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ME465 - Sound and Space

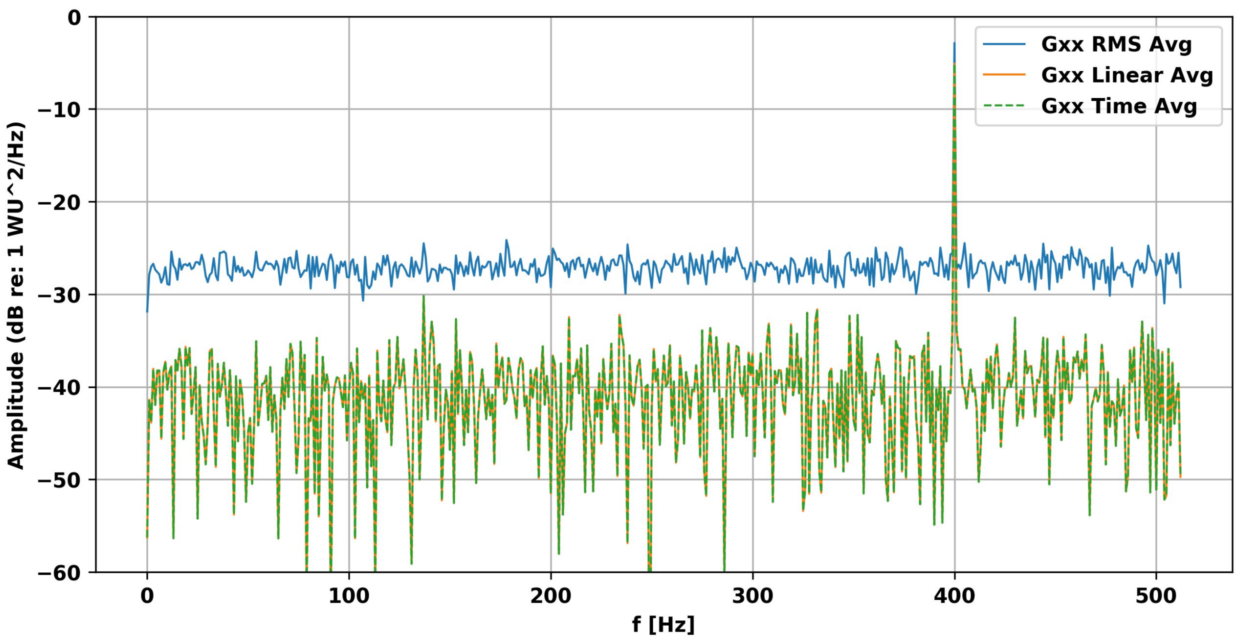
02/20/2020

HW3: RMS, Linear and Time Averaging

Exercise 1

The task was to create three functions: one that performed asynchronous averaging (RMS Average of Spectral Density) and two that performed asynchronous averaging (Linear and Time-Average). The asynchronous RMS averaging involved breaking the signal into smaller time chunks, computing the spectral density of each piece, and then averaging the amplitude at the each frequency. The synchronous linear averaging involved breaking the signal into evenly sized pieces, computing the FFT to convert into the frequency-domain, averaging the complex numbers at each frequency, and then computing the spectral density of the averaged frequency-domain signal. Similarly, the synchronous time-average signal involved breaking the signal into components and immediately averaging them in the time domain before computing the power spectral density. The two synchronous methods are expected to be identical since the same phase information exists when the averaging takes place (and the random phase cancels out). To test the functions, a 16 second sine wave (fs: 1024 Hz) with a frequency of 400 Hz hidden in noise was analyzed, and revealed a peak at the expected signal frequency (Figure 1).

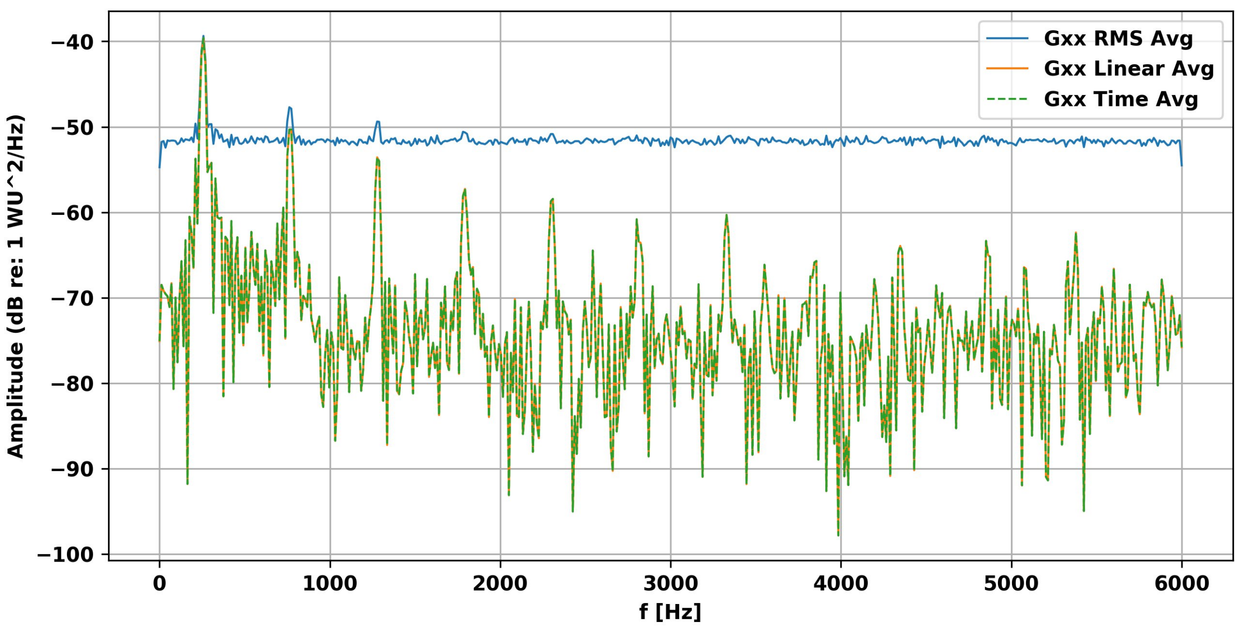
Figure 1: Power Spectral Density of 400 Hz Sine Wave



Exercise 2

The task was to analyze the three power spectral density averages of a repeating pulse hidden in noise (pulsenoise.wav). 256 averages were used to perform all three signal averages because the original signal (pulse.wav) contained 1024 samples, and the noisy signal contained a little over 256*1024 samples — necessitating a small truncation of about 50 samples at the end of the signal. Figure 2 plots the results of the three signal averages: the RMS averaged signal (asynchronous) has the least amplitude variability but only the first four overtones are easily visible. In contrast, the linear and time averaged signals (synchronous) have greater variability in amplitude, but more overtones are visible (at least eleven). Furthermore, the linear and time averaged signals were identical (as expected) and all three signals indicated the fundamental frequency at 257.8 Hz with the same amplitude.

Figure 2: Pulse-noise Power Spectral Density (256 Averages)



Concluding Remarks

The first major take away is that to compare the different averaging methods it is necessary to use the same number of averages; in this assignment, it was necessary to leverage knowledge of the length of one pulse and the fact that it was repeated in the noisy signal. The second major insight is that synchronous averaging preserves the signal's phase information during the averaging process, thereby allowing the random phase from the noise to cancel out. RMS averaging does not allow for the random noise information to cancel because calculating the power spectral density (Sxx and Gxx) involves taking the magnitude of the linear spectrum (X), expressed with complex numbers (and therefore contains phase information), thereby converting it to real numbers that don't contain have phase information.

tzavelis_hw3

February 19, 2020

[24]:
```
import numpy as np
import pandas as pd
import soundfile as sf
import simpleaudio as sa
import sounddevice as sd
from scipy.io import wavfile
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns

plt.rcParams['font.weight'] = 'bold'
plt.rcParams['axes.labelweight'] = 'bold'
plt.rcParams['lines.linewidth'] = 1
plt.rcParams['axes.titleweight'] = 'bold'

class SignalTB:
    """
        My signal toolbox (SignalTB)!
    """

    def __init__(self, x, fs):
        """
        Arguments:
            x: Time Series
            fs: Sample Frequency
        """
        self.fs = fs; # [hz]
        self.x = x    # time domain series
        self.X = None # frequency domain series
        self.sxx = None
        self.gxx = None
        self.gxx_rms_a = None
        self.gxx_linear_a = None
        self.signals = [self.x] #useful container

        self.N = self.x.shape[0]   # number of samples
        self.L = self.x.index[-1] - self.x.index[0]   # total time of signal [s]
```

```
        self.dt = self.L/self.N  # [s]
        self.df = self.fs/self.N

    def get_signals():
        return filter(lambda x: x is not None, [self.X, self.x, self.sxx, self.
↪gxx])

    def my_fft(self):
        """
        Description:
            This method calculates the fft of a time domain signal using␣
↪numpy's fft function and
        adjusting it appropriately to multiplies it by dt.

        Returns:
            Series of frequency domain signal
        """
        freq = np.arange(-np.ceil(self.N/2)+1,
                         np.floor(self.N/2)+1) * self.df
        X = np.fft.fft(a=self.x.values, n=None, axis=-1, norm=None) * self.dt
        X = np.concatenate((X[self.N//2+1:],
                            X[0:self.N//2+1])) # rearrange the frequencies from␣
↪standard form to sequential. Remember that 1:self.N//2 does not grab that␣
↪second index value
        X = pd.Series(data=X,
                      index=freq,
                      name='X')
        self.X = X
        self.parseval_thrm(self.x,self.X)  #check Parsevals thrm
        self.signals.append(self.X)
        return X

    def my_ifft(self):
        """
        Description:
            This method calculates the ifft of a time domain signal using␣
↪numpy's ifft function and
        adjusting it appropriately to multiplies it by dt.

        Returns:
            Series of frequency domain signal
        """
        t = np.linspace(start=self.x.index[0], stop=self.x.index[-1], num=self.
↪N, endpoint=True)
        X = self.X.values # these are in sequential, non standard form
        X = np.concatenate((X[int(np.ceil(self.N/2))-1:],
```

```
                            X[0:int(np.ceil(self.N/2))-1])) #put the fft values␣
↪in standard form so ifft can accept it
        x = np.fft.ifft(a=X, n=None, axis=-1, norm=None) / self.dt
        self.parseval_thrm(x,X)  #check Parsevals thrm
        x = pd.Series(data=x,
                      index=t,
                      name='x2')
        self.signals.append(x)
        return x

    def parseval_thrm(self, x, X):
        """
        Description:
            Checks to make sure Parseval's Theorem holds between a time domain␣
↪and FFT holds true

        Arguments:
            x: time domain signal
            X: frequency domain signal
        """
        td = round((x**2).sum() * self.dt, 1)
        fd = round((np.absolute(X)**2).sum() * self.df, 1)
        assert td == fd , "Parseval Theorem not satisfied: {} != {}".
↪format(td,fd)

    def sd(self):
        """
        Descrition:
            Spectral Density
        """
        sxx = np.abs(self.X)**2 / self.L; sxx.name = 'S_xx'; #display('sxx',sxx)
        # mean squared check
        X_ms = round(1/self.L * np.sum(np.abs(self.X)**2)*self.df,1)
        sxx_ms = round(np.sum(sxx)*self.df,1)
        assert X_ms == sxx_ms, 'Mean Squared Value Error: {} != {}'.
↪format(X_ms,sxx_ms)
        self.sxx = sxx
        self.signals.append(self.sxx)

        #gxx
        freq = np.arange(0, np.floor(self.N/2)+1) * self.df;␣
↪#display('freq',freq)
        i_zero = int(np.ceil(self.N/2)-1); #display('i_zero',i_zero)

        X = self.sxx.values[i_zero:] * 2 #grab from the center value all the␣
↪way to the end and double it
```

```
        X[0] = X[0]/2
        if self.N%2 == 0: X[-1] = X[-1]/2 #even
        gxx = pd.Series(data=X,
                        index=freq,
                        name='G_xx')

        # mean squared check
        gxx_ms = round(np.sum(gxx) * self.df,1)
        assert sxx_ms == gxx_ms, 'Mean Squared Value Error: {} != {}'.
↪format(sxx_ms,gxx_ms)
        self.gxx = gxx # uts of db
        self.signals.append(self.gxx)
        return self.sxx, self.gxx

    def rms_a(self, n_intervals = 16):
        """
            RMS Averaging for Gxx
        """
        frames=[]
        for i in range(1,n_intervals+1):
            x = self.x.iloc[int((i-1)*self.N/n_intervals):int(i*self.N/
↪n_intervals)]
            m = SignalTB(x=x, fs=self.fs)
            m.my_fft(); #calc fft
            m.sd() #calculate sxx and gxx
            frames.append(m.gxx) #save each gxx for averaging
        assert len(frames) == n_intervals, 'Could not perfectly cut the number␣
↪of samples by the n_interval: {}'.format(n_intervals)
        gxx_rms_a = pd.concat(frames,axis='columns').mean(axis='columns') #␣
↪calculates the mean of at each row (frequency)
        gxx_rms_a.name = 'G_xx_rms_a'
        self.gxx_rms_a = gxx_rms_a
        self.signals.append(gxx_rms_a)
        return gxx_rms_a

    def linear_a(self, n_intervals = 16):
        """
            Linear Averaging for X, then calculation of Gxx
        """
        frames=[]
        for i in range(1,n_intervals+1):
            x = self.x.iloc[int((i-1)*self.N/n_intervals):int(i*self.N/
↪n_intervals)]
            m = SignalTB(x=x, fs=self.fs)
            m.my_fft(); #calc fft
            frames.append(m.X) #save the fft
```

```python
        assert len(frames) == n_intervals, 'Could not perfectly cut the number of samples by the n_interval: {}'.format(n_intervals)
        X_a = pd.concat(frames,axis='columns').mean(axis='columns') #average all the X's at each frequency

        #generate a temporary object so that you can perform computations
        m = SignalTB(x=self.x.iloc[int((i-1)*self.N/n_intervals):int(i*self.N/n_intervals)], fs=self.fs) # the time signal passed in doesn't mean anything, its just necessary to instatiate object
        m.X = X_a #set the new averaged X_a as the frequency domain signal in the temporary object
        m.sd()
        m.gxx.name = 'G_xx_linear_a'
        self.gxx_linear_a = m.gxx
        self.signals.append(m.gxx)
        return m.gxx

    def time_a(self, n_intervals = 16):
        """
            Time Averaging for x, then calculation of Gxx
        """
        frames=[]
        for i in range(1,n_intervals+1):
            x = self.x.iloc[int((i-1)*self.N/n_intervals):int(i*self.N/n_intervals)]
            x = pd.Series(data=x.values,
                          index=self.x.index[0:int(self.N/n_intervals)]) # make sure that all the objects have the same time index. This is important for taking the average and when we instatiate a new object.
            frames.append(x) #save the fft
        assert len(frames) == n_intervals, 'Could not perfectly cut the number of samples by the n_interval: {}'.format(n_intervals)
        x_a = pd.concat(frames,axis='columns').mean(axis='columns'); #display(x_a);

        m = SignalTB(x=x_a, fs=self.fs) #generate a temporary object
        m.my_fft()
        m.sd()
        m.gxx.name = 'G_xx_time_a'
        self.gxx_time_a = m.gxx
        self.signals.append(m.gxx)
        return m.gxx

    def spectrogram(self, n_intervals = 16):
        """
            Spectrogram!
```

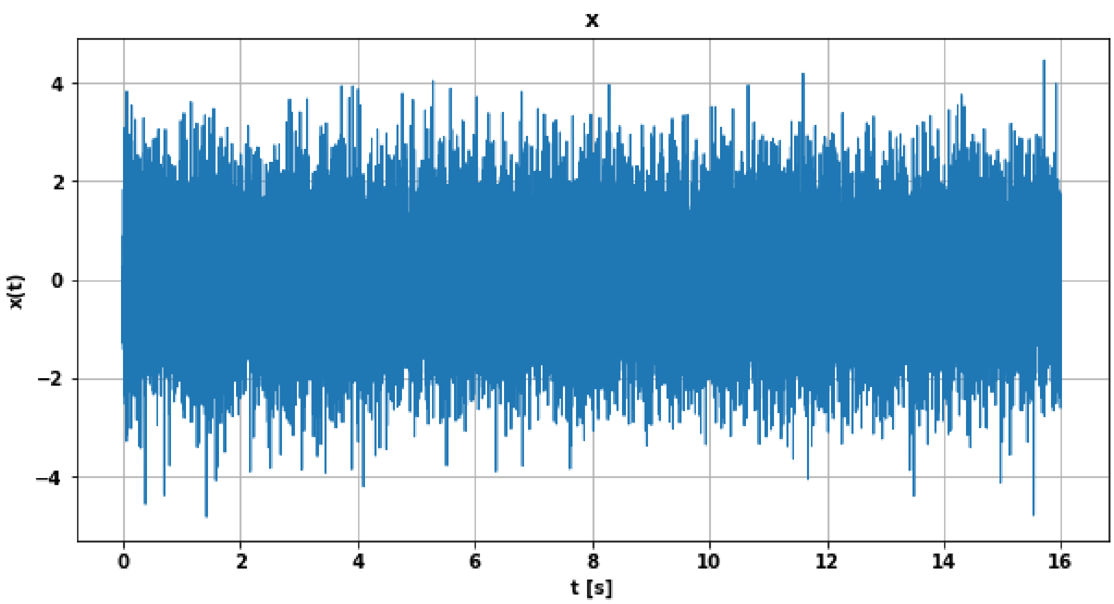
```
        """
        frames=[]
        for i in range(1,n_intervals+1):
            x = self.x.iloc[int((i-1)*self.N/n_intervals):int(i*self.N/
↪n_intervals)]
            m = SignalTB(x=x, fs=self.fs)
            m.my_fft();
            m.sd()
            m.gxx.name = round(self.x.index[(int((i-1)*self.N/n_intervals) +␣
↪int(i*self.N/n_intervals))/2],1)  # name the slice at the middle
            frames.append(m.gxx)
        assert len(frames) == n_intervals, 'Could not perfectly cut the number␣
↪of samples by the n_interval: {}'.format(n_intervals)
        gxx_df = pd.concat(frames,axis='columns').sort_index(ascending=False)
        gxx_df.name = 'Gxx_spectro'
        self.gxx_df = gxx_df
        self.signals.append(self.gxx_df)
        return gxx_df

    def plot_signals(self, xrange=None):
        """
        Description:
            Plots all of the signals in the self.signals container

        Returns:
            Nothing
        """

        for i, sig in enumerate(self.signals):
            if type(sig) != pd.DataFrame:
                if sig.dtype == complex: sig = np.absolute(sig) # ALWAYS the␣
↪magnitude of this in case its a complex number
            fig = plt.figure(figsize=(10,5))
            plt.title(sig.name)
            if sig.name in ['x','x2','time domain signal']:
                plt.ylabel('x(t)'); plt.xlabel('t [s]')
            elif sig.name in␣
↪['X','S_xx','G_xx','G_xx_rms_a','G_xx_linear_a','G_xx_time_a']:
                sig = 10*np.log10(sig); plt.ylabel('X(f)'); plt.xlabel('f␣
↪[Hz]'); #plt.ylim([-30:])
            elif sig.name in ['Gxx_spectro']:
                sns.heatmap(sig, cmap="jet"); plt.ylabel('f [Hz]'); plt.
↪xlabel('t [s]')
                continue
            if xrange != None:
                sig[xrange[0]:xrange[1]].plot();
```

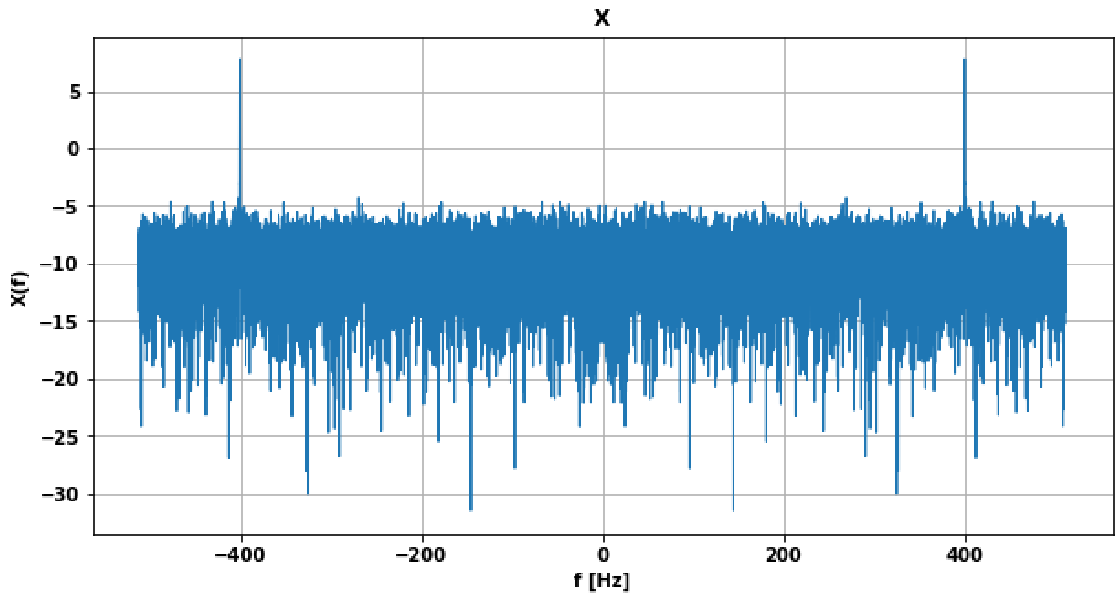
```
            else:
                sig.plot();
            plt.grid()

    #Useful functions to generate signals
    @staticmethod
    def sin(A,f,L,N):
        """
        Arguments:
            A: Amplitude
            f: Frequency of signal [hz]
            L: Total length of time [s]
            N: Number of points

        Returns:
            Series
        """
        t = np.linspace(start=0, stop=L, num=N, endpoint=True, dtype=float)
        return pd.Series(data=A*np.sin(2*np.pi*f*t),
                         index=t,
                         name='x')

    @staticmethod
    def randn_sig(L,N):
        """
        Arguments:
            L : Total length of time [s]
            N : Number of points

        Returns:
            Series
        """
        return pd.Series(data=np.random.randn(N,),
                         index=np.linspace(start=0, stop=L, num=N,␣
↪endpoint=True),
                         name='x')
```

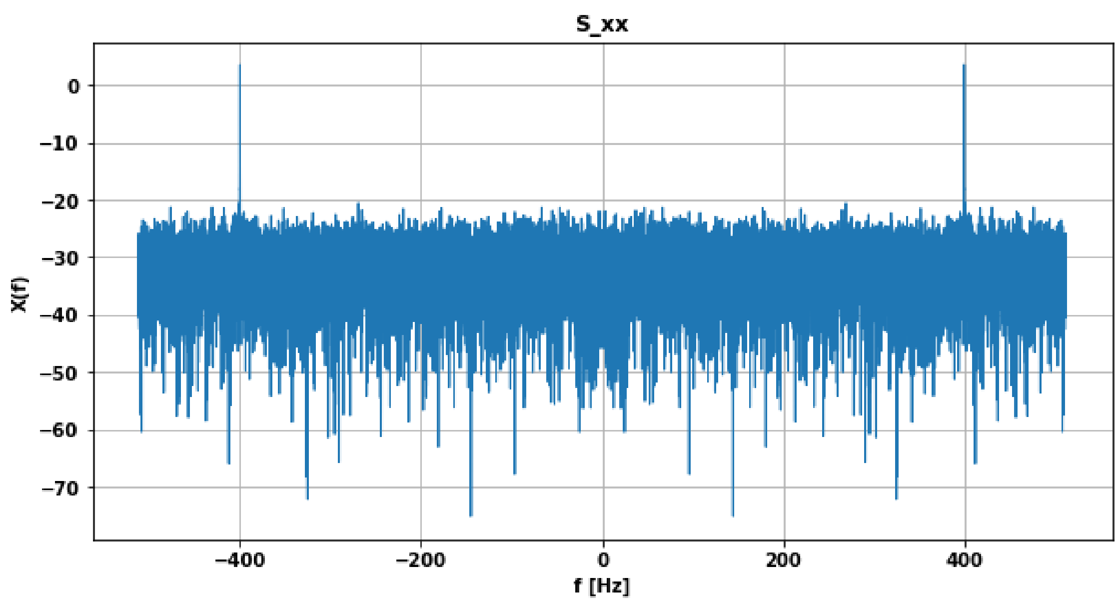
1 HW 3 - Testing

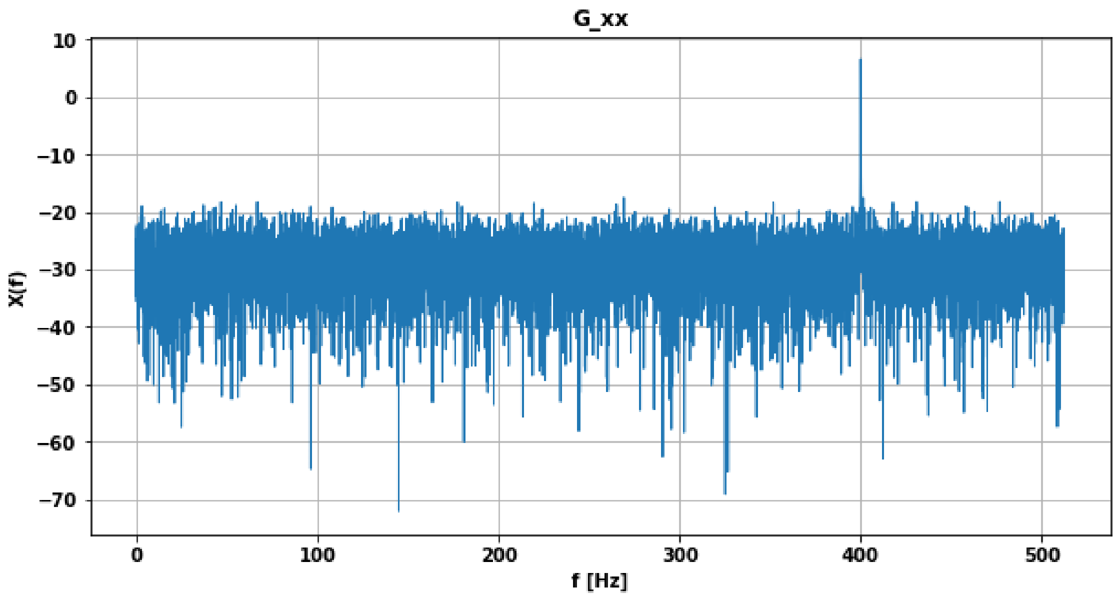
[25]:
```
#1.3.1
L = 16 # [s]
fs = 1024 #[hz]
N = int(L/(1/fs));  # generate the number of points based on the sampling␣
↪frequency which is higher than the actual signal frequency
f_sin = 400;
```

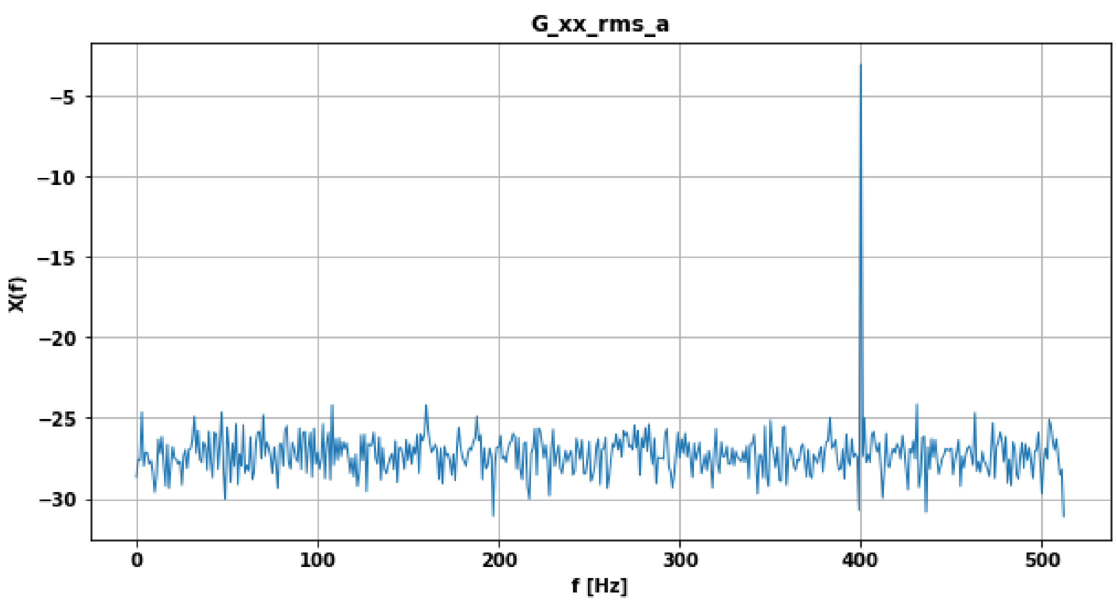
```
x = SignalTB.sin(A = 1, f = f_sin, N = N, L = L) + SignalTB.randn_sig(L=L,N=N)

s = SignalTB(x=x, fs=fs)
s.my_fft();
s.sd()
n_intervals = 16
a = s.rms_a(n_intervals = n_intervals)
b = s.linear_a(n_intervals = n_intervals)
c = s.time_a(n_intervals = n_intervals)
#s.spectrogram(n_intervals = 16);
s.plot_signals();
```

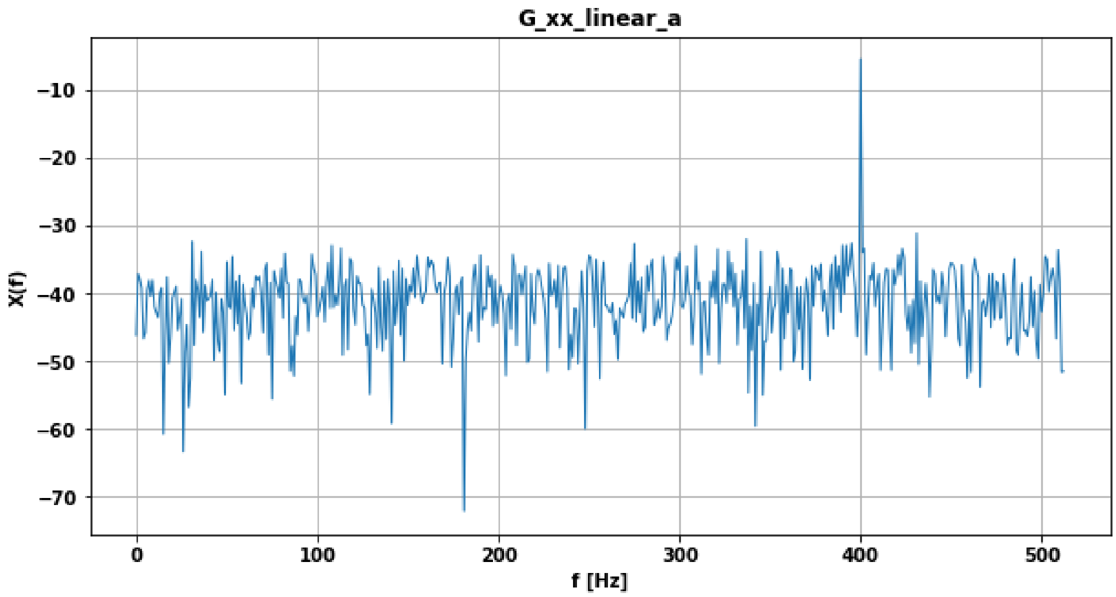


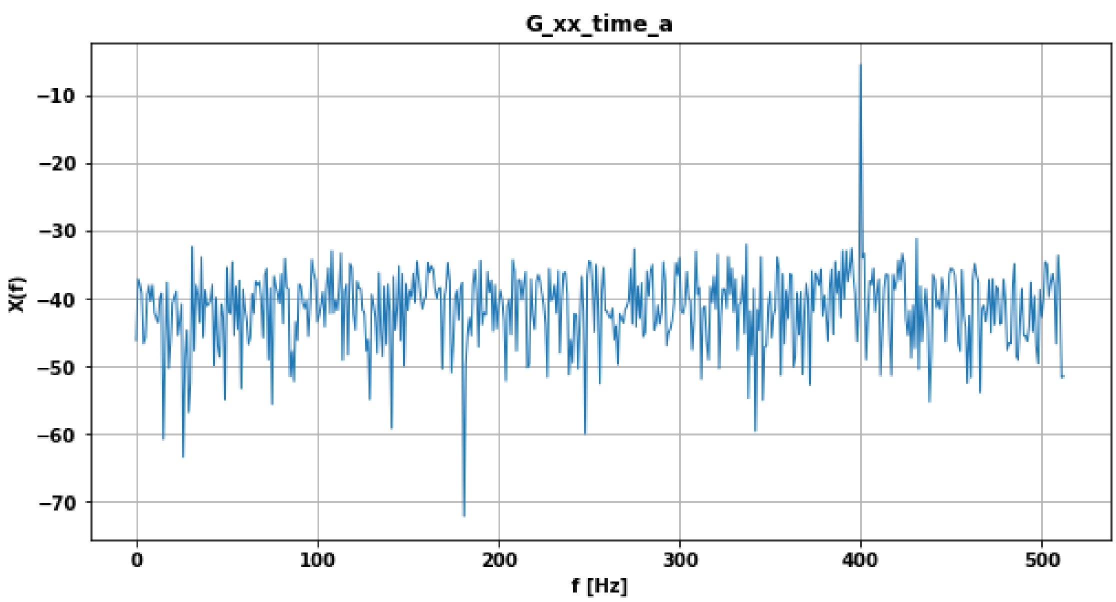

X
5
0
−5
−10
−15
−20
−25
−30
X(f)
−400
−200
0
200
400
f [Hz]


S_xx
0
−10
−20
−30
−40
−50
−60
−70
X(f)
−400
−200
0
200
400
f [Hz]


G_xx
10
0
−10
−20
−30
−40
−50
−60
−70
X(f)
0
100
200
300
400
500
f [Hz]

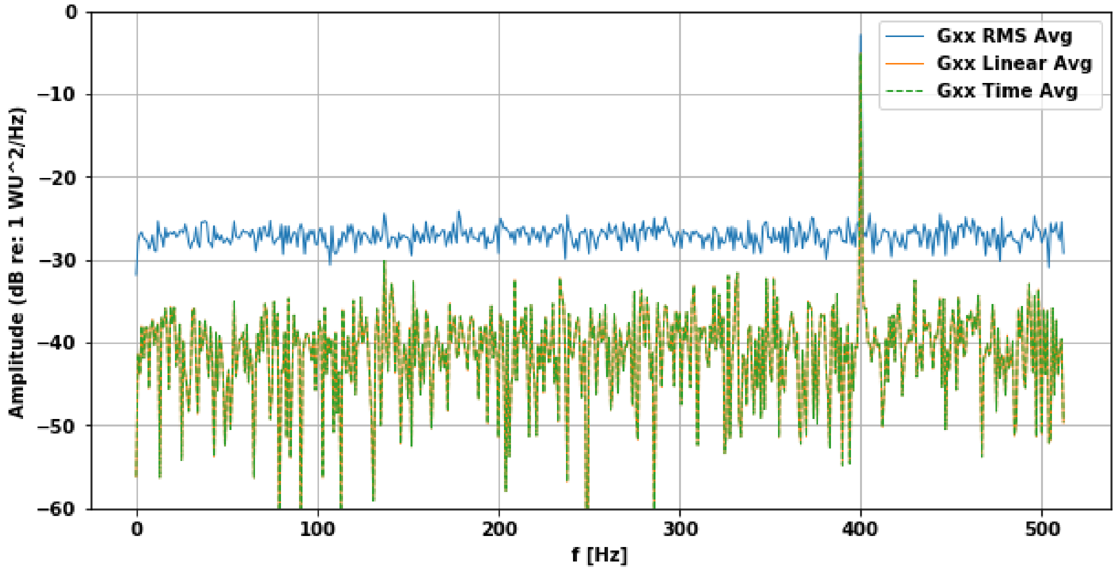

G_xx_rms_a
−5
−10
−15
−20
−25
−30
X(f)
0
100
200
300
400
500
f [Hz]





[18]:
```
fig = plt.figure(figsize=(10,5))
for myplot in [a,b]:
    plt.plot(10*np.log10(np.abs(myplot)));
plt.plot(10*np.log10(np.abs(c)),'--');
plt.ylabel('Amplitude (dB re: 1 WU^2/Hz)'); plt.xlabel('f [Hz]');plt.
 ↪ylim([-60,0])
plt.legend(['Gxx RMS Avg','Gxx Linear Avg','Gxx Time Avg'])
```

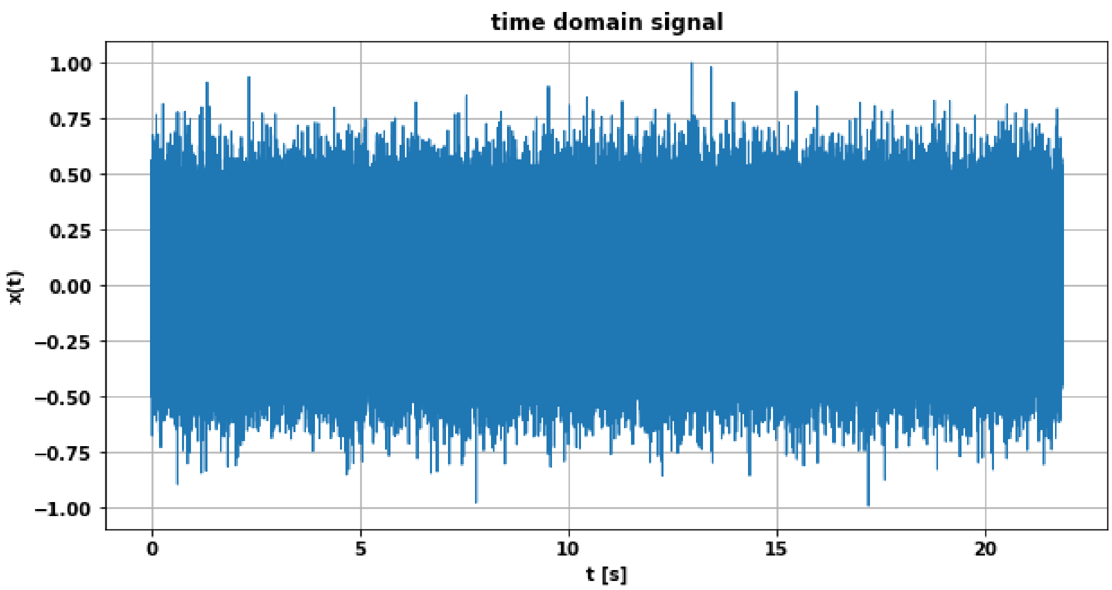
```
plt.grid()
fig.savefig('./plots/gxx_averages.png', dpi=300, bbox_inches='tight');
```

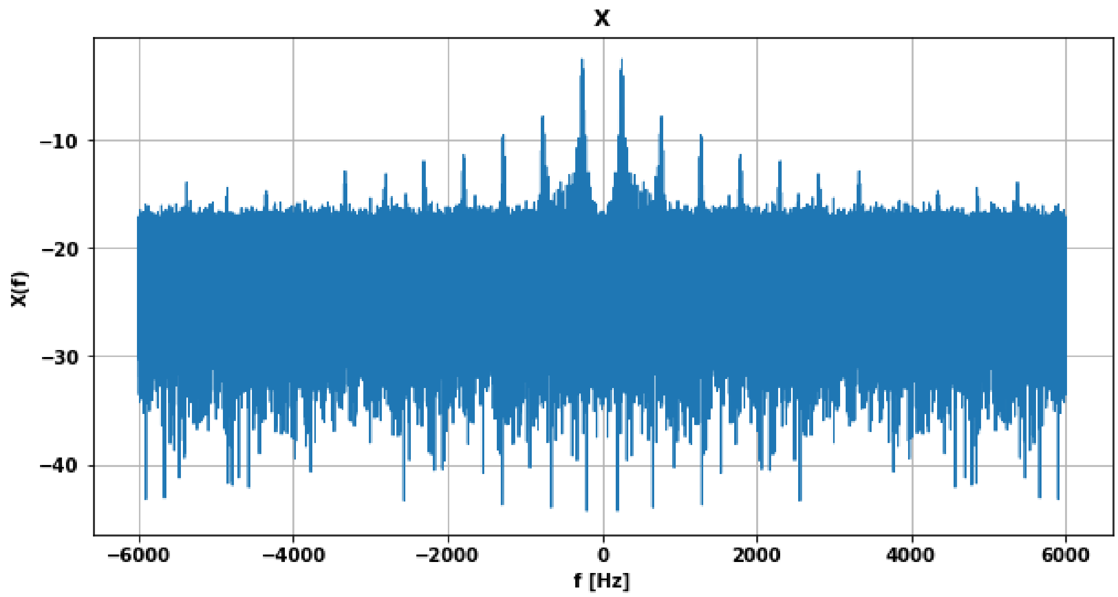


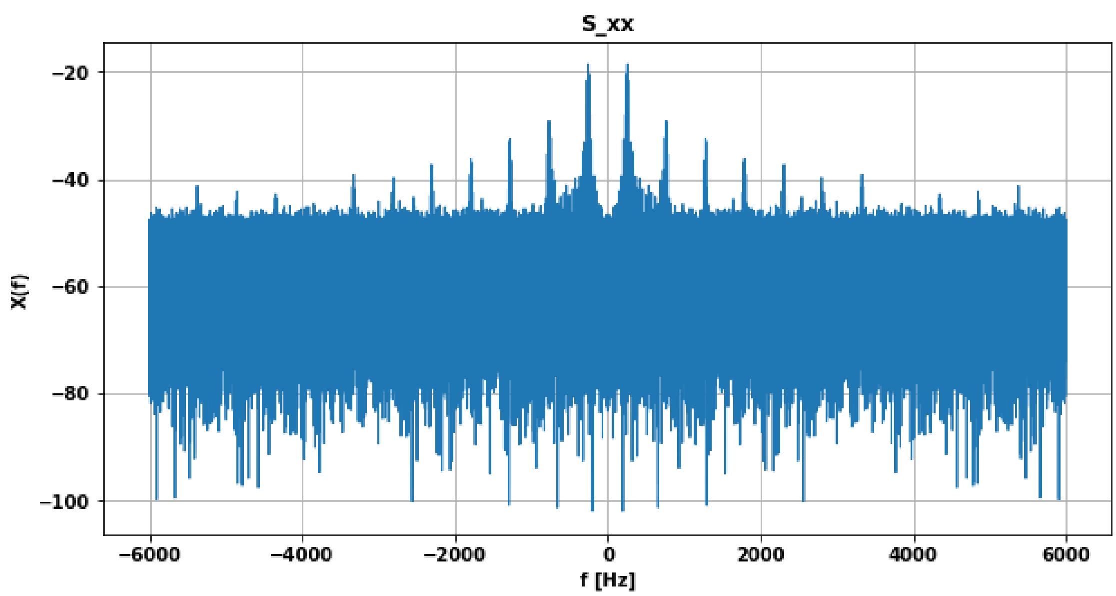
2 HW 3 - Analyzing Pulse Noise

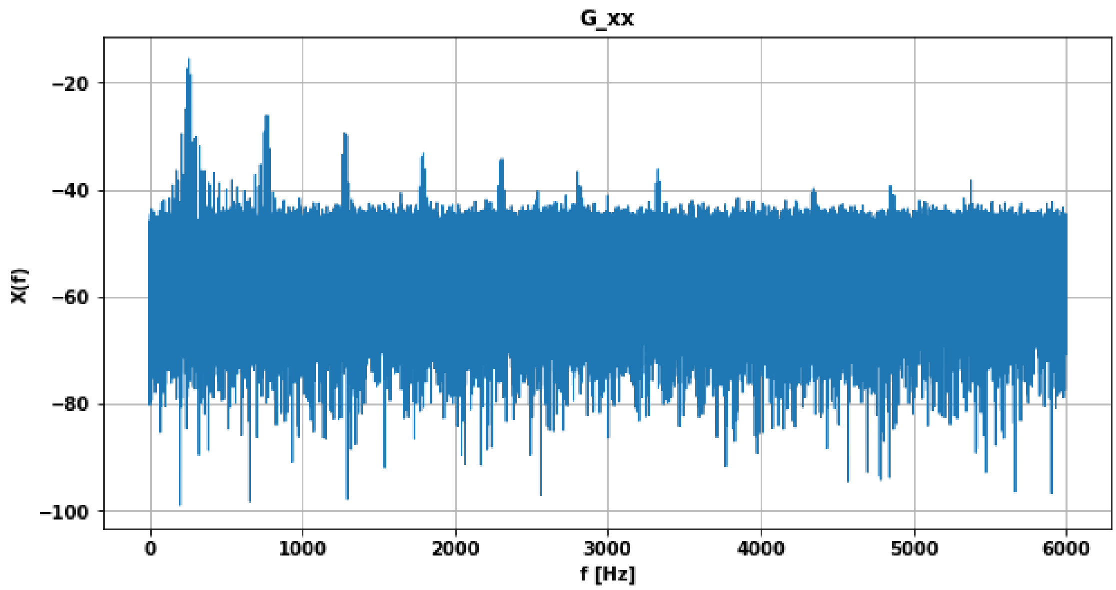
[66]:
```
data, fs = sf.read('./hw3_files/HW2_pulsenoise.wav');
N = len(data); display('N: ', N) # 1024
L = N/fs;      display('L: ', L)

x = pd.Series(data=data[:-(N%1024)],
              index=np.linspace(start=0, stop=L, num=N, endpoint=True)[:
 ↪-(N%1024)],
              name='time domain signal')
n_intervals = int(len(x)/1024); display('intervals: ', n_intervals) # make sure␣
 ↪to divide

#display(x)              # 0.085333s
s = SignalTB(x=x, fs=fs)
s.my_fft();
s.sd(); display('max freq',s.gxx.idxmax())
a = s.rms_a(n_intervals = n_intervals)
b = s.linear_a(n_intervals = n_intervals)
c = s.time_a(n_intervals = n_intervals)
#s.spectrogram(n_intervals = 16);
s.plot_signals();
```

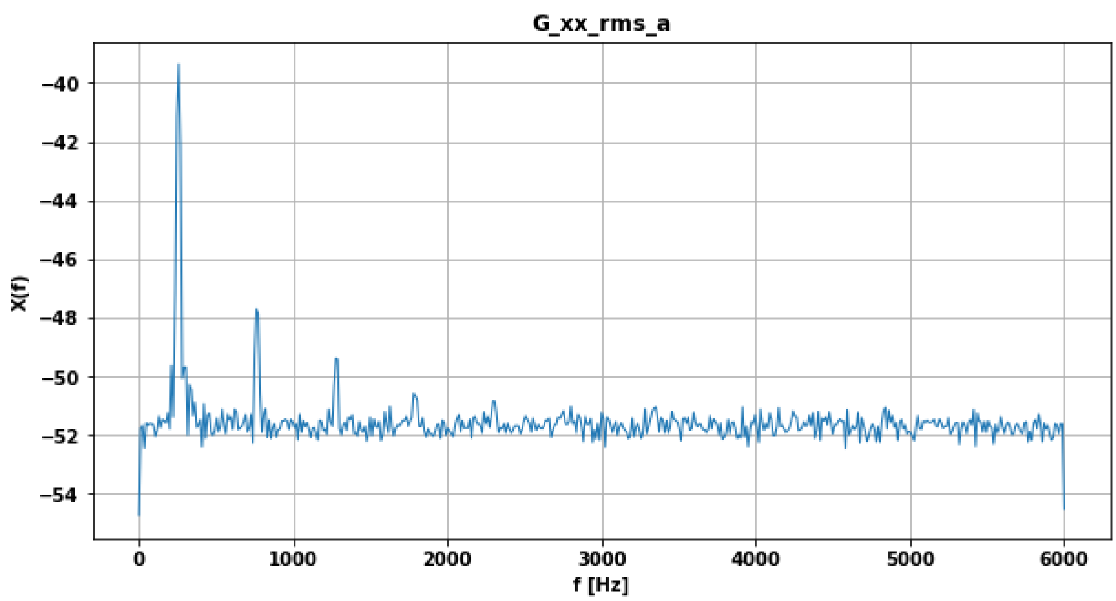
```
'N: '

262196

'L: '

21.849666666666668

'intervals: '

256

'max freq'

257.8125
```

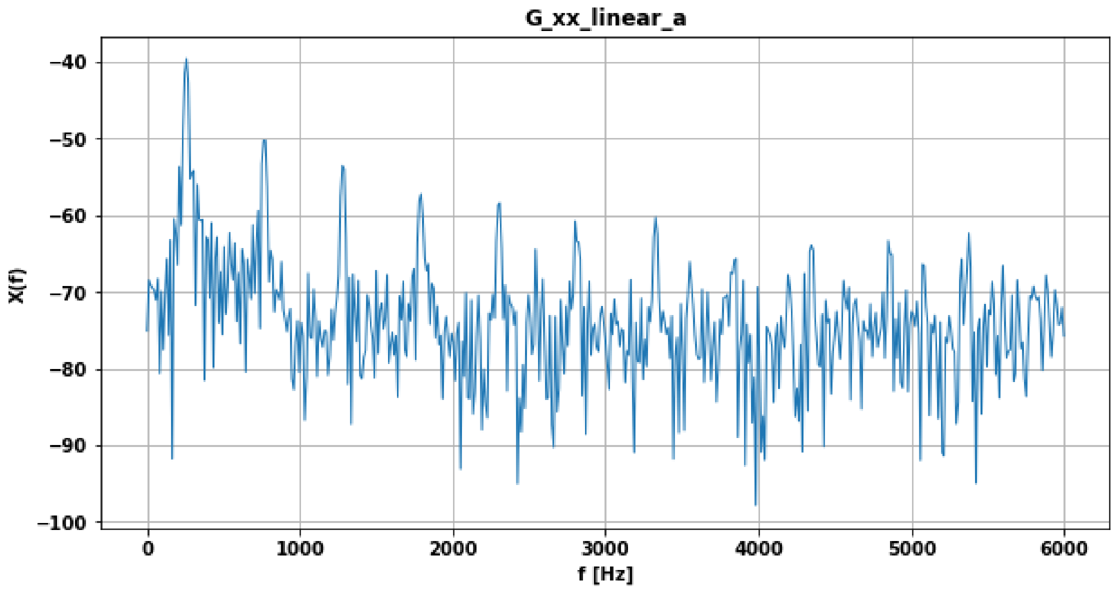


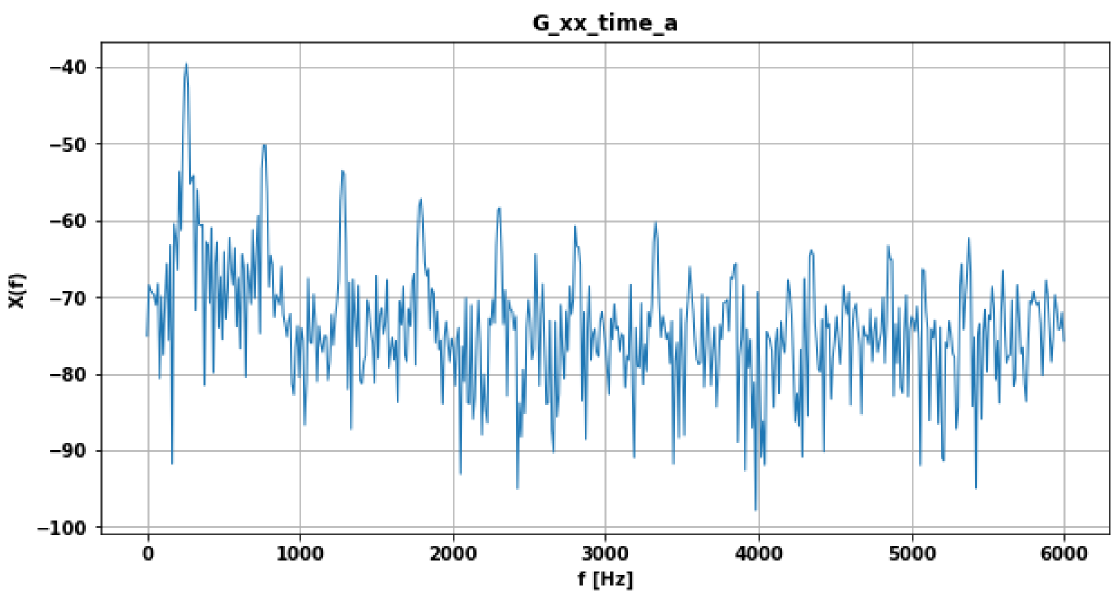

X
−10
−20
−30
−40
X(f)
−6000
−4000
−2000
0
2000
4000
6000
f [Hz]


S_xx
−20
−40
−60
−80
−100
X(f)
−6000
−4000
−2000
0
2000
4000
6000
f [Hz]


G_xx
−20
−40
−60
−80
−100
X(f)
0
1000
2000
3000
4000
5000
6000
f [Hz]

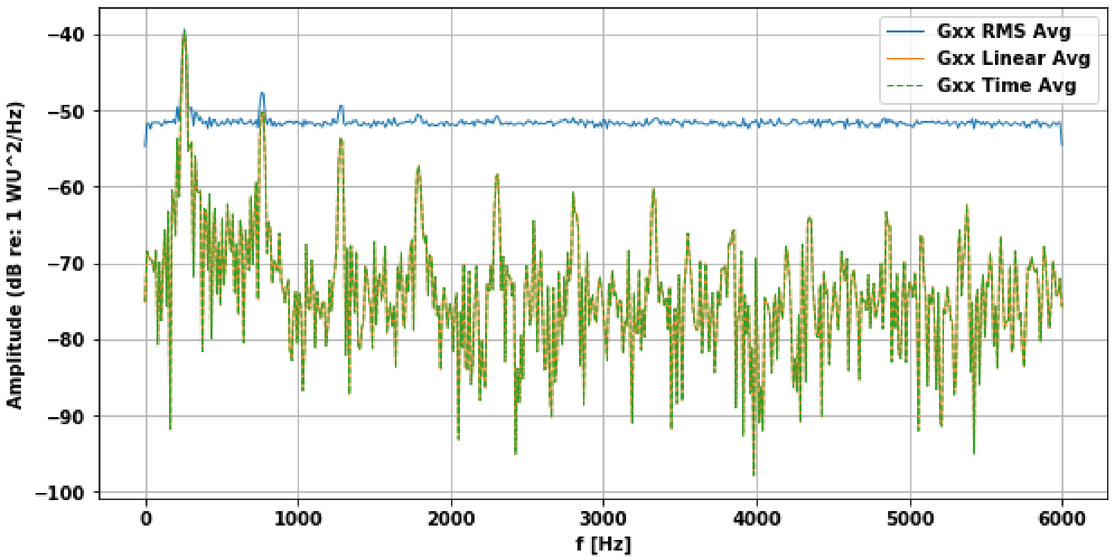

G_xx_rms_a
−40
−42
−44
−46
−48
−50
−52
−54
X(f)
0
1000
2000
3000
4000
5000
6000
f [Hz]





[67]:
```
fig = plt.figure(figsize=(10,5))
for myplot in [a,b]:
    plt.plot(10*np.log10(np.abs(myplot)));
plt.plot(10*np.log10(np.abs(c)),'--');
plt.ylabel('Amplitude (dB re: 1 WU^2/Hz)'); plt.xlabel('f [Hz]');#plt.
↪xlim([250,260])
plt.legend(['Gxx RMS Avg','Gxx Linear Avg','Gxx Time Avg'])
```

```python
plt.grid()
fig.savefig('./plots/hw3_2_gxx_averages.png', dpi=300, bbox_inches='tight');
```



[ ]:

```python

```