Comprehensive Integral Calculus Cheat Sheet

By Magnus Hviid Sigurdsson

Basic Techniques

1. Power Rule

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$

where $n \neq -1$.

Examples:

1. $\int x^2 \, dx = \frac{x^3}{3} + C$

2. $\int x^{-1} \, dx = \ln|x| + C$

2. Integration by Substitution

$$\int f(g(x))g'(x) \, dx = \int f(u) \, du$$

where $u = g(x)$.

Examples:

1. $\int 2x \cos(x^2) \, dx = \sin(x^2) + C$

2. $\int e^{x^2} x \, dx = \frac{1}{2}e^{x^2} + C$

3. Integration by Parts

$$\int u \, dv = uv - \int v \, du$$

Examples:

1. $\int x \sin(x) \, dx = -x \cos(x) + \int \cos(x) \, dx = -x \cos(x) + \sin(x) + C$

2. $\int x^2 e^x \, dx = x^2 e^x - 2 \int x e^x \, dx$

Intermediate Techniques

4. Trigonometric Integrals

$$\int \sin(x)\,dx = -\cos(x) + C$$

$$\int \cos(x)\,dx = \sin(x) + C$$

Examples:

1. $\int \tan(x)\,dx = \ln|\sec(x)| + C$
2. $\int \sec^2(x)\,dx = \tan(x) + C$

5. Partial Fractions

$$\frac{A}{x-a} + \frac{B}{x-b} = \frac{A(x-b) + B(x-a)}{(x-a)(x-b)}$$

Examples:

1. $\int \frac{1}{x^2-1}\,dx = \int \left(\frac{1}{2(x-1)} - \frac{1}{2(x+1)} \right)\,dx = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$

2. $\int \frac{x+3}{x^2+x-6}\,dx = \int \left(\frac{1}{x-2} - \frac{1}{x+3} \right)\,dx$

6. Integration by Trigonometric Substitution

$$\sqrt{a^2 - x^2}, \quad \sqrt{x^2 + a^2}, \quad \sqrt{x^2 - a^2}$$

Examples:

1. $\int \frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\left(\frac{x}{3}\right) + C$

2. $\int \frac{dx}{\sqrt{x^2+4}} = \ln|x + \sqrt{x^2+4}| + C$

Advanced Techniques

7. Improper Integrals

$$\int_a^\infty f(x)\,dx, \quad \int_{-\infty}^b f(x)\,dx, \quad \int_{-\infty}^\infty f(x)\,dx$$

Examples:

1. $\int_1^\infty \frac{1}{x^2}\,dx = 1$

2. $\int_{-\infty}^\infty e^{-x^2}\,dx = \sqrt{\pi}$

8. Line Integrals

$$\int_C f(x, y)\, ds, \quad \int_C \mathbf{F}(\mathbf{r}) \cdot d\mathbf{r}$$

Examples:

1. $\int_C x\, dy + y\, dx$
2. $\int_C \langle y^2, x^2 \rangle \cdot d\mathbf{r}$

9. Double Integrals

$$\iint_R f(x, y)\, dA$$

Examples:

1. $\iint_R x^2 y\, dA$
2. $\iint_R e^{x+y}\, dA$

10. Triple Integrals

$$\iiint_V f(x, y, z)\, dV$$

Examples:

1. $\iiint_V x^2 y\, dV$
2. $\iiint_V z\, dV$