\{}section*{Axiomatic Set Theory} \{}subsection*{Domain} Mathematical Logic \{} Foundations \{}subsection*{Subfield} Set Theory

\{}subsection*{Definition} Axiomatic Set Theory is a foundational framework in mathematics that formulates the principles of set membership and operations using a collection of axioms. The most widely accepted system is Zermelo-Fraenkel Set Theory (ZF), often supplemented with the Axiom of Choice (ZFC), which formalizes how sets are constructed and manipulated.

\{}subsection*{Core Principles} \{}begin{itemize} \{}item The Axiom of Extensionality: Two sets are equal if and only if they have the same elements. \{}item The Axiom of Regularity: Every non-empty set has a member that is disjoint from it. \{}item The Axiom of Power Set: For any set, there exists a set of all its subsets. \{}item The Axiom of Union: For any set, there exists a set that contains all the elements of the subsets of the original set. \{}end{itemize}

\{}subsection*{Key Formulas or Symbolic Representations} \{}begin{align*} \{} A = B \{}iff \{}forall x (x \{}in A \{}iff x \{}in B) \{}\{} \{} P(A) = \{}{ B \{}mid B \{}subseteq A \{}} \{}\{} \{} \{}bigcup A = \{}{ x \{}mid \{}exists B \{}in A (x \{}in B) \{}} \{}end{align*}

\{}subsection*{Worked Example} Consider the set \{}(A = \{}{1, 2, 3\{}} \{}). The power set \{}(P(A) \{}) consists of all subsets of \{}(A \{}): \{}[P(A) = \{}{\{}emptyset, \{}{1\{}}, \{}{2\{}}, \{}{3\{}}, \{}{1, 2\{}}, \{}{1, 3\{}}, \{}{2, 3\{}}, \{}{1, 2, 3\{}}\{}} \{}] This construction illustrates the Axiom of Power Set.

\{}subsection*{Common Pitfalls} Learners often confuse the concepts of a set and its elements, mistakenly believing that a set can contain itself as an element, which is prohibited in standard set theory.

\{}subsection*{Connections} Axiomatic Set Theory operates as the foundation for various branches of mathematics, notably in areas such as topology, algebra, and analysis, where the concept of a set underpins the structure of mathematical objects.

\{}subsection*{Further Reading} For comprehensive coverage, see: - Enderton, H. B. (1977). \{}textit{Elements of Set Theory}. Academic Press. - Jech, T. J. (2003). \{}textit{Set Theory: The Third Millennium Edition, Revised and Expanded}. Springer.