

关于 Maple Algebra 的这一路

枫聆 (maplegra)

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目录

1	Equivalence of Program	2
1.1	Reactive Systems	2

Equivalence of Program

Reactive Systems

Definition 1.1. A labelled transition system is a tuple $(S, \Lambda, \rightarrow)$ where S is set of states, Λ is set of labels, and \rightarrow is relation of labelled transitions (i.e., a subset of $S \times \Lambda \times S$). A $(p, \alpha, q) \in \rightarrow$ is written as $p \xrightarrow{\alpha} q$.

Annotation 1.2. TODO: categorical semantics: F -coalgebra

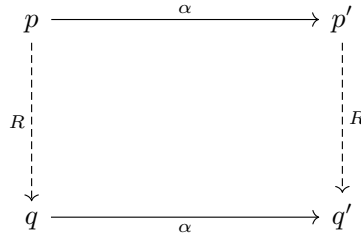
Definition 1.3. Let $T = (S, \Lambda, \rightarrow)$ be a labelled transition system. The set of traces $Tr(s)$, for $s \in S$ is the minimal set satisfying

- $\varepsilon \in Tr(s)$.
- $\alpha \sigma \in Tr(s)$ if $\{ s' \in S \mid s \xrightarrow{\alpha} s' \text{ and } \sigma \in Tr(s') \}$.

Definition 1.4. Two states p, q are trace equivalent iff $Tr(p) = Tr(q)$.

Definition 1.5. (Simulation) Given two labelled transition system $(S_1, \Lambda, \rightarrow_1)$ and $(S_2, \Lambda, \rightarrow_2)$, relation $R \subseteq S_1 \times S_2$ is a simulation iff, for all $(p, q) \in R$ and $\alpha \in \Lambda$ satisfies

for any $p \xrightarrow{\alpha}_1 p'$, then there exists q' such that $q \xrightarrow{\alpha}_2 q'$ and $(p', q') \in R$



Definition 1.6. We say q simulates p if there exists a simulation R includes (p, q) (i.e., $(p, q) \in R$).

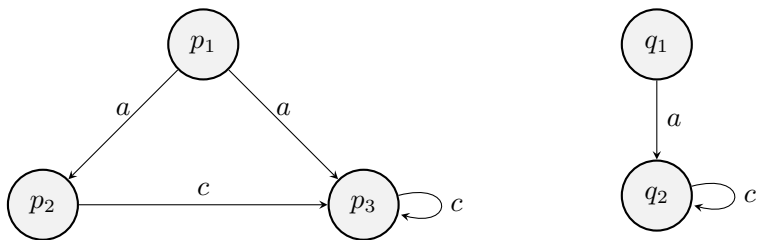
Lemma 1.7. The simulation is reflexive and transitive.

Annotation 1.8. 最有意思的是我们应该如何找到这样 simulation 来满足 $(p, q) \in R$, 更进一步我们更希望找到 the minimal relation.

Definition 1.9. (Bisimulation) Given two labelled transition system $(S_1, \Lambda, \rightarrow_1)$ and $(S_2, \Lambda, \rightarrow_2)$, relation $R \subseteq S_1 \times S_2$ is a bisimulation iff both R and its converse \bar{R} are simulations, for all $(p, q) \in R$ and $\alpha \in \Lambda$ satisfies

- for any $p \xrightarrow{\alpha}_1 p'$, then there exists q' such that $q \xrightarrow{\alpha}_2 q'$ and $(p', q') \in R$
- for any $q \xrightarrow{\alpha}_2 q'$, then there exists p' such that $p \xrightarrow{\alpha}_1 p'$ and $(p', q') \in R$

Example 1.10. 一些 bisimulation 的例子



关于上面两个 transition system 的 bisimulation 为 $R = \{(p_1, q_1), (p_2, q_2), (p_3, q_2)\}$. 还有一个比较有特点的例子

