

Project 2: Grammar Analysis and Parsing

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1 Introduction

This report contains our implementation of a scanner and parser for context free grammars, a series of hygiene functions for sterilizing the grammar, and finally a parser *for* the grammar specified in the context free grammar.

It is divided up into several sections, roughly corresponding to the problems given in the specification, each a Haskell module. The work was split up evenly amongst the group members, and approximately 40 man hours went into the final preparation of this document, the source code, unit testing, and related work.

2 Context Free Grammar

In this section we provide the context free grammar data type.

At its heart, a grammar it consists of a list of productions, where each production consists of a constructor and two arguments; the first a parameterized nonterminal, and the second a parameterized right hand side.

An *RHS* is either empty, a terminal, which takes two arguments — the parameterized object representing a terminal, and another *RHS*; or a non-terminal, which similarly takes two arguments.

```
{-# LANGUAGE FlexibleInstances, MultiParamTypeClasses #-}
module ContextFreeGrammar
  (Grammar, Production (.), RHS (.), module Dropable,
   nonTerminals, terminals, Terminal (..)) where

  import Dropable
  import Filterable
  import Prelude hiding (drop, filter)
  type Grammar nt t = [Production nt t]
```

```

data Terminal t = Epsilon | EOF | Terminal t deriving (Show, Eq, Ord)
instance (Eq nt)  $\Rightarrow$  Dropable nt (Grammar nt t) where
    drop x grammar = map (drop x) grammar
instance Filterable (nt  $\rightarrow$  Bool) (Grammar nt t) where
    filter pred grammar = map (filter pred) grammar
data Production nt t = Production { nonterminal :: nt,
    rhs :: RHS nt t } deriving (Eq, Ord)
instance Show (Production String String) where
    show (Production nt rhs) = nt ++ " ->" ++ show rhs
instance Show (Production Char Char) where
    show (Production nt rhs) = show nt ++ " -> " ++ show rhs
instance (Eq nt)  $\Rightarrow$  Dropable nt (Production nt t) where
    drop x (Production nt rhs) = Production nt (drop x rhs)
instance Filterable (nt  $\rightarrow$  Bool) (Production nt t) where
    filter pred (Production nt rhs) = Production nt (filter pred rhs)
data RHS nt t = Empty
    | Term t (RHS nt t)
    | NonT nt (RHS nt t) deriving (Eq, Ord)
instance Show (RHS String String) where
    show Empty = ""
    show (Term t rhs) = " " ++ t ++ (show rhs)
    show (NonT nt rhs) = " " ++ nt ++ (show rhs)
instance Show (RHS Char Char) where
    show Empty = ""
    show (Term t rhs) = show t ++ (show rhs)
    show (NonT nt rhs) = show nt ++ (show rhs)
instance (Eq nt)  $\Rightarrow$  Dropable nt (RHS nt t) where
    drop x (NonT nt rhs)
        | x  $\equiv$  nt = drop x rhs
        | otherwise = (NonT nt (drop x rhs))
    drop x (Term t rhs) = Term t (drop x rhs)
    drop _ Empty = Empty
instance Filterable (nt  $\rightarrow$  Bool) (RHS nt t) where
    filter _ Empty = Empty
    filter pred (Term t rhs) = (Term t (filter pred rhs))
    filter pred (NonT nt rhs) =
        if pred nt then (NonT nt (filter pred rhs)) else (filter pred rhs)

```

`nonTerminals` takes the RHS of a Production and returns a list of all Non Terminals

```

nonTerminals :: RHS nt t → [nt]
nonTerminals (NonT nt rhs) = nt : nonTerminals rhs
nonTerminals (Term _ rhs) = nonTerminals rhs
nonTerminals Empty = []

```

`terminals` takes the RHS of a Production and returns a list of all Terminals

```

terminals :: RHS nt t → [t]
terminals (Term t rhs) = t : terminals rhs
terminals (NonT _ rhs) = terminals rhs
terminals Empty = []

simpleGrammar :: Grammar String String
simpleGrammar = [a, b, c, d] where
  a = Production "A" (Term "a" Empty)
  b = Production "B" (NonT "B" Empty)
  c = Production "C" (Term "a" (NonT "B" Empty))
  d = Production "D" (NonT "B" (Term "a" Empty))

```

3 Scanner and Parser for context-free grammars

In this section we provide code for a simple scanner and parser for a textual representation of a context free grammar.

The grammar for the concrete representation follows the suggestion in the assignment, with one minor difference:

```

Grammar -> Grammar Production
Grammar -> Production
Production -> UpperSymbol Arrow RHS
RHS -> RHS Symbol
RHS ->
Symbol -> UpperSymbol
Symbol -> AlphaNumSymbol

```

In other words, non-terminals are restricted to their first letter being upper case, terminals are sequences of alphanumeric characters where the first character cannot be upper-case, and right hand side terminals and non-terminals are delimited by spaces.

A couple helper functions are initially defined, in addition to the grammar token data structure, which is as follows:

```
module ScanAndParse (sparse) where
import ContextFreeGrammar
import Data.Char (isUpper, isSpace, isAlphaNum, isAlpha, isDigit)
data GrammarToken =
    Symbol String |
    ArrowToken |
    NewLineToken deriving (Show, Eq)
alphanumeric = takeWhile isAlphaNum
drop' _ [] = []
drop' i (x : xs) =
    if i ≤ 0 then (x : xs)
    else
        drop' (i - 1) xs
```

The scanner is a simple function that checks for two special characters, the arrow, `->` and the newline character, `\n`, scans symbols for nonterminals or terminals, and returns their appropriate tokens.

If a non alphanumeric character is found, the scanner returns an error.

```
scan :: String → [GrammarToken]
scan [] = []
scan ('->' : '>' : cs) = ArrowToken : scan cs
scan ('\n' : cs) = NewLineToken : scan cs
scan (c : cs) | isSpace c = scan cs
scan s@(c : cs) | isAlphaNum c =
    let name = alphanumeric s
        len = length name in
        (Symbol name) : scan (drop' len s)
scan s@(c : cs) =
    error ("lexical error; " ++ c : " is an unrecognized character.")
```

The parser generates a list of productions, i.e., a “grammar”, from a list of grammar tokens. The helper function, *parseRHS*, will throw a syntax error if an arrow token is found on the right hand side.

The function *parse* will throw an error if multiple non-terminals occur on the left-hand side, or an arrow is missing.

```

parseRHS :: [GrammarToken] → ((RHS String String), [GrammarToken])
parseRHS [] =
  (Empty, [])
parseRHS (NewLineToken : rhs) =
  (Empty, rhs)
parseRHS (ArrowToken : rhs) =
  error "syntax error; arrow token found on right hand side"
parseRHS ((Symbol (c : cs)) : rhs) =
  let (term, rhs') = parseRHS rhs in
  if isUpper c then
    ((NonT (c : cs) term), rhs')
  else
    ((Term (c : cs) term), rhs')
parse :: [GrammarToken] → Grammar String String
parse [] = []
parse (NewLineToken : p) = parse p
parse ((Symbol s) : ArrowToken : rhs) =
  let (production, rhs') = parseRHS rhs in
  (Production s (production)) : parse rhs'
parse ((Symbol s) : rhs) =
  error "Missing arrow or multiple non-terminals on left-hand side."
sparse = parse ∘ scan

```

4 Hygiene Module

In this module, we perform basic hygiene checks on the grammar, remove unreachable non terminals, etc.

```

module BadHygiene (computeReachable,
  eliminateUnreachable,
  computeGenerating,
  eliminateNonGenerating,
  eliminateUseless,
  isEmptyGrammar) where
import ContextFreeGrammar
import qualified Data.Set as S
import Filterable
import ScanAndParse

```

{-BEGIN CLEANING FUNCTIONS -}

computeReachable finds the Set of all Non Terminals of a Grammar that can be reached from the start node.

```
computeReachable :: Ord nt => Grammar nt t -> S.Set nt
computeReachable [] = S.empty
computeReachable ps = go (S.singleton o nonterminal o head $ ps) (concat o replicate (length
go marked [] = marked
go marked ((Production nt rhs) : prs) = if S.member nt marked
    then go marked' prs
    else go marked prs
    where marked' = S.union marked o S.fromList o nonTerminals $ rhs
```

eliminateUnreachable removes all unreachable Non Terminals from a Grammar.

```
eliminateUnreachable :: Ord nt => Grammar nt t -> Grammar nt t
eliminateUnreachable g = cleanGrammar where
    reachable = computeReachable $ g
    -- unnecessary? By definition, the unreachable non-terminals cannot be in any
    -- other production list.
    -- cleanProductions = Filterable.filter ('S.member' reachable) g
    cleanGrammar = Prelude.filter (\(Production nt rhs) -> S.member nt reachable) g
```

computeGenerating finds the Set of all Non Terminals of a Grammar that can produce a string of Terminals.

```
computeGenerating :: (Ord nt, Ord t) => Grammar nt t -> S.Set nt
computeGenerating [] = S.empty
computeGenerating ps = go S.empty (concat o replicate (length ps) $ ps) where
    allTerms = S.fromList o concatMap (terminals o rhs) $ ps
    go markedNT [] = markedNT
    go markedNT ((Production nt rhs) : prs) = if (all ('S.member' allTerms) o terminals $ rhs)
        (all ('S.member' markedNT) o nonTerminals $ rhs)
        then go (S.insert nt markedNT) prs
        else go markedNT prs
```

eliminateNonGenerating removes all non Generating Non Terminals from a Grammar.

```
eliminateNonGenerating :: (Ord nt, Ord t) => Grammar nt t -> Grammar nt t
eliminateNonGenerating g = cleanGrammar where
```

```

generating = computeGenerating g
cleanProductions = Filterable.filter ('S.member' generating) g
cleanGrammar = Prelude.filter (\(Production nt rhs) → S.member nt generating) cleanPr

```

eliminateUseless removes all non Generating and unreachable Non Terminals from a Grammar.

```

eliminateUseless :: (Ord nt, Ord t) ⇒ Grammar nt t → Grammar nt t
eliminateUseless = eliminateUnreachable ∘ eliminateNonGenerating

```

isEmptyGrammar determines if a Grammar will produce any strings at all.

```

isEmptyGrammar :: (Ord t, Ord nt) ⇒ Grammar nt t → Bool
isEmptyGrammar [] = True
isEmptyGrammar g = ¬ ∘ elem nt ∘ map nonterminal $ g' where
  g' = eliminateNonGenerating g
  (Production nt _) = head g
{-END CLEANING FUNCTIONS -}

```

5 Nullable, First, and Follow

In this section, we provide several modules for computing the nullable, first and follow sets of a given context free grammar, respectively.

5.1 Nullable

Here we compute whether a production is nullable or not.

```

module Nullable (nullable) where
import ContextFreeGrammar
import qualified Data.Set as S
import Prelude hiding (drop)
type Set = S.Set
nullable :: (Ord nt) ⇒ Grammar nt t → Set nt
nullable = nullable' S.empty
nullable' :: (Ord nt) ⇒ Set nt → Grammar nt t → Set nt
nullable' set grammar = set'' where
  set'' = if nulls ≡ set then set else set'

```

```

    set' = nullable' nulls (S.fold drop grammar nulls)
    nulls = S.fromList ◦ map nonterminal ◦ filter isNull $ grammar
isNull :: Production nt t → Bool
isNull (Production _ Empty) = True
isNull _ = False

simpleGrammar :: Grammar String String
simpleGrammar = [a] where
    a = Production "A" (Term "ab" Empty)

simpleGrammar2 :: Grammar String String
simpleGrammar2 = [a, a', b, b', c] where
    a = Production "A" (Term "ab" Empty)
    a' = Production "A" Empty
    b = Production "B" (NonT "A" (NonT "A" Empty))
    b' = Production "B" (NonT "A" (Term "b" Empty))
    c = Production "C" (Term "cdef" Empty)

```

5.2 First

In this section, we compute the first set for a context-free grammar.

```

module First (first) where
import ContextFreeGrammar
import Control.Monad
import Control.Monad.State
import Data.Functor
import Data.List
import qualified Data.Map as M
import Data.Maybe
import Nullable
import ScanAndParse
import qualified Data.Set as S
import Test.HUnit hiding (State)

```

first is the interface function exported for general use. Given a Grammar, first computes the First Set for each Production, and returns the Sets in a Map from a Non-terminal to it's First Set.

```

first :: (Ord nt, Ord t)
      ⇒ Grammar nt t

```



```

→ M.Map nt (S.Set (Terminal t))
first g = firsts ∘ execState state ∘ FS M.empty ∘ nullable $ g where
state = mapM first' ∘ concat ∘ replicate (length g) $ g

```

The FirstState Data Type stores the map of Sets of First Terminals that is modified and returned at the end of a call to first. It also stores the set of Non-terminals which are nullable.

```

data FirstState nt t = FS {
  firsts :: M.Map nt (S.Set (Terminal t)),
  nulls :: S.Set nt
}
type Environment nt t a = State (FirstState nt t) a

```

first' does the work for the first function. Given a production, first' will calculate the set of first terminals and store it in the implicit FirstState.

```

first' :: (Ord nt, Ord t) ⇒ Production nt t → Environment nt t ()
first' (Production nt rhs) = do
  fs ← get
  let mp = firsts fs
  case rhs of
    Empty → case M.lookup nt mp of
      Nothing → put fs {firsts = M.insert nt (S.singleton Epsilon) mp}
      Just _ → put fs {firsts = M.adjust (S.insert Epsilon) nt mp}
    _ → do
      sets ← firstRHS rhs
      let s = fromMaybe S.empty (M.lookup nt mp)
      put fs {firsts = M.insert nt (S.unions (s : sets)) mp}

```

firstRHS is a helper function which, given a RHS will return the first sets of every terminal and non-terminal until a non nullable terminal/non-terminal is found.

```

firstRHS :: (Ord nt, Ord t)
⇒ RHS nt t
→ Environment nt t [S.Set (Terminal t)]
firstRHS Empty = return []
firstRHS (Term y _) = return [S.singleton ∘ Terminal $ y]
firstRHS (NonT y ys) = do
  nlls ← gets nulls

```

```

case S.member y nlls of
  True  $\rightarrow$  do
    set  $\leftarrow$  getFirsts y
    sets  $\leftarrow$  firstRHS ys
    return (set : sets)
  False  $\rightarrow$   $(:[])$   $< \$ >$  getFirsts y

```

getFirsts is a helper function which given a Non-terminal will return its current first set.

```

getFirsts :: Ord nt  $\Rightarrow$  nt  $\rightarrow$  Environment nt t (S.Set (Terminal t))
getFirsts nt = do
  set  $\leftarrow$   $(M.lookup\ nt) < \$ > gets\ firsts$ 
  case set of
    Nothing  $\rightarrow$  return S.empty
    Just set  $\rightarrow$  return set
{- BEGIN TESTS - -}
makeTestM ::  $(Eq\ a, Show\ a)$ 
   $\Rightarrow$  String
   $\rightarrow$  FilePath
   $\rightarrow$  String
   $\rightarrow$  a
   $\rightarrow$   $(Grammar\ String\ String \rightarrow a)$ 
   $\rightarrow$  Test
makeTestM name file forF e f = TestLabel name  $\circ$  TestCase $ do
  grammar  $\leftarrow$  fmap sparse  $\circ$  readFile $ file
  assertEqual forF e (f grammar)
testFirst = makeTestM "testFirst"
  "tests\\test1.txt"
  "for first with test1"
  expected
  first where
  expected = M.fromList [  $($  "A", S.singleton  $\circ$  Terminal $ "a"  $)$ ,
     $($  "B", S.fromList [ Terminal "b",
      Terminal "a",
      Epsilon  $]$   $)$ ,
     $($  "C", S.fromList [ Terminal "a",
      Terminal "b",
      Epsilon  $]$   $)$ ,
     $($  "D", S.fromList [ Terminal "a",

```

```

        Terminal "b",
        Epsilon)])]
testFirst2 = makeTestM "testFirst2"
    "tests\\39.txt"
    "for first with 39"
    expected
    first where
        expected = M.fromList [("T", S.fromList [Epsilon, Terminal "a", Terminal "b"]),
            ("R", S.fromList [Epsilon, Terminal "b"])]
tests = TestList [testFirst,
    testFirst2]
runTests :: IO Counts
runTests = runTestTT tests
doTestsPass :: IO Bool
doTestsPass = do
    counts ← runTests
    let errs = errors counts
        fails = failures counts
    return $ (errs == 0) ∧ (fails == 0)
{- END TESTS - -}

```

5.3 Follow

In this section, we implement a function *follow* which calculates the follow set for our data structure of production grammars.

```

module Follow (follow) where
import ContextFreeGrammar
import Control.Monad.State
import Data.Functor
import qualified Data.Map as M
import Data.Maybe
import qualified Data.Set as S
import First
import Nullable
import ScanAndParse
import Test.HUnit hiding (State)

```

A GrammarState holds data that the follow' function requires to work.

```

data GrammarState nt t = GS {
  grammar :: Grammar nt t,
  follows :: M.Map nt (S.Set (Terminal t)),
  firsts :: M.Map nt (S.Set (Terminal t))
}
type Environment nt t a = State (GrammarState nt t) a

```

follow is the interface function exported for general use. Given a Grammar, follow computes the Follow Set for each Production, and returns the Sets in a Map from a Non-terminal to it's Follow Set.

```

follow :: (Ord nt, Ord t)
  => Grammar nt t
  -> M.Map nt (S.Set (Terminal t))
follow [] = M.empty
follow g@((Production s rhs) : ps) = fs where
  state = mapM follow' o concat o replicate (length g) $ g
  fs = follows o execState state o GS g initial o first $ g
  initial = M.singleton s (S.singleton EOF)

```

follow' is where the main work of the follow function is done. For a given production, follow' will add an entry into the GrammarState passed along. This function is meant to be mapM'd across the Grammar you want to compute the follow sets of.

```

follow' :: (Ord nt, Ord t)
  => Production nt t
  -> Environment nt t ()
follow' (Production a _) = do
  fllostate <- get
  let g = grammar fllostate
  flow = follows fllostate
  ps = filter (elem a o nonTerminals o rhs) g
  sets <- forM ps $ \ (Production x rhs) -> do
    case after a rhs of
      Empty -> return o fromMaybe S.empty o M.lookup x $ flow
      Term t _ -> return o S.singleton o Terminal $ t
      NonT nt rest -> do
        let frstb = (firsts fllostate) M.! nt
        flowx = case S.member Epsilon frstb of
          False -> S.empty

```

```

    True → fromMaybe S.empty ∘ M.lookup x $ flow
    return $ S.union (S.delete Epsilon frstb) (flowx)
let s = fromMaybe S.empty (M.lookup a flow)
    newS = S.union s (S.unions sets)
    put fllstate { follows = M.insert a newS flow }

```

after is a helper function which removes all Terminals and Non-terminals from a RHS until a specific Non-terminal is reached. Then the rest of the RHS is returned.

```

after :: (Eq nt) ⇒ nt → RHS nt t → RHS nt t
after nt Empty = Empty
after nt (Term t rhs) = after nt rhs
after nt (NonT nt2 rhs) = if nt ≡ nt2
    then rhs else after nt rhs
{- BEGIN TESTS - -}
makeTestM :: (Eq a, Show a)
    ⇒ String
    → FilePath
    → String
    → a
    → (Grammar String String → a)
    → Test
makeTestM name file forF e f = TestLabel name ∘ TestCase $ do
    grammar ← sparse < $ > readFile file
    assertEquals forF e (f grammar)
testFollow = makeTestM "testFollow"
    "tests\\39.txt"
    "for first with 39"
    expected
    follow where
    expected = M.fromList [("R", S.fromList [EOF, Terminal "c"]),
        ("T", S.fromList [EOF, Terminal "c"])]
tests = TestList [testFollow]
runTests :: IO Counts
runTests = runTestTT tests
doTestsPass :: IO Bool
doTestsPass = do
    counts ← runTests

```

```

let errs = errors counts
      fails = failures counts
      return $ (errs == 0) ^& (fails == 0)
{- END TESTS - -}

```

6 Generating a Parse Table

In this section we generate a parse table for a given grammar, assuming it has been properly scanned, parsed, and thoroughly cleansed.

```

module Table where
import ContextFreeGrammar
import Filterable
import Nullable
import First
import Follow
import System.Environment
import Data.List
import qualified Data.Map as M
import qualified Data.Set as S
import ScanAndParse
import BadHygiene
type Table nt t = M.Map nt (M.Map t (Production nt t))
foo1 :: Grammar String String
foo1 = [a, b, c, d] where
  a = Production "A" (Term "a" Empty)
  b = Production "B" (NonT "B" Empty)
  c = Production "C" (Term "a" (NonT "B" Empty))
  d = Production "D" (NonT "B" (Term "a" Empty))
getFeature feature productions = loop productions []
where
  loop [] acc = sort o nub o concat $ acc
  loop ((Production s rhs) : xs) acc = loop xs ((feature rhs) : acc)
  -- need to generate S' → S$
getNewStart [] = error "getNewStart run on empty grammar --- a new low point"
getNewStart (Production nt rhs : ps) =
  Production (nt ++ "PRIME") (NonT nt (Term "$" Empty))

```

```

isRHSNullable :: Ord a => S.Set a -> RHS a t -> Bool
isRHSNullable _ Empty = True
isRHSNullable _ (Term t _) = False
isRHSNullable m (NonT nt rhs) =
  if S.member nt m then
    isRHSNullable m rhs
  else False

firstRHS _ Empty = S.empty
firstRHS i (Term t rhs) = S.insert t (firstRHS i rhs)
firstRHS i (NonT nt rhs) =
  if (S.member nt (nulls i)) then
    S.union firstsnt (firstRHS i rhs)
  else
    firstsnt
  where firstsnt = ((firsts i)M.! nt)

data GrammarInfo nt t = GI {
  firsts :: M.Map nt (S.Set (Terminal t)),
  follows :: M.Map nt (S.Set (Terminal t)),
  nulls :: S.Set nt
} deriving Show

```

A production $N \rightarrow \alpha$ is in the table (N, a) iff a is in $\text{first}(\alpha) \vee (\text{nullable}(\alpha) \wedge a \text{ is in } \text{follow}(\alpha))$. This condition is readily translateable into our code:

```

validEntry (Production _  $\alpha$ ) term nterm gi = firstalpha  $\vee$  (nullablea  $\wedge$  follown)
  where
    firstalpha = S.member term (firstRHS gi  $\alpha$ )
    nullablea = isRHSNullable (nulls gi)  $\alpha$ 
    follown = S.member term ((follows gi)M.! nterm)

toTerminal [] = []
toTerminal (" $" : ss) = EOF : toTerminal ss
toTerminal (s : ss) = Terminal s : toTerminal ss

fromTerminal' Epsilon = ""
fromTerminal' EOF     = "$"
fromTerminal' (Terminal t) = t
fromTerminalSet ts = map fromTerminal' $ S.toList ts

columns p t [] gi = []
columns p t (nt : nts) gi =
  if validEntry p t nt gi then

```

```

      (t, p) : (columns p t (nts) gi)
    else
      columns p t (nts) gi
  rows p [] nts gi = []
  rows p (t : ts) nts gi = (columns p t nts gi) : (rows p ts nts gi)
  buildTable' [] terms nterms gi = []
  buildTable' (p : ps) terms nterms gi = ⊥
  -- ((rows p terms nterms gi):buildTable' ps terms nterms gi
  buildTable grammar =
    let grammar' = (getNewStart grammar) : grammar in
    let terms = toTerminal ∘ getFeature terminals $ grammar' in
    let nterms = getFeature nonTerminals grammar' in
    let gi = GI (first grammar') (follow grammar') (nullable grammar') in
    let table = buildTable' grammar' terms nterms gi in
    table
  man = do
  contents ← readFile "tests/39.txt"
  let g = sparse contents
  let g' = eliminateUseless ∘ sparse $ contents
  let gnull = nullable ∘ sparse $ contents
  let gfollow = follow ∘ sparse $ contents
  let gfirst = first ∘ sparse $ contents
  let gi = GI gfirst gfollow gnull
  putStrLn $ show (getFeature terminals g')
  putStrLn $ show (getFeature nonTerminals g')
  putStrLn $ show gi

```

7 Main module

The main module puts everything together, takes a textual representation of a context-free grammar as input, scans, parses, and performs the rest of the duties that are required.

```

module Main where
import ContextFreeGrammar
import ScanAndParse
import BadHygiene
import Table

```



```
import System.Environment

main = do
  -- [file] i- getArgs
  -- contents i- readFile file
  contents ← readFile "tests/39.txt"
  -- contents i- readFile "tests/ir.txt"
  let g = sparse contents
  let g' = eliminateUseless ∘ sparse $ contents
  putStrLn $ show g
  putStrLn $ show g'
```