

[Axiomatic Set Theory]

\section*{Axiomatic Set Theory}

\subsection*{Domain} Mathematical Logic & Foundations

\subsection*{Subfield} Set Theory

\subsection*{Definition}

Axiomatic set theory is a branch of mathematical logic that formalizes the concept of set through a collection of axioms.

\subsection*{Core Principles}

\begin{itemize}

\item **Axiom of Extensionality**: Two sets are equal if they have the same elements.

\item **Axiom of Pairing**: For any two sets, there exists a set that contains exactly those two sets.

\item **Axiom of Union**: For any set, there exists a set that contains all elements of the elements of the set.

\item **Axiom of Infinity**: There exists a set that contains the empty set and is closed under the operation of taking the successor.

\item **Axiom of Power Set**: For any set, there exists a set of all its subsets.

\end{itemize}

\subsection*{Key Formulas or Symbolic Representations}

\begin{align*}

& \text{Let } x, y \in \mathcal{P}(A) \implies x = y \text{ if } \forall z (z \in x \iff z \in y) \quad \text{Axiom of Extensionality}

& \{x, y\} = \{z \mid z = x \vee z = y\} \quad \text{Axiom of Pairing}

& \bigcup A = \{x \mid \exists y (x \in y \wedge y \in A)\} \quad \text{Axiom of Union}

& \mathcal{P}(A) = \{B \mid B \subseteq A\} \quad \text{Axiom of Power Set}

\end{align*}

\subsection*{Worked Example}

Consider the set $A = \{1, 2\}$. According to the Axiom of Pairing, the set $\{1, 2\}$ can be constructed.

\subsection*{Common Pitfalls}

- Confusing the notion of a set with its elements, leading to incorrect applications of the Axiom of Extensionality.
- Misapplying axioms such as assuming the existence of infinite sets without using the Axiom of Infinity.
- Overlooking the distinction between a set and the property defining its members.

\subsection*{Connections}

Axiomatic set theory is foundational to various domains in mathematics, including number theory, topology, and algebra.

\subsection*{Further Reading}

- Cohen, P. J. (1963). *Set Theory and the Continuum Hypothesis*.
- Zermelo, E. (1908). "Über die Grundlagen der Mengenlehre." *Mathematische Annalen*.
- Enderton, H. B. (1977). *Elements of Set Theory*.