point of the input signal and the 50%-point of the output signal when both signals make a transition. There are two delay times depending on whether the output signal is going from L to H (delay notation t_{PLH}) or from H to L (delay notation t_{PHL}). The subscript P stands for "propagation."

Note that the rise time and the fall time are defined using a single waveform (the output waveform), while the delay time is defined between two waveforms: the input waveform and the corresponding output waveform.

5.2 Elmore Delay Estimation

Figure 5.2.1 depicts a multi-element configuration. The resistor R_1 in this figure charges all N capacitors downstream of its own position. The Elmore estimated delay τ_1 from point x_0 to x_1 is therefore

$$\tau_1 = R_1 \sum_{m=1}^{N} C_m$$

Resistor R_2 charges only capacitors numbered 2 through N, so the estimated delay from point x_1 to x_2 is

$$\tau_2 = R_2 \sum_{m=2}^{N} C_m$$

Working down the row, the total delay for the whole circuit is then estimated as:

$$\tau = \sum_{n=1}^{N} R_n \sum_{m=n}^{N} C_m$$

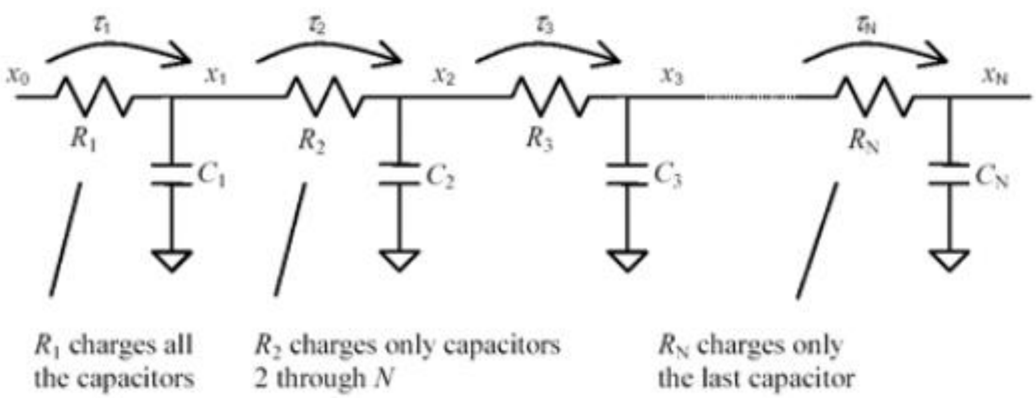

Figure 5.2.1: N-stage RC circuit delay estimation.

5.3 Frequency Response of a Circuit System

An analog circuit system has different responses for sine waves with different frequencies. The magnitude of the output voltage always changes in terms of frequencies if the magnitude of the input sine wave stays the same. Therefore, the frequency response is the quantitative measure to characterize the system.

Since any input signal can be regarded as the sum of a set of sinusoidal waves, the output signal will have different responses to input waves with the set of frequencies. If the circuit has high magnitude for low frequencies, and close to zero magnitude for high frequencies, the high frequencies will be removed by the circuit in the output signal, and vice versa.

The frequency response is one of the main characteristics of the system, and you will explore methods of analyzing the frequency response in the following labs.

5.4 Parameter Extraction via Linear Least-Squares-Fit Technique

The important parameters of $V_{\text{out}}(t)$ are the maximum amplitude and the time constant τ. The maximum amplitude is easily measured by using the oscilloscope. Measuring the time constant directly and accurately is more difficult, since the waveform is an exponential function of time. A linear least-squares-fit procedure can be used in the lab to extract the time constant from measured voltage and time values as follows.

The equation for $V_{\text{out}}(t)$ during the time interval when $V_{\text{out}}(t)$ falls with time, which you can write based on what you learned in prerequisite courses, can be manipulated to provide a linear function in terms of the time t. The slope of this line is then used to extract the time constant τ.

Alternatively, the equation for $V_{\text{out}}(t)$ during the time interval when $V_{\text{out}}(t)$ rises with time can also be manipulated to provide a linear function in terms of the time t. The slope of this line is then used to extract the time constant τ.

In the lab, you will measure a set of data points (t, V_{out}). These values, after the appropriate manipulation as above, can be used to plot a straight line, whose slope is a function of τ. You can use any procedure or a calculator to plot and extract the slop. The slope value will then be used to calculate the time constant τ.

Make sure you understand this procedure and be ready to use it in the lab. Note that the more points you measure, the more accurate the extracted value for τ.