

Basic Set Operations (union, intersection, complement) ID: MATH.1.2 Domain: Foundations & Preliminaries
Topic: Set Theory & Foundations

[Axiomatic Set Theory]

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\section*[Axiomatic Set Theory]
\subsection*[Domain] Mathematical Logic & Foundations
\subsection*[Subfield] Set Theory
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\subsection*[Definition]

Axiomatic Set Theory is a formal framework for defining sets and their properties through a series of axioms.

\subsection*[Core Principles]

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\begin{itemize}
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- \item Axiom of Extensionality: Two sets are equal if they have the same elements.
- \item Axiom of Pairing: For any two sets, there exists a set that contains exactly those two sets.
- \item Axiom of Union: For any set, there exists a set that contains all elements of the sets in that set.
- \item Axiom of Power Set: For any set, there exists a set of all its subsets.
- \item Axiom of Infinity: There exists a set that contains the empty set and is closed under the operation of taking unions of pairs.

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\end{itemize}
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\subsection*[Key Formulas or Symbolic Representations]

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\begin{align*}
A &\coloneqq \{x \mid P(x)\} \text{ (Set builder notation)} \\
A \subseteq B &\text{ if } \forall x (x \in A \Rightarrow x \in B) \\
\mathcal{P}(A) &\text{ (Power set of } A\text{)}
\end{align*}
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\subsection*[Worked Example]

Consider the set \$ A = \{1, 2, 3\} \$. By the Axiom of Pairing, one can form the set \$ B = \{A, \{A\}\} \$.

\subsection*[Common Pitfalls]

Students may confuse the Axiom of Extensionality with mere compositional understanding, mistakenly believing that

\subsection*[Connections]

Axiomatic Set Theory underpins various branches of mathematics including algebra and topology. It also provides a foundation for logic and computer science.

\subsection*[Further Reading]

For a thorough examination of Axiomatic Set Theory, refer to "Set Theory: An Introduction to Independence Proofs" by Thomas Jech.