Basics of Quantum Physics

Magnus Hviid Sigurdson

Introduction to Quantum Physics

Quantum physics, also known as quantum mechanics, is the branch of physics that deals with the behavior of particles at the smallest scales, such as atoms and subatomic particles. It revolutionized our understanding of the physical world by introducing concepts such as wave-particle duality, quantization of energy, and uncertainty principles. In this document, we explore the basics of quantum physics, from foundational principles to key concepts.

Foundational Principles

1. Wave-Particle Duality

One of the central principles of quantum physics is wave-particle duality, which states that particles, such as electrons and photons, exhibit both wave-like and particle-like behavior depending on the experimental setup. This duality is described by wave functions, which represent the probability amplitudes of finding a particle at a given position and time.

Example: Consider the famous double-slit experiment, where electrons are fired one at a time toward a screen with two slits. When observed, electrons behave like particles, creating two distinct bands on the screen. However, when unobserved, they exhibit interference patterns characteristic of waves, indicating their wave-like nature.

2. Uncertainty Principle

The uncertainty principle, formulated by Werner Heisenberg, states that certain pairs of physical properties, such as position and momentum, cannot be simultaneously measured with arbitrary precision. There is a fundamental limit to the precision with which these complementary properties can be known.

Example: Suppose we try to measure the position and momentum of an electron simultaneously. The more precisely we measure its position, the less precisely we can know its momentum, and vice versa. This inherent uncertainty is a fundamental aspect of quantum physics.

3. Quantization of Energy

Quantization of energy refers to the discretization of energy levels in certain physical systems, such as atoms and molecules. In these systems, energy is quantized, meaning it can only take on discrete values rather than any arbitrary value. This concept is essential for understanding phenomena such as atomic spectra and electronic transitions.

Example: In the Bohr model of the hydrogen atom, electrons are restricted to certain quantized energy levels. When an electron transitions between these levels, it emits or absorbs photons with energies corresponding to the energy difference between the levels.

Key Concepts

4. Schrödinger Equation

The Schrödinger equation is the fundamental equation of quantum mechanics, describing how the wave function of a quantum system evolves over time. It is a partial differential equation that governs the behavior of particles at the microscopic scale.

Example: Consider a particle in a one-dimensional box with infinite potential barriers at the edges. The time-independent Schrödinger equation for this system is given by:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

where $\hbar$ is the reduced Planck constant, m is the mass of the particle, $V(x)$ is the potential energy function, $\psi(x)$ is the wave function, and E is the energy eigenvalue.

Calculation Rules:

- The Schrödinger equation is a second-order differential equation that describes the wave function of a quantum system.
- The eigenvalues of the Schrödinger equation correspond to the allowed energy levels of the system.
- Solutions to the Schrödinger equation provide information about the probability amplitudes of finding the particle in different states.

Calculation: Suppose we have a particle of mass m in a one-dimensional box of length L. The potential energy inside the box is zero, and outside the box, it is infinite. The Schrödinger equation for this system is:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$$

Solving this equation yields the energy eigenvalues E_n and corresponding wave functions $\psi_n(x)$ for each quantum state n.

5. Wave Functions

Wave functions are mathematical functions that describe the quantum state of a particle or system. They encode information about the probability amplitudes of different possible states of the system. The square of the wave function, $|\psi(x)|^2$, represents the probability density of finding the particle at a given position x.

Example: Consider a particle in a one-dimensional box. The wave function describing the particle's state within the box can be expressed as a superposition of stationary states, each corresponding to a quantized energy level.

Calculation Rules:

- Wave functions must be normalized, meaning that the integral of the square of the wave function over all space is equal to 1.
- The probability of finding the particle within a specific region of space is given by the integral of the square of the wave function over that region.

Calculation: Suppose we have a particle in a one-dimensional box of length L. The normalized wave function for the ground state ($n = 1$) is given by:

$$\psi_1(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

The probability of finding the particle in the first half of the box ($0 \leq x \leq \frac{L}{2}$) is then:

$$P = \int_0^{\frac{L}{2}} |\psi_1(x)|^2 \, dx$$

6. Quantum Operators

Quantum operators are mathematical entities that represent physical observables, such as position, momentum, and energy, in quantum mechanics. Operators act on wave functions to extract information about the system's properties through measurements.

Example: The position operator $\hat{x}$ acts on a wave function $\psi(x)$ to determine the position of a particle along the x-axis. Similarly, the momentum operator $\hat{p}$ acts on $\psi(x)$ to extract information about the particle's momentum.

Calculation Rules:

- Quantum operators are represented by Hermitian matrices in matrix mechanics or differential operators in wave mechanics.
- The expectation value of an observable corresponds to the average measurement obtained from multiple measurements of the same system.
- Commutation relations between operators provide information about the compatibility of measurements and the uncertainty principle.

Calculation: Consider a particle in a one-dimensional box. The position operator $\hat{x}$ and momentum operator $\hat{p}$ are given by:

$$\hat{x} = x$$

$$\hat{p} = -i\hbar\frac{d}{dx}$$

The expectation value of the position operator for the ground state wave function $\psi_1(x)$ is then:

$$\langle\hat{x}\rangle = \int_0^L \psi_1^*(x)\hat{x}\psi_1(x)\,dx$$

Conclusion

Quantum physics is a profound and fascinating field that has revolutionized our understanding of the universe. Its principles challenge our classical intuition and provide a deeper insight into the fundamental nature of reality. By exploring the basics of quantum physics, we gain a foundation for delving into more advanced topics and applications in various scientific disciplines.

References

1. Griffiths, D. J. (2005). *Introduction to Quantum Mechanics.* Pearson Education.

2. Sakurai, J. J., & Napolitano, J. (2017). *Modern Quantum Mechanics.* Cambridge University Press.