

Propositional Logic ID: MATH.1.1 Domain: Foundations & Preliminaries Topic: Mathematical Logic & Proof Techniques

[Axiomatic Set Theory]

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\section*{Axiomatic Set Theory}
\subsection*{Domain} Mathematical Logic & Foundations
\subsection*{Subfield} Set Theory
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\subsection*{Definition}

Axiomatic set theory is a branch of mathematical logic that formalizes the concept of set through a collection of axioms.

\subsection*{Core Principles}

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\begin{itemize}
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- \item **Axiom of Extensionality**: Two sets are equal if they have the same elements.
- \item **Axiom of Pairing**: For any two sets, there exists a set that contains exactly those two sets.
- \item **Axiom of Union**: For any set, there exists a set that contains all elements of the elements of the set.
- \item **Axiom of Infinity**: There exists a set that contains the empty set and is closed under the operation of taking unions of pairs.
- \item **Axiom of Power Set**: For any set, there exists a set of all its subsets.

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\end{itemize}
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\subsection*{Key Formulas or Symbolic Representations}

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\begin{aligned}
& \text{Let } x, y \in \mathcal{P}(A) \implies x = y \text{ if } \forall z (z \in x \iff z \in y) \\
& \{x, y\} = \{z \mid z = x \lor z = y\} \quad \text{(Pairing)} \\
& \bigcup A = \{x \mid \exists y (x \in y \land y \in A)\} \quad \text{(Union)} \\
& \mathcal{P}(A) = \{B \mid B \subseteq A\} \quad \text{(Power Set)}
\end{aligned}
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\subsection*{Worked Example}

Consider the set $A = \{1, 2\}$. According to the Axiom of Pairing, the set $\{1, 2\}$ can be constructed as follows:

\subsection*{Common Pitfalls}

- Confusing the notion of a set with its elements, leading to incorrect applications of the Axiom of Extensionality.
- Misapplying axioms such as assuming the existence of infinite sets without using the Axiom of Infinity.
- Overlooking the distinction between a set and the property defining its members.

\subsection*{Connections}

Axiomatic set theory is foundational to various domains in mathematics, including number theory, topology, and analysis.

\subsection*{Further Reading}

- Cohen, P. J. (1963). *Set Theory and the Continuum Hypothesis.*
- Zermelo, E. (1908). "Über die Grundlagen der Mengenlehre." *Mathematische Annalen.*
- Enderton, H. B. (1977). *Elements of Set Theory.*