

Lattice

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Ordered Sets

Definition 1.1. **Partially ordered set** is a system $\mathcal{P} = (P, \leq)$ where P is a nonempty set and \leq is a binary relation on P satisfying, for all $x, y, z \in P$,

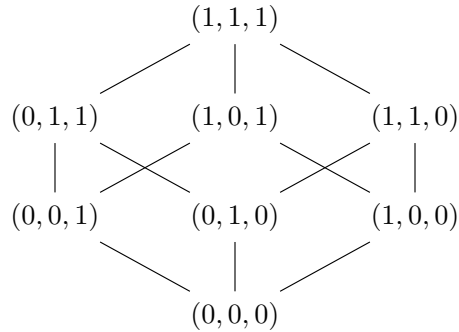
1. $x \leq x$, (reflexivity)
2. if $x \leq y$ and $y \leq x$, then $x = y$, (antisymmetry)
3. if $x \leq y$ and $y \leq z$, then $x \leq z$. (transitivity)

Definition 1.2. \mathcal{C} is a **chain** if for every $x, y \in \mathcal{C}$, either $x \leq y$ or $y \leq x$.

chain 上的元素都可以相互比较.

Definition 1.3. We say that x is **covered** by y in \mathcal{P} , written $x \prec y$, if $x \leq y$ and there is no $z \in P$ with $x \leq z \leq y$.

Definition 1.4. **Hasse diagram** for a finite partially order set \mathcal{P} : the elements of P are represented by points in the plane, and a line is drawn from a up to b precisely when $a \prec b$.



Definition 1.5. Given a partially order set, f is a **order preserving map** satisfying the condition $x \leq y$ implies $f(x) \leq f(y)$.