OpenManipulatorIK

Teddy W. Aaron T.

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For context, we are using the Paden-Kahan subproblems.

$$e_1 e_2 e_3 e_4 g(0) = g_d$$

$$e_1 e_2 e_3 e_4 = g_d g^{-1}(0)$$

Let $g_d g^{-1}(0) = g$ and choose a point p on axis 4

$$e_1 e_2 e_3 e_4 p = gp \rightarrow e_1 e_2 e_3 p = gp$$

Now choose a point q on the intersection of axes 1 and 2. We get:

$$e_1 e_2 e_3 p - e_1 e_2 q = gp - e_1 e_2 q$$

Since $q = e_1 e_2 q$, we have:

$$e_1 e_2 e_3 p - e_1 e_2 q = gp - q$$

Distributing, we get:

$$e_1 e_2 (e_3 p - q) = gp - q$$

Take the magnitude of both sides to cancel $e_1 e_2$ to get:

$$||e_3 p - q|| = ||gp - q||$$

Apply subproblem 3 (rotation to a given distance) to find θ_3 from e_3.

Now that we have θ_3 we return to the form $e_1 e_2 e_3 p = gp$. Since θ_3 (and thus e_3) is known, we can use subproblem 2 (rotation about two subsequent axes) to solve for e_1 and e_2. We do so by choosing $e_3 p$ and gp as our 2 points and using e_1 and e_2 as our two axes. We get θ_1 and θ_2, leaving θ_4 as the last unknown; it can be solved for by algebra using $\theta_{sum} = \theta_2 + \theta_3 + \theta_4$.