

Predicate Logic ID: MATH.1.1 Domain: Foundations & Preliminaries Topic: Mathematical Logic & Proof Techniques

## [Axiomatic Set Theory]

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\section*[Axiomatic Set Theory]
\subsection*[Domain] Mathematical Logic \& Foundations
\subsection*[Subfield] Set Theory

\subsection*[Definition]
Axiomatic Set Theory is a formal framework for set theory that defines set membership and operations th

\subsection*[Core Principles]
\begin{itemize}
\item Axiom of Extensionality: Two sets are equal if they have the same elements.
\item Axiom of Pairing: For any two sets, there exists a set containing exactly those two sets.
\item Axiom of Union: For any set, there exists a set that is the union of its elements.
\item Axiom of Power Set: For any set, there exists a set of all of its subsets.
\item Axiom of Infinity: There exists a set that contains the natural numbers.
\end{itemize}

\subsection*[Key Formulas or Symbolic Representations]
\begin{align*}
& \text{Extensionality: } \forall A \forall B (\forall x (x \in A \iff x \in B) \implies A = B \quad \text{r} \\
& \text{Power Set: } \mathcal{P}(A) = \{ B \mid B \subseteq A \}
\end{align*}

\subsection*[Worked Example]
Consider the set $ A = \{1, 2\} $. By the Axiom of Pairing, the set $ \{ A, \{A\} \} $ is formed containi

\subsection*[Common Pitfalls]
- Confusing the notion of a set with that of its elements.
- Assuming the existence of sets without reference to the axioms that guarantee them.
- Misunderstanding the concept of infinite sets and their construction.

\subsection*[Connections]
Axiomatic Set Theory serves as the foundation for much of modern mathematics, linking disparate fields t

\subsection*[Further Reading]
- Kunen, K. (1980). *Set Theory: An Introduction to Independence*.
- Jech, T. (2003). *Set Theory*.
- Halmos, P. R. (1960). *Naive Set Theory*.
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