## tzavelis hw5

## March 6, 2020

```
[2]: import numpy as np
     import pandas as pd
     from scipy import signal
     import soundfile as sf
     import simpleaudio as sa
     import sounddevice as sd
     from scipy.io import wavfile
     import matplotlib.pyplot as plt
     %matplotlib inline
     import seaborn as sns
     plt.rcParams['font.weight'] = 'bold'
     plt.rcParams['axes.labelweight'] = 'bold'
     plt.rcParams['lines.linewidth'] = 1
     plt.rcParams['axes.titleweight'] = 'bold'
     class SignalTB:
         11 11 11
            My signal toolbox (SignalTB)!
         def __init__(self, x, fs):
             Arguments:
                 x: Time Series
                 fs: Sample Frequency
             self.fs = fs; # [hz]
             self.x = x # time domain series
             self.X = None # frequency domain series
             self.sxx = None
             self.gxx = None
             self.gxx_rms_a = None
             self.gxx_linear_a = None
             self.signals = [self.x] #useful container
             self.N = self.x.shape[0]
                                        # number of samples
```

```
self.L = self.x.index[-1] - self.x.index[0] # total time of signal [s]
       self.dt = self.L/self.N # [s]
       self.df = self.fs/self.N
   def get_signals():
       return filter(lambda x: x is not None, [self.X, self.x, self.xx, self.
\rightarrowgxx])
   def my_fft(self):
       11 11 11
       Description:
           This method calculates the fft of a time domain signal using \Box
→numpy's fft function and
       adjusting it appropriately to multiplies it by dt.
       Returns:
           Series of frequency domain signal
       freq = np.arange(-np.ceil(self.N/2)+1,
                         np.floor(self.N/2)+1) * self.df
       X = np.fft.fft(a=self.x.values, n=None, axis=-1, norm=None) * self.dt
       X = np.concatenate((X[int(np.floor(self.N/2))+1:],
                            X[0:int(np.floor(self.N/2))+1])) # rearrange the
\rightarrow frequencies from standard form to sequential. Remember that 1:self.N//2 does_\sqcup
→not grab that second index value
       X = pd.Series(data=X,
                      index=freq,
                      name='X')
       self.X = X
       SignalTB.parseval_thrm(self.x,self.X,self.df,self.dt) #check Parsevals_
\hookrightarrow thrm
       self.signals.append(self.X)
       return X
   Ostaticmethod
   def my_ifft(X, N, df, dt):
       Description:
           This method calculates the ifft of a time domain signal using using
\rightarrow numpy's ifft function and
       adjusting it appropriately to multiplies it by dt.
       Returns:
           Series of frequency domain signal
       t = np.linspace(start=0, stop=N*dt, num=N, endpoint=True)
       X = X.values # these are in sequential, non standard form
```

```
X = np.concatenate((X[int(np.ceil(N/2))-1:],
                           X[0:int(np.ceil(N/2))-1])) #put the fft values in
⇒standard form so ifft can accept it
       x = np.fft.ifft(a=X, n=None, axis=-1, norm=None) / dt
       SignalTB.parseval_thrm(x,X,df,dt) #check Parsevals thrm
       return pd.Series(data=x,
                         index=t,
                        name='x2')
   Ostaticmethod
   def parseval_thrm(x, X, df, dt):
       Description:
            Checks to make sure Parseval's Theorem holds between a time domain_
\hookrightarrow and FFT holds true
       Arguments:
           x: time domain signal
           X: frequency domain signal
       x = np.absolute(x) #qet rid of complex numbers to do calc
       X = np.absolute(X)
       td = round((x**2).sum() * dt, 1)
       fd = round((X**2).sum() * df, 1)
       assert td == fd , "Parseval Theorem not satisfied: {} != {}".
\rightarrowformat(td,fd)
   def sd(self):
       .....
       Descrition:
           Spectral Density
       sxx = np.abs(self.X)**2 / self.L; sxx.name = 'S_xx'; #display('sxx',sxx)
       # mean squared check
       X_ms = round(1/self.L * np.sum(np.abs(self.X)**2)*self.df,1)
       sxx_ms = round(np.sum(sxx)*self.df,1)
       assert X_ms == sxx_ms, 'Mean Squared Value Error: {} != {}'.
→format(X ms,sxx ms)
       self.sxx = sxx
       self.signals.append(self.sxx)
       #qxx
       freq = np.arange(0, np.floor(self.N/2)+1) * self.df;__
→#display('freq', freq)
       i_zero = int(np.ceil(self.N/2)-1); #display('i_zero',i_zero)
```

```
X = self.sxx.values[i_zero:] * 2 #qrab from the center value all the_
→way to the end and double it
       X[0] = X[0]/2
       if self. N\%2 == 0: X[-1] = X[-1]/2 #even
       gxx = pd.Series(data=X,
                         index=freq,
                         name='G xx')
       # mean squared check
       gxx_ms = round(np.sum(gxx) * self.df,1)
       assert sxx_ms == gxx_ms, 'Mean Squared Value Error: {} != {}'.
→format(sxx_ms,gxx_ms)
       self.gxx = gxx # uts of db
       self.signals.append(self.gxx)
       return self.sxx, self.gxx
   def rms_a(self, n_intervals = 16):
           RMS Averaging for Gxx
       ,, ,, ,,
       frames=[]
       for i in range(1,n_intervals+1):
           x = self.x.iloc[int((i-1)*self.N/n_intervals):int(i*self.N/
→n_intervals)]
           m = SignalTB(x=x, fs=self.fs)
           m.my_fft(); #calc fft
           m.sd() #calculate sxx and qxx
           frames.append(m.gxx) #save each gxx for averaging
       assert len(frames) == n_intervals, 'Could not perfectly cut the number_
→of samples by the n_interval: {}'.format(n_intervals)
       gxx_rms_a = pd.concat(frames,axis='columns').mean(axis='columns') #__
→calculates the mean of at each row (frequency)
       gxx rms a.name = 'G xx rms a'
       self.gxx_rms_a = gxx_rms_a
       self.signals.append(gxx_rms_a)
       return gxx_rms_a
   def linear_a(self, n_intervals = 16):
           Linear Averaging for X, then calculation of Gxx
       11 11 11
       frames=[]
       for i in range(1,n_intervals+1):
           x = self.x.iloc[int((i-1)*self.N/n_intervals):int(i*self.N/
→n_intervals)]
           m = SignalTB(x=x, fs=self.fs)
```

```
m.my_fft(); #calc fft
           frames.append(m.X) #save the fft
       assert len(frames) == n intervals, 'Could not perfectly cut the number_
→of samples by the n_interval: {}'.format(n_intervals)
       X_a = pd.concat(frames,axis='columns').mean(axis='columns') #average_
\rightarrow all the X's at each frequency
       #generate a temporary object so that you can perform computations
       m = SignalTB(x=self.x.iloc[int((i-1)*self.N/n_intervals):int(i*self.N/
→n intervals)], fs=self.fs) # the time signal passed in doesn't mean
→anything, its just necessary to instatiate object
       m.X = X_a #set the new averaged X_a as the frequency domain signal in_{\sqcup}
→ the temporary object
       m.sd()
       m.gxx.name = 'G_xx_linear_a'
       self.gxx_linear_a = m.gxx
       self.signals.append(m.gxx)
       return m.gxx
   def time a(self, n intervals = 16):
           Time Averaging for x, then calculation of Gxx
       11 11 11
       frames=[]
       for i in range(1,n_intervals+1):
           x = self.x.iloc[int((i-1)*self.N/n_intervals):int(i*self.N/
→n_intervals)]
           x = pd.Series(data=x.values,
                         index=self.x.index[0:int(self.N/n_intervals)]) # make_
→sure that all the objects have the same time index. This is important for
→ taking the average and when we instatiate a new object.
           frames.append(x) #save the fft
       assert len(frames) == n_intervals, 'Could not perfectly cut the number_
→of samples by the n_interval: {}'.format(n_intervals)
       x_a = pd.concat(frames,axis='columns').mean(axis='columns');
\rightarrow#display(x a);
       m = SignalTB(x=x_a, fs=self.fs) #qenerate a temporary object
       m.my_fft()
       m.sd()
       m.gxx.name = 'G_xx_time_a'
       self.gxx_time_a = m.gxx
       self.signals.append(m.gxx)
       return m.gxx
   def spectrogram(self, n_intervals = 16, overlap = 0.25):
```

```
11 11 11
           Spectrogram!
       p_size = int(np.floor(self.N/n_intervals));#print('psize: {}'.
→ format(p_size));print('n_intervals*p_size: {}'.format(n_intervals*p_size))
       x = self.x.iloc[0:int(n intervals*p size)]
       frames=[]
       for i in range(1,n_intervals+1):
           if i == 1:
               f = 0
               1 = p_size
           else:
               f = 1 - int(np.floor(overlap*p_size))
               l = f + p_size
           sig = x.iloc[f:1]
           m = SignalTB(x=sig, fs=self.fs)
           m.my_fft();
           m.sd()
           r = (int((i-1)*self.N/n_intervals) + int(i*self.N/n_intervals))/2
           m.gxx.name = round(r*m.dt,2) # name the slice at the middle
           frames.append(m.gxx)
       assert len(frames) == n_intervals, 'Could not perfectly cut the number_
→of samples by the n_interval: {}'.format(n_intervals)
       gxx df = pd.concat(frames,axis='columns').sort_index(ascending=False)
       gxx_df.name = 'Gxx_spectro'
       gxx_df.index = gxx_df.index.values.round(decimals=1)
       self.gxx_df = gxx_df
       self.signals.append(self.gxx_df)
       return gxx_df
   def plot_signals(self, xrange=None):
       Description:
           Plots all of the signals in the self.signals container
       Returns:
           Nothing
       for i, sig in enumerate(self.signals):
           if type(sig) != pd.DataFrame:
               if sig.dtype == complex: sig = np.absolute(sig) # ALWAYS the_
→magnitude of this in case its a complex number
           fig = plt.figure(figsize=(10,5))
```

```
plt.title(sig.name)
          if sig.name in ['x','x2','time domain signal']:
              plt.ylabel('x(t)'); plt.xlabel('t [s]')
          elif sig.name in_
sig = 10*np.log10(sig); plt.ylabel('X(f)'); plt.xlabel('full)
\rightarrow [Hz]'); #plt.ylim([-30:])
          elif sig.name in ['Gxx_spectro']:
              sns.heatmap(sig, cmap="jet"); plt.ylabel('f [Hz]'); plt.
→xlabel('t [s]')
              continue
          if xrange != None:
              sig[xrange[0]:xrange[1]].plot();
          else:
              sig.plot();
          plt.grid()
  #Useful functions to generate signals
  Ostaticmethod
  def impulse(N,L,Tp):
      x = np.zeros(N)
      x[0:int(N*Tp/L)] = 1
      x = pd.Series(data=x,
                    index=np.linspace(start=0, stop=L, num=N, endpoint=True,__
→dtype=float),
                    name='x')
      return x
  Ostaticmethod
  def cw_sin(A,f,N,L,Tp):
       11 11 11
      Arguments:
          A: Amplitude
          f: Frequency of signal [hz]
          L: Total length of time [s]
          N: Number of points
          Tp: Pulse length
      Returns:
          Series
      t = np.linspace(start=0, stop=L, num=N, endpoint=True, dtype=float)
      x = np.zeros(N)
      t_s = t[0:int(N*Tp/L)];
      x[0:int(N*Tp/L)] = A*np.sin(2*np.pi*f*t_s)
```

```
return pd.Series(data=x,
                     index=t,
                     name='x')
  Ostaticmethod
  def lin_sin_s(A,N,L,Tp,f1,f2):
       Arguments:
           A: Amplitude
           f: Frequency of signal [hz]
           L: Total length of time [s]
           N: Number of points
           Tp: Pulse length
       Returns:
           Series
       t = np.linspace(start=0, stop=L, num=N, endpoint=True, dtype=float)
       x = np.zeros(N)
       t_s = t[0:int(N*Tp/L)];
       phi = 2*np.pi*( (f2-f1)/(2*Tp)*t_s**2+f1*t_s)
       x[0:int(N*Tp/L)] = A*np.sin(phi)
       return pd.Series(data=x,
                     index=t.
                     name='x')
  @staticmethod
  def log_sin_s(A,N,L,Tp,f1,f2):
       n n n
       Arguments:
           A: Amplitude
           L: Total length of time [s]
           N: Number of points
           Tp: Pulse length
       Returns:
           Series
       t = np.linspace(start=0, stop=L, num=N, endpoint=True, dtype=float)
       x = np.zeros(N)
       t_s = t[0:int(N*Tp/L)];
       phi = 2*np.pi*f1*(f2/f1)**(t_s/Tp)*Tp/(np.log(f2/f1))-2*np.pi*f1*Tp/np.
\rightarrowlog(f2/f1)
       x[0:int(N*Tp/L)] = A*np.sin(phi)
```

```
x = pd.Series(data=x,
                      index=t,
                      name='x')
       return x
   Ostaticmethod
   def white_noise(N,L,Tp):
       HHHH
       Arguments:
           L : Total length of time [s]
           N : Number of points
       Returns:
           Series
       fs = N/L
       df = fs/N
       p = np.random.uniform(0,2*np.pi,int(N/2)-1) #random reals
       Z = 1*np.sin(p) + 1*np.cos(p)*1j #random complex numbers with magnitude_
\hookrightarrow of 1
       Z = np.concatenate((Z, np.array([0]), np.flip(np.conj(Z))))
       if N\%2 == 0: Z = np.concatenate((Z, np.array([0])))
       freq = np.arange(-np.ceil(N/2)+1, np.floor(N/2)+1) * df
       X = pd.Series(data=Z,index=freq); #construct the full freq response
       x = SignalTB.my_ifft(X=X, N=N, df=df, dt=dt) # get the full time noise
       y = np.zeros(N,dtype=complex)
       y[0:int(N*Tp/L)] = x.values[0:int(N*Tp/L)] # take only a subset of the
→noisey signal and put it infront of all the zeros
       return pd.Series(data=np.real(y),
                         index=x.index,
                        name='x')
   Ostaticmethod
   def sin(A,f,L,N):
       11 11 11
       Arguments:
           A: Amplitude
           f: Frequency of signal [hz]
           L: Total length of time [s]
           N: Number of points
```

```
Returns:
           Series
       t = np.linspace(start=0, stop=L, num=N, endpoint=True, dtype=float)
       return pd.Series(data=A*np.sin(2*np.pi*f*t),
                         index=t,
                         name='x')
   Ostaticmethod
   def randn_sig(L,N):
       Arguments:
           L : Total length of time [s]
           N: Number of points
       Returns:
           Series
        ,, ,, ,,
       return pd.Series(data=np.random.randn(N,),
                         index=np.linspace(start=0, stop=L, num=N,__
→endpoint=True),
                         name='x')
   Ostaticmethod
   def csd(s0,s1):
       11 11 11
       Descrition:
           Cross Spectral Density
       #calculate the fft of the objects
       s0.my_fft(); s1.my_fft()
       #sxy
       sxy = np.conj(s0.X)*s1.X / s0.L; sxy.name = 'S_xy';
       #qxy
       freq = np.arange(0, np.floor(s0.N/2)+1) * s0.df; #display('freq', freq)
       i_zero = int(np.ceil(s0.N/2)-1); #display('i_zero', i_zero)
       X = \text{sxy.values}[i\_\text{zero:}] * 2 #grab from the center value all the way to_{\bot}
\rightarrow the end and double it
       X[0] = X[0]/2
       if s0.N\%2 == 0: X[-1] = X[-1]/2 #even
       gxy = pd.Series(data=X,
                        index=freq,
                        name='G_xy')
       return sxy, gxy
```

```
Ostaticmethod
   def cross_corr(s0,s1):
       HHHH
       Description:
           Cross correlation F^-1(Sxy)
       sxy, gxy = SignalTB.csd(s0,s1)
       X = sxy.values # these are in sequential, non standard form
       X = np.concatenate((X[int(np.ceil(s0.N/2))-1:],
                           X[0:int(np.ceil(s0.N/2))-1])) #put the fft values
\rightarrow in standard form so ifft can accept it
       x = np.fft.ifft(a=X, n=None, axis=-1, norm=None) / s0.dt
       SignalTB.parseval_thrm(x,X, s0.df, s0.dt) #check Parsevals thrm
       x = np.concatenate((x[int(np.floor(s0.N/2))+1:],
                           x[0:int(np.floor(s0.N/2))+1])) #put the fft values
→ in standard form so ifft can accept it
       t = np.arange(-np.ceil(s0.N/2)+1,np.floor(s0.N/2)+1) * s0.dt
       cross_corr = pd.Series(data=x,
                              index=t,
                              name='Cross Correlation')
       return cross_corr, sxy, gxy
```

```
[3]: # QUESTION 1
L = 10 # [s]
fs = 1024 #[hz]
N = int(L/(1/fs)); # generate the number of points based on the sampling

→ frequency which is higher than the actual signal frequency
f_sin = 400;
df = fs/N
dt = L/N

Tp = 3
f1 = 1
f2 = 200

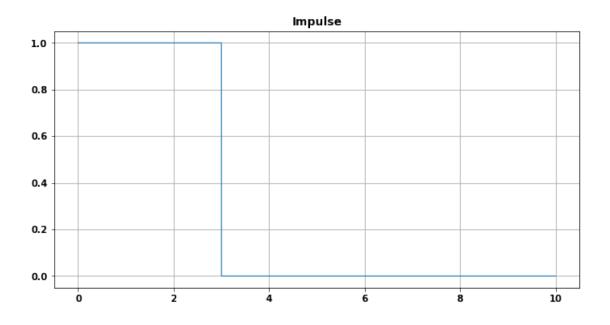
x, Z = SignalTB.white_noise(N,L,Tp)
#plt.plot(np.angle(Z))
plt.plot(x)
```

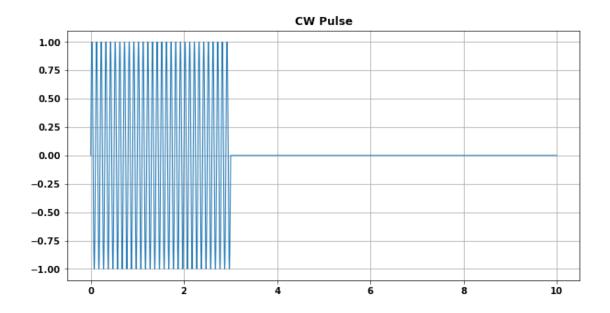
```
ValueError Traceback (most recent call<sub>□</sub>
```

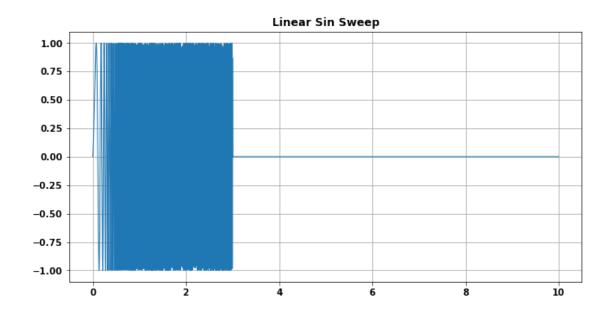
```
<ipython-input-3-Oafc105afb8d> in <module>
    11 f2 = 200
    12
---> 13 x, Z = SignalTB.white_noise(N,L,Tp)
    14 #plt.plot(np.angle(Z))
    15 plt.plot(x)
```

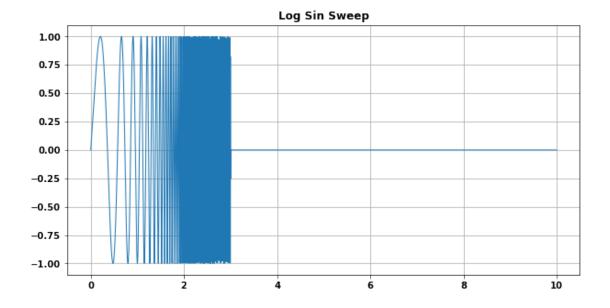
ValueError: too many values to unpack (expected 2)

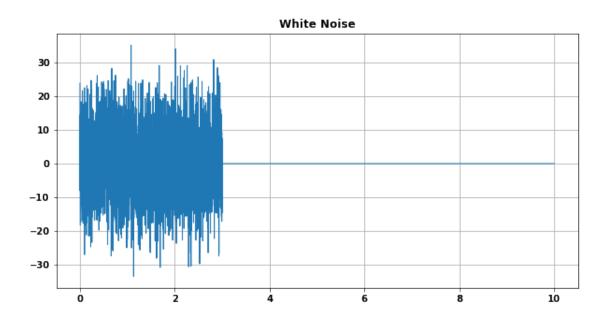
```
[11]: # QUESTION 1
      L = 10 \# [s]
      fs = 1024 \#[hz]
      N = int(L/(1/fs)); # generate the number of points based on the sampling
      → frequency which is higher than the actual signal frequency
      f_sin = 400;
      df = fs/N
      dt = L/N
      Tp = 3
      f1 = 1
      f2 = 200
      sigs = [SignalTB.impulse(N=N,L=L,Tp=Tp),
              SignalTB.cw_sin(A=1,f=10, N=N,L=L,Tp=Tp),
              SignalTB.lin_sin_s(A=1, N=N, L=L, Tp=Tp, f1=f1, f2=f2),
              SignalTB.log\_sin\_s(A=1,N=N,L=L,Tp=Tp,f1=f1,f2=f2),
              SignalTB.white_noise(N=N,L=L,Tp=Tp)]
      signals = [SignalTB(x=x, fs=fs) for x in sigs]
      titles = ['Impulse', 'CW Pulse', 'Linear Sin Sweep', 'Log Sin Sweep', 'White
      →Noise']
      #q1
      for i, s in enumerate(signals):
          plt.figure(figsize=(10,5))
          plt.title(titles[i])
          s.x.plot(); plt.grid()
          #s.my_fft()
          #plt.plot(np.abs(SignalTB.my_ifft(s.X, N, df, dt)),'--')
```



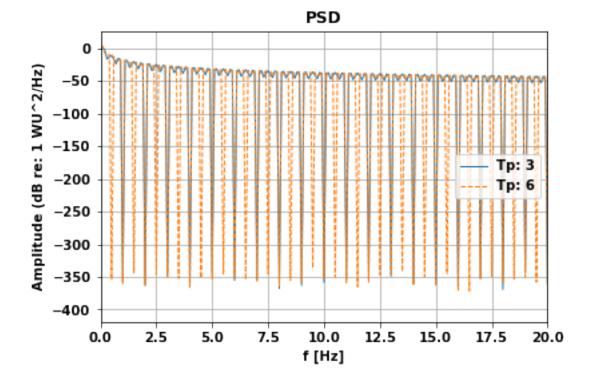






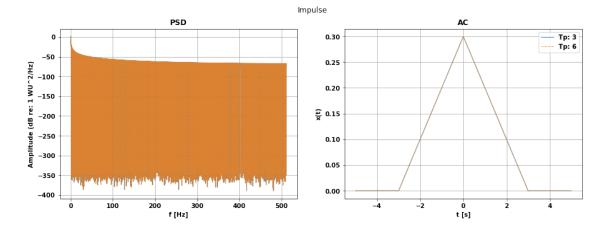


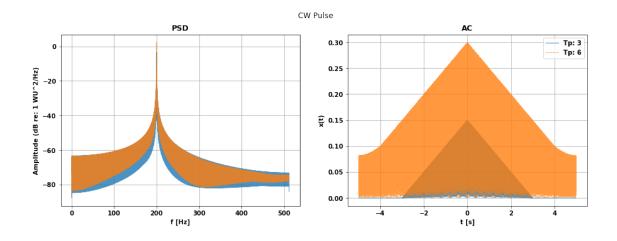
```
[37]: # QUESTION 1
      L = 10 \# [s]
      fs = 1024 \#[hz]
      N = int(L/(1/fs)); # generate the number of points based on the sampling
      → frequency which is higher than the actual signal frequency
      f \sin = 400;
      df = fs/N
      dt = L/N
      Tp = 3
      f1 = 1
      f2 = 200
      x = SignalTB.impulse(L=L,N=N,Tp=Tp)
      s = SignalTB(x=x,fs=fs)
      s.my_fft() #must calc fft to do sd
      #power spectral density
      sxx, gxx = s.sd()
      fig = plt.figure()
      plt.plot(10*np.log10(np.abs(gxx)));
      Tp = 6
      f1 = 1
      f2 = 200
      x = SignalTB.impulse(L=L,N=N,Tp=Tp)
```

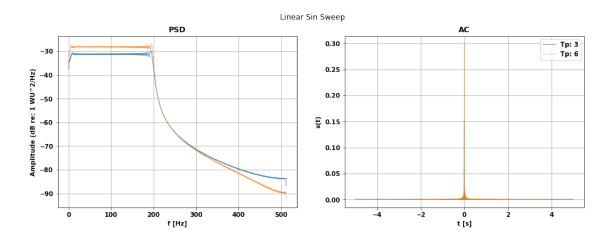


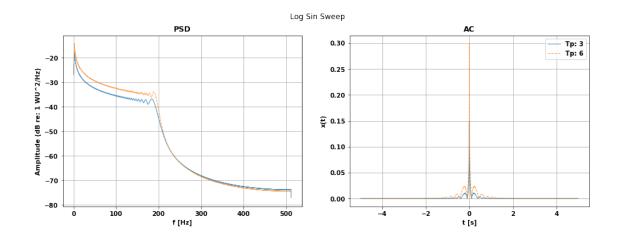
```
sigs = [SignalTB.impulse(L=L,N=N,Tp=Tp),
               SignalTB.cw_sin(A=1,f=f2, N=N,L=L,Tp=length),
               SignalTB.lin_sin_s(A=1,N=N,L=L,Tp=length,f1=f1,f2=f2),
               SignalTB.log_sin_s(A=1, N=N, L=L, Tp=length, f1=f1, f2=f2),
               SignalTB.white_noise(N=N,L=L,Tp=Tp),
       signals = [SignalTB(x=x, fs=fs) for x in sigs]
       s = signals[i] #select the actual signal we care about
       s.my_fft() #must calc fft to do sd
       #power spectral density
       sxx, gxx = s.sd()
       axs[0].plot(10*np.log10(np.abs(gxx)),alpha=0.8,linestyle=linestyles[j]);
       axs[0].set_title('PSD'); axs[0].set_ylabel('Amplitude (dB re: 1 WU^2/
→Hz)'); axs[0].set_xlabel('f [Hz]'); axs[0].grid(True)
       #autocorrelation
       cross corr, sxy, gxy = SignalTB.cross corr(s,s)
       axs[1].plot(np.abs(cross_corr),alpha=0.8,linestyle=linestyles[j]);
       axs[1].set_title('AC'); axs[1].set_ylabel('x(t)'); axs[1].set_xlabel('tu
\rightarrow [s]'); axs[1].grid(True);
   plt.legend(['Tp: {}'.format(times[0]),'Tp: {}'.format(times[1])])
   fig.savefig('./plots/hw5_'+t+'.png', dpi=300, bbox_inches='tight');
```

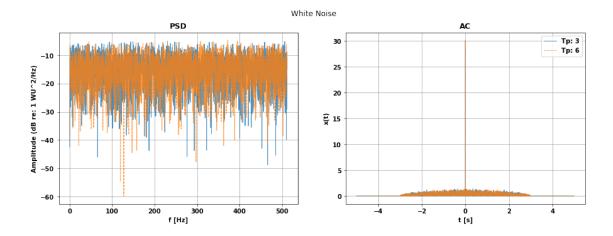
/home/m4rz910/anaconda3/lib/python3.7/site-packages/pandas/core/series.py:679:
RuntimeWarning: divide by zero encountered in log10
 result = getattr(ufunc, method)(\*inputs, \*\*kwargs)



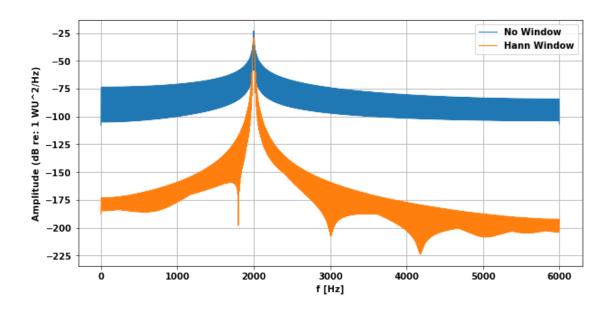








```
[40]: # 3. TAPER AND WINDOW
      L = 1
      Tp=0.1
      fs=12000
      f=2000
      N = int(L/(1/fs));
      fig = plt.figure(figsize=(10,5));plt.grid()
      #without window
      x = SignalTB.cw_sin(A=1,f=f,N=N,L=L,Tp=Tp)
      s0 = SignalTB(x,fs)
      s0.my_fft();
      s0.sd();
      #plt.plot(s0.x); #plt.xlim([0,.01])
      plt.plot(10*np.log10(np.abs(s0.gxx)))
      #with window
      w = np.zeros(N)
      w[0:int(N*Tp/L)] = np.hanning(int(N*Tp/L))
      x = SignalTB.cw_sin(A=1,f=f,N=N,L=L,Tp=Tp)*w
      s1 = SignalTB(x,fs)
      s1.my_fft();
      s1.sd();
      plt.plot(10*np.log10(np.abs(s1.gxx)))
      #plt.plot(s1.x);
      #plt.xlim([0,5000])
      plt.ylabel('Amplitude (dB re: 1 WU^2/Hz)'); plt.xlabel('f [Hz]');
      plt.legend(['No Window', 'Hann Window']);
      fig.savefig('./plots/hw5_'+'q3taper'+'.png', dpi=300, bbox_inches='tight');
```



```
[42]: # 4. Sine Sweep
      # 3. TAPER AND WINDOW
      L = 1
      Tp=0.1
      fs=12000
      f1=100
      f2=5000
      N = int(L/(1/fs));
      fig = plt.figure(figsize=(10,5));plt.grid()
      #without window
      x=SignalTB.lin_sin_s(A=1,N=N,L=L,Tp=Tp,f1=f1,f2=f2)
      \#x=SignalTB.log\_sin\_s(A=1,N=N,L=L,Tp=Tp,f1=f1,f2=f2)
      \#x = SignalTB.cw\_sin(A=1, f=f, N=N, L=L, Tp=Tp)
      s0 = SignalTB(x,fs)
      s0.my_fft();
      s0.sd();
      #plt.plot(s0.x); #plt.xlim([0,.01])
      plt.plot(10*np.log10(np.abs(s0.gxx)))
      #with window
      w = np.zeros(N)
      w[0:int(N*Tp/L)] = np.hanning(int(N*Tp/L))
      x=SignalTB.lin_sin_s(A=1,N=N,L=L,Tp=Tp,f1=f1,f2=f2)*w
      \#x = SignalTB. log_sin_s(A=1, N=N, L=L, Tp=Tp, f1=f1, f2=f2)*w
      \#x = SignalTB.cw\_sin(A=1, f=f, N=N, L=L, Tp=Tp)*w
      s1 = SignalTB(x,fs)
```

```
s1.my_fft();
s1.sd();
plt.plot(10*np.log10(np.abs(s1.gxx)))
#plt.plot(s1.x);
#plt.xlim([0,5000])

plt.ylabel('Amplitude (dB re: 1 WU^2/Hz)'); plt.xlabel('f [Hz]');
plt.legend(['No Window', 'Hann Window']);
fig.savefig('./plots/hw5_'+'q3taper'+'.png', dpi=300, bbox_inches='tight');
```

