

XCPC 算法模板

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目录

1	图论	5
1.1	确定有限状态自动机最小化	5
1.2	拓扑排序	6
1.3	欧拉回路	8
1.4	最小生成树	10
1.5	Dijkstra 算法	11
1.6	SPFA 算法	13
1.7	一般图最大权匹配	14
1.8	有向图的传递闭包	19
1.9	最小树形图	20
1.10	支配树	22
2	树算法	24
2.1	限定距离的子树问题	24
2.2	树的欧拉序	26
2.3	倍增求 LCA	28
2.4	点分治	29
2.5	动态点分治	30
2.6	快速求树中与结点 x 距离不超过 k 的点权和	32
2.7	动态维护树中白色结点的最长距离	37
2.8	虚树	41
2.9	$O(1)$ -LCA	43
3	基础算法	44
3.1	Java 快速读入	44
3.2	C++ 快速读入	44
3.3	C++ 光速读入（交互版）	45
3.4	C++17 万能输出	46
3.5	C++20 万能输出	48
3.6	RMQ 算法	50
3.7	哈希表	50
3.8	哈希值组合	52
3.9	基数排序	52
3.10	快速离散化	54
3.11	带删除的优先队列	55
3.12	排列组合枚举	56
3.13	动态维护连续相同数字区间	57
3.14	二分查找	58
4	匹配算法	59
4.1	匈牙利算法	59
4.2	KM 算法	61
4.3	带花树算法	64

目录	2
5 高精度	67
5.1 大整数类	67
5.2 分数类	71
6 动态规划	72
6.1 斯坦纳树（点权）	72
6.2 斯坦纳树（边权）	75
6.3 插头 DP	78
6.4 最长上升子序列	80
7 莫队算法	81
7.1 莫队算法	81
7.2 回滚莫队	82
7.3 带修改莫队	83
8 数据结构	85
8.1 可修改优先队列	85
8.2 AVL Tree	88
8.3 TopTree	89
8.4 可持久化数组	96
8.5 四分树	98
8.6 Treap	102
8.7 link-cut-tree	105
8.8 link-cut-tree（指针）	107
8.9 link-cut-tree（边权）	111
8.10 link-cut-tree（维护子树）	114
8.11 可持久化 Treap	118
8.12 树链剖分	129
8.13 树链剖分求 LCA 和距离	132
8.14 树状数组	134
8.15 李超线段树	135
8.16 整数集合	137
8.17 可持久化整数集合	144
8.18 线段树	153
8.19 动态开点线段树	156
8.20 主席树	158
8.21 动态主席树	162
8.22 动态开点线段树（单点加-区间求和-合并）	166
8.23 可持久化线段树	168
8.24 线段树（历史最大值）	170
8.25 zkw 线段树（单点加-区间加-单点查询-区间求和）	173
8.26 SplayTree	175
8.27 可持久化并查集	177
8.28 KD-Tree	180
8.29 Euler-Tour-Tree	185
8.30 rope	189
8.31 pb-ds 平衡树	190

9 数学算法	190
9.1 SG 函数	190
9.2 自适应辛普森积分	191
9.3 高斯消元	191
9.4 稀疏矩阵的高斯消元	195
9.5 求解异或方程组	201
9.6 矩阵与状态转移	201
9.7 递推式求解	202
9.8 快速傅里叶变换	204
9.9 快速幂运算	205
9.10 莫比乌斯反演	209
9.11 逆元	210
9.12 欧拉函数	211
9.13 线性筛素数	212
9.14 三分求极值	212
9.15 多项式拟合（辛普森积分）	213
9.16 多项式拟合（泰勒展开）	215
9.17 雅可比方法	216
9.18 矩阵求逆	218
9.19 牛顿迭代求非线性方程组	220
9.20 QR 迭代	222
9.21 行列式计算	224
9.22 二元一次不定方程	225
10 网络流	226
10.1 Dinic	226
10.2 ISAP	228
10.3 HLPP	230
10.4 MCMF-spfa	232
10.5 MCMF-dijkstra	234
11 线性规划	236
11.1 线性规划	236
12 字符串算法	237
12.1 后缀树	237
12.2 扩展 KMP	243
12.3 AC 自动机	244
12.4 KMP 算法	246
12.5 Manacher 算法	247
12.6 后缀数组	248
12.7 后缀自动机	252
12.8 回文自动机	261
12.9 回文串 Border	263
12.10 双端回文自动机	264
12.11 非势能分析回文自动机	266
12.12 序列自动机	268

12.13 hash	269
12.14 LCT 维护隐式后缀树	270
12.15 区间本质不同子串个数	272
12.16 Lyndon 分解	277
12.17 后缀平衡树	278
12.18 模糊匹配	281
12.19 基于后缀自动机构建后缀树	284
13 人工智能	286
13.1 主成分分析	286
13.2 Adam 算法	288
13.3 Dyna-Q	289
13.4 kmeans 聚类	291

1 图论

1.1 确定有限状态自动机最小化

```

const int maxn = 110000, sigma_size = 20;
struct Automaton {
    int n, m, cur, G[maxn][sigma_size];
    int inq[maxn], cls[maxn];
    vector<int> from[maxn][sigma_size];
    unordered_set<int> equiv[maxn]; //equiv[i] 表示编号为 i 的等价类的集合
    void init(int n, int m, vector<bool> final) { //状态数、输入字符集大小、每个结点是否为终态
        //如果要多次使用该算法，需要在此处先将状态清空。
        this->n = n;
        this->m = m;
        this->cur = 2;
        for (int i = 0; i < n; ++i) {
            int st = final[i] ^ 1;
            equiv[st].insert(i);
            cls[i] = st;
        }
    }
    void add_edge(int x, int c, int y) { //添加一条从 x 指向 y 的边，字符为 c
        G[x][c] = y;
    }
    int minimize() { //调用该方法之前应该先将不可达状态从自动机中删除
        for (int i = 0; i < n; ++i)
            for (int c = 0; c < m; ++c)
                from[G[i][c]][c].push_back(i);
        queue<int> Q;
        Q.push(0); inq[0] = true;
        while (!Q.empty()) {
            int x = Q.front(); Q.pop(); //x 是一个等价类
            inq[x] = false;
            for (int c = 0; c < m; ++c) {
                unordered_map<int, vector<int>> par; //class -> set of states
                for (auto i : equiv[x]) for (auto u : from[i][c])
                    //if (cls[u] != x)
                    par[cls[u]].push_back(u);
                for (auto &[id, member] : par) if (member.size() != equiv[id].size()) {
                    int now = cur++;
                    for (auto y : member) {
                        equiv[id].erase(y);
                        equiv[now].insert(y);
                        cls[y] = now;
                    }
                    if (inq[id] || equiv[now].size() < equiv[id].size())
                        Q.push(now), inq[now] = true;
                }
            }
        }
    }
}

```

```

        else
            Q.push(id), inq[id] = true;
    }
}
}
return cur; //返回合并之后的总状态数
}
} am;
int main() {
    int b, m;
    scanf("%d %d", &b, &m);
    vector<bool> final(m, 0); final[0] = 1;
    am.init(m, b, move(final));
    for (int i = 0; i < m; ++i)
        for (int j = 0; j < b; ++j)
            am.add_edge(i, j, (i * b + j) % m);
    int sz = am.minimize();
    printf("%d 0\n", sz);
    printf("G");
    for (int i = 1; i < sz; ++i)
        printf(" B");
    printf("\n");
    for (int i = 0; i < sz; ++i) {
        for (int j = 0; j < b; ++j) {
            int first = *am.equiv[i].begin();
            printf("%d ", am.cls[(first * b + j) % m]);
        }
        printf("\n");
    }
    return 0;
}

```

1.2 拓扑排序

```

const int maxn = 110000;
int n, m, degree[maxn];
vector<int> G[maxn];
vector<int> toposort1() {
    queue<int> Q;
    for (int i = 1; i <= n; ++i)
        degree[i] = 0;
    for (int x = 1; x <= n; ++x)
        for (auto y : G[x])
            degree[y]++;
    for (int i = 1; i <= n; ++i) if (degree[i] == 0)
        Q.push(i);
    vector<int> res;

```

```

while (!Q.empty()) {
    int x = Q.front(); Q.pop();
    res.push_back(x);
    for (auto y : G[x]) {
        degree[y]--;
        if (degree[y] == 0)
            Q.push(y);
    }
}
return res;
}

vector<int> result;
int vis[maxn];
void dfs(int x) {
    vis[x] = true;
    for (auto y : G[x]) if (!vis[y])
        dfs(y);
    result.push_back(x);
}

vector<int> toposort2() { //必须保证是有向无环图
    memset(vis, 0, sizeof(vis));
    for (int i = 1; i <= n; ++i) if (!vis[i])
        dfs(i);
    reverse(result.begin(), result.end());
    return result;
}

int main() { //uva 10305
    //freopen("in.txt", "r", stdin);
    while (scanf("%d %d", &n, &m) == 2) {
        if (n == 0 && m == 0)
            break;
        for (int i = 1; i <= n; ++i)
            G[i].clear();
        for (int i = 0; i < m; ++i) {
            int x, y;
            scanf("%d %d", &x, &y);
            G[x].push_back(y);
        }
        auto ans = toposort2();
        printf("%d", ans[0]);
        for (int i = 1; i < n; ++i)
            printf(" %d", ans[i]);
        printf("\n");
    }
    return 0;
}

```


1.3 欧拉回路

/*

1. 无向图有欧拉路径的充要条件是：图联通且至多有两个奇点。

若不存在奇点，则有欧拉回路。

2. 有向图有欧拉路径的充要条件是：至多有两个点的入度不等于出度，且必须是一个点的出度比入度大 1（起点），一个点的入度比出度大 1（终点），且忽略边的方向后图必须连通。

*/

```
const int maxn = 110000;
```

```
int t, n, m;
```

```
namespace direct_graph {
```

```
    //若有多组数据应将 G 和 result 清空，调用完 euler 之后 G 中的边会被删除
```

```
    vector<int> G[maxn];
```

```
    vector<pair<int, int>> result;
```

```
    void dfs(int x) {
```

```
        while (G[x].size()) {
```

```
            auto y = G[x].back();
```

```
            G[x].pop_back();
```

```
            dfs(y);
```

```
            result.emplace_back(x, y);
```

```
        }
```

```
    }
```

```
    bool euler(int s) {
```

```
        dfs(s);
```

```
        reverse(result.begin(), result.end());
```

```
        if (result.size() != m || (result.size() && result.back().second != s))
```

```
            return false; //若只要求有欧拉道路则: if (result.size() != m) return false;
```

```
        for (int i = 1; i < result.size(); ++i) if (result[i - 1].second != result[i].first)
```

```
            return false;
```

```
        return true;
```

```
    }
```

```
    void solve() {
```

```
        multimap<pair<int, int>, int> id;
```

```
        for (int i = 1; i <= m; ++i) {
```

```
            int x, y;
```

```
            scanf("%d %d", &x, &y);
```

```
            G[x].push_back(y);
```

```
            id.emplace(make_pair(x, y), i);
```

```
        }
```

```
        int s = 1; while (G[s].empty() && s <= n) s++;
```

```
        if (!euler(s)) {
```

```
            puts("NO");
```

```
            return;
```

```
        }
```

```
        else {
```

```
            puts("YES");
```

```

        for (auto pr : result) {
            int x = pr.first, y = pr.second;
            auto iter = id.find({ x, y });
            if (iter == id.end()) {
                iter = id.find({ y, x });
                printf("%d ", -iter->second);
            }
            else
                printf("%d ", iter->second);
            id.erase(iter);
        }
    }
}

namespace undirect_graph {
    //若有多组数据应将 G 和 result 清空, 调用完 euler 之后 G 中的边会被删除
    multiset<int> G[maxn];
    vector<pair<int, int>> result;
    void dfs(int x) {
        while (G[x].size()) {
            auto iter = G[x].begin();
            auto y = *iter;
            G[x].erase(iter);
            G[y].erase(G[y].find(x));
            dfs(y);
            result.emplace_back(x, y);
        }
    }
    bool euler(int s) {
        dfs(s);
        reverse(result.begin(), result.end());
        if (result.size() != m || (result.size() && result.back().second != s))
            return false; //若只要求有欧拉道路则: if (result.size() != m) return false;
        for (int i = 1; i < result.size(); ++i) if (result[i - 1].second != result[i].first)
            return false;
        return true;
    }
    void solve() {
        multimap<pair<int, int>, int> id;
        for (int i = 1; i <= m; ++i) {
            int x, y;
            scanf("%d %d", &x, &y);
            G[x].insert(y);
            G[y].insert(x);
            id.emplace(make_pair(x, y), i);
        }
    }
}

```

```

    int s = 1; while (G[s].empty() && s <= n) s++;
    if (!euler(s)) {
        puts("NO");
        return;
    }
    else {
        puts("YES");
        for (auto pr : result) {
            int x = pr.first, y = pr.second;
            auto iter = id.find({ x, y });
            if (iter == id.end()) {
                iter = id.find({ y, x });
                printf("%d ", -iter->second);
            }
            else
                printf("%d ", iter->second);
            id.erase(iter);
        }
    }
}

int main() { //uoj 117
    //freopen("in.txt", "r", stdin);
    scanf("%d %d %d", &t, &n, &m);
    if (t == 2) { //有向图
        direct_graph::solve();
    }
    else {
        undirect_graph::solve();
    }
    return 0;
}

```

1.4 最小生成树

```

const int maxn = 210000;
int n, m, from[maxn], to[maxn], weight[maxn];
int p[maxn], r[maxn];
int find(int x) {
    return p[x] == x ? x : p[x] = find(p[x]);
}

int kruskal() {
    for (int i = 1; i <= n; ++i)
        p[i] = i;
    for (int i = 1; i <= m; ++i)
        r[i] = i;
    sort(r + 1, r + m + 1, [](int a, int b) {

```

```

        return weight[a] < weight[b];
    });
    int ans = 0, edges = 0;
    for (int i = 1; i <= m; ++i) {
        int e = r[i];
        int x = find(from[e]), y = find(to[e]);
        if (x != y) {
            p[x] = y;
            ans += weight[e];
            edges += 1;
        }
    }
    if (edges != n - 1)
        return -1; //无解
    else
        return ans;
}

int main() { //洛谷 P3366
    //freopen("in.txt", "r", stdin);
    scanf("%d %d", &n, &m);
    for (int i = 1; i <= m; ++i)
        scanf("%d %d %d", &from[i], &to[i], &weight[i]);
    int ans = kruskal();
    if (ans == -1)
        puts("orz");
    else
        printf("%d\n", ans);
    return 0;
}

```

1.5 Dijkstra 算法

```

int n, m, s; //点数、边数、起点
namespace simple_dijkstra { //  $O(n^2)$ 
    const int maxn = 1001;
    int vis[maxn], d[maxn], w[maxn][maxn];
    void dijkstra() {
        memset(vis, 0, sizeof(vis));
        memset(d, 0x3f, sizeof(d));
        d[s] = 0;
        for (int i = 1; i <= n; ++i) {
            int x = -1;
            for (int y = 1; y <= n; ++y) if (!vis[y])
                if (x == -1 || d[y] < d[x])
                    x = y;
            vis[x] = true;
            for (int y = 1; y <= n; ++y)

```

```

        d[y] = min(d[y], d[x] + w[x][y]);
    }
}

void solve() {
    scanf("%d %d %d", &n, &m, &s);
    memset(w, 0x3f, sizeof(w));
    for (int i = 0; i < m; ++i) {
        int x, y, z;
        scanf("%d %d %d", &x, &y, &z);
        w[x][y] = min(w[x][y], z);
    }
    dijkstra();
    for (int i = 1; i <= n; ++i)
        printf("%d ", d[i] == 0x3f3f3f3f ? INT_MAX : d[i]);
    printf("\n");
}

}

namespace fast_dijkstra {
    const int maxn = 210000;
    vector<pair<int, int>> G[maxn];
    int d[maxn];
    void dijkstra() {
        using node = pair<int, int>;
        priority_queue<node, vector<node>, greater<node>> Q;
        memset(d, 0x3f, sizeof(d));
        d[s] = 0;
        Q.emplace(0, s);
        while (!Q.empty()) {
            auto [dist, x] = Q.top(); Q.pop();
            if (dist != d[x])
                continue;
            for (auto [y, w] : G[x]) {
                if (d[y] > d[x] + w) {
                    d[y] = d[x] + w;
                    Q.emplace(d[y], y);
                    //p[y] = x;
                }
            }
        }
    }
}

void solve() {
    scanf("%d %d %d", &n, &m, &s);
    for (int i = 0; i < m; ++i) {
        int x, y, z;
        scanf("%d %d %d", &x, &y, &z);
        G[x].emplace_back(y, z);
    }
}

```

```

    }
    dijkstra();
    for (int i = 1; i <= n; ++i)
        printf("%d ", d[i] == 0x3f3f3f3f ? INT_MAX : d[i]);
    printf("\n");
}
}

int main() {
    //freopen("in.txt", "r", stdin);
    fast_dijkstra::solve();
    return 0;
}

```

1.6 SPFA 算法

```

int n, m, s; //点数、边数、起点
namespace BellmanFord { // O(nm)
    const int maxn = 1001;
    const int maxedges = 110000;
    int d[maxn], x[maxedges], y[maxedges], w[maxedges];
    void BellmanFord() {
        memset(d, 0x3f, sizeof(d));
        d[s] = 0;
        for (int k = 1; k < n; ++k)
            for (int i = 1; i <= m; ++i)
                d[y[i]] = min(d[y[i]], d[x[i]] + w[i]);
    }
    void solve() {
        scanf("%d %d %d", &n, &m, &s);
        for (int i = 1; i <= m; ++i)
            scanf("%d %d %d", &x[i], &y[i], &w[i]);
        BellmanFord();
        for (int i = 1; i <= n; ++i)
            printf("%d ", d[i] == 0x3f3f3f3f ? INT_MAX : d[i]);
        printf("\n");
    }
}

namespace SPFA {
    const int maxn = 210000;
    int inq[maxn], cnt[maxn], d[maxn];
    vector<pair<int, int>> G[maxn];
    bool spfa() {
        queue<int> Q;
        memset(inq, 0, sizeof(inq));
        memset(cnt, 0, sizeof(cnt));
        memset(d, 0x3f, sizeof(d));
        d[s] = 0;
    }
}

```

```

    inq[s] = true;
    Q.push(s);
    while (!Q.empty()) {
        int x = Q.front(); Q.pop();
        inq[x] = false;
        for (auto [y, w] : G[x]) {
            if (d[y] > d[x] + w) {
                d[y] = d[x] + w;
                //p[y] = x;
                if (!inq[y]) {
                    Q.push(y);
                    inq[y] = true;
                    if (++cnt[y] > n)
                        return false;
                }
            }
        }
    }
    return true;
}

void solve() {
    scanf("%d %d %d", &n, &m, &s);
    for (int i = 1; i <= m; ++i) {
        int x, y, w;
        scanf("%d %d %d", &x, &y, &w);
        G[x].emplace_back(y, w);
    }
    spfa();
    for (int i = 1; i <= n; ++i)
        printf("%d ", d[i] == 0x3f3f3f3f ? INT_MAX : d[i]);
    printf("\n");
}

int main() {
    //freopen("in.txt", "r", stdin);
    SPFA::solve();
    return 0;
}

```

1.7 一般图最大权匹配

```

#define dist(e) (lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2)
const int maxn = 1023, inf = 1e9;
struct edge {
    int u, v, w;
} g[maxn][maxn];
int n, m, num, lab[maxn], match[maxn], slack[maxn], st[maxn], pa[maxn];

```

```

int from[maxn][maxn], S[maxn], vis[maxn];
vector<int> flower[maxn];
deque<int> q;
void update(int u, int x) {
    if (!slack[x] || dist(g[u][x]) < dist(g[slack[x]][x]))
        slack[x] = u;
}
void set_slack(int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
        if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)
            update(u, x);
}
void push(int x) {
    if (x <= n)
        return q.push_back(x);
    for (int i = 0; i < flower[x].size(); i++)
        push(flower[x][i]);
}
void set_st(int x, int b) {
    st[x] = b;
    if (x <= n) return;
    for (int i = 0; i < flower[x].size(); ++i)
        set_st(flower[x][i], b);
}
int get(int b, int xr) {
    int pr = find(flower[b].begin(), flower[b].end(), xr) - flower[b].begin();
    if (pr % 2 == 1) {
        reverse(flower[b].begin() + 1, flower[b].end());
        return (int)flower[b].size() - pr;
    }
    else
        return pr;
}
void set_match(int u, int v) {
    match[u] = g[u][v].v;
    if (u <= n) return;
    edge e = g[u][v];
    int xr = from[u][e.u], pr = get(u, xr);
    for (int i = 0; i < pr; ++i)
        set_match(flower[u][i], flower[u][i ^ 1]);
    set_match(xr, v);
    rotate(flower[u].begin(), flower[u].begin() + pr, flower[u].end());
}
void augment(int u, int v) {
    int xnv = st[match[u]];

```



```

    set_match(u, v);
    if (!xnv) return;
    set_match(xnv, st[pa[xnv]]);
    augment(st[pa[xnv]], xnv);
}

int get_lca(int u, int v) {
    static int t = 0;
    for (++t; u || v; swap(u, v)) {
        if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
    }
    return 0;
}

void add(int u, int lca, int v) {
    int b = n + 1;
    while (b <= num && st[b]) ++b;
    if (b > num) ++num;
    lab[b] = 0, S[b] = 0;
    match[b] = match[lca];
    flower[b].clear();
    flower[b].push_back(lca);
    for (int x = u, y; x != lca; x = st[pa[y]])
        flower[b].push_back(x), flower[b].push_back(y = st[match[x]]), push(y);
    reverse(flower[b].begin() + 1, flower[b].end());
    for (int x = v, y; x != lca; x = st[pa[y]])
        flower[b].push_back(x), flower[b].push_back(y = st[match[x]]), push(y);
    set_st(b, b);
    for (int x = 1; x <= num; ++x)
        g[b][x].w = g[x][b].w = 0;
    for (int x = 1; x <= n; ++x)
        from[b][x] = 0;
    for (int i = 0; i < flower[b].size(); ++i) {
        int xs = flower[b][i];
        for (int x = 1; x <= num; ++x)
            if (g[b][x].w == 0 || dist(g[xs][x]) < dist(g[b][x]))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        for (int x = 1; x <= n; ++x)
            if (from[xs][x])
                from[b][x] = xs;
    }
    set_slack(b);
}

void expand(int b) {

```

```

    for (int i = 0; i < flower[b].size(); ++i)
        set_st(flower[b][i], flower[b][i]);
    int xr = from[b][g[b][pa[b]].u], pr = get(b, xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flower[b][i], xns = flower[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack(xns);
        push(xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (int i = pr + 1; i < flower[b].size(); ++i) {
        int xs = flower[b][i];
        S[xs] = -1, set_slack(xs);
    }
    st[b] = 0;
}

bool found(const edge& e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, push(nu);
    }
    else if (S[v] == 0) {
        int lca = get_lca(u, v);
        if (!lca) return augment(u, v), augment(v, u), true;
        else add(u, lca, v);
    }
    return false;
}

bool matching() {
    fill(S, S + num + 1, -1);
    fill(slack, slack + num + 1, 0);
    q.clear();
    for (int x = 1; x <= num; ++x)
        if (st[x] == x && !match[x]) pa[x] = 0, S[x] = 0, push(x);
    if (q.empty()) return false;
    for (;;) {
        while (q.size()) {
            int u = q.front();
            q.pop_front();
            if (S[st[u]] == 1) continue;
            for (int v = 1; v <= n; ++v) {
                if (g[u][v].w > 0 && st[u] != st[v]) {

```

```

        if (dist(g[u][v]) == 0) {
            if (found(g[u][v]))
                return true;
        }
        else update(u, st[v]);
    }
}
}
int d = inf;
for (int b = n + 1; b <= num; ++b)
    if (st[b] == b && S[b] == 1) d = min(d, lab[b] / 2);
for (int x = 1; x <= num; ++x)
    if (st[x] == x && slack[x])
    {
        if (S[x] == -1) d = min(d, dist(g[slack[x]][x]));
        else if (S[x] == 0) d = min(d, dist(g[slack[x]][x]) / 2);
    }
for (int u = 1; u <= n; ++u) {
    if (S[st[u]] == 0) {
        if (lab[u] <= d)
            return false;
        lab[u] -= d;
    }
    else if (S[st[u]] == 1) lab[u] += d;
}
for (int b = n + 1; b <= num; ++b)
    if (st[b] == b) {
        if (S[st[b]] == 0) lab[b] += d * 2;
        else if (S[st[b]] == 1) lab[b] -= d * 2;
    }
q.clear();
for (int x = 1; x <= num; ++x)
    if (st[x] == x && slack[x] && st[slack[x]] != x && dist(g[slack[x]][x]) == 0)
        if (found(g[slack[x]][x]))
            return true;
for (int b = n + 1; b <= num; ++b)
    if (st[b] == b && S[b] == 1 && lab[b] == 0)
        expand(b);
}
return false;
}
pair<long long, int> weight_blossom() {
    fill(match, match + n + 1, 0);
    num = n;
    int matches = 0;
    long long tot_weight = 0;

```

```

for (int u = 0; u <= n; ++u) st[u] = u, flower[u].clear();
int w_max = 0;
for (int u = 1; u <= n; ++u)
    for (int v = 1; v <= n; ++v) {
        from[u][v] = (u == v ? u : 0);
        w_max = max(w_max, g[u][v].w);
    }
for (int u = 1; u <= n; ++u) lab[u] = w_max;
while (matching()) ++matches;
for (int u = 1; u <= n; ++u)
    if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
return make_pair(tot_weight, matches);
}

int main() { //边权必须是正数
    cin >> n >> m;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v)
            g[u][v] = edge{ u,v,0 };
    for (int i = 0, u, v, w; i < m; ++i) {
        cin >> u >> v >> w;
        g[u][v].w = g[v][u].w = w;
    }
    cout << weight_blossom().first << '\n';
    for (int u = 1; u <= n; ++u) cout << match[u] << ' ';
}

```

1.8 有向图的传递闭包

```

namespace Closure { //求有向图的传递闭包
    const int n = 4000; //对于  $n = 4000$ , 可以在大约 1s 的时间内求解
    bitset<n> A[n];
    void solve(int M[][n]) { //M 是一个 01 矩阵,  $M[i][j]$  为 1 当且仅当有一条从  $i$  到  $j$  的有向边
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                A[i][j] = M[i][j];
        for (int k = 0; k < n; ++k)
            for (int i = 0; i < n; ++i)
                if (A[i][k])
                    A[i] |= A[k];
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                M[i][j] = A[i][j]; //将结果复制回输入矩阵中
    }
}

const int maxn = 4000;
int A[maxn][maxn], B[maxn][maxn];

```

```

int main() {
    for (int i = 0; i < maxn; ++i)
        for (int j = 0; j < maxn; ++j)
            A[i][j] = B[i][j] = rand() % 2;
    clock_t start = clock();
    for (int k = 0; k < maxn; ++k)
        for (int i = 0; i < maxn; ++i)
            for (int j = 0; j < maxn; ++j)
                B[i][j] = (B[i][j] || (B[i][k] && B[k][j]));
    clock_t end = clock();
    printf("%.5f\n", (double)(end - start) / CLOCKS_PER_SEC);
    start = clock();
    Closure::solve(A);
    end = clock();
    printf("%.5f\n", (double)(end - start) / CLOCKS_PER_SEC);
    for (int i = 0; i < maxn; ++i)
        for (int j = 0; j < maxn; ++j)
            if (A[i][j] != B[i][j])
                abort();
    return 0;
}

```

1.9 最小树形图

// 固定根的最小树型图，邻接矩阵写法

```

struct MDST {
    int n;
    int w[maxn][maxn]; // 边权
    int vis[maxn];      // 访问标记，仅用来判断无解
    int ans;            // 计算答案
    int removed[maxn]; // 每个点是否被删除
    int cid[maxn];      // 所在圈编号
    int pre[maxn];      // 最小入边的起点
    int iw[maxn];       // 最小入边的权值
    int max_cid;        // 最大圈编号
    void init(int n) {
        this->n = n;
        for(int i = 0; i < n; i++)
            for(int j = 0; j < n; j++) w[i][j] = INF;
    }
    void AddEdge(int u, int v, int cost) {
        w[u][v] = min(w[u][v], cost); // 重边取权最小的
    }
    // 从 s 出发能到达多少个结点
    int dfs(int s) {
        vis[s] = 1;
        int ans = 1;

```

```

    for(int i = 0; i < n; i++)
        if(!vis[i] && w[s][i] < INF) ans += dfs(i);
    return ans;
}
// 从 u 出发沿着 pre 指针找圈
bool cycle(int u) {
    max_cid++;
    int v = u;
    while(cid[v] != max_cid) { cid[v] = max_cid; v = pre[v]; }
    return v == u;
}
// 计算 u 的最小入弧, 入弧起点不得在圈 c 中
void update(int u) {
    iw[u] = INF;
    for(int i = 0; i < n; i++)
        if(!removed[i] && w[i][u] < iw[u]) {
            iw[u] = w[i][u];
            pre[u] = i;
        }
}
// 根结点为 s, 如果失败则返回 false
bool solve(int s) {
    memset(vis, 0, sizeof(vis));
    if(dfs(s) != n) return false;
    memset(removed, 0, sizeof(removed));
    memset(cid, 0, sizeof(cid)); //注意: 是将 cid 清空, 而不是 pre
    for(int u = 0; u < n; u++) update(u);
    pre[s] = s; iw[s] = 0; // 根结点特殊处理
    ans = max_cid = 0;
    for(;;) {
        bool have_cycle = false;
        for(int u = 0; u < n; u++) if(u != s && !removed[u] && cycle(u)){
            have_cycle = true;
            // 以下代码缩圈, 圈上除了 u 之外的结点均删除
            int v = u;
            do {
                if(v != u) removed[v] = 1;
                ans += iw[v];
                // 对于圈外点 i, 把边 i->v 改成 i->u (并调整权值); v->i 改为 u->i
                // 注意圈上可能还有一个 v'使得 i->v'或者 v'->i 存在, 因此只保留权值最小的 i->u 和 u->i
                for(int i = 0; i < n; i++) if(cid[i] != cid[u] && !removed[i]) {
                    if(w[i][v] < INF) w[i][u] = min(w[i][u], w[i][v]-iw[v]);
                    w[u][i] = min(w[u][i], w[v][i]);
                    if(pre[i] == v) pre[i] = u;
                }
            } while(v = pre[v]);
        }
    }
}

```

```

        } while(v != u);
        update(u);
        break;
    }
    if(!have_cycle) break;
}
for(int i = 0; i < n; i++)
    if(!removed[i]) ans += iw[i];
return true;
}
};

```

1.10 支配树

/*
 在调用 `dominator_tree::add_edge` 之前要先调用 `init` 函数进行初始化, `init` 的参数 `n` 表示支配树中有 `n` 的
 ↪ 点, 编号为 `1-n`。
`idom[x]` 表示点 `x` 的最近支配点, `idom[root] == 0`。
`head` 保存为原图的链表。
`back` 保存为反图的链表。
`dfn[x]` 表示结点 `x` 的 DFS 序。
`id[i]` 表示 DFS 序为 `i` 的结点编号。
`tarjan(s)` 可以求出以 `s` 为起点的支配树。
 性质: $dfn[x] > dfn[idom[x]]$, 即结点 `x` 的 DFS 序一定大于它的支配点的 DFS 序
 因此可以按照 DFS 序从小到大枚举点 `x`, 此时它在支配树上的父结点已经计算完毕。

```

    for (int i = 1; i <= n; ++i) {
        int x = id[i];
        ans[x] = F(ans[idom[x]], x);
    }
*/
const int maxn = 210000;
const int maxedges = 310000;
struct dominator_tree {
    int n, cnt, tot, head[maxn], fa[maxn], p[maxn], top[maxn], back[maxn], val[maxn], dfn[maxn],
    ↪ id[maxn], semi[maxn], idom[maxn];
    struct edge {
        int to, next;
    } e[maxedges * 3]; //min: maxedges * 2 + maxn
    inline bool cmp(int x, int y) {
        return dfn[semi[x]] > dfn[semi[y]];
    }
    inline void ins(int* first, int from, int to) {
        e[++cnt].to = to;
        e[cnt].next = first[from];
        first[from] = cnt;
    }
    void add_edge(int x, int y) {

```

```

    ins(head, x, y);
    ins(back, y, x);
}

void dfs(int x) {
    id[dfn[x] = ++tot] = x;
    for (int i = head[x]; i; i = e[i].next)
        if (!dfn[e[i].to])
            fa[e[i].to] = x, dfs(e[i].to);
}

int find(int x) {
    if (p[x] == x) return x;
    int y = find(p[x]);
    if (cmp(val[x], val[p[x]]))
        val[x] = val[p[x]];
    return p[x] = y;
}

void tarjan(int s) {
    dfs(s);
    for (int i = tot; i > 1; --i) {
        int u = id[i];
        for (int j = back[u]; j; j = e[j].next) {
            if (dfn[e[j].to]) {
                find(e[j].to);
                if (cmp(u, val[e[j].to]))
                    semi[u] = semi[val[e[j].to]];
            }
        }
        ins(top, semi[u], u);
        p[u] = fa[u];
        u = fa[u];
        for (int j = top[u]; j; j = e[j].next) {
            find(e[j].to);
            if (semi[val[e[j].to]] == semi[u])
                idom[e[j].to] = u;
            else
                idom[e[j].to] = val[e[j].to];
        }
        top[u] = 0;
    }
    for (int i = 2; i <= tot; ++i) {
        int x = id[i];
        if (idom[x] != semi[x])
            idom[x] = idom[idom[x]];
    }
}

void init(int n) {

```



```

    this->n = n;
    cnt = tot = 0;
    memset(head, 0, sizeof(int) * (n + 2));
    memset(back, 0, sizeof(int) * (n + 2));
    memset(top, 0, sizeof(int) * (n + 2));
    memset(dfn, 0, sizeof(int) * (n + 2));
    memset(idom, 0, sizeof(int) * (n + 2));
    for (int i = 1; i <= n; ++i)
        val[i] = semi[i] = p[i] = i;
}
}tree;
long long ans[maxn];
int main() {
    //freopen("in.txt", "r", stdin);
    int n, m;
    scanf("%d %d", &n, &m);
    tree.init(n);
    for (int i = 1; i <= m; ++i) {
        int u, v;
        scanf("%d %d", &u, &v);
        tree.add_edge(u, v);
    }
    tree.tarjan(1);
    for (int i = 1; i <= n; ++i)
        ans[i] = 1;
    for (int i = n; i >= 1; --i) {
        int x = tree.id[i];
        ans[tree.idom[x]] += ans[x];
    }
    for (int i = 1; i <= n; ++i)
        printf("%d ", ans[i]);
    printf("\n");
    return 0;
}

```

2 树算法

2.1 限定距离的子树问题

/*

该算法用来求解有根树中结点 v 的子树中与 v 距离不超过 d 的所有结点的权值和（也可以求最值等），时间复杂度 $O(d)$ ，若用倍增可以优化到 $O(\log d)$ 。

调用 `init` 函数之前应当输入 n （结点标号从 1 开始），以及用 `add_edge` 建图。

*/

```

const int maxn = 1010000;
int n, q, w[maxn], head[maxn], nxt[maxn * 2], to[maxn * 2], id[maxn], l[maxn], r[maxn];
int father[maxn], depth[maxn], seq[maxn], cur = 0;

```

```

long long sum[maxn];
void add_edge(int u, int v) {
    to[++cur] = v;
    nxt[cur] = head[u];
    head[u] = cur;
}
void init() {
    int sz = 0;
    queue<int> Q;
    father[1] = depth[1] = seq[n + 1] = 0;
    Q.push(1);
    while (!Q.empty()) {
        int x = Q.front(); Q.pop();
        seq[++sz] = x;
        id[x] = sz;
        l[x] = r[x] = 0;
        for (int i = head[x]; i; i = nxt[i]) if (to[i] != father[x]) {
            int y = to[i];
            l[x] = (l[x] ? l[x] : y);
            r[x] = y;
            father[y] = x;
            depth[y] = depth[x] + 1;
            Q.push(y);
        }
    }
    for (int j = 1; j <= n; ++j)
        sum[j] = sum[j - 1] + w[seq[j]];
    for (int j = 1; j <= n; ++j) if (!r[seq[j]])
        r[seq[j]] = r[seq[j - 1]];
    for (int j = n; j >= 1; --j) if (!l[seq[j]])
        l[seq[j]] = l[seq[j + 1]];
}
long long travel(int x, int d) { //求出以  $x$  为根的子树中与  $x$  距离不超过  $d$  的所有结点的权值和 (可改
    ↪ 为求最值或种类数)
    if (d < 0) return 0; //时间复杂度  $O(d)$ , 可以改成倍增使得时间复杂度变为  $O(\log d)$ 
    long long ret = 0;
    int L = x, R = x;
    for (int i = 0; i <= d; ++i) {
        int left = id[L], right = id[R];
        if (L == 0 || R == 0 || left > right) break;
        long long s = sum[right] - sum[left - 1];
        ret += s;
        L = l[L];
        R = r[R];
    }
    return ret;
}

```

```

}
int main() {
    //freopen("in.txt", "r", stdin);
    scanf("%d", &n);
    for (int i = 1; i <= n; ++i)
        scanf("%d", &w[i]);
    for (int i = 0; i < n - 1; ++i) {
        int u, v;
        scanf("%d %d", &u, &v);
        add_edge(u, v);
        add_edge(v, u);
    }
    init();
    scanf("%d", &q);
    while (q--) { //每次询问求出树中与  $v$  距离不超过  $k$  的所有点的权值和, 复杂度  $O(k^2)$ 
        int v, k;
        scanf("%d %d", &v, &k);
        long long ans = travel(v, k);
        int last = v;
        for (int u = father[v], t = k - 1; u != 0 && t >= 0; u = father[u], --t) {
            ans += travel(u, t) - travel(last, t - 1);
            last = u;
        }
        printf("%lld\n", ans);
    }
    return 0;
}

```

2.2 树的欧拉序

/*
 seq 下标从 1 开始, 表示欧拉序列。欧拉序列记录的是点的标号, 一段欧拉序列中若一个点出现了两次则不予处理。
 树上的一条路径一定对应一段连续的欧拉序列 (可能还要并上一个额外的点)。
 */

```

const int maxn = 210000;
const int maxlog = 25;
int head[maxn], nxt[maxn], to[maxn], seq[maxn], depth[maxn], anc[maxn][maxlog], first[maxn],
    ↪ last[maxn], cur, sz;
struct item { //下标区间  $[L, R]$  的欧拉序列, 若  $lca \neq 0$  则还要将  $lca$  考虑进来
    int L, R, lca;
    item(){}
    item(int L, int R, int lca) : L(L), R(R), lca(lca){}
};
void init() {
    cur = sz = 0;
    memset(head, 0, sizeof(head));
    memset(depth, 0, sizeof(depth));

```

```

    memset(anc, 0, sizeof(anc));
}
void add_edge(int x, int y) { //对于树中的每条边要调用两次 add_edge
    to[++cur] = y;
    nxt[cur] = head[x];
    head[x] = cur;
}
void dfs(int x, int fa) { //dfs(1, 0)
    seq[++sz] = x;
    first[x] = sz;
    for (int i = head[x]; i; i = nxt[i]) if (to[i] != fa) {
        int y = to[i];
        depth[y] = depth[x] + 1;
        anc[y][0] = x;
        for (int i = 1; (1 << i) <= depth[y]; ++i)
            anc[y][i] = anc[anc[y][i - 1]][i - 1];
        dfs(y, x);
    }
    seq[++sz] = x;
    last[x] = sz;
}
int getlca(int x, int y) {
    if (depth[x] < depth[y])
        swap(x, y);
    for (int i = maxlog - 1; i >= 0; --i)
        if (depth[x] - (1 << i) >= depth[y])
            x = anc[x][i];
    if (x == y)
        return x;
    for (int i = maxlog - 1; i >= 0; --i)
        if (anc[x][i] != anc[y][i])
            x = anc[x][i], y = anc[y][i];
    return anc[x][0];
}
item path(int x, int y) { //返回树上 x 到 y 的路径对应的欧拉序列
    if (first[x] > first[y])
        swap(x, y);
    int lca = getlca(x, y);
    if (lca == x)
        return item(first[x], first[y], 0);
    else
        return item(last[x], first[y], lca);
}
int main() {
    return 0;
}

```

2.3 倍增求 LCA

```

const int maxn = 510000;
const int maxlog = 20;
vector<int> G[maxn];
int anc[maxn][maxlog], dep[maxn];
void dfs(int x, int fa, int d) {
    anc[x][0] = fa;
    dep[x] = d;
    for (auto y : G[x]) if (y != fa)
        dfs(y, x, d + 1);
}

void preprocess(int n) { //点的编号从 1 开始
    for (int j = 1; j < maxlog; ++j)
        for (int i = 0; i <= n; ++i)
            anc[i][j] = 0;
    dfs(1, 0, 0);
    for (int j = 1; j < maxlog; ++j)
        for (int i = 1; i <= n; ++i)
            anc[i][j] = anc[anc[i][j - 1]][j - 1];
}

//返回结点 x 向上走 d 步到达的结点
int moveup(int x, int d) {
    for (int i = 0; d >> i; ++i)
        if (d >> i & 1)
            x = anc[x][i];
    return x;
}

int lca(int x, int y) {
    if (dep[x] < dep[y])
        swap(x, y);
    x = moveup(x, dep[x] - dep[y]);
    if (x == y)
        return x;
    for (int i = maxlog - 1; i >= 0; --i)
        if (anc[x][i] != anc[y][i])
            x = anc[x][i], y = anc[y][i];
    return anc[x][0];
}

int dist(int x, int y) { //返回结点 x 和 y 之间的距离
    return dep[x] + dep[y] - 2 * dep[lca(x, y)];
}

int move(int x, int y, int d) { //返回从结点 x 向结点 y 走 d 步到达的结点
    int p = lca(x, y);
    int h = dep[x] - dep[p];
    if (h >= d)
        return moveup(x, d);

```

```

    else
        return moveup(y, dep[x] + dep[y] - d - 2 * dep[p]);
}

int main() {
    //freopen("in.txt", "r", stdin);
    int n, m, s;
    scanf("%d %d %d", &n, &m, &s);
    for (int i = 1; i < n; ++i) {
        int x, y;
        scanf("%d %d", &x, &y);
        G[x].push_back(y);
        G[y].push_back(x);
    }
    dfs(s, 0, 0);
    for (int j = 1; j < maxlog; ++j)
        for (int i = 1; i <= n; ++i)
            anc[i][j] = anc[anc[i][j - 1]][j - 1];
    for (int i = 0; i < m; ++i) {
        int x, y;
        scanf("%d %d", &x, &y);
        printf("%d\n", lca(x, y));
    }
    return 0;
}

```

2.4 点分治

```

#define dfs1(x, fa, d)
#define dfs2(x, fa, d)
const int maxn = 110000;
vector<pair<int, int>> G[maxn]; //to, weight
int vis[maxn], siz[maxn], f[maxn], pos, sum;
void init(int n) {
    for (int i = 0; i <= n; ++i) {
        vis[i] = false;
        G[i].clear();
    }
}

void getroot(int x, int fa) {
    f[x] = 0; siz[x] = 1;
    for (auto [y, w] : G[x]) if (y != fa && !vis[y]) {
        getroot(y, x);
        f[x] = max(f[x], siz[y]);
        siz[x] += siz[y];
    }
    f[x] = max(f[x], sum - siz[x]);
    if (f[x] < f[pos])

```

```

        pos = x;
    }
    void calc(int x) { //统计经过结点 x 的所有答案
        for (auto [y, w] : G[x]) if (!vis[y]) {
            //注意在 dfs 中枚举子结点 y 的时候要判两个条件: y != fa && !vis[y]
            dfs1(y, x, w); //计算当前子树与之前子树以及 结点 x 连接构成的路径。
            dfs2(y, x, w); //维护处理过的子树的信息
        }
        //维护信息的时候有两种方案:
        //1. 在每次 calc 调用的时候新建一个数据结构维护。
        //2. 维护一个全局的数据结构, 此时在 calc 末尾应当加一个循环来撤销之前的所有操作。
    }
    void solve(int x, int cnt) { //cnt 为子树 x 中的结点总数
        pos = maxn - 1; f[pos] = sum = cnt;
        getroot(x, -1);
        int root = pos; //因为 pos 是全局变量, 递归的时候值会改变, 所以此处存为局部变量。
        vis[root] = 1;
        calc(root);
        for (auto [y, w] : G[root]) if (!vis[y])
            solve(y, siz[y] < siz[root] ? siz[y] : cnt - siz[root]);
    }
    int main() {
        int n;
        scanf("%d", &n);
        init(n);
        for (int i = 1; i < n; ++i) {
            int x, y, w;
            scanf("%d %d %d", &x, &y, &w);
            G[x].emplace_back(y, w);
            G[y].emplace_back(x, w);
        }
        solve(1, n);
        return 0;
    }
}

```

2.5 动态点分治

```

#define dfs(x, fa, d)
const int maxn = 110000;
vector<pair<int, int>> G[maxn]; //to, weight
int vis[maxn], siz[maxn], f[maxn], pa[maxn], pos, sum; //pa[x] 表示在点分树上结点 x 的父亲
int son[maxn], dep[maxn], father[maxn], top[maxn];
void DFS1(int x, int fa, int d) {
    dep[x] = d;
    siz[x] = 1;
    son[x] = 0;
    father[x] = fa;
}

```

```

    for (auto [y, w] : G[x]) if (y != fa) {
        DFS1(y, x, d + 1);
        siz[x] += siz[y];
        if (siz[son[x]] < siz[y])
            son[x] = y;
    }
}

void DFS2(int x, int tp) {
    top[x] = tp;
    if (son[x])
        DFS2(son[x], tp);
    for (auto [y, w] : G[x])
        if (y != father[x] && y != son[x])
            DFS2(y, y);
}

int lca(int x, int y) {
    while (top[x] != top[y]) {
        if (dep[top[x]] < dep[top[y]])
            swap(x, y);
        x = father[top[x]];
    }
    return dep[x] < dep[y] ? x : y;
}

int dist(int x, int y) {
    return dep[x] + dep[y] - 2 * dep[lca(x, y)];
}

void init(int n) {
    for (int i = 0; i <= n; ++i) {
        vis[i] = false;
        G[i].clear();
    }
}

void getroot(int x, int fa) {
    f[x] = 0; siz[x] = 1;
    for (auto [y, w] : G[x]) if (y != fa && !vis[y]) {
        getroot(y, x);
        f[x] = max(f[x], siz[y]);
        siz[x] += siz[y];
    }
    f[x] = max(f[x], sum - siz[x]);
    if (f[x] < f[pos])
        pos = x;
}

void prepare(int x) { //pa[x] 为结点 x 在点分树中的父结点，没有的话为-1。
    for (auto [y, w] : G[x]) if (!vis[y]) {
        //注意在 dfs 中枚举子结点 y 的时候要判两个条件: y != fa && !vis[y]
    }
}

```



```

        dfs(y, x, w);
    }
}

void solve(int x, int cnt, int pre = -1) { //cnt 为子树 x 中的结点总数
    pos = maxn - 1; f[pos] = sum = cnt;
    getroot(x, -1);
    int root = pos; //因为 pos 是全局变量，递归的时候值会改变，所以此处存为局部变量。
    vis[root] = true;
    pa[root] = pre;
    for (auto [y, w] : G[root]) if (!vis[y]) {
        int total = siz[y] < siz[root] ? siz[y] : cnt - siz[root];
        solve(y, total, root);
    }
    vis[root] = false;
    prepare(root);
}

int main() {
    int n;
    scanf("%d", &n);
    init(n);
    for (int i = 1; i < n; ++i) {
        int x, y, w;
        scanf("%d %d %d", &x, &y, &w);
        G[x].emplace_back(y, w);
        G[y].emplace_back(x, w);
    }
    DFS1(1, 0, 1);
    DFS2(1, 1);
    solve(1, n);
    return 0;
}

```

2.6 快速求树中与结点 x 距离不超过 k 的点权和

```

const int maxn = 110000;
const int inf = 1 << 30;
int head[maxn * 2], nxt[maxn * 2], to[maxn * 2];
int vis[maxn], pa[maxn], pos, sz, cur; //pa[x] 表示在点分树上结点 x 的父亲
int value[maxn], root[maxn], root2[maxn];
int dep[maxn], Size[maxn], father[maxn], son[maxn], top[maxn];
void DFS1(int u, int fa, int d) {
    dep[u] = d;
    Size[u] = 1;
    son[u] = 0;
    father[u] = fa;
    for (int i = head[u]; i; i = nxt[i]) {
        int v = to[i];

```

```

    if (v != fa) {
        DFS1(v, u, d + 1);
        Size[u] += Size[v];
        if (Size[son[u]] < Size[v])
            son[u] = v;
    }
}

void DFS2(int u, int tp) {
    top[u] = tp;
    if (son[u])
        DFS2(son[u], tp);
    for (int i = head[u]; i; i = nxt[i]) {
        int v = to[i];
        if (v != father[u] && v != son[u])
            DFS2(v, v);
    }
}

int lca(int x, int y) {
    while (top[x] != top[y]) {
        if (dep[top[x]] < dep[top[y]])
            swap(x, y);
        x = father[top[x]];
    }
    return dep[x] < dep[y] ? x : y;
}

int dist(int x, int y) {
    return dep[x] + dep[y] - 2 * dep[lca(x, y)];
}

struct TreeSet {
    struct node {
        int l, r, v;
    } T[maxn * 150];
    int left, right, sz; //空的线段树标号为 0
    void init(int L, int R) { //所有线段树对应区间都为 [L, R]
        left = L;
        right = R;
        sz = 1;
    }
    int newnode() {
        T[sz].l = T[sz].r = T[sz].v = 0;
        return sz++;
    }
    void insert(int x, int pos, int value) { //编号为 x 的线段树的 pos 位置上的值加上 value
        int L = left, R = right;
        while (L < R) {

```

```

    int M = (L + R) >> 1;
    T[x].v += value;
    if (pos <= M) {
        if (!T[x].l)
            T[x].l = newnode();
        x = T[x].l;
        R = M;
    }
    else {
        if (!T[x].r)
            T[x].r = newnode();
        x = T[x].r;
        L = M + 1;
    }
}
T[x].v += value;
}

int sum(int x, int pos) { //在编号为 x 的线段树上计算 pos 位置的前缀和
    int L = left, R = right, res = 0;
    while (x && L < R) {
        int M = (L + R) >> 1, t = T[T[x].l].v;
        if (pos <= M)
            x = T[x].l, R = M;
        else
            x = T[x].r, L = M + 1, res += t;
    }
    res += T[x].v;
    return res;
}

}tree;

void init() {
    cur = 0;
    memset(head, 0, sizeof(head));
    memset(vis, 0, sizeof(vis));
}

void insert(int u, int v) {
    nxt[++cur] = head[u];
    head[u] = cur;
    to[cur] = v;
}

int dfs0(int u, int fa) { //求出以 u 为根的子树的大小
    int tot = 1;
    for (int i = head[u]; i; i = nxt[i]) {
        int v = to[i];
        if (v != fa && !vis[v])
            tot += dfs0(v, u);
    }
}

```

```

    }
    return tot;
}

int dfs1(int u, int fa, int cnt) { //求出以 u 为根的子树的重心
    int tot = 1, num = 0;
    for (int i = head[u]; i; i = nxt[i]) {
        int v = to[i];
        if (v != fa && !vis[v]) {
            int result = dfs1(v, u, cnt);
            tot += result;
            num = max(num, result);
        }
    }
    num = max(num, cnt - tot);
    if (num < sz) {
        sz = num;
        pos = u;
    }
    return tot;
}

void dfs2(int u, int fa, int host) {
    tree.insert(root[host], dist(u, host), value[u]);
    for (int i = head[u]; i; i = nxt[i]) {
        int v = to[i];
        if (v != fa && !vis[v]) {
            dfs2(v, u, host);
        }
    }
}

void dfs3(int u, int fa, int host) {
    tree.insert(root2[host], dist(u, pa[host]), value[u]);
    for (int i = head[u]; i; i = nxt[i]) {
        int v = to[i];
        if (v != fa && !vis[v]) {
            dfs3(v, u, host);
        }
    }
}

void build(int u, int cnt, int pre = 0) { //递归处理子树 u, cnt 为子树 u 中的结点总数
    sz = inf;
    dfs1(u, -1, cnt); //求出以 u 为根的子树的重心 root, 作为新的根结点
    int root = pos;
    vis[root] = 1; //标记 root 结点不能再被访问
    pa[root] = pre;
    dfs2(root, -1, root);
    if (pre) dfs3(root, -1, root);
}

```

```

    for (int i = head[root]; i; i = nxt[i]) {
        int v = to[i];
        if (!vis[v]) {
            build(v, dfs0(v, -1), root);
        }
    }
}

long long query(int x, int k) {
    long long res = tree.sum(root[x], k);
    for (int i = x; pa[i]; i = pa[i]) {
        int d = dist(x, pa[i]);
        if (k - d >= 0) {
            res += tree.sum(root[pa[i]], k - d);
            res -= tree.sum(root2[i], k - d);
        }
    }
    return res;
}

void update(int x, int delta) {
    tree.insert(root[x], 0, delta);
    for (int i = x; pa[i]; i = pa[i]) {
        int d = dist(pa[i], x);
        tree.insert(root[pa[i]], d, delta);
        tree.insert(root2[i], d, delta);
    }
}

int main() {
    // freopen("in.txt", "r", stdin);
    int n, m;
    scanf("%d %d", &n, &m);
    init();
    tree.init(0, 110000);
    for (int i = 1; i <= n; ++i)
        root[i] = tree.newnode();
    for (int i = 1; i <= n; ++i)
        root2[i] = tree.newnode();
    for (int i = 1; i <= n; ++i)
        scanf("%d", &value[i]);
    for (int i = 1; i < n; ++i) {
        int u, v;
        scanf("%d %d", &u, &v);
        insert(u, v);
        insert(v, u);
    }
    DFS1(1, 0, 1);
    DFS2(1, 1);
}

```

```

build(1, n);
long long ans = 0;
while (m--) {
    long long tp, x, y;
    scanf("%lld %lld %lld", &tp, &x, &y);
    x ^= ans; y ^= ans;
    if (tp == 0) {
        ans = query(x, y);
        printf("%lld\n", ans);
    }
    else {
        int delta = y - value[x];
        value[x] = y;
        update(x, delta);
    }
}
return 0;
}

```

2.7 动态维护树中白色结点的最长距离

```

/*
P2056 [ZJOI2007] 捉迷藏
动态维护树中白色结点的最长距离。
操作 C 翻转一个结点的颜色，操作 G 进行一次查询（树中白色结点的直径）。
*/
const int maxn = 101000;
template<typename T> class heap {
private:
    priority_queue<T> Q, R;
public:
    void maintain() {
        while (!R.empty() && Q.top() == R.top())
            Q.pop(), R.pop();
    }
    void push(const T& val) {
        Q.push(val);
    }
    void erase(const T& val) { //只能删除还在优先队列中的值，不能删除不存在的值。
        R.push(val);
        maintain();
    }
    void pop() {
        Q.pop();
        maintain();
    }
    T top() {

```

```

    return Q.top();
}
T top2() { //返回第二大的值, 调用之前必须保证有 size() >= 2。
    auto val = Q.top(); pop();
    auto sec = Q.top(); push(val);
    return sec;
}
auto size() const {
    return Q.size() - R.size();
}
bool empty() const {
    return size() == 0;
}
template<typename ...F> void emplace(F&&... args) {
    Q.emplace(std::forward<F>(args)...);
}
};
vector<int> G[maxn]; //to, weight
int vis[maxn], siz[maxn], f[maxn], pa[maxn], pos, sum; //pa[x] 表示在点分树上结点 x 的父亲
int state[maxn], res[maxn], rt, n;
int son[maxn], dep[maxn], father[maxn], top[maxn];
heap<int> head[maxn], Q[maxn], answer;
void DFS1(int x, int fa, int d) {
    dep[x] = d;
    siz[x] = 1;
    son[x] = 0;
    father[x] = fa;
    for (auto y : G[x]) if (y != fa) {
        DFS1(y, x, d + 1);
        siz[x] += siz[y];
        if (siz[son[x]] < siz[y])
            son[x] = y;
    }
}
void DFS2(int x, int tp) {
    top[x] = tp;
    if (son[x])
        DFS2(son[x], tp);
    for (auto y : G[x])
        if (y != father[x] && y != son[x])
            DFS2(y, y);
}
int lca(int x, int y) {
    while (top[x] != top[y]) {
        if (dep[top[x]] < dep[top[y]])
            swap(x, y);
    }
}

```

```

        x = father[top[x]];
    }
    return dep[x] < dep[y] ? x : y;
}

int dist(int x, int y) {
    return dep[x] + dep[y] - 2 * dep[lca(x, y)];
}

void getroot(int x, int fa) {
    f[x] = 0; siz[x] = 1;
    for (auto y : G[x]) if (y != fa && !vis[y]) {
        getroot(y, x);
        f[x] = max(f[x], siz[y]);
        siz[x] += siz[y];
    }
    f[x] = max(f[x], sum - siz[x]);
    if (f[x] < f[pos])
        pos = x;
}

void dfs(int x, int fa) {
    Q[rt].push(dist(x, pa[rt]));
    for (auto y : G[x]) if (y != fa && !vis[y]) {
        dfs(y, x);
    }
}

void prepare(int x) {
    head[x].push(0);
    res[x] = (head[x].size() == 1 ? 0 : head[x].top() + head[x].top2());
    answer.emplace(res[x]);
    if (pa[x] < 0) return;
    Q[x].push(dist(x, pa[x]));
    for (auto y : G[x]) if (!vis[y]) {
        ::rt = x;
        dfs(y, x);
    }
    head[pa[x]].push(Q[x].top());
}

void update(int x) {
    state[x] ^= 1;
    for (int y = -1, p = x; p >= 1; y = p, p = pa[y]) {
        if (y == -1) {
            if (state[x] == 0)
                head[x].push(0);
            else
                head[x].erase(0);
        }
        else {

```



```

        auto& q = Q[y];
        if (q.size())
            head[p].erase(q.top());
        if (state[x] == 0)
            q.emplace(dist(x, p));
        else
            q.erase(dist(x, p));
        if (q.size())
            head[p].push(q.top());
    }
    int last = res[p];
    if (head[p].size() >= 2)
        res[p] = head[p].top() + head[p].top2();
    else if (head[p].size() >= 1)
        res[p] = (state[p] == 0 ? head[p].top() : 0);
    else
        res[p] = -1;
    if (last != res[p]) {
        answer.erase(last);
        answer.emplace(res[p]);
    }
}
}

void solve(int x, int cnt, int pre = -1) { //cnt 为子树 x 中的结点总数
    pos = maxn - 1; f[pos] = sum = cnt;
    getroot(x, -1);
    int root = pos; //因为 pos 是全局变量，递归的时候值会改变，所以此处存为局部变量。
    vis[root] = true;
    pa[root] = pre;
    for (auto y : G[root]) if (!vis[y]) {
        int total = siz[y] < siz[root] ? siz[y] : cnt - siz[root];
        solve(y, total, root);
    }
    vis[root] = false;
    prepare(root);
}

int main() {
    //freopen("in.txt", "r", stdin);
    scanf("%d", &n);
    for (int i = 1; i < n; ++i) {
        int x, y;
        scanf("%d %d", &x, &y);
        G[x].emplace_back(y);
        G[y].emplace_back(x);
    }
    DFS1(1, 0, 1);
}

```

```

DFS2(1, 1);
solve(1, n);
int Q, x; scanf("%d", &Q);
while (Q--) {
    char tp; scanf(" %c ", &tp);
    if (tp == 'G') {
        printf("%d\n", answer.top());
    }
    else {
        scanf("%d", &x);
        update(x);
    }
}
return 0;
}

```

2.8 虚树

```

/*
ins 函数会在每个结点  $i$  第一次入栈的时候，将  $E[i]$  清空。这样在最坏情况下空间复杂度可达  $O(n \log n)$ ，
可以改成在每次询问后用一个  $DFS$  清空  $E$ ，这样也  $E$  就不会占用额外内存了。
*/
const int maxn = 1110000;
int dep[maxn], sz[maxn], pa[maxn], son[maxn], top[maxn], stk[maxn], dfn[maxn], clk, tp;
vector<int> G[maxn], E[maxn];
void dfs1(int u, int fa, int d) {
    dfn[u] = ++clk;
    dep[u] = d;
    sz[u] = 1;
    son[u] = 0;
    pa[u] = fa;
    for (auto v : G[u]) if (v != fa) {
        dfs1(v, u, d + 1);
        sz[u] += sz[v];
        if (sz[son[u]] < sz[v])
            son[u] = v;
    }
}
void dfs2(int u, int tp) {
    top[u] = tp;
    if (son[u])
        dfs2(son[u], tp);
    for (auto v : G[u]) if (v != pa[u] && v != son[u])
        dfs2(v, v);
}
int lca(int x, int y) {
    while (top[x] != top[y]) {

```

```

        if (dep[top[x]] < dep[top[y]])
            swap(x, y);
        x = pa[top[x]];
    }
    return dep[x] < dep[y] ? x : y;
}

int dist(int x, int y) {
    return dep[x] + dep[y] - 2 * dep[lca(x, y)];
}

void init(int root) {
    clk = 0; stk[0] = -1;
    dfs1(root, 0, 1);
    dfs2(root, root);
}

void ins(int x) {
    if (tp > 0) {
        int fa = lca(stk[tp], x);
        if (fa != stk[tp]) {
            while (tp > 1 && dep[fa] < dep[stk[tp - 1]]) {
                E[stk[tp - 1]].push_back(stk[tp]);
                --tp;
            }
            int last = stk[tp--];
            if (fa != stk[tp]) {
                E[fa].clear();
                stk[++tp] = fa;
            }
            E[fa].push_back(last);
        }
    }
    E[x].clear();
    stk[++tp] = x;
}

int build(vector<int> nodes) { //对集合 nodes 中的结点建立虚树，并返回根结点。
    sort(nodes.begin(), nodes.end(), [](int x, int y) {
        return dfn[x] < dfn[y];
    });
    for (auto i : nodes)
        ins(i);
    while (--tp)
        E[stk[tp]].push_back(stk[tp + 1]);
    return stk[1];
}

int main() {
    return 0;
}

```

2.9 O(1)-LCA

```

const int maxn = 1110000; //maxn 至少为最大结点数的两倍
vector<int> G[maxn], seq;
int pos[maxn], dep[maxn], lg[maxn], a[maxn][20];
void dfs(int x, int fa, int d) {
    dep[x] = d;
    pos[x] = seq.size();
    seq.push_back(x);
    for (auto y : G[x]) if (y != fa) {
        dfs(y, x, d + 1);
        seq.push_back(x);
    }
}
void init(int s) { //根结点为 s, 调用之前 G 中应当已经保存了整棵树。
    seq.resize(1);
    dfs(s, -1, 1);
    const int n = seq.size() - 1;
    lg[1] = 0;
    for (int i = 2; i <= n; ++i)
        lg[i] = lg[i >> 1] + 1;
    for (int i = 1; i <= n; ++i)
        a[i][0] = seq[i];
    for (int j = 1; j <= lg[n]; ++j) {
        for (int i = 1; i + (1 << j) - 1 <= n; ++i) {
            int x = a[i][j - 1], y = a[i + (1 << (j - 1))][j - 1];
            a[i][j] = (dep[x] < dep[y] ? x : y);
        }
    }
}
inline int lca(int x, int y) {
    int L = pos[x], R = pos[y];
    if (L > R)
        swap(L, R);
    int k = lg[R - L + 1];
    x = a[L][k];
    y = a[R - (1 << k) + 1][k];
    return dep[x] < dep[y] ? x : y;
}
inline int dist(int x, int y) {
    return dep[x] + dep[y] - 2 * dep[lca(x, y)];
}
int main() {
    //freopen("in.txt", "r", stdin);
    int n, m, s;
    scanf("%d %d %d", &n, &m, &s);
    for (int i = 1; i < n; ++i) {

```

```

    int x, y;
    scanf("%d %d", &x, &y);
    G[x].push_back(y);
    G[y].push_back(x);
}
init(s);
for (int i = 0; i < m; ++i) {
    int x, y;
    scanf("%d %d", &x, &y);
    printf("%d\n", lca(x, y));
}
return 0;
}

```

3 基础算法

3.1 Java 快速读入

```

import java.io.*;
class InputReader {
    public BufferedReader reader;
    public StringTokenizer tokenizer;
    public InputReader(InputStream stream) {
        reader = new BufferedReader(new InputStreamReader(stream), 32768);
        tokenizer = null;
    }
    public String next() {
        while (tokenizer == null || !tokenizer.hasMoreTokens()) {
            try {
                tokenizer = new StringTokenizer(reader.readLine());
            } catch (IOException e) {
                throw new RuntimeException(e);
            }
        }
        return tokenizer.nextToken();
    }
    public int nextInt() {
        return Integer.parseInt(next());
    }
}

```

3.2 C++ 快速读入

```

inline int read_positive() { //读取一个正整数
    int x = 0;
    char c = getchar();
    while (c < '0' || c > '9')

```

```

        c = getchar();
    while (c >= '0' && c <= '9') {
        x = x * 10 + c - '0';
        c = getchar();
    }
    return x;
}

inline int read_negative() { //可以读取负数
    int x = 0, y = 1;
    char c = getchar();
    while (c < '0' || c > '9') {
        if (c == '-')
            y = -1;
        c = getchar();
    }
    while (c >= '0' && c <= '9') {
        x = x * 10 + c - '0';
        c = getchar();
    }
    return x * y;
}

int main() {
}

```

3.3 C++ 光速读入（交互版）

// 每次读取/输出一行，适用于处理交互题。

```

namespace io {
    constexpr int MAXBUFFER = 1024 * 1024 * 8;
    char ibuffer[MAXBUFFER], *iptr, obuffer[MAXBUFFER], *optr;
    inline void start_reading() { // 开始读取新的一行
        fgets(ibuffer, sizeof(ibuffer), stdin);
        iptr = ibuffer;
    }
    inline void start_writing() { //开始输出新的一行
        optr = obuffer;
    }
    inline int read_int() { //读入有符号整数
        char* nxt;
        int ret = strtol(iptr, &nxt, 10);
        iptr = nxt;
        return ret;
    }
    inline double read_double() noexcept { // 读入浮点数
        char* nxt;
        double ret = strtod(iptr, &nxt);
        iptr = nxt;
    }
}

```

```

    return ret;
}

inline void write_int(int val) { //输出有符号整数，输出完一行后需要调用 flush。
    char tmp[32], *now = tmp + 20;
    int length = 1;
    if (val < 0) {
        *optr++ = '-';
        val *= -1;
    }
    *now = ' ';
    do {
        *--now = '0' + val % 10;
        val /= 10;
        length += 1;
    } while (val > 0);
    memcpy(optr, now, length);
    optr += length;
}

inline void flush() {
    if (optr != obuffer) {
        optr[-1] = '\n';
    }
    fwrite(obuffer, 1, optr - obuffer, stdout);
    fflush(stdout);
}

}

int main() {
    io::start_reading();
    double val = io::read_double();
    printf("%.6f\n", val);
    return 0;
}

```

3.4 C++17 万能输出

```

template<typename T> void print_element(const T& param);
template<typename A, typename B> void print_element(const std::pair<A, B>& param);
template<typename ...T> void print_element(const std::tuple<T...>& param);
template<typename T, int SIZE> void print_element(const T(&param)[SIZE]);
template<typename T> void print_element(const std::vector<T>& param);
template<typename T> void print_element(const std::set<T>& param);
template<typename K, typename V> void print_element(const std::map<K, V>& param);
template<typename T> void print_element(const std::unordered_set<T>& param);
template<typename K, typename V> void print_element(const std::unordered_map<K, V>& param);
template<typename T> void print_element(const T& param) {
    std::cout << param;
}

```

```

template<typename A, typename B> void print_element(const std::pair<A, B>& param) {
    std::cout << '(';
    print_element(param.first);
    print_element(param.second);
    std::cout << ')';
}

template<typename ...T> void print_element(const std::tuple<T...>& param) {
    std::cout << '(';
    std::apply([](auto&&... args) {
        (... , (print_element(args), std::cout << ' '));
    }, param);
    std::cout << "\b";
}

template<typename T, int SIZE> void print_element(const T(&param)[SIZE]) {
    std::cout << '[';
    for (int i = 0; i < SIZE; ++i) {
        if (i) {
            std::cout << ' ';
        }
        print_element(param[i]);
    }
    std::cout << ']';
}

template<typename T> void print_element(const std::vector<T>& param) {
    std::cout << '[';
    for (int i = 0; i < param.size(); ++i) {
        if (i) {
            std::cout << ' ';
        }
        print_element(param[i]);
    }
    std::cout << ']';
}

template<typename T> void print_element(const std::set<T>& param) {
    std::cout << '{';
    for (auto iter = param.begin(); iter != param.end(); ++iter) {
        if (iter != param.begin()) {
            std::cout << ' ';
        }
        print_element(*iter);
    }
    std::cout << '}';
}

template<typename K, typename V> void print_element(const std::map<K, V>& param) {
    std::cout << '{';
    for (auto iter = param.begin(); iter != param.end(); ++iter) {

```



```

        if (iter != param.begin()) {
            std::cout << ' ';
        }
        print_element(*iter);
    }
    std::cout << '}' ;
}

template<typename T> void print_element(const std::unordered_set<T>& param) {
    std::cout << '{';
    for (auto iter = param.begin(); iter != param.end(); ++iter) {
        if (iter != param.begin()) {
            std::cout << ' ';
        }
        print_element(*iter);
    }
    std::cout << '}' ;
}

template<typename K, typename V> void print_element(const std::unordered_map<K, V>& param) {
    std::cout << '{';
    for (auto iter = param.begin(); iter != param.end(); ++iter) {
        if (iter != param.begin()) {
            std::cout << ' ';
        }
        print_element(*iter);
    }
    std::cout << '}' ;
}

template<typename ...T> void print(const T&... params) {
    (... , (print_element(params), std::cout << " "));
    std::cout << std::endl;
}

int main() {
    auto PR = std::make_tuple(123, "aa", std::vector<float>(3, 0.5));
    std::unordered_set<int> S{ 6, 7, 8 };
    std::vector<std::unordered_set<int>> V(2, S);
    print(PR, V);
    return 0;
}

```

3.5 C++20 万能输出

```

template<class> inline constexpr bool always_false_v = false;
auto output(auto&& param) {
    if constexpr (requires {std::cout << param; }) { //general
        std::cout << param;
    }
    else if constexpr (requires {param.first; param.second; }) { //std::pair

```

```

    std::cout << '(' << param.first << ' ' << param.second << ')';
}
else if constexpr (requires {std::get<0>(param); }) { //std::tuple and std::variant
    if constexpr (requires {param.index(); }) { //std::variant
        std::visit([](auto&& arg) {
            output(arg);
        }, param);
    }
    else { //std::tuple
        std::cout << '(';
        std::apply([](auto&&... args) {
            (... , (output(args), std::cout << ' '));
        }, param);
        std::cout << "\b";
    }
}
else if constexpr (requires {param.push_back(param[0]); }) {
    std::cout << '[';
    for (auto iter = param.begin(); iter != param.end(); ++iter) {
        if (iter != param.begin())
            std::cout << ' ';
        output(*iter);
    }
    std::cout << ']';
}
else if constexpr (requires {param.count(*param.begin()); }) {
    std::cout << '{';
    for (auto iter = param.begin(); iter != param.end(); ++iter) {
        if (iter != param.begin())
            std::cout << ' ';
        output(*iter);
    }
    std::cout << '}';
}
else {
    static_assert(always_false_v<decltype(param)>, " 无法输出的类型");
}
}

auto print(auto&&... params) {
    (... , (output(std::forward<decltype(params)>(params))(params)), std::cout << ' ');
    std::cout << std::endl;
}

auto render(auto&&... params) {
    std::cout << "\033[1;32m";
    print(std::forward<decltype(params)>(params)(params)...);
    std::cout << "\033[0m";
}

```

```

}

int main() {
    std::variant<int, std::string> var = "zzzzz";
    std::vector vec{ 1, 2, 3, 4, 5 };
    render(123, std::make_tuple(123, "asd", "zxc"), 123, var, vec);
    auto PR = std::make_tuple(123, "aa", std::vector<float>(3, 0.5));
    std::unordered_set<int> S{ 6, 7, 8 };
    std::vector<std::unordered_set<int>> V(2, S);
    print(PR, V);
    return 0;
}

```

3.6 RMQ 算法

```

const int maxn = 110000, maxlog = 20;
int d[maxn][maxlog], lg[maxn], A[maxn];
void preprocess(int n) { //下标范围 [1, n], 调用之前应当完成数组 A 的赋值
    lg[1] = 0;
    for (int i = 2; i <= n; ++i)
        lg[i] = lg[i >> 1] + 1;
    for (int i = 1; i <= n; ++i)
        d[i][0] = A[i];
    for (int j = 1; j <= lg[n]; ++j)
        for (int i = 1; i + (1 << j) - 1 <= n; ++i)
            d[i][j] = max(d[i][j - 1], d[i + (1 << (j - 1))][j - 1]);
}

inline int rmq(int L, int R) {
    int k = lg[R - L + 1];
    return max(d[L][k], d[R - (1 << k) + 1][k]);
}

int main() {
    return 0;
}

```

3.7 哈希表

//key_t 应当为整数类型，且实际值必须非负

```

template<typename key_t, typename type> struct hash_table {
    static const int maxn = 1000010;
    static const int table_size = 11110007;
    int first[table_size], nxt[maxn], sz; //init: memset(first, 0, sizeof(first)), sz = 0
    key_t id[maxn];
    type data[maxn];
    type& operator[] (key_t key) {
        const int h = key % table_size;
        for (int i = first[h]; i; i = nxt[i])
            if (id[i] == key)

```

```

        return data[i];
    int pos = ++sz;
    nxt[pos] = first[h];
    first[h] = pos;
    id[pos] = key;
    return data[pos] = type();
}

bool count(key_t key) {
    for (int i = first[key % table_size]; i; i = nxt[i])
        if (id[i] == key)
            return true;
    return false;
}

type get(key_t key) { //如果 key 对应的值不存在, 则返回 type()。
    for (int i = first[key % table_size]; i; i = nxt[i])
        if (id[i] == key)
            return data[i];
    return type();
}

};

unordered_map<long long, long long> A;
hash_table<long long, long long> B;
const int maxn = 1000000;
int main() {
    default_random_engine e;
    uniform_int_distribution<long long> d(0, LLONG_MAX);
    for (int i = 0; i < maxn; ++i) {
        long long key = d(e);
        long long value = d(e);
        A[key] = value;
        B[key] = value;
    }
    for (int i = 0; i < maxn * 10; ++i) {
        long long key = d(e);
        if (A.count(key) != B.count(key))
            abort();
        if (A.count(key)) {
            if (A[key] != B.get(key))
                abort();
        }
    }
    return 0;
}

```

3.8 哈希值组合

```
template<typename F, typename ...T> inline size_t hash_combine(const F& first, const T&...
↪ params) {
    if constexpr (sizeof...(params) == 0) {
        return std::hash<F>()(first);
    }
    else {
        size_t seed = hash_combine(params...);
        return std::hash<F>()(first) + 0x9e3779b9 + (seed << 6) + (seed >> 2);
    }
}

template<typename A, typename B>
struct std::hash<std::pair<A, B>> {
    inline size_t operator() (const std::pair<A, B>& pr) const {
        return hash_combine(pr.first, pr.second);
    }
};

template<typename ...T>
struct std::hash<std::tuple<T...>> {
    inline size_t operator() (const std::tuple<T...>& tp) const {
        return apply(hash_combine<T...>, tp); // 需要 C++17
    }
};

int main() {
    std::unordered_set<std::tuple<int, int, std::string>> S;
    S.insert({ 1, 2, "asd"s });
    S.insert({ 4, 5, "zxc"s });
    for (auto [a, b, c] : S) {
        std::cout << a << " " << b << " " << c << std::endl;
    }
    // 1 2 asd
    // 4 5 zxc
    return 0;
}
```

3.9 基数排序

```
const int maxn = 1100000; //maxn 应当大于 data 数组的大小
namespace RadixSort_Int {
    const int base = (1 << 17) - 1;
    int c[base + 10], tmp[maxn];
    void sort(int* data, int n) { //只能用于对非负数离散化, 若要处理负数,
        for (int i = 0; i < 32; i += 16) { //可以将所有的值都加上一个特别大的数使其变为非负数然
↪ 后再离散化
            memset(c, 0, sizeof(c));
            for (int j = 0; j < n; ++j)
```

```

        c[(data[j] >> i) & base]++;
    for (int j = 1; j <= base; ++j)
        c[j] += c[j - 1];
    for (int j = n - 1; j >= 0; --j)
        tmp[--c[(data[j] >> i) & base]] = data[j];
    for (int j = 0; j < n; ++j)
        data[j] = tmp[j];
}
}
}

namespace RadixSort_LLong {
    const int base = (1 << 17) - 1;
    int c[base + 10];
    long long tmp[maxn];
    void sort(long long* data, int n) { //只能用于对非负数离散化，若要处理负数，
        for (int i = 0; i < 64; i += 16) { //可以将所有的值都加上一个特别大的数使其变为非负数然
            ↪ 后再离散化
                memset(c, 0, sizeof(c));
                for (int j = 0; j < n; ++j)
                    c[(data[j] >> i) & base]++;
                for (int j = 1; j <= base; ++j)
                    c[j] += c[j - 1];
                for (int j = n - 1; j >= 0; --j)
                    tmp[--c[(data[j] >> i) & base]] = data[j];
                for (int j = 0; j < n; ++j)
                    data[j] = tmp[j];
            }
        }
    }

    int A[maxn], B[maxn];
    int main() {
        int n = 1000000;
        default_random_engine e;
        uniform_int_distribution<int> d(0, INT_MAX);
        for (int i = 0; i < n; ++i)
            A[i] = B[i] = d(e);
        printf("start....\n");
        auto start = clock();
        sort(A, A + n);
        printf("std::sort: %f\n", static_cast<double>(clock() - start) / CLOCKS_PER_SEC);
        start = clock();
        RadixSort_Int::sort(B, n);
        printf("std::sort: %f\n", static_cast<double>(clock() - start) / CLOCKS_PER_SEC);
        for (int i = 0; i < n; ++i)
            if (A[i] != B[i])
                abort();
    }
}

```

```

    return 0;
}

```

3.10 快速离散化

`const int maxn = 11000000;` //maxn 应当大于 data 数组的大小

```

namespace Discretization_Int {
    const int base = (1 << 17) - 1;
    int c[base + 10];
    pair<int, int> data[maxn], tmp[maxn];
    void discretization(int* input, int n) { //只能用于对非负数离散化, 若要处理负数,
        for (int i = 0; i < n; ++i) //可以将所有的值都加上一个特别大的数使其变为非负

```

→ 数然后再离散化

```

        data[i] = make_pair(input[i], i);
        for (int i = 0; i < 32; i += 16) {
            memset(c, 0, sizeof(c));
            for (int j = 0; j < n; ++j)
                c[(data[j].first >> i) & base]++;
            for (int j = 1; j <= base; ++j)
                c[j] += c[j - 1];
            for (int j = n - 1; j >= 0; --j)
                tmp[--c[(data[j].first >> i) & base]] = data[j];
            for (int j = 0; j < n; ++j)
                data[j] = tmp[j];
        }
        for (int i = 0, j = -1; i < n; ++i) {
            if (i == 0 || data[i].first != data[i - 1].first)
                ++j;
            input[data[i].second] = j;
        }
    }
}

```

```

namespace Discretization_LLong {
    const int base = (1 << 17) - 1;
    int c[base + 10];
    pair<long long, int> data[maxn], tmp[maxn];
    void discretization(long long* input, int n) { //只能用于对非负数离散化, 若要处理负数,
        for (int i = 0; i < n; ++i) //可以将所有的值都加上一个特别大的数使其变为

```

→ 非负数然后再离散化

```

        data[i] = make_pair(input[i], i);
        for (int i = 0; i < 64; i += 16) {
            memset(c, 0, sizeof(c));
            for (int j = 0; j < n; ++j)
                c[(data[j].first >> i) & base]++;
            for (int j = 1; j <= base; ++j)
                c[j] += c[j - 1];
            for (int j = n - 1; j >= 0; --j)

```

```

        tmp[--c[(data[j].first >> i) & base]] = data[j];
    for (int j = 0; j < n; ++j)
        data[j] = tmp[j];
}
for (int i = 0, j = -1; i < n; ++i) {
    if (i == 0 || data[i].first != data[i - 1].first)
        ++j;
    input[data[i].second] = j;
}
}

long long A[maxn], B[maxn], tmp[maxn];
int main() {
    int n = 100000000;
    default_random_engine e;
    uniform_int_distribution<long long> d(0, LLONG_MAX);
    e.seed(time(0));
    for (int i = 0; i < n; ++i)
        A[i] = B[i] = tmp[i] = d(e);
    printf("start....\n");
    auto start = clock();
    sort(tmp, tmp + n);
    int sz = unique(tmp, tmp + n) - tmp;
    for (int i = 0; i < n; ++i)
        A[i] = lower_bound(tmp, tmp + sz, A[i]) - tmp;
    printf("std::sort: %f\n", static_cast<double>(clock() - start) / CLOCKS_PER_SEC);
    start = clock();
    Discretization_LLong::discretization(B, n);
    printf("std::sort: %f\n", static_cast<double>(clock() - start) / CLOCKS_PER_SEC);
    for (int i = 0; i < n; ++i)
        if (A[i] != B[i])
            abort();
    return 0;
}

```

3.11 带删除的优先队列

```

template<typename T> class heap {
private:
    priority_queue<T> Q, R;
public:
    void maintain() {
        while (!R.empty() && Q.top() == R.top())
            Q.pop(), R.pop();
    }
    void push(const T& val) {
        Q.push(val);
    }

```



```

}
void erase(const T& val) { //只能删除还在优先队列中的值，不能删除不存在的值。
    R.push(val);
    maintain();
}
void pop() {
    Q.pop();
    maintain();
}
T top() {
    return Q.top();
}
T top2() { //返回第二大的值，调用之前必须保证有 size() >= 2。
    auto val = Q.top(); pop();
    auto sec = Q.top(); push(val);
    return sec;
}
auto size() const {
    return Q.size() - R.size();
}
bool empty() const {
    return size() == 0;
}
template<typename ...F> void emplace(F&&... args) {
    Q.emplace(std::forward<F>(args)...);
}
};
int main() {
    return 0;
}

```

3.12 排列组合枚举

```

void enumerate(int n, int k) {
    //枚举 [0, n) 的所有的子集的子集
    for (int s = 0; s < (1 << n); ++s) {
        for (int s0 = s; s0; s0 = (s0 - 1) & s) { //枚举集合 s 的所有子集
        }
    }
    //枚举 [0, n) 中所有大小为 k 的子集
    for (int x, y, s = (1 << k) - 1; s < (1 << n); x = s & -s, y = s + x, s = (((s & ~y) / x) >>
↪ 1) | y) {
        //cout << bitset<5>(s) << endl;
    }
    //枚举 [0, n) 中所有大小为 k 的排列
    for (int x, y, s = (1 << k) - 1; s < (1 << n); x = s & -s, y = s + x, s = (((s & ~y) / x) >>
↪ 1) | y) {

```

```

    vector<int> P;
    for (int i = 0; i < n; ++i) if (s & (1 << i))
        P.push_back(i);
    do {
        //for (int i = 0; i < k; ++i) printf("%d ", P[i]); printf("\n");
    } while (next_permutation(P.begin(), P.end()));
}
}

int main() {
    enumerate(4, 3);
    return 0;
}

```

3.13 动态维护连续相同数字区间

```

/*
动态维护由相同数字构成的区间， $A$  是原始数组。将  $A$  中相同数字只保留最左侧，其余设为 -1 得到  $val$  数组。
该算法动态维护  $val$  数组，时间复杂度  $O(1)$ 。
*/
const int maxn = 210000;
int A[maxn], val[maxn], n;
void build() {
}

inline void add(int index) { //将  $A[index]$  添加到  $val[index]$  中构成一个新的区间
    val[index] = A[index];
    //此处添加代码维护数据结构
}

inline void del(int index) { //删除  $val[index]$ ，也就是删除下标为  $index$  的区间
    val[index] = -1;
    //此处添加代码维护数据结构
}

void init() { //调用  $init$  函数初始化之前需要读入  $n$  和  $A$ 
    A[0] = A[n + 1] = -1;
    val[1] = A[1];
    for (int i = 2; i <= n; ++i)
        val[i] = (A[i] == A[i - 1] ? -1 : A[i]);
    build(); //用  $val$  数组构建数据结构
}

void change(int index, int v) { //将  $A$  中下标为  $index$  的位置的元素修改为  $v$ ，函数会维护  $A$  和  $val$  数
↪ 组
    if (A[index] == v)
        return;
    if (A[index] == A[index + 1])
        add(index + 1);
    if (val[index] != -1)
        del(index);
    if (val[index + 1] == v)

```

```

        del(index + 1);
    A[index] = v;
    if (A[index - 1] != v)
        add(index);
}
int main() {
    freopen("in.txt", "r", stdin);
    n = 5;
    for (int i = 1; i <= n; ++i)
        A[i] = 1;
    init();
    for (int i = 1; i <= n; ++i) printf("%d ", val[i]); printf("\n");
    change(3, 2); for (int i = 1; i <= n; ++i) printf("%d ", val[i]); printf("\n");
    change(4, 2); for (int i = 1; i <= n; ++i) printf("%d ", val[i]); printf("\n");
    change(2, 2); for (int i = 1; i <= n; ++i) printf("%d ", val[i]); printf("\n");
    change(2, 1); for (int i = 1; i <= n; ++i) printf("%d ", val[i]); printf("\n");
    change(1, 2); for (int i = 1; i <= n; ++i) printf("%d ", val[i]); printf("\n");
    change(2, 2); for (int i = 1; i <= n; ++i) printf("%d ", val[i]); printf("\n");
    change(3, 1); for (int i = 1; i <= n; ++i) printf("%d ", val[i]); printf("\n");
    change(3, 2); for (int i = 1; i <= n; ++i) printf("%d ", val[i]); printf("\n");
    return 0;
}

```

3.14 二分查找

```

#define MAXN 1000000
int BSearch_Upper(function<bool(int)> ok) //求满足 ok 函数的值的上界（满足 ok 函数的最大值）
{
    int L = 0, R = MAXN;
    while (L < R)
    {
        int M = L + (R - L + 1) / 2;
        if (ok(M)) L = M;
        else R = M - 1;
    }
    return L;
}
int BSearch_Lower(function<bool(int)> ok) //求满足 ok 函数的值的下界（满足 ok 函数的最小值）
{
    int L = 0, R = MAXN;
    while (L < R)
    {
        int M = L + (R - L) / 2;
        if (ok(M)) R = M;
        else L = M + 1;
    }
    return L;
}

```

```

}

double BSearch_Upper_Double(function<bool(double)> ok) //精度型二分法（满足 ok 函数的最大值）
{
    double L = 0, R = MAXN;
    while (R - L > 1e-5)
    {
        double M = (L + R) / 2;
        if (ok(M)) L = M;
        else R = M;
    }
    return L;
}

double BSearch_Lower_Double(function<bool(double)> ok) //精度型二分法（满足 ok 函数的最小值）
{
    double L = 0, R = MAXN;
    while (R - L > 1e-5)
    {
        double M = (L + R) / 2;
        if (ok(M)) R = M;
        else L = M;
    }
    return L;
}

int main()
{
    std::cout << BSearch_Upper([](int v) {return v <= 5; }) << std::endl;
    std::cout << BSearch_Lower([](int v) {return v >= 5; }) << std::endl;
    std::cout << BSearch_Upper_Double([](double v) {return v <= 5; }) << std::endl;
    std::cout << BSearch_Lower_Double([](double v) {return v >= 5; }) << std::endl;
    return 0;
}

```

4 匹配算法

4.1 匈牙利算法

```

// UVa11419 SAM I AM
// Rujia Liu

const int maxn = 1000 + 5; // 单侧顶点的最大数目
// 二分图最大基数匹配

struct BPM {
    int n, m; // 左右顶点个数
    vector<int> G[maxn]; // 邻接表
    int left[maxn]; // left[i] 为右边第 i 个点的匹配点编号, -1 表示不存在
    bool T[maxn]; // T[i] 为右边第 i 个点是否已标记
    int right[maxn]; // 求最小覆盖用
    bool S[maxn]; // 求最小覆盖用
}

```

```

void init(int n, int m) {
    this->n = n;
    this->m = m;
    for(int i = 0; i < n; i++) G[i].clear();
}
void AddEdge(int u, int v) {
    G[u].push_back(v);
}
bool match(int u){
    S[u] = true;
    for(int i = 0; i < G[u].size(); i++) {
        int v = G[u][i];
        if (!T[v]){
            T[v] = true;
            if (left[v] == -1 || match(left[v])){
                left[v] = u;
                right[u] = v;
                return true;
            }
        }
    }
    return false;
}
// 求最大匹配
int solve() {
    memset(left, -1, sizeof(left));
    memset(right, -1, sizeof(right));
    int ans = 0;
    for(int u = 0; u < n; u++) { // 从左边结点 u 开始增广
        memset(S, 0, sizeof(S));
        memset(T, 0, sizeof(T));
        if(match(u)) ans++;
    }
    return ans;
}
// 求最小覆盖。X 和 Y 为最小覆盖中的点集（最大独立集与最小覆盖集互补）
int mincover(vector<int>& X, vector<int>& Y) {
    int ans = solve();
    memset(S, 0, sizeof(S));
    memset(T, 0, sizeof(T));
    for(int u = 0; u < n; u++)
        if(right[u] == -1) match(u); // 从所有 X 未盖点出发增广
    for(int u = 0; u < n; u++)
        if(!S[u]) X.push_back(u); // X 中的未标记点
    for(int v = 0; v < m; v++)
        if(T[v]) Y.push_back(v); // Y 中的已标记点
}

```

```

    return ans;
}
};
BPM solver;
int R, C, N;
int main(){
    int kase = 0;
    while(scanf("%d%d%d", &R, &C, &N) == 3 && R && C && N) {
        solver.init(R, C);
        for(int i = 0; i < N; i++) {
            int r, c;
            scanf("%d%d", &r, &c); r--; c--;
            solver.AddEdge(r, c);
        }
        vector<int> X, Y;
        int ans = solver.mincover(X, Y);
        printf("%d", ans);
        for(int i = 0; i < X.size(); i++) printf(" r%d", X[i]+1);
        for(int i = 0; i < Y.size(); i++) printf(" c%d", Y[i]+1);
        printf("\n");
    }
    return 0;
}

```

4.2 KM 算法

```

const long long inf = 1LL << 60;
const int maxn = 505;
//若要求不完美匹配，需要把所有的-inf 替换成 0。
struct KM {
    int n, py[maxn], vy[maxn], pre[maxn];
    long long G[maxn][maxn], slk[maxn], kx[maxn], ky[maxn];
    void init(int n) { //左右两侧各有 n 个结点，结点编号从 1 开始
        this->n = n;
        for (int i = 1; i <= n; ++i)
            for (int j = 1; j <= n; ++j)
                G[i][j] = -inf;
        for (int i = 1; i <= n; ++i)
            ky[i] = py[i] = 0;
    }
    void add_edge(int x, int y, long long w) {
        G[y][x] = max(G[y][x], w);
    }
    long long solve() {
        int k, p;
        for (int i = 1; i <= n; ++i)
            kx[i] = *max_element(G[i] + 1, G[i] + n + 1);
    }
}

```

```

    for (int i = 1; i <= n; i++) {
        for (int j = 0; j <= n; ++j)
            vy[j] = pre[j] = 0, slk[j] = inf;
        for (py[k = 0] = i; py[k]; k = p) {
            long long d = inf;
            int x = py[k];
            vy[k] = 1;
            for (int j = 1; j <= n; j++) if (!vy[j]) {
                long long t = kx[x] + ky[j] - G[x][j];
                if (t < slk[j])
                    slk[j] = t, pre[j] = k;
                if (slk[j] < d)
                    d = slk[j], p = j;
            }
            for (int j = 0; j <= n; j++) {
                if (vy[j])
                    kx[py[j]] -= d, ky[j] += d;
                else
                    slk[j] -= d;
            }
        }
        for (; k; k = pre[k])
            py[k] = py[pre[k]];
    }
    long long ans = 0;
    for (int i = 1; i <= n; i++) if (G[py[i]][i] > -inf)
        ans += kx[i] + ky[i];
    return ans;
}

vector<int> result() { //返回左侧每个结点对应的右侧匹配点, 没有则为 0
    vector<int> res(1); //调用该函数之前, 应当先调用 solve 函数
    for (int i = 1; i <= n; ++i)
        res.push_back(G[py[i]][i] > -inf ? py[i] : 0);
    return res;
}

}km;

struct KM_old {
    static const int inf = 1 << 30;
    int n;
    vector<int> G[maxn];
    int W[maxn][maxn];
    int lx[maxn], ly[maxn];
    int left[maxn];
    bool S[maxn], T[maxn];
    void init(int n) {
        this->n = n;
    }

```

```

    for (int i = 0; i < n; ++i)
        G[i].clear();
    memset(W, 0, sizeof(W));
}

void add_edge(int u, int v, int w) {
    G[u].push_back(v);
    W[u][v] = w;
}

bool match(int u) {
    S[u] = true;
    for (unsigned i = 0; i < G[u].size(); ++i) {
        int v = G[u][i];
        if (lx[u] + ly[v] == W[u][v] && !T[v]) {
            T[v] = true;
            if (left[v] == -1 || match(left[v])) {
                left[v] = u;
                return true;
            }
        }
    }
    return false;
}

void update() {
    int a = inf;
    for (int u = 0; u < n; ++u) if (S[u]) {
        for (unsigned i = 0; i < G[u].size(); ++i) {
            int v = G[u][i];
            if (!T[v]) a = min(a, lx[u] + ly[v] - W[u][v]);
        }
    }
    for (int i = 0; i < n; ++i) {
        if (S[i]) lx[i] -= a;
        if (T[i]) ly[i] += a;
    }
}

int solve() {
    for (int i = 0; i < n; ++i) {
        lx[i] = *max_element(W[i], W[i] + n);
        left[i] = -1;
        ly[i] = 0;
    }
    for (int u = 0; u < n; ++u) {
        for (;;) {
            for (int i = 0; i < n; ++i) S[i] = T[i] = false;
            if (match(u)) break; else update();
        }
    }
}

```



```

    }
    int ans = 0;
    for (int i = 0; i < n; ++i) if (left[i] != -1)
        ans += W[left[i]][i];
    return ans;
}
};

int main() {
    int n, m;
    scanf("%d %d", &n, &m);
    km.init(n);
    for (int i = 1; i <= m; ++i) {
        int x, y, w;
        scanf("%d %d %d", &x, &y, &w);
        km.add_edge(y, x, w);
    }
    printf("%lld\n", km.solve());
    auto res = km.result();
    for (int i = 1; i <= n; ++i)
        printf("%d ", res[i]);
    printf("\n");
    return 0;
}

```

4.3 带花树算法

```

const int maxn = 1005;
struct Blossom {
    int mate[maxn], nxt[maxn], pa[maxn], st[maxn], vis[maxn], t, n;
    int ban[maxn];
    vector<int> G[maxn];
    queue<int> Q;
    void init(int n) { //编号从 0 开始
        this->n = n;
        this->t = 0;
        memset(mate, -1, sizeof(mate));
        memset(ban, 0, sizeof(ban));
        for (int i = 0; i < n; ++i)
            G[i].clear();
    }
    inline void add_edge(int x, int y) { //添加双向边 (x, y)
        G[x].push_back(y);
        G[y].push_back(x);
    }
    inline int find(int x) {
        return pa[x] == x ? x : pa[x] = find(pa[x]);
    }
}

```

```

inline void merge(int a, int b) {
    pa[find(a)] = find(b);
}

int lca(int x, int y) {
    for (t++;;swap(x, y)) if (~x) {
        if (vis[x = find(x)] == t)
            return x;
        vis[x] = t;
        x = ~mate[x] ? nxt[mate[x]] : -1;
    }
}

void group(int a, int p) {
    for (int b, c; a != p; merge(a, b), merge(b, c), a = c) {
        b = mate[a], c = nxt[b];
        if (find(c) != p)
            nxt[c] = b;
        if (st[b] == 2)
            st[b] = 1, Q.push(b);
        if (st[c] == 2)
            st[c] = 1, Q.push(c);
    }
}

void augment(int s) {
    for (int i = 0; i < n; ++i)
        nxt[i] = vis[i] = -1, pa[i] = i, st[i] = 0;
    Q = queue<int>();
    Q.push(s);
    st[s] = 1;
    while (mate[s] == -1 && !Q.empty()) {
        int x = Q.front(); Q.pop();
        for (auto y : G[x]) {
            if (!ban[y] && y != mate[x] && find(x) != find(y) && st[y] != 2) {
                if (st[y] == 1) {
                    int p = lca(x, y);
                    if (find(x) != p)
                        nxt[x] = y;
                    if (find(y) != p)
                        nxt[y] = x;
                    group(x, p);
                    group(y, p);
                }
                else if (mate[y] == -1) {
                    nxt[y] = x;
                    for (int j = y, k, i; ~j; j = i)
                        k = nxt[j], i = mate[k], mate[j] = k, mate[k] = j;
                    break;
                }
            }
        }
    }
}

```

```

        }
        else
            nxt[y] = x, Q.push(mate[y]), st[mate[y]] = 1, st[y] = 2;
    }
}
}

void solve() { //求最大匹配
    for (int i = 0; i < n; ++i) if (mate[i] == -1) //从所有的点开始增广
        augment(i);
    int ans = 0;
    for (int i = 0; i < n; ++i) if (mate[i] != -1)
        ans++;
    printf("%d\n", ans / 2); //匹配数
    for (int i = 0; i < n; ++i) //打印每个点的匹配点, -1 表示未匹配
        printf("%d ", mate[i]);
    printf("\n");
}

vector<int> unnecessary() { //求出所有的非必要匹配点 (存在一个最大匹配不包含这个点)
    for (int i = 0; i < n; ++i) if (mate[i] == -1) //先求出最大匹配
        augment(i);
    vector<int> ret;
    for (int x = 0; x < n; ++x) {
        if (mate[x] == -1)
            ret.push_back(x);
        else {
            int y = mate[x];
            mate[y] = -1;
            mate[x] = -1;
            ban[x] = 1;
            augment(y);
            ban[x] = 0;
            if (mate[y] != -1)
                ret.push_back(x);
            else
                augment(y);
        }
    }
    return ret;
}

}mp;

int main() {
    int n, m;
    scanf("%d %d", &n, &m);
    mp.init(n);
    for (int i = 0; i < m; ++i) {

```

```

    int x, y;
    scanf("%d %d", &x, &y); --x; --y;
    mp.add_edge(x, y);
}
mp.solve();
return 0;
}

```

5 高精度

5.1 大整数类

```

typedef long long ll;
const int base = 1000000000;
const int num_digit = 8;
const int maxn = 1000;
ll mul_mod (ll x, ll y, ll n){
    ll T = floor(sqrt(n) + 0.5);
    ll t = T * T - n;
    ll a = x / T; ll b = x % T;
    ll c = y / T; ll d = y % T;
    ll e = a * c / T; ll f = a * c % T;
    ll v = ((a * d + b * c) % n + e * t) % n;
    ll g = v / T; ll h = v % T;
    ll ans = (((f + g) * t % n + b * d) % n + h * T) % n;
    while (ans < 0) ans += n;
    return ans;
}

struct bign {
    int len;
    int s[maxn];
    bign(const char *str = "0"){ (*this) = str; }
    bign operator= (const char *str){
        int i;
        int j = strlen(str) - 1;
        len = j / num_digit + 1;
        for(i = 0; i <= len ; i++) s[i] = 0;
        for(i = 0; i <= j; i++){
            int k = (j - i) / num_digit + 1;
            s[k] = s[k] * 10 + str[i] - '0';
        }
        return *this ;
    }
};

void print(const bign &a){
    printf("%d", a.s[a.len]);
    for(int i = a.len - 1; i >= 1; i--)

```

```

        printf("%0*d", num_digit, a.s[i]);
    }
    //比较的前提是整数没有前导 0
    int compare (const bign &a, const bign &b){
        if(a.len > b.len) return 1;
        if(a.len < b.len) return -1;
        int i = a.len;
        while ((i > 1) && (a.s[i] == b.s[i])) i--;
        return a.s[i] - b.s[i];
    }
    inline bool operator< (const bign &a, const bign &b)
    {
        return compare(a, b) < 0;
    }
    inline bool operator<= (const bign &a, const bign &b)
    {
        return compare(a, b) <= 0;
    }
    inline bool operator== (const bign &a, const bign &b)
    {
        return compare(a, b) == 0;
    }
    //加法和减法很容易写出，只需注意不要忽略前导 0
    bign operator+ (const bign &a, const bign &b){
        bign c;
        int i;
        for(i = 1; i <= a.len || i <= b.len || c.s[i]; i++){
            if(i <= a.len) c.s[i] += a.s[i];
            if(i <= b.len) c.s[i] += b.s[i];
            c.s[i+1] = c.s[i] / base;
            c.s[i] %= base;
        }
        c.len = i-1;
        if(c.len == 0) c.len = 1;
        return c;
    }
    //减法的前提是 a > b
    bign operator- (const bign &a, const bign &b){
        bign c;
        int i, j;
        for(i = 1, j = 0; i <= a.len; i++){
            c.s[i] = a.s[i] - j;
            if(i <= b.len) c.s[i] -= b.s[i];
            if(c.s[i] < 0){ j = 1; c.s[i] += base; }
            else j = 0;
        }
    }

```

```

    c.len = a.len ;
    while (c.len > 1 && ! c.s[c.len]) c.len--;
    return c;
}

bign operator* (const bign &a, const bign &b){
    bign c;
    ll g = 0;
    int i, k;
    c.len = a.len + b.len;
    c.s[0] = 0;
    for(i = 1; i <= c.len; i++) c.s[i] = 0;
    for(k = 1; k <= c.len; k++){
        ll tmp = g;
        i = k + 1 - b.len;
        if(i < 1) i = 1;
        for (; i <= k && i <= a.len; i++)
            tmp += (ll)a.s[i] * (ll)b.s[k+1-i];
        g = tmp / base;
        c.s[k] = tmp % base;
    }
    while (c.len > 1 && !c.s[c.len]) c.len--;
    return c;
}

bign operator/ (const bign &a, int n)
{
    ll g = 0;
    bign c;
    c.len = a.len;
    for (int i = a.len; i > 0; --i)
    {
        ll tmp = g * base + a.s[i];
        c.s[i] = tmp / n;
        g = tmp % n;
    }
    while (c.len > 1 && !c.s[c.len]) c.len--;
    return c;
}

bign operator/ (const bign &a, const bign &b)
{
    bign L = "0", R = a;
    while (L < R)
    {
        bign M = L + (R - L + "1") / 2;
        if (M * b <= a) L = M;
        else R = M - "1";
    }
}

```

```

    return L;
}
ll bigmod(const bign &a, ll m){
    ll d = 0;
    for(int i = a.len; i > 0; --i){
        d = mul_mod(d, base, m);
        d = (d + a.s[i]) % m;
    }
    return d;
}
bign sqrt(const bign &n){
    bign c, d, x, y = n;
    do
    {
        x = y;
        y = (x + n / x) / 2;
    }
    while (y < x);
    return x;
}
bign gcd(bign a, bign b)
{
    bign c = "1";
    for (;;)
    {
        if (a == b)
            return a * c;
        else if (a.s[1] % 2 == 0 && b.s[1] % 2 == 0)
        {
            a = a / 2;
            b = b / 2;
            c = c * "2";
        }
        else if (a.s[1] % 2 == 0)
        {
            a = a / 2;
        }
        else if (b.s[1] % 2 == 0)
        {
            b = b / 2;
        }
        else if (b < a)
        {
            a = a - b;
        }
        else

```

```

        {
            b = b - a;
        }
    }
}

int main()
{
    bign a("345345345436546"), b("26768"), c, d;
    //divide(a, b, c, d);
    c = gcd(a, b);
    print(c);
    //print(a*b);
    //cout << '\n' << 3445453953435LL * 897676LL;
    return 0;
}

```

5.2 分数类

```

using integer = long long;
struct frac {
    integer num, den;
    frac() : num(0), den(1){}
    frac(integer val) {
        num = val;
        den = 1;
    }
    frac(integer a, integer b) {
        integer g = gcd(a, b);
        num = a / g;
        den = b / g;
        if (den < 0) {
            num = -num;
            den = -den;
        }
    }
};

frac operator+ (frac x, frac y) {
    return frac(x.num * y.den + y.num * x.den, x.den * y.den);
}

frac operator- (frac x, frac y) {
    return frac(x.num * y.den - y.num * x.den, x.den * y.den);
}

frac operator* (frac x, frac y) {
    return frac(x.num * y.num, x.den * y.den);
}

frac operator/ (frac x, frac y) {
    return frac(x.num * y.den, x.den * y.num);
}

```



```

}
frac operator+ (frac x, integer val) {
    return frac(x.num + x.den * val, x.den);
}
frac operator* (frac x, integer val) {
    return frac(x.num * val, x.den);
}
frac operator/ (frac x, integer val) {
    return frac(x.num, x.den * val);
}
bool operator== (frac x, frac y) {
    return x.num == y.num && x.den == y.den;
}
bool operator!= (frac x, frac y) {
    return x.num != y.num || x.den != y.den;
}
bool operator< (frac x, frac y) {
    return x.num * y.den < y.num * x.den;
}
frac abs(frac x) {
    if (x.num < 0)
        return frac(-x.num, x.den);
    return x;
}
int main() {
    printf("%d\n", gcd(0, 6));
    return 0;
}

```

6 动态规划

6.1 斯坦纳树（点权）

```

/*
BZOJ2595
输入一张带有点权的图，求将所有权值为 0 的结点连接起来所需要的最小费用（最小权值和）。
*/
const int inf = 0x3f3f3f3f; //inf + inf 必须小于 INT_MAX
                        //如果此处修改 inf, 那么也应当在 init 函数中修改 dp 数组的初始值。
const int maxn = 130;
const int maxedges = 1100;
const int maxstate = 11000;
struct SteinerTree {
    int n, k; //n 为图中点的个数, k 为斯坦纳树的结点数
    int dp[maxn][maxstate], st[maxn], val[maxn]; //若点 i 为点集中的点则 st[i] 为该点对应的状态, 否
    ↪ 则为 0
    int first[maxn], nxt[maxedges * 2], to[maxedges * 2], cur; //图的邻接表

```

```

pair<int, int> pre[maxn][maxstate];
bool inq[maxn][maxstate], vis[maxn];
queue<int> Q;
void init(int n, int* w, vector<int> c) { //n 为图中的所有结点个数, c 为斯坦纳树的结点集合
    this->n = n; this->k = c.size(); //w 为点权
    memset(dp, 0x3f, sizeof(dp));
    memset(pre, 0x3f, sizeof(pre));
    memset(st, 0, sizeof(st));
    for (int i = 0; i < k; ++i)
        st[c[i]] = (1 << i);
    for (int i = 1; i <= n; ++i)
        dp[i][st[i]] = 0;
    for (int i = 1; i <= n; ++i)
        val[i] = w[i];
    memset(inq, 0, sizeof(inq));
    memset(vis, 0, sizeof(vis));
    while (!Q.empty()) Q.pop();
    memset(first, 0, sizeof(first));
    cur = 0;
}
void add_edge(int u, int v) {
    nxt[++cur] = first[u];
    first[u] = cur;
    to[cur] = v;
}
void spfa(int s) { //对当前点集状态为 s 的 dp 值进行松弛
    while (!Q.empty()) {
        int u = Q.front(); Q.pop();
        for (int i = first[u]; i; i = nxt[i]) {
            int v = to[i];
            if (dp[v][s] > dp[u][s] + val[v]) {
                dp[v][s] = dp[u][s] + val[v];
                pre[v][s] = make_pair(u, s);
                if (!inq[v][s]) {
                    Q.push(v);
                    inq[v][s] = true;
                }
            }
        }
    }
    inq[u][s] = false;
}
}
void solve() { //斯坦纳树的权值和为  $\min\{dp[i][(1 \ll k) - 1]\}$ ,  $1 \leq i \leq n$ 
    for (int j = 1; j < (1 << k); ++j) {
        for (int i = 1; i <= n; ++i) {
            for (int sub = (j - 1) & j; sub; sub = (sub - 1) & j) {

```

```

        int x = sub, y = j - sub;
        int t = dp[i][x] + dp[i][y] - val[i];
        if (dp[i][j] > t) {
            dp[i][j] = t;
            pre[i][j] = make_pair(i, sub);
        }
    }
    if (dp[i][j] < inf) {
        Q.push(i);
        inq[i][j] = true;
    }
}
spfa(j);
}
}

void dfs(int i, int state) {
    if (i == inf || pre[i][state].second == 0)
        return;
    vis[i] = 1; //vis[i] 表示结点 i 存在于斯坦纳树中
    pair<int, int> pr = pre[i][state];
    dfs(pr.first, pr.second);
    if (pr.first == i)
        dfs(i, state - pr.second);
}

}tree;

int A[maxn][maxn], weight[maxn], dr[] = { -1, 1, 0, 0 }, dc[] = { 0, 0, -1, 1 };
int main() {
#define node(i, j) ((i) * m + (j) + 1)
    //freopen("in.txt", "r", stdin);
    int n, m, nd = -1;
    scanf("%d %d", &n, &m);
    vector<int> c;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m; ++j) {
            scanf("%d", &A[i][j]);
            weight[node(i, j)] = A[i][j];
            if (A[i][j] == 0) {
                c.push_back(node(i, j));
                if (nd == -1) nd = node(i, j);
            }
        }
    }

    tree.init(n * m, weight, c);
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < m; ++j) {
            for (int k = 0; k < 4; ++k) {

```

```

        int x = i + dr[k], y = j + dc[k];
        if (x < 0 || y < 0 || x >= n || y >= m) continue;
        tree.add_edge(node(i, j), node(x, y));
    }
}
}
tree.solve();
printf("%d\n", tree.dp[nd][(1 << c.size()) - 1]);
tree.dfs(nd, (1 << c.size()) - 1);
for (int i = 0; i < n; i++) {
    for (int j = 0; j < m; j++) {
        if (!A[i][j]) printf("x");
        else if (tree.vis[node(i, j)]) printf("o");
        else printf("_");
        if (j == m - 1) printf("\n");
    }
}
return 0;
}

```

6.2 斯坦纳树（边权）

/*

HDU4085

给定 n 个点，前 k 个点属于集合 A ，后 k 个点属于集合 B 。点之间通过 m 条带权边相连。

要求选出权值和最小的边集使得 A 集合中的每个点都可以与 B 中的某个点进行配对（若 a 能走到 b 则可以考虑 \rightarrow 是使其配对）。

解决方案：将集合 A 与集合 B 中的点作为关键点求出斯坦纳树，因为最后的结果可以是一个森林，所以我们在进行 \rightarrow 一次 DP。

*/

```
const int inf = 0x3f3f3f3f; //inf + inf 必须小于 INT_MAX
```

//如果此处修改 inf ，那么也应当在 $init$ 函数中修改 dp 数组的初始值。

```
const int maxn = 110;
```

```
const int maxedges = 1100;
```

```
const int maxstate = 11000;
```

```
struct SteinerTree{
```

```
    int n, k; //n 为图中点的个数，k 为斯坦纳树的结点数
```

```
    int dp[maxn][maxstate], st[maxn]; //若点  $i$  为点集中的点则  $st[i]$  为该点对应的状态，否则为 0
```

```
    int first[maxn], nxt[maxedges * 2], to[maxedges * 2], weight[maxedges * 2], cur; //图的邻接表
```

```
    bool inq[maxn][maxstate];
```

```
    queue<int> Q;
```

```
    void init(int n, vector<int> c) { //n 为图中的所有结点数，c 为斯坦纳树的结点集合
```

```
        this->n = n; this->k = c.size();
```

```
        memset(dp, 0x3f, sizeof(dp));
```

```
        memset(st, 0, sizeof(st));
```

```
        for(int i = 0; i < k; ++i)
```

```
            st[c[i]] = (1 << i);
```

```

    for(int i = 1; i <= n; ++i)
        dp[i][st[i]] = 0;
    memset(inq, 0, sizeof(inq));
    while(!Q.empty()) Q.pop();
    memset(first, 0, sizeof(first));
    cur = 0;
}

void add_edge(int u, int v, int w) {
    nxt[++cur] = first[u];
    first[u] = cur;
    to[cur] = v;
    weight[cur] = w;
}

void spfa(int s) { //对当前点集状态为 s 的 dp 值进行松弛
    while (!Q.empty()) {
        int u = Q.front(); Q.pop();
        inq[u][s] = false;
        for (int i = first[u]; i; i = nxt[i]) {
            int v = to[i], w = weight[i];
            int state = st[v] | s;
            if (dp[v][state] > dp[u][s] + w) {
                dp[v][state] = dp[u][s] + w;
                if (state != s || inq[v][s])
                    continue;
                Q.push(v);
                inq[v][s] = true;
            }
        }
    }
}

void solve() { //斯坦纳树的权值和为  $\min\{dp[i][(1 \ll k) - 1]\}$ ,  $1 \leq i \leq n$ 
    for (int j = 1; j < (1 << k); ++j) {
        for (int i = 1; i <= n; ++i) {
            if (st[i] && (st[i] & j) == 0)
                continue;
            for (int sub = (j - 1) & j; sub; sub = (sub - 1) & j) {
                int x = st[i] | sub, y = st[i] | (j - sub);
                dp[i][j] = min(dp[i][j], dp[i][x] + dp[i][y]);
            }
            if (dp[i][j] != inf) {
                Q.push(i);
                inq[i][j] = true;
            }
        }
        spfa(j);
    }
}

```

```

    /*
    int ans = inf;
    for (int i = 1; i <= n; ++i)
        ans = min(ans, tree.dp[i][(1 << k) - 1]);
    */
}
}tree;
int n, m, k, d[maxstate];
bool check(int s) { //当且仅当状态 s 中 A 集合与 B 集合结点个数相等时, 才是合法状态
    int res = 0;
    for (int i = 0; s; i++, s >>= 1)
        res += (s & 1) * (i < k ? 1 : -1);
    return (res == 0 ? true : false);
}
int main() {
    //freopen("in.txt", "r", stdin);
    int T;
    scanf("%d", &T);
    while (T--) {
        scanf("%d %d %d", &n, &m, &k);
        vector<int> c;
        for (int i = 1; i <= k; ++i)
            c.push_back(i);
        for (int i = 1; i <= k; ++i)
            c.push_back(n - k + i);
        tree.init(n, c);
        for (int i = 0; i < m; ++i) {
            int u, v, w;
            scanf("%d %d %d", &u, &v, &w);
            tree.add_edge(u, v, w);
            tree.add_edge(v, u, w);
        }
        tree.solve();
        const int mask = (1 << (2 * k)) - 1;
        for (int s = 0; s <= mask; s++) {
            d[s] = inf;
            for (int i = 1; i <= n; i++)
                d[s] = min(d[s], tree.dp[i][s]); //初始值为状态 s 中的点连成一棵树的情况
        }
        for (int s = 1; s <= mask; s++) if (check(s))
            for (int p = (s - 1) & s; p; p = (p - 1) & s) if (check(p))
                d[s] = min(d[s], d[p] + d[s - p]); //考虑将树分解为森林
        if (d[mask] >= inf)
            puts("No solution");
        else
            printf("%d\n", d[mask]);
    }
}

```

```

    }
    return 0;
}

```

6.3 插头 DP

```

const int INF = 1000000000;
int nrows, ncols;
int G[10][10];
// 插头编号: 0 表示无插头, 1 表示和数字 2 连通, 2 表示和数字 3 连通
struct State {
    int up[9]; // up[i] (0 ≤ i < m) 表示第 i 列处轮廓线上方的插头编号
    int left; // 当前格 (即下一个要放置的方格) 左侧的插头
    // 三进制编码
    int encode() const {
        int key = left;
        for (int i = 0; i < ncols; i++)
            key = key * 3 + up[i];
        return key;
    }
    // 在 (row, col) 处放一个新方格。UDLR 分别为该方格上下左右四个边界上的插头编号
    // 产生的新状态存放在 T 里, 成功返回 true, 失败返回 false
    bool next(int row, int col, int U, int D, int L, int R, State& T) const {
        if (row == nrows - 1 && D != 0) return false; // 最下行下方不能有插头
        if (col == ncols - 1 && R != 0) return false; // 最右列右边不能有插头
        int must_left = (col > 0 && left != 0); // 是否必须要有左插头
        int must_up = (row > 0 && up[col] != 0); // 是否必须要有上插头
        if ((must_left && L != left) || (!must_left && L != 0)) return false; // 左插头不匹配
        if ((must_up && U != up[col]) || (!must_up && U != 0)) return false; // 上插头不匹配
        if (must_left && must_up && left != up[col]) return false; // 若左插头和上插头都存在, 二者
        ⇨ 必须匹配
        // 产生新状态。实际上只有当前列的下插头和 left 插头有变化
        for (int i = 0; i < ncols; i++) T.up[i] = up[i];
        T.up[col] = D;
        T.left = R;
        return true;
    }
};

int memo[9][9][59049]; // 3^10
// 当前要放置格子 (row, col), 状态为 S。返回最小总长度
int rec(int row, int col, const State& S) {
    if (col == ncols) { col = 0; row++; }
    if (row == nrows) return 0;
    int key = S.encode();
    int& res = memo[row][col][key];
    if (res >= 0) return res;
    res = INF;
}

```

```

State T;
if (G[row][col] <= 1) { // 空格 (0) 或者障碍格 (1)
    if (S.next(row, col, 0, 0, 0, 0, T)) res = min(res, rec(row, col + 1, T)); // 整个格子里
    ↪ 都不连线
    if (G[row][col] == 0) // 如果是空格, 可以连线。由于线不能分叉, 所以这条线一定连接格子的某两
    ↪ 个边界 (6 种情况)
        for (int t = 1; t <= 2; t++) { // 枚举线的种类。t=1 表示 2 线, t=2 表示 3 线
            if (S.next(row, col, t, t, 0, 0, T)) res = min(res, rec(row, col + 1, T) + 2); //
            ↪ 上 <-> 下
            if (S.next(row, col, t, 0, t, 0, T)) res = min(res, rec(row, col + 1, T) + 2); //
            ↪ 上 <-> 左
            if (S.next(row, col, t, 0, 0, t, T)) res = min(res, rec(row, col + 1, T) + 2); //
            ↪ 上 <-> 右
            if (S.next(row, col, 0, t, t, 0, T)) res = min(res, rec(row, col + 1, T) + 2); //
            ↪ 下 <-> 左
            if (S.next(row, col, 0, t, 0, t, T)) res = min(res, rec(row, col + 1, T) + 2); //
            ↪ 下 <-> 右
            if (S.next(row, col, 0, 0, t, t, T)) res = min(res, rec(row, col + 1, T) + 2); //
            ↪ 左 <-> 右
        }
    }
    else {
        int t = G[row][col] - 1; // 数字为 2 和 3, 但插头类型是 1 和 2, 所以要减 1
        // 由于线不能分叉, 所以这条线一定连接格子中间的数字和某一个边界 (4 种情况)
        if (S.next(row, col, t, 0, 0, 0, T)) res = min(res, rec(row, col + 1, T) + 1); // 从上边
        ↪ 界出来
        if (S.next(row, col, 0, t, 0, 0, T)) res = min(res, rec(row, col + 1, T) + 1); // 从下边
        ↪ 界出来
        if (S.next(row, col, 0, 0, t, 0, T)) res = min(res, rec(row, col + 1, T) + 1); // 从左边
        ↪ 界出来
        if (S.next(row, col, 0, 0, 0, t, T)) res = min(res, rec(row, col + 1, T) + 1); // 从右边
        ↪ 界出来
    }
    return res;
}

int main() {
    while (scanf("%d%d", &nrows, &ncols) == 2 && nrows && ncols) {
        for (int i = 0; i < nrows; i++)
            for (int j = 0; j < ncols; j++)
                scanf("%d", &G[i][j]);

        State S;
        memset(&S, 0, sizeof(S));
        memset(memo, -1, sizeof(memo));
        int ans = rec(0, 0, S);
        if (ans == INF) ans = 0;
        printf("%d\n", ans / 2);
    }
}

```



```

    }
    return 0;
}
/*
void normalize() {
    int rep[maxn] = {}, num = 0;
    for (int i = 0; i < m; ++i) if (cp[i] > 0) {
        if (rep[cp[i]] <= 0)
            rep[cp[i]] = ++num;
        cp[i] = rep[cp[i]];
    }
    if (left > 0) {
        if (rep[left] <= 0)
            rep[left] = ++num;
        left = rep[left];
    }
}
// 把所有编号为 b 的连通分量改成 a
void merge(int a, int b) {
    if (a == b) return;
    for (int i = 0; i < ncols; i++)
        if (comp[i] == b) comp[i] = a;
}
*/

```

6.4 最长上升子序列

```

const int maxn = 110000;
const int inf = 1 << 30;
//d[i] 表示以下标 i 结尾的最长上升子序列长度
int a[maxn], g[maxn], d[maxn], n;
int main() {
    scanf("%d", &n);
    for (int i = 1; i <= n; ++i)
        scanf("%d", &a[i]);
    for (int i = 1; i <= n; ++i)
        g[i] = inf;
    for (int i = 1; i <= n; ++i) {
        int k = lower_bound(g + 1, g + n + 1, a[i]) - g;
        g[k] = a[i];
        d[i] = k;
    }
    int ans = *max_element(d + 1, d + n + 1);
    printf("%d\n", ans);
    return 0;
}

```

7 莫队算法

7.1 莫队算法

```

const int maxn = 110000; // maxn > n + sqrt(n)
inline void del(int index) { //删除下标为 index 的元素
}
inline void add(int index) { //添加下标为 index 的元素
}
inline int get_ans() { //用小于  $O(\sqrt{n})$  的时间复杂度求出当前区间的解
    return 0;
}
namespace MoDui {
    struct query { //查询区间为 [l, r], id 表示是第几次查询
        int l, r, id;
    };
    int belong[maxn], ans[maxn];
    bool operator<(query a, query b) {
        if (belong[a.l] != belong[b.l])
            return belong[a.l] < belong[b.l];
        else if (belong[a.l] & 1)
            return a.r < b.r;
        else
            return a.r > b.r;
    }
    void work(int n, vector<query> q) { //n 指定数组大小 (下标从 1 开始)
        int size = sqrt(n);
        int number = ceil((double)n / size);
        for (int i = 1; i <= number; ++i)
            for (int j = (i - 1) * size + 1; j <= i * size; ++j)
                belong[j] = i;
        sort(q.begin(), q.end());
        int l = 1, r = 0;
        for (auto item : q) {
            while (l < item.l) del(l++);
            while (l > item.l) add(--l);
            while (r < item.r) add(++r);
            while (r > item.r) del(r--);
            ans[item.id] = get_ans();
        }
    }
}
int main() {
    return 0;
}

```

7.2 回滚莫队

```

/*
left[i] 表示第 i 块的左端点编号（块编号从 1 开始）
right[i] 表示第 i 块的右端点编号
belong[i] 表示下标 i 所在的块编号（下标从 1 开始）
*/
const int maxn = 110000;
void init() { //用 O(1) 的时间复杂度创建一个空的区间
}
void add_right(int index) { //将输入数组中下标 index 的元素添加到当前区间的右边
}
void add_left(int index) { //将输入数组中下标 index 的元素添加到当前区间的左边
    //该函数可能需要保存原状态，以便于之后状态的恢复
}
void breakpoint() { //记录下当前的状态以便于之后恢复
    //调用该函数之后会调用 add_left 修改状态，最后用 resume 恢复到调用 breakpoint
    ↪ 之前的状态
}
void resume() { //撤销所有的 add_left 操作，恢复到调用 breakpoint 之前的状态（一次性撤销所有的
    ↪ add_left）
}
int get_ans() { //用小于 O(sqrt(n)) 的时间复杂度求出当前区间的解
    return 0;
}
namespace MoDui {
    struct query { //查询区间为 [l, r], id 表示是第几次查询
        int l, r, id;
    };
    int left[maxn], right[maxn], belong[maxn], ans[maxn]; // ans 可能要用 long long 存
    bool operator< (query a, query b) {
        return belong[a.l] != belong[b.l] ? belong[a.l] < belong[b.l] : a.r < b.r;
    }
    void work(int n, vector<query> q) { //n 指定数组大小（下标从 1 开始）
        int size = sqrt(n);
        int number = ceil((double)n / size);
        for (int i = 1; i <= number; ++i) {
            left[i] = size * (i - 1) + 1;
            right[i] = size * i;
            for (int j = left[i]; j <= right[i]; ++j)
                belong[j] = i;
        }
        right[number] = n;
        sort(q.begin(), q.end());
        for (auto item : q) if (belong[item.l] == belong[item.r]) {
            init();
            for (int i = item.l; i <= item.r; ++i)

```

```

        add_right(i);
        ans[item.id] = get_ans();
    }
    for (int i = 0, k = 1; k <= number; ++k) {
        init();
        for (int r = right[k]; i < q.size() && belong[q[i].l] == k; ++i) {
            int ql = q[i].l, qr = q[i].r;
            int l = right[k] + 1;
            if (belong[ql] != belong[qr]) {
                while (r < qr) {
                    ++r;
                    add_right(r);
                }
                breakpoint();
                while (l > ql) {
                    --l;
                    add_left(l);
                }
                ans[q[i].id] = get_ans();
                resume();
            }
        }
    }
}

int main() {
    return 0;
}

```

7.3 带修改莫队

```

const int maxn = 210000; //  $maxn > n + pow(n, 2.0 / 3.0)$ 
struct query {
    int tp, l, r, id, t; //  $tp == 0$  对应查询操作: 查询区间为  $[l, r]$ 
    //  $tp == 1$  对应修改操作: 将下标为  $l$  的位置修改为  $r$ 
};

inline void del(int index) { // 删除下标为  $index$  的元素
}

inline void add(int index) { // 添加下标为  $index$  的元素
}

inline void change(query& q, bool flag) { // 将输入数组下标为  $q.l$  的值修改为  $q.r$ ,  $flag$  表示这次修改
    ↪ 是否位于当前区间内
    //  $flag$  为  $true$  则考虑这次修改对答案的影响, 否则不考虑
}

inline void undo(query& q, bool flag) { // 撤销修改操作,  $flag$  表示这次撤销是否位于当前处理的区间内
}

inline int get_ans() { // 用小于  $O(\sqrt{n})$  的时间复杂度求出当前区间的解
    return 0;
}

```

```

}
int belong[maxn], ans[maxn];
bool operator< (query a, query b) {
    if (belong[a.l] != belong[b.l])
        return belong[a.l] < belong[b.l];
    else if (belong[a.r] != belong[b.r])
        return belong[a.r] < belong[b.r];
    else
        return a.t < b.t;
}

void work(int n, vector<query> q) { //n 指定数组大小 (下标从 1 开始)
    int size = pow(n, 2.0 / 3.0);
    int number = ceil((double)n / size);
    for (int i = 1; i <= number; ++i)
        for (int j = (i - 1) * size + 1; j <= i * size; ++j)
            belong[j] = i;
    vector<query> a, b; //a 记录查询, b 记录修改
    for (auto item : q) {
        if (item.tp == 0) {
            item.id = a.size();
            item.t = b.size();
            a.push_back(item);
        }
        else {
            b.push_back(item);
        }
    }
    sort(a.begin(), a.end());
    int l = 1, r = 0, t = 0;
    for (auto item : a) {
        while (l < item.l) del(l++);
        while (l > item.l) add(--l);
        while (r < item.r) add(++r);
        while (r > item.r) del(r--);
        while (t < item.t) {
            change(b[t], item.l <= b[t].l && item.r >= b[t].l);
            ++t;
        }
        while (t > item.t) {
            --t;
            undo(b[t], item.l <= b[t].l && item.r >= b[t].l);
        }
        ans[item.id] = get_ans();
    }
    /*
    for (int i = 0; i < a.size(); ++i)

```

```

        printf("%d\n", ans[i]);
    */
}
int main() {
    return 0;
}

```

8 数据结构

8.1 可修改优先队列

```

template<typename T, int maxsize> struct ModifiablePriorityQueue { // 大根堆
    int data[maxsize * 2];
    int pos[maxsize * 2];
    T value[maxsize * 2];
    int sz;
    ModifiablePriorityQueue() : pos(), sz(0) {}
    void up(int i) { // 值变大
        auto index = data[i];
        auto val = value[index];
        while (i > 1) {
            int fa = i / 2;
            if (val > value[data[fa]]) {
                data[i] = data[fa];
                pos[data[i]] = i;
                i = fa;
            }
            else {
                break;
            }
        }
        data[i] = index;
        pos[index] = i;
    }
    void down(int i) { // 值变小
        auto index = data[i];
        auto val = value[index];
        while (i * 2 <= sz) {
            int child = i * 2;
            if (i * 2 + 1 <= sz && value[data[i * 2 + 1]] > value[data[child]]) {
                child = i * 2 + 1;
            }
            if (val < value[data[child]]) {
                data[i] = data[child];
                pos[data[i]] = i;
                i = child;
            }
        }
    }
}

```

```

        else {
            break;
        }
    }
    data[i] = index;
    pos[index] = i;
}

void push(int index, const T& val) {
    sz += 1;
    data[sz] = index;
    value[index] = val;
    up(sz);
}

void pop() {
    pos[data[1]] = 0;
    data[1] = data[sz];
    sz -= 1;
    if (sz > 0) {
        down(1);
    }
}

void erase(int index) {
    index = pos[index];
    pos[data[index]] = 0;
    data[index] = data[sz];
    sz -= 1;
    if (index <= sz) {
        down(index);
    }
}

void modify(int index, const T& val) {
    if (pos[index] == 0) {
        push(index, val);
    }
    else if (val > value[index]) {
        value[index] = val;
        up(pos[index]);
    }
    else {
        value[index] = val;
        down(pos[index]);
    }
}

int top() const {
    return data[1];
}

```

```

    T get(int index) const {
        return value[index];
    }
    T maximum() const {
        return value[data[1]];
    }
    bool contains(int index) const {
        return pos[index] != 0;
    }
    int size() const {
        return sz;
    }
    bool empty() const {
        return sz == 0;
    }
};

int main() { // 洛谷 P4779
    static int d[210000];
    static vector<pair<int, int>> G[210000];
    int n, m, s;
    scanf("%d %d %d", &n, &m, &s);
    for (int i = 0; i < m; ++i) {
        int x, y, z;
        scanf("%d %d %d", &x, &y, &z);
        G[x].emplace_back(y, z);
    }

    static ModifiablePriorityQueue<int, 210000> Q;
    memset(d, 0x3f, sizeof(d));
    d[s] = 0;
    Q.push(s, 0);
    while (!Q.empty()) {
        auto x = Q.top(); Q.pop();
        for (auto [y, w] : G[x]) {
            if (d[y] > d[x] + w) {
                d[y] = d[x] + w;
                Q.modify(y, -d[y]);
            }
        }
    }

    for (int i = 1; i <= n; ++i) {
        printf("%d ", d[i] == 0x3f3f3f3f ? INT_MAX : d[i]);
    }
    printf("\n");
    return 0;
}

```


8.2 AVL Tree

```

struct node {
    node *ch[2]; // 左右子树
    int height, value;
    node() : height(0), value(0), ch() {}
    void maintain() {
        height = max(ch[0]->height, ch[1]->height) + 1;
    }
}*nil = new node;

void rotate(node* &o, int d) {
    node* k = o->ch[d ^ 1];
    o->ch[d ^ 1] = k->ch[d];
    k->ch[d] = o;
    o->maintain(); k->maintain();
    o = k;
}

node *Insert(int X, node *T)
{
    if (T == nil) {
        T = new node;
        T->value = X;
        T->height = 0;
        T->ch[0] = T->ch[1] = nil;
    }
    else if (X < T->value) {
        T->ch[0] = Insert(X, T->ch[0]);
        if (T->ch[0]->height - T->ch[1]->height == 2) {
            if (X >= T->ch[0]->value)
                rotate(T->ch[0], 0);
            rotate(T, 1);
        }
    }
    else if (X > T->value) {
        T->ch[1] = Insert(X, T->ch[1]);
        if (T->ch[1]->height - T->ch[0]->height == 2){
            if (X <= T->ch[1]->value)
                rotate(T->ch[1], 1);
            rotate(T, 0);
        }
    }
    T->height = max(T->ch[0]->height, T->ch[1]->height) + 1;
    return T;
}

int main() {
    // freopen("in.txt", "r", stdin);
    int n;

```

```

scanf("%d", &n);
node *root = nil;
for (int i = 0; i < n; ++i) {
    int val;
    scanf("%d", &val);
    root = Insert(val, root);
}
printf("%d\n", root->value);
return 0;
}

```

8.3 TopTree

```

/*
K=0 表示子树修改, 后面 x,y, 表示以 x 为根的子树的点权值改成 y
K=1 表示换根, 后面 x, 表示把这棵树的根变成 x
K=2 表示链修改, 后面 x,y,z, 表示把这棵树中 x-y 的路径上点权值改成 z
K=3 表示子树询问 min, 后面 x, 表示以 x 为根的子树中点的权值 min
K=4 表示子树询问 max, 后面 x, 表示以 x 为根的子树中点的权值 max
K=5 表示子树加, 后面 x,y, 表示 x 为根的子树中点的权值 +y
K=6 表示链加, 后面 x,y,z, 表示把这棵树中 x-y 的路径上点权值改成 +z
K=7 表示链询问 min, 后面 x,y, 表示把这棵树中 x-y 的路径上点的 min
K=8 表示链询问 max, 后面 x,y, 表示把这棵树中 x-y 的路径上点的 max
K=9 表示换父亲, 后面 x,y, 表示把 x 的父亲换成 y, 如果 y 在 x 子树里不操作。
K=10 表示链询问 sum, 后面 x,y,z, 表示把这棵树中 x-y 的路径上点的 sum
K=11 表示子树询问 sum, 后面 x, 表示以 x 为根的子树的点权 sum
*/
//const int E_ALL = 0, E_CH = 1, E_RE = 2;
//const int S_LCT = 0, S_AAA = 2;
const int maxn = 110000 << 2, inf = INT_MAX;
struct mark { //splay 树上的标记类
    int k, b;
    mark(int k = 1, int b = 0) : k(k), b(b) {} //k == 0 表示设置值, k == 1 表示增加值, b 表示修改或
    ↪ 增加的值
    int operator()(int x, int sz = 1) { //将当前标记类作用于值 x (sz 指定值的个数)
        return k * x + sz * b;
    }
    mark& operator +=(mark a) { //将当前标记与 a 结合
        b = b * a.k + a.b; //计算新的标记值
        k *= a.k; //若两个标记中有一个为 set 则结果为 set
        return *this;
    }
    bool operator !=(mark a) {
        return k != a.k || b != a.b;
    }
}none;
struct info { //splay 树上的信息类

```

```

    int minv, maxv, sumv, sz;
    info(int minv = inf, int maxv = -inf, int sumv = 0, int sz = 0) :minv(minv), maxv(maxv),
    ↪ sumv(sumv), sz(sz) {}
    info operator +(info rhs) { //将信息进行合并
        return info(min(minv, rhs.minv), max(maxv, rhs.maxv), sumv + rhs.sumv, sz + rhs.sz);
    }
    info& operator +=(mark b) { //将标记作用于信息
        if (sz) {
            minv = b(minv);
            maxv = b(maxv);
            sumv = b(sumv, sz);
        }
        return *this;
    }
};

struct node {
    node* son[4], * fa;
    info se, re, ch, all;
    mark mch, mre;
    bool rev, imag;
    node() { //construction for null node
        fill(son, son + 4, fa = 0);
        se = re = ch = all = info();
        mch = mre = none;
        rev = imag = 0;
    }
    int kind() {
        for (int i = 0; i < 4; ++i)
            if (fa->son[i] == this)
                return i;
    }
    bool isroot(int type) {
        return type ? (!fa->imag || !imag) : (fa->son[0] != this && fa->son[1] != this);
    }
    void link(int d, node* s) {
        son[d] = s;
        s->fa = this;
    }
    void set(node* s, int d) {
        down();
        son[d] = s;
        s->fa = this;
        up();
    }
    void reverse() {
        swap(son[0], son[1]);
    }
};

```

```

    rev ^= 1;
}
void edit(mark f, int type) {
    //if (this == null) return;
    if (type == 0 || type == 1) {
        se += f;
        mch += f;
        ch += f;
    }
    if (type == 0 || type == 2) {
        mre += f;
        re += f;
    }
    all = ch + re;
}
void up() {
    ch = son[0]->ch + son[1]->ch + se;
    re = son[0]->re + son[1]->re + son[2]->all + son[3]->all;
    all = ch + re;
}
void down() {
    if (rev) {
        son[0]->reverse();
        son[1]->reverse();
        rev = 0;
    }
    if (mre != none) {
        son[0]->edit(mre, 2);
        son[1]->edit(mre, 2);
        son[2]->edit(mre, 0);
        son[3]->edit(mre, 0);
        mre = none;
    }
    if (mch != none) {
        son[0]->edit(mch, 1);
        son[1]->edit(mch, 1);
        mch = none;
    }
}
}pool[maxn], *me = pool, *bin[maxn], **ptr = bin, *null = new(me++) node, *nd[maxn], *root;
node* newnode(int val = inf) { //val 表示结点的初始权值
    node* cur = *ptr ? *ptr-- : me++;
    fill(cur->son, cur->son + 4, cur->fa = null);
    cur->re = info();
    cur->mch = cur->mre = none;
    cur->rev = 0;
}

```

```

    if (val == inf) {
        cur->se = cur->all = cur->ch = info();
        cur->imag = 1;
    }
    else {
        cur->se = cur->all = cur->ch = info(val, val, val, 1);
        cur->imag = 0;
    }
    return cur;
}

void delnode(node*& pos) {
    *++ptr = pos;
    pos = 0;
}

void trans(node* pos) {
    node* fa = pos->fa, * grf = fa->fa;
    fa->down(), pos->down();
    int d = pos->kind();
    if (grf != null)
        grf->son[fa->kind()] = pos;
    pos->fa = grf;
    fa->link(d, pos->son[d ^ 1]);
    pos->link(d ^ 1, fa);
    fa->up();
}

void splay(node* pos, int type) {
    while (!pos->isroot(type)) {
        node* fa = pos->fa;
        if (!fa->isroot(type))
            trans(fa->kind() == pos->kind() ? fa : pos);
        trans(pos);
    }
    pos->up();
}

void addvirt(node* pos, node* fa) {
    if (pos == null) return;
    fa->down();
    for (int i = 2; i < 4; ++i) {
        if (fa->son[i] == null) {
            fa->set(pos, i);
            return;
        }
    }
    while (fa->son[2]->imag)
        fa = fa->son[2];
    splay(fa, 2);
}

```

```

    node* vi = newnode();
    vi->link(2, fa->son[2]);
    vi->link(3, pos);
    fa->set(vi, 2);
    splay(vi, 2);
}

void delvirt(node* pos) {
    if (pos == null) return;
    static node** top = (node** )malloc(maxn * sizeof(node*));
    for (node* p = pos->fa; p->imag; p = p->fa) * ++top = p;
    if (*top) (*top)->fa->down();
    for (; *top; top--) (*top)->down();
    node* fa = pos->fa;
    int d = pos->kind();
    if (fa->imag) {
        fa->fa->set(fa->son[d ^ 1], fa->kind());
        splay(fa->fa, 2);
        delnode(fa);
    }
    else {
        fa->set(null, d);
        splay(fa, 2);
    }
    pos->fa = null;
}

void access(node* pos) {
    node* pred = null;
    while (pos != null) {
        splay(pos, 0);
        addvirt(pos->son[1], pos);
        delvirt(pred);
        pos->set(pred, 1);
        pred = pos;
        for (pos = pos->fa; pos->imag; pos = pos->fa);
    }
}

void touch(node* pos) {
    access(pos);
    splay(pos, 0);
}

void beroot(node* pos) { //将结点 pos 设置为根结点
    touch(pos);
    pos->reverse();
}

void link(node* x, node* y) {
    beroot(x);

```

```

    touch(y);
    addvirt(x, y);
}

void cut(node* x, node* y) {
    beroot(x);
    touch(y);
    y->set(null, 0);
    x->fa = null;
}

node* findroot(node* pos) {
    touch(pos);
    while (pos->son[0] != null) {
        pos->down();
        pos = pos->son[0];
    }
    splay(pos, 0);
    return pos;
}

node* findfa(node* pos) {
    touch(pos);
    pos = pos->son[0];
    if (pos == null) return 0;
    while (pos->son[1] != null) {
        pos->down();
        pos = pos->son[1];
    }
    splay(pos, 0);
    return pos;
}

void changefather(node *pos, node *father) {
    if (pos == root) return;
    node* fa = findfa(pos);
    cut(fa, pos);
    if (findroot(pos) != findroot(father))
        link(pos, father);
    else
        link(pos, fa);
}

namespace chain {
    info query(node *x, node *y) { //对 x 到 y 的链进行查询
        beroot(x);
        touch(y);
        return y->ch;
    }

    void edit(node *x, node *y, mark f) //对 x 到 y 的链进行修改
    {

```

```

        beroot(x);
        touch(y);
        y->edit(f, 1);
    }
}

namespace sub {
    info query(node *rt) { //在以 root 为根的情况下, 对 rt 子树进行查询
        touch(rt);
        return rt->se + rt->son[2]->all + rt->son[3]->all;
    }
    void edit(node *rt, mark f) { //在以 root 为根的情况下, 对 rt 子树进行修改
        touch(rt);
        rt->se += f;
        rt->son[2]->edit(f, 0);
        rt->son[3]->edit(f, 0);
    }
}

int main() {
#define get(x) scanf("%d",&x)
#define put(x) printf("%d",x)
    static int w[maxn], edge[maxn][2];
    int n, m, tmp; get(n), get(m);
    for (int i = 1; i <= n - 1; ++i)
        get(edge[i][0]), get(edge[i][1]);
    for (int i = 1; i <= n; ++i) {
        get(w[i]);
        nd[i] = newnode(w[i]);
    }
    get(tmp); root = nd[tmp];
    for (int i = 1; i <= n - 1; ++i) link(nd[edge[i][0]], nd[edge[i][1]]);
    for (int i = 1; i <= m; ++i) {
        int opt, x, y, z, Ans; get(opt);
        info res;
        beroot(root);
        switch (opt)
        {
            case 0:
            case 5: //子树修改
                get(x), get(y);
                sub::edit(nd[x], mark(opt == 0 ? 0 : 1, y)); //0 set 5 add
                break;
            case 1:
                get(tmp); root = nd[tmp]; //换根
                break;
            case 2:
            case 6: //链修改

```



```

        get(x), get(y), get(z);
        chain::edit(nd[x], nd[y], mark(opt == 2 ? 0 : 1, z));
        break;
    case 3://min
    case 4://max
    case 11://sum
        get(x);
        res = sub::query(nd[x]);
        switch (opt)
        {
            case 3:Ans = res.minv; break;
            case 4:Ans = res.maxv; break;
            case 11:Ans = res.sumv; break;
        }
        put(Ans), putchar('\n');
        break;
    case 7://min
    case 8://max
    case 10://sum
        get(x), get(y);
        res = chain::query(nd[x], nd[y]);
        switch (opt)
        {
            case 7:Ans = res.minv; break;
            case 8:Ans = res.maxv; break;
            case 10:Ans = res.sumv; break;
        }
        put(Ans), putchar('\n');
        break;
    case 9: //换父亲
        get(x), get(y);
        changefather(nd[x], nd[y]);
        break;
    }
}
return 0;
}

```

8.4 可持久化数组

```

#define lc t[p].lchild
#define rc t[p].rchild
template<typename T, int begin, int end, int factor = 30>
struct Array {
    static const int size = end - begin + 1;
    struct segment {
        int lchild, rchild;

```

```

    T val;
}t[size * factor];
int sz;
void ins(int& p, int x, int y, T A[]) {
    p = ++sz;
    if (x == y)
        t[p].val = A[x];
    else {
        int mid = (x + y) >> 1;
        ins(lc, x, mid, A);
        ins(rc, mid + 1, y, A);
    }
}
//将数组 A 中的值作为初始值, 并返回初始数组的版本号
int init(T A[]) {
    int root = sz = 0;
    ins(root, begin, end, A);
    return root;
}
//将 val 作为可持久化数组的初始值, 并返回初始数组的版本号 (0)
//采用此初始化方案时, 数组的区间范围可以很大,
//且可以在 get 函数中加入 p==0 就直接返回 t[0].val 的优化
int init(T val) {
    t[0].val = val;
    return 0;
}
//获取版本号 p 的下标为 index 的值
T get(int p, int index, int x = begin, int y = end) {
    if (x == y)
        return t[p].val;
    int mid = (x + y) >> 1;
    if (index <= mid)
        return get(lc, index, x, mid);
    else
        return get(rc, index, mid + 1, y);
}
//修改版本号为 p 的数组的下标 index 位置的值为 v
//使用方法为: root[i] = root[i-1]; set(root[i], idx, v);
void set(int &p, int index, const T& v, int x = begin, int y = end) {
    t[++sz] = t[p]; p = sz; //如果类型 T 的复制开销较大, 此处可以考虑只复制左右子树的编号
    if (x == y)
        t[p].val = v;
    else {
        int mid = (x + y) >> 1;
        if (index <= mid)
            set(lc, index, v, x, mid);
    }
}

```

```

        else
            set(rc, index, v, mid + 1, y);
    }
}
};
const int maxn = 1000;
Array<int, 0, maxn, 100> arr;
int A[maxn];
int main() {
    for (int i = 0; i < maxn; ++i)
        A[i] = -1;
    int p = arr.init(-1);
    for (int i = 0; i < maxn; ++i)
        if (arr.get(p, i) != A[i])
            abort();
    vector<int> total[maxn];
    vector<int> root; root.push_back(p);
    total[0] = vector<int>(A, A + maxn);
    int T = 10000;
    while (T--) {
        int tp = rand() % 10;
        int pos = rand() % maxn, nd = rand() % root.size();
        int value = rand();
        if (tp != 0) {
            if (arr.get(root[nd], pos) != total[nd][pos]) {
                abort();
            }
        }
        else {
            int sz = root.size();
            total[sz] = total[nd];
            total[sz][pos] = value;
            root.push_back(root[nd]);
            arr.set(root.back(), pos, value);
        }
    }
    return 0;
}

```

8.5 四分树

```

const int maxn = 1100;
const int inf = 1 << 30;
int A[2000][2000];
int cur = 0, tot, mn, mx;
struct area {
    int x1, x2, y1, y2;
}

```

```

    int a, b, c, d;
    int sum, min, max, set, add; //set 必须是非负数
}t[maxn * maxn * 4];
void init() {
    cur = 0;
    t[0].min = inf;
    t[0].max = -inf;
    t[0].sum = 0;
}
void maintain(int p) { //maintain 函数用来维护结点信息, 应当保证多次对一个结点调用 maintain 结果不变
    if (p == 0) return;
    int a = t[p].a, b = t[p].b, c = t[p].c, d = t[p].d;
    t[p].sum = t[p].min = t[p].max = 0;
    if (t[p].x1 != t[p].x2 || t[p].y1 != t[p].y2) {
        t[p].sum = t[a].sum + t[b].sum + t[c].sum + t[d].sum;
        t[p].min = min({ t[a].min, t[b].min, t[c].min, t[d].min });
        t[p].max = max({ t[a].max, t[b].max, t[c].max, t[d].max });
    }
    if (t[p].set >= 0) {
        t[p].min = t[p].max = t[p].set;
        t[p].sum = t[p].set * (t[p].x2 - t[p].x1 + 1) * (t[p].y2 - t[p].y1 + 1);
    }
    if (t[p].add) {
        t[p].min += t[p].add;
        t[p].max += t[p].add;
        t[p].sum += t[p].add * (t[p].x2 - t[p].x1 + 1) * (t[p].y2 - t[p].y1 + 1);
    }
}
void pushdown(int p) { //pushdown 将标记传递给子结点, 不影响当前结点的信息。
    int a = t[p].a, b = t[p].b, c = t[p].c, d = t[p].d;
    if (t[p].set >= 0) {
        t[a].set = t[b].set = t[c].set = t[d].set = t[p].set;
        t[a].add = t[b].add = t[c].add = t[d].add = 0;
        t[p].set = -1;
    }
    if (t[p].add) {
        t[a].add += t[p].add;
        t[b].add += t[p].add;
        t[c].add += t[p].add;
        t[d].add += t[p].add;
        t[p].add = 0;
    }
}
int build(int x1, int x2, int y1, int y2) {
    int p = ++cur;
    t[p].x1 = x1;

```

```

    t[p].x2 = x2;
    t[p].y1 = y1;
    t[p].y2 = y2;
    t[p].a = t[p].b = t[p].c = t[p].d = 0;
    t[p].add = 0; t[p].set = -1; //清空结点标记
    if (x1 == x2 && y1 == y2) {
        t[p].add = A[x1][y1];
    }
    else {
        int xm = (x1 + x2) >> 1;
        int ym = (y1 + y2) >> 1;
        t[p].a = build(x1, xm, y1, ym);
        if (ym < y2)
            t[p].b = build(x1, xm, ym + 1, y2);
        if (xm < x2)
            t[p].c = build(xm + 1, x2, y1, ym);
        if (xm < x2 && ym < y2)
            t[p].d = build(xm + 1, x2, ym + 1, y2);
    }
    maintain(p);
    return p;
}

void update(int p, int U, int D, int L, int R, int op, int v) { //[U, D] 对应行号区间, [L, R] 对应
↪ 列号区间。
    if (U <= t[p].x1 && D >= t[p].x2 && L <= t[p].y1 && R >= t[p].y2) {
        if (op == 0) t[p].add += v;
        else {
            t[p].set = v;
            t[p].add = 0;
        }
    }
    else {
        pushdown(p);
        int xm = (t[p].x1 + t[p].x2) >> 1;
        int ym = (t[p].y1 + t[p].y2) >> 1;
        if (U <= xm && L <= ym)
            update(t[p].a, U, D, L, R, op, v);
        else
            maintain(t[p].a);
        if (U <= xm && ym < R && ym < t[p].y2)
            update(t[p].b, U, D, L, R, op, v);
        else
            maintain(t[p].b);
        if (xm < D && L <= ym && xm < t[p].x2)
            update(t[p].c, U, D, L, R, op, v);
        else

```

```

        maintain(t[p].c);
    if (xm < D && ym < R && xm < t[p].x2 && ym < t[p].y2)
        update(t[p].d, U, D, L, R, op, v);
    else
        maintain(t[p].d);
}
maintain(p);
}

void query(int p, int U, int D, int L, int R) { //调用之前要设置: mn = inf; mx = -inf; tot = 0;
    if (U <= t[p].x1 && D >= t[p].x2 && L <= t[p].y1 && R >= t[p].y2) {
        tot += t[p].sum;
        mn = min(mn, t[p].min);
        mx = max(mx, t[p].max);
    }
    else {
        pushdown(p);
        maintain(t[p].a);
        maintain(t[p].b);
        maintain(t[p].c);
        maintain(t[p].d);
        int xm = (t[p].x1 + t[p].x2) >> 1;
        int ym = (t[p].y1 + t[p].y2) >> 1;
        if (U <= xm && L <= ym)
            query(t[p].a, U, D, L, R);
        if (U <= xm && ym < R && ym < t[p].y2)
            query(t[p].b, U, D, L, R);
        if (xm < D && L <= ym && xm < t[p].x2)
            query(t[p].c, U, D, L, R);
        if (xm < D && ym < R && xm < t[p].x2 && ym < t[p].y2)
            query(t[p].d, U, D, L, R);
    }
}

int main() {
#define ran engine(e)
    int n = 500, m = 500, q = 10000;
    default_random_engine e;
    e.seed(123);
    uniform_int_distribution<unsigned> engine(1, n);
    for (int i = 1; i <= n; ++i)
        for (int j = 1; j <= m; ++j)
            A[i][j] = ran;
    init();
    int root = build(1, n, 1, m);
    while (q--) {
        int a = ran, b = ran, c = ran, d = ran;
        int x1 = min(a, b), x2 = max(a, b), y1 = min(c, d), y2 = max(c, d);

```

```

    int tp = rand() % 3, v = ran;
    if (tp == 0) {
        for (int i = x1; i <= x2; ++i)
            for (int j = y1; j <= y2; ++j)
                A[i][j] += v;
        update(root, x1, x2, y1, y2, 0, v);
    }
    else if (tp == 1) {
        for (int i = x1; i <= x2; ++i)
            for (int j = y1; j <= y2; ++j)
                A[i][j] = v;
        update(root, x1, x2, y1, y2, 1, v);
    }
    else {
        mn = inf; mx = -inf; tot = 0;
        int MN = inf, MX = -inf, TOT = 0;
        for (int i = x1; i <= x2; ++i)
            for (int j = y1; j <= y2; ++j)
                MN = min(MN, A[i][j]), MX = max(MX, A[i][j]), TOT += A[i][j];
        query(root, x1, x2, y1, y2);
        if (MN != mn)
            printf("mn: %d %d\n", MN, mn);
        if (mx != MX)
            printf("mx: %d %d\n", MX, mx);
        if (tot != TOT)
            printf("sum: %d %d\n", TOT, tot);
    }
}
}
return 0;
}

```

8.6 Treap

```

const int maxn = 5000000;
struct node {
    node *ch[2]; // 左右子树
    int r; // 随机优先级
    int v; // 值
    int s; // 结点总数
    int cmp(int x) const {
        if (x == v) return -1;
        return x < v ? 0 : 1;
    }
    void maintain() {
        s = ch[0]->s + ch[1]->s + 1;
    }
}nodes[maxn], *cur, *nil;

```

```

node *newnode(int x)
{
    node *o = cur++;
    o->ch[0] = o->ch[1] = nil;
    o->r = rand();
    o->v = x;
    o->s = 1;
    return o;
}

void init()
{
    cur = nodes;
    nil = newnode(0);
    nil->s = 0;
}

void rotate(node* &o, int d) {
    node* k = o->ch[d^1]; o->ch[d^1] = k->ch[d]; k->ch[d] = o;
    o->maintain(); k->maintain(); o = k;
}

void insert(node* &o, int x) {
    if(o == nil) o = newnode(x);
    else {
        int d = (x < o->v ? 0 : 1); // 不要用 cmp 函数, 因为可能会有相同结点
        insert(o->ch[d], x);
        if(o->ch[d]->r > o->r) rotate(o, d^1);
    }
    o->maintain();
}

node *find(node* o, int x) {
    while (o != nil)
    {
        int d = o->cmp(x);
        if (d == -1) return o;
        else o = o->ch[d];
    }
    return nil;
}

// 要确保结点存在
void remove(node* &o, int x) {
    int d = o->cmp(x);
    if (d == -1)
    {
        if (o->ch[0] == nil)
            o = o->ch[1];
        else if (o->ch[1] == nil)
            o = o->ch[0];
    }
}

```



```

        else
        {
            int d2 = (o->ch[0]->r > o->ch[1]->r ? 1 : 0);
            rotate(o, d2); remove(o->ch[d2], x);
        }
    }
    else
        remove(o->ch[d], x);
    if(o != nil) o->maintain();
}

int kth(node* o, int k) {
    if(o == nil || k <= 0 || k > o->s) return 0;
    if(k == o->ch[0]->s+1) return o->v;
    else if(k <= o->ch[0]->s) return kth(o->ch[0], k);
    else return kth(o->ch[1], k - o->ch[0]->s - 1);
}

// 返回在以 o 为根的子树中 x 的排名, 没有 x 则返回 x 的后继的排名
int rank(node* o, int x) {
    if(o == nil) return 1;
    if(x <= o->v) return rank(o->ch[0], x);
    else return rank(o->ch[1], x) + o->ch[0]->s + 1;
}

// 返回在以 o 为根的子树中 x 的前驱的排名
int rank1(node *o, int x)
{
    if (o == nil)
        return 0;
    if (x <= o->v)
        return rank1(o->ch[0], x);
    else return rank1(o->ch[1], x) + o->ch[0]->s + 1;
}

// 返回在以 o 为根的子树中 x 的后继的排名
int rank2(node *o, int x)
{
    if (o == nil)
        return 1;
    if (x >= o->v)
        return rank2(o->ch[1], x) + o->ch[0]->s + 1;
    else return rank2(o->ch[0], x);
}
}
/*

```

对于 *rank*、*rank1*、*rank2* 的理解:

给定序列 $A = \{1, 2, 3, 3, 4, 4, 5\}$

下标 (排名): 1 2 3 4 5 6 7

将序列 A 插入到平衡树 *root* 中。

`rank(root, 3)`: 因为只要 $x \leq o \rightarrow v$ 就一直往左走, 所以会走到排名 2~3 的位置, 到达 `nil` 结点返回 1, 所以
 \hookrightarrow 最外层 `rank` 返回值为 3

`rank1(root, 3)`: 因为只要 $x \leq o \rightarrow v$ 就一直往左走, 所以会走到排名 2~3 的位置, 到达 `nil` 结点返回 0, 所以
 \hookrightarrow 最外层 `rank` 返回值为 2

`rank2(root, 3)`: 因为只要 $x \geq o \rightarrow v$ 就一直往右走, 所以会走到排名 4~5 的位置, 到达 `nil` 结点返回 1, 所以
 \hookrightarrow 最外层 `rank` 返回值为 5

```
*/
int main() {
    init();
    node *root = nil;
    int A[] = {1, 2, 3, 3, 4, 4, 5};
    for (unsigned i = 0; i < sizeof(A) / sizeof(*A); ++i)
        insert(root, A[i]);
    printf("%d\n", rank(root, 3));
    printf("%d\n", rank1(root, 3));
    printf("%d\n", rank2(root, 3));
    return 0;
}
```

8.7 link-cut-tree

```
const int maxn = 110000;
```

//若要修改一个点的点权, 应当先将其 `splay` 到根, 然后修改, 最后还要调用 `pushup` 维护。

```
namespace lct {
    int ch[maxn][2], fa[maxn], stk[maxn], rev[maxn];
    void init() { //初始化 link-cut-tree
        memset(ch, 0, sizeof(ch));
        memset(fa, 0, sizeof(fa));
        memset(rev, 0, sizeof(rev));
    }
    inline bool son(int x) {
        return ch[fa[x]][1] == x;
    }
    inline bool isroot(int x) {
        return ch[fa[x]][1] != x && ch[fa[x]][0] != x;
    }
    inline void reverse(int x) { //给结点 x 打上反转标记
        swap(ch[x][1], ch[x][0]);
        rev[x] ^= 1;
    }
    inline void pushup(int x) { }
    inline void pushdown(int x) {
        if (rev[x]) {
            reverse(ch[x][0]);
            reverse(ch[x][1]);
            rev[x] = 0;
        }
    }
}
```

```

}

void rotate(int x) {
    int y = fa[x], z = fa[y], c = son(x);
    if (!isroot(y))
        ch[z][son(y)] = x;
    fa[x] = z;
    ch[y][c] = ch[x][!c];
    fa[ch[y][c]] = y;
    ch[x][!c] = y;
    fa[y] = x;
    pushup(y);
}

void splay(int x) {
    int top = 0;
    stk[++top] = x;
    for (int i = x; !isroot(i); i = fa[i])
        stk[++top] = fa[i];
    while (top)
        pushdown(stk[top--]);
    for (int y = fa[x]; !isroot(x); rotate(x), y = fa[x]) if (!isroot(y))
        son(x) ^ son(y) ? rotate(x) : rotate(y);
    pushup(x);
}

void access(int x) {
    for (int y = 0; x; y = x, x = fa[x]) {
        splay(x);
        ch[x][1] = y;
        pushup(x);
    }
}

void makeroot(int x) { //将 x 变为树的新的根结点
    access(x);
    splay(x);
    reverse(x);
}

int findroot(int x) { //返回 x 所在树的根结点
    access(x);
    splay(x);
    while (ch[x][0])
        pushdown(x), x = ch[x][0];
    return x;
}

void split(int x, int y) { //提取出来 y 到 x 之间的路径, 并将 y 作为根结点
    makeroot(x);
    access(y);
    splay(y);
}

```

```

}

void cut(int x) { //断开结点 x 与它的父结点之间的边
    access(x);
    splay(x);
    ch[x][0] = fa[ch[x][0]] = 0;
    pushup(x);
}

void cut(int x, int y) { //切断 x 与 y 相连的边（必须保证 x 与 y 在一棵树中）
    makeroot(x); //将 x 置为整棵树的根
    cut(y); //删除 y 与其父结点之间的边
}

void link(int x, int y) { //连接 x 与 y（必须保证 x 和 y 属于不同的树）
    makeroot(x);
    fa[x] = y;
}

bool sametree(int x, int y) { //判断结点 x 与 y 是否属于同一棵树
    makeroot(x);
    return findroot(y) == x;
}

}

int main() {
    return 0;
}

```

8.8 link-cut-tree（指针）

```

const int maxn = 400000;
const int inf = 1 << 30;
struct node {
    node* p, * ch[2];
    int mx, rev, val, add;
}nodes[maxn], * cur, * nil;
node* newnode(int key) {
    cur->p = cur->ch[0] = cur->ch[1] = nil;
    cur->mx = cur->val = key;
    cur->add = cur->rev = 0;
    return cur++;
}

void init() {
    cur = nodes;
    nil = newnode(-inf);
}

bool isroot(node* x) {
    return x->p == nil || x->p->ch[0] != x && x->p->ch[1] != x;
}

void increase(node* x, int v) {
    x->val += v;
}

```

```

    x->add += v;
    x->mx += v;
}

void pushup(node* x) {
    x->mx = x->val;
    if (x->ch[0] != nil)
        x->mx = max(x->mx, x->ch[0]->mx);
    if (x->ch[1] != nil)
        x->mx = max(x->mx, x->ch[1]->mx);
}

void pushdown(node* x) {
    if (x->rev) {
        x->rev = 0;
        if (x->ch[0] != nil) x->ch[0]->rev ^= 1;
        if (x->ch[1] != nil) x->ch[1]->rev ^= 1;
        swap(x->ch[0], x->ch[1]);
    }
    if (x->add) {
        if (x->ch[0] != nil)
            increase(x->ch[0], x->add);
        if (x->ch[1] != nil)
            increase(x->ch[1], x->add);
        x->add = 0;
    }
}

void rotate(node* x, int d) {
    if (isroot(x)) return;
    node* y = x->p;
    y->ch[!d] = x->ch[d];
    x->ch[d]->p = y;
    x->p = y->p;
    if (!isroot(y))
        y->p->ch[y == y->p->ch[1]] = x;
    x->ch[d] = y;
    y->p = x;
    pushup(y);
}

void splay(node* x) { //若要修改一个点的权值则要先将其伸展到根
    static node* sta[maxn];
    int top = 1;
    sta[0] = x;
    for (node* y = x; !isroot(y); y = y->p)
        sta[top++] = y->p;
    while (top) pushdown(sta[--top]);
    while (!isroot(x)) {
        node* y = x->p;

```

```

    if (isroot(y))
        rotate(x, x == y->ch[0]);
    else {
        int d = y == y->p->ch[0];
        if (x == y->ch[d])
            rotate(x, !d);
        else
            rotate(y, d);
        rotate(x, d);
    }
}
pushup(x);
}

node* access(node* x) { //打通结点 x 到根结点的路径
    node* y = nil;
    while (x != nil) {
        splay(x);
        y->p = x;
        x->ch[1] = y;
        pushup(x);
        y = x;
        x = x->p;
    }
    return y;
}

void changeroot(node* x) { //将 x 置为整棵树的根结点
    access(x)->rev ^= 1;
}

void link(node* x, node* y) { //将结点 x 与 y 连接起来（必须保证 x 与 y 属于不同的树）
    access(x);
    splay(x);
    x->rev ^= 1;
    x->p = y;
}

void cut(node* x) { //断开结点 x 与它的父结点之间的边
    access(x);
    splay(x);
    x->ch[0] = x->ch[0]->p = nil;
    pushup(x);
}

void cut(node* x, node* y) { //断开结点 x 与 y 之间的边
    changeroot(x); //将 x 置为整棵树的根
    cut(y); //删除 y 与其父结点之间的边
}

node* getroot(node* x) { //返回结点 x 所在的树当前的根结点
    access(x);

```

```

    splay(x);
    while (pushdown(x), x->ch[0] != nil)
        x = x->ch[0];
    splay(x);
    return x;
}

bool sametree(node* x, node* y) { //判断结点 x 与 y 是否属于同一棵树
    changeroot(x);
    return getroot(y) == x;
}

int n, m, x, y, w;
int eu[maxn], ev[maxn];
int main()
{
    //freopen("in.txt", "r", stdin);
    while (scanf("%d", &n) != -1)
    {
        init();
        for (int i = 1; i < n; i++)
            scanf("%d%d", &eu[i], &ev[i]);
        for (int i = 1; i <= n; i++)
        {
            int a;
            scanf("%d", &a);
            newnode(a);
        }
        for (int i = 1; i < n; i++)
            link(nodes + eu[i], nodes + ev[i]);
        scanf("%d", &m);
        for (int i = 1; i <= m; i++)
        {
            scanf("%d", &x);
            if (x == 1)
            {
                scanf("%d%d", &x, &y);
                if (sametree(nodes + x, nodes + y))
                {
                    printf("-1\n");
                    continue;
                }
                link(nodes + x, nodes + y);
            }
            else if (x == 2)
            {
                scanf("%d%d", &x, &y);
                if (x == y || !sametree(nodes + x, nodes + y))

```

```

    {
        printf("-1\n");
        continue;
    }
    changeroot(nodes + x);
    cut(nodes + y);
}
else if (x == 3)
{
    scanf("%d%d%d", &w, &x, &y);
    if (!sametree(nodes + x, nodes + y))
    {
        printf("-1\n");
        continue;
    }
    changeroot(nodes + x);
    access(nodes + y);
    splay(nodes + y);
    node* q = nodes + y;
    increase(q, w);
}
else
{
    scanf("%d%d", &x, &y);
    if (!sametree(nodes + x, nodes + y))
    {
        printf("-1\n");
        continue;
    }
    changeroot(nodes + x);
    access(nodes + y);
    splay(nodes + y);
    printf("%d\n", (nodes + y)->mx);
}
}
printf("\n");
}
return 0;
}

```

8.9 link-cut-tree (边权)

```

const int maxn = 400000;
const int inf = 1 << 30;
struct node { //mn 记录最小值, pos 记录最小值所在的结点
    node* p, *ch[2], *pos;
    int mn, rev, val;

```



```

}nodes[maxn], * cur, * nil;
pair<node*, node*> edge[maxn];
node* newnode(int key) {
    cur->p = cur->ch[0] = cur->ch[1] = nil;
    cur->mn = cur->val = key;
    cur->rev = 0;
    return cur++;
}
void init() {
    cur = nodes;
    nil = newnode(inf);
}
bool isroot(node* x) {
    return x->p == nil || x->p->ch[0] != x && x->p->ch[1] != x;
}
void pushup(node* x) {
    x->mn = x->val;
    x->pos = x;
    if (x->ch[0] != nil && x->ch[0]->mn < x->mn) {
        x->mn = x->ch[0]->mn;
        x->pos = x->ch[0]->pos;
    }
    if (x->ch[1] != nil && x->ch[1]->mn < x->mn) {
        x->mn = x->ch[1]->mn;
        x->pos = x->ch[1]->pos;
    }
}
void pushdown(node* x) {
    if (x->rev) {
        x->rev = 0;
        if (x->ch[0] != nil) x->ch[0]->rev ^= 1;
        if (x->ch[1] != nil) x->ch[1]->rev ^= 1;
        swap(x->ch[0], x->ch[1]);
    }
}
void rotate(node* x, int d) {
    if (isroot(x)) return;
    node* y = x->p;
    y->ch[!d] = x->ch[d];
    x->ch[d]->p = y;
    x->p = y->p;
    if (!isroot(y))
        y->p->ch[y == y->p->ch[1]] = x;
    x->ch[d] = y;
    y->p = x;
    pushup(y);
}

```

```

}

void splay(node* x) {
    static node* sta[maxn];
    int top = 1;
    sta[0] = x;
    for (node* y = x; !isroot(y); y = y->p)
        sta[top++] = y->p;
    while (top) pushdown(sta[--top]);
    while (!isroot(x)) {
        node* y = x->p;
        if (isroot(y))
            rotate(x, x == y->ch[0]);
        else {
            int d = y == y->p->ch[0];
            if (x == y->ch[d])
                rotate(x, !d);
            else
                rotate(y, d);
            rotate(x, d);
        }
    }
    pushup(x);
}

node* access(node* x) { //打通结点 x 到根结点的路径
    node* y = nil;
    while (x != nil) {
        splay(x);
        y->p = x;
        x->ch[1] = y;
        pushup(x);
        y = x;
        x = x->p;
    }
    return y;
}

void changeroot(node* x) { //将 x 置为整棵树的根结点
    access(x)->rev ^= 1;
}

void link(node* x, node* y) {
    access(x);
    splay(x);
    x->rev ^= 1;
    x->p = y;
}

node *link(node* x, node* y, int value) { //在结点 x 与结点 y 之间连接一条权值为 v 的边 (必须保证 x
    ↪ 与 y 属于不同的树)

```

```

    node* e = newnode(value);
    link(x, e);
    link(y, e);
    edge[e - nodes] = make_pair(x, y);
    return e; //返回新建的边对应的结点
}

void cut(node* x, node* y) { //断开结点 x 与 y 之间的边
    changeroot(x); //将 x 置为整棵树的根
    access(y);
    splay(y);
    y->ch[0] = y->ch[0]->p = nil;
    pushup(y);
}

void cut(node* x) { //删除结点 x 所代表的边
    pair<node*, node*> pr = edge[x - nodes];
    cut(pr.first, x);
    cut(pr.second, x);
    edge[x - nodes] = pair<node *, node*>(0, 0);
}

bool exist(node* x) { //检查点 x 所对应的边是否还存在于树中（没有被删除）
    return edge[x - nodes].first != 0;
}

node* getroot(node* x) { //返回结点 x 所在的树当前的根结点
    access(x);
    splay(x);
    while (pushdown(x), x->ch[0] != nil)
        x = x->ch[0];
    splay(x);
    return x;
}

bool sametree(node* x, node* y) { //判断结点 x 与 y 是否属于同一棵树
    changeroot(x);
    return getroot(y) == x;
}

int main() {
}

```

8.10 link-cut-tree（维护子树）

/*

用 *link-cut-tree* 维护子树信息：对于维护的信息，每个点要开两个属性，其中一个是在原树中的子树信息，另一个是在 *LCT* 中虚子树的信息。除了要在 *pushup* 中维护这些信息外，还要在 *access* 和 *link* 函数中维护。若要访问一个结点 *x* 的子树信息，应当先将其 *access* 到根，此时 *x->size* 表示 *x* 的子树中除去 *x* 外所有结点 \hookrightarrow 的信息（大小），

与 *x* 自身的信息结合即可得到 *x* 的子树信息（*x->size + 1* 即为子树大小）。

若要访问整棵树的信息，可以考虑结合 *changeroot* 函数，并使用结点的 *sum* 信息。

*/

```

const int maxn = 400000;
struct node {
    node* p, * ch[2];
    int rev, sum, size; //sum 维护当前点的虚子树与实子树大小之和, size 是在 LCT 中虚子树的大小
}nodes[maxn], * cur, * nil;
node* newNode() {
    cur->p = cur->ch[0] = cur->ch[1] = nil;
    cur->sum = 1;
    cur->size = 0;
    cur->rev = 0;
    return cur++;
}
void init() {
    cur = nodes;
    nil = newNode();
    nil->sum = 0;
}
bool isroot(node* x) {
    return x->p == nil || x->p->ch[0] != x && x->p->ch[1] != x;
}
void pushup(node* x) {
    x->sum = x->ch[0]->sum + x->ch[1]->sum + x->size + 1;
}
void pushdown(node* x) {
    if (x->rev) {
        x->rev = 0;
        if (x->ch[0] != nil) x->ch[0]->rev ^= 1;
        if (x->ch[1] != nil) x->ch[1]->rev ^= 1;
        swap(x->ch[0], x->ch[1]);
    }
}
void rotate(node* x, int d) {
    if (isroot(x)) return;
    node* y = x->p;
    y->ch[!d] = x->ch[d];
    x->ch[d]->p = y;
    x->p = y->p;
    if (!isroot(y))
        y->p->ch[y == y->p->ch[1]] = x;
    x->ch[d] = y;
    y->p = x;
    pushup(y);
}
void splay(node* x) { //若要修改一个点的权值则要先将其伸展到根
    static node* sta[maxn];
    int top = 1;

```

```

    sta[0] = x;
    for (node* y = x; !isroot(y); y = y->p)
        sta[top++] = y->p;
    while (top) pushdown(sta[--top]);
    while (!isroot(x)) {
        node* y = x->p;
        if (isroot(y))
            rotate(x, x == y->ch[0]);
        else {
            int d = y == y->p->ch[0];
            if (x == y->ch[d])
                rotate(x, !d);
            else
                rotate(y, d);
            rotate(x, d);
        }
    }
    pushup(x);
}

node* access(node* x) { //打通结点 x 到根结点的路径
    node* y = nil;
    while (x != nil) {
        splay(x);
        x->size += x->ch[1]->sum - y->sum; //动态维护虚子树信息
        y->p = x;
        x->ch[1] = y;
        pushup(x);
        y = x;
        x = x->p;
    }
    return y;
}

void changeroot(node* x) { //将 x 置为整棵树的根结点
    access(x)->rev ^= 1;
    splay(x); //将 x 伸展到 splay 的根结点, 这样才能保证 link 的正确性
}

void link(node* x, node* y) { //将结点 x 与 y 连接起来 (必须保证 x 与 y 属于不同的树且不为 nil 结点)
    ↪
    changeroot(x);
    changeroot(y);
    x->p = y;
    y->size += x->sum; //动态维护虚子树信息
    pushup(y);
}

void cut(node* x) { //断开结点 x 与它的父结点之间的边
    access(x);

```

```

    splay(x);
    x->ch[0] = x->ch[0]->p = nil;
    pushup(x);
}

void cut(node* x, node* y) { //断开结点 x 与 y 之间的边
    changeroot(x); //将 x 置为整棵树的根
    cut(y); //删除 y 与其父结点之间的边
}

node* getroot(node* x) { //返回结点 x 所在的树当前的根结点
    access(x);
    splay(x);
    while (pushdown(x), x->ch[0] != nil)
        x = x->ch[0];
    splay(x);
    return x;
}

bool sametree(node* x, node* y) { //判断结点 x 与 y 是否属于同一棵树
    changeroot(x);
    return getroot(y) == x;
}

node* nd[maxn];
int main() {
    //freopen("in.txt", "r", stdin);
    int n, q;
    scanf("%d %d", &n, &q);
    init();
    for (int i = 1; i <= n; ++i)
        nd[i] = newnode();
    while (q--) {
        char tp;
        int x, y;
        scanf(" %c %d %d", &tp, &x, &y);
        if (tp == 'A') {
            link(nd[x], nd[y]);
        }
        else {
            changeroot(nd[x]);
            access(nd[y]);
            long long L = nd[y]->size + 1; //将虚子树的大小加 1, 即为子树大小。
            changeroot(nd[y]);
            access(nd[x]);
            long long R = nd[x]->size + 1;
            printf("%lld\n", L * R);
        }
    }
}

return 0;

```

```
}
```

8.11 可持久化 Treap

```
#define rank Rank
const int maxn = 510000;
namespace treap_old {
    //在每次更新标记的时候对被更新节点的属性进行维护。
    //在 pushdown 的时候不修改根节点的值。
    //在 pushup 的时候直接访问子树的属性，而无需考虑 flip 等特殊标记。
    struct node {
        node* lch, * rch;
        int r;
        int v;
        int s;
        void up() {
            s = lch->s + rch->s + 1;
        }
        void down() {
            /*
            if (rev)
            {
                rev = 0;
                lch->rev ^= 1;
                rch->rev ^= 1;
                swap(lch, rch);
            }
            */
        }
    }
    nodes[maxn], * cur, * nil;
    node* newnode(int x) {
        node* o = cur++;
        o->lch = o->rch = nil;
        o->r = rand();
        o->v = x;
        o->s = 1;
        return o;
    }
    void init() {
        cur = nodes;
        nil = newnode(0);
        nil->s = 0;
    }
    node* copy(node* o) {
        *cur = *o;
        return cur++;
    }
}
```

//如果有 *down* 操作则可持久化操作放在 *down* 中, *down* 中复制左右子树后进行标记下传

```
void merge(node*& o, node* a, node* b) {
    if (a == nil) o = b;
    else if (b == nil) o = a;
    else if (a->r > b->r) {
        o = a; //o = copy(a);
        o->down();
        merge(o->rch, a->rch, b);
        //o->rch->fa = o;
        o->up();
    }
    else {
        o = b; //o = copy(b);
        o->down();
        merge(o->lch, a, b->lch);
        //o->lch->fa = o;
        o->up();
    }
}

void split(node* o, node*& a, node*& b, int k) {
    if (k == 0)
        a = nil, b = o;
    else if (o->s <= k)
        a = o, b = nil;
    else if (o->lch->s >= k) { //如果有序列的翻转操作则应当将 o->down() 放在该 if 语句之前, 以
        ↪ 确保正确访问左子树的大小
        o->down();
        b = o; //b = copy(o);
        split(o->lch, a, b->lch, k);
        //a->fa = nil;
        //b->lch->fa = b;
        b->up();
    }
    else {
        o->down();
        a = o; //a = copy(o);
        split(o->rch, a->rch, b, k - o->lch->s - 1);
        //a->rch->fa = a;
        //b->fa = nil;
        a->up();
    }
}

bool find(node* o, int x) {
    while (o != nil) {
        if (x == o->v)
            return true;
    }
}
```



```

        else if (x < o->v)
            o = o->lch;
        else
            o = o->rch;
    }
    return false;
}

int rank(node* o, int x) {
    if (o == nil)
        return 1;
    if (x <= o->v)
        return rank(o->lch, x);
    else return rank(o->rch, x) + o->lch->s + 1;
}

int kth(node* o, int k) {
    if (o == nil || k <= 0 || k > o->s) return 0;
    if (k == o->lch->s + 1) return o->v;
    else if (k <= o->lch->s) return kth(o->lch, k);
    else return kth(o->rch, k - o->lch->s - 1);
}

//insert(root, rank(root, x), x);
void insert(node*& o, int k, int x) {
    node* a, * b;
    split(o, a, b, k - 1);
    merge(a, a, newnode(x));
    merge(o, a, b);
}

//del(root, rank(root, x));
void del(node*& o, int k) {
    node* a, * b, * c;
    split(o, a, b, k - 1);
    split(b, b, c, 1);
    merge(o, a, c);
    //freenode(b);
}

//node *root = nil; //创建一棵空树
}

namespace treap {
    struct node {
        node* lch, * rch;
        int v;
        int s;
        void up() {
            s = lch->s + rch->s + 1;
        }
    }nodes[maxn], * cur, * nil;
}

```

```

node* newnode(int v) {
    node* x = cur++;
    x->lch = x->rch = nil;
    x->v = v;
    x->s = 1;
    return x;
}

void init() {
    cur = nodes;
    nil = newnode(0);
    nil->s = 0;
}

node* copy(node* x) {
    *cur = *x;
    return cur++;
}

node* merge(node* a, node* b) {
    if (a == nil) return b;
    if (b == nil) return a;
    if (rand() % (a->s + b->s) < a->s) { // 必须保证 rand 产生的随机数足够大
        node* x = copy(a);
        x->rch = merge(a->rch, b);
        x->up();
        return x;
    }
    else {
        node* x = copy(b);
        x->lch = merge(a, b->lch);
        x->up();
        return x;
    }
}

node* merge(node* a, node* b, node* c) {
    return merge(a, merge(b, c));
}

pair<node*, node*> split(node* x, int k) {
    if (k == 0)
        return { nil, x };
    if (x->s <= k)
        return { x, nil };
    if (x->lch->s >= k) {
        node* b = copy(x);
        auto [a, m] = split(x->lch, k);
        b->lch = m;
        b->up();
        return { a, b };
    }
}

```

```

    }
    else {
        node* a = copy(x);
        auto [m, b] = split(x->rch, k - x->lch->s - 1);
        a->rch = m;
        a->up();
        return { a, b };
    }
}

tuple<node*, node*, node*> split(node* x, int L, int R) {
    auto [a, m] = split(x, L - 1);
    auto [b, c] = split(m, R - L + 1);
    return { a, b, c };
}

bool find(node* x, int v) {
    while (x != nil) {
        if (v == x->v)
            return true;
        else if (v < x->v)
            x = x->lch;
        else
            x = x->rch;
    }
    return false;
}

int rank(node* x, int v) { //v 在 x 中是第几大的值
    if (x == nil)
        return 1;
    if (v <= x->v)
        return rank(x->lch, v);
    else
        return rank(x->rch, v) + x->lch->s + 1;
}

int kth(node* x, int k) {
    if (x == nil || k <= 0 || k > x->s) return 0;
    if (k == x->lch->s + 1) return x->v;
    else if (k <= x->lch->s) return kth(x->lch, k);
    else return kth(x->rch, k - x->lch->s - 1);
}

//insert(root, rank(root, x), x);
[[nodiscard]] node* insert(node* x, int k, int v) {
    auto [a, b] = split(x, k - 1);
    return merge(a, newnode(v), b);
}

//del(root, rank(root, x));
[[nodiscard]] node* del(node* x, int k) {

```

```

    auto [a, m] = split(x, k - 1);
    auto [v, b] = split(m, 1);
    return merge(a, b);
}
//node *root = nil; //创建一棵空树
}

namespace shared_treap { //用智能指针优化可持久化 treap 的内存
    struct node;
    struct pointer {
        node* ptr;
        pointer() : ptr(nullptr) {}
        pointer(node* ptr);
        pointer(const pointer& ptr);
        pointer& operator= (const pointer& rhs);
        operator node* () const;
        node* operator-> ();
        node& operator* ();
        ~pointer();
        void del();
    };
    struct node {
        pointer lch, rch;
        int v, s, ref;
        void up();
    } nodes[maxn];
    node* const nil = nodes;
    vector<node*> pool;
    inline pointer::pointer(const pointer& ptr) : pointer(ptr.ptr) {}
    inline pointer::pointer(node* ptr) : ptr(ptr) {
        if (ptr)
            ptr->ref += 1;
    }
    inline pointer& pointer::operator= (const pointer& rhs) {
        if (ptr != rhs.ptr) {
            del();
            ptr = rhs.ptr;
            if (ptr)
                ptr->ref += 1;
        }
        return *this;
    }
    inline node* pointer::operator-> () {
        return ptr;
    }
    inline pointer::operator node* () const {
        return ptr;
    }

```

```

}

inline void pointer::del() {
    if (ptr) {
        ptr->ref -= 1;
        if (ptr->ref == 0) {
            ptr->lch = ptr->rch = nullptr;
            pool.push_back(ptr);
        }
    }
}

inline pointer::~~pointer() {
    del();
}

inline void node::up() {
    s = lch->s + rch->s + 1;
}

void init() {
    pool.clear();
    for (int i = 1; i < maxn; ++i)
        pool.push_back(nodes + i);
}

pointer newnode(int v) {
    node* x = pool.back(); //类型是 node* !!!!
    pool.pop_back();
    x->lch = x->rch = nil;
    x->v = v;
    x->s = 1;
    x->ref = 0;
    return x;
}

inline node* copy(node* x) {
    if (x == nil) return x;
    node* y = pool.back(); pool.pop_back();
    *y = *x;
    y->ref = 0;
    return y;
}

pointer merge(node* a, node* b) {
    if (a == nil) return b;
    if (b == nil) return a;
    if (rand() % (a->s + b->s) < a->s) { // 必须保证 rand 产生的随机数足够大
        auto x = copy(a);
        x->rch = merge(a->rch, b);
        x->up();
        return x;
    }
}

```

```

    else {
        auto x = copy(b);
        x->lch = merge(a, b->lch);
        x->up();
        return x;
    }
}

pointer merge(node* a, node* b, node* c) {
    return merge(a, merge(b, c));
}

pair<pointer, pointer> split(node* x, int k) {
    if (k == 0)
        return { nil, x };
    if (x->s <= k)
        return { x, nil };
    if (x->lch->s >= k) {
        pointer b = copy(x);
        auto [a, m] = split(x->lch, k);
        b->lch = m;
        b->up();
        return { a, b };
    }
    else {
        pointer a = copy(x);
        auto [m, b] = split(x->rch, k - x->lch->s - 1);
        a->rch = m;
        a->up();
        return { a, b };
    }
}

tuple<pointer, pointer, pointer> split(node* x, int L, int R) {
    auto [a, m] = split(x, L - 1);
    auto [b, c] = split(m, R - L + 1);
    return { a, b, c };
}

bool find(node* x, int v) {
    while (x) {
        if (v == x->v)
            return true;
        else if (v < x->v)
            x = x->lch;
        else
            x = x->rch;
    }
    return false;
}

```

```

int rank(node* x, int v) { //v 在 x 中是第几大的值
    if (x == nil)
        return 1;
    if (v <= x->v)
        return rank(x->lch, v);
    else
        return rank(x->rch, v) + x->lch->s + 1;
}

int kth(node* x, int k) {
    if (x == nil || k <= 0 || k > x->s) return 0;
    if (k == x->lch->s + 1) return x->v;
    else if (k <= x->lch->s) return kth(x->lch, k);
    else return kth(x->rch, k - x->lch->s - 1);
}

//insert(root, rank(root, x), x);
[[nodiscard]] pointer insert(node* x, int k, int v) {
    auto [a, b] = split(x, k - 1);
    return merge(a, newnode(v), b);
}

//del(root, rank(root, x));
[[nodiscard]] pointer del(node* x, int k) {
    auto [a, m] = split(x, k - 1);
    auto [v, b] = split(m, 1);
    return merge(a, b);
}

//pointer root = nil; //创建一棵空树
}

namespace shared_treap_pushdown {
    struct node;
    struct pointer {
        node* ptr;
        pointer() : ptr(nullptr) {}
        pointer(node* ptr);
        pointer(const pointer& ptr);
        pointer& operator= (const pointer& rhs);
        operator node* () const;
        node* operator-> ();
        node& operator* ();
        ~pointer();
        void del();
    };
    struct node {
        pointer lch, rch;
        int s, ref;
        int val, add;
        long long sum;
    };
}

```

```

    void up();
    void down();
    void mark(int tag) { //在 mark 之前要先 copy 当前结点!!!
        add += tag;
        val += tag;
        sum += 1LL * tag * s;
    }
}nodes[maxn];
node* const nil = nodes;
vector<node*> pool;
inline pointer::pointer(const pointer& ptr) : pointer(ptr.ptr) {}
inline pointer::pointer(node* ptr) : ptr(ptr) {
    if (ptr)
        ptr->ref += 1;
}
inline pointer& pointer::operator= (const pointer& rhs) {
    if (ptr != rhs.ptr) {
        del();
        ptr = rhs.ptr;
        if (ptr)
            ptr->ref += 1;
    }
    return *this;
}
inline node* pointer::operator-> () {
    return ptr;
}
inline pointer::operator node* () const {
    return ptr;
}
inline void pointer::del() {
    if (ptr) {
        ptr->ref -= 1;
        if (ptr->ref == 0) {
            ptr->lch = ptr->rch = nullptr;
            pool.push_back(ptr);
        }
    }
}
inline pointer::~pointer() {
    del();
}
void init() {
    pool.clear();
    for (int i = 1; i < maxn; ++i)
        pool.push_back(nodes + i);
}

```



```

}
pointer newnode(int v) {
    node* x = pool.back(); //类型是 node* !!!!
    pool.pop_back();
    x->lch = x->rch = nil;
    x->val = x->sum = v;
    x->add = 0;
    x->s = 1;
    x->ref = 0;
    return x;
}

inline node* copy(node* x) {
    if (x == nil) return x;
    node* y = pool.back(); pool.pop_back();
    *y = *x;
    y->ref = 0;
    return y;
}

pointer merge(node* a, node* b) {
    if (a == nil) return b;
    if (b == nil) return a;
    if (rand() % (a->s + b->s) < a->s) { // 必须保证 rand 产生的随机数足够大
        auto x = copy(a);
        x->down();
        x->rch = merge(x->rch, b);
        x->up();
        return x;
    }
    else {
        auto x = copy(b);
        x->down();
        x->lch = merge(a, x->lch);
        x->up();
        return x;
    }
}

inline pointer merge(node* a, node* b, node* c) {
    return merge(a, merge(b, c));
}

pair<pointer, pointer> split(node* x, int k) {
    if (k == 0)
        return { nil, x };
    if (x->s <= k)
        return { x, nil };
    x = copy(x);
    x->down();

```

```

    if (x->lch->s >= k) {
        auto [a, m] = split(x->lch, k);
        x->lch = m;
        x->up();
        return { a, x };
    }
    else {
        auto [m, b] = split(x->rch, k - x->lch->s - 1);
        x->rch = m;
        x->up();
        return { x, b };
    }
}

inline tuple<pointer, pointer, pointer> split(node* x, int L, int R) {
    auto [a, m] = split(x, L - 1);
    auto [b, c] = split(m, R - L + 1);
    return { a, b, c };
}

inline void node::up() {
    s = lch->s + rch->s + 1;
    sum = lch->sum + rch->sum + val;
}

inline void node::down() {
    if (add) {
        if (lch.ptr != nil) {
            lch = copy(lch.ptr);
            lch->mark(add);
        }
        if (rch.ptr != nil) {
            rch = copy(rch.ptr);
            rch->mark(add);
        }
        add = 0;
    }
}

int main() {
    using namespace shared_treap_pushdown;
    init();
    pointer root = nil;
    return 0;
}

```

8.12 树链剖分

/*

1. 如果权值在边上的话，则先将权值压到深度大的点上，再建立线段树。

2. 结点 x 的子树对应的 DFS 序区间为 $[id[x], id[x] + siz[x] - 1]$ 。

```

*/
#define lc t[p].lchild
#define rc t[p].rchild
const int maxn = 110000;
int cur, root, dfn, dep[maxn], siz[maxn], pa[maxn], id[maxn], son[maxn], top[maxn];
int val[maxn], A[maxn], ans;
vector<int> G[maxn];
struct segment {
    int l, r, lchild, rchild;
    int sum, add;
}t[maxn * 4];
inline void maintain(int p) {
    t[p].sum = t[lc].sum + t[rc].sum;
}
inline void mark(int p, int addv) { //给结点打标记
    if (addv) {
        t[p].add += addv;
        t[p].sum += addv * (t[p].r - t[p].l + 1);
    }
}
inline void pushdown(int p) { //pushdown 将标记传递给子结点，不影响当前结点的信息。
    mark(lc, t[p].add);
    mark(rc, t[p].add);
    t[p].add = 0;
}
int build(int L, int R) {
    int p = ++cur;
    t[p].l = L;
    t[p].r = R;
    t[p].add = 0;
    if (L == R) {
        mark(p, val[L]);
    }
    else {
        int mid = (L + R) >> 1;
        lc = build(L, mid);
        rc = build(mid + 1, R);
        maintain(p);
    }
    return p;
}
void update(int p, int L, int R, int v) {
    if (L <= t[p].l && R >= t[p].r) {
        mark(p, v);
    }
}

```

```

else {
    pushdown(p); //如果没有 pushdown 只需要在最后调用一次 maintain 即可。
    int mid = (t[p].l + t[p].r) >> 1;
    if (L <= mid)
        update(lc, L, R, v);
    if (R > mid)
        update(rc, L, R, v);
    maintain(p);
}
}

void query(int p, int L, int R) {
    if (L <= t[p].l && R >= t[p].r) {
        ans += t[p].sum;
    }
    else {
        pushdown(p);
        int mid = (t[p].l + t[p].r) >> 1;
        if (L <= mid)
            query(lc, L, R);
        if (R > mid)
            query(rc, L, R);
    }
}

void dfs1(int x, int fa, int d) {
    dep[x] = d;
    siz[x] = 1;
    son[x] = 0;
    pa[x] = fa;
    for (auto y : G[x]) if (y != fa) {
        dfs1(y, x, d + 1);
        siz[x] += siz[y];
        if (siz[son[x]] < siz[y])
            son[x] = y;
    }
}

void dfs2(int x, int tp) {
    top[x] = tp;
    id[x] = ++dfn; //id[x] 表示结点 x 的 DFS 序编号
    if (son[x])
        dfs2(son[x], tp);
    for (auto y : G[x])
        if (y != pa[x] && y != son[x])
            dfs2(y, y);
}

int ask(int x, int y) {
    ::ans = 0;

```

```

while (top[x] != top[y]) {
    if (dep[top[x]] < dep[top[y]])
        swap(x, y);
    query(root, id[top[x]], id[x]);
    x = pa[top[x]];
}
//如果权值在边上则加上: if (x == y) return ans;
if (dep[x] > dep[y]) swap(x, y);
query(root, id[x], id[y]); //如果权值在边上则查询 id[son[x]]
return ans;
}

void add(int x, int y, int v) {
    while (top[x] != top[y]) {
        if (dep[top[x]] < dep[top[y]])
            swap(x, y);
        update(root, id[top[x]], id[x], v);
        x = pa[top[x]];
    }
    //如果权值在边上则加上: if (x == y) return ans;
    if (dep[x] > dep[y]) swap(x, y);
    update(root, id[x], id[y], v); //如果权值在边上则改为 id[son[x]]
}

void init(int n) { //调用该函数之前应当先完成建图
    cur = dfn = siz[0] = 0;
    dfs1(1, -1, 1); //根结点为 s 的话, 改为: dfs1(s, -1, 1), dfs2(s, s);
    dfs2(1, 1);
    for (int i = 1; i <= n; ++i)
        val[id[i]] = A[i]; //A[i] 表示结点 i 的权值, 将其复制到 DFS 序上。
    ::root = build(1, n);
}

int main() {
    return 0;
}

```

8.13 树链剖分求 LCA 和距离

```

const int maxn = 510000;
const int inf = 1 << 30;
int dep[maxn], sz[maxn], pa[maxn], son[maxn], top[maxn];
vector<int> G[maxn];
void dfs1(int u, int fa, int d) {
    dep[u] = d;
    sz[u] = 1;
    son[u] = 0;
    pa[u] = fa;
    for (auto v : G[u]) if (v != fa) {
        dfs1(v, u, d + 1);
    }
}

```

```

        sz[u] += sz[v];
        if (sz[son[u]] < sz[v])
            son[u] = v;
    }
}

void dfs2(int u, int tp) {
    top[u] = tp;
    if (son[u])
        dfs2(son[u], tp);
    for (auto v : G[u]) if (v != pa[u] && v != son[u])
        dfs2(v, v);
}

int lca(int x, int y) {
    while (top[x] != top[y]) {
        if (dep[top[x]] < dep[top[y]])
            swap(x, y);
        x = pa[top[x]];
    }
    return dep[x] < dep[y] ? x : y;
}

int dist(int x, int y) {
    return dep[x] + dep[y] - 2 * dep[lca(x, y)];
}

void init(int root) {
    dfs1(root, 0, 1);
    dfs2(root, root);
}

int main() {
    int n, m, s;
    scanf("%d %d %d", &n, &m, &s);
    for (int i = 0; i < n - 1; ++i) {
        int x, y;
        scanf("%d %d", &x, &y);
        G[x].push_back(y);
        G[y].push_back(x);
    }
    init(s);
    while (m--) {
        int x, y;
        scanf("%d %d", &x, &y);
        printf("%d\n", dist(x, y));
    }
    return 0;
}

```

8.14 树状数组

```

#define lowbit(x) ((x)&(-x))
namespace Fenwick_Tree
{
    const int maxn = 110000;
    int C[maxn], n;
    void add(int x, int d)
    {
        for (int i = x; i <= n; i += lowbit(i))
            C[i] += d;
    }
    int sum(int x)
    {
        int ret = 0;
        for (int i = x; i > 0; i -= lowbit(i))
            ret += C[i];
        return ret;
    }
}

namespace Fenwick_Tree_2D
{
    const int maxn = 1100;
    int C[maxn][maxn], n, m;
    void add(int x, int y, int d)
    {
        for (int i = x; i <= n; i += lowbit(i))
            for (int j = y; j <= m; j += lowbit(j))
                C[i][j] += d;
    }
    int sum(int x, int y)
    {
        int ret = 0;
        for (int i = x; i > 0; i -= lowbit(i))
            for (int j = y; j > 0; j -= lowbit(j))
                ret += C[i][j];
        return ret;
    }
}

int main()
{
    return 0;
}

```

8.15 李超线段树

```

const int maxn = 200005;
const int inf = 1 << 30;
int cur = 0;
struct segment {
    int l, r, lc, rc;
    double k, b;
}t[maxn * 4];
void init() {
    cur = 0;
}
int build(int L, int R) { //新建一棵线段树，并返回根结点的编号
    int p = ++cur;
    t[p].l = L;
    t[p].r = R;
    if (L < R) {
        int mid = (L + R) >> 1;
        t[p].lc = build(L, mid);
        t[p].rc = build(mid + 1, R);
    }
    t[p].k = 0; t[p].b = -inf;
    return p;
}
void pushdown(int p, double k, double b) {
    double l1 = k * t[p].l + b, r1 = k * t[p].r + b;
    double l2 = t[p].k * t[p].l + t[p].b, r2 = t[p].k * t[p].r + t[p].b;
    if (l1 >= l2 && r1 >= r2)
        t[p].k = k, t[p].b = b;
    else if (l2 < l1 || r2 < r1) {
        double pos = (b - t[p].b) / (t[p].k - k);
        int mid = (t[p].l + t[p].r) >> 1;
        if (pos <= mid) {
            if (r1 > r2)
                swap(t[p].k, k), swap(t[p].b, b);
            pushdown(t[p].lc, k, b);
        }
        else {
            if (l1 > l2)
                swap(t[p].k, k), swap(t[p].b, b);
            pushdown(t[p].rc, k, b);
        }
    }
}
void insert(int p, int L, int R, double k, double b) { //在区间 [L, R] 中插入直线  $y = kx + b$ 
    if (L <= t[p].l && R >= t[p].r)
        pushdown(p, k, b);

```



```

    else {
        int mid = (t[p].l + t[p].r) >> 1;
        if (L <= mid)
            insert(t[p].lc, L, R, k, b);
        if (R > mid)
            insert(t[p].rc, L, R, k, b);
    }
}

double query(int p, int x) { //p 是线段树的根结点, x 是查询的横坐标, 返回所有直线在 x 处的最大值
    double ans = t[p].k * x + t[p].b;
    if (t[p].l < t[p].r) {
        int mid = (t[p].l + t[p].r) >> 1;
        if (x <= mid)
            ans = max(ans, query(t[p].lc, x));
        else
            ans = max(ans, query(t[p].rc, x));
    }
    return ans;
}

const double eps = 1e-9;
struct Seg {
    Seg() : k(), b(), id(1) {}
    Seg(double k, double b) : k(k), b(b) {}
    double k, b;
    int id;
}A[maxn];

int main() {
    srand(time(0));
    int n = 200000, m = 300;
    for (int i = 1; i <= m; ++i)
        A[i].k = rand() % 10 + 1, A[i].b = rand() % 1000 - 500, A[i].id = i;
    init();
    int root = build(1, n);
    for (int i = 1; i <= m; ++i)
        insert(root, 1, n, A[i].k, A[i].b);
    for (int i = 1; i <= n; ++i) {
        long long ans = query(root, i) + eps;
        long long res = -inf;
        for (int j = 1; j <= m; ++j)
            res = max(res, (long long)(A[j].k * i + A[j].b + eps));
        if (res != ans)
            printf("%lld %lld\n", ans, res);
    }
    return 0;
}

```

8.16 整数集合

```

const int inf = 1 << 30;
static const int maxn = 210000; //maxn 应当大于所有集合大小的总和
struct node {          //该算法实现的整数集合为可重集合 (multiset)
    int l, r, v;        //左节点编号 右节点编号 值
} T[maxn * 25];
int len, sz, info[maxn]; //len 是值域线段树的长度
int ranking(int v) {
    //如果不需要数据离散化直接 return v 即可, 同时还要将所有的 info[v] 改为 v, 此时线段树的值域为
    ↪ [1, len]。
    return lower_bound(info + 1, info + len + 1, v) - info;
}
void ins(int& i, int l, int r, int p) { //在区间 [l, r] 中插入 p
    if (!i) {
        i = ++sz;
        T[i].v = T[i].l = T[i].r = 0;
    }
    int m = (l + r) / 2;
    T[i].v++; //递增当前区间内值的个数
    if (l == r) return;
    if (p <= m) ins(T[i].l, l, m, p);
    else ins(T[i].r, m + 1, r, p);
}
void insert(int& x, int v) { //向集合 x 中添加数值 v
    ins(x, 1, len, ranking(v));
}
void init(int* A, int length) { //输入数组 A 的下标从 1 开始, length 表示 A 的长度
    sz = 0;                      //A 记录集合中所有可能出现的整数
    copy(A + 1, A + length + 1, info + 1);
    sort(info + 1, info + length + 1); //如果没有离散化需要对 len 进行赋值来指明线段树的值域。
    len = unique(info + 1, info + length + 1) - info - 1; //对 info 数组进行排序去重
}
int kth(int x, int k) { //查找集合 x 中的第 k 大元素
    int l = 1, r = len;
    while (l < r) {
        int m = (l + r) / 2, t = T[T[x].l].v;
        if (k <= t)
            x = T[x].l, r = m;
        else
            x = T[x].r, l = m + 1, k -= t;
    }
    if (k > T[x].v) //如果 k 大于集合 x 的大小, 则返回 inf
        return inf;
    return info[r];
}
int ask(int x, int v) { //返回集合 x 中比 v 小的数的个数

```

```

    int l = 1, r = len, k = 0;
    int p = ranking(v) - 1;
    if (p <= 0) return 0;
    while (l < r) {
        int m = (l + r) / 2, t = T[T[x].l].v;
        if (p <= m)
            x = T[x].l, r = m;
        else
            x = T[x].r, l = m + 1, k += t;
    }
    k += T[x].v;
    return k;
}

int pre(int x, int v) { //在集合 x 中查询 v 的前驱, 找不到返回 -inf
    int k = ask(x, v);
    if (k == 0) return -inf;
    return kth(x, k);
}

int next(int x, int v) { //在集合 x 中查询 v 的后继, 找不到返回 inf
    int k = ask(x, v + 1) + 1;
    return kth(x, k);
}

int count(int x, int v) { //返回集合 x 中数值 v 的个数
    int l = 1, r = len;
    int p = ranking(v);
    if (p > len || info[p] != v)
        return 0;
    while (l < r && T[x].v > 0) {
        int m = (l + r) / 2;
        if (p <= m)
            x = T[x].l, r = m;
        else
            x = T[x].r, l = m + 1;
    }
    return T[x].v;
}

bool find(int x, int v) { //返回集合 x 中是否存在 v
    return count(x, v) >= 1;
}

int maximum(int x) { //返回集合 x 中的最大值
    int l = 1, r = len;
    if (T[x].v == 0) //如果集合为空则返回 -inf
        return -inf;
    while (l < r) {
        int m = (l + r) / 2, t = T[T[x].r].v;
        if (t == 0)

```

```

        x = T[x].l, r = m;
    else
        x = T[x].r, l = m + 1;
}
return info[r];
}

int minimum(int x) { //返回集合  $x$  中的最小值
    int l = 1, r = len;
    if (T[x].v == 0) //如果集合为空则返回  $inf$ 
        return inf;
    while (l < r) {
        int m = (l + r) / 2, t = T[T[x].l].v;
        if (t > 0)
            x = T[x].l, r = m;
        else
            x = T[x].r, l = m + 1;
    }
    return info[r];
}

int merge(int x, int y, int l = 1, int r = len) { //将集合  $x$  与集合  $y$  合并成一个新的集合后返回
    if (!x) return y; //合并之后  $x$ 、 $y$  均会失效
    if (!y) return x;
    if (l == r) {
        T[x].v += T[y].v;
    }
    else {
        int m = (l + r) / 2;
        T[x].l = merge(T[x].l, T[y].l, l, m);
        T[x].r = merge(T[x].r, T[y].r, m + 1, r);
        T[x].v = T[T[x].l].v + T[T[x].r].v;
    }
    return x;
}

int diff(int x, int y, int l = 1, int r = len) { //返回集合  $x$  与集合  $y$  的差集 ( $x - y$ )，执行之
    ↪ 后  $x$ 、 $y$  均会失效 //时间复杂度  $size(y)\log(len)$ 
    if (l == r) {
        T[x].v = max(0, T[x].v - T[y].v);
    }
    else {
        int m = (l + r) / 2;
        if (T[y].l)
            T[x].l = diff(T[x].l, T[y].l, l, m);
        if (T[y].r)
            T[x].r = diff(T[x].r, T[y].r, m + 1, r);
        T[x].v = T[T[x].l].v + T[T[x].r].v;
    }
}

```

```

    return x;
}
int intersect(int x, int y, int l = 1, int r = len) {    //返回集合 x 与集合 y 的交集, 执行之后 x、
↪ y 均会失效
    if (l == r) {                                       //时间复杂度  $\min\{size(x), size(y)\} * \log(len)$ 
        T[x].v = min(T[x].v, T[y].v);
    }
    else {
        int m = (l + r) / 2;
        T[x].l = (!T[x].l || !T[y].l ? 0 : intersect(T[x].l, T[y].l, l, m));
        T[x].r = (!T[x].r || !T[y].r ? 0 : intersect(T[x].r, T[y].r, m + 1, r));
        T[x].v = T[T[x].l].v + T[T[x].r].v;
    }
    return T[x].v == 0 ? 0 : x;
}
int symmetric(int x, int y, int l = 1, int r = len) {    //返回集合 x 与集合 y 的对称差分, 执行之
↪ 后 x、y 均会失效
    if (!x) return y;                                  //时间复杂度  $\min\{size(x), size(y)\} * \log(len)$ 
    if (!y) return x;
    if (l == r) {
        T[x].v = abs(T[x].v - T[y].v);
    }
    else {
        int m = (l + r) / 2;
        T[x].l = symmetric(T[x].l, T[y].l, l, m);
        T[x].r = symmetric(T[x].r, T[y].r, m + 1, r);
        T[x].v = T[T[x].l].v + T[T[x].r].v;
    }
    return x;
}
void left(vector<int>& X, vector<int>& Y) {
    for (auto& i : X)
        i = T[i].l;
    for (auto& i : Y)
        i = T[i].l;
}
void right(vector<int>& X, vector<int>& Y) {
    for (auto& i : X)
        i = T[i].r;
    for (auto& i : Y)
        i = T[i].r;
}
int left_value(vector<int>& X, vector<int>& Y) {
    int tot = 0;
    for (auto i : X)

```

```

        tot += T[T[i].l].v;
    for (auto i : Y)
        tot -= T[T[i].l].v;
    return tot;
}

int right_value(vector<int>& X, vector<int>& Y) {
    int tot = 0;
    for (auto i : X)
        tot += T[T[i].r].v;
    for (auto i : Y)
        tot -= T[T[i].r].v;
    return tot;
}

int value(vector<int>& X, vector<int>& Y) {
    int tot = 0;
    for (auto i : X)
        tot += T[i].v;
    for (auto i : Y)
        tot -= T[i].v;
    return tot;
}

int kth(vector<int> X, vector<int> Y, int k) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得集
↪ 合的第 k 大
    int l = 1, r = len;                          //必须保证 Y 的并集 是 X 的并集 的子集
    while (l < r) {
        int m = (l + r) / 2, t = left_value(X, Y);
        if (k <= t)
            left(X, Y), r = m;
        else
            right(X, Y), l = m + 1, k -= t;
    }
    if (k > value(X, Y))
        return inf;
    return info[r];
}

int ask(vector<int> X, vector<int> Y, int v) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得集
↪ 合中比 v 小的数的个数
    int l = 1, r = len, k = 0;                    //必须保证 Y 的并集 是 X 的并集 的子集
    int p = ranking(v) - 1;
    if (p <= 0) return 0;
    while (l < r) {
        int m = (l + r) / 2;
        if (p <= m)
            left(X, Y), r = m;
        else {
            k += left_value(X, Y);

```

```

        right(X, Y);
        l = m + 1;
    }
}
k += value(X, Y);
return k;
}

int pre(vector<int> X, vector<int> Y, int v) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得集
↪ 合中 v 的前驱
    int k = ask(X, Y, v);                        //必须保证 Y 的并集 是 X 的并集 的子集
    if (k == 0) return -inf; //找不到返回-inf
    return kth(X, Y, k);
}

int next(vector<int> X, vector<int> Y, int v) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得
↪ 集合中 v 的后继
    int k = ask(X, Y, v + 1) + 1;                //必须保证 Y 的并集 是 X 的并集 的子集
    return kth(X, Y, k); //找不到返回 inf
}

int count(vector<int> X, vector<int> Y, int v) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得
↪ 集合中 v 的个数
    int l = 1, r = len;                          //必须保证 Y 的并集 是 X 的并集 的子集
    int p = ranking(v);
    if (p > len || info[p] != v)
        return 0;
    while (l < r) {
        int m = (l + r) / 2;
        if (p <= m)
            left(X, Y), r = m;
        else
            right(X, Y), l = m + 1;
    }
    return value(X, Y);
}

int maximum(vector<int> X, vector<int> Y) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得集合中
↪ 的最大值
    int l = 1, r = len;                          //必须保证 Y 的并集 是 X 的并集 的子集
    if (value(X, Y) == 0) //如果集合为空则返回-inf
        return -inf;
    while (l < r) {
        int m = (l + r) / 2, t = right_value(X, Y);
        if (t == 0)
            left(X, Y), r = m;
        else
            right(X, Y), l = m + 1;
    }
    return info[r];
}

```

```

}

int minimum(vector<int> X, vector<int> Y) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得集合中
↪ 的最小值
    int l = 1, r = len;                      //必须保证 Y 的并集 是 X 的并集 的子集
    if (value(X, Y) == 0) //如果集合为空则返回 inf
        return inf;
    while (l < r) {
        int m = (l + r) / 2, t = left_value(X, Y);
        if (t > 0)
            left(X, Y), r = m;
        else
            right(X, Y), l = m + 1;
    }
    return info[r];
}

#define VALUE_TYPE 100
int tmp[maxn], root[maxn];
vector<int> A[maxn], B[maxn], C[maxn];
int main() {
    int n = 10, q = 400000;
    default_random_engine e;
    uniform_int_distribution<int> d(1, VALUE_TYPE);
    uniform_int_distribution<int> big(-1000000000, 1000000000);
    e.seed(time(0));
    for (int i = 1; i <= VALUE_TYPE; ++i)
        tmp[i] = big(e);
    init(tmp, VALUE_TYPE);
    int rt = 0;
    vector<int> vec;
    for (int i = 1; i <= 100000; ++i) {
        int val = d(e); val = tmp[val];
        if (rand() % 2)
            insert(root[0], val), A[0].push_back(val);
        else
            insert(root[1], val), A[1].push_back(val);
    }
    rt = merge(root[0], root[1]);
    sort(A[0].begin(), A[0].end());
    sort(A[1].begin(), A[1].end());
    //std::set_union(A[0].begin(), A[0].end(), A[1].begin(), A[1].end(), back_inserter(vec));
    vec = A[0]; for (auto v : A[1]) vec.push_back(v);
    vec.push_back(-inf); vec.push_back(inf);
    sort(vec.begin(), vec.end());
    printf("%d %d\n", vec.size(), T[rt].v);
    for (int q = 0; q < 100000; ++q) {
        int tp = rand() % 8, val = d(e);

```



```

    val = tmp[val];
    if (tp == 0) {
        insert(rt, val);
        vec.push_back(val);
        sort(vec.begin(), vec.end());
    }
    else if (tp == 1) {
        int k = rand() % T[rt].v + 1;
        if (vec[k] != kth(rt, k))
            abort();
    }
    else if (tp == 2) {
        if (lower_bound(vec.begin(), vec.end(), val) - vec.begin() - 1 != ask(rt, val))
            abort();
    }
    else if (tp == 3) {
        int p = lower_bound(vec.begin(), vec.end(), val) - vec.begin() - 1;
        if (vec[p] != pre(rt, val))
            abort();
    }
    else if (tp == 4) {
        int p = upper_bound(vec.begin(), vec.end(), val) - vec.begin();
        if (vec[p] != next(rt, val))
            abort();
    }
    else if (tp == 5) {
        if (count(rt, val) != count(vec.begin(), vec.end(), val))
            abort();
    }
    else if (tp == 6) {
        if (*-- --vec.end() != maximum(rt))
            abort();
    }
    else if (tp == 7) {
        if (*++vec.begin() != minimum(rt))
            abort();
    }
}
return 0;
}

```

8.17 可持久化整数集合

```

const int inf = 1 << 30;
static const int maxn = 210000; //maxn 应当大于所有集合大小的总和
struct node {                //该算法实现的整数集合为可重集合 (multiset)
    int l, r, v;              //左节点编号 右节点编号 值

```

```

} T[maxn * 25];
int len, sz, info[maxn]; //len 是值域线段树的长度
int ranking(int v) {
    //如果不需要数据离散化直接 return v 即可, 同时还要将所有的 info[v] 改为 v, 此时线段树的值域为
    ↪ [1, len]。
    return lower_bound(info + 1, info + len + 1, v) - info;
}
void ins(int& i, int l, int r, int p) { //在区间 [l, r] 中插入 p
    T[++sz] = T[i]; i = sz;
    T[i].v++; //递增当前区间内值的个数
    if (l == r) return;
    int m = (l + r) / 2;
    if (p <= m) ins(T[i].l, l, m, p);
    else ins(T[i].r, m + 1, r, p);
}
int insert(int x, int v) { //向集合 x 中添加数值 v, 返回新的集合的编号, 原来的集合不会改变
    ins(x, 1, len, ranking(v));
    return x;
}
void init(int* A, int length) { //输入数组 A 的下标从 1 开始, length 表示 A 的长度
    sz = 0; //A 记录集合中所有可能出现的整数
    copy(A + 1, A + length + 1, info + 1);
    sort(info + 1, info + length + 1); //如果没有离散化需要对 len 进行赋值来指明线段树的值域。
    len = unique(info + 1, info + length + 1) - info - 1; //对 info 数组进行排序去重
}
int kth(int x, int k) { //查找集合 x 中的第 k 大元素
    int l = 1, r = len;
    while (l < r) {
        int m = (l + r) / 2, t = T[T[x].l].v;
        if (k <= t)
            x = T[x].l, r = m;
        else
            x = T[x].r, l = m + 1, k -= t;
    }
    if (k > T[x].v) //如果 k 大于集合 x 的大小, 则返回 inf
        return inf;
    return info[r];
}
int ask(int x, int v) { //返回集合 x 中比 v 小的数的个数
    int l = 1, r = len, k = 0;
    int p = ranking(v) - 1;
    if (p <= 0) return 0;
    while (l < r) {
        int m = (l + r) / 2, t = T[T[x].l].v;
        if (p <= m)
            x = T[x].l, r = m;
    }

```

```

        else
            x = T[x].r, l = m + 1, k += t;
    }
    k += T[x].v;
    return k;
}

int pre(int x, int v) { //在集合 x 中查询 v 的前驱, 找不到返回 -inf
    int k = ask(x, v);
    if (k == 0) return -inf;
    return kth(x, k);
}

int next(int x, int v) { //在集合 x 中查询 v 的后继, 找不到返回 inf
    int k = ask(x, v + 1) + 1;
    return kth(x, k);
}

int count(int x, int v) { //返回集合 x 中数值 v 的个数
    int l = 1, r = len;
    int p = ranking(v);
    if (p > len || info[p] != v)
        return 0;
    while (l < r && T[x].v > 0) {
        int m = (l + r) / 2;
        if (p <= m)
            x = T[x].l, r = m;
        else
            x = T[x].r, l = m + 1;
    }
    return T[x].v;
}

bool find(int x, int v) { //返回集合 x 中是否存在 v
    return count(x, v) >= 1;
}

int maximum(int x) { //返回集合 x 中的最大值
    int l = 1, r = len;
    if (T[x].v == 0) //如果集合为空则返回 -inf
        return -inf;
    while (l < r) {
        int m = (l + r) / 2, t = T[T[x].r].v;
        if (t == 0)
            x = T[x].l, r = m;
        else
            x = T[x].r, l = m + 1;
    }
    return info[r];
}

int minimum(int x) { //返回集合 x 中的最小值

```

```

    int l = 1, r = len;
    if (T[x].v == 0) //如果集合为空则返回 inf
        return inf;
    while (l < r) {
        int m = (l + r) / 2, t = T[T[x].l].v;
        if (t > 0)
            x = T[x].l, r = m;
        else
            x = T[x].r, l = m + 1;
    }
    return info[r];
}

int merge(int x, int y, int l = 1, int r = len) { //将集合 x 与集合 y 合并成一个新的集合后返回
    //不会改变 x 和 y
    if (!x) return y;
    if (!y) return x;
    int i = ++sz;
    if (l == r) {
        T[i].v = T[x].v + T[y].v;
        T[i].l = T[i].r = 0;
    }
    else {
        int m = (l + r) / 2;
        T[i].l = merge(T[x].l, T[y].l, l, m);
        T[i].r = merge(T[x].r, T[y].r, m + 1, r);
        T[i].v = T[T[i].l].v + T[T[i].r].v;
    }
    return i;
}

int diff(int x, int y, int l = 1, int r = len) { //返回集合 x 与集合 y 的差集 (x - y), 不会改变
    ↪ x 和 y //时间复杂度 size(y)log(len)
    int i = ++sz; T[i] = T[x];
    if (l == r) {
        T[i].v = max(0, T[x].v - T[y].v);
    }
    else {
        int m = (l + r) / 2;
        if (T[y].l)
            T[i].l = diff(T[x].l, T[y].l, l, m);
        if (T[y].r)
            T[i].r = diff(T[x].r, T[y].r, m + 1, r);
        T[i].v = T[T[i].l].v + T[T[i].r].v;
    }
    return i;
}

int intersect(int x, int y, int l = 1, int r = len) { //返回集合 x 与集合 y 的交集 (不会改变 x
    ↪ 和 y)

```

```

    int i = ++sz; //时间复杂度  $\min\{size(x), size(y)\} * \log(len)$ 
    ↪ log(len)
    if (l == r) {
        T[i].v = min(T[x].v, T[y].v);
        T[i].l = T[i].r = 0;
    }
    else {
        int m = (l + r) / 2;
        T[i].l = (!T[x].l || !T[y].l ? 0 : intersect(T[x].l, T[y].l, l, m));
        T[i].r = (!T[x].r || !T[y].r ? 0 : intersect(T[x].r, T[y].r, m + 1, r));
        T[i].v = T[T[i].l].v + T[T[i].r].v;
    }
    return T[i].v == 0 ? 0 : i;
}

int symmetric(int x, int y, int l = 1, int r = len) { //返回集合  $x$  与集合  $y$  的对称差分 (不会改变  $x$  和  $y$ )
    ↪ 变  $x$  和  $y$ 
    if (!x) return y; //时间复杂度  $\min\{size(x), size(y)\} * \log(len)$ 
    ↪ log(len)
    if (!y) return x;
    int i = ++sz;
    if (l == r) {
        T[i].v = abs(T[x].v - T[y].v);
        T[i].l = T[i].r = 0;
    }
    else {
        int m = (l + r) / 2;
        T[i].l = symmetric(T[x].l, T[y].l, l, m);
        T[i].r = symmetric(T[x].r, T[y].r, m + 1, r);
        T[i].v = T[T[i].l].v + T[T[i].r].v;
    }
    return i;
}

void left(vector<int>& X, vector<int>& Y) {
    for (auto& i : X)
        i = T[i].l;
    for (auto& i : Y)
        i = T[i].l;
}

void right(vector<int>& X, vector<int>& Y) {
    for (auto& i : X)
        i = T[i].r;
    for (auto& i : Y)
        i = T[i].r;
}

int left_value(vector<int>& X, vector<int>& Y) {
    int tot = 0;

```

```

    for (auto i : X)
        tot += T[T[i].l].v;
    for (auto i : Y)
        tot -= T[T[i].l].v;
    return tot;
}

int right_value(vector<int>& X, vector<int>& Y) {
    int tot = 0;
    for (auto i : X)
        tot += T[T[i].r].v;
    for (auto i : Y)
        tot -= T[T[i].r].v;
    return tot;
}

int value(vector<int>& X, vector<int>& Y) {
    int tot = 0;
    for (auto i : X)
        tot += T[i].v;
    for (auto i : Y)
        tot -= T[i].v;
    return tot;
}

int kth(vector<int> X, vector<int> Y, int k) { //计算 X 中集合的并集 减去 Y 中集合的并集 所得集
    ↪ 合的第 k 大 //必须保证 Y 的并集 是 X 的并集 的子集
    int l = 1, r = len;
    while (l < r) {
        int m = (l + r) / 2, t = left_value(X, Y);
        if (k <= t)
            left(X, Y), r = m;
        else
            right(X, Y), l = m + 1, k -= t;
    }
    if (k > value(X, Y))
        return inf;
    return info[r];
}

int ask(vector<int> X, vector<int> Y, int v) { //计算 X 中集合的并集 减去 Y 中集合的并集 所得集
    ↪ 合中比 v 小的数的个数 //必须保证 Y 的并集 是 X 的并集 的子集
    int l = 1, r = len, k = 0;
    int p = ranking(v) - 1;
    if (p <= 0) return 0;
    while (l < r) {
        int m = (l + r) / 2;
        if (p <= m)
            left(X, Y), r = m;
        else {

```

```

        k += left_value(X, Y);
        right(X, Y);
        l = m + 1;
    }
}
k += value(X, Y);
return k;
}

int pre(vector<int> X, vector<int> Y, int v) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得集
↪ 合中 v 的前驱
    int k = ask(X, Y, v);                        //必须保证 Y 的并集 是 X 的并集 的子集
    if (k == 0) return -inf; //找不到返回-inf
    return kth(X, Y, k);
}

int next(vector<int> X, vector<int> Y, int v) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得
↪ 集合中 v 的后继
    int k = ask(X, Y, v + 1) + 1;                //必须保证 Y 的并集 是 X 的并集 的子集
    return kth(X, Y, k); //找不到返回 inf
}

int count(vector<int> X, vector<int> Y, int v) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得
↪ 集合中 v 的个数
    int l = 1, r = len;                          //必须保证 Y 的并集 是 X 的并集 的子集
    int p = ranking(v);
    if (p > len || info[p] != v)
        return 0;
    while (l < r) {
        int m = (l + r) / 2;
        if (p <= m)
            left(X, Y), r = m;
        else
            right(X, Y), l = m + 1;
    }
    return value(X, Y);
}

int maximum(vector<int> X, vector<int> Y) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得集合中
↪ 的最大值
    int l = 1, r = len;                          //必须保证 Y 的并集 是 X 的并集 的子集
    if (value(X, Y) == 0) //如果集合为空则返回-inf
        return -inf;
    while (l < r) {
        int m = (l + r) / 2, t = right_value(X, Y);
        if (t == 0)
            left(X, Y), r = m;
        else
            right(X, Y), l = m + 1;
    }
}

```

```

    return info[r];
}
int minimum(vector<int> X, vector<int> Y) {    //计算 X 中集合的并集 减去 Y 中集合的并集 所得集合中
↪ 的最小值
    int l = 1, r = len;                      //必须保证 Y 的并集 是 X 的并集 的子集
    if (value(X, Y) == 0) //如果集合为空则返回 inf
        return inf;
    while (l < r) {
        int m = (l + r) / 2, t = left_value(X, Y);
        if (t > 0)
            left(X, Y), r = m;
        else
            right(X, Y), l = m + 1;
    }
    return info[r];
}
#define VALUE_TYPE 5000
int tmp[maxn], root[maxn];
vector<int> A[maxn];
int main() {
    int n = 10, q = 400000;
    default_random_engine e;
    uniform_int_distribution<int> d(1, VALUE_TYPE);
    uniform_int_distribution<int> big(-1000000000, 1000000000);
    //e.seed(time(0));
    for (int i = 1; i <= VALUE_TYPE; ++i)
        tmp[i] = big(e);
    init(tmp, VALUE_TYPE);
    int rt = 0;
    vector<int> vec;
    for (int i = 1; i <= 100000; ++i) {
        int val = d(e); val = tmp[val];
        if (rand() % 2)
            root[0] = insert(root[0], val), A[0].push_back(val);
        else
            root[1] = insert(root[1], val), A[1].push_back(val);
    }
    rt = symmetric(root[0], root[1]);
    sort(A[0].begin(), A[0].end());
    sort(A[1].begin(), A[1].end());
    std::set_symmetric_difference(A[0].begin(), A[0].end(), A[1].begin(), A[1].end(),
↪ back_inserter(vec));
    //rt = root[1]; vec = A[1];
    vec.push_back(-inf); vec.push_back(inf);
    sort(vec.begin(), vec.end());
    printf("%d %d\n", vec.size(), T[rt].v);
}

```



```

for (int q = 0; q < 100000; ++q) {
    if (q % 1000 == 0) printf("%d\n", q);
    int tp = rand() % 8, val = d(e);
    val = tmp[val];
    if (tp == 0) {
        rt = insert(rt, val);
        vec.push_back(val);
        sort(vec.begin(), vec.end());
    }
    else if (tp == 1) {
        int k = rand() % T[rt].v + 1;
        if (vec[k] != kth(rt, k))
            abort();
    }
    else if (tp == 2) {
        if (lower_bound(vec.begin(), vec.end(), val) - vec.begin() - 1 != ask(rt, val))
            abort();
    }
    else if (tp == 3) {
        int p = lower_bound(vec.begin(), vec.end(), val) - vec.begin() - 1;
        if (vec[p] != pre(rt, val))
            abort();
    }
    else if (tp == 4) {
        int p = upper_bound(vec.begin(), vec.end(), val) - vec.begin();
        if (vec[p] != next(rt, val))
            abort();
    }
    else if (tp == 5) {
        if (count(rt, val) != count(vec.begin(), vec.end(), val))
            abort();
    }
    else if (tp == 6) {
        if (*-- --vec.end() != maximum(rt))
            abort();
    }
    else if (tp == 7) {
        if (*++vec.begin() != minimum(rt))
            abort();
    }
}
return 0;
}

```

8.18 线段树

```

const int maxn = 210000, offset = 210000;
const int inf = 1 << 30;
struct tmp {
    int data[maxn * 5];
    int& operator[] (int idx) {
        return data[idx + offset];
    }
}A;
#define lc t[p].lchild
#define rc t[p].rchild
int cur = 0, tot, mn, mx;
struct segment {
    int l, r, lchild, rchild;
    int sum, min, max, set, add;
}t[maxn * 4];
void init() {
    cur = 0;
}
inline void maintain(int p) {
    t[p].sum = t[lc].sum + t[rc].sum;
    t[p].min = min(t[lc].min, t[rc].min);
    t[p].max = max(t[lc].max, t[rc].max);
}
inline void mark(int p, int setv, int addv) { //给结点打标记
    if (setv >= 0) {
        t[p].set = setv; t[p].add = 0;
        t[p].min = t[p].max = setv;
        t[p].sum = setv * (t[p].r - t[p].l + 1);
    }
    if (addv) {
        t[p].add += addv;
        t[p].min += addv;
        t[p].max += addv;
        t[p].sum += addv * (t[p].r - t[p].l + 1);
    }
}
inline void pushdown(int p) { //pushdown 将标记传递给子结点，不影响当前结点的信息。
    mark(lc, t[p].set, t[p].add);
    mark(rc, t[p].set, t[p].add);
    t[p].set = -1;
    t[p].add = 0;
}
int build(int L, int R) { //只要计算 mid 的方式是 (L + R) >> 1 而不是 (L + R) / 2, 就可以建立负坐标
    ↪ 线段树。
    int p = ++cur;

```

```

    t[p].l = L;
    t[p].r = R;
    t[p].add = 0; t[p].set = -1; //清空结点标记
    if (t[p].l == t[p].r) {
        mark(p, 0, A[L]);
    }
    else {
        int mid = (t[p].l + t[p].r) >> 1;
        lc = build(L, mid);
        rc = build(mid + 1, R);
        maintain(p);
    }
    return p;
}

void update(int p, int L, int R, int op, int v) {
    if (L <= t[p].l && R >= t[p].r) {
        if (op == 0)
            mark(p, -1, v);
        else
            mark(p, v, 0);
    }
    else {
        pushdown(p); //如果没有 pushdown 只需要在最后调用一次 maintain 即可。
        int mid = (t[p].l + t[p].r) >> 1;
        if (L <= mid)
            update(lc, L, R, op, v);
        if (R > mid)
            update(rc, L, R, op, v);
        maintain(p);
    }
}

void update(int p, int pos, int v) { //单点修改
    if (t[p].l == t[p].r) {
        mark(p, -1, v);
    }
    else {
        pushdown(p);
        int mid = (t[p].l + t[p].r) >> 1;
        if (pos <= mid)
            update(lc, pos, v);
        else
            update(rc, pos, v);
        maintain(p);
    }
}

void query(int p, int L, int R) { //调用之前要设置: mn = inf; mx = -inf; tot = 0;

```

```

    if (L <= t[p].l && R >= t[p].r) {
        tot += t[p].sum;
        mn = min(mn, t[p].min);
        mx = max(mx, t[p].max);
    }
    else {
        pushdown(p);
        int mid = (t[p].l + t[p].r) >> 1;
        if (L <= mid)
            query(lc, L, R);
        if (R > mid)
            query(rc, L, R);
    }
}

int main() {
    default_random_engine e;
    int n = 100000, m = 100000;
    uniform_int_distribution<int> d(-n, n);
    for (int i = -n; i <= n; ++i)
        A[i] = d(e);
    init();
    int root = build(-n, n);
    for (int i = 1; i <= m; ++i) {
        int op = rand() % 4, a = d(e), b = d(e), v = rand();
        int L = min(a, b), R = max(a, b);
        if (op == 0) {
            for (int i = L; i <= R; ++i)
                A[i] += v;
            update(root, L, R, op, v);
        }
        else if (op == 1) {
            for (int i = L; i <= R; ++i)
                A[i] = v;
            update(root, L, R, op, v);
        }
        else if (op == 2) {
            mn = inf; mx = -inf; tot = 0;
            query(root, L, R);
            if (mn != *min_element(A.data + offset + L, A.data + offset + R + 1))
                abort();
            if (mx != *max_element(A.data + offset + L, A.data + offset + R + 1))
                abort();
            if (tot != accumulate(A.data + offset + L, A.data + offset + R + 1, 0))
                abort();
        }
    }
    else {

```

```

        A[L] += v;
        update(root, L, v);
    }
}
return 0;
}

```

8.19 动态开点线段树

```

const int maxn = 210000, offset = 210000;
const int inf = 1 << 30;
struct tmp {
    int data[maxn * 5];
    int& operator[] (int idx) {
        return data[idx + offset];
    }
}A;
#define lc t[p].lchild
#define rc t[p].rchild
int cur = 0, tot, mn, mx;
struct segment {
    int lchild, rchild;
    int sum, min, max, set, add;
}t[maxn * 4];
void init() {
    cur = 0;
}
inline void newnode(int& p) {
    if (p == 0) {
        p = ++cur;
        memset(&t[p], 0, sizeof(t[p]));
        t[p].set = -1;
    }
}
inline void maintain(int p) {
    t[p].sum = t[lc].sum + t[rc].sum;
    t[p].min = min(t[lc].min, t[rc].min);
    t[p].max = max(t[lc].max, t[rc].max);
}
inline void mark(int &p, int setv, int addv, int x, int y) { //给结点打标记
    newnode(p);
    if (setv >= 0) {
        t[p].set = setv; t[p].add = 0;
        t[p].min = t[p].max = setv;
        t[p].sum = setv * (y - x + 1);
    }
    if (addv) {

```

```

        t[p].add += addv;
        t[p].min += addv;
        t[p].max += addv;
        t[p].sum += addv * (y - x + 1);
    }
}

inline void pushdown(int p, int x, int y) { //pushdown 将标记传递给子结点，不影响当前结点的信息。
    int mid = (x + y) >> 1;
    mark(lc, t[p].set, t[p].add, x, mid);
    mark(rc, t[p].set, t[p].add, mid+1, y);
    t[p].set = -1;
    t[p].add = 0;
}

void update(int &p, int L, int R, int op, int v, int x, int y) {
    newnode(p);
    if (L <= x && R >= y) {
        if (op == 0)
            mark(p, -1, v, x, y);
        else
            mark(p, v, 0, x, y);
    }
    else {
        pushdown(p, x, y); //如果没有 pushdown 只需要在最后调用一次 maintain 即可。
        int mid = (x + y) >> 1;
        if (L <= mid)
            update(lc, L, R, op, v, x, mid);
        if (R > mid)
            update(rc, L, R, op, v, mid+1, y);
        maintain(p);
    }
}

void query(int &p, int L, int R, int x, int y) { //调用之前要设置: mn = inf; mx = -inf; tot = 0;
    newnode(p);
    if (L <= x && R >= y) {
        tot += t[p].sum;
        mn = min(mn, t[p].min);
        mx = max(mx, t[p].max);
    }
    else {
        pushdown(p, x, y);
        int mid = (x + y) >> 1;
        if (L <= mid)
            query(lc, L, R, x, mid);
        if (R > mid)
            query(rc, L, R, mid+1, y);
    }
}

```

```

}

int main() {
    default_random_engine e;
    int n = 100000, m = 100000;
    uniform_int_distribution<int> d(-n, n);
    for (int i = -n; i <= n; ++i)
        A[i] = 0;
    init();
    int root = 0;
    for (int i = 1; i <= m; ++i) {
        int op = rand() % 3, a = d(e), b = d(e), v = rand();
        int L = min(a, b), R = max(a, b);
        if (op == 0) {
            for (int i = L; i <= R; ++i)
                A[i] += v;
            update(root, L, R, op, v, -n, n);
        }
        else if (op == 1) {
            for (int i = L; i <= R; ++i)
                A[i] = v;
            update(root, L, R, op, v, -n, n);
        }
        else {
            mn = inf; mx = -inf; tot = 0;
            query(root, L, R, -n, n);
            if (mn != *min_element(A.data + offset + L, A.data + offset + R + 1))
                abort();
            if (mx != *max_element(A.data + offset + L, A.data + offset + R + 1))
                abort();
            if (tot != accumulate(A.data + offset + L, A.data + offset + R + 1, 0))
                abort();
        }
    }
    return 0;
}

```

8.20 主席树

/*

动态区间颜色数:

把主席树第 i 个位置的权值设为这个数上一次出现的位置, 然后查询区间 $[L, R]$ 中的颜色数转化为查询区间 $[L, R]$ 中比 L 小的值有多少个。(可以直接用 `ask` 函数查询)

更快的方法: 直接在主席树的第 R 个版本中查询比 L 小的数有多少个, 然后减去 $(L - 1)$ 即可。

*/

```
const int inf = 1 << 30;
```

```
struct ZXTree
```

```
{
```

```

static const int maxn = 100010;
struct node
{
    int l, r, v; //左节点编号 右节点编号 值
} T[maxn * 25];
int n, sz, root[maxn], data[maxn]; //n 是值域线段树的长度
void ins(int &i, int l, int r, int p) //在区间 [l, r] 中插入 p
{
    int m = (l + r) / 2;
    T[++sz] = T[i]; i = sz;
    T[i].v++; //递增当前区间内值的个数
    if (l == r) return;
    if (p <= m) ins(T[i].l, l, m, p);
    else ins(T[i].r, m + 1, r, p);
}
int rank(int v)
{
    //如果不需要数据离散化直接 return v 即可, 同时还要将所有的 data[v] 改为 v, 此时主席树的值域
    ↪ 为 [1, n]。
    return lower_bound(data + 1, data + n + 1, v) - data;
}
void init(int *A, int length) //输入数组 A 的下标从 1 开始, length 表示 A 的长度
{
    root[0] = sz = 0;
    copy(A + 1, A + length + 1, data + 1);
    sort(data + 1, data + length + 1); //如果没有离散化需要对 n 进行赋值来指明主席树的值域。
    this->n = unique(data + 1, data + length + 1) - data - 1; //对 data 数组进行排序去重
    for (int i = 1; i <= length; ++i) //依次把原数组中每个位置的排名插入到可持久化线段树中
        ins(root[i] = root[i - 1], 1, n, rank(A[i])); //插入 rank[i], 如果没有对 A 数组进行离
    ↪ 散化, 此处直接插入 A[i] 即可。
    //root[i] 表示用 A[1] 到 A[i] 的所有值构
    ↪ 建的权值线段树。
}
int kth(int x, int y, int k) //查询原数组中区间 [x, y] 中的第 k 小的值, 如果 k 大于区间长度会
    ↪ 返回无效值!!!
{
    int l = 1, r = n;
    x = root[x - 1], y = root[y];
    while (l < r)
    {
        int m = (l + r) / 2, t = T[T[y].l].v - T[T[x].l].v;
        if (k <= t)
            x = T[x].l, y = T[y].l, r = m;
        else
            x = T[x].r, y = T[y].r, l = m + 1, k -= t;
    }
}

```



```

    return data[r];
}
int ask(int x, int y, int v) //查询原数组中区间  $[x, y]$  中比  $v$  小的值的个数
{
    int l = 1, r = n, k = 0;
    x = root[x - 1], y = root[y];
    int p = rank(v) - 1;
    if (p <= 0) return 0;
    while (l < r)
    {
        int m = (l + r) / 2, t = T[T[y].l].v - T[T[x].l].v;
        if (p <= m)
            x = T[x].l, y = T[y].l, r = m;
        else
            x = T[x].r, y = T[y].r, l = m + 1, k += t;
    }
    k += T[y].v - T[x].v;
    return k;
}
int pre(int x, int y, int l, int r, int p)
{
    int m = (l + r) / 2, v = T[y].v - T[x].v;
    if (l == r) return v > 0 ? data[r] : -inf;
    int t = T[T[y].r].v - T[T[x].r].v;
    if (p <= m || t == 0) return pre(T[x].l, T[y].l, l, m, p); //如果  $p$  在中点的左侧或者右子树
    ↪ 为空, 则直接在左子树查找
    int k = pre(T[x].r, T[y].r, m + 1, r, p);
    if (k != -inf) return k;
    return pre(T[x].l, T[y].l, l, m, p);
}
int pre(int x, int y, int v) //在区间  $[x, y]$  中查询  $v$  的前驱, 找不到返回  $-inf$ 
{
    int p = rank(v) - 1;
    if (p <= 0) return -inf;
    return pre(root[x - 1], root[y], 1, n, p);
}
int pre2(int x, int y, int v) //在区间  $[x, y]$  中查询  $v$  的前驱, 找不到返回  $-inf$ 
{
    int k = ask(x, y, v);
    if (k == 0) return -inf;
    return kth(x, y, k);
}
int next(int x, int y, int l, int r, int p)
{
    int m = (l + r) / 2, v = T[y].v - T[x].v;
    if (l == r) return v > 0 ? data[r] : inf;

```

```

    int t = T[T[y].l].v - T[T[x].l].v;
    if (p > m || t == 0) return next(T[x].r, T[y].r, m + 1, r, p);
    int k = next(T[x].l, T[y].l, 1, m, p);
    if (k != inf) return k;
    return next(T[x].r, T[y].r, m + 1, r, p);
}

int next(int x, int y, int v) //在区间 [x, y] 中查询 v 的后继, 找不到返回 inf
{
    int p = rank(v + 1);
    if (p > n) return inf;
    return next(root[x - 1], root[y], 1, n, p);
}

int next2(int x, int y, int v) //在区间 [x, y] 中查询 v 的后继, 找不到返回 inf
{
    int k = ask(x, y, v + 1) + 1;
    if (k > y - x + 1) return inf;
    return kth(x, y, k);
}

int count(int x, int y, int v) //返回区间 [x, y] 中 v 的个数
{
    int l = 1, r = n;
    x = root[x - 1], y = root[y];
    int p = rank(v);
    if (p > n || data[p] != v)
        return 0;
    while (l < r && T[y].v - T[x].v > 0)
    {
        int m = (l + r) / 2;
        if (p <= m)
            x = T[x].l, y = T[y].l, r = m;
        else
            x = T[x].r, y = T[y].r, l = m + 1;
    }
    return T[y].v - T[x].v;
}

bool find(int x, int y, int v) //查询原数组的区间 [x, y] 中是否有元素 v
{
    return count(x, y, v) >= 1;
}

}zxtree;

const int maxn = 100000;
int A[maxn], n, q;
int main()
{
    freopen("in.txt", "r", stdin);
    freopen("out.txt", "w", stdout);

```

```

scanf("%d %d", &n, &q);
for (int i = 1; i <= n; ++i)
    scanf("%d", &A[i]);
zxtree.init(A, n);
for (int i = 0; i < q; ++i)
{
    int tp, L, R, v;
    scanf("%d %d %d %d", &tp, &L, &R, &v);
    if (tp == 0)
    {
        printf("%d\n", zxtree.pre(L, R, v));
    }
    else if (tp == 1)
    {
        printf("%d\n", zxtree.next(L, R, v));
    }
    else if (tp == 2)
    {
        printf("%d\n", zxtree.kth(L, R, v));
    }
    else if (tp == 3)
    {
        printf("%d\n", zxtree.ask(L, R, v));
    }
    else if (tp == 4)
    {
        printf("%d\n", zxtree.count(L, R, v));
    }
    else
    {
        bool result = zxtree.find(L, R, v);
        printf("%d\n", (int)result);
    }
}
return 0;
}

```

8.21 动态主席树

```

#define lowbit(x) ((x) & (-x))
const int inf = 1 << 30;
struct Dynamic_ZXTree {
    static const int maxn = 51000;
    struct node {
        int l, r, v; //左节点编号 右节点编号 值
    } T[maxn * 100]; //size(T) = maxn * logmaxn * logmaxn, size(data) = maxn + maxq

```

```

int n, sz, length, nx, ny, data[maxn * 3], seq[maxn], X[maxn], Y[maxn], C[maxn]; //n 是值域线
↪ 段树的长度
void sum(int x, int y) {
    nx = ny = 0;
    for (int i = x; i > 0; i -= lowbit(i))
        X[nx++] = C[i];
    for (int i = y; i > 0; i -= lowbit(i))
        Y[ny++] = C[i];
}
void add(int x, int value, int v) { //内部函数, 不要在类外访问!
    int p = rank(value);
    for (int i = x; i <= length; i += lowbit(i))
        ins(C[i], 1, n, p, v);
}
void set(int x, int value) { //将序列中下标为 x 的位置的数修改为 value
    add(x, seq[x], -1);
    add(x, value, 1);
    seq[x] = value;
}
void ins(int& i, int l, int r, int p, int v = 1) {
    int m = (l + r) / 2;
    if (i == 0) {
        T[++sz] = T[i];
        i = sz;
    }
    T[i].v += v;
    if (l == r) return;
    if (p <= m) ins(T[i].l, l, m, p, v);
    else ins(T[i].r, m + 1, r, p, v);
}
int rank(int v) {
    //如果不需要数据离散化直接 return v 即可, 同时还要将所有的 data[v] 改为 v, 此时线段树的值域
↪ 为 [1, n]。
    return lower_bound(data + 1, data + n + 1, v) - data;
}
void init(int* A, int length, int* all, int size) //输入数组 A 的下标从 1 开始, length 表示
↪ A 的长度
{
    //数组 all 记录所有可能出现的值 (包括可能
↪ 修改为的值), 下标从 1 开始, size 为数组 all 的长度
    this->length = length;
    copy(A + 1, A + length + 1, seq + 1);
    copy(all + 1, all + size + 1, data + 1); //data[i] 储存原序列中所有可能出现的值 (包括可
↪ 能修改成的值) 离散化之后第 i 小的值
    sort(data + 1, data + size + 1); //如果不用离散化, 赋值 data[i] = i
    n = unique(data + 1, data + size + 1) - data - 1;
    for (int i = 1; i <= length; ++i)

```

```

        add(i, rank(seq[i]), 1);
    }
    void left() {
        for (int i = 0; i < ny; ++i)
            Y[i] = T[Y[i]].l;
        for (int i = 0; i < nx; ++i)
            X[i] = T[X[i]].l;
    }
    void right() {
        for (int i = 0; i < ny; ++i)
            Y[i] = T[Y[i]].r;
        for (int i = 0; i < nx; ++i)
            X[i] = T[X[i]].r;
    }
    int left_value() {
        int tot = 0;
        for (int i = 0; i < ny; ++i)
            tot += T[T[Y[i]].l].v;
        for (int i = 0; i < nx; ++i)
            tot -= T[T[X[i]].l].v;
        return tot;
    }
    int value() {
        int tot = 0;
        for (int i = 0; i < ny; ++i)
            tot += T[Y[i]].v;
        for (int i = 0; i < nx; ++i)
            tot -= T[X[i]].v;
        return tot;
    }
    int kth(int x, int y, int k) { //查询原数组中区间  $[x, y]$  中的第  $k$  小的值, 如果  $k$  大于区间长度会
    ↪ 返回无效值!!!
        int l = 1, r = n;
        sum(x - 1, y);
        while (l < r) {
            int m = (l + r) / 2, t = left_value();
            if (k <= t)
                left(), r = m;
            else
                right(), l = m + 1, k -= t;
        }
        return data[r];
    }
    int ask(int x, int y, int v) { //查询原数组中区间  $[x, y]$  中比  $v$  小的值的个数
        int l = 1, r = n, k = 0;
        sum(x - 1, y);

```

```

    int p = rank(v) - 1;
    if (p <= 0) return 0;
    while (l < r) {
        int m = (l + r) / 2;
        if (p <= m)
            left(), r = m;
        else {
            k += left_value();
            right();
            l = m + 1;
        }
    }
    k += value();
    return k;
}

int pre(int x, int y, int v) { //在区间 [x, y] 中查询 v 的前驱, 找不到返回 -inf
    int k = ask(x, y, v);
    if (k == 0) return -inf;
    return kth(x, y, k);
}

int next(int x, int y, int v) { //在区间 [x, y] 中查询 v 的后继, 找不到返回 inf
    int k = ask(x, y, v + 1) + 1;
    if (k > y - x + 1) return inf;
    return kth(x, y, k);
}

int count(int x, int y, int v) { //返回区间 [x, y] 中 v 的个数
    int l = 1, r = n;
    sum(x - 1, y);
    int p = rank(v);
    if (p > n || data[p] != v)
        return 0;
    while (l < r) {
        int m = (l + r) / 2;
        if (p <= m)
            left(), r = m;
        else
            right(), l = m + 1;
    }
    return value();
}

bool find(int x, int y, int v) { //查询原数组的区间 [x, y] 中是否有元素 v
    return count(x, y, v) >= 1;
}

}tree;

const int maxn = 100000;
int A[maxn], B[maxn], n, q;

```

```

int main() {
    freopen("in.txt", "r", stdin);
    freopen("out.txt", "w", stdout);
    scanf("%d %d", &n, &q);
    for (int i = 1; i <= n; ++i)
        scanf("%d", &A[i]);
    for (int i = 1; i <= n; i++)
        B[i] = i;
    tree.init(A, n, B, n);
    for (int i = 0; i < q; ++i) {
        int tp, L, R, v;
        scanf("%d %d %d %d", &tp, &L, &R, &v);
        if (tp == 0) {
            printf("%d\n", tree.pre(L, R, v));
        }
        else if (tp == 1) {
            printf("%d\n", tree.next(L, R, v));
        }
        else if (tp == 2) {
            printf("%d\n", tree.kth(L, R, v));
        }
        else if (tp == 3) {
            printf("%d\n", tree.ask(L, R, v));
        }
        else if (tp == 4) {
            printf("%d\n", tree.count(L, R, v));
        }
        else if (tp == 5) {
            bool result = tree.find(L, R, v);
            printf("%d\n", (int)result);
        }
        else {
            tree.set(L, v);
        }
    }
    return 0;
}

```

8.22 动态开点线段树（单点加-区间求和-合并）

```

const int inf = 1 << 30;
const int maxn = 110000;
struct TreeSet {
    struct node {
        int l, r, v;
    } T[maxn * 25];
    int left, right, sz; //空的线段树标号为 0
}

```

```

void init(int L, int R) { //所有线段树对应区间都为 [L, R]
    left = L;
    right = R;
    sz = 1;
}
int newnode() {
    T[sz].l = T[sz].r = T[sz].v = 0;
    return sz++;
}
void insert(int x, int pos, int value) { //编号为 x 的线段树的 pos 位置上的值加上 value
    int L = left, R = right;
    while (L < R) {
        int M = (L + R) >> 1;
        T[x].v += value;
        if (pos <= M) {
            if (!T[x].l)
                T[x].l = newnode();
            x = T[x].l;
            R = M;
        }
        else {
            if (!T[x].r)
                T[x].r = newnode();
            x = T[x].r;
            L = M + 1;
        }
    }
    T[x].v += value;
}
int sum(int x, int pos) { //在编号为 x 的线段树上计算 pos 位置的前缀和
    int L = left, R = right, res = 0;
    while (x && L < R) {
        int M = (L + R) >> 1, t = T[T[x].l].v;
        if (pos <= M)
            x = T[x].l, R = M;
        else
            x = T[x].r, L = M + 1, res += t;
    }
    res += T[x].v;
    return res;
}
int merge(int x, int y, int l, int r) { //将集合 x 与集合 y 合并成一个新的集合后返回
    if (!x) return y; //合并之后 x、y 均会失效
    if (!y) return x;
    if (l == r)
        T[x].v += T[y].v;
}

```



```

        else {
            int m = (l + r) >> 1;
            T[x].l = merge(T[x].l, T[y].l, l, m);
            T[x].r = merge(T[x].r, T[y].r, m + 1, r);
            T[x].v = T[T[x].l].v + T[T[x].r].v;
        }
        return x;
    }
    int merge(int x, int y) {
        return merge(x, y, left, right);
    }
}tree;
int A[210000];
int main() {
    default_random_engine e;
    uniform_int_distribution<int> d(0, 100000), v(1, 10);
    e.seed(334);
    int n = 100000;
    tree.init(0, n);
    int root = tree.newnode();
    for (int i = 0; i < 100000; ++i) {
        int tp = rand() % 2;
        int pos = d(e), value = v(e);
        if (tp == 0) {
            int tmp = tree.newnode();
            tree.insert(tmp, pos, value);
            root = tree.merge(root, tmp);
            A[pos] += value;
        }
        else {
            int ans = tree.sum(root, pos), res = accumulate(A, A + pos + 1, 0);
            if (ans != res) {
                printf("%d %d\n", ans, res);
                abort();
            }
        }
    }
    return 0;
}

```

8.23 可持久化线段树

/*

可持久化线段树：区间取 *min*

每次 *update* 之后返回新版本的线段树的根结点。

查询操作传入相应版本的线段树的根结点即可。

因为要可持久化，所以必须标记永久化，不能 *pushdown*。

```

*/
const int maxn = 110000;
const int inf = 1 << 30;
struct node {
    int l, r, lc, rc;
    int minv, tag;
} t[maxn * 20];
int cur = 0, mn;
void init() {
    cur = 0;
}
void maintain(int p) {
    int lc = t[p].lc, rc = t[p].rc;
    t[p].minv = t[p].tag;
    if (t[p].r > t[p].l) {
        t[p].minv = min({ t[lc].minv, t[rc].minv, t[p].tag });
    }
}
int build(int L, int R) { //只要计算 mid 的方式是 (L + R) >> 1 而不是 (L + R) / 2, 就可以建立负坐标
↪ 线段树。
    int p = ++cur;
    t[p].l = L;
    t[p].r = R;
    t[p].tag = inf; //重置结点标记
    if (L < R) {
        int mid = (L + R) >> 1;
        t[p].lc = build(L, mid);
        t[p].rc = build(mid + 1, R);
    }
    maintain(p);
    return p;
}
int update(int i, int L, int R, int v) { //区间取 min
    int p = ++cur; t[p] = t[i];
    if (L <= t[p].l && R >= t[p].r) {
        t[p].tag = min(t[p].tag, v);
    }
    else {
        int mid = (t[p].l + t[p].r) >> 1;
        if (L <= mid)
            t[p].lc = update(t[p].lc, L, R, v);
        if (R > mid)
            t[p].rc = update(t[p].rc, L, R, v);
    }
    maintain(p);
    return p;
}

```

```

}

void query(int p, int L, int R) { //调用之前设置 mn = inf, 查询区间最小值
    mn = min(mn, t[p].tag);
    if (L <= t[p].l && R >= t[p].r) {
        mn = min(mn, t[p].minv);
    }
    else {
        int mid = (t[p].l + t[p].r) >> 1;
        if (L <= mid)
            query(t[p].lc, L, R);
        if (R > mid)
            query(t[p].rc, L, R);
    }
}

int main() {
    init();
    int root[10], n = 10;
    root[0] = build(1, n);
    root[1] = update(root[0], 1, 7, 10);
    root[2] = update(root[1], 5, 8, 0);
    mn = inf; query(root[2], 7, 7);
    printf("%d\n", mn);
    return 0;
}

```

8.24 线段树（历史最大值）

```

/*
1. 标记 (a, b) 表示对区间中的所有值 v 执行: v = max(v + a, b)
2. 将标记 (c, d) 合并到 (a, b) 上得到: (a + c, max(b + c, d))
3. 标记 (a, b) 与 (c, d) 取最大得到: (max(a, c), max(b, d))
4. 必须满足 inf > 操作次数 * 单次修改的最大增加值
*/
#define lc t[p].lchild
#define rc t[p].rchild
const int maxn = 510000;
const long long inf = 1LL << 50;
long long A[maxn], mx, hmax;
int cur = 0;
struct segment {
    int l, r, lchild, rchild;
    long long max, hmax; //最大值、历史最大值
    long long a, b, x, y; //(a, b) 为当前标记, (x, y) 为历史最大标记
}t[maxn * 4];

inline void maintain(int p) {
    t[p].max = max(t[lc].max, t[rc].max);
    t[p].hmax = max(t[p].hmax, t[p].max);
}

```

```

}

inline void mark(int p, long long c, long long d) {
    t[p].a += c;
    t[p].b = max(t[p].b + c, d);
    t[p].max = max(t[p].max + c, d);
    t[p].x = max(t[p].x, t[p].a);
    t[p].y = max(t[p].y, t[p].b);
    t[p].hmax = max(t[p].hmax, t[p].max);
    t[p].a = max(t[p].a, -inf); //防止出现: (操作次数 * inf) > LLONG_MAX
}

inline void change(int p, long long c, long long d) {
    long long i = t[p].a + c, j = max(t[p].b + c, d);
    t[p].x = max(t[p].x, i);
    t[p].y = max(t[p].y, j);
    t[p].hmax = max(t[p].hmax, max(t[p].max + c, d));
}

inline void pushdown(int p) {
    change(lc, t[p].x, t[p].y);
    change(rc, t[p].x, t[p].y);
    mark(lc, t[p].a, t[p].b);
    mark(rc, t[p].a, t[p].b);
    t[p].x = t[p].a = 0;
    t[p].y = t[p].b = -inf;
}

int build(int L, int R) {
    int p = ++cur;
    t[p].l = L;
    t[p].r = R;
    t[p].x = t[p].a = 0;
    t[p].y = t[p].b = -inf;
    t[p].hmax = -inf;
    if (t[p].l == t[p].r) {
        t[p].hmax = t[p].max = A[L];
    }
    else {
        int mid = (t[p].l + t[p].r) >> 1;
        lc = build(L, mid);
        rc = build(mid + 1, R);
        maintain(p);
    }
    return p;
}

void query(int p, int L, int R) {
    if (L <= t[p].l && R >= t[p].r) {
        mx = max(mx, t[p].max);
        hmax = max(hmax, t[p].hmax);
    }
}

```

```

    }
    else {
        pushdown(p);
        int mid = (t[p].l + t[p].r) >> 1;
        if (L <= mid)
            query(lc, L, R);
        if (R > mid)
            query(rc, L, R);
    }
}

void update(int p, int L, int R, long long a, long long b) {
    if (L <= t[p].l && R >= t[p].r) {
        mark(p, a, b);
    }
    else {
        pushdown(p);
        int mid = (t[p].l + t[p].r) >> 1;
        if (L <= mid)
            update(lc, L, R, a, b);
        if (R > mid)
            update(rc, L, R, a, b);
        maintain(p);
    }
}

int main() {
    //freopen("in.txt", "r", stdin);
    int n, m;
    scanf("%d", &n);
    for (int i = 1; i <= n; ++i)
        scanf("%lld", &A[i]);
    int root = build(1, n);
    scanf("%d", &m);
    while (m--) {
        char tp;
        int L, R;
        long long v;
        scanf(" %c %d %d\n", &tp, &L, &R);
        if (tp == 'Q') { //查询区间最大值
            mx = -inf;
            query(root, L, R);
            printf("%lld\n", mx);
        }
        else if (tp == 'A') { //查询区间历史最大值
            hmax = -inf;
            query(root, L, R);
            printf("%lld\n", hmax);
        }
    }
}

```

```

    }
    else if (tp == 'P') { //区间 add
        scanf("%lld", &v);
        update(root, L, R, v, -inf);
    }
    else { //区间 set
        scanf("%lld", &v);
        update(root, L, R, -inf, v);
    }
}
return 0;
}

```

8.25 zkw 线段树（单点加-区间加-单点查询-区间求和）

```

const int maxn = 110000;
long long A[maxn];
struct {
    int n, m;
    long long sum[maxn * 4], add[maxn * 4];
    void build(int size) {
        this->n = size;
        for (m = 1; m <= n; m <= 1);
        memset(add, 0, sizeof(long long) * (n + m + 1));
        for (int i = 1; i <= n; ++i)
            sum[m + i] = A[i];
        for (int i = m - 1; i; --i)
            sum[i] = sum[i << 1] + sum[i << 1 | 1];
    }
    void update(int pos, int v) { //单点加 v
        for (int x = pos + m; x > 1; x >>= 1)
            sum[x] += v;
    }
    void update(int s, int t, int v) { //区间加 v
        long long lc = 0, rc = 0, len = 1;
        for (s += m - 1, t += m + 1; s ^ t ^ 1; s >>= 1, t >>= 1, len <= 1) {
            if (~s & 1)
                add[s ^ 1] += v, lc += len;
            if (t & 1)
                add[t ^ 1] += v, rc += len;
            sum[s >> 1] += v * lc, sum[t >> 1] += v * rc;
        }
        for (lc += rc; s; s >>= 1)
            sum[s >> 1] += v * lc;
    }
    long long query(int pos) { //单点查询
        long long ans = sum[pos + m];
    }
}

```

```

    for (int x = pos + m; x; x >>= 1)
        ans += add[x];
    return ans;
}

long long query(int s, int t) { //计算区间和
    long long ans = 0, lc = 0, rc = 0, len = 1;
    for (s += m - 1, t += m + 1; s ^ t ^ 1; s >>= 1, t >>= 1, len <= 1) {
        if (s & 1 ^ 1)
            ans += sum[s ^ 1] + len * add[s ^ 1], lc += len;
        if (t & 1)
            ans += sum[t ^ 1] + len * add[t ^ 1], rc += len;
        ans += add[s >> 1] * lc;
        ans += add[t >> 1] * rc;
    }
    for (lc += rc, s >>= 1; s; s >>= 1)
        ans += add[s] * lc;
    return ans;
}

}tree;

int main() {
    default_random_engine e;
    uniform_int_distribution<int> d(1, 100000);
    e.seed(35345);
    int n = 100000;
    for (int i = 1; i <= n; ++i)
        A[i] = d(e);
    auto start = clock();
    tree.build(n);
    for (int i = 0; i < 1000000; ++i) {
        int tp = d(e) % 4, a = d(e) % n + 1, b = d(e) % n + 1;
        int L = min(a, b), R = max(a, b);
        if (tp == 0) {
            tree.update(L, R, 1);
            for (int i = L; i <= R; ++i)
                A[i] += 1;
        }
        else if (tp == 1) {
            long long ans = tree.query(L, R), res = accumulate(A + L, A + R + 1, 0LL);
            if (ans != res)
                abort();
        }
        else if (tp == 2) {
            tree.update(L, 7);
            A[L] += 7;
        }
        else {

```

```

        long long ans = tree.query(L);
        if (ans != A[L])
            abort();
    }
}

auto end = clock();
printf("time: %.2f\n", static_cast<double>(end - start) / CLOCKS_PER_SEC);
for (int i = 1; i <= 100000; ++i)
    if (A[i] != tree.query(i))
        abort();
return 0;
}

```

8.26 SplayTree

```

const int maxn = 110000;
//若要修改一个点的点权，应当先将其 splay 到根，然后修改，最后还要调用 pushup 维护。
//调用完 splay 之后根结点会改变，应该用 splay 的返回值更新根结点。
namespace splay_tree {
    int ch[maxn][2], fa[maxn], stk[maxn], rev[maxn], sz[maxn], key[maxn], cur;
    void init() {
        cur = 0;
    }
    int newnode(int val) {
        int x = ++cur;
        ch[x][0] = ch[x][1] = fa[x] = rev[x] = 0;
        sz[x] = 1;
        key[x] = val;
        return x;
    }
    inline bool son(int x) {
        return ch[fa[x]][1] == x;
    }
    inline void pushup(int x) {
        sz[x] = sz[ch[x][0]] + sz[ch[x][1]] + 1;
    }
    inline void pushdown(int x) {
        if (rev[x]) {
            rev[x] = 0;
            swap(ch[x][0], ch[x][1]);
            rev[ch[x][0]] ^= 1;
            rev[ch[x][1]] ^= 1;
        }
    }
    void rotate(int x) {
        int y = fa[x], z = fa[y], c = son(x);
        if (fa[y])

```



```

        ch[z][son(y)] = x;
    fa[x] = z;
    ch[y][c] = ch[x][!c];
    fa[ch[y][c]] = y;
    ch[x][!c] = y;
    fa[y] = x;
    pushup(y);
}

void ascend(int x) {
    for (int y = fa[x]; y; rotate(x), y = fa[x]) if (fa[y])
        son(x) ^ son(y) ? rotate(x) : rotate(y);
    pushup(x);
}

int splay(int x) { //没有 pushdown 操作时, 可以直接用 ascend 替换 splay
    int top = 0;
    for (int i = x; i; i = fa[i])
        stk[++top] = i;
    while (top)
        pushdown(stk[top--]);
    ascend(x);
    return x;
}

int splay(int x, int k) { //将以 x 为根的子树中的第 k 个结点旋转到根结点
    while (pushdown(x), k != sz[ch[x][0]] + 1) {
        if (k <= sz[ch[x][0]])
            x = ch[x][0];
        else
            k -= sz[ch[x][0]] + 1, x = ch[x][1];
    }
    if (x) ascend(x);
    return x;
}

template<typename ...T> int merge(int x, int y, T... args) {
    if constexpr (sizeof...(args) == 0) {
        if (x == 0) return y; //swap(x, y);
        x = splay(x, sz[x]);
        ch[x][1] = y; fa[y] = x;
        pushup(x);
        return x;
    }
    else {
        return merge(merge(x, y), args...);
    }
}

auto split(int x, int pos) { //分成两个区间 [1, pos - 1] 和 [pos, n]
    if (pos == sz[x] + 1)

```

```

        return make_pair(x, 0);
    x = splay(x, pos);
    int y = ch[x][0];
    fa[y] = ch[x][0] = 0;
    pushup(x);
    return make_pair(y, x);
}

auto extract(int x, int L, int R) {
    auto [left, y] = split(x, L);
    auto [mid, right] = split(y, R - L + 2);
    return make_tuple(left, mid, right);
}

void traverse(int x) {
    if (x != 0) {
        traverse(ch[x][0]);
        printf("%d ", key[x]);
        //printf("%d (left: %d, right: %d) sz(%d) key(%d)\n", x, ch[x][0], ch[x][1], sz[x],
        ↪ key[x]);
        traverse(ch[x][1]);
    }
}

}

using namespace splay_tree;

int main() {
    init();
    int nd[50], root = 0;
    for (int i = 1; i <= 10; ++i) {
        nd[i] = newnode(i);
        root = merge(root, nd[i]);
    }
    traverse(get<1>(extract(root, 3, 10))); printf("\n");
    return 0;
}

```

8.27 可持久化并查集

```

const int maxn = 210000;
struct node {
    int lc, rc, fa, size;
} t[maxn * 40]; //若内存不够, 可以考虑改成按深度合并
int cur, length;
void init(int n) {
    cur = 0;
    length = n;
}

int build(int L = 1, int R = length) {
    int p = ++cur;

```

```

    t[p].fa = L;
    t[p].size = 1;
    if (L < R) {
        int mid = (L + R) >> 1;
        t[p].lc = build(L, mid);
        t[p].rc = build(mid + 1, R);
    }
    return p;
}

int link(int i, int pos, int fa, int L = 1, int R = length) {
    int p = ++cur; t[p] = t[i];
    t[p].fa = fa;
    if (L < R) {
        int mid = (L + R) >> 1;
        if (pos <= mid)
            t[p].lc = link(t[p].lc, pos, fa, L, mid);
        else
            t[p].rc = link(t[p].rc, pos, fa, mid + 1, R);
    }
    return p;
}

int update(int i, int pos, int size, int L = 1, int R = length) {
    int p = ++cur; t[p] = t[i];
    if (L == R)
        t[p].size += size;
    else {
        int mid = (L + R) >> 1;
        if (pos <= mid)
            t[p].lc = update(t[p].lc, pos, size, L, mid);
        else
            t[p].rc = update(t[p].rc, pos, size, mid + 1, R);
    }
    return p;
}

int query(int p, int pos, int L = 1, int R = length) {
    if (L == R)
        return p;
    else {
        int mid = (L + R) >> 1;
        if (pos <= mid)
            return query(t[p].lc, pos, L, mid);
        else
            return query(t[p].rc, pos, mid + 1, R);
    }
}

int find(int i, int pos) {

```

```

    int p = query(i, pos);
    return t[p].fa == pos ? p : find(i, t[p].fa);
}

int join(int i, int x, int y) { //在版本为 i 的并查集中连接点 x 和点 y, 返回新的并查集
    int a = find(i, x);
    int b = find(i, y);
    if (t[a].fa != t[b].fa) { //如果 x 和 y 已经在同一个集合中了, 则不修改直接返回
        if (t[a].size > t[b].size)
            swap(a, b);
        int p = link(i, t[a].fa, t[b].fa);
        p = update(p, t[b].fa, t[a].size);
        return p;
    }
    return i;
}

bool same(int i, int x, int y) { //判断在版本为 i 的并查集中, 点 x 和点 y 是否属于相同的集合
    int a = find(i, x);
    int b = find(i, y);
    return t[a].fa == t[b].fa;
}

int main() {
    //freopen("in.txt", "r", stdin);
    static int root[maxn];
    int n, m;
    scanf("%d %d", &n, &m);
    init(n);
    int r = root[0] = build();
    for (int i = 1; i <= m; ++i) {
        int tp, x, y, k;
        scanf("%d", &tp);
        if (tp == 1) {
            scanf("%d %d", &x, &y);
            r = join(r, x, y);
        }
        else if (tp == 2) {
            scanf("%d", &k);
            r = root[k];
        }
        else {
            scanf("%d %d", &x, &y);
            int ans = same(r, x, y);
            printf("%d\n", ans);
        }
        root[i] = r;
    }
    return 0;
}

```

```
}
```

8.28 KD-Tree

```
#define sqr(x) ((long long)(x)*(x))
const int maxn = 210000;
const int inf = 1 << 30;
const int k = 2;
using point = array<int, k>;
namespace kdt {
    int cur, sz[maxn], ch[maxn][2]; //如果没有插入操作, 只需要询问最近点, 则可以不维护 sz 属性
    int mark[maxn], real[maxn]; //没有删除操作时可以不记录这两个属性
    point mn[maxn], mx[maxn], val[maxn];
    int ans;
    void init() {
        cur = 0;
        for (int i = 0; i < k; ++i)
            mn[0][i] = inf, mx[0][i] = -inf;
    }
    inline int newnode(point pt) {
        int x = ++cur;
        sz[x] = 1;
        ch[x][0] = ch[x][1] = 0;
        mn[x] = mx[x] = val[x] = pt;
        mark[x] = real[x] = 1; //如果是静态 KD-Tree 则不需要记录 mark 标记和 real 标记
        return x;
    }
    inline void pushup(int x) {
        sz[x] = sz[ch[x][0]] + sz[ch[x][1]] + 1;
        real[x] = real[ch[x][0]] + real[ch[x][1]] + mark[x];
        for (int i = 0; i < k; ++i) {
            mn[x][i] = min(val[x][i], min(mn[ch[x][0]][i], mn[ch[x][1]][i]));
            mx[x][i] = max(val[x][i], max(mx[ch[x][0]][i], mx[ch[x][1]][i]));
        }
        //要么特判不合并 0 号结点的值, 要么将 0 号结点的值设为极限值
    }
    inline void pushdown(int x) {
        //如果 pushdown 会将标记传给 0 号结点的话, 则要在 pushup 中要判断不能合并 0 号结点的值
    }
    //A 序列是已经用 newnode 分配好的结点序列, 参数区间 [L, R] 是闭区间
    template<int d = 0> int build(int L, int R, int* A) {
        if (L > R) return 0;
        int mid = (L + R) >> 1;
        nth_element(A + L, A + mid, A + R + 1, [](int x, int y) {
            return val[x][d] < val[y][d];
        });
        int x = A[mid];
```

```

    ch[x][0] = build<(d + 1) % k>(L, mid - 1, A);
    ch[x][1] = build<(d + 1) % k>(mid + 1, R, A);
    pushup(x);
    return x;
}

inline bool in(int x, const point& lower, const point& upper) { //x in [lower, upper]
    for (int i = 0; i < k; ++i)
        if (mn[x][i] < lower[i] || mx[x][i] > upper[i])
            return false;
    return true;
}

inline bool out(int x, const point& lower, const point& upper) { //x out of [lower, upper]
    for (int i = 0; i < k; ++i)
        if (mn[x][i] > upper[i] || mx[x][i] < lower[i])
            return true;
    return false;
}

inline bool contain(int x, const point& lower, const point& upper) { //val[x] in [lower,
↪ upper]
    for (int i = 0; i < k; ++i)
        if (val[x][i] < lower[i] || val[x][i] > upper[i])
            return false;
    return true;
}

//-----区间查询和修改-----
int query(int x, const point& lower, const point& upper) {
    if (x == 0 || out(x, lower, upper))
        return 0;
    if (in(x, lower, upper))
        return sz[x];
    pushdown(x);
    return contain(x, lower, upper) + query(ch[x][0], lower, upper) + query(ch[x][1], lower,
↪ upper);
}

void update(int x, const point& lower, const point& upper) {
    if (x == 0 || out(x, lower, upper))
        return;
    if (in(x, lower, upper)) {
        ; //对区间打上标记
        return;
    }
    if (contain(x, lower, upper))
        ; //直接修改当前结点的值
    pushdown(x);
    update(ch[x][0], lower, upper);
    update(ch[x][1], lower, upper);
}

```

```

    pushup(x);
}
//-----圆（球）形查询-----
inline bool contain(int x, const point& center, long long r) { //val[x] 是否在以 center 为圆
↪ 心半径为 r 的圆里
    long long dist = 0;
    for (int i = 0; i < k; ++i)
        dist += sqr(val[x][i] - center[i]);
    return dist <= r * r;
}
int query(int x, const point& center, long long r) {
    if (x == 0 || gmin(x, center) > r * r) //当前区域与查询圆没有交
        return 0;
    if (gmax(x, center) <= r * r) //当前区域在圆内
        return sz[x];
    pushdown(x);
    return contain(x, center, r) + query(ch[x][0], center, r) + query(ch[x][1], center, r);
}
//-----查询与 a 曼哈顿距离最近和最远的点-----
inline int manhattan(const point& a, const point& b) {
    int dist = 0;
    for (int i = 0; i < k; ++i)
        dist += abs(a[i] - b[i]);
    return dist;
}
inline int fmin(int x, const point& a) {
    int ret = 0;
    for (int i = 0; i < k; ++i) //如果坐标范围可以到 2e9, 下面的加法会溢出
        ret += max(mn[x][i] - a[i], 0) + max(a[i] - mx[x][i], 0);
    return ret;
}
inline int fmax(int x, const point& a) {
    int ret = 0;
    for (int i = 0; i < k; ++i) //如果坐标范围可以到 2e9, 下面的加法会溢出
        ret += max(abs(mn[x][i] - a[i]), abs(mx[x][i] - a[i]));
    return ret;
}
void querymin(int x, const point& a) { //查询之前应先将 ans 设为无穷大
    ans = min(ans, manhattan(val[x], a));
    int f[2];
    f[0] = ch[x][0] ? fmin(ch[x][0], a) : inf;
    f[1] = ch[x][1] ? fmin(ch[x][1], a) : inf;
    int d = f[0] >= f[1];
    if (f[d] < ans)
        querymin(ch[x][d], a);
    if (f[!d] < ans)

```

```

        querymin(ch[x][!d], a);
    }
    void querymax(int x, const point& a) { //查询之前应先将 ans 设为无穷小
        ans = max(ans, manhattan(val[x], a));
        int f[2];
        f[0] = ch[x][0] ? fmax(ch[x][0], a) : -inf;
        f[1] = ch[x][1] ? fmax(ch[x][1], a) : -inf;
        int d = f[0] <= f[1];
        if (f[d] > ans)
            querymax(ch[x][d], a);
        if (f[!d] > ans)
            querymax(ch[x][!d], a);
    }
    //-----查询与 a 欧几里得距离最近和最远的点-----
    //如果要查询欧几里得距离, 应当将 point 的类型改为 array<long long, k>, 同时调大 inf 的值
    inline long long euclid(const point& a, const point& b) {
        long long dist = 0;
        //如果坐标范围可以到 2e9 且 k>=3, 则 dist 就会超出 long long 的表示范围
        //可以考虑用 double 或 long double 存储
        for (int i = 0; i < k; ++i)
            dist += sqr((long long)a[i] - b[i]);
        return dist;
    }
    inline long long gmin(int x, const point& a) { //点 a 到 x 表示区域的最近欧氏距离
        long long ret = 0;
        for (int i = 0; i < k; ++i) //可能超出 long long 的表示范围
            ret += sqr(max(mn[x][i] - a[i], 0)) + sqr(max(a[i] - mx[x][i], 0));
        return ret;
    }
    inline long long gmax(int x, const point& a) { //点 a 到 x 表示区域的最远欧氏距离
        long long ret = 0;
        for (int i = 0; i < k; ++i) //可能超出 long long 的表示范围
            ret += max(sqr(mn[x][i] - a[i]), sqr(mx[x][i] - a[i]));
        return ret;
    }
    //注意 ans 保存的是距离的平方
    void findmin(int x, const point& a) { //查询之前应先将 ans 设为无穷大
        //此处乘 1LL, 只用于与前面的模板同步, 没有实际意义
        ans = min(1LL * ans, euclid(val[x], a)); //ans 应当用 long long 或 double 存储
        long long f[2];
        f[0] = ch[x][0] ? gmin(ch[x][0], a) : inf;
        f[1] = ch[x][1] ? gmin(ch[x][1], a) : inf;
        int d = f[0] >= f[1];
        if (f[d] < ans)
            findmin(ch[x][d], a);
        if (f[!d] < ans)

```



```

        findmin(ch[x][!d], a);
    }
    void findmax(int x, const point& a) { //查询之前应先将 ans 设为无穷小
        ans = max(1LL * ans, euclid(val[x], a)); //ans 应当用 long long 或 double 存储
        long long f[2];
        f[0] = ch[x][0] ? gmax(ch[x][0], a) : -inf;
        f[1] = ch[x][1] ? gmax(ch[x][1], a) : -inf;
        int d = f[0] <= f[1];
        if (f[d] > ans)
            findmax(ch[x][d], a);
        if (f[!d] > ans)
            findmax(ch[x][!d], a);
    }
    //-----动态 KD-Tree: 插入删除操作-----
    //考虑到算法整体的时间复杂度瓶颈不在插入删除上, 此处直接重用了前面的 build 函数来重构,
    //单次时间复杂度  $O(n \log n)$ , 可以通过直接取中间的位置优化为单次  $O(n)$ 
    const double alpha = 0.6;
    int* pos, length, arr[maxn];
    decltype(build<0>)* func;
    void dfs(int x) {
        if (!x) return;
        pushdown(x);
        dfs(ch[x][0]);
        if (mark[x]) //被标记为删除的点不参与重构, 若没有删除则直接执行下面的操作, 不需要 if 判断
            arr[++length] = x; //如果结点还有权值等属性的话也要一起参与重构
        dfs(ch[x][1]);
    }
    template<int d = 0> void add(int& x, point a) {
        if (!x) {
            x = newnode(a);
            return;
        }
        pushdown(x);
        if (a[d] < val[x][d])
            add<(d + 1) % k>(ch[x][0], a);
        else
            add<(d + 1) % k>(ch[x][1], a);
        pushup(x);
        if (sz[ch[x][0]] > sz[x] * alpha || sz[ch[x][1]] > sz[x] * alpha)
            pos = &x, func = build<d>;
    }
    void insert(int& x, point a) { //应当保证没有重复的点
        pos = nullptr;
        add(x, a);
        if (pos) {
            length = 0;

```

```

        dfs(*pos);
        *pos = func(1, length, arr);
    }
}

template<int d = 0> void del(int& x, point a) {
    if (!x)
        return;
    pushdown(x);
    if (a == val[x])
        mark[x] = 0; //如果还有其他标记的话也要一起清空
    else if (a[d] < val[x][d])
        del<(d + 1) % k>(ch[x][0], a);
    else
        del<(d + 1) % k>(ch[x][1], a);
    pushup(x);
}

void remove(int& x, point a) { //注意在查询操作的时候不要考虑已经被删掉的点
    del(x, a);
    if (real[x] >= sz[x] / 2) { //超过一半的结点被删除则重建树
        length = 0;
        dfs(x);
        x = func(1, length, arr);
    }
}

};

point a[maxn];
int node[maxn];
int main() {
    freopen("in.txt", "r", stdin);
    kdt::init();
    int n;
    scanf("%d", &n);
    for (int i = 0; i < n; ++i)
        scanf("%d %d", &a[i][0], &a[i][1]);
    for (int i = 0; i < n; ++i)
        node[i] = kdt::newnode(a[i]);
    int root = kdt::build(0, n, node);
    return 0;
}

```

8.29 Euler-Tour-Tree

```

const int maxn = 310000; //maxn 至少为结点数 + 操作数的两倍
namespace ett {
    //-----Splay Tree-----
    //若要修改一个点的点权, 应当先将其 splay 到根, 然后修改, 最后还要调用 pushup 维护。
    //调用完 splay 之后根结点会改变, 应该用 splay 的返回值更新根结点。

```

```

int ch[maxn][2], fa[maxn], stk[maxn], sum[maxn], key[maxn], cur;
int alloc(int val) {
    int x = ++cur;
    ch[x][0] = ch[x][1] = fa[x] = 0;
    sum[x] = key[x] = val;
    return x;
}
inline bool son(int x) {
    return ch[fa[x]][1] == x;
}
inline void pushup(int x) {
    sum[x] = sum[ch[x][0]] + sum[ch[x][1]] + key[x];
}
inline void pushdown(int x) {}
void rotate(int x) {
    int y = fa[x], z = fa[y], c = son(x);
    if (fa[y])
        ch[z][son(y)] = x;
    fa[x] = z;
    ch[y][c] = ch[x][!c];
    fa[ch[y][c]] = y;
    ch[x][!c] = y;
    fa[y] = x;
    pushup(y);
}
void ascend(int x) {
    for (int y = fa[x]; y; rotate(x), y = fa[x]) if (fa[y])
        son(x) ^ son(y) ? rotate(x) : rotate(y);
    pushup(x);
}
int splay(int x) { //没有 pushdown 操作时, 可以直接用 ascend 替换 splay
    int top = 0;
    for (int i = x; i; i = fa[i])
        stk[++top] = i;
    while (top)
        pushdown(stk[top--]);
    ascend(x);
    return x;
}
template<typename ...T> int merge(int x, int y, T... args) {
    if constexpr (sizeof...(args) == 0) {
        if (x == 0) return y;
        while (pushdown(x), ch[x][1])
            x = ch[x][1];
        ascend(x);
        ch[x][1] = y; fa[y] = x;
    }
}

```

```

        pushup(x);
        return x;
    }
    else {
        return merge(merge(x, y), args...);
    }
}

inline int split(int x, int d) {
    int y = ch[x][d];
    fa[y] = ch[x][d] = 0;
    pushup(x);
    return y;
}

//-----Euler Tour Tree-----

int idx;
unordered_map<long long, int> locate;
#define edge(x, y) (((1LL * (x)) << 30) + (y))
void init() {
    cur = idx = 0;
    locate.clear();
}

inline int newnode(int val) {
    int x = ++idx;
    locate[edge(x, x)] = alloc(val);
    return x;
}

inline int newedge(int x, int y) {
    return locate[edge(x, y)] = alloc(0);
}

inline int changeroot(int x) {
    int a = locate[edge(x, x)];
    splay(a);
    int b = split(a, 0);
    return merge(a, b);
}

inline int gettree(int x) {
    return splay(locate[edge(x, x)]);
}

inline bool sametree(int x, int y) {
    if (x == y) return true;
    auto a = locate[edge(x, x)];
    auto b = locate[edge(y, y)];
    splay(a);
    splay(b);
    return fa[a] != 0;
}

```

```

void link(int x, int y) {
    auto a = newedge(x, y);
    auto b = newedge(y, x);
    merge(changeroot(x), a, changeroot(y), b);
}

void cut(int x, int y) {
    auto a = locate[edge(x, y)];
    auto b = locate[edge(y, x)];
    splay(a);
    auto i = split(a, 0), j = split(a, 1);
    splay(b);
    bool flag = (i != 0 && fa[i] != 0) || b == i;
    auto L = split(b, 0), R = split(b, 1);
    if (flag)
        merge(L, j);
    else
        merge(i, R);
}
}

using namespace ett;
int node[maxn];
//访问结点 node[x] 所在树的权值和: sum[gettree(node[x])]
//修改结点 node[x] 的权值为 v: int a = gettree(node[x]); key[a] = v; pushup(a);
int main() { //洛谷 P4219
    //freopen("in.txt", "r", stdin);
    int n, q;
    scanf("%d %d", &n, &q);
    init();
    for (int i = 1; i <= n; ++i)
        node[i] = newnode(1);
    for (int i = 0; i < q; ++i) {
        char tp;
        int x, y;
        scanf(" %c %d %d", &tp, &x, &y);
        if (tp == 'A') {
            link(node[x], node[y]);
        }
        else {
            cut(node[x], node[y]);
            long long a = sum[gettree(node[x])];
            long long b = sum[gettree(node[y])];
            printf("%lld\n", a * b);
            link(node[x], node[y]);
        }
    }
}

return 0;

```

```
}
```

8.30 rope

```
/*
// #include <ext/rope>
// using namespace __gnu_cxx;
rope<int> r;
1. r.push_back(a) 往后插入 a
   r.pop_back() 删除最后一个元素
2. r.insert(idx, a) 在下标 idx 处插入 a
   r.insert(idx, rope) 在下标 idx 处插入 rope 对象，时间复杂度  $O(\log(n))$ 
3. r.erase(idx, length) 删除从 idx 开始的 length 个字符
4. r.replace(idx, a) 将下标 idx 的位置设为 a
5. r.substr(pos, length) 返回从 pos 开始长度为 length 的子串，时间复杂度  $O(\log(n))$ ，
   修改 substr 返回的 rope 不会改变原 rope。
6. 支持基本的字符串运算比如 ==、+=、<，拼接的时间复杂度为  $O(\log(n))$ 
7. rope 的拷贝是  $O(1)$  的，修改是  $O(\log(n))$  的。
8. 注意 r[i] 返回值是左值，不能修改。修改需调用 replace 函数。
*/
using namespace __gnu_cxx;
const int maxn = 110000;
rope<int> version[maxn];
int main() {
    const int n = 100000;
    for (int i = 0; i < n; ++i)
        version[0].push_back(i);
    for (int i = 1; i < n; ++i) {
        version[i] = version[i - 1];
        version[i].replace(i, i * 10);
    }
    const int pos = 88888;
    printf("%d %d %d\n", version[pos][pos], version[pos][pos + 1], version[pos + 1][pos + 1]);
    rope<int> r;
    r.push_back(0); r.push_back(1);
    for (int i = 0; i < 32; ++i)
        r += r;
    int total = 0;
    for (int i = 0; i < n; ++i)
        total += r[998244353 + i];
    printf("%d\n", total);
    return 0;
}
```

8.31 pb-ds 平衡树

```

/*
// #include <ext/pb_ds/assoc_container.hpp>
// #include <ext/pb_ds/tree_policy.hpp>
// using namespace __gnu_pbds;
tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update> t;
1. 第二个模板参数为值域的类型, 如果只要实现 set 的功能, 则用 null_type 即可
2. 基本操作与 std::map/std::set 兼容
3. t.erase(val) 从平衡树中删除值为 val 的结点
4. t.order_of_key(val) 返回平衡树中比 val 小的数有多少个
5. t.find_by_order(k) 返回排名为 k 的位置的迭代器 (排名从 0 开始)
6. 不支持多重值, 如果需要多重值, 可以再开一个 unordered_map 来记录值出现的次数。
   将 val<<32 后加上出现的次数后插入 tree, 注意此时应该为 long long 类型。
*/
using namespace __gnu_pbds;
const int maxn = 110000;
tree<int, null_type, less<int>, rb_tree_tag, tree_order_statistics_node_update> t;
int main() {
    for (int i = 1; i <= 5; ++i)
        t.insert(i);
    printf("=>%d\n", *t.find_by_order(2));
    return 0;
}

```

9 数学算法

9.1 SG 函数

//注意: 异或运算符的优先级小于比较运算符!!!

```

const int maxn = 1000;
int sg[maxn], vis[maxn];
void getSG()
{
    memset(sg, 0, sizeof(sg));
    for (int i = 1; i < maxn; ++i)
    {
        memset(vis, 0, sizeof(vis));
        for (int j = 1; j <= i; ++j)
            vis[sg[i - j]] = true;
        for (int j = 0; j < maxn; ++j)
        {
            if (!vis[j])
            {
                sg[i] = j;
                break;
            }
        }
    }
}

```

```

    }
}
}
int main()
{
    getSG();
    printf("%d\n", sg[5]);
    return 0;
}

```

9.2 自适应辛普森积分

```

double f(double x) { //任意一个自定义的函数
    return x * x;
}

double simpson(double l, double r) { //计算函数  $f$  在区间  $[l, r]$  上的辛普森积分
    return (f(l) + f(r) + 4 * f((l + r) / 2)) * (r - l) / 6;
}

double solve(double l, double r, double eps, double val) { //递归求解积分
    double mid = (l + r) / 2;
    double L = simpson(l, mid), R = simpson(mid, r);
    if (fabs(L + R - val) <= 15 * eps)
        return L + R + (L + R - val) / 15;
    return solve(l, mid, eps / 2, L) + solve(mid, r, eps / 2, R);
}

double asme(double l, double r, double eps = 1e-7) { //自适应辛普森积分
    return solve(l, r, eps, simpson(l, r)); //求函数  $f$  在区间  $[l, r]$  上的自适应辛普森
    ↪ 积分,  $eps$  指定精度
}

int main() {
    printf("%.15f\n", asme(0, 1)); //ans = 0.333333333333333
    return 0;
}

```

9.3 高斯消元

```

const int maxn = 100;
using matrix = double[maxn][maxn];
using vect = array<double, maxn>;
//当方程有唯一解的时候, 算出唯一解
//矩阵  $A$  的大小为  $n * (n + 1)$ 
void gauss_elimination(matrix A, int n) {
    for (int i = 0; i < n; ++i) {
        int r = i;
        for (int j = i + 1; j < n; ++j)
            if (fabs(A[j][i]) > fabs(A[r][i]))
                r = j;
    }
}

```



```

    if (r != i) for (int j = 0; j <= n; ++j)
        swap(A[r][j], A[i][j]);
    for (int k = i + 1; k < n; ++k)
        for (int j = n; j >= i; --j)
            A[k][j] -= A[k][i] / A[i][i] * A[i][j];
}
for (int i = n - 1; i >= 0; --i) {
    for (int j = i + 1; j < n; ++j)
        A[i][n] -= A[j][n] * A[i][j];
    A[i][n] /= A[i][i];
}
}

//无解返回-1, 有唯一解返回 0, 有无穷多解返回 1。
//在有解的情况下通过 ans 返回任意一个解。
//矩阵 A 的大小为  $n * (m + 1)$ , 表示有  $n$  个方程,  $m$  个变量。
const double eps = 1e-8;
int row[maxn], var[maxn];
int one_possible(matrix A, int n, int m, vect& ans) {
    memset(row, -1, sizeof(row));
    int r = 0;
    for (int c = 0; c < m && r < n; ++c) {
        int x = r;
        for (int i = x + 1; i < n; ++i)
            if (fabs(A[i][c]) > fabs(A[x][c]))
                x = i;
        if (x != r) for (int j = 0; j <= m; ++j)
            swap(A[x][j], A[r][j]);
        if (fabs(A[r][c]) < eps)
            continue;
        for (int k = r + 1; k < n; ++k)
            for (int j = m; j >= c; --j)
                A[k][j] -= A[k][c] / A[r][c] * A[r][j];
        row[c] = r++;
    }
    for (int i = r; i < n; ++i) if (fabs(A[i][m]) > eps)
        return -1;
    for (int c = m - 1; c >= 0; --c) {
        int x = row[c];
        if (x < 0)
            ans[c] = 0;
        else {
            for (int i = x - 1; i >= 0; --i)
                A[i][m] -= A[i][c] / A[x][c] * A[x][m];
            ans[c] = A[x][m] / A[x][c];
        }
    }
}

```

```

    return r < m;
}
//计算方程的最小二乘解
void least_square(matrix A, int n, int m, vect& ans) {
    static matrix T;
    for (int i = 0; i < n; ++i)
        for (int j = 0; j <= m; ++j)
            T[i][j] = A[i][j];
    for (int i = 0; i < m; ++i) {
        for (int j = 0; j <= m; ++j) {
            A[i][j] = 0;
            for (int k = 0; k < n; ++k)
                A[i][j] += T[k][i] * T[k][j];
        }
    }
    one_possible(A, m, m, ans);
}
//将矩阵 A 化为简化阶梯形
//ans 为方程组的一个特解, basis 为齐次方程组解空间的一组基。
//该算法针对稀疏矩阵进行了优化, 在矩阵中有大量 0 元素时, 时间复杂度会小于  $O(n^3)$ 。
int row_simplify(matrix A, int n, int m, vect& ans, vector<vect>& basis) {
    memset(row, -1, sizeof(row));
    int r = 0;
    for (int c = 0; c < m && r < n; ++c) {
        int x = r;
        for (int i = x + 1; i < n; ++i)
            if (fabs(A[i][c]) > fabs(A[x][c]))
                x = i;
        if (x != r) for (int j = 0; j <= m; ++j)
            swap(A[x][j], A[r][j]);
        if (fabs(A[r][c]) < eps)
            continue;
        for (int j = m; j >= c; --j)
            A[r][j] /= A[r][c];
        for (int k = r + 1; k < n; ++k) if (fabs(A[k][c]) > eps)
            for (int j = m; j >= c; --j)
                A[k][j] -= A[k][c] * A[r][j];
        var[r] = c;
        row[c] = r++;
    }
    for (int i = r; i < n; ++i) if (fabs(A[i][m]) > eps)
        return -1;
    for (int c = m - 1; c >= 0; --c) {
        int x = row[c];
        if (x < 0)
            ans[c] = 0;
    }
}

```

```

    else {
        for (int i = x - 1; i >= 0; --i) if (fabs(A[i][c]) > eps)
            for (int j = m; j >= c; --j)
                A[i][j] -= A[i][c] * A[x][j];
        ans[c] = A[x][m];
    }
}

//求出基础解系
for (int c = m - 1; c >= 0; --c) if (row[c] < 0) {
    vect now = {};
    for (int i = 0; i < r; ++i)
        now[var[i]] = -A[i][c];
    now[c] = 1;
    basis.push_back(now);
}

return r < m;
}

namespace rectangle_mod {
    const int mod = 998244353;
    using matrix = int[maxn][maxn];
    inline int pow_mod(int a, int n) {
        int ans = 1;
        while (n) {
            if (n & 1)
                ans = 1LL * ans * a % mod;
            n >>= 1;
            a = 1LL * a * a % mod;
        }
        return ans;
    }

    inline int inv(int n) {
        return pow_mod(n, mod - 2);
    }
}

//对 n 行 m 列的矩阵进行取模意义下的高斯消元，必须保证矩阵行满秩
inline void gauss(matrix A, int n, int m) {
    for (int i = 0; i < n; ++i) {
        int r = i;
        for (int j = i; j < n; ++j) {
            if (A[j][i] != 0) {
                r = j;
                break;
            }
        }

        if (r != i) for (int j = 0; j < m; ++j)
            swap(A[r][j], A[i][j]);
        int pivot = inv(A[i][i]);

```

```

        for (int k = i + 1; k < n; ++k) {
            int val = 1LL * A[k][i] * pivot % mod;
            for (int j = m - 1; j >= i; --j)
                A[k][j] = (A[k][j] - 1LL * A[i][j] * val % mod + mod) % mod;
        }
    }
    for (int i = n - 1; i >= 0; --i) {
        for (int j = i + 1; j < n; ++j) {
            for (int k = n; k < m; ++k)
                A[i][k] = (A[i][k] - 1LL * A[j][k] * A[i][j] % mod + mod) % mod;
        }
        int pivot = inv(A[i][i]);
        for (int k = n; k < m; ++k)
            A[i][k] = 1LL * A[i][k] * pivot % mod;
    }
}

int main() { //3 -5 -2
    matrix A = {
        {1, 1, 0, 0, -3},
        {1, 1, 0, 0, -1},
        {1, 0, 1, 0, 0},
        {1, 0, 1, 0, 2},
        {1, 0, 0, 1, 5},
        {1, 0, 0, 1, 1}
    };
    vect ans;
    vector<vect> basis;
    least_square(A, 6, 4, ans);
    for (int i = 0; i < 4; ++i)
        printf("%f\n", ans[i]);
    printf("\n");
    return 0;
}

```

9.4 稀疏矩阵的高斯消元

```

const int maxn = 110000;
const long long mod = 1e9 + 7;
const double eps = 1e-9;
long long pow_mod(long long a, long long n, long long p) {
    long long ans = 1;
    while (n) {
        if (n & 1)
            ans = ans * a % p;
        a = a * a % p;
        n >>= 1;
    }
}

```

```

    }
    return ans;
}

long long inv(long long a) {
    return pow_mod(a, mod - 2, mod);
}

struct fast_gauss_mod { //基于十字链表的高斯消元
    static const int maxstate = 3000000;
    static const int table_size = 5110007;
    struct {
        long long val;
        int r, c, next, right, down;
    } node[maxstate];
    int first[table_size], row[maxn], column[maxn], vis[maxn], length[maxn], sz, n;
    void init(int n) { //待消元的矩阵大小是  $n * (n + 1)$  的
        this->n = n;
        sz = 0;
        memset(first, 0, sizeof(first));
        memset(row, 0, sizeof(row));
        memset(column, 0, sizeof(column));
        memset(vis, 0, sizeof(vis));
        memset(length, 0, sizeof(length));
    }

    long long& A(int r, int c) {
        const int h = ((long long)r << 20 | c) % table_size;
        for (int i = first[h]; i; i = node[i].next)
            if (node[i].r == r && node[i].c == c)
                return node[i].val;

        int i = ++sz;
        node[i].next = first[h];
        first[h] = i;
        node[i].right = row[r];
        row[r] = i;
        node[i].down = column[c];
        column[c] = i;
        node[i].r = r;
        node[i].c = c;
        node[i].val = 0;
        length[r] += 1;
        return node[i].val;
    }

    void insert(int r, int c, long long v) { //在矩阵的第  $r$  行第  $c$  列填上  $v$ 
        A(r, c) = v;
    }

    vector<long long> solve() { //无解或用无穷解时返回空 vector
        vector<long long> ans(n);
    }
}

```

```

    for (int i = 0; i < n; ++i) {
        int r = -1;
        for (int x = column[i]; x; x = node[x].down) {
            int j = node[x].r;
            if (!vis[j] && node[x].val != 0)
                if (r < 0 || length[j] < length[r])
                    r = j;
        }
        if (r == -1)
            return {};
        int* last = &row[r];
        for (int y = row[r]; y; y = node[y].right) {
            if (node[y].val == 0)
                *last = node[y].right;
            else
                last = &node[y].right;
        }
        auto pivot = mod - inv(A(r, i));
        for (int x = column[i]; x; x = node[x].down) {
            int j = node[x].r;
            if (!vis[j] && j != r) {
                auto ratio = node[x].val * pivot % mod;
                for (int y = row[r]; y; y = node[y].right) {
                    int k = node[y].c;
                    A(j, k) = (A(j, k) + ratio * node[y].val) % mod;
                }
                length[j] -= 1;
            }
        }
        ans[i] = r;
        vis[r] = true;
    }
    for (int i = n - 1; i >= 0; --i) {
        int r = ans[i];
        auto pivot = (mod - A(r, n)) * inv(A(r, i)) % mod;
        for (int x = column[i]; x; x = node[x].down) {
            int j = node[x].r;
            if (j != r)
                A(j, n) = (A(j, n) + node[x].val * pivot) % mod;
        }
    }
    for (int i = 0; i < n; ++i)
        ans[i] = A(ans[i], n) * inv(A(ans[i], i)) % mod;
    return ans;
}
}gauss;

```

```

struct fast_gauss_double {
    static const int maxstate = 2000000;
    static const int table_size = 5110007;
    struct {
        double val;
        int r, c, next, right, down;
    } node[maxstate];
    int first[table_size], row[maxn], column[maxn], vis[maxn], length[maxn], sz, n;
    void init(int n) { //待消元的矩阵大小是  $n * (n + 1)$  的
        this->n = n;
        sz = 0;
        memset(first, 0, sizeof(first));
        memset(row, 0, sizeof(row));
        memset(column, 0, sizeof(column));
        memset(vis, 0, sizeof(vis));
        memset(length, 0, sizeof(length));
    }
    double& A(int r, int c) {
        const int h = ((long long)r << 20 | c) % table_size;
        for (int i = first[h]; i; i = node[i].next)
            if (node[i].r == r && node[i].c == c)
                return node[i].val;
        int i = ++sz;
        node[i].next = first[h];
        first[h] = i;
        node[i].right = row[r];
        row[r] = i;
        node[i].down = column[c];
        column[c] = i;
        node[i].r = r;
        node[i].c = c;
        node[i].val = 0;
        length[r] += 1;
        return node[i].val;
    }
    void insert(int r, int c, double v) { //在矩阵的第  $r$  行第  $c$  列填上  $v$ 
        A(r, c) = v;
    }
    vector<double> solve() { //无解或用无穷解时返回空 vector
        vector<double> ans(n);
        vector<int> res(n);
        for (int i = 0; i < n; ++i) {
            int r = -1;
            for (int x = column[i]; x; x = node[x].down) {
                int j = node[x].r;
                if (!vis[j] && fabs(node[x].val) > eps)

```

```

        if (r < 0 || length[j] < length[r])
            r = j;
    }
    if (r == -1)
        return {};
    int* last = &row[r];
    for (int y = row[r]; y; y = node[y].right) {
        if (fabs(node[y].val) < eps)
            *last = node[y].right;
        else
            last = &node[y].right;
    }
    auto pivot = -A(r, i);
    for (int x = column[i]; x; x = node[x].down) {
        int j = node[x].r;
        if (!vis[j] && j != r) {
            auto ratio = node[x].val / pivot;
            for (int y = row[r]; y; y = node[y].right) {
                int k = node[y].c;
                A(j, k) += ratio * node[y].val;
            }
            length[j] -= 1;
        }
    }
    res[i] = r;
    vis[r] = true;
}
for (int i = n - 1; i >= 0; --i) {
    int r = res[i];
    auto pivot = -A(r, n) / A(r, i);
    for (int x = column[i]; x; x = node[x].down) {
        int j = node[x].r;
        if (j != r)
            A(j, n) += node[x].val * pivot;
    }
}
for (int i = 0; i < n; ++i)
    ans[i] = A(res[i], n) / A(res[i], i);
return ans;
}
}gas;
namespace gauss_mod { //稀疏矩阵时, 复杂度接近  $O(n^2)$ 
    const int maxn = 1000;
    using matrix = int[maxn][maxn];
    inline int pow_mod(int a, int n) {
        int ans = 1;

```



```

    while (n) {
        if (n & 1)
            ans = 1LL * ans * a % mod;
        n >>= 1;
        a = 1LL * a * a % mod;
    }
    return ans;
}

inline int inv(int n) {
    return pow_mod(n, mod - 2);
}

inline void gauss(matrix A, int n) {
    int column[maxn];
    for (int i = 0; i < n; ++i) {
        int r = i;
        for (int j = i; j < n; ++j) {
            if (A[j][i] != 0) {
                r = j;
                break;
            }
        }
        if (r != i) for (int j = 0; j <= n; ++j)
            swap(A[r][j], A[i][j]);
        int pivot = inv(A[i][i]), sz = 0;
        for (int j = n; j >= i; --j) if (A[i][j])
            column[sz++] = j;
        for (int k = i + 1; k < n; ++k) if (A[k][i]) {
            int val = 1LL * A[k][i] * pivot % mod;
            for (int t = 0; t < sz; ++t) {
                int j = column[t];
                A[k][j] = (A[k][j] - 1LL * A[i][j] * val % mod + mod) % mod;
            }
        }
    }
}

for (int i = n - 1; i >= 0; --i) {
    for (int j = i + 1; j < n; ++j) if (A[i][j]) {
        A[i][n] = (A[i][n] - 1LL * A[j][n] * A[i][j] % mod + mod) % mod;
    }
    A[i][n] = 1LL * A[i][n] * inv(A[i][i]) % mod;
}
}

int main() {
    int n;
    scanf("%d", &n);
    gas.init(n);
}

```

```

    for (int i = 0; i < n; ++i) {
        for (int v, j = 0; j <= n; ++j) {
            scanf("%d", &v);
            if (v) gas.insert(i, j, v);
        }
    }
    auto ans = gas.solve();
    if (ans.empty()) {
        puts("No Solution");
        return 0;
    }
    for (int i = 0; i < n; ++i)
        printf("%.2f\n", ans[i]);
    return 0;
}

```

9.5 求解异或方程组

```

const int maxn = 110;
bitset<maxn> a[maxn];
void Gauss(int n) {
    int now = 0; //now 记录当前正在处理的行号
    for (int i = 0; i < n; ++i) { //i 记录当前正在处理的列号
        int j = now;
        while (j < n && !a[j][i]) ++j;
        if (j == n) continue;
        if (j != now) swap(a[now], a[j]);
        for (int k = 0; k < n; ++k)
            if (k != now && a[k][i])
                a[k] ^= a[now];
        ++now;
    }
}
int main() {
    return 0;
}

```

9.6 矩阵与状态转移

```

/*
对于状态图中的每一条边  $(i, j)$ , 设置矩阵  $A[i][j] = 1$ 。
令  $B = \text{pow}(A, n)$ ,
则从  $i$  开始走  $n$  步到达  $j$  的方案数为  $B[i][j]$ 。
*/
const int maxn = 100;
int n;
struct matrix

```

```

{
    int data[maxn][maxn];
    matrix() : data(){}
    int *operator[] (int idx)
    {
        return data[idx];
    }
};

matrix operator*(matrix &A, matrix &B)
{
    matrix C;
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            for (int k = 0; k < n; ++k)
                C[i][j] += A[i][k] * B[k][j];
    return C;
}

matrix pow(matrix A, int t)
{
    matrix ans;
    for (int i = 0; i < n; ++i)
        ans[i][i] = 1;
    while (t)
    {
        if (t & 1)
            ans = ans * A;
        t >>= 1;
        A = A * A;
    }
    return ans;
}

int main()
{
    return 0;
}

```

9.7 递推式求解

```

const int mod = 1e9 + 7;
int a[2100], c[2100], tmp[2100], tmpa[2100], res[2100], k;
long long n;
void mul(int* p, int* q, int* c)
{
    memset(tmp, 0, sizeof(tmp));
    for (int i = 0; i < k; i++)
        for (int j = 0; j < k; j++)
            tmp[i + j] = (1ll * p[i] * q[j] + tmp[i + j]) % mod;
}

```

```

    for (int i = 2 * k - 2; i >= k; i--)
        for (int j = 0; j < k; j++)
            tmp[i - j - 1] = (1ll * tmp[i] * c[j] + tmp[i - j - 1]) % mod;
    for (int i = 0; i < k; i++)
        p[i] = tmp[i];
}

int solve(int* a, int* c, long long n) //a 为初始值数组, c 为系数矩阵
{
    if (n < k) return a[n];
    memset(tmpa, 0, sizeof(tmpa));
    memset(res, 0, sizeof(res));
    tmpa[1] = res[0] = 1;
    while (n)
    {
        if (n & 1)
            mul(res, tmpa, c);
        mul(tmpa, tmpa, c);
        n >>= 1;
    }
    int ans = 0;
    for (int i = 0; i < k; i++)
        ans = (ans + 1ll * a[i] * res[i]) % mod;
    return ans;
}

int pow_mod(int a, long long N) {
    int ans = 1;
    a %= mod;
    while (N) {
        if (N & 1)
            ans = 1ll * ans * a % mod;
        a = 1ll * a * a % mod;
        N >>= 1;
    }
    return ans;
}

int main() {
    int T;
    scanf("%d", &T);
    while (T--) {
        scanf("%d %lld", &k, &n);
        memset(a, 0, sizeof(a));
        if (k == 1) {
            printf("1\n");
            continue;
        }
        if (n == -1) {

```

```

        printf("%d\n", 2 * pow_mod(k + 1, mod - 2) % mod);
        continue;
    }
    int rev = pow_mod(k, mod - 2);
    for (int i = 0; i < k; ++i)
        c[i] = rev;
    a[0] = 1;
    for (int i = 1; i < k; ++i) {
        for (int j = i - k; j < i; ++j) if (j >= 0) {
            a[i] += 1ll * rev * a[j] % mod;
            if (a[i] >= mod) a[i] -= mod;
        }
    }
    int ans = solve(a, c, n);
    printf("%d\n", ans);
}
return 0;
}

```

9.8 快速傅里叶变换

```

const int maxn = 1 << 22; //必须是 2 的幂
const double pi = acos(-1);
complex<double> w[maxn];
void init() {
    for (int i = 0; i < maxn; ++i)
        w[i] = { cos(pi * 2 * i / maxn), sin(pi * 2 * i / maxn) };
}
void fft(vector<complex<double>> &a, bool inverse) {
    for (int i = 0, j = 0; i < a.size(); ++i) {
        if (i < j)
            std::swap(a[i], a[j]);
        for (int k = a.size() >> 1; (j ^= k) < k; k >>= 1);
    }
    for (int step = 2; step <= a.size(); step *= 2) {
        int h = step / 2, d = maxn / step;
        for (int i = 0; i < a.size(); i += step)
            for (int j = 0; j < h; ++j) {
                auto &x = a[i + j];
                auto &y = a[i + j + h];
                auto t = w[d * j] * y;
                y = x - t;
                x = x + t;
            }
    }
    if (inverse) {
        std::reverse(a.begin() + 1, a.end());
    }
}

```

```

        for (auto& x : a)
            x /= a.size();
    }
}

vector<double> operator* (const vector<double> &a, const vector<double> &b) {
    int size = 2;
    while (size < a.size() + b.size()) size <= 1;
    vector<complex<double>> x(size), y(size);
    copy(a.begin(), a.end(), x.begin());
    copy(b.begin(), b.end(), y.begin());
    fft(x, false);
    fft(y, false);
    for (int i = 0; i < size; i++)
        x[i] *= y[i];
    fft(x, true);
    vector<double> res(a.size() + b.size() - 1);
    for (int i = 0; i < res.size(); ++i)
        res[i] = x[i].real();
    return res;
}

int main() {
    int n, m;
    init();
    scanf("%d %d", &n, &m);
    vector<double> a(n + 1), b(m + 1);
    for (int i = 0; i <= n; ++i)
        scanf("%lf", &a[i]);
    for (int i = 0; i <= m; ++i)
        scanf("%lf", &b[i]);
    auto ans = a * b;
    for (int i = 0; i < ans.size(); ++i)
        printf("%.0f ", ans[i] + 1e-8);
    return 0;
}

```

9.9 快速幂运算

```

const int maxn = 100;
const int mod = 1e9 + 7;
int n;
struct matrix
{
    int data[maxn][maxn];
    matrix() : data() {}
    int *operator[] (int idx)
    {
        return data[idx];
    }
}

```

```

    }
    const int *operator[] (int idx) const
    {
        return data[idx];
    }
};

matrix operator*(const matrix &A, const matrix &B)
{
    matrix C;
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            for (int k = 0; k < n; ++k)
                C[i][j] += A[i][k] * B[k][j];
    return C;
}

matrix operator+ (const matrix &A, const matrix &B)
{
    matrix C;
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            C[i][j] = A[i][j] + B[i][j];
    return C;
}

matrix pow(matrix A, int m)
{
    matrix ans;
    for (int i = 0; i < n; ++i)
        ans[i][i] = 1;
    while (m)
    {
        if (m & 1)
            ans = ans * A;
        A = A * A;
        m >>= 1;
    }
    return ans;
}

long long pow_mod(long long a, int m)
{
    long long ans = 1;
    while (m)
    {
        if (m & 1)
            ans = ans * a % mod;
        a = a * a % mod;
        m >>= 1;
    }
}

```

```

    }
    return ans;
}

matrix pow_sum(matrix A, int m) //pow_sum(A, m) = sigma pow(A, i)
{
    // 1<=i<=m
    matrix ans, B = A;
    while (m)
    {
        if (m & 1)
            ans = ans * A + B;
        B = B * A + B;
        A = A * A;
        m >>= 1;
    }
    return ans;
}

long long pow_sum(long long a, int m) //pow_sum(A, m) = sigma pow(A, i)
{
    // 1<=i<=m
    long long ans = 0, b = a;
    while (m)
    {
        if (m & 1)
            ans = (ans * a + b) % mod;
        b = (b * a + b) % mod;
        a = (a * a) % mod;
        m >>= 1;
    }
    return ans;
}

//通过预处理, 可以在 O(1) 的时间内求出 x 的 n 次幂
struct Power_Int { //指数在 int 范围内
    static const int base = (1 << 16) - 1;
    long long f[2][base + 1];
    void init(int x) {
        f[0][0] = f[1][0] = 1;
        for (int i = 1; i <= base; ++i)
            f[0][i] = f[0][i - 1] * x % mod;
        f[1][1] = f[0][base] * x % mod;
        for (int i = 2; i <= base; ++i)
            f[1][i] = f[1][i - 1] * f[1][1] % mod;
    }
    long long pow(int n) {
        return f[0][n & base] * f[1][(n >> 16) & base] % mod;
    }
};

struct Power_LLong { //指数在 long long 范围内

```



```

static const int base = (1 << 16) - 1;
long long f[4][base + 1];
void init(int x) {
    for (int i = 0; i < 4; ++i)
        f[i][0] = 1;
    for (int i = 1; i <= base; ++i)
        f[0][i] = f[0][i - 1] * x % mod;
    for (int j = 1; j <= 3; ++j) {
        f[j][1] = f[j - 1][base] * f[j - 1][1] % mod;
        for (int i = 2; i <= base; ++i)
            f[j][i] = f[j][i - 1] * f[j][1] % mod;
    }
}
long long pow(long long n) {
    return f[0][n & base] * f[1][(n >> 16) & base] % mod
        * f[2][(n >> 32) & base] % mod * f[3][(n >> 48) & base] % mod;
}
}power;
int main()
{
    long long ans = pow_sum(2, 10);
    printf("%lld\n", ans);
    n = 2;
    matrix A; A[0][0] = 4; A[0][1] = 2; A[1][1] = 7; A[1][0] = 1;
    matrix B = pow_sum(A, 3);
    A = A * A * A + A * A + A;
    for (int i = 0; i < n; ++i)
    {
        for (int j = 0; j < n; ++j)
        {
            printf("%d ", B[i][j]);
        }
        printf("\n");
    }
    for (int i = 0; i < n; ++i)
    {
        for (int j = 0; j < n; ++j)
        {
            printf("%d ", A[i][j]);
        }
        printf("\n");
    }
    system("pause");
    return 0;
}

```

9.10 莫比乌斯反演

```

/*
莫比乌斯函数性质:
 $\sigma \mu(d) = [n == 1]$ 
 $d/n$ 
*/
const int maxn = 1000000;
int vis[maxn], prime[maxn], mu[maxn], cnt;
void init()
{
    memset(vis, 0, sizeof(vis));
    mu[1] = 1;
    cnt = 0;
    for (int i = 2; i < maxn; ++i)
    {
        if (!vis[i])
        {
            prime[cnt++] = i;
            mu[i] = -1;
        }
        for (int j = 0; j < cnt && i * prime[j] < maxn; ++j)
        {
            int t = i * prime[j];
            vis[t] = 1;
            if (i % prime[j] == 0)
            {
                mu[t] = 0;
                break;
            }
            else
            {
                mu[t] = -mu[i];
            }
        }
    }
}

int main()
{
    init();
    printf("%d\n", mu[3*7*11*13]);
    return 0;
}

```

9.11 逆元

```

const long long maxn = 1000005, mod = 1000000007;
long long pow(long long a, long long n, long long p)
{
    long long ans = 1;
    while (n)
    {
        if (n & 1)
            ans = ans * a % p;
        a = a * a % p;
        n >>= 1;
    }
    return ans;
}

long long inverse1(long long a, long long n)    //费马小定理求逆元
{
    return pow(a, n - 2, n);
}

void extgcd(long long a, long long b, long long& d, long long& x, long long& y)
{
    if (!b) { d = a; x = 1; y = 0; }
    else { extgcd(b, a % b, d, y, x); y -= x * (a / b); }
}

long long inverse2(long long a, long long n)
{
    long long d, x, y;
    extgcd(a, n, d, x, y);
    return d == 1 ? (x + n) % n : -1;
}

long long inv[maxn];
void inverse3(long long n, long long p)
{
    inv[1] = 1;
    for (long long i = 2; i <= n; ++i)
        inv[i] = (p - p / i) * inv[p % i] % p;
}

int main()
{
    int number = 888;
    inverse3(1000005, mod);
    printf("%lld %lld %lld\n", inverse1(number, mod), inverse2(number, mod), inv[number]);
    return 0;
}

```

9.12 欧拉函数

// $\phi(n)$ 表示小于 n 且与 n 互素的整数个数

```
const int maxn = 1000000;
int vis[maxn], prime[maxn], phi[maxn], cnt;
void init() {
    memset(vis, 0, sizeof(vis));
    phi[1] = 1;
    cnt = 0;
    for (int i = 2; i < maxn; i++) {
        if (!vis[i]) {
            prime[cnt++] = i;
            phi[i] = i - 1;
        }
        for (int j = 0; j < cnt && i * prime[j] < maxn; j++) {
            int t = i * prime[j];
            vis[t] = 1;
            if (i % prime[j] == 0) {
                phi[t] = phi[i] * prime[j];
                break;
            }
            else {
                phi[t] = phi[i] * phi[prime[j]];
            }
        }
    }
}

int euler(int n) { //时间复杂度  $O(\sqrt{n})$ 
    int ans = n;
    for (int i = 2; i * i <= n; ++i) if (n % i == 0) {
        ans = ans / i * (i - 1);
        while (n % i == 0)
            n /= i;
    }
    if (n > 1)
        ans = ans / n * (n - 1);
    return ans;
}

int main() {
    init();
    for (int i = 0; i <= 10000; ++i) if (phi[i] != euler(i))
        printf("%d\n", i);
    return 0;
}
```

9.13 线性筛素数

```

const int maxn = 1000000;
int vis[maxn], prime[maxn], cnt;
void init()
{
    memset(vis, 0, sizeof(vis));
    cnt = 0;
    for (int i = 2; i < maxn; i++)
    {
        if (!vis[i])
            prime[cnt++] = i;
        for (int j = 0; j < cnt && i * prime[j] < maxn; j++)
        {
            int t = i * prime[j];
            vis[t] = 1;
            if (i % prime[j] == 0)
                break;
        }
    }
}

int main()
{
    init();
    for (int i = 0; i < 10; ++i)
        printf("%d\n", prime[i]);
    return 0;
}

```

9.14 三分求极值

```

#define f(x) fabs(x - 8)
const double eps = 1e-6;
int maximum_int(int L, int R) { //三分求 f 函数的最大值 (定义域为整数)
    while (R > L) {
        int m1 = (2 * L + R) / 3;
        int m2 = (2 * R + L + 2) / 3;
        if (f(m1) > f(m2))
            R = m2 - 1;
        else
            L = m1 + 1;
    }
    return L; //f(L) 为最大值
}

int minimum_int(int L, int R) { //三分求 f 函数的最小值 (定义域为整数)
    while (R > L) {
        int m1 = (2 * L + R) / 3;

```

```

    int m2 = (2 * R + L + 2) / 3;
    if (f(m1) < f(m2))
        R = m2 - 1;
    else
        L = m1 + 1;
}
return L; //f(L) 为最小值
}

double maximum_double(double L, double R) { //三分求 f 函数的最大值 (定义域为实数)
    while (R - L > eps) { // for i in range(100):
        double m1 = (2 * L + R) / 3;
        double m2 = (2 * R + L) / 3;
        if (f(m1) > f(m2))
            R = m2;
        else
            L = m1;
    }
    return L; //f(L) 为最大值
}

double minimun_double(double L, double R) { //三分求 f 函数的最小值 (定义域为实数)
    while (R - L > eps) { // for i in range(100):
        double m1 = (2 * L + R) / 3;
        double m2 = (2 * R + L) / 3;
        if (f(m1) < f(m2))
            R = m2;
        else
            L = m1;
    }
    return L; //f(L) 为最小值
}

int main() {
    double pos = minimun_double(-1, 100);
    printf("%f\n", pos);
    return 0;
}

```

9.15 多项式拟合 (辛普森积分)

```

const int maxn = 110;
const long double pi = acos(-1.0L);
const long double L = -pi, R = pi; //函数 f(x) 的定义域为 [L, R]
const int T = 15; //用不超过 T 次的多项式对函数 f(x) 进行拟合
long double a[maxn][maxn]; //a[i] 表示第 i 个规范正交基, a[i][j] 表示第 i 个基的  $x^j$  的系数
long double pl[maxn], pr[maxn]; //pl[i] = pow(L, i), pr[i] = pow(R, i)
long double c[maxn]; //拟合出来的多项式系数
int pos = 0;
long double f(long double x) { //待拟合函数

```

```

    return sin(x);
}
long double calc(long double x) {
    long double result = 0, y = 1;
    for (int i = 0; i <= pos; ++i) {
        result += a[pos][i] * y;
        y *= x;
    }
    return result * f(x);
}
long double simpson(long double l, long double r) { //计算函数 calc 在区间 [l, r] 上的辛普森积分
    return (calc(l) + calc(r) + 4 * calc((l + r) / 2)) * (r - l) / 6;
}
long double solve(long double l, long double r, long double eps, long double val) { //递归求解积分
    ↪ 分
    long double mid = (l + r) / 2;
    long double L = simpson(l, mid), R = simpson(mid, r);
    if (fabs(L + R - val) <= 15 * eps)
        return L + R + (L + R - val) / 15;
    return solve(l, mid, eps / 2, L) + solve(mid, r, eps / 2, R);
}
//eps 设置太小可能会导致死循环
long double asme(long double l, long double r, long double eps = 1e-16l) { //自适应辛普森积分
    return solve(l, r, eps, simpson(l, r)); //求函数 calc 在区间 [l, r] 上的自适应辛普森积分,
    ↪ eps 指定精度
}
int main() {
    pl[0] = pr[0] = 1;
    for (int i = 1; i < maxn; ++i) {
        pl[i] = pl[i - 1] * L;
        pr[i] = pr[i - 1] * R;
    }
    a[0][0] = 1 / sqrt(R - L);
    for (int i = 1; i <= T; ++i) { //计算第 i 个规范正交基
        a[i][i] = 1;
        for (int j = 0; j < i; ++j) {
            long double sum = 0; //sum = <x^i, a[j]>
            for (int k = 0; k <= j; ++k)
                sum += a[j][k] * (pr[i + k + 1] - pl[i + k + 1]) / (i + k + 1);
            for (int k = 0; k <= j; ++k)
                a[i][k] -= sum * a[j][k];
        }
        long double total = 0;
        for (int j = 0; j <= i; ++j)
            for (int k = 0; k <= i; ++k)
                total += a[i][j] * a[i][k] * (pr[j + k + 1] - pl[j + k + 1]) / (j + k + 1);
    }
}

```

```

    long double length = sqrt(total);
    for (int j = 0; j <= i; ++j)
        a[i][j] /= length;
}
for (::pos = 0; pos <= T; ++pos) {
    long double res = asme(L, R);
    for (int i = 0; i <= pos; ++i)
        c[i] += a[pos][i] * res;
}
for (int i = 0; i <= T; ++i)
    printf("%.17le", c[i]);
return 0;
}

```

9.16 多项式拟合（泰勒展开）

该程序展示了用 15 次多项式去拟合函数 $f(x) = \int_0^x e^{-x^2} dx$

这个积分是求不出来原函数的，但是我们可以将 e^{-x^2} 泰勒展开，然后再对得到的每个多项式积分就得到了一个和

↪ 式：

$$\# \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(2n+1)}$$

这个和式就是 $f(x)$ 的泰勒展开式，我们用其前 50 项作为对 $f(x)$ 的近似，并求出拟合函数。

```

from decimal import *
getcontext().prec = 100
maxn = 210
b = Decimal(1)
F = [0, 1]
for i in range(1, 50): #F 为待求函数的泰勒展开式的系数，F[i] 为  $x^i$  的系数，F 中的项越多近似越好
    b *= -1
    F.append(0)
    F.append(1 / b / Decimal(2 * i + 1))
def asme(pos):
    length = pos + len(F) + 2
    t = [Decimal(0) for i in range(length)]
    for i in range(pos + 1):
        for j in range(len(F)):
            t[i + j] += a[pos][i] * F[j]
    res = Decimal(0)
    for i in range(length):
        res += t[i] * (pr[i + 1] - pl[i + 1]) / (i + 1)
    return res
L, R, T = Decimal(0), Decimal(2), 15 # 待拟合区间为 [L, R]，所用多项式的最高次数为 T
a = [[Decimal(0) for j in range(maxn)] for i in range(maxn)]
c = [Decimal(0) for i in range(maxn)]
pl = [Decimal(1)]
pr = [Decimal(1)]
for i in range(maxn):
    pl.append(pl[-1] * L)

```



```

    pr.append(pr[-1] * R)
a[0][0] = 1 / (R - L).sqrt()
for i in range(1, T + 1):
    a[i][i] = Decimal(1)
    for j in range(i):
        sum = Decimal(0)
        for k in range(j + 1):
            sum += a[j][k] * (pr[i + k + 1] - pl[i + k + 1]) / (i + k + 1)
        for k in range(j + 1):
            a[i][k] -= sum * a[j][k]
    total = Decimal(0)
    for j in range(i + 1):
        for k in range(i + 1):
            total += a[i][j] * a[i][k] * (pr[j + k + 1] - pl[j + k + 1]) / (j + k + 1)
    length = total.sqrt()
    for j in range(i + 1):
        a[i][j] /= length
for pos in range(T + 1):
    res = asme(pos);
    for i in range(pos + 1):
        c[i] += a[pos][i] * res;
for i in range(T + 1):
    print("%.20e" % c[i], end=", ")

```

9.17 雅可比方法

/*

用雅可比方法求出实对称矩阵的特征值和特征向量，注意必须是实对称矩阵。

时间复杂度 $O(n^3)$ ，大常数。

*/

```

const int maxn = 210;
const double eps = 1e-8;
using matrix = double[maxn][maxn];
using vec = array<double, maxn>;
using pair_t = pair<double, vec>;
struct {
    matrix A, V;
    int column[maxn], n;
    void update(int r, int c, double v) {
        A[r][c] = v;
        if (column[r] == c || fabs(A[r][c]) > fabs(A[r][column[r]])) {
            for (int i = 0; i < n; ++i) if (i != r)
                if (fabs(A[r][i]) > fabs(A[r][column[r]]))
                    column[r] = i;
        }
    }
}
void Jacobi() {

```

```

    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j)
            V[i][j] = 0;
        V[i][i] = 1;
        column[i] = (i == 0 ? 1 : 0);
    }
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            if (j != i && fabs(A[i][j]) > fabs(A[i][column[i]]))
                column[i] = j;
    for (int T = 0; ; ++T) { //迭代次数限制
        int x, y;
        double val = 0;
        for (int i = 0; i < n; ++i)
            if (fabs(A[i][column[i]]) > val)
                val = fabs(A[i][column[i]]), x = i, y = column[i];
        if (val < eps) //精度限制
            break;
        double phi = atan2(-2 * A[x][y], A[y][y] - A[x][x]) / 2;
        double sinp = sin(phi), cosp = cos(phi);
        for (int i = 0; i < n; ++i) if (i != x && i != y) {
            double a = A[x][i] * cosp + A[y][i] * sinp;
            double b = A[x][i] * -sinp + A[y][i] * cosp;
            update(x, i, a);
            update(y, i, b);
        }
        for (int i = 0; i < n; ++i) if (i != x && i != y) {
            double a = A[i][x] * cosp + A[i][y] * sinp;
            double b = A[i][x] * -sinp + A[i][y] * cosp;
            update(i, x, a);
            update(i, y, b);
        }
        for (int i = 0; i < n; ++i) {
            double a = V[i][x] * cosp + V[i][y] * sinp;
            double b = V[i][x] * -sinp + V[i][y] * cosp;
            V[i][x] = a, V[i][y] = b;
        }
        double a = A[x][x] * cosp * cosp + A[y][y] * sinp * sinp + 2 * A[x][y] * cosp * sinp;
        double b = A[x][x] * sinp * sinp + A[y][y] * cosp * cosp - 2 * A[x][y] * cosp * sinp;
        double tmp = (A[y][y] - A[x][x]) * sin(2 * phi) / 2 + A[x][y] * cos(2 * phi);
        update(x, y, tmp);
        update(y, x, tmp);
        A[x][x] = a, A[y][y] = b;
    }
}

```

//返回特征值和特征向量组成的 *pair*, 按照特征值从大到小排序

```

    auto solve(const matrix& input, int n) {
        this->n = n;
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                A[i][j] = input[i][j];
        Jacobi();
        vector<pair_t> result;
        for (int i = 0; i < n; ++i)
            result.emplace_back(A[i][i], vec());
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                result[i].second[j] = V[j][i];
        sort(result.begin(), result.end(), greater<pair_t>());
        return result;
    }
}jacobi;
matrix tmp = { {15.980000, 3.400000, -10.370000, },
               {3.400000, 8.492000, -8.062000, },
               {-10.370000, -8.062000, 11.572000, }, },
int main() {
    auto result = jacobi.solve(tmp, 3);
    for (int i = 0; i < 3; ++i) {
        auto eigenvalue = result[i].first;
        auto eigenvector = result[i].second;
        printf("(%.f)", eigenvalue);
        for (int i = 0; i < 3; ++i)
            printf(" %.f", eigenvector[i]);
        printf("\n");
    }
    return 0;
}

```

9.18 矩阵求逆

```

const int maxn = 555;
namespace inverse_double {
    using matrix = double[maxn][maxn];
    double temp[maxn][maxn * 2];
    void inverse(const matrix& A, matrix& res, int n) {
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j)
                temp[i][j] = A[i][j], temp[i][n + j] = 0;
            temp[i][n + i] = 1;
        }
        for (int i = 0; i < n; ++i) {
            int r = i;
            for (int j = i + 1; j < n; ++j)

```

```

        if (fabs(temp[j][i]) > fabs(temp[r][i]))
            r = j;
    if (r != i)
        for (int j = 0; j < 2 * n; ++j)
            swap(temp[i][j], temp[r][j]);
    assert(fabs(temp[i][i]) > 1e-10);
    for (int k = i + 1; k < n; ++k)
        for (int j = 2 * n - 1; j >= i; --j)
            temp[k][j] -= temp[k][i] / temp[i][i] * temp[i][j];
}
for (int i = n - 1; i >= 0; --i) {
    for (int j = i + 1; j < n; ++j)
        for (int k = n; k < 2 * n; ++k)
            temp[i][k] -= temp[j][k] * temp[i][j];
    for (int k = n; k < 2 * n; ++k)
        temp[i][k] /= temp[i][i];
}
for (int i = 0; i < n; ++i)
    for (int j = 0; j < n; ++j)
        res[i][j] = temp[i][n + j];
}
}

namespace inverse_mod {
    const long long mod = 998244353;
    using matrix = long long[maxn][maxn];
    matrix temp;
    long long pow_mod(long long a, long long n) {
        long long ans = 1;
        while (n) {
            if (n & 1)
                ans = ans * a % mod;
            n >>= 1;
            a = a * a % mod;
        }
        return ans;
    }
    long long inv(long long x) {
        return pow_mod(x, mod - 2);
    }
}

void inverse(const matrix& A, matrix& res, int n) {
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j)
            temp[i][j] = A[i][j], temp[i][n + j] = 0;
        temp[i][n + i] = 1;
    }
    for (int i = 0; i < n; ++i) {

```

```

    int r = i;
    for (int j = i; j < n; ++j) {
        if (temp[j][i] != 0) {
            r = j;
            break;
        }
    }
    if (r != i) for (int j = 0; j < n * 2; ++j)
        swap(temp[i][j], temp[r][j]);
    auto pivot = inv(temp[i][i]);
    for (int k = i + 1; k < n; ++k)
        for (int j = 2 * n - 1; j >= i; --j)
            temp[k][j] = (temp[k][j] - temp[k][i] * pivot % mod * temp[i][j] % mod + mod)
↪ % mod;
    }
    for (int i = n - 1; i >= 0; --i) {
        for (int j = i + 1; j < n; ++j) {
            for (int k = n; k < n * 2; ++k) {
                temp[i][k] = (temp[i][k] - temp[j][k] * temp[i][j] % mod + mod) % mod;
            }
        }
        auto pivot = inv(temp[i][i]);
        for (int k = n; k < n * 2; ++k)
            temp[i][k] = temp[i][k] * pivot % mod;
    }
    for (int i = 0; i < n; ++i)
        for (int j = 0; j < n; ++j)
            res[i][j] = temp[i][n + j];
}
}
using namespace inverse_double;
int main() {
    matrix res, A = { {0, 1, 2}, {1, 0, 3}, {4, -3, 8} };
    inverse(A, res, 3);
    for (int i = 0; i < 3; ++i) {
        for (int j = 0; j < 3; ++j)
            printf("%.4f ", res[i][j]);
        printf("\n");
    }
    return 0;
}

```

9.19 牛顿迭代求非线性方程组

```

//const int maxn = 111;
//using matrix = double[maxn][maxn];
using vec = array<double, 3>; //用 vector 的话会很慢

```

//x 为初始解向量

//calc 函数返回一个 $n * (n + 1)$ 的矩阵，左边是雅可比矩阵，右边是函数值向量

```
vec solve(vec x, function<void(vec, matrix&)> calc) {
    static matrix A;
    static vect res;
    int n = x.size();
    for (int T = 0; T < 50; ++T) { //不同的问题应当设置不同的迭代次数
        calc(x, A);
        least_square(A, n, n, res);
        for (int i = 0; i < n; ++i)
            x[i] -= res[i];
    }
    return x;
}
```

//求向量值函数 f 的零点

const double h = 1e-5; //h 指定差分的幅度

```
vec solve(vec x, function<vec(vec)> f) {
    static matrix A;
    static vect res;
    int n = x.size();
    for (int T = 0; T < 50; ++T) { //不同的问题应当设置不同的迭代次数
        vec y = f(x);
        for (int i = 0; i < n; ++i) { //差分法近似求雅可比矩阵
            vec u = x, v = x;
            u[i] += h / 2; v[i] -= h / 2;
            vec a = f(u), b = f(v);
            for (int j = 0; j < n; ++j)
                A[j][i] = (a[j] - b[j]) / h;
            A[i][n] = y[i];
        }
        least_square(A, n, n, res);
        for (int i = 0; i < n; ++i)
            x[i] -= res[i];
    }
    return x;
}
```

```
int main() {
    //  $(x - 1)^2 + y^2 + z^2 - 1 = 0$ 
    //  $x^2 + (y - 1)^2 + z^2 - 1 = 0$ 
    //  $x^2 + y^2 + (z - 1)^2 - 1 = 0$ 
    auto f = [](vec arg) ->vec {
        auto x = arg[0], y = arg[1], z = arg[2];
        return vec{
            (x - 1) * (x - 1) + y * y + z * z - 1,
            x * x + (y - 1) * (y - 1) + z * z - 1,
            x * x + y * y + (z - 1) * (z - 1) - 1,
        };
    };
}
```

```

    };
};
auto calc = [&](vec x, matrix& A) {
    //Jacobi
    for (int i = 0; i < x.size(); ++i) {
        for (int j = 0; j < x.size(); ++j) {
            if (i == j)
                A[i][j] = 2 * (x[i] - 1);
            else
                A[i][j] = 2 * x[j];
        }
    }
    //f(x)
    auto res = f(x);
    for (int i = 0; i < x.size(); ++i)
        A[i][x.size()] = res[i];
};
vec x = { -122, 100, 30 };
auto res = solve(x, f);
for (auto i : res)
    printf("%.12f ", i);
printf("\n");
return 0;
}

```

9.20 QR 迭代

```

const int maxn = 111;
const double eps = 1e-10;
using matrix = double[maxn][maxn];
matrix Q, R;
double u[maxn], w[maxn];
vector<double> eigenvalues(matrix& A, int n) {
    for (int T = 0; T < 1000000; ++T) {
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j) {
                Q[i][j] = 0;
                R[i][j] = A[i][j];
            }
            Q[i][i] = 1;
        }
        for (int i = 0; i < n; ++i) {
            double sum = 0;
            for (int j = i; j < n; ++j)
                sum += fabs(R[i][j]);
            if (sum < eps)
                continue;

```

```

    double sigma = R[i][i] <= 0 ? 1 : -1;
    double tot = 0;
    for (int j = i; j < n; ++j) {
        tot += R[j][i] * R[j][i];
        w[j] = -sigma * R[j][i];
    }
    w[i] += sqrt(tot);
    tot = sqrt(tot - R[i][i] * R[i][i] + w[i] * w[i]);
    for (int j = i; j < n; ++j)
        u[j] = w[j] / tot;
    for (int j = 0; j < n; ++j) {
        double product = 0;
        for (int k = i; k < n; ++k)
            product += u[k] * Q[k][j];
        for (int k = i; k < n; ++k)
            Q[k][j] = sigma * (Q[k][j] - 2 * product * u[k]);
    }
    for (int j = i; j < n; ++j) {
        double product = 0;
        for (int k = i; k < n; ++k)
            product += u[k] * R[k][j];
        for (int k = i; k < n; ++k)
            R[k][j] = sigma * (R[k][j] - 2 * product * u[k]);
    }
}
}
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j) {
        A[i][j] = 0;
        for (int k = 0; k < n; ++k)
            A[i][j] += R[i][k] * Q[j][k];
    }
}
}
vector<double> ret;
for (int i = 0; i < n; ++i)
    ret.push_back(R[i][i]);
for (int i = 0; i < n; ++i) {
    for (int j = 0; j < n; ++j)
        printf("%.10f ", R[i][j]);
    printf("\n");
}
return ret;
}
matrix A = { {-1, 1, 0}, {-4, 3, 0 }, {1, 0, 2} };
int main() {
    eigenvalues(A, 3);
}

```



```

    return 0;
}

```

9.21 行列式计算

```
/*
```

矩阵树定理与部分扩展：

1. 给出一个无向无权图，设 A 为邻接矩阵， D 为度数矩阵 ($D[i][i]$ = 结点 i 的度数，其他的无值)。则基尔霍夫 (Kirchhoff) 矩阵即为： $K = D - A$ ， K 的任意一个 $n - 1$ 阶子式即为该图的生成树个数。
2. 把度数矩阵重新定义为 $D[i][i]$ = 与结点 i 相连的所有边的权值和，把邻接矩阵重新定义为 $A[i][j]$ = 结点 i 与结点 j 之间所有边的权值和，那么矩阵树定理求的就是：所有生成树边权乘积的总和。
3. 有向图的情况也是可以做的，若 $D[i][i]$ = 结点 i 的入边的权值和，此时求的就是外向树 (从根向外)，若 $D[i][i]$ = 结点 i 的出边的权值和，此时求的就是内向树 (从外向根)，既然是有向的，那么就需要指定根，求行列式的时候去掉哪一行就是那一个元素为根。

```
*/
```

```

const long long mod = 1e9 + 7;
const int maxn = 500;
using matrix = long long[maxn][maxn];
//时间复杂度  $n^3 \log m$ ，如果超时可换成一般的高斯消元
//注意矩阵  $A$  的下标从 1 开始
long long det(matrix A, int n) {
    long long res = 1;
    for (int i = 0; i < n; ++i) {
        for (int j = i + 1; j < n; ++j) {
            while (A[j][i]) { //辗转相除法
                long long t = A[i][i] / A[j][i];
                for (int k = i; k < n; ++k) {
                    A[i][k] = (A[i][k] - t * A[j][k] % mod + mod) % mod;
                    swap(A[i][k], A[j][k]);
                }
                res = -res;
            }
        }
        if (!A[i][i]) return 0;
        res = (res * A[i][i] % mod + mod) % mod;
    }
    return res;
}

int main() { //洛谷 P6178
    static matrix A;
    int n, m, t;
    scanf("%d %d %d", &n, &m, &t);
    for (int i = 0; i < m; ++i) {
        int x, y, w;
        scanf("%d %d %d", &x, &y, &w);
    }
}

```

```

--x; --y;
if (t == 0) {
    A[x][y] = (A[x][y] - w + mod) % mod;
    A[y][x] = (A[y][x] - w + mod) % mod;
    A[x][x] = (A[x][x] + w) % mod;
    A[y][y] = (A[y][y] + w) % mod;
}
else {
    if (x == 0) x = n - 1; else if (x == n - 1) x = 0;
    if (y == 0) y = n - 1; else if (y == n - 1) y = 0;
    A[x][y] = (A[x][y] - w + mod) % mod;
    A[y][y] = (A[y][y] + w) % mod;
}
}
printf("%lld\n", det(A, n - 1));
return 0;
}

```

9.22 二元一次不定方程

```

void gcd(long long a, long long b, long long& d, long long& x, long long& y) {
    if (!b) {
        d = a;
        x = 1;
        y = 0;
    }
    else {
        gcd(b, a % b, d, y, x);
        y -= x * (a / b);
    }
}

int main() {
    //freopen("in.txt", "r", stdin);
    int T;
    scanf("%d", &T);
    while (T--) {
        long long a, b, c, g, x, y;
        scanf("%lld %lld %lld", &a, &b, &c);
        gcd(a, b, g, x, y);
        if (c % g != 0) { //ax + by = c 无解
            puts("-1");
            continue;
        }
        x = x * c / g, y = y * c / g; //算出一组特解
        long long dx = b / g, dy = a / g;
        //通解的形式为 (x + s * dx, y - s * dy)
        //若要求 x 和 y 都大于 0, 则 s 的取值范围为 [L, R]
    }
}

```

```

    long long L = x > 0 ? (1 - x) / dx : -x / dx + 1;
    long long R = y > 0 ? (y - 1) / dy : y / dy - 1;
    if (L <= R)
        printf("%lld %lld %lld %lld %lld\n", R - L + 1, x + L * dx, y - R * dy,
            x + R * dx, y - L * dy);
    else //无正整数数解时, 分别输出  $x$  和  $y$  的最小正整数值
        printf("%lld %lld\n", x + L * dx, y - R * dy);
}
return 0;
}

```

10 网络流

10.1 Dinic

```

const int maxn = 100000 + 10;
const int inf = 1 << 30;
struct edge{
    int from, to, cap, flow;
    edge(int u, int v, int c, int f) : from(u), to(v), cap(c), flow(f) {}
};
struct Dinic {
    int n, m, s, t;
    vector<edge> edges; // 边数的两倍
    vector<int> G[maxn]; // 邻接表,  $G[i][j]$  表示结点  $i$  的第  $j$  条边在  $e$  数组中的序号
    bool vis[maxn]; // BFS 使用
    int d[maxn]; // 从起点到  $i$  的距离
    int cur[maxn]; // 当前弧指针
    void init(int n) {
        this->n = n;
        for (int i = 0; i < n; i++)
            G[i].clear();
        edges.clear();
    }
    void clear() {
        for (int i = 0; i < edges.size(); i++)
            edges[i].flow = 0;
    }
    void reduce() {
        for (int i = 0; i < edges.size(); i++)
            edges[i].cap -= edges[i].flow;
    }
    void addedge(int from, int to, int cap) {
        edges.push_back(edge(from, to, cap, 0));
        edges.push_back(edge(to, from, 0, 0));
        m = edges.size();
        G[from].push_back(m - 2);
    }
};

```

```

    G[to].push_back(m - 1);
}

bool BFS() {
    memset(vis, 0, sizeof(vis));
    queue<int> Q;
    Q.push(s);
    vis[s] = 1;
    d[s] = 0;
    while (!Q.empty()) {
        int x = Q.front(); Q.pop();
        for (int i = 0; i < G[x].size(); i++) {
            edge& e = edges[G[x][i]];
            if (!vis[e.to] && e.cap > e.flow) {
                vis[e.to] = 1;
                d[e.to] = d[x] + 1;
                Q.push(e.to);
            }
        }
    }
    return vis[t];
}

int DFS(int x, int a) {
    if (x == t || a == 0) return a;
    int flow = 0, f;
    for (int& i = cur[x]; i < G[x].size(); i++) {
        edge& e = edges[G[x][i]];
        if (d[x] + 1 == d[e.to] && (f = DFS(e.to, min(a, e.cap - e.flow))) > 0) {
            e.flow += f;
            edges[G[x][i] ^ 1].flow -= f;
            flow += f;
            a -= f;
            if (a == 0) break;
        }
    }
    return flow;
}

int Maxflow(int s, int t) {
    this->s = s; this->t = t;
    int flow = 0;
    while (BFS()) {
        memset(cur, 0, sizeof(cur));
        flow += DFS(s, inf);
    }
    return flow;
}

vector<int> Mincut() { // call this after maxflow
```

```

    vector<int> ans;
    for (int i = 0; i < edges.size(); i++) {
        edge& e = edges[i];
        if (vis[e.from] && !vis[e.to] && e.cap > 0)
            ans.push_back(i);
    }
    return ans;
}
}dinic;
int main()
{
    freopen("D:\\in.txt", "r", stdin);
    int n, m;
    scanf("%d %d", &n, &m);
    dinic.init(n + 5);
    while (m--) {
        int s, t, u;
        scanf("%d %d %d", &s, &t, &u);
        dinic.addedge(s, t, u);
    }
    auto start = clock();
    printf("%d\n", dinic.Maxflow(1, n));
    double tot = static_cast<double>(clock() - start) / CLOCKS_PER_SEC;
    printf("Dinic: %f\n", tot);
    return 0;
}

```

10.2 ISAP

```

const int maxn = 1100;
const int maxedges = 51000;
const int inf = 1 << 30;
struct edge {
    int to, flow;
    edge *next, *pair;
    edge() {}
    edge(int to, int flow, edge *next) : to(to), flow(flow), next(next) {}
    void *operator new(unsigned, void *p) { return p; }
};
struct ISAP {
    int gap[maxn], h[maxn], n, s, t;
    edge *cur[maxn], *first[maxn], edges[maxedges * 2], *ptr;
    void init(int n) {
        this->n = n;
        ptr = edges;
        memset(first, 0, sizeof(first));
        memset(gap, 0, sizeof(gap));
    }

```

```

    memset(h, 0, sizeof(h));
    gap[0] = n;
}

void add_edge(int from, int to, int cap) {
    first[from] = new(ptr++)edge(to, cap, first[from]);
    first[to] = new(ptr++)edge(from, 0, first[to]);
    first[from]->pair = first[to];
    first[to]->pair = first[from];
}

int augment(int x, int limit) {
    if (x == t)
        return limit;
    int rest = limit;
    for (edge*& e = cur[x]; e; e = e->next) if (e->flow && h[e->to] + 1 == h[x]) {
        int d = augment(e->to, min(rest, e->flow));
        e->flow -= d, e->pair->flow += d, rest -= d;
        if (h[s] == n || !rest)
            return limit - rest;
    }
    int minh = n;
    for (edge *e = cur[x] = first[x]; e; e = e->next) if (e->flow)
        minh = min(minh, h[e->to] + 1);
    if (--gap[h[x]] == 0)
        h[s] = n;
    else
        ++gap[h[x] = minh];
    return limit - rest;
}

int solve(int s, int t, int limit = inf) {
    this->s = s; this->t = t;
    memcpy(cur, first, sizeof(first)); // memcpy!
    int flow = 0;
    while (h[s] < n && flow < limit)
        flow += augment(s, limit - flow);
    return flow;
}

}isap;

int main()
{
    freopen("D:\\in.txt", "r", stdin);
    int n, m;
    scanf("%d %d", &n, &m);
    isap.init(n + 5);
    while (m--)
    {
        int s, t, u;

```

```

        scanf("%d %d %d", &s, &t, &u);
        isap.add_edge(s, t, u);
    }
    auto start = clock();
    printf("%d\n", isap.solve(1, n));
    double tot = static_cast<double>(clock() - start) / CLOCKS_PER_SEC;
    printf("Isap: %f\n", tot);
    return 0;
}

```

10.3 HLPP

```

const int maxn = 2e5 + 5, maxedges = 4e6 + 5, inf = 0x3f3f3f3f;
int n, m, s, t, tot;
int v[maxedges * 2], w[maxedges * 2], first[maxn], nxt[maxedges * 2];
int h[maxn], e[maxn], gap[maxn * 2], inq[maxn]; //节点高度是可以到达  $2n-1$  的
struct cmp{
    inline bool operator()(int a, int b) const {
        return h[a] < h[b]; //因为在优先队列中的节点高度不会改变，所以可以直接比较
    }
};
queue<int> Q;
priority_queue<int, vector<int>, cmp> heap;
inline void add_edge(int from, int to, int flow) {
    tot += 2;
    v[tot + 1] = from; v[tot] = to; w[tot] = flow; w[tot + 1] = 0;
    nxt[tot] = first[from]; first[from] = tot;
    nxt[tot + 1] = first[to]; first[to] = tot + 1;
    return;
}
inline bool bfs() {
    memset(h + 1, 0x3f, sizeof(int) * n);
    h[t] = 0;
    Q.push(t);
    while (!Q.empty())
    {
        int now = Q.front(); Q.pop();
        for (int go = first[now]; go; go = nxt[go])
            if (w[go ^ 1] && h[v[go]] > h[now] + 1)
                h[v[go]] = h[now] + 1, Q.push(v[go]);
    }
    return h[s] != inf;
}
inline void push(int now) {
    for (int go = first[now]; go; go = nxt[go]) {
        if (w[go] && h[v[go]] + 1 == h[now]) {
            int d = min(e[now], w[go]);

```

```

        w[go] -= d; w[go ^ 1] += d; e[now] -= d; e[v[go]] += d;
        if (v[go] != s && v[go] != t && !inq[v[go]])
            heap.push(v[go]), inq[v[go]] = 1;
        if (!e[now])//已经推送完毕可以直接退出
            break;
    }
}
}
inline void relabel(int now) {
    h[now] = inf;
    for (int go = first[now]; go; go = nxt[go])
        if (w[go] && h[v[go]] + 1 < h[now])
            h[now] = h[v[go]] + 1;
    return;
}
inline int hlpp() {
    int now, d;
    if (!bfs()) //s 和 t 不连通
        return 0;
    h[s] = n;
    memset(gap, 0, sizeof(int) * (n * 2));
    for (int i = 1; i <= n; i++)
        if (h[i] < inf)
            ++gap[h[i]];
    for (int go = first[s]; go; go = nxt[go]) {
        if (d = w[go]) {
            w[go] -= d; w[go ^ 1] += d; e[s] -= d; e[v[go]] += d;
            if (v[go] != s && v[go] != t && !inq[v[go]])
                heap.push(v[go]), inq[v[go]] = 1;
        }
    }
    while (!heap.empty()) {
        inq[now = heap.top()] = 0; heap.pop(); push(now);
        if (e[now]) {
            if (!--gap[h[now]]) //gap 优化, 因为当前节点是最高的所以修改的节点一定不在优先队列中, 不必担心修改对优先队列会造成影响
                for (int i = 1; i <= n; i++)
                    if (i != s && i != t && h[i] > h[now] && h[i] < n + 1)
                        h[i] = n + 1;
            relabel(now); ++gap[h[now]];
            heap.push(now); inq[now] = 1;
        }
    }
    return e[t];
}
int main()

```



```

{
    freopen("D:\\in.txt", "r", stdin);
    scanf("%d %d", &n, &m); s = 1; t = n;
    while (m--)
    {
        int s, t, u;
        scanf("%d %d %d", &s, &t, &u);
        add_edge(s, t, u);
    }
    auto start = clock();
    printf("%d\n", hlpp());
    double tot = static_cast<double>(clock() - start) / CLOCKS_PER_SEC;
    printf("HLPP: %f\n", tot);
    return 0;
}

```

10.4 MCMF-spfa

```

const int maxn = 20100;
const int inf = 1 << 30;
struct edge {
    int from, to, cap, flow, cost;
    edge(int u, int v, int c, int f, int w) : from(u), to(v), cap(c), flow(f), cost(w) {}
};
struct MCMF {
    int n, m;
    vector<edge> edges;
    vector<int> G[maxn];
    int inq[maxn];
    int d[maxn];
    int p[maxn];
    int a[maxn];
    void init(int n) {
        this->n = n;
        for (int i = 0; i < n; ++i)
            G[i].clear();
        edges.clear();
    }
    void add_edge(int from, int to, int cap, int cost) {
        edges.push_back(edge(from, to, cap, 0, cost));
        edges.push_back(edge(to, from, 0, 0, -cost));
        m = edges.size();
        G[from].push_back(m - 2);
        G[to].push_back(m - 1);
    }
    bool BellmanFord(int s, int t, int &flow, int &cost, int limit) {
        for (int i = 0; i < n; ++i)

```

```

        d[i] = inf;
    memset(inq, 0, sizeof(inq));
    d[s] = 0; inq[s] = 1; p[s] = 0; a[s] = inf;
    queue<int> Q;
    Q.push(s);
    while (!Q.empty()) {
        int u = Q.front(); Q.pop();
        inq[u] = false;
        for (unsigned i = 0; i < G[u].size(); ++i) {
            edge &e = edges[G[u][i]];
            if (e.cap > e.flow && d[e.to] > d[u] + e.cost) {
                d[e.to] = d[u] + e.cost;
                p[e.to] = G[u][i];
                a[e.to] = min(a[u], e.cap - e.flow);
                if (!inq[e.to]) {
                    Q.push(e.to);
                    inq[e.to] = true;
                }
            }
        }
    }
}

if (d[t] == inf)
    return false;
a[t] = min(a[t], limit - flow);
flow += a[t];
cost += d[t] * a[t];
for (int u = t; u != s; u = edges[p[u]].from) {
    edges[p[u]].flow += a[t];
    edges[p[u] ^ 1].flow -= a[t];
}
return true;
}

int solve(int s, int t, int limit = inf) {
    int flow = 0, cost = 0;
    while (flow < limit && BellmanFord(s, t, flow, cost, limit));
    return cost;
}

}mcmf;

int main()
{
    freopen("D:\\in.txt", "r", stdin);
    int n, m, k;
    scanf("%d %d %d", &n, &m, &k);
    mcmf.init(n + 10);
    for (int i = 1; i <= m; ++i) {
        int x, y, c, w;

```

```

        scanf("%d %d %d %d", &x, &y, &c, &w);
        mcmf.add_edge(x, y, c, w);
    }
    auto start = clock();
    printf("%d\n", mcmf.solve(1, n, k));
    double tot = static_cast<double>(clock() - start) / CLOCKS_PER_SEC;
    printf("MCMF-spfa: %f\n", tot);
    return 0;
}

```

10.5 MCMF-dijkstra

```

const int maxn = 21000;
const int inf = 1 << 30;
struct edge {
    int to, cap, cost, rev;
    edge() {}
    edge(int to, int cap, int cost, int rev) : to(to), cap(cap), cost(cost), rev(rev) {}
};
struct MCMF {
    int n, h[maxn], d[maxn], pre[maxn], num[maxn];
    vector<edge> G[maxn];
    void init(int n) {
        this->n = n;
        for (int i = 0; i <= n; ++i)
            G[i].clear();
    }
    void add_edge(int from, int to, int cap, int cost) {
        G[from].push_back(edge(to, cap, cost, G[to].size()));
        G[to].push_back(edge(from, 0, -cost, G[from].size() - 1));
    }
    //flow 是自己传进去的变量，就是最后的最大流，返回的是最小费用
    int solve(int s, int t, int &flow, int limit = inf) {
        int cost = 0;
        memset(h, 0, sizeof(h));
        while (limit) {
            priority_queue<pair<int, int>, vector<pair<int, int>>, greater<pair<int, int>>> Q;
            for (int i = 0; i <= n; ++i)
                d[i] = inf;
            d[s] = 0;
            Q.emplace(0, s);
            while (!Q.empty()) {
                auto now = Q.top(); Q.pop();
                int u = now.second;
                if (d[u] < now.first) continue;
                for (int i = 0; i < G[u].size(); ++i) {
                    edge &e = G[u][i];

```

```

        if (e.cap > 0 && d[e.to] > d[u] + e.cost + h[u] - h[e.to]) {
            d[e.to] = d[u] + e.cost + h[u] - h[e.to];
            pre[e.to] = u;
            num[e.to] = i;
            Q.emplace(d[e.to], e.to);
        }
    }
}

if (d[t] == inf) break;
for (int i = 0; i <= n; ++i)
    h[i] += d[i];
int a = limit;
for (int u = t; u != s; u = pre[u])
    a = min(a, G[pre[u]][num[u]].cap);
limit -= a; flow += a; cost += a * h[t];
for (int u = t; u != s; u = pre[u]) {
    edge &e = G[pre[u]][num[u]];
    e.cap -= a;
    G[u][e.rev].cap += a;
}
}

return cost;
}

}mcmf;

int main()
{
    freopen("D:\\in.txt", "r", stdin);
    int n, m, k, flow = 0;
    scanf("%d %d %d", &n, &m, &k);
    mcmf.init(n + 10);
    for (int i = 1; i <= m; ++i) {
        int x, y, c, w;
        scanf("%d %d %d %d", &x, &y, &c, &w);
        mcmf.add_edge(x, y, c, w);
    }
    auto start = clock();
    printf("%d\n", mcmf.solve(1, n, flow, k));
    printf("flow: %d\n", flow);
    double tot = static_cast<double>(clock() - start) / CLOCKS_PER_SEC;
    printf("MCMF-dijkstra: %f\n", tot);
    return 0;
}

```

11 线性规划

11.1 线性规划

// 改进单纯型法的实现

// 参考: http://en.wikipedia.org/wiki/Simplex_algorithm

// 输入矩阵 a 描述线性规划的标准形式。 a 为 $m+1$ 行 $n+1$ 列, 其中行 $0\sim m-1$ 为不等式, 行 m 为目标函数 (最
 \rightarrow 大化)。列 $0\sim n-1$ 为变量 $0\sim n-1$ 的系数, 列 n 为常数项

// 第 i 个约束为 $a[i][0]*x[0] + a[i][1]*x[1] + \dots \leq a[i][n]$

// 目标为 $\max(a[m][0]*x[0] + a[m][1]*x[1] + \dots + a[m][n-1]*x[n-1] - a[m][n])$

// 注意: 变量均有非负约束 $x[i] \geq 0$

const int maxm = 500; // 约束数目上限

const int maxn = 500; // 变量数目上限

const double INF = 1e100;

const double eps = 1e-10;

struct Simplex {

int n; // 变量个数

int m; // 约束个数

double a[maxm][maxn]; // 输入矩阵

int f[maxm], d[maxn]; // 算法辅助变量

void pivot(int r, int c)

{

swap(d[c], f[r]);

a[r][c] = 1 / a[r][c];

for (int j = 0; j <= n; j++) if (j != c) a[r][j] *= a[r][c];

for (int i = 0; i <= m; i++) if (i != r)

{

for (int j = 0; j <= n; j++) if (j != c) a[i][j] -= a[i][c] * a[r][j];

a[i][c] = -a[i][c] * a[r][c];

}

}

bool feasible()

{

for (;;)

{

int r, c;

double p = INF;

for (int i = 0; i < m; i++) if (a[i][n] < p) p = a[r = i][n];

if (p > -eps) return true;

p = 0;

for (int i = 0; i < n; i++) if (a[r][i] < p) p = a[r][c = i];

if (p > -eps) return false;

p = a[r][n] / a[r][c];

for (int i = r + 1; i < m; i++) if (a[i][c] > eps)

{

double v = a[i][n] / a[i][c];

if (v < p) r = i, p = v;

```

    }
    pivot(r, c);
}
}
// 解有界返回 1, 无解返回 0, 无界返回 -1。f[i] 为 x[i] 的值, ret 为目标函数的值
int simplex(int n, int m, double x[maxn], double& ret)
{
    this->n = n;
    this->m = m;
    for (int i = 0; i < n; i++) d[i] = i;
    for (int i = 0; i < m; i++) f[i] = n + i;
    if (!feasible()) return 0;
    for (;;)
    {
        int r, c;
        double p = 0;
        for (int i = 0; i < n; i++) if (a[m][i] > p) p = a[m][c = i];
        if (p < eps)
        {
            for (int i = 0; i < n; i++) if (d[i] < n) x[d[i]] = 0;
            for (int i = 0; i < m; i++) if (f[i] < n) x[f[i]] = a[i][n];
            ret = -a[m][n];
            return 1;
        }
        p = INF;
        for (int i = 0; i < m; i++) if (a[i][c] > eps)
        {
            double v = a[i][n] / a[i][c];
            if (v < p) r = i, p = v;
        }
        if (p == INF) return -1;
        pivot(r, c);
    }
}
};
int main()
{
    return 0;
}

```

12 字符串算法

12.1 后缀树

- /*
1. $ch[u][c]$ 表示从结点 u 出发走字符 c 后到达的结点 (0 为空结点)。
 2. $len[u]$ 表示从结点 u 的父结点走到 u 的这条边的长度 (可能为 0)。

3. 从结点 u 的父结点走到 u 的字符串的下标区间为 $[start[u], start[u] + len[u] - 1]$ 。
4. $link[u]$ 记录结点 u 的后缀链接, 若内部结点 u 表示的字符串为 $[s, t]$, 则其后缀链接的结点对应字符串 \rightarrow 为 $[s+1, t]$ 。
5. $depth[u]$ 表示结点 u 的加权深度, $dep[u]$ 表示结点的不加权深度 (根结点到 u 经过的边数)。
6. s 数组记录输入的字符串。
7. 对于叶结点 $pos[i]$ 表示结点 i 对应的后缀的下标, 非叶结点 $pos[i]$ 为 -1 。
8. $node$ 是 pos 的反函数, $node[i]$ 表示后缀 $s[i, m]$ 对应的结点 (下标从 1 开始)。

*/

```

const int sigma_size = 26;
const int maxlog = 20;
const int maxn = 4e5 + 10;
const int inf = 1e9;
struct Suffix_Tree {
public: //后缀树基本结构
    int ch[maxn][sigma_size + 1], len[maxn], start[maxn], link[maxn], depth[maxn], s[maxn], cur,
    ↪ n, rem, now;
    int pos[maxn], node[maxn];
    void init() {
        memset(ch, 0, sizeof(ch));
        cur = now = 1;
        n = rem = 0;
        len[0] = inf;
        depth[0] = 0;
    }
    int newnode(int p, int w, int d) {
        start[++cur] = p;
        len[cur] = w;
        depth[cur] = d;
        link[cur] = 1;
        return cur;
    }
    void extend(int x) { //0 <= x <= sigma_size
        s[++n] = x, ++rem;
        for (int last = 1; rem; now == 1 ? --rem : now = link[now]) {
            while (rem > len[ch[now][s[n - rem + 1]])
                rem -= len[now = ch[now][s[n - rem + 1]]];
            int ed = s[n - rem + 1];
            int& v = ch[now][ed];
            int c = s[start[v] + rem - 1];
            link[last] = now;
            if (!v) {
                v = newnode(n - rem + 1, inf, n - rem + 1 - depth[now]);
                last = now;
            }
            else if (x == c) {
                last = now;
            }
        }
    }
};

```

```

        break;
    }
    else {
        int u = newnode(start[v], rem - 1, depth[now] + rem - 1);
        ch[u][x] = newnode(n, inf, n - depth[u]);
        ch[u][c] = v;
        start[v] += rem - 1;
        if (len[v] != inf)
            len[v] -= rem - 1;
        link[last] = v = u;
        last = v;
    }
}

}

void maintain(int m) { //m 为输入字符串的长度
//对后缀树进行调整, 并消除最后 extend(sigma_size) 对后缀树结构的影响。
//调整后的后缀树的最后没有字符 sigma_size, 但是树边的长度可能为 0,
//并且遍历的时候要枚举所有的字符: 0 <= c <= sigma_size
    ch[1][sigma_size] = 0;
    for (int i = 1; i <= cur; ++i) {
        if (len[i] == inf) {
            len[i] = m - start[i] + 1;
            pos[i] = depth[i];
            node[pos[i]] = i;
            depth[i] = m - depth[i] + 1;
        }
        else
            pos[i] = -1;
    }
}

void insert(const char* str) { //插入的字符串的下标从 1 开始
    assert(str[0] == 0);
    int m = strlen(str + 1);
    for (int i = 1; i <= m; ++i)
        extend(str[i] - 'a');
    extend(sigma_size);
    maintain(m);
}

int go(const char* str, int nd = 1) { //如果终止在边内则会到子结点
    int m = strlen(str);
    for (int i = 0; i < m; i += len[nd]) {
        int c = str[i] - 'a';
        if (!ch[nd][c])
            return 0;
        nd = ch[nd][c];
        for (int j = 0; j < len[nd] && i + j < m; ++j)

```



```

        if (str[i + j] - 'a' != s[start[nd] + j])
            return 0;
    }
    //如果有长度为 0 的边则直接走过去。在处理子树问题时应当直接返回 nd。
    return ch[nd][sigma_size] != 0 ? ch[nd][sigma_size] : nd;
}
public: //后缀树上倍增
    int L[maxn], R[maxn], dep[maxn], anc[maxn][maxlog]; //anc[i][j] 表示结点 i 往上走  $2^j$  个点到达
    ↪ 的点
    vector<int> seq;
    void dfs(int x, int fa) {
        dep[x] = dep[fa] + 1;
        anc[x][0] = fa;
        L[x] = seq.size();
        if (pos[x] > 0)
            seq.push_back(pos[x]);
        for (int i = 0; i <= sigma_size; ++i) if (ch[x][i])
            dfs(ch[x][i], x);
        R[x] = seq.size() - 1;
    }
    void preprocess() { //后缀树上倍增初始化
        for (int i = 1; i <= cur; ++i) {
            for (int j = 1; (1 << (j - 1)) <= cur; ++j)
                anc[i][j] = 0;
        }
        for (int j = 1; (1 << j) <= cur; ++j) {
            for (int i = 1; i <= cur; ++i) {
                if (anc[i][j - 1] != 0) {
                    int a = anc[i][j - 1];
                    anc[i][j] = anc[a][j - 1];
                }
            }
        }
    }
    void build() { //求出后缀树的 DFS 序
        seq.clear(); //可以在此处向 seq 中加入一个元素 0, 从而使得 DFS 序区间从 1 开始
        dep[0] = -1;
        dfs(1, 0); //求后缀树的 DFS 序
        preprocess(); //后缀树上倍增初始化
    }
    int ascend(int x, int length) { //将结点上升到加权深度为 length 的位置。
        for (int i = maxlog - 1; i >= 0; --i)
            if (depth[anc[x][i]] >= length) //调用该函数之前需要先调用 preprocess 初始化
                x = anc[x][i];
        return x;
    }
}

```

```

int query(int L, int R) { //返回从根结点开始沿着字符串  $s[L, R]$  走到的结点 (若最后结束在边上, 则
↪ 走到子结点)
    int x = node[L], length = R - L + 1;
    return ascend(x, length);
}
int lca(int x, int y) { //返回后缀树中结点  $x$  与  $y$  的最近公共祖先。
    if (dep[x] < dep[y])
        swap(x, y);
    for (int i = maxlog - 1; i >= 0; --i)
        if (dep[x] - (1 << i) >= dep[y])
            x = anc[x][i];
    if (x != y) {
        for (int i = maxlog - 1; i >= 0; --i)
            if (anc[x][i] && anc[x][i] != anc[y][i])
                x = anc[x][i], y = anc[y][i];
        x = anc[x][0];
    }
    return x;
}
int lcp(int i, int j) { //返回后缀  $s[i, m]$  与后缀  $s[j, m]$  的最长公共前缀。
    return depth[lca(node[i], node[j])];
}
public: //后缀树上快速下降
    int sz[maxn], leaf[maxn];
    void travel(int x) { //调用  $travel(1)$  对后缀树进行树剖, 之后才能调用  $go(x, L, R)$ 。
        sz[x] = 1;
        leaf[x] = pos[x];
        for (int z = 0, i = 0; i <= sigma_size; ++i) {
            int y = ch[x][i];
            if (y) travel(y);
            sz[x] += sz[y];
            if (sz[y] > sz[z]) {
                z = y;
                leaf[x] = leaf[y];
            }
        }
    }
    int go(int x, int L, int R) { //从结点  $x$  开始往下走字符串  $s[L, R]$ , 时间复杂度  $O(\log^2 n)$ 
        while (x && L <= R) {
            if (s[L] == s[leaf[x] + depth[x]]) {
                int length = min(R - L + 1, lcp(leaf[x] + depth[x], L));
                L += length;
                int y = ascend(node[leaf[x]], depth[x] + length);
                if (L <= R && depth[y] != depth[x] + length)
                    return 0;
                x = y;
            }
        }
    }
}

```

```

    }
    else {
        int y = ch[x][s[L]];
        if (lcp(start[y], L) < min(len[y], R - L + 1))
            return 0;
        L += len[y];
        x = y;
    }
}
return x;
//return ch[x][sigma_size] != 0 ? ch[x][sigma_size] : x;
}
int recognise(int l, int r, int L, int R) { //返回从根结点开始先走 s[l, r] 再走 s[L, R] 所到的
↪ 结点。
    int x = query(l, r);
    int diff = depth[x] - (r - l + 1);
    if (diff > 0 && lcp(start[x] + len[x] - diff, L) < min(diff, R - L + 1))
        return 0;
    else
        return go(x, L + diff, R);
    //因为树中有长度为 0 的边，所以即使要识别的字符串为原串的后缀也不一定走到叶结点，
    //若要保证在这种情况下走到叶结点则应有：x = (ch[x][sigma_size] != 0 ? ch[x][sigma_size] :
↪ x);
}
} st;
int main() {
    static char s[maxn];
    srand(time(0));
    int n = 200000;
    for (int i = 1; i <= n; ++i)
        s[i] = 'a' + (rand() % 2 == 0);
    auto start = clock();
    st.init();
    st.insert(s);
    st.build();
    st.travel(1);
    for (int i = 1; i <= n - 10; ++i) {
        int mid = (i + n) / 2;
        int x = st.recognise(i, mid, mid + 1, n);
        x = (st.ch[x][sigma_size] != 0 ? st.ch[x][sigma_size] : x);
        auto j = st.pos[x];
        if (j != i || st.depth[x] != n - i + 1) {
            printf("%d %d (%d)\n", j, i, x);
            abort();
        }
    }
}

```

```

    auto end = clock();
    printf("time: %f\n", double(end - start) / CLOCKS_PER_SEC);
    return 0;
}

```

12.2 扩展 KMP

```

char T[] = "aaababa", P[] = "aa";
const int maxn = 10000;
int nxt[maxn], extend[maxn];
//注意: 字符串 T 与字符串 P 必须以不同的特殊字符结尾 (该特殊字符不算在字符串内, 故不能算入字符串的长度
→ 里)
void getnext(char P[])
{
    int po = 1, m = strlen(P);
    nxt[0] = m;
    nxt[1] = mismatch(P + 1, P + m, P).second - P;
    for (int i = 2; i < m; ++i)
    {
        if (nxt[i - po] + i < nxt[po] + po)
            nxt[i] = nxt[i - po];
        else
        {
            int j = max(nxt[po] + po - i, 0);
            nxt[i] = mismatch(P + j + i, P + m, P + j).second - P;
            po = i;
        }
    }
}

void exkmp(char T[], char P[])
{
    int po = 0, n = strlen(T), m = strlen(P);
    P[m] = 0; T[n] = 1;
    extend[0] = mismatch(T, T + n, P).second - P;
    getnext(P); //注意: 若字符串 P 最后的特殊字符不是 0, 则字符串的长度会加 1, next[0] 会与实际不符。
    for (int i = 1; i < n; i++)
    {
        if (nxt[i - po] + i < extend[po] + po)
            extend[i] = nxt[i - po];
        else
        {
            int j = max(extend[po] + po - i, 0);
            extend[i] = mismatch(T + i + j, T + n, P + j).second - P;
            po = i;
        }
    }
}

```

```
int main()
{
    exkmp(T, P);
    for (int i = 0; i < 15; ++i)
        printf("%d\n", extend[i]);
    return 0;
}
```

12.3 AC 自动机

```
/*
通过 insert 将字符串插入 Trie 之后不要忘记调用 getfail 建立 AC 自动机！
对于 AC 自动机中的状态  $u$ ，不能仅仅通过  $val[u] \neq 0$  来判断  $u$  是否能匹配输入字符串，还要结合  $last$ 。
对于 AC 自动机中的状态  $u$ ，其不能匹配任何一个字符串的条件为  $val[u] == 0$  且  $last[u] == 0$ 。
*/
const int maxnode = 100000;
const int sigma_size = 26;
struct trie
{
    int ch[maxnode][sigma_size];
    int f[maxnode]; // fail 函数
    int val[maxnode]; // 每个字符串的结尾结点都有一个非 0 的 val
    int last[maxnode]; // 输出链表的下一个结点
    int sz;
    void init()
    {
        sz = 1;
        memset(ch[0], 0, sizeof(ch[0]));
    }
    // 字符 c 的编号
    int idx(char c)
    {
        return c - 'a';
    }
    // 插入字符串。v 必须非 0
    void insert(const char *s, int v)
    {
        int u = 0, n = strlen(s);
        for (int i = 0; i < n; i++)
        {
            int c = idx(s[i]);
            if (!ch[u][c])
            {
                memset(ch[sz], 0, sizeof(ch[sz]));
                val[sz] = 0;
                ch[u][c] = sz++;
            }
        }
    }
}
```

```

        u = ch[u][c];
    }
    val[u] = v;
}
// 计算 fail 函数
void getfail()
{
    queue<int> Q;
    f[0] = 0;
    // 初始化队列
    for (int c = 0; c < sigma_size; c++)
    {
        int u = ch[0][c];
        if (u) { f[u] = 0; Q.push(u); last[u] = 0; }
    }
    // 按 BFS 顺序计算 fail
    while (!Q.empty())
    {
        int r = Q.front(); Q.pop();
        for (int c = 0; c < sigma_size; c++)
        {
            int u = ch[r][c];
            if (!u)
            {
                ch[r][c] = ch[f[r]][c];
                continue;
            }
            Q.push(u);
            int v = f[r];
            while (v && !ch[v][c]) v = f[v];
            f[u] = ch[v][c];
            last[u] = val[f[u]] ? f[u] : last[f[u]];
        }
    }
}
// 在 T 中找模板
void find(const char *T)
{
    int n = strlen(T);
    int j = 0; // 当前结点编号, 初始为根结点
    for (int i = 0; i < n; i++)
    {
        // 文本串当前指针
        int c = idx(T[i]);
        j = ch[j][c];
        if (val[j]) print(j);
    }
}

```

```

        else if (last[j]) print(last[j]); // 找到了!
    }
}
// 递归打印以结点 j 结尾的所有字符串
void print(int j)
{
    if (j)
    {
        //对查找到的编号为 val[j] 的字符串进行操作
        printf("%d\n", val[j]);
        print(last[j]);
    }
}
}ac;
int main()
{
    ac.init();
    ac.insert("aabbcc", 1);
    ac.insert("bbcff", 2);
    ac.insert("bb", 3);
    ac.getfail();
    ac.find("aabbccffbbbop");
    return 0;
}

```

12.4 KMP 算法

```

char T[] = "abcdefabc", P[] = "abc";
const int maxn = 10000;
int f[maxn];    //f[i] 表示字符串 s[0, i-1] 的后缀与前缀的最长公共部分（后缀与前缀均不包含字符串本
               ↪ 身）
               //若 f[i] = k 则，字符串 s[0, k-1] 与字符串 s[i-k, i-1] 相同
void getfail(char *P, int *f)
{
    int m = strlen(P);
    f[0] = 0; f[1] = 0;
    for (int i = 1; i < m; ++i)
    {
        int j = f[i];
        while (j && P[j] != P[i])
            j = f[j];
        f[i + 1] = P[j] == P[i] ? j + 1 : 0;
    }
}
void find(char *T, char *P, int *f)
{
    int n = strlen(T), m = strlen(P);

```

```

getfail(P, f);
int j = 0;
for (int i = 0; i < n; ++i)
{
    while (j && P[j] != T[i])
        j = f[j];
    if (P[j] == T[i])
        ++j;
    if (j == m)
        printf("%d\n", i - m + 1); //在串 T 中找到了 P, 下标为 i - m + 1
}
}
int main()
{
    getfail(P, f);
    find(T, P, f);
    return 0;
}

```

12.5 Manacher 算法

```

const int maxn = 1.1e7 * 2 + 100; //maxn 应当大于 原字符串长度的两倍加二
char s[maxn];
int len[maxn];
//回文串在原字符串中的长度为 len[i] - 1
void manacher(char* str) {
    int n = strlen(str), m = 0;
    for (int i = 0; i < n; ++i) {
        s[++m] = '#';
        s[++m] = str[i];
    }
    s[++m] = '#'; s[0] = '$'; //s 是加入新字符后的字符串
    len[1] = 1;
    int r = 1, po = 1; //r 是当前极长回文子串的最右的端点 po 为 r 对应的回文子串的中心
    for (int i = 1; i <= m; ++i) {
        if (i < r)
            len[i] = min(len[2 * po - i], r - i + 1); //2*po-i 为 i 在当前这个极长回文子串中在左边
            ↪ 相对应的位置
        else
            len[i] = 1;
        while (s[i + len[i]] == s[i - len[i]])
            ++len[i];
        if (i + len[i] > r) {
            r = i + len[i] - 1;
            po = i;
        }
    }
}

```



```

//原字符串的最长回文子串为  $\max\{len[i] - 1\}$ 
//以  $i$  为中心的奇回文串的长度为  $len[2 * i + 2] - 1$ 
//以  $i$  为左中心的偶回文串的长度为  $len[2 * i + 3] - 1$ 
}
char a[maxn / 2];
int main() {
    scanf("%s", a);
    const int n = strlen(a);
    manacher(a);
    int ans = *max_element(len, len + n * 2 + 4) - 1;
    printf("%d\n", ans);
    return 0;
}

```

12.6 后缀数组

```

/*
1.  rank[i] 表示下标 i 的排名 (排名从 0 开始)。
2.  sa[i] 表示第 i 小的后缀的下标 (i 从 0 开始)。
3.  height[i] 表示 sa[i - 1] 与 sa[i] 的最长公共前缀。
*/
const int maxn = 210000; //maxn 应当开到最大字符串长度的两倍, 否则 (1) 处下标访问可能越界。
const int maxlog = 20;
struct Suffix_Array {
    char s[maxn];
    int sa[maxn], rank[maxn], height[maxn];
    int t[maxn], t2[maxn], c[maxn], n;
    void init(const char* str) {
        strcpy(s, str);
        n = strlen(s);
        memset(t, 0, sizeof(int) * (2 * n + 10)); //为了保证 (1) 处访问越界时得到的数组值恒为 0,
        ↪ 应当将 t 和 t2 数组清空
        memset(t2, 0, sizeof(int) * (2 * n + 10));
    }
    void build_sa(int m = 256) {
        int* x = t, * y = t2;
        for (int i = 0; i < m; ++i) c[i] = 0;
        for (int i = 0; i < n; ++i) c[x[i] = s[i]]++;
        for (int i = 1; i < m; ++i) c[i] += c[i - 1];
        for (int i = n - 1; i >= 0; --i) sa[--c[x[i]]] = i;
        for (int k = 1; k <= n; k <= 1) {
            int p = 0;
            for (int i = n - 1; i >= n - k; --i) y[p++] = i;
            for (int i = 0; i < n; ++i) if (sa[i] >= k) y[p++] = sa[i] - k;
            for (int i = 0; i < m; ++i) c[i] = 0;
            for (int i = 0; i < n; ++i) c[x[y[i]]]++;
            for (int i = 1; i < m; ++i) c[i] += c[i - 1];

```

```

    for (int i = n - 1; i >= 0; --i) sa[--c[x[y[i]]]] = y[i];
    swap(x, y);
    p = 1; x[sa[0]] = 0;
    for (int i = 1; i < n; ++i)
        x[sa[i]] = y[sa[i - 1]] == y[sa[i]] && y[sa[i - 1] + k] == y[sa[i] + k] ? p - 1 :
↪ p++; //(1)
    if (p >= n) break;
    m = p;
}
}

void getheight() {
    int k = 0;
    for (int i = 0; i < n; ++i) rank[sa[i]] = i;
    for (int i = 0; i < n; ++i) if (rank[i] > 0) {
        if (k) k--;
        int j = sa[rank[i] - 1];
        while (i + k < n && j + k < n && s[i + k] == s[j + k]) k++;
        height[rank[i]] = k;
    }
}

int d[maxn][maxlog], log[maxn];
void RMQ_init() {
    log[0] = -1;
    for (int i = 1; i <= n; ++i)
        log[i] = log[i / 2] + 1;
    for (int i = 0; i < n; ++i)
        d[i][0] = height[i];
    for (int j = 1; j <= log[n]; ++j)
        for (int i = 0; i + (1 << j) - 1 < n; ++i)
            d[i][j] = min(d[i][j - 1], d[i + (1 << (j - 1))][j - 1]);
}

int lcp(int i, int j) { //返回下标 i 开始的后缀与下标 j 开始的后缀的最长公共前缀。
    if (i == j)
        return n - i;
    if (rank[i] > rank[j])
        swap(i, j);
    int x = rank[i] + 1, y = rank[j];
    int k = log[y - x + 1];
    return min(d[x][k], d[y - (1 << k) + 1][k]);
}

pair<int, int> locate(int l, int r) {
    //返回一个最长的区间 [L, R] 使得 sa 中下标从 L 到 R 的所有后缀都以 s[l, r] 为前缀。
    int pos = rank[l], length = r - l + 1;
    int L = 0, R = pos;
    while (L < R) {
        int M = L + (R - L) / 2;

```

```

        if (lcp(l, sa[M]) >= length) R = M;
        else L = M + 1;
    }
    int tmp = L;
    L = pos, R = n - 1;
    while (L < R) {
        int M = L + (R - L + 1) / 2;
        if (lcp(l, sa[M]) >= length) L = M;
        else R = M - 1;
    }
    return make_pair(tmp, L);
}

pair<int, int> locate(vector<pair<int, int>> ranges) {
    //将 ranges 中的所有下标区间对应的子串拼接到一起, 得到字符串 T,
    //返回一个最长的区间 [L, R] 使得 sa 中下标从 L 到 R 的所有后缀都以 T 为前缀,
    //无解返回 { 0, -1 }。
    int l = 0, r = n - 1, pos = 0;
    for (auto [x, y] : ranges) {
        int L = l, R = r, len = y - x + 1;
        while (L < R) {
            int M = L + (R - L) / 2;
            int pre = sa[M] + pos < n ? lcp(sa[M] + pos, x) : 0;
            int ch1 = s[sa[M] + pos + pre], ch2 = s[x + pre];
            if (pre >= len || ch1 > ch2) R = M;
            else L = M + 1;
        }
        int left = L;
        L = l, R = r;
        while (L < R) {
            int M = L + (R - L + 1) / 2;
            int pre = sa[M] + pos < n ? lcp(sa[M] + pos, x) : 0;
            int ch1 = s[sa[M] + pos + pre], ch2 = s[x + pre];
            if (pre >= len || ch1 < ch2) L = M;
            else R = M - 1;
        }
        int right = L;
        if (sa[left] + pos > n || lcp(sa[left] + pos, x) < len)
            return make_pair(0, -1);
        else
            l = left, r = right;
        pos += len;
    }
    return make_pair(l, r);
}

pair<int, int> go(int x, int y, int h, char c) {
    //设 sa 数组中区间 [x, y] 的最长公共前缀至少为 h, 则返回区间中的后缀 i 满足 s[i + h] == c

```

```

//若要识别  $s$  中所有以  $c$  开头的后缀, 则调用  $go(0, n - 1, 0, c)$ 
int L = x, R = y;
while (L < R) {
    int M = L + (R - L) / 2;
    if (sa[M] + h < n && s[sa[M] + h] >= c) R = M;
    else L = M + 1;
}
int tmp = L;
R = y;
while (L < R) {
    int M = L + (R - L + 1) / 2;
    if (s[sa[M] + h] == c) L = M;
    else R = M - 1;
}
if (sa[L] + h >= n || s[sa[L] + h] != c)
    return { 0, -1 }; //空区间
return { tmp, L };
}

pair<int, int> go(int x, int y, int h, const string &str) {
    //设  $sa$  数组中区间  $[x, y]$  的最长公共前缀至少为  $h$ , 则返回区间中的后缀  $i$  满足  $s[i + h + j] ==$ 
    ↪  $str[j]$ 
    //若要识别  $s$  中所有以  $str$  开头的后缀, 则调用  $go(0, n - 1, 0, str)$ 
    for (auto c : str) {
        int L = x, R = y;
        while (L < R) {
            int M = L + (R - L) / 2;
            if (sa[M] + h < n && s[sa[M] + h] >= c) R = M;
            else L = M + 1;
        }
        x = L, R = y;
        while (L < R) {
            int M = L + (R - L + 1) / 2;
            if (s[sa[M] + h] == c) L = M;
            else R = M - 1;
        }
        if (sa[L] + h >= n || s[sa[L] + h] != c)
            return { 0, -1 }; //空区间
        y = L;
        h += 1;
    }
    return { x, y };
}

pair<int, int> go_rev(int x, int y, char c) {
    //设区间  $[x, y]$  表示  $sa$  中的一个区间, 将  $[x, y]$  中的每个后缀都往前走一个字符  $c$ ,
    //得到一个新的连续区间并返回, 注意这个新区间不一定是  $[x, y]$  的子区间
    //注意: 因为  $sa[0, n-1]$  不包含空串, 所以  $go(0, n - 1, 0, c) != go\_rev(0, n - 1, c)$ 

```

```

    int L = 0, R = n - 1;
    while (L < R) {
        int M = L + (R - L) / 2;
        if (s[sa[M]] < c || s[sa[M]] == c && (sa[M] == n - 1 || rank[sa[M] + 1] < x))
            L = M + 1;
        else
            R = M;
    }
    int tmp = L;
    R = n - 1;
    while (L < R) {
        int M = L + (R - L + 1) / 2;
        if (s[sa[M]] > c || s[sa[M]] == c && rank[sa[M] + 1] > y)
            R = M - 1;
        else
            L = M;
    }
    if (s[sa[L]] != c || rank[sa[L] + 1] < x || rank[sa[L] + 1] > y)
        return { 0, -1 }; //空区间
    return { tmp, L };
}
}arr;
int main() {
    static char s[maxn];
    scanf("%s", s);
    int n = strlen(s), Q;
    scanf("%d", &Q);
    arr.init(s);
    arr.build_sa();
    arr.getheight();
    arr.RMQ_init();
    while (Q--) {
        int x, y, a, b;
        scanf("%d %d %d %d", &x, &y, &a, &b); --x; --y; --a; --b;
        auto [L, R] = arr.locate(vector<pair<int, int>>{make_pair(x, y), make_pair(a, b)});
        printf("%d\n", R - L + 1);
    }
}

```

12.7 后缀自动机

/*

在 *Trie* 树中建立后缀自动机:

只需 DFS 一遍 *Trie* 树, 对于 *Trie* 中的当前结点 u , 它的父结点为 p , p 在后缀自动机中对应的结点为 v 。

则将 v 作为 *last*, 将 u 对应的字符插入后缀自动机中即可。(一定要先插入结点 u , 再插入结点 u 在 *Trie* 树中 \hookrightarrow 的子结点)

在建立广义后缀自动机时:

对于当前插入的字符串 s ，若 s 的前缀 $s[0, i]$ 也是之前某个字符串的前缀，则在插入 $s[0, i]$ 时，字符 $s[0]$ \hookrightarrow 对应的结点出度为 0，
 而对于 $s[1, i]$ 中的每个字符 $s[t]$ ，它对应的结点在后缀自动机中入度为 1，来源于 $s[t-1]$ 。所以若从根结点开 \hookrightarrow 始走，不会走到这些结点，
 故在后缀自动机中插入多个字符串不会破坏自动机的结构。同时对于 $s[0, i]$ 中的每一个字符 $s[t]$ ，它对应的结 \hookrightarrow 点虽然不可达，
 但是会在 *parent* 树中出现，所以广义后缀自动机中记录的信息是完全的。
 设在自动机中输入字符串 *str* 后到达结点 u ，并设 u 的 *right* 集合为 R （一个结点 *right* 集合为其子结点 \hookrightarrow *right* 集合的并集），
 则 R 记录了 *str* 在每个字符串中的所有的结束位置。

*/

//结论：从空串开始不断随机字符放到结尾直到 s 作为子串出现的期望次数 = 对 s 的所有 *border* i 计算 $f(i)$ \hookrightarrow 的和， $f(i)$ 表示从头开始直接随机出 i 的概率的倒数

const int maxn = 210000; //maxn 应当大于最大字符串长度的 2 倍

//后缀自动机的基本数据

int par[maxn], len[maxn], go[maxn][26]; //min[s] = len[par[s]] + 1, 当前结点表示的最小长度等于父结 \hookrightarrow 点的最大长度加一

int cur, root, last;

//用于求 *parent* 树的 DFS 序

int first[maxn], nxt[maxn], info[maxn], id[maxn], L[maxn], R[maxn]; //right 集合中的下标是子串 ** \hookrightarrow 右 ** 端点的位置

vector<int> seq;

//用于对后缀自动机进行拓扑排序

int cnt[maxn], arr[maxn];

int newstate(int length, int index = -1) {

len[cur] = length;

id[cur] = index;

return cur++;

}

void init() {

memset(go, 0, sizeof(go)); //如果数据组数很多，可能超时，应当改为 *memset(go, 0, sizeof(go[0]) * \hookrightarrow (2 * n + 10));*

cur = 1;

root = last = newstate(0);

}

void extend(int w, int index) {

int x = last;

int nx = newstate(len[x] + 1, index);

while (x && !go[x][w])

go[x][w] = nx, x = par[x];

if (!x)

par[nx] = root;

else {

int y = go[x][w];

if (len[x] + 1 == len[y])

par[nx] = y;

```

        else {
            int ny = newstate(len[x] + 1);
            memcpy(go[ny], go[y], sizeof(go[y]));
            par[ny] = par[y];
            par[y] = ny;
            par[nx] = ny;
            while (x && go[x][w] == y)
                go[x][w] = ny, x = par[x];
        }
    }
    last = nx;
    //每加入一个新的字符, 新产生的本质不同的子串个数为 len[last] - len[par[last]]
}

set<int> travel(int x) { //采用按秩合并的方式处理得到结点 x 的 right 集合, 最坏时间复杂度  $n \log^2 n$ 
    set<int> right;
    vector<set<int> > child;
    for (int i = first[x]; i != -1; i = nxt[i]) {
        child.push_back(travel(info[i]));
    }
    sort(child.begin(), child.end(), [](const auto& A, const auto& B) {
        return A.size() > B.size();
    });
    if (!child.empty())
        right = std::move(child[0]);
    for (int i = 1; i < child.size(); ++i)
        for (auto item : child[i])
            right.insert(item);
    if (id[x] != -1)
        right.insert(id[x]);
    //此处通过前面得到的 right 集合来计算答案, 注意此处代码的时间复杂度不能达到  $O(n)$ ,
    //否则总的时间复杂度可能会到  $O(n^2)$  甚至更高。
    return right;
}

void dfs(int x) {
    L[x] = seq.size();
    if (id[x] != -1)
        seq.push_back(id[x]);
    for (int i = first[x]; i != -1; i = nxt[i])
        dfs(info[i]);
    R[x] = seq.size() - 1;
}

void build() { //建树
    int sz = 0;
    seq.clear(); //可以在此处向 seq 中加入一个元素 0, 从而使得 DFS 序区间从 1 开始
    memset(first, -1, sizeof(first));
    for (int i = 1; i < cur; ++i) {

```

```

    int p = par[i];
    info[sz] = i;
    nxt[sz] = first[p];
    first[p] = sz++;
}
//travel(root); //遍历子树求解问题
dfs(root); //求 parent 树的 DFS 序
}

void topsort() { //对后缀自动机进行拓扑排序
    for (int i = 1; i < cur; ++i)
        cnt[i] = 0;
    for (int i = 1; i < cur; ++i)
        cnt[len[i]]++;
    for (int i = 1; i < cur; ++i)
        cnt[i] += cnt[i - 1];
    for (int i = cur - 1; i >= 1; --i) //arr: [1, cur)
        arr[cnt[len[i]]--] = i;
    for (int i = 2; i < cur; ++i) //只有建立广义后缀自动机的时候才需要这个循环
        if (par[arr[i - 1]] == arr[i])
            swap(arr[i - 1], arr[i]);
    /*
    拓扑排序后从叶结点开始递推更新 id 数组, 注意循环完成后 id[0] 为所有结点 id 的或。
    for (int i = cur - 1; i >= 1; --i) {
        int j = arr[i];
        if (id[par[j]] == -1)
            id[par[j]] = id[j];
        else
            id[par[j]] |= id[j];
    }
    */
}

void insert(char* str, int id) { //通过多次调用 insert 函数向自动机中插入多个字符串, 来建立广义后缀
    ↪ 自动机。
    int n = strlen(str);
    last = root; //!!!
    for (int i = 0; i < n; ++i)
        extend(str[i] - 'a', id * 100 + i); //可以将第二个参数改为 pair, 来记录插入字符来源于的字符
    ↪ 串编号和下标。
}

const int maxlog = 25;
int anc[maxn][maxlog]; //anc[i][j] 表示结点 i 往上走 2^j 个点到达的点
map<int, int> trans; //id 函数的反函数
void preprocess() { //parent 树上倍增初始化
    trans.clear();
    for (int i = 1; i < cur; ++i)
        if (id[i] != -1)

```



```

        trans[id[i]] = i;
    for (int i = 1; i < cur; ++i) {
        anc[i][0] = par[i];
        for (int j = 1; (1 << (j - 1)) < cur; ++j)
            anc[i][j] = 0;
    }
    for (int j = 1; (1 << j) < cur; ++j) {
        for (int i = 1; i < cur; ++i) {
            if (anc[i][j - 1] != 0) {
                int a = anc[i][j - 1];
                anc[i][j] = anc[a][j - 1];
            }
        }
    }
}

int query(int id, int length) { //从标号 id 的结点往上走到 len[p] >= length 的深度最小的结点
    int p = trans[id]; //若要识别字符串的子串 [L, R], 则调用 query(id(R), R-L+1), 其中
    ↪ id(R) 表示下标 R 对应的 id
    for (int i = maxlog - 1; i >= 0; --i)
        if (len[anc[p][i]] >= length)
            p = anc[p][i];
    return p;
}

namespace Suffix_Tree { //建立反串的后缀树 (后缀自动机的 parent 树是反串的后缀树)
    int pos[maxn]; //pos[x] 表示结点 x 对应的字符串在原串中的结束下标。
    int ch[maxn][26]; //ch[x][w] 表示结点 x 在后缀树中沿着字符 w 走到达的结点。
    char s[maxn];
    void extend(int w, int index) { //比之前的 extend 多了维护 pos 数组的代码。
        int x = last;
        int nx = newstate(len[x] + 1, index);
        pos[nx] = index;
        while (x && !go[x][w])
            go[x][w] = nx, x = par[x];
        if (!x)
            par[nx] = root;
        else {
            int y = go[x][w];
            if (len[x] + 1 == len[y])
                par[nx] = y;
            else {
                int ny = newstate(len[x] + 1);
                memcpy(go[ny], go[y], sizeof(go[y]));
                pos[ny] = pos[y];
                par[ny] = par[y];
                par[nx] = par[y] = ny;
                while (x && go[x][w] == y)

```

```

        go[x][w] = ny, x = par[x];
    }
}
last = nx;
}
void build(const char* str) { //对字符串 str 建立后缀自动机和反串的后缀树。
    strcpy(s, str);
    int n = strlen(s);
    last = root;
    for (int i = 0; i < n; ++i)
        extend(s[i] - 'a', i);
    for (int i = 2; i < cur; ++i) {
        int w = s[pos[i] - len[par[i]]] - 'a';
        ch[par[i]][w] = i;
    }
}
int walk(int x, int length, const char* str) { //从结点 x 开始沿着后缀树走字符串 str, 如果终止
→ 在边内则会到子结点。
    int m = strlen(str);
    for (int i = 0; i < m && i < len[x] - length; ++i)
        if (str[i] != s[pos[x] - length - i])
            return 0;
    for (int i = len[x] - length; i < m; i += len[x] - len[par[x]]) {
        int c = str[i] - 'a';
        if (!ch[x][c])
            return 0;
        x = ch[x][c];
        for (int j = 0; j < len[x] - len[par[x]] && i + j < m; ++j)
            if (str[i + j] != s[pos[x] - len[par[x]] - j])
                return 0;
    }
    return x;
}
int expand(int L, int R, const char* str) {
    int length = R - L + 1;
    int x = query(R, length);
    return walk(x, length, str);
}
/*

```

建立的后缀树是后缀自动机的辅助数据结构, 假设对字符串 s 建立后缀自动机, 令结点 x 表示字符串 $s[L, R]$,

则 $go[x][c]$ 表示字符串 $s[L, R] + c$ 对应的结点, $ch[x][c]$ 表示字符串 $c + s[L, R]$ 对应的结点。

注意因为 ch 对应的边是后缀树中的边, 边上是压缩的字符串而不是一个字符, 所以连续向左扩展的时候应该调用 $walk$ 函数或 $expand$ 函数, 而不是直接走 ch 。

$walk$ 函数用字符串 str 向左扩展状态 x , 因为一个状态可能对应多个字符串, 所以用 $length$ 来明确指明字符串的长度,

length 应满足条件: $\text{len}[\text{par}[x]] < \text{length} \leq \text{len}[x]$

注意该函数会先扩展 *str*[0] 然后 *str*[1]、*str*[2]..., 最终结果是将 *str* 的反串拼接在 *x* 所表示的字符串的左边得到的结果。

expand 函数返回将 *str* 反向拼接在 *s*[*L*, *R*] 的左边所得到的状态。

*/

}

namespace MultiString {

//label[i] 表示结点 *i* 的标记, *times*[*i*] 记录结点 *i* 在多少个字符串中出现

//real[i] 表示结点 *i* 所能表示的最大长度 (所有字符串在结点 *i* 所能表示的最大长度的最小值)

int label[maxn], mx[maxn], times[maxn], real[maxn];

void preprocess() { //初始化数据结构, 要在 *match* 之前 *extend* 之后调用 (必须在后缀自动机建立完成后调用)

memset(label, -1, sizeof(label));

memset(times, 0, sizeof(times));

for (int i = 0; i <= cur; ++i)

real[i] = len[i];

}

void match(const char* str, int index) {

vector<pair<int, int>> nodes;

int x = root, length = 0;

for (const char* s = str; *s; ++s) {

int c = *s - 'a';

while (x != root && !go[x][c]) {

x = par[x];

length = len[x];

}

if (go[x][c]) {

x = go[x][c];

length += 1;

}

nodes.emplace_back(x, length);

}

sort(nodes.begin(), nodes.end(), [](auto a, auto b) {

return len[a.first] > len[b.first];

});

vector<int> vec;

for (auto pr : nodes) {

int x = pr.first;

while (x && label[x] != index) {

vec.push_back(x);

label[x] = index;

mx[x] = 0;

x = par[x];

}

mx[pr.first] = max(mx[pr.first], pr.second);

}

```

    sort(vec.begin(), vec.end(), [](int a, int b) {
        return len[a] > len[b];
    });
    for (auto x : vec) {
        mx[par[x]] = max(mx[par[x]], len[par[x]]);
        real[x] = min(real[x], mx[x]);
        times[x] += 1;
    }
}

int longest_common_string(int n) {
    //设有 n 个字符串, 任选一个构建后缀自动机, 对其余的串调用 match 函数,
    //然后调用该函数, 返回这些字符串的最长公共子串长度。
    int ans = 0;
    for (int i = 1; i < cur; ++i) if (times[i] == n - 1)
        ans = max(ans, real[i]);
    return ans;
}

long long number_of_common_string(int n) {
    //设有 n 个字符串, 任选一个构建后缀自动机, 对其余的串调用 match 函数,
    //然后调用该函数, 返回这些字符串的本质不同的公共子串个数。
    long long ans = 0;
    for (int i = 1; i < cur; ++i) if (times[i] == n - 1)
        ans += max(0, real[i] - len[par[i]]);
    return ans;
}
}

namespace Generalized_Suffix_Automaton { //广义后缀自动机
    //pos[i] 表示结点 i 对应的子串在 index==0 的字符串中最左侧出现的下标
    //times[i] 表示在 parent 树中结点 i 的子树中包含的类型数
    int par[maxn], len[maxn], go[maxn][26];
    int label[maxn], times[maxn];
    int n, cur, root, last, pos[maxn];
    int newstate(int length, int index = maxn) {
        len[cur] = length;
        pos[cur] = index;
        return cur++;
    }
}

void init() { //n 表示字符串的个数, 需要在外部初始化
    memset(label, -1, sizeof(label));
    cur = 1;
    root = last = newstate(0);
}

void extend(int w, int index) {
    int x = last;
    int nx = newstate(len[x] + 1, index);
    while (x && !go[x][w])

```

```

        go[x][w] = nx, x = par[x];
    if (!x)
        par[nx] = root;
    else {
        int y = go[x][w];
        if (len[x] + 1 == len[y])
            par[nx] = y;
        else {
            int ny = newstate(len[x] + 1, pos[y]);
            memcpy(go[ny], go[y], sizeof(go[y]));
            times[ny] = times[y];
            par[ny] = par[y];
            par[y] = ny;
            par[nx] = ny;
            while (x && go[x][w] == y)
                go[x][w] = ny, x = par[x];
        }
    }
    last = nx;
}
//多次调用 insert 建立广义自动机, index 从 0 开始
void insert(const string& s, int index) {
    last = root;
    int pre = cur;
    for (int i = 0; i < s.size(); ++i)
        extend(s[i] - 'a', index == 0 ? i : maxn);
    for (int i = pre; i < cur; ++i) {
        for (int x = i; x && label[x] != index; x = par[x]) {
            label[x] = index;
            times[x] += 1;
            pos[par[x]] = min(pos[par[x]], pos[x]);
        }
    }
}
//dp[i] 表示从结点 i 对应的任意字符串开始, 能走到的本质不同的子串个数
long long dp[maxn];
long long dfs(int x) {
    if (dp[x] > 0)
        return dp[x];
    dp[x] = 0;
    for (int i = 0; i < 26; ++i) if (go[x][i] && times[go[x][i]] == n)
        dp[x] += dfs(go[x][i]) + 1;
    return dp[x];
}
void output(int x, long long k) { //输出第 k 小的本质不同公共子串
    if (k == 0) return;

```

```

    for (int i = 0; i < 26; ++i) if (go[x][i] && times[go[x][i]] == n) {
        if (k - dp[go[x][i]] - 1 > 0)
            k -= dp[go[x][i]] + 1;
        else {
            putchar('a' + i);
            output(go[x][i], k - 1);
            return;
        }
    }
}

int main() {
    //freopen("in.txt", "r", stdin);
    static char s[maxn];
    init();
    scanf("%s", s);
    insert(s, 0);
    int n = 1;
    MultiString::preprocess();
    while (scanf("%s", s) == 1)
        MultiString::match(s, ++n);
    printf("%d\n", MultiString::longest_common_string(n));
    return 0;
}

```

12.8 回文自动机

/*

1. $len[i]$ 表示编号为 i 的节点对应的回文串的长度（一个节点表示一个回文串）。
2. $fail[i]$ 表示结点 i 失配以后跳转到的最长后缀回文串对应的结点。
3. $par[i]$ 表示结点 i 删除掉最外层的一个字符后得到的回文串对应的结点。
例如结点 i 表示的回文串为 *cabbac*，则结点 $par[i]$ 对应的回文串为 *abba*。
4. $node[i]$ 表示输入字符串中以下标 i 结尾的最长回文串对应的回文树结点。
5. $cnt[i]$ 表示结点 i 对应的字符串出现的次数（建树时求出的不是完全的，最后 *calc()* 函数跑一遍以后才是正确的）。
6. $num[i]$ 表示以结点 i 表示的最长回文串的最右端点为回文串结尾的回文串个数。
7. 若要判断回文自动机中是否包含回文串 $P[0, m - 1]$ ，则根据 P 的长度是奇数还是偶数来选择根结点（长度为偶数的回文串以 0 为根结点，长度为奇数的回文串以 1 为根结点），
然后将字符串 $P[m / 2, m - 1]$ 输入自动机，到达了结点 $node$ ，自动机包含回文串 P 当且仅当 $len[node] == m$ 。
8. 一般来说不需要对回文自动机进行 *dfs*，因为循环 *for (int i = 2; i <= cur; ++i)* 就可以按照 *fail* 树的拓扑序（也是 *par* 树的拓扑序）访问结点。

*/

```

const int maxn = 110000;
const int sigma_size = 26;
int info[maxn], ch[maxn][sigma_size], fail[maxn], len[maxn];

```

```

int cnt[maxn], num[maxn], par[maxn], node[maxn], last, cur, sz;
void init() {
    memset(ch, 0, sizeof(ch));
    memset(cnt, 0, sizeof(cnt));
    fail[0] = 1;
    info[0] = -1;
    len[1] = -1;
    cur = last = 1;
    sz = 0;
}
void extend(int w) {
    int p = last;
    info[++sz] = w;
    while (info[sz - len[p] - 1] != w)
        p = fail[p];
    if (!ch[p][w]) {
        int u = ++cur, x = fail[p];
        while (info[sz - len[x] - 1] != w)
            x = fail[x];
        par[u] = p;
        len[u] = len[p] + 2;
        fail[u] = ch[x][w]; //(*)
        ch[p][w] = u;
        num[u] = num[fail[u]] + 1;
    }
    last = ch[p][w];
    cnt[last]++;
}
void insert(char* str) { //输入字符串下标从 0 开始
    int n = strlen(str);
    for (int i = 0; i < n; ++i) {
        extend(str[i] - 'a');
        node[i] = last;
    }
}
void calc() {
    //更新 fail 的地方只有 (*) 处, 此时将新建的结点 u 连接到之前的结点上而不改变之前的结点的连接状态,
    //这样, 对于从 cur 到 2 的循环, 就是对 fail 树按拓扑序逆序循环 (也是对 par 树按拓扑序逆序循环)。
    for (int i = cur; i >= 2; --i)
        cnt[fail[i]] += cnt[i];
}
int main() {
    int A[] = { 1, 2, 3, 2, 1, 3, 3 }, B[] = { 3, 2, 1 };
    init();
    for (int i = 0; i < sizeof(A) / sizeof(*A); ++i)

```

```

    extend(A[i]);
    int st = 0; //0 偶数, 1 奇数
    for (int i = 0; i < sizeof(B) / sizeof(*B); ++i)
        st = ch[st][B[i]];
    printf("%d\n", len[st]);
    return 0;
}

```

12.9 回文串 Border

```

/*
定义  $dif[x] = len[x] - len[fail[x]]$ ,
 $slink[x]$  为  $x$  后缀链接路径上第一个  $dif[x] dif[fail[x]]$  的祖先。
*/
const int maxn = 110000;
const int sigma_size = 26;
int info[maxn], ch[maxn][sigma_size], fail[maxn], len[maxn], dif[maxn], slink[maxn];
int node[maxn], last, cur, sz;
void init() {
    memset(ch, 0, sizeof(ch));
    fail[0] = 1;
    info[0] = -1;
    len[1] = -1;
    cur = last = 1;
    sz = 0;
}
void extend(int w) {
    int p = last;
    info[++sz] = w;
    while (info[sz - len[p] - 1] != w)
        p = fail[p];
    if (!ch[p][w]) {
        int u = ++cur, x = fail[p];
        while (info[sz - len[x] - 1] != w)
            x = fail[x];
        len[u] = len[p] + 2;
        fail[u] = ch[x][w]; //(*)
        ch[p][w] = u;
        dif[u] = len[u] - len[fail[u]];
        if (dif[u] != dif[fail[u]])
            slink[u] = fail[u];
        else
            slink[u] = slink[fail[u]];
    }
    last = ch[p][w];
}
void insert(char* str) { //输入字符串下标从 0 开始

```



```

    int n = strlen(str);
    for (int i = 0; i < n; ++i) {
        extend(str[i] - 'a');
        node[i] = last;
    }
}

int main() {
    return 0;
}

```

12.10 双端回文自动机

```

/*
len[i] 表示编号为 i 的节点表示的回文串的长度（一个节点表示一个回文串）
fail[i] 表示结点 i 失配以后跳转到的最长后缀回文串对应的结点
cnt[i] 表示结点 i 对应的字符串出现的次数（建树时求出的不是完全的，最后 calc() 函数跑一遍以后才是正确
↪ 的）
num[i] 表示以结点 i 表示的最长回文串的最右端点为回文串结尾的回文串个数
num[i] 也表示结点 i 在 fail 树中的深度
若要判断回文自动机中是否包含回文串 P[0, m - 1]，则根据 P 的长度是奇数还是偶数来选择根结点（长度为偶数
↪ 的回文串以 0 为根结点，长度为奇数的回文串以 1 为根结点），
然后将字符串 P[m / 2, m - 1] 输入自动机，到达了结点 node，自动机包含回文串 P 当且仅当 len[node] ==
↪ m。
*/

const int maxn = 300010; //maxn 至少为字符串最大可能长度的两倍
long long ans = 0; //统计当前的字符串中有多少个回文串（位置不同即不同）
struct Palindromic_Tree {
    int ch[maxn][26], fail[maxn], info[maxn];
    int cnt[maxn], num[maxn], len[maxn];
    int last[2], cur, L, R;
    void init() {
        memset(ch, 0, sizeof(ch));
        memset(cnt, 0, sizeof(cnt));
        memset(info, -1, sizeof(info)); //info 的初始值为不会出现在输入字符串中的值即可
        len[1] = -1;
        cur = fail[0] = last[1] = 1;
        last[0] = 0;
        L = maxn / 2;
        R = L - 1;
    }
    int get_fail(int x, int back) {
        int k = back ? R : L;
        int tp = back ? 1 : -1;
        while (info[k - (len[x] + 1) * tp] != info[k])
            x = fail[x];
        return x;
    }
}

```

```

void insert(int w, int back) { //back 为 1 则插入 w 到字符串的结尾, 为 0 则插入到开头。
    if (back) info[++R] = w;
    else info[--L] = w;
    int p = get_fail(last[back], back);
    if (!ch[p][w]) {
        int u = ++cur;
        fail[u] = ch[get_fail(fail[p], back)][w];
        len[u] = len[p] + 2;
        ch[p][w] = u;
        num[u] = num[fail[u]] + 1;
    }
    last[back] = ch[p][w];
    if (len[last[back]] == R - L + 1)
        last[back ^ 1] = last[back];
    ans += num[last[back]]; //当前结点的插入会使得字符串中新增 num[last[back]] 个新的回文串
    cnt[last[back]]++;
}

void calc() {
    for (int i = cur; i >= 2; --i)
        cnt[fail[i]] += cnt[i];
}

}tree;

int main(){
    //freopen("in.txt", "r", stdin);
    int n;
    while (scanf("%d", &n) == 1) {
        tree.init(); ans = 0;
        while (n--) {
            int tp;
            scanf("%d", &tp);
            if (tp == 1) {
                char c;
                scanf(" %c", &c);
                tree.insert(c - 'a', false);
            }
            else if (tp == 2) {
                char c;
                scanf(" %c", &c);
                tree.insert(c - 'a', true);
            }
            else if (tp == 3) {
                printf("%d\n", tree.cur - 1);
            }
            else {
                printf("%lld\n", ans);
            }
        }
    }
}

```

```

    }
}
}

```

12.11 非势能分析回文自动机

```
/*
```

每次插入时间复杂度都为 $O(1)$ 的回文自动机（不基于势能分析），可用于回滚莫队。

```
*/
```

```

const int maxn = 300007;
struct Palindromic_Tree {
    int cur, cnt[maxn], fail[maxn], ch[maxn][26], len[maxn], quick[maxn][26];
    int last[2], info[maxn * 2], L, R;
    void init() {
        fail[0] = cur = 1;
        len[1] = -1;
        last[0] = last[1] = 0;
        L = maxn;
        R = L - 1;
        memset(ch, 0, sizeof(ch)); //多组数据时应当在新建结点的时候清空
        memset(cnt, 0, sizeof(cnt));
        memset(info, -1, sizeof(info));
        for (int i = 0; i < 26; i++)
            quick[1][i] = quick[0][i] = 1;
    }
    void extend(int w, int back = 1) { //back 为 1 则插入 w 到字符串的结尾，为 0 则插入到开头。
        int tp = back ? -1 : 1;
        int x = back ? ++R : --L;
        int p = last[back];
        info[x] = w;
        if (info[x + tp * (len[p] + 1)] != info[x])
            p = quick[p][w];
        if (!ch[p][w]) {
            int u = ++cur;
            len[u] = len[p] + 2;
            int now = fail[p];
            if (info[x + tp * (len[now] + 1)] != info[x])
                now = quick[now][w];
            fail[u] = ch[now][w];
            memcpy(quick[u], quick[fail[u]], sizeof(quick[u]));
            quick[u][info[x + tp * len[fail[u]]]] = fail[u];
            ch[p][w] = u;
        }
        last[back] = ch[p][w];
        if (len[last[back]] == R - L + 1)
            last[back ^ 1] = last[back]; //注意这里
        cnt[last[back]]++;
    }
};

```

```

    }
} tree;
namespace Palindromic_Tree_Back {
    int info[maxn], ch[maxn][26], quick[maxn][26], fail[maxn], len[maxn];
    int last, cur, sz, n, m;
    vector<int> node;
    void init() {
        node.clear();
        memset(ch, 0, sizeof(ch[0]) * 5);
        fail[0] = 1;
        info[0] = len[1] = -1;
        cur = last = 1;
        sz = 0;
        for (int i = 0; i < 26; ++i)
            quick[1][i] = quick[0][i] = 1;
    }
    void extend(int w) { // 0 <= w <= sigma_size
        int p = last;
        info[++sz] = w;
        if (info[sz - len[p] - 1] != w)
            p = quick[p][w];
        if (!ch[p][w]) {
            int u = ++cur, x = fail[p];
            memset(ch[u], 0, sizeof(ch[u]));
            if (info[sz - len[x] - 1] != w)
                x = quick[x][w];
            len[u] = len[p] + 2;
            fail[u] = ch[x][w];
            ch[p][w] = u;
            memcpy(quick[u], quick[fail[u]], sizeof(quick[u]));
            quick[u][info[sz - len[fail[u]]]] = fail[u];
            //value[u] = (value[fail[u]] + (n - len[u] + 1) * pow_mod(26, n - len[u])) % mod;
        }
        last = ch[p][w];
        //answer = (answer + value[last]) % mod;
    }
    void rollback() { //回滚
        int x = node.back();
        node.pop_back();
        last = node.back();
        sz -= 1;
        //answer = (answer - value[x] + mod) % mod;
    }
}
char A[maxn];
int c[maxn], id[maxn];

```

```

int main() {
    scanf("%s", A + 1);
    int L = strlen(A + 1);
    tree.init();
    long long ans = 0;
    int T = L >> 1;
    for (int i = T + 1; i <= L; i++) tree.extend(A[i] - 'a', 1);
    for (int i = T; i; i--) tree.extend(A[i] - 'a', 0);
    for (int i = 2; i <= tree.cur; i++) c[tree.len[i]]++;
    for (int i = 1; i <= L; i++) c[i] += c[i - 1];
    for (int i = 2; i <= tree.cur; i++) id[c[tree.len[i]]--] = i;
    for (int i = tree.cur - 1; i; i--) tree.cnt[tree.fail[id[i]]] += tree.cnt[id[i]];
    for (int i = 2; i <= tree.cur; i++) ans = max(ans, 1ll * tree.len[i] * tree.cnt[i]);
    printf("%lld\n", ans);
}

```

12.12 序列自动机

/*

序列自动机：能够识别给定序列的所有子序列（非连续子序列）

last[c] 表示字符 *c* 在序列中最后一次出现的位置对应的自动机结点

par[i] 表示下标 *i* 处的字符的上一次的出现位置对应的自动机结点

ch[i][x] 表示以下标 *i* 为起点，字符 *x* 的第一次出现位置对应的自动机结点

对于用 *build* 建立的自动机，自动机结点与原数组中的位置一一对应（自动机中下标 *i* 的位置对应原数组中下标 $\mapsto i$ 的位置）。

对于用 *extend* 函数建立的自动机，自动机结点与原数组中的位置有常数值的偏移。

基本原理：

对于给定序列 *A*、*B* 判断 *B* 是否为 *A* 的子序列，只要从左往右找到字符 *B[0]* 在 *A* 中的第一次出现位置 *A[i]*，

\mapsto 然后再从 *i + 1* 开始找到 *B[1]* 的第一次出现位置，

以此类推。若按照如上算法在找到 *B* 之前已经到达了 *A* 的结尾，则 *B* 不是 *A* 的子序列，否则 *B* 是 *A* 的子序列。

*/

```

const int maxn = 110000;
const int sigma_size = 26;
int cur, root, par[maxn], last[sigma_size], ch[maxn][sigma_size];
void init()
{
    cur = root = 1;
    memset(ch, 0, sizeof(ch));
    for (int i = 0; i < sigma_size; ++i)
        last[i] = root;
}
void extend(int x) {
    par[++cur] = last[x];
    for (int c = 0; c < sigma_size; ++c)
        for (int i = last[c]; i && !ch[i][x]; i = par[i])
            ch[i][x] = cur;
    last[x] = cur;
}

```

```

}

void build(int A[], int n)    //若数组已知，则可以直接构造序列自动机，此时 root = 0.
{
    //注意：数组 A 的下标从 1 开始
    for (int i = n; i > 0; --i)
    {
        memcpy(ch[i - 1], ch[i], sizeof(ch[i]));
        ch[i - 1][A[i]] = i;
    }
}

int main()
{
    int A[] = { 0, 1, 1, 2, 3, 5, 1, 2 }, B[] = { 1, 1, 2, 5, 1 };
    //init();
    //for (int i = 0; i < sizeof(A) / sizeof(*A); ++i)
    //    extend(A[i]);
    //int st = root;
    build(A, sizeof(A) / sizeof(*A) - 1);
    int st = 0;
    for (int i = 0; i < sizeof(B) / sizeof(*B); ++i)
        st = ch[st][B[i]];
    printf("%d\n", st);
    return 0;
}

```

12.13 hash

```

const int maxn = 210000;
const unsigned long long x = 123;
unsigned long long H[maxn], xp[maxn];
char s[maxn];
int n;
void init()
{
    n = strlen(s);
    H[n] = 0;
    for (int i = n - 1; i >= 0; --i)
        H[i] = H[i + 1] * x + s[i] - 'a' + 1;
    xp[0] = 1;
    for (int i = 1; i <= n; ++i)
        xp[i] = xp[i - 1] * x;
}

unsigned long long Hash(int i, int L)
{
    return H[i] - H[i + L] * xp[L];
}

int main()
{

```

```

    strcpy(s, "abcdabcfg");
    init();
    cout << Hash(0, 3) << endl;
    cout << Hash(4, 3) << endl;
    cout << Hash(1, 3) << endl;
    return 0;
}

```

12.14 LCT 维护隐式后缀树

```

/*
在每次调用 extend 函数构建后缀自动机的时候, extend 函数会  $\log n$  次调用 solve 函数,
这些次调用的  $[L, R]$  构成了连续区间  $[1, index]$ 。solve 函数的参数意思是对于以下标 index 结尾的字符串,
长度区间从  $[L, R]$  的这些子串上一次出现的位置都是  $x$  ( $x == -1$  表示之前没有出现过)。
可以通过在 LCT 上多维护一个标记, 来维护上上次出现的位置。
对于询问母串中一个区间子串的问题, 可以考虑用这个模型来解决。每次 extend(w, index) 的时候,
处理所有下标以 index 结尾的询问, 并且动态维护左端点的询问值。
当前以 index 结尾的询问应该以 index - 1 的询问为基础, 通过 solve 函数更新产生。
solve 函数每次会对一个左端点区间的询问值进行更新, 可以考虑用线段树维护。
*/

const int maxn = 210000; //maxn 应当大于最大字符串长度的 2 倍
int par[maxn], len[maxn], go[maxn][26];
int ch[maxn][2], fa[maxn], stk[maxn], pos[maxn];
int cur, root, last;
long long ans = 0, base = 0;
void solve(int L, int R, int x, int index) {
    //当前算法对于一个字符串中的所有子串 求出了其本质不同的子串个数 的和。
    if (x == -1) {
        int t = R - L + 1;
        base += t * (t + 1) / 2;
    }
    else {
        base += (index - x) * (R - L + 1);
    }
}

inline bool son(int x) {
    return ch[fa[x]][1] == x;
}

inline bool isroot(int x) {
    return ch[fa[x]][1] != x && ch[fa[x]][0] != x;
}

inline void pushdown(int x) {
    pos[ch[x][0]] = pos[ch[x][1]] = pos[x];
}

void rotate(int x) {
    int y = fa[x], z = fa[y], c = son(x);
    if (!isroot(y))

```

```

        ch[z][son(y)] = x;
    fa[x] = z;
    ch[y][c] = ch[x][!c];
    fa[ch[y][c]] = y;
    ch[x][!c] = y;
    fa[y] = x;
}

void splay(int x) {
    int top = 0;
    stk[++top] = x;
    for (int i = x; !isroot(i); i = fa[i])
        stk[++top] = fa[i];
    while (top)
        pushdown(stk[top--]);
    for (int y = fa[x]; !isroot(x); rotate(x), y = fa[x]) if (!isroot(y))
        son(x) ^ son(y) ? rotate(x) : rotate(y);
}

void access(int nd, int index) {
    for (int y = nd, x = fa[nd]; y > 1; y = x, x = fa[x]) {
        if (x) {
            splay(x);
            ch[x][1] = y;
        }
        solve(len[x] + 1, len[y], pos[y], index);
    }
}

void cut(int x) { //断开结点 x 与它的父结点之间的边
    splay(x);
    fa[ch[x][0]] = fa[x];
    fa[x] = ch[x][0] = 0;
}

int newstate(int length) {
    len[cur] = length;
    return cur++;
}

void init() {
    memset(ch, 0, sizeof(ch));
    memset(fa, 0, sizeof(fa));
    memset(go, 0, sizeof(go));
    memset(pos, -1, sizeof(pos));
    cur = 1;
    root = last = newstate(0);
}

void extend(int w, int index) {
    int x = last;
    int nx = newstate(len[x] + 1);

```



```

while (x && !go[x][w])
    go[x][w] = nx, x = par[x];
if (!x)
    par[nx] = root, fa[nx] = root;
else {
    int y = go[x][w];
    if (len[x] + 1 == len[y])
        par[nx] = y, fa[nx] = y;
    else {
        int ny = newstate(len[x] + 1);
        memcpy(go[ny], go[y], sizeof(go[y]));
        fa[ny] = par[y], par[ny] = par[y];
        cut(y);
        pos[ny] = pos[y];
        fa[y] = ny, par[y] = ny;
        fa[nx] = ny, par[nx] = ny;
        while (x && go[x][w] == y)
            go[x][w] = ny, x = par[x];
    }
}
last = nx;
access(last, index);
splay(last);
pos[last] = index;
}

char A[maxn];
int main() {
    freopen("in.txt", "r", stdin);
    init();
    scanf("%s", A);
    int n = strlen(A);
    for (int i = 0; i < n; ++i) {
        extend(A[i] - 'A', i);
        ans += base;
    }
    printf("ans: %lld\n", ans);
    return 0;
}

```

12.15 区间本质不同子串个数

```

const int maxn = 210000; //maxn 应当大于最大字符串长度的 2 倍
int par[maxn], len[maxn], go[maxn][26];
int ch[maxn][2], fa[maxn], stk[maxn], pos[maxn];
int cur, root, last;
struct stnode {
    int l, r;
}

```

```

    int lc, rc;
    long long a, d;
};

struct segment {
    stnode tree[maxn * 4];
    int tc;
    int root;
    int init_tree(int l, int r) {
        int mid = (l + r) / 2;
        int pos = ++tc;
        tree[pos].l = l;
        tree[pos].r = r;
        tree[pos].a = tree[pos].d = 0;
        if (l != r) {
            tree[pos].lc = init_tree(l, mid);
            tree[pos].rc = init_tree(mid + 1, r);
        }
        return pos;
    }
    long long query_tree(int pos, int p) {
        int mid = (tree[pos].l + tree[pos].r) / 2;
        if (tree[pos].l == tree[pos].r) {
            return tree[pos].a;
        }
        else {
            if (p <= mid) {
                return query_tree(tree[pos].lc, p) + tree[pos].a + (p - tree[pos].l) *
↪ tree[pos].d;
            }
            else {
                return query_tree(tree[pos].rc, p) + tree[pos].a + (p - tree[pos].l) *
↪ tree[pos].d;
            }
        }
    }
    void update_tree(int pos, int l, int r, long long a, long long d) {
        int mid = (tree[pos].l + tree[pos].r) / 2;
        if (tree[pos].l == l and r == tree[pos].r) {
            tree[pos].a += a;
            tree[pos].d += d;
        }
        else {
            if (r <= mid) {
                update_tree(tree[pos].lc, l, r, a, d);
            }
            else if (l <= mid) {

```

```

        update_tree(tree[pos].lc, l, mid, a, d);
        update_tree(tree[pos].rc, mid + 1, r, a + (mid + 1 - l) * d, d);
    }
    else {
        update_tree(tree[pos].rc, l, r, a, d);
    }
}
}
long long query(int p) {
    return query_tree(root, p + 1);
}
void update(int l, int r, long long a, long long d) {
    if (l <= r)
        update_tree(root, l + 1, r + 1, a, d); //下标从 1 开始
}
void init(int x) {
    tc = 0;
    root = init_tree(1, x);
}
} tree;
void solve(int L, int R, int x, int index) {
    if (x == -1) {
        tree.update(index - R + 1, index - L + 1, R - L + 1, -1);
    }
    else {
        int length = min(R - L + 1, index - x);
        tree.update(x - R + 2, x - R + length, 1, 1);
        tree.update(index - L - length + 3, index - L + 1, length - 1, -1);
        tree.update(x - R + length + 1, index - L - length + 2, length, 0);
    }
}
inline bool son(int x) {
    return ch[fa[x]][1] == x;
}
inline bool isroot(int x) {
    return ch[fa[x]][1] != x && ch[fa[x]][0] != x;
}
inline void pushdown(int x) {
    pos[ch[x][0]] = pos[ch[x][1]] = pos[x];
}
void rotate(int x) {
    int y = fa[x], z = fa[y], c = son(x);
    if (!isroot(y))
        ch[z][son(y)] = x;
    fa[x] = z;
    ch[y][c] = ch[x][!c];
}

```

```

    fa[ch[y][c]] = y;
    ch[x][!c] = y;
    fa[y] = x;
}

void splay(int x) {
    int top = 0;
    stk[++top] = x;
    for (int i = x; !isroot(i); i = fa[i])
        stk[++top] = fa[i];
    while (top)
        pushdown(stk[top--]);
    for (int y = fa[x]; !isroot(x); rotate(x), y = fa[x]) if (!isroot(y))
        son(x) ^ son(y) ? rotate(x) : rotate(y);
}

void access(int nd, int index) {
    for (int y = nd, x = fa[nd]; y > 1; y = x, x = fa[x]) {
        if (x) {
            splay(x);
            ch[x][1] = y;
        }
        solve(len[x] + 1, len[y], pos[y], index);
    }
}

void cut(int x) { //断开结点 x 与它的父结点之间的边
    splay(x);
    fa[ch[x][0]] = fa[x];
    fa[x] = ch[x][0] = 0;
}

int newstate(int length) {
    len[cur] = length;
    return cur++;
}

void init() {
    memset(par, 0, sizeof(par));
    memset(ch, 0, sizeof(ch));
    memset(fa, 0, sizeof(fa));
    memset(go, 0, sizeof(go));
    memset(pos, -1, sizeof(pos));
    cur = 1;
    root = last = newstate(0);
}

void extend(int w, int index) {
    int x = last;
    int nx = newstate(len[x] + 1);
    while (x && !go[x][w])
        go[x][w] = nx, x = par[x];
}

```

```

if (!x)
    par[nx] = root, fa[nx] = root;
else {
    int y = go[x][w];
    if (len[x] + 1 == len[y])
        par[nx] = y, fa[nx] = y;
    else {
        int ny = newstate(len[x] + 1);
        memcpy(go[ny], go[y], sizeof(go[y]));
        fa[ny] = par[y], par[ny] = par[y];
        cut(y);
        pos[ny] = pos[y];
        fa[y] = ny, par[y] = ny;
        fa[nx] = ny, par[nx] = ny;
        while (x && go[x][w] == y)
            go[x][w] = ny, x = par[x];
    }
}
last = nx;
access(last, index);
splay(last);
pos[last] = index;
}

char s[maxn];
vector<pair<int, int>> Q[maxn]; //多组数据时要清空
long long answer[maxn];
int main() {
    //freopen("in.txt", "r", stdin);
    scanf("%s", s);
    int n = strlen(s);
    init();
    tree.init(n + 10);
    int m;
    scanf("%d", &m);
    for (int i = 0; i < m; ++i) {
        int L, R;
        scanf("%d %d", &L, &R);
        --L; --R;
        Q[R].emplace_back(L, i);
    }
    for (int i = 0; i < n; ++i) {
        extend(s[i] - 'a', i);
        for (auto [L, id] : Q[i])
            answer[id] = tree.query(L);
    }
    for (int i = 0; i < m; ++i)

```

```

        printf("%lld\n", answer[i]);
    return 0;
}

```

12.16 Lyndon 分解

//Lyndon 分解将一个字符串分解为若干个连续子串 S_1, S_2, \dots, S_n , 并且满足

// $S_1 \geq S_2 \geq \dots \geq S_n$, 并且所有这些子串都满足最小后缀是其本身。

//该函数返回 Lyndon 分解成的所有子串的起始下标。

```

vector<int> Lyndon(const char* s) {
    int n = strlen(s);
    vector<int> res;
    for (int i = 0; i < n; ) {
        int j = i, k = i + 1;
        while (k < n && s[j] <= s[k])
            j = (s[j] == s[k++]) ? j + 1 : i;
        while (i <= j) {
            res.push_back(i);
            i += k - j;
        }
    }
    return res;
}

const int maxn = 1110000;
int pos[maxn]; //pos[i] 表示前缀 s[0, i] 的最小后缀的下标
void preprocess(const char* s) {
    int n = strlen(s);
    for (int i = 0; i < n; ) {
        int j = i, k = i + 1;
        pos[i] = i;
        while (k < n && s[j] <= s[k]) {
            if (s[j] < s[k]) {
                pos[k] = j = i;
            }
            else {
                pos[k] = pos[j] + k - j;
                j += 1;
            }
            k += 1;
        }
        while (i <= j)
            i += k - j;
    }
}

char s[maxn];
int main() { //hdu6761
    const int mod = 1e9 + 7;

```

```

int T;
scanf("%d", &T);
while (T--) {
    scanf("%s", s);
    preprocess(s);
    int n = strlen(s);
    long long ans = 0, val = 1;
    for (int i = 0; i < n; ++i) {
        ans = (ans + (pos[i] + 1) * val) % mod;
        val = val * 1112 % mod;
    }
    printf("%lld\n", ans);
}
return 0;
}

```

12.17 后缀平衡树

```

const int maxn = 1110000;
const double alpha = 0.6;
struct SuffixBalancedTree {
    string data;
    double val[maxn], left, right;
    int sz[maxn], ch[maxn][2], root;
    int* pos, length, A[maxn];
    void init() {
        data.resize(1);
        root = 0;
    }
    void pushup(int x) {
        sz[x] = sz[ch[x][0]] + sz[ch[x][1]] + 1;
    }
    void dfs(int x) {
        if (!x) return;
        dfs(ch[x][0]);
        A[++length] = x;
        dfs(ch[x][1]);
    }
    int build(int a, int b, double L, double R) {
        if (a > b) return 0;
        int mid = (a + b) >> 1;
        int x = A[mid];
        sz[x] = b - a + 1;
        val[x] = (L + R) / 2;
        ch[x][0] = build(a, mid - 1, L, val[x]);
        ch[x][1] = build(mid + 1, b, val[x], R);
        return x;
    }
}

```

```

}

void add(int& x, double L, double R) {
    if (!x) {
        x = data.size() - 1;
        ch[x][0] = ch[x][1] = 0;
        val[x] = (L + R) / 2;
        sz[x] = 1;
        return;
    }
    int y = data.size() - 1;
    if (data[y] < data[x] || data[y] == data[x] && val[y - 1] < val[x - 1])
        add(ch[x][0], L, val[x]);
    else
        add(ch[x][1], val[x], R);
    pushup(x);
    if (sz[ch[x][0]] > sz[x] * alpha || sz[ch[x][1]] > sz[x] * alpha)
        pos = &x, left = L, right = R;
}

void push_front(char c) { //向开头添加一个新的字符之后会改变其他位置的下标
    data.push_back(c);
    pos = nullptr;
    add(root, 0, 1);
    if (pos) {
        length = 0;
        dfs(*pos);
        *pos = build(1, length, left, right);
    }
}

int merge(int x, int y) {
    if (!x || !y)
        return x | y;
    if (sz[x] > sz[y]) {
        ch[x][1] = merge(ch[x][1], y);
        pushup(x);
        return x;
    }
    else {
        ch[y][0] = merge(x, ch[y][0]);
        pushup(y);
        return y;
    }
}

void del(int& x) {
    const int y = data.size() - 1;
    sz[x] -= 1;
    if (x == y)

```



```

        x = merge(ch[x][0], ch[x][1]);
    else if (val[y] < val[x])
        del(ch[x][0]);
    else
        del(ch[x][1]);
}

void pop_front() {
    del(root);
    data.pop_back();
}

double weight(int index) { //返回下标 index 的权值, 权值越小, 字典序越小 (下标从 1 开始)
    return val[data.size() - index];
}

int rank(int index) { //返回下标 index 的排名 (下标从 1 开始)
    double key = weight(index);
    int ret = 0, x = root;
    while (x) {
        if (key < val[x])
            x = ch[x][0];
        else
            ret += sz[ch[x][0]] + 1, x = ch[x][1];
    }
    return ret;
}

int rank(const char *s) { //返回字典序小于 s 的后缀的个数
    int ret = 0, x = root, n = strlen(s);
    while (x) {
        int L = min(x, n) + 1;
        int flag = 0;
        for (int i = 0; i < L; ++i) {
            if (s[i] != data[x - i]) {
                flag = s[i] - data[x - i];
                break;
            }
        }
        if (flag <= 0)
            x = ch[x][0];
        else
            ret += sz[ch[x][0]] + 1, x = ch[x][1];
    }
    return ret;
}

}tree;

int main() {
    return 0;
}

```

12.18 模糊匹配

```

/*
init(s) 用 s 作为主串来初始化
query(t) 查询 t 在 s 中出现的次数（只要不同字符的个数不超过一个就算匹配）
*/
const int maxn = 410000;
int n, sz, root[maxn];
pair<int, int> range[maxn];
struct SuffixArray {
    int sa[maxn], rank[maxn];
    int t[maxn], t2[maxn], c[maxn];
    char s[maxn];
    void build_sa(const char *str, int m = 256) {
        strcpy(s, str);
        memset(t, 0, sizeof(int) * (2 * n + 10));
        memset(t2, 0, sizeof(int) * (2 * n + 10));
        int* x = t, * y = t2;
        for (int i = 0; i < m; ++i) c[i] = 0;
        for (int i = 0; i < n; ++i) c[x[i] = s[i]]++;
        for (int i = 1; i < m; ++i) c[i] += c[i - 1];
        for (int i = n - 1; i >= 0; --i) sa[--c[x[i]]] = i;
        for (int k = 1; k <= n; k <= 1) {
            int p = 0;
            for (int i = n - 1; i >= n - k; --i) y[p++] = i;
            for (int i = 0; i < n; ++i) if (sa[i] >= k) y[p++] = sa[i] - k;
            for (int i = 0; i < m; ++i) c[i] = 0;
            for (int i = 0; i < n; ++i) c[x[y[i]]]++;
            for (int i = 1; i < m; ++i) c[i] += c[i - 1];
            for (int i = n - 1; i >= 0; --i) sa[--c[x[y[i]]]] = y[i];
            swap(x, y);
            p = 1; x[sa[0]] = 0;
            for (int i = 1; i < n; ++i)
                x[sa[i]] = y[sa[i - 1]] == y[sa[i]] && y[sa[i - 1] + k] == y[sa[i] + k] ? p - 1 :
↪ p++;
            if (p >= n) break;
            m = p;
        }
    }
}

void calc() { //将空串放在 sa 数组的最前面, 并计算 rank 数组
    for (int i = n; i >= 1; --i)
        sa[i] = sa[i - 1];
    sa[0] = n;
    for (int i = 0; i <= n; ++i)
        rank[sa[i]] = i;
}

pair<int, int> advance(pair<int, int> range, char c) {

```

```

//设区间  $[x, y]$  表示  $sa$  中的一个区间, 返回将  $[x, y]$  中的每个后缀都往前走一个字符  $c$  得到的区
↪ 间
auto [x, y] = range;
int L = 1, R = n;
while (L < R) {
    int M = L + (R - L) / 2;
    if (s[sa[M]] < c || s[sa[M]] == c && rank[sa[M] + 1] < x)
        L = M + 1;
    else
        R = M;
}
int tmp = L;
R = n;
while (L < R) {
    int M = L + (R - L + 1) / 2;
    if (s[sa[M]] > c || s[sa[M]] == c && rank[sa[M] + 1] > y)
        R = M - 1;
    else
        L = M;
}
if (s[sa[L]] != c || rank[sa[L] + 1] < x || rank[sa[L] + 1] > y)
    return { 0, -1 }; //空区间
return { tmp, L };
}
}pre, suf;
struct node {
    int l, r, v;
} T[maxn * 25];
void ins(int& i, int l, int r, int p) {
    int m = (l + r) >> 1;
    T[++sz] = T[i]; i = sz;
    T[i].v++;
    if (l == r) return;
    if (p <= m) ins(T[i].l, l, m, p);
    else ins(T[i].r, m + 1, r, p);
}
int ask(int x, int y, int v) {
    int l = 0, r = n, k = 0;
    x = root[x - 1], y = root[y];
    int p = v - 1;
    if (p < 0) return 0;
    while (l < r) {
        int m = (l + r) >> 1, t = T[T[y].l].v - T[T[x].l].v;
        if (p <= m)
            x = T[x].l, y = T[y].l, r = m;
        else

```

```

        x = T[x].r, y = T[y].r, l = m + 1, k += t;
    }
    k += T[y].v - T[x].v;
    return k;
}

void init(string s) {
    ::sz = 0; ::n = s.size();
    pre.build_sa(s.c_str());
    pre.calc();
    reverse(s.begin(), s.end());
    suf.build_sa(s.c_str());
    suf.calc();
    int tree = 0;
    for (int i = 0; i <= n; ++i) {
        int j = n - suf.sa[i] - 1;
        if (j + 2 <= n)
            ins(tree, 0, n, pre.rank[j + 2]);
        root[i + 1] = tree;
    }
}

long long query(string t) {
    long long ans = 0;
    range[t.size()] = { 0, n };
    for (int i = t.size() - 1; i >= 0; --i) {
        if (range[i + 1].first > range[i + 1].second)
            range[i] = range[i + 1];
        else
            range[i] = pre.advance(range[i + 1], t[i]);
    }
    pair<int, int> now(0, n);
    for (int i = 0; i < t.size() && now.first <= now.second; ++i) {
        if (range[i + 1].first <= range[i + 1].second) {
            ans += ask(now.first + 1, now.second + 1, range[i + 1].second + 1);
            ans -= ask(now.first + 1, now.second + 1, range[i + 1].first);
        }
        now = suf.advance(now, t[i]);
    }
    if (now.first <= now.second)
        ans -= (now.second - now.first + 1) * (t.size() - 1);
    return ans;
}

int main() {
    //freopen("in.txt", "r", stdin);
    int T, Q;
    cin >> T;
    while (T--) {

```

```

    string s, t;
    cin >> s;
    init(s);
    cin >> Q;
    while (Q--) {
        cin >> t;
        printf("%lld\n", query(t));
    }
}
return 0;
}

```

12.19 基于后缀自动机构建后缀树

```

const int maxn = 2010000;
int par[maxn], len[maxn], cur, root, last; //par[x] 是后缀树上 x 的父结点
int ch[maxn][26]; //ch[x][w] 表示结点 x 在后缀树中沿着字符 w 走到达的结点。
bool issuf[maxn]; //issuf[x] 记录结点 x 是否表示一个后缀，若是则 pos[x] 就是后缀的下标。
int pos[maxn]; //pos[x] 表示结点 x 对应的字符串在原串中的起始下标。
int node[maxn]; //node[i] 表示下标 i 开始的后缀在树中的结点编号。
int newstate(int length, int index) {
    len[cur] = length;
    pos[cur] = index;
    return cur++;
}
void init() {
    memset(ch, 0, sizeof(ch));
    cur = 1;
    root = last = newstate(0, -1);
}
void extend(int w, int index) {
    int x = last;
    int nx = newstate(len[x] + 1, index);
    node[index] = nx;
    issuf[nx] = true;
    while (x && !ch[x][w])
        ch[x][w] = nx, x = par[x];
    if (!x)
        par[nx] = root;
    else {
        int y = ch[x][w];
        if (len[x] + 1 == len[y])
            par[nx] = y;
        else {
            int ny = newstate(len[x] + 1, pos[y]);
            memcpy(ch[ny], ch[y], sizeof(ch[y]));
            par[ny] = par[y];

```

```

        par[nx] = par[y] = ny;
        while (x && ch[x][w] == y)
            ch[x][w] = ny, x = par[x];
    }
}
last = nx;
}

void build(const char* s) {
    int n = strlen(s);
    for (int i = n - 1; i >= 0; --i)
        extend(s[i] - 'a', i);
    memset(ch, 0, sizeof(ch[0]) * (cur + 10));
    for (int i = 2; i < cur; ++i) {
        int w = s[pos[i] + len[par[i]]] - 'a';
        ch[par[i]][w] = i;
    }
}

const int maxlog = 23;
int anc[maxn][maxlog]; //anc[i][j] 表示结点 i 往上走 2^j 个点到达的点
void preprocess() { //后缀树上倍增初始化
    for (int i = 1; i < cur; ++i) {
        anc[i][0] = par[i];
        for (int j = 1; (1 << (j - 1)) < cur; ++j)
            anc[i][j] = 0;
    }
    for (int j = 1; (1 << j) < cur; ++j) {
        for (int i = 1; i < cur; ++i) {
            if (anc[i][j - 1] != 0) {
                int a = anc[i][j - 1];
                anc[i][j] = anc[a][j - 1];
            }
        }
    }
}

int query(int index, int length) { //从下标 index 的后缀往上走到 len[p] >= length 的深度最小的结点
    int p = node[index]; //若要识别字符串的子串 [L, R], 则调用 query(L, R-L+1)
    for (int i = maxlog - 1; i >= 0; --i)
        if (len[anc[p][i]] >= length)
            p = anc[p][i];
    return p;
}

int main() {
    return 0;
}

```

13 人工智能

13.1 主成分分析

```

const int maxn = 210000;
const int maxdim = 1001;
const double eps = 1e-8;
using matrix = double[maxdim][maxdim];
using vec = array<double, maxdim>;
using pair_t = pair<double, vec>;
struct PCA {
    matrix A, V;
    int column[maxdim], n;
    void update(int r, int c, double v) {
        A[r][c] = v;
        if (column[r] == c || fabs(A[r][c]) > fabs(A[r][column[r]])) {
            for (int i = 0; i < n; ++i) if (i != r)
                if (fabs(A[r][i]) > fabs(A[r][column[r]]))
                    column[r] = i;
        }
    }
    void Jacobi() {
        for (int i = 0; i < n; ++i) {
            for (int j = 0; j < n; ++j)
                V[i][j] = 0;
            V[i][i] = 1;
            column[i] = (i == 0 ? 1 : 0);
        }
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                if (j != i && fabs(A[i][j]) > fabs(A[i][column[i]]))
                    column[i] = j;
        for (int T = 0; ; ++T) { //迭代次数限制
            int x, y;
            double val = 0;
            for (int i = 0; i < n; ++i)
                if (fabs(A[i][column[i]]) > val)
                    val = fabs(A[i][column[i]]), x = i, y = column[i];
            if (val < eps) //精度限制
                break;
            double phi = atan2(-2 * A[x][y], A[y][y] - A[x][x]) / 2;
            double sinp = sin(phi), cosp = cos(phi);
            for (int i = 0; i < n; ++i) if (i != x && i != y) {
                double a = A[x][i] * cosp + A[y][i] * sinp;
                double b = A[x][i] * -sinp + A[y][i] * cosp;
                update(x, i, a);
                update(y, i, b);
            }
        }
    }
};

```

```

    }
    for (int i = 0; i < n; ++i) if (i != x && i != y) {
        double a = A[i][x] * cosp + A[i][y] * sinp;
        double b = A[i][x] * -sinp + A[i][y] * cosp;
        update(i, x, a);
        update(i, y, b);
    }
    for (int i = 0; i < n; ++i) {
        double a = V[i][x] * cosp + V[i][y] * sinp;
        double b = V[i][x] * -sinp + V[i][y] * cosp;
        V[i][x] = a, V[i][y] = b;
    }
    double a = A[x][x] * cosp * cosp + A[y][y] * sinp * sinp + 2 * A[x][y] * cosp * sinp;
    double b = A[x][x] * sinp * sinp + A[y][y] * cosp * cosp - 2 * A[x][y] * cosp * sinp;
    double tmp = (A[y][y] - A[x][x]) * sin(2 * phi) / 2 + A[x][y] * cos(2 * phi);
    update(x, y, tmp);
    update(y, x, tmp);
    A[x][x] = a, A[y][y] = b;
}
}

//a 为输入向量组
//n 为向量的维数
//center 指针用来保存输入向量组的中心点
//返回特征值和特征向量的 pair, 按照特征值从大到小排序
//特征值是各个点在对应特征向量方向的坐标平方和, 除以 (a.size() - 1) 为方差。
auto solve(vector<vec> a, int n, vec* center = nullptr) {
    this->n = n;
    vec s = {};
    for (int i = 0; i < a.size(); ++i)
        for (int j = 0; j < n; ++j)
            s[j] += a[i][j];
    for (int j = 0; j < n; ++j)
        s[j] /= a.size();
    for (int i = 0; i < a.size(); ++i)
        for (int j = 0; j < n; ++j)
            a[i][j] -= s[j];
    if (center) *center = s;
    for (int i = 0; i < n; ++i) {
        for (int j = 0; j < n; ++j) {
            A[i][j] = 0;
            for (int k = 0; k < a.size(); ++k)
                A[i][j] += a[k][i] * a[k][j];
        }
    }
}

Jacobi();
vector<pair_t> result;

```



```

        for (int i = 0; i < n; ++i)
            result.emplace_back(A[i][i], vec());
        for (int i = 0; i < n; ++i)
            for (int j = 0; j < n; ++j)
                result[i].second[j] = V[j][i];
        sort(result.begin(), result.end(), greater<pair_t>());
        return result;
    }
}pca;
int main() { //1329070654.526
    freopen("in.txt", "r", stdin);
    vector<vec> a;
    int n, m;
    scanf("%d %d", &n, &m);
    for (int i = 0; i < n; ++i) {
        vec v = {};
        for (int j = 0; j < m; ++j)
            scanf("%lf", &v[j]);
        a.push_back(v);
    }
    auto now = clock();
    auto result = pca.solve(a, m);
    for (int i = 0; i < m; ++i)
        printf("%.3f\n", result[i].first);
    printf("time: %f\n", double(clock() - now) / CLOCKS_PER_SEC);
    return 0;
}

```

13.2 Adam 算法

```

const int dim = 2;
using vec = array<double, dim>;
/*
* grad: 计算梯度的函数
* pos: 搜索的起始点
* lr: 学习率 (移动的步长)
* decay: 学习率的衰减指数
* exit: 当学习率小于 exit 时算法结束
* 注意该算法求出的是 函数的极 ** 小 ** 值点
*/
vec Adam(function<vec(vec)> grad, vec pos, double lr, double decay, double exit,
    const double beta1 = 0.9, const double beta2 = 0.9) {
    double pw1 = 1, pw2 = 1;
    vec m = {}, v = {};
    for (int t = 1; lr >= exit; ++t) {
        vec g = grad(pos);
        pw1 *= beta1;

```

```

    pw2 *= beta2;
    for (int i = 0; i < dim; ++i) {
        m[i] = m[i] * beta1 + g[i] * (1 - beta1);
        v[i] = v[i] * beta2 + g[i] * g[i] * (1 - beta2);
        double update = m[i] / (1 - pw1);
        double factor = lr / (sqrt(v[i] / (1 - pw2)) + 1e-8);
        pos[i] -= update * factor;
    }
    lr *= decay;
}
return pos;
}

int main() {
    auto grad = [] (vec A) {
        double x = A[0], y = A[1];
        return vec{ 2 * y + -4 / (x * x), 2 * x + -4 / (y * y) };
    };
    auto ans = Adam(grad, vec{ 1, 1 }, 0.01, 0.999, 1e-8);
    printf("%.10f %.10f\n", ans[0], ans[1]); //1.2599210498948732
    return 0;
}

```

13.3 Dyna-Q

```

const int nrow = 10, ncol = 10, max_state = nrow * ncol, max_action = 4;
const int dr[4] = { -1, 1, 0, 0 };
const int dc[4] = { 0, 0, -1, 1 };
struct CliffWalkingEnv {
    char s[11][11];
    int x, y;
    vector<pair<int, int>> path;
    CliffWalkingEnv() {
        reset();
    }
    int reset() {
        x = 0, y = 0;
        path.clear();
        path.emplace_back(x, y);
        memset(s, 0, sizeof(s));
        for (int i = 0; i < nrow; ++i) {
            for (int j = 0; j < ncol; ++j) {
                s[i][j] = '.';
            }
        }
        for (int i = 0; i < 5; ++i) {
            for (int j = 1; j < ncol - 1; ++j) {
                s[i][j] = '#';
            }
        }
    }

```

```

    }
}
return x * ncol + y;
}
void print() {
    char t[11][11];
    memcpy(t, s, sizeof(s));
    for (auto [x, y] : path) {
        if (t[x][y] >= '1' && t[x][y] < '9')
            t[x][y] += 1;
        else
            t[x][y] = '1';
    }
    for (int i = 0; i < nrow; ++i)
        printf("%s\n", t[i]);
    printf("\n");
}
tuple<int, int, int> step(int number) {
    int nx = x + dr[number], ny = y + dc[number];
    if (nx >= 0 && nx < nrow && ny >= 0 && ny < ncol)
        x = nx, y = ny;
    int reward = -1, done = false;
    if (s[x][y] == '#') {
        reward = -100;
        done = true;
    }
    if (x == 0 && y == ncol - 1)
        done = true;
    path.emplace_back(x, y);
    return make_tuple(x * ncol + y, reward, done);
}
} env;
const double alpha = 1e-1; /* 学习率 */
const double gamma = 0.9; /* 折扣因子 */
const double epsilon = 1e-2; /*epsilon-greedy*/
const int N = 10; //Q-Planning 的次数
const int max_episode = 200; //训练多少轮
uniform_real_distribution<double> p;
uniform_int_distribution<int> d;
default_random_engine e;
double Q[max_state][max_action];
map<pair<int, int>, int> id;
vector<tuple<int, int, int, int>> model;
int epsilon_greedy(int state) { /* 基于 epsilon-greedy 选择动作 */
    if (p(e) < epsilon)
        return d(e) % max_action;
}

```

```

    return max_element(Q[state], Q[state] + max_action) - Q[state];
}

void learn(int s0, int a0, int r, int s1) {
    auto td_error = r + gamma * *max_element(Q[s1], Q[s1] + max_action) - Q[s0][a0];
    Q[s0][a0] += alpha * td_error;
}

void update(int s0, int a0, int r, int s1) {
    learn(s0, a0, r, s1);
    pair<int, int> pr(s0, a0);
    if (!id.count(pr)) {
        id[pr] = model.size();
        model.emplace_back(s0, a0, r, s1);
    }
    else {
        model[id[pr]] = make_tuple(s0, a0, r, s1);
    }
    for (int i = 0; i < N; ++i) {
        int idx = d(e) % model.size();
        auto [s0, a0, r, s1] = model[idx];
        learn(s0, a0, r, s1);
    }
}

void DynaQ() {
    for (int i = 0; i < max_episode; ++i) {
        int state = env.reset();
        for (;;) {
            int action = epsilon_greedy(state);
            auto [next_state, reward, done] = env.step(action);
            update(state, action, reward, next_state);
            state = next_state;
            if (done)
                break;
        }
        env.print();
    }
}

int main() {
    DynaQ();
    return 0;
}

```

13.4 kmeans 聚类

```

#pragma GCC optimize("Ofast")
#pragma GCC target("avx512bw,avx512vl")
#pragma GCC optimization("unroll-loops")
/*

```

vec: 待聚类的向量组

n: 向量个数

dimension: 向量维度

belong: 向量属于哪一个簇

center: 算法计算出的每个簇的中心向量

k: 簇的个数

iterations: 迭代次数

**/*

```
template<typename T, int MAXN, int MAXM, int MAXD>
inline void kmeans(const T(&vec)[MAXN][MAXD], int n, int dimension, int(&belong)[MAXN],
↪ T(&center)[MAXM][MAXD], int k, int iterations) {
    mt19937 engine;
    vector<int> indices(n), choices;
    iota(indices.begin(), indices.end(), 0);
    sample(indices.begin(), indices.end(), back_inserter(choices), k, engine);
    for (int i = 0; i < k; ++i) {
        copy(vec[choices[i]], vec[choices[i]] + dimension, center[i]);
    }
    for (int t = 0; t < iterations; ++t) { //迭代 iterations 轮
        for (int i = 0; i < n; ++i) {
            T best = numeric_limits<T>::max();
            for (int j = 0; j < k; ++j) {
                T distance = 0;
                for (int d = 0; d < dimension; ++d) {
                    distance += (vec[i][d] - center[j][d]) * (vec[i][d] - center[j][d]);
                }
                if (distance < best) {
                    best = distance;
                    belong[i] = j;
                }
            }
        }
        if (t + 1 != iterations) {
            int sz[MAXM] = {};
            for (int j = 0; j < k; ++j) {
                for (int d = 0; d < dimension; ++d) {
                    center[j][d] = 0;
                }
            }
            for (int i = 0; i < n; ++i) {
                sz[belong[i]] += 1;
                for (int d = 0; d < dimension; ++d) {
                    center[belong[i]][d] += vec[i][d];
                }
            }
            for (int j = 0; j < k; ++j) {
```

```

        for (int d = 0; d < dimension; ++d) {
            center[j][d] /= sz[j];
        }
    }
}

template<typename T, int MAXN, int MAXM, int MAXD>
inline void kmeans_plusplus(const T(&vec)[MAXN][MAXD], int n, int dimension, int(&belong)[MAXN],
    ↪ T(&center)[MAXM][MAXD], int k, int iterations) {
    mt19937 engine;
    vector<T> distance(n, numeric_limits<T>::max());
    vector<bool> mask(n);
    mask[0] = true;
    copy(vec[0], vec[0] + dimension, center[0]);
    for (int j = 1; j < k; ++j) {
        for (int i = 0; i < n; ++i) {
            T dist = 0;
            for (int d = 0; d < dimension; ++d) {
                dist += (vec[i][d] - center[j - 1][d]) * (vec[i][d] - center[j - 1][d]);
            }
            distance[i] = min(distance[i], dist);
        }
        vector<T> sum(n);
        for (int i = 1; i < n; ++i) {
            sum[i] = sum[i - 1] + (mask[i] ? 0.0 : distance[i]);
        }
        uniform_real_distribution<T> gen(0, sum[n - 1]);
        auto index = lower_bound(sum.begin(), sum.end(), gen(engine)) - sum.begin();
        mask[index] = true;
        copy(vec[index], vec[index] + dimension, center[j]);
    }
    for (int t = 0; t < iterations; ++t) { //迭代 iterations 轮
        for (int i = 0; i < n; ++i) {
            T best = numeric_limits<T>::max();
            for (int j = 0; j < k; ++j) {
                T distance = 0;
                for (int d = 0; d < dimension; ++d) {
                    distance += (vec[i][d] - center[j][d]) * (vec[i][d] - center[j][d]);
                }
                if (distance < best) {
                    best = distance;
                    belong[i] = j;
                }
            }
        }
    }
}

```

```

    if (t + 1 != iterations) {
        int sz[MAXM] = {};
        for (int j = 0; j < k; ++j) {
            for (int d = 0; d < dimension; ++d) {
                center[j][d] = 0;
            }
        }
        for (int i = 0; i < n; ++i) {
            sz[belong[i]] += 1;
            for (int d = 0; d < dimension; ++d) {
                center[belong[i]][d] += vec[i][d];
            }
        }
        for (int j = 0; j < k; ++j) {
            for (int d = 0; d < dimension; ++d) {
                center[j][d] /= sz[j];
            }
        }
    }
}

constexpr int MAXN = 50000, MAXD = 1024, MAXM = 100;
double vec[MAXN][MAXD], center[MAXM][MAXD];
int belong[MAXN];
int main() {
    mt19937 engine;
    normal_distribution<double> generate(0, 0.5), gen(0, 0.1);
    vector<array<double, MAXD>> cluster(MAXM);
    for (int i = 0; i < MAXM; ++i) {
        for (int j = 0; j < MAXD; ++j) {
            cluster[i][j] = generate(engine);
        }
    }
    for (int i = 0; i < MAXN; ++i) {
        int c = i % MAXM;
        for (int j = 0; j < MAXD; ++j) {
            vec[i][j] = cluster[c][j] + gen(engine);
        }
    }
    kmeans(vec, MAXN, MAXD, belong, center, MAXM, 10); // Error: 103956
    //kmeans_plusplus(vec, MAXN, MAXD, belong, center, MAXM, 10); // Error: 59772.6
    double error = 0;
    for (int i = 0; i < MAXN; ++i) {
        double dist = 0;
        for (int j = 0; j < MAXM; ++j) {
            dist += (vec[i][j] - center[belong[i]][j]) * (vec[i][j] - center[belong[i]][j]);
        }
    }
}

```

```
    }  
    error += sqrt(dist);  
}  
int sz[MAXM] = {};  
for (int i = 0; i < MAXN; ++i) {  
    sz[belong[i]] += 1;  
}  
for (int i = 0; i < MAXM; ++i) {  
    cout << sz[i] << " ";  
}  
cout << endl << "Error: " << error << endl;  
return 0;  
}
```