【チェック問題：解答】

(1) $a_n = c$

(2) $a_n = nc$

(3) $a_n = c^n$

(4) $a_n = \begin{cases} \dfrac{c^{n+1}-c}{c-1} & (c \neq 1), \\ n & (c = 1). \end{cases}$

(5) $a_n = c$

(6) $\begin{cases} \text{解はない} & (c \neq 0), \\ a_n = 0 & (c = 0). \end{cases}$

【おまけ：入試問題 解答】

(1)

$$
\begin{aligned}
(\text{左辺}) &= \frac{k}{k+l-1} \cdot \frac{2k-1}{2k}\, a_k a_l + \frac{l}{k+l-1}\, a_k \cdot \frac{2l-1}{2l}\, a_l \\
&= a_k a_l = (\text{右辺})
\end{aligned}
$$

(2)

ある正の整数mについて，　$S_m = \displaystyle\sum_{k=1}^{m} a_k a_{m-k+1} = 1$　とすると，(1) より

$$
\begin{aligned}
1 = S_m &= \sum_{k=1}^{m} \left(\frac{k}{m}\, a_{k+1} a_{m-k+1} + \frac{m-k+1}{m}\, a_k a_{m-k+2} \right) \\
&= \sum_{k=1}^{m-1} \frac{k}{m}\, a_{k+1} a_{m-k+1} + a_{m+1} a_1 + a_1 a_{m+1} + \sum_{k=2}^{m} \frac{m-k+1}{m}\, a_k a_{m-k+2} \\
&= a_1 a_{m+1} + \sum_{k=2}^{m} \frac{k-1}{m}\, a_k a_{m-k+2} + \sum_{k=2}^{m} \frac{m-k+1}{m}\, a_k a_{m-k+2} + a_{m+1} a_1 \\
&= a_1 a_{m+1} + \sum_{k=2}^{m} a_k a_{m-k+2} + a_{m+1} a_1 \\
&= \sum_{k=1}^{m+1} a_k a_{(m+1)-k+1} \\
&= S_{m+1}
\end{aligned}
$$

$S_1 = a_1 a_1 = 1$ なので、すべての正の整数 m に対して $S_m = 1$ が成り立つ。