Open Manipulator IK

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(For context, we are using the Paden-Kahan subproblems.)

$$\begin{aligned} e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2} e_3^{\widehat{\xi}_3\theta_3} e_4^{\widehat{\xi}_4\theta_4} g(0) =& g_d \\ e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2} e_3^{\widehat{\xi}_3\theta_3} e_4^{\widehat{\xi}_4\theta_4} =& g_d g^{-1}(0) \end{aligned}$$

Let $g_d g^{-1}(0) = g$ and choose a point p on axis 4

$$e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2} e_3^{\widehat{\xi}_3\theta_3} e_4^{\widehat{\xi}_4\theta_4} p = gp \rightarrow e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2} e_3^{\widehat{\xi}_3\theta_3} p = gp$$

Now choose a point q on the intersection of axes 1 and 2. We get:

$$e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2} e_3^{\widehat{\xi}_3\theta_3} p - e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2} q = gp - e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2} q$$

Since $q = e_1 e_2 q$, we have:

$$e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2} e_3^{\widehat{\xi}_3\theta_3} - e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2} q = gp - q$$

Distributing, we get:

$$e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2} (e_3^{\widehat{\xi}_3\theta_3} p - q) = gp - q$$

Take the magnitude of both sides to cancel $e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2}$ to get:

$$||e_3^{\widehat{\xi}_3\theta_3} p - q|| = ||gp - q||$$

Apply subproblem 3 (rotation to a given distance) to find θ_3 from e_3.

Now that we have θ_3 we return to the form $e_1^{\widehat{\xi}_1\theta_1} e_2^{\widehat{\xi}_2\theta_2} e_3^{\widehat{\xi}_3\theta_3} p = gp$. Since θ_3 (and thus $e_3^{\widehat{\xi}_3\theta_3}$) is known, we can use subproblem 2 (rotation about two subsequent axes) to solve for $e_1^{\widehat{\xi}_1\theta_1}$ and $e_2^{\widehat{\xi}_2\theta_2}$. We do so by choosing $e_3^{\widehat{\xi}_3\theta_3} p$ and gp as our 2 points and using $e_1^{\widehat{\xi}_1\theta_1}$ and $e_2^{\widehat{\xi}_2\theta_2}$ as our two axes. We get θ_1 and θ_2, leaving θ_4 as the last unknown; it can be solved for by algebra using $\theta_{sum} = \theta_2 + \theta_3 + \theta_4$.