关于 Maple Algebra 的这一路

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Definition 1.1. A labelled transition system is a tuple $(S, \Lambda, \rightarrow)$ where S is set of states, Λ is set of labels, and $\rightarrow$ is relation of labelled transitions (i.e., a subset of $S \times \Lambda \times S$). A $(p, \alpha, q) \in\rightarrow$ is written as $p \xrightarrow{\alpha} q$.

Annotation 1.2. TODO: categorical semantics: F-coalgebra

Definition 1.3. Let $T = (S, \Lambda, \rightarrow)$ be a labelled transition system. The set of traces $Tr(s)$, for $s \in S$ is the minimal set satisfying

- $\varepsilon \in Tr(s)$.
- $\alpha\ \sigma \in Tr(s)$ if $\{\, s' \in S \mid s \ \xrightarrow{\alpha}\ s'$ and $\sigma \in Tr(s')\,\}$.

Definition 1.4. Two states p, q are trace equivalent iff $Tr(p) = Tr(q)$.

Definition 1.5. (Simultation) Given two labelled transition system $(S_1, \Lambda, \rightarrow_1)$ and $(S_2, \Lambda, \rightarrow_2)$, relation $R \subseteq S_1 \times S_2$ is a simulation iff, for all $(p, q) \in R$ and $\alpha \in \lambda$ satisfies

$$\text{for any } p \xrightarrow{\alpha}_1 p',\ \text{then there exists } q' \text{ such that } q \xrightarrow{\alpha}_2 q' \text{ and } (p', q') \in R$$

$$\begin{tikzcd}
p \arrow[r, "\alpha"] \arrow[d, dashed, "R"'] & p' \arrow[d, dashed, "R"] \\
q \arrow[r, "\alpha"'] & q'
\end{tikzcd}$$

Definition 1.6. We say q simulates p if there exists a simulation R includes (p, q) (i.e., $(p, q) \in R$).

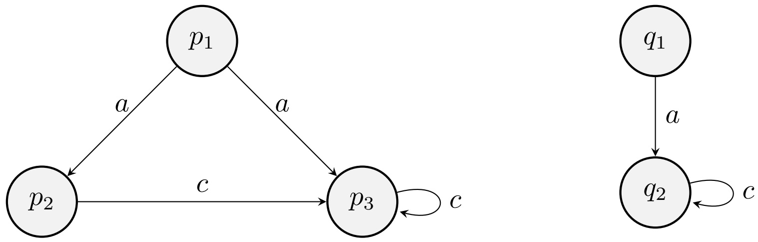
Lemma 1.7. The simulation is reflexive and transitive.

Annotation 1.8. 最有意思的是我们应该如何找到这样 simulation 来满足 $(p, q) \in R$, 更一步我们更希望找到 the minimal relation.

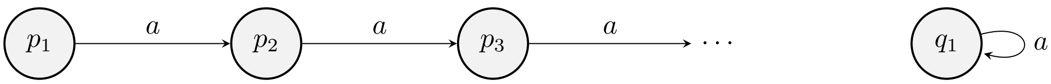
Definition 1.9. (Bisimultation) Given two labelled transition system $(S_1, \Lambda, \rightarrow_1)$ and $(S_2, \Lambda, \rightarrow_2)$, relation $R \subseteq S_1 \times S_2$ is a bisimulation iff both R and its converse $\overline{R}$ are simulations, for all $(p, q) \in R$ and $\alpha \in \lambda$ satisfies

$$\text{for any } p \xrightarrow{\alpha}_1 p',\ \text{then there exists } q' \text{ such that } q \xrightarrow{\alpha}_2 q' \text{ and } (p', q') \in R$$
$$\text{for any } q \xrightarrow{\alpha}_2 q',\ \text{then there exists } p' \text{ such that } p \xrightarrow{\alpha}_1 p' \text{ and } (p', q') \in R$$

Example 1.10. 一些 bisimulation 的例子



关于上面两个 transition system 的 bisimultaion 为 $R = \{(p_1, q_1), (p_2, q_2), (p_3, q_2)\}$. 还有一个比较有点特别的例子



如果关于上图这样 bisimulation R 存在，那么 $(p_i, q_1) \in R$ for every i.