Types and Programming Language

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目录

Introduction

Definition 1.1. A type system is a tractable syntactic method for proving the absence of certain program behaviors by classlying phrases according to the kinds of value they compute.

type system 是一种用于证明某些确定的程序行为不会发生的方法，它怎么做呢？通过它们计算出值的类型来分类, 有点抽象... 我想知道 the kinds of value they compute 是什么？如何分类？分类之后接下来该怎么做？

Annotation 1.2. Being static, type systems are necessarily also conservative: they can categorically prove the absence of some bad program behaviors，but they cant prove their presence.

Example 1.3.

```
if <complex test> then 5 else <type error>
```

上面这个 annotation 在说 type system 只能证明它看到的一些 bad program behavior 不会出现，但是它们可能会 reject 掉一些 runtime time 阶段运行良好的程序，例如在 runtime 阶段上面的 else 可能永远都不会进. 即 type system 无法证明它是否真的存在.

Untyped Systems

Syntax

Definition 2.1. The set of terms is the smallest set $\mathcal{T}$ such that

1. $\{\text{true}, \text{false}, 0\} \subseteq \mathcal{T}$;

2. if $t_1 \in \mathcal{T}$, then $\{\text{succ } t_1, \text{pred } t_1, \text{iszero } t_1\} \subseteq \mathcal{T}$;

3. if $t_1 \in \mathcal{T}, t_2 \in \mathcal{T}, t_3 \in \mathcal{T}$, then $\texttt{if } t_1 \texttt{ then } t_2 \texttt{ else } t_3 \in \mathcal{T}$.

Definition 2.2. The set of terms is defined by the following rules:

$$\text{true} \in \mathcal{T} \qquad \text{false} \in \mathcal{T} \qquad 0 \in \mathcal{T}$$
$$\frac{t_1 \mathcal{T}}{\text{succ} t_1 \in \mathcal{T}} \qquad \frac{t_1 \mathcal{T}}{\text{succ} t_1 \in \mathcal{T}} \qquad \frac{t_1 \mathcal{T}}{\text{succ} t_1 \in \mathcal{T}}$$
$$\frac{t_1 \in \mathcal{T} \quad t_2 \in \mathcal{T} \quad t_3 \in \mathcal{T}}{\texttt{if} t_1 \texttt{then} t_2 \texttt{else} t_3}$$

Definition 2.3. For each natural number i, define a S(X) as follow:

$$\begin{aligned} S_0(X) &= X \\ S_1(X) &= \{\, \text{succ } t, \text{prev } t, \text{iszero } t \mid t \in X \,\} \cup \{\, \texttt{if} t_1 \texttt{then} t_2 \texttt{else} t_3 \mid t_1, t_2, t_3 \in X \,\} \\ &\vdots \\ S_{i+1}(X) &= S(S_i(X)). \end{aligned}$$

Proposition 2.4. $\mathcal{T} = \bigcup_{i=0}^{\omega} S_i(\{\text{true}, \text{false}, 0\})$.

证明. $\square$