

[Axiomatic Set Theory]

\section*{Axiomatic Set Theory}

\subsection*{Domain} Mathematical Logic \& Foundations

\subsection*{Subfield} Set Theory

\subsection*{Definition}

Axiomatic Set Theory is a formal framework for set theory that defines set membership and operations th

\subsection*{Core Principles}

\begin{itemize}

\item Axiom of Extensionality: Two sets are equal if they have the same elements.

\item Axiom of Pairing: For any two sets, there exists a set containing exactly those two sets.

\item Axiom of Union: For any set, there exists a set that is the union of its elements.

\item Axiom of Power Set: For any set, there exists a set of all of its subsets.

\item Axiom of Infinity: There exists a set that contains the natural numbers.

\end{itemize}

\subsection*{Key Formulas or Symbolic Representations}

\begin{align*}

& \text{Extensionality: } \forall A \forall B \left(\forall x (x \in A \iff x \in B) \implies A = B \right)

& \text{Power Set: } \mathcal{P}(A) = \{ B \mid B \subseteq A \}

\end{align*}

\subsection*{Worked Example}

Consider the set $A = \{1, 2\}$. By the Axiom of Pairing, the set $\{A, \{A\}\}$ is formed contain

\subsection*{Common Pitfalls}

- Confusing the notion of a set with that of its elements.
- Assuming the existence of sets without reference to the axioms that guarantee them.
- Misunderstanding the concept of infinite sets and their construction.

\subsection*{Connections}

Axiomatic Set Theory serves as the foundation for much of modern mathematics, linking disparate fields

\subsection*{Further Reading}

- Kunen, K. (1980). *Set Theory: An Introduction to Independence*.
- Jech, T. (2003). *Set Theory*.
- Halmos, P. R. (1960). *Naive Set Theory*.