

[Axiomatic Set Theory]

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\section*{Axiomatic Set Theory}
\subsection*{Domain} Mathematical Logic \& Foundations
\subsection*{Subfield} Set Theory
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\subsection*{Definition}
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Axiomatic Set Theory is a branch of mathematical logic that formalizes the concept of sets through a set

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\subsection*{Core Principles}
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\begin{itemize}
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- \item \textbf{Axiom of Extensionality:} Two sets are equal if they have the same elements.
- \item \textbf{Axiom of Empty Set:} There exists a set with no elements, denoted as \varnothing .
- \item \textbf{Axiom of Pairing:} For any two sets A and B , there exists a set that contains exactly A and B .
- \item \textbf{Axiom of Union:} For any set A , there exists a set containing all elements of the elements of A .

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\end{itemize}
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\subsection*{Key Formulas or Symbolic Representations}
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\begin{align*}
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& \text{Let } A = \{ x \in U \mid P(x) \} \text{ denote the set of all elements } x \text{ in } U \text{ such that } P(x) \text{ holds.} \\
& A \cap B = \{ x \mid x \in A \text{ and } x \in B \} \\
& A \cup B = \{ x \mid x \in A \text{ or } x \in B \}
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\end{align*}
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\subsection*{Worked Example}
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Consider the sets $A = \{1, 2\}$ and $B = \{2, 3\}$. The intersection $A \cap B = \{2\}$ and the union $A \cup B = \{1, 2, 3\}$.

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\subsection*{Common Pitfalls}
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- Confusing the concepts of subsets and elements, where $a \in A$ does not imply $\{a\} \subseteq A$.
- Misapplying the Axiom of Choice in contexts where it is not necessary or relevant.

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\subsection*{Connections}
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Axiomatic Set Theory serves as the foundation for various mathematical disciplines, including topology, algebra, and analysis.

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\subsection*{Further Reading}
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- Cohen, P. J. (1963). *Set Theory and the Continuum Hypothesis*.
- Jech, T. (2003). *Set Theory*.
- Halmos, P. R. (1960). *Naive Set Theory*.