

Formal Systems and Axiomatic Methods ID: MATH.1.1 Domain: Foundations & Preliminaries Topic: Mathematical Logic & Proof Techniques

[Axiomatic Set Theory]

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\section*{Axiomatic Set Theory}
\subsection*{Domain} Mathematical Logic \& Foundations
\subsection*{Subfield} Set Theory

\subsection*{Definition}
Axiomatic Set Theory is a branch of mathematical logic that formalizes the concept of sets through a series of axioms. It provides a foundation for all of mathematics by defining what constitutes a set and how sets can be manipulated.

\subsection*{Core Principles}
\begin{itemize}
    \item \textbf{Axiom of Extensionality:} Two sets are equal if they have the same elements.
    \item \textbf{Axiom of Empty Set:} There exists a set with no elements, denoted as  $\varnothing$ .
    \item \textbf{Axiom of Pairing:} For any two sets  $a$  and  $b$ , there exists a set that contains exactly  $\{a, b\}$ .
    \item \textbf{Axiom of Union:} For any set  $A$ , there exists a set containing all elements of the elements of  $A$ .
\end{itemize}

\subsection*{Key Formulas or Symbolic Representations}
\begin{aligned}
& \text{\text{Let } } A = \{ x \mid P(x) \} \text{ denote the set of all elements } x \text{ in } U \text{ such that } P(x) \text{ is true.} \\
& \text{and } A \cap B = \{ x \mid x \in A \text{ and } x \in B \} \\
& \text{or } A \cup B = \{ x \mid x \in A \text{ or } x \in B \}
\end{aligned}

\subsection*{Worked Example}
Consider the sets  $A = \{1, 2\}$  and  $B = \{2, 3\}$ . The intersection  $A \cap B = \{2\}$  and the union  $A \cup B = \{1, 2, 3\}$ .
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\subsection*{Common Pitfalls}

- Confusing the concepts of subsets and elements, where $a \in A$ does not imply $a \subseteq A$.
- Misapplying the Axiom of Choice in contexts where it is not necessary or relevant.

\subsection*{Connections}

Axiomatic Set Theory serves as the foundation for various mathematical disciplines, including topology, analysis, and algebra.

\subsection*{Further Reading}

- Cohen, P. J. (1963). **Set Theory and the Continuum Hypothesis**.
- Jech, T. (2003). **Set Theory**.
- Halmos, P. R. (1960). **Naive Set Theory**.