Determining the Required Travel Speed to Perceive Red Light as Green Using the Doppler Effect

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The colour green is characterized by wavelengths in the approximate range of 495 to 570 [1]. nanometres (nm). For the purpose of this discussion, we will utilize a representative wavelength of 500 nm.

The colour red is associated with wavelengths approximately in the range of 625 to 740 nanometres (nm). [2], For the purposes of this analysis, we will use a representative wavelength of 700 nm.

The formula for the relativistic Doppler effect is given by: [3]

$$\lambda_{obs} = \lambda_{src} \cdot \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$$

In the given context, it is established that the observed wavelength corresponds to the colour green, while the wavelength emitted by the source aligns with the colour red. The variable v represents the requisite velocity for a perceived alteration in wavelength, and c denotes the fundamental speed of light.

$$c = 299{,}792{,}458\ \mathrm{ms}^{-1}$$ [4]

Now, the given values can be substituted into the equation.

$$500\ nm = 700\ nm \cdot \sqrt{\frac{1 - \frac{v}{299{,}792{,}458\ \mathrm{ms}^{-1}}}{1 + \frac{v}{299{,}792{,}458\ \mathrm{ms}^{-1}}}}$$

To determine the value of v (velocity), the equation needs to be rearranged to isolate v . The equation is presented as follows:

$$v = \frac{\lambda_{src}^2 - \lambda_{obs}^2}{\frac{\lambda_{src}^2}{c} + \frac{\lambda_{obs}^2}{c}} = \frac{(700\ nm)^2 - (500\ nm)^2}{\frac{(700\ nm)^2}{299{,}792{,}458\ \mathrm{ms}^{-1}} + \frac{(500\ nm)^2}{299{,}792{,}458\ \mathrm{ms}^{-1}}} \approx 9{,}722999 \cdot 10^7$$

[1] https://web.archive.org/web/20171027012933/http://hyperphysics.phy-astr.gsu.edu/hbase/vision/specol.html
[2] Ibid
[3] The relativistic doppler effect formula
[4] https://www.britannica.com/science/speed-of-light

To simplify, we can divide the value$9{,}722999 \cdot 10^7$ by the speed of light and then multiply by 100% to obtain the percentage value representing the velocity needed for the observed wavelength shift.

$$\frac{9{,}722999 \cdot 10^7}{299792458} \cdot 100 \approx 32{,}43243$$

To perceive green light as red, one would need to travel at approximately 32.43% of the speed of light.