

Basic Set Operations (union, intersection, complement) ID: MATH.1.2 Domain: Foundations & Preliminaries
Topic: Set Theory & Foundations

[Axiomatic Set Theory]

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\section*{Axiomatic Set Theory}
\subsection*{Domain} Mathematical Logic & Foundations
\subsection*{Subfield} Set Theory
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\subsection*{Definition}
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Axiomatic Set Theory is a formal framework for defining sets and their properties through a series of axioms.

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\subsection*{Core Principles}
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\begin{itemize}
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 \item Axiom of Extensionality: Two sets are equal if they have the same elements.

 \item Axiom of Pairing: For any two sets, there exists a set that contains exactly those two sets.

 \item Axiom of Union: For any set, there exists a set that contains all elements of the sets in that set.

 \item Axiom of Power Set: For any set, there exists a set of all its subsets.

 \item Axiom of Infinity: There exists a set that contains the empty set and is closed under the operation of successor.

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\end{itemize}
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\subsection*{Key Formulas or Symbolic Representations}
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\begin{align*}
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$A = \{x \mid P(x)\}$ \text{ (Set builder notation)} \\\

$A \subseteq B$ \text{ if } \forall x (x \in A \rightarrow x \in B) \\\

$\mathcal{P}(A)$ \text{ (Power set of } A\text{)}

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\end{align*}
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\subsection*{Worked Example}
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Consider the set $A = \{1, 2, 3\}$. By the Axiom of Pairing, one can form the set $B = \{A, \{A\}\}$.

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\subsection*{Common Pitfalls}
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Students may confuse the Axiom of Extensionality with mere compositional understanding, mistakenly believing that sets are defined by their elements alone.

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\subsection*{Connections}
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Axiomatic Set Theory underpins various branches of mathematics including algebra and topology. It also provides the foundation for modern logic.

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\subsection*{Further Reading}
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For a thorough examination of Axiomatic Set Theory, refer to "Set Theory: An Introduction to Independence Proofs" by Kenneth Kunen.