Power Series Method and Recurrences - Internals

Ian Smith

1 Introduction

This is the situation for $k = 4$:

2 The Algorithm

$$\begin{aligned}
u(t) &= U_0.t^0 &+ U_1.t^1 &+U_2.t^2 &+U_3.t^3 &+ U_4.t^4\\
\frac{du(t)}{dt} &= 1U_1.t^0 &+ 2U_2.t^1 &+3U_3.t^2 &+4U_4.t^3 &\\
v(t) &= V_0.t^0 &+ V_1.t^1 &+V_2.t^2 &+V_3.t^3 &\cancel{+V_4.t^4}
\end{aligned}$$

3 Cauchy

$$\begin{aligned}
P_4 = &V_0.U_4\\
&+ V_1.U_3\\
&+ V_2.U_2\\
&+ V_3.U_1\\
&+ V_4.U_0
\end{aligned}$$

4 chain rule

power series for time derivative of a composed function

$$\begin{aligned}
F'_3 = &dFdU_0.U'_3\\
&+ dFdU_1.U'_2\\
&+ dFdU_2.U'_1\\
&+ dFdU_3.U'_0
\end{aligned}$$

replace time derivatives

$$\begin{aligned} 4.F_4 = & \, dFdU_0.U_4.4 \\ & + dFdU_1.U_3.3 \\ & + dFdU_2.U_2.2 \\ & + dFdU_3.U_1.1 \end{aligned}$$

5 reverse chain

continuing from above, extract term containing U_4

$$\begin{aligned} dFdU_0.U_4.4 = & \, 4.F_4 - dFdU_1.U_3.3 \\ & - dFdU_2.U_2.2 \\ & - dFdU_3.U_1.1 \end{aligned}$$

isolate U_4

$$\begin{aligned} U_4 = \frac{1}{dFdU_0}.(F_4 & - dFdU_1.U_3.3/4 \\ & - dFdU_2.U_2.2/4 \\ & - dFdU_3.U_1.1/4) \end{aligned}$$

THE END