

Power Series Method and Recurrences - Internals

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1 Introduction

This is the situation for $k = 4$:

2 The Algorithm

$$\begin{array}{rcccccc} u(t) = & U_0.t^0 & + & U_1.t^1 & + & U_2.t^2 & + & U_3.t^3 & + & U_4.t^4 \\ \frac{du(t)}{dt} = & 1U_1.t^0 & + & 2U_2.t^1 & + & 3U_3.t^2 & + & 4U_4.t^3 & & \\ v(t) = & V_0.t^0 & + & V_1.t^1 & + & V_2.t^2 & + & V_3.t^3 & + & \cancel{V_4.t^4} \end{array}$$

3 Cauchy

$$\begin{array}{l} P_4 = V_0.U_4 \\ + V_1.U_3 \\ + V_2.U_2 \\ + V_3.U_1 \\ + V_4.U_0 \end{array}$$

4 chain rule

power series for time derivative of a composed function

$$\begin{array}{l} F'_3 = dFdU_0.U'_3 \\ + dFdU_1.U'_2 \\ + dFdU_2.U'_1 \\ + dFdU_3.U'_0 \end{array}$$

replace time derivatives

$$\begin{aligned}4.F_4 &= dFdU_0.U_4.4 \\ &+ dFdU_1.U_3.3 \\ &+ dFdU_2.U_2.2 \\ &+ dFdU_3.U_1.1\end{aligned}$$

5 reverse chain

continuing from above, extract term containing U_4

$$\begin{aligned}dFdU_0.U_4.4 &= 4.F_4 - dFdU_1.U_3.3 \\ &- dFdU_2.U_2.2 \\ &- dFdU_3.U_1.1\end{aligned}$$

isolate U_4

$$\begin{aligned}U_4 &= \frac{1}{dFdU_0}.(F_4 - dFdU_1.U_3.3/4 \\ &- dFdU_2.U_2.2/4 \\ &- dFdU_3.U_1.1/4)\end{aligned}$$

THE END