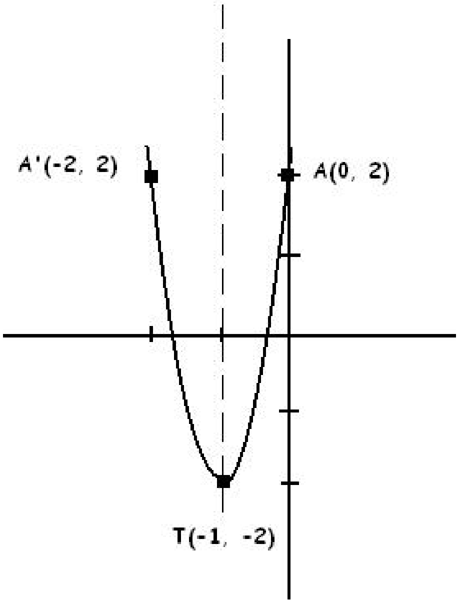
2. DZ

1.



1. za A(0, 2) $\Rightarrow \quad 2 \; = \; c \; \Rightarrow \quad c \; = \; 2$
2. za A'(-2, 2) $\Rightarrow 2 = 4a - 4b + 2 \Rightarrow 4a - 4b = 0$
3. za T(-1, -2) $\Rightarrow -2 = a - b + 2 \Rightarrow a - b = -4$

iz 2. i 3. dobivamo sustav jednadžbi:

$a - b = -4$

$4a - 4b = 0$

čije rješenje je a = 4 i b = 8.

Jednadžba parabole je: $y = 4x^2 + 8x + 2$

Nultočke:

$4x^2 + 8x + 2 = 0$

$$x_{1,2} = \frac{-8 \pm \sqrt{64 - 32}}{8} = \frac{-8 \pm 4\sqrt{2}}{8}$$

2. $f(x)=2\sin\left(\frac{\pi}{4}x+a\right)$

f-ja je parna: $f(x)=f(-x)$

$$f(x)-f(-x)=0$$

$$2\sin\left(\frac{\pi}{4}x+a\right)-2\sin\left(-\frac{\pi}{4}x+a\right)=0$$

$$\sin\left(\frac{\pi}{4}x+a\right)-\sin\left(-\frac{\pi}{4}x+a\right)=0$$

$$2\cos\frac{\frac{\pi}{4}x+a-\frac{\pi}{4}x+a}{2}\sin\frac{\frac{\pi}{4}x+a+\frac{\pi}{4}x-a}{2}=0$$

$$2\cos a\sin\frac{\pi}{4}x=0$$

Zanima nas dio sa a:

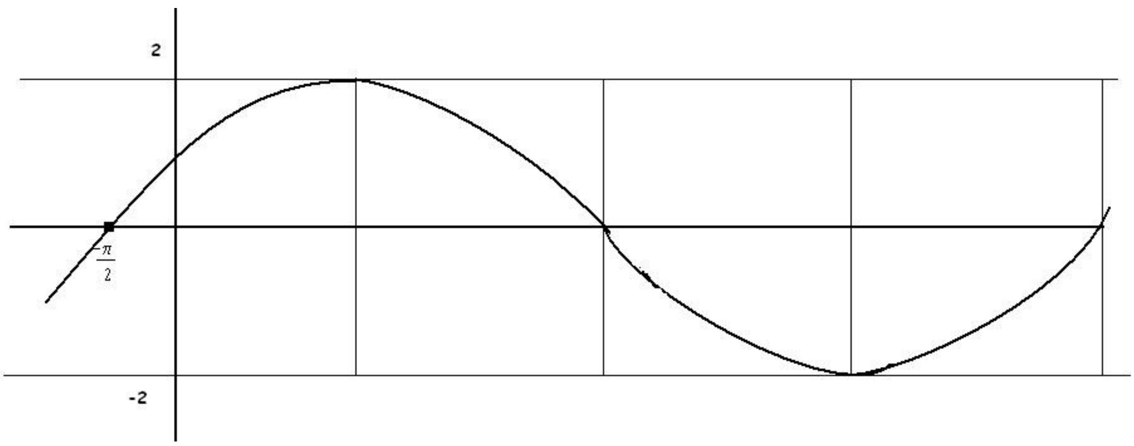
$$\cos a=0$$

$$a=\frac{\pi}{2}+k\pi$$

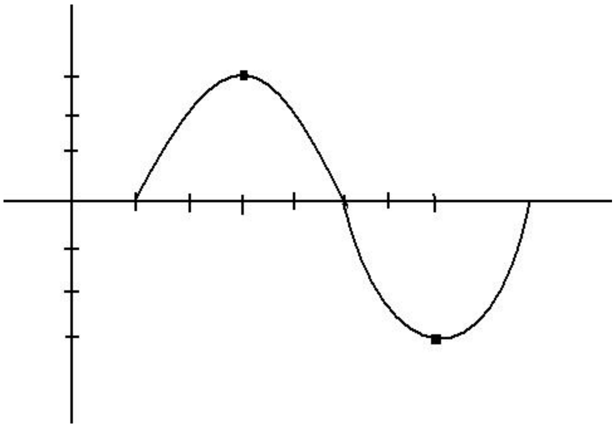
Iz uvjeta da je za x = 0, f (x) > 0, imamo da je $a=\frac{\pi}{2}$

$$y=2\sin\left(\frac{\pi}{4}x+\frac{\pi}{2}\right)$$

$$T=\frac{2\pi}{\omega}=\frac{2\pi}{\frac{\pi}{4}}=8$$



3.



$T = 8$

$$T = \frac{2\pi}{\omega} \Rightarrow \omega = \frac{2\pi}{8} = \frac{\pi}{4}$$

$$3 = 3\sin\left(\frac{\pi}{4}3 + \varphi\right)$$

$$\sin\left(\frac{3\pi}{4} + \varphi\right) = 1$$

$$\frac{3\pi}{4} + \varphi = \frac{\pi}{2} + k\pi$$

$$\varphi = -\frac{\pi}{4} + k\pi$$

Jednadžba: $y = 3\sin\left(\frac{\pi}{4}x - \frac{\pi}{4}\right)$

4.

$y = 2x + 1$

1) za 1 udesno $\Rightarrow y = 2(x-1)^2 + 1 = 2(x^2 - 2x + 1) + 1 = 2x^2 - 4x + 3$
2) za 2 prema dolje $\Rightarrow y + 2 = 2x^2 + 1 \Rightarrow y = 2x^2 - 1$
3) zrcalimo s obzirom na x-os $y = -2x^2 - 1$

5.

$y = \dfrac{1}{x}$

1) za 2 ulijevo $\Rightarrow y = \dfrac{1}{x+2}$
2) zatim za 1 prema gore $\Rightarrow y - 1 = \dfrac{1}{x+2} \Rightarrow y = \dfrac{1}{x+2} + 1$
3) i na kraju zrcalimo s obzirom na y-os $\Rightarrow y = \dfrac{1}{-x+2} + 1$

6.

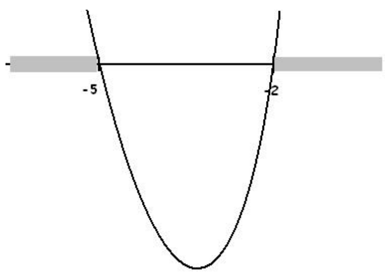
a) $f(x) = \sqrt{x^2 + 7x + 10}$

$x^2 + 7x + 10 \geq 0$

$x^2 + 7x + 10 = 0$

$x_{1,2} = -\dfrac{7}{2} \pm \sqrt{\dfrac{49}{4} - \dfrac{40}{4}} = -\dfrac{7}{2} \pm \dfrac{3}{2}$

$x_1 = -5$
$x_2 = -2$



$x \in (-\infty, -5] \cup [-2, \infty)$

b) $f(x)=\sqrt{x^3-3x^2-10x+24}$

$x^3-3x^2-10x+24\geq 0$

$x^3-3x^2-10x+24=0$
$x=\pm 1,\pm 2,\pm 3,\pm 4,\pm 6,\pm 8,\pm 12,\pm 24$

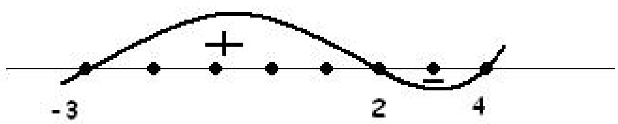
Rješenja su cjelobrojni višekratnici slobodnog člana

Pogodimo za $x_1 = 2$

$(x^3-3x^2-10x+24):(x-2)=x^2-x-12$

Odredimo rješenja dobivenog polinoma:

$x^2-x-12=0$
$x_{2,3}=\frac{1}{2}\pm\sqrt{\frac{1}{4}+12}$
$x_2=-3$
$x_3=4$



Rješenje: $x\in[-3,2]\cup[4,\infty)$

c) $f(x)=\sqrt{x^3+3x^2+3x+2}$

$x^3+3x^2+3x+2\geq 0$
$(x+1)^3+1\geq 0$

Rješenje: $x\in[0,\infty)$

7. a)

1. uvjet:

$x^2 - 3x \geq 0$

$x(x-3) \geq 0$

$x_1 = 0$

$x_2 = 3$

$x \in (-\infty, 0] \cup [3, \infty)$

2. uvjet:

$2 - \sqrt{x^2 - 3x} \neq 0$

$\sqrt{x^2 - 3x} \neq 2$

$x^2 - 3x \neq 4$

$x^2 - 3x - 4 \neq 0$

$x_1 = -1$

$x_2 = 4$

$x \neq \{-1, 4\}$

Iz 1. i 2. uvjeta: $x \in (-\infty, 0] \cup [3, \infty) / \{-1, 4\}$

b)

1. uvjet:

$\ln(x^2 - 1) \neq 0$

$x^2 - 1 \neq e^0$

$x^2 - 1 \neq 1$

$x^2 \neq 2$

$x \neq \pm\sqrt{2}$

2. uvjet:

$x^2 - 1 > 0$

$x \in (-\infty, -1) \cup (1, \infty)$

Rješenje: $x \in (-\infty, -1) \cup (1, \infty) / \{\pm\sqrt{2}\}$

c)

$$\frac{x-1}{x^2-x-6} \geq 0$$

1. uvjet:

1. slučaj:	2. slučaj:
$x-1 \geq 0$	$x-1 \leq 0$
$x^2-x-6>0$	$x^2-x-6<0$
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$x \geq 1$	$x \leq 1$
$x_1=-2$	$x_1=-2$
$x_2=3$	$x_2=3$
-------------------	--------------------
$x>3$	$x \in (-2,1]$

2. uvjet:

$x^2-x-6 \neq 0$

$x \neq \{-2,3\}$

Rješenje: $x \in (-2,1] \cup (3,\infty)$

8. a)

1. uvjet:

$$\ln\left(\frac{x^2-15}{x-9}\right)\geq 0$$

$$\frac{x^2-15}{x-9}\geq e^0$$

$$\frac{x^2-15}{x-9}\geq 1$$

$$\frac{x^2-x-6}{x-9}\geq 0$$

<u>1. slučaj:</u>

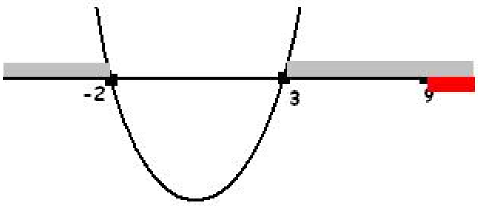
$x^2-x-6\geq 0$

$x-9>0$

$x_1=-2$

$x_2=3$

$x>9$



***x* > 9**

<u>2. slučaj:</u>

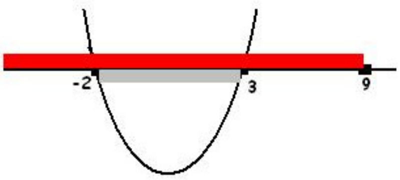
$x^2-x-6\leq 0$

$x-9<0$

$x_1=-2$

$x_2=3$

$x<9$



x $\in$ **[-2, 3]**

2. uvjet:

$$\frac{x^2-15}{x-9}>0$$

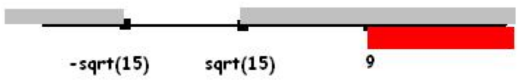
<u>1. slučaj:</u>

$x^2-15>0$

$x-9>0$

$x^2>15\Rightarrow\sqrt{15}<x<-\sqrt{15}$

$x>9$



***x* > 9**

<u>2. slučaj:</u>

$x^2-15<0$

$x-9<0$

$x^2<15\Rightarrow-\sqrt{15}<x<\sqrt{15}$

$x<9$

x $\in$ **(**$-\sqrt{15},\sqrt{15}$ **)**

Iz 1. uvjeta je: $x \in [-2,3] \cup (9,\infty)$

Iz 2. uvjeta je: $x \in \left(-\sqrt{15},\sqrt{15}\right) \cup (9,\infty)$

Rješenje: $x \in [-2,3] \cup (9,\infty)$

b) $\dfrac{\ln^2 x - 1}{\ln^2 x - 4} \geq 0$

1.uvjet:

1. slučaj:

$\ln^2 x - 1 \geq 0$

$\ln^2 x - 4 > 0$

$t^2 - 1 \geq 0$

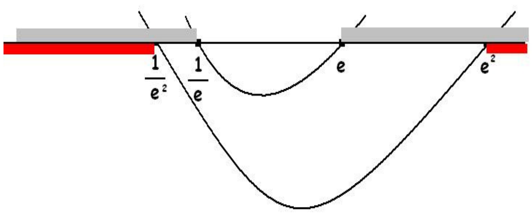
$u^2 - 4 > 0$

$t_1 = -1 \Rightarrow \ln x_1 = -1 \Rightarrow x_1 = e^{-1} = \dfrac{1}{e}$

$t_2 = 1 \Rightarrow \ln x_2 = 1 \Rightarrow x_2 = e$

$u_1 = -2 \Rightarrow \ln x_3 = -2 \Rightarrow x_3 = e^{-2} = \dfrac{1}{e^2}$

$u_2 = 2 \Rightarrow \ln x_4 = 2 \Rightarrow x_4 = e^2$



2. slučaj:

$\ln^2 x - 1 \leq 0$

$\ln^2 x - 4 < 0$

$t^2 - 1 \leq 0$

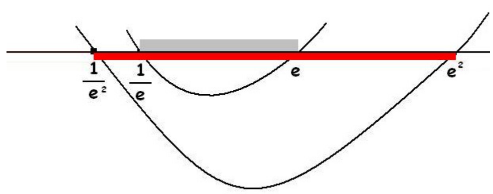
$u^2 - 4 < 0$

$t_1 = -1 \Rightarrow \ln x_1 = -1 \Rightarrow x_1 = e^{-1} = \dfrac{1}{e}$

$t_2 = 1 \Rightarrow \ln x_2 = 1 \Rightarrow x_2 = e$

$u_1 = -2 \Rightarrow \ln x_3 = -2 \Rightarrow x_3 = e^{-2} = \dfrac{1}{e^2}$

$u_2 = 2 \Rightarrow \ln x_4 = 2 \Rightarrow x_4 = e^2$



2. uvjet: $x > 0$

Rješenje: $x \in \left(0,\dfrac{1}{e^2}\right) \cup \left[\dfrac{1}{e},e\right] \cup \left(e^2,\infty\right)$