11 - 13 - INTEGRALNI RAČUN

11.1. Definicije određenog i neodređenog integrala

(Pr.) Tijelo se giba brzinom $v(t) = 3t^2$ $(s(0)=0)$.
Odredi put koje tijelo prevali u 10 s.

$$v = \frac{ds}{dt} = s'(t)$$

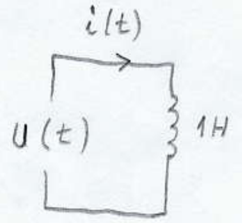
diferencijalna jednadžba $\rightarrow$ $3t^2 = s'(t)$ $\longrightarrow$ Što treba derivirati da bi se dobilo $3x^2$?

$$s(t) = t^3 + C$$

$$s(0) = 0^3 + C = 0 \implies C = 0$$

$$\underline{s(10) = 10^3 + 0 = 1000}$$

(Pr.) Odredi struju kroz zavojnicu induktiviteta $L = 1H$ priključenu na izvor $u(t) = \cos(2t)$.



$$u_L(t) = L \cdot \frac{di}{dt} = L \cdot i'(t)$$

$$i'(t) = \cos 2t$$

$$i(t) = \frac{1}{2}\sin 2t \quad \left[\left(\frac{1}{2}\sin 2t\right)' = \frac{1}{\cancel{2}} \cdot \cos 2t \cdot \cancel{2} = \cos 2t\right]$$

def. Funkciju $F(x)$ zovemo PRIMITIVNA FUNKCIJA OD $f(x)$ na $\langle a,b \rangle$ ako $\forall x \in \langle a,b \rangle : F'(x) = f(x)$

(Pr.) $f(x) = 3x^2$

$$\left.\begin{aligned} F(x) &= x^3 \\ F(x) &= x^3 + 5 \\ F(x) &= x^3 + \pi \end{aligned}\right\} F(x) = x^3 + C$$

TM Neka su F_1 i F_2 primitivne funkcije od f.
Tada se one razlikuju za konstantu, tj. ako je F_1 primitivna funkcija od f, tada je i $F_2 = F_1 + C$ primitivna funkcija od f.

DOKAZ: Direktno po korolaru Lagrangeovog teorema srednje vrijednosti
↳ 2) Ako je $f'(x) = g'(x)$, tada je $f(x) = g(x) + C$.

def. Skup svih primitivnih funkcija od $f(x)$ nazivamo
NEODREĐENI INTEGRAL OD $f(x)$.

$$\int \underbrace{f(x)}_{\text{podintegralna funkcija}} \underbrace{dx}_{\text{diferencijal}}$$

$$\int f(x)\,dx = \left\{ F(x) + C,\ C \in \mathbb{R} \right\}$$

npr. $\int 3x^2\,dx = x^3 + C$.

$$\int f(x)\,dx = F(x) \iff F'(x) = f(x)$$

na intervalu I

Pr.

a) $\int \frac{dx}{\cos^2 x} = \int \frac{1}{\cos^2 x}\,dx = \operatorname{tg} x + C, \quad I = \langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle$

b) $\int \frac{dx}{x} = \ln|x| + C, \quad I = \mathbb{R} \setminus \{0\}$

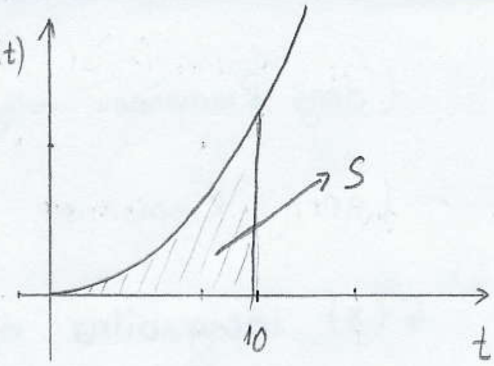
c) $\int x^{\alpha}\,dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \quad \alpha \in \mathbb{R} \setminus \{-1\} \rightarrow x^{-1} = \frac{1}{x} \quad \int \frac{dx}{x} = \ln|x| + C$

$\int \frac{\sin x}{x}\,dx, \quad \int e^{x^2}\,dx \longrightarrow$ neelementarni integrali
- nerješivi!

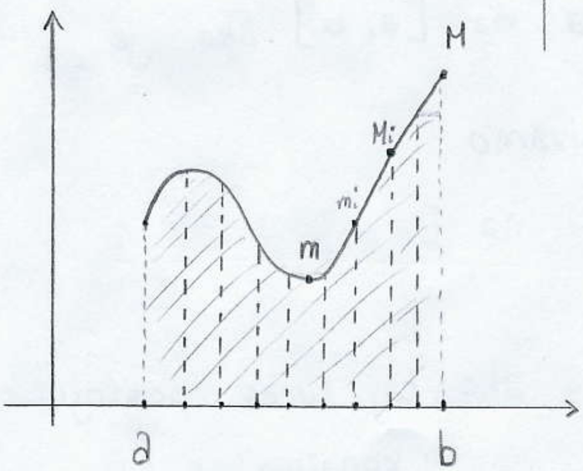
Pr. [prvi primjer]

$v(t) = 3t^2$

Površina pod krivuljom na vt grafu je put.



Pr.



$P = ?$

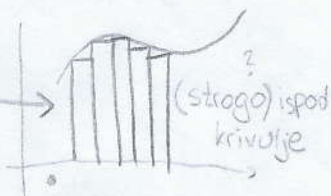
- napravili smo SUBDIVIZIJU intervala (Δ)

$$a = x_0 < x_1 < \dots < x_i < \dots < x_n = b$$

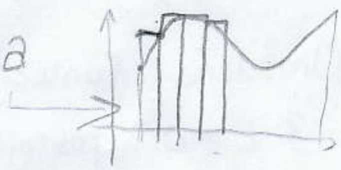
$$\Delta x_i = x_{i+1} - x_i$$

$$m_i = \inf f(x)$$
$$M_i = \sup f(x)$$

$s_\Delta = \sum_{i=1}^{n} m_i \Delta x_i$ — donja integralna suma



$S_\Delta = \sum_{i=1}^{n} M_i \Delta x_i$ — gornja integralna suma

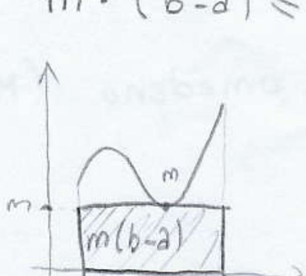


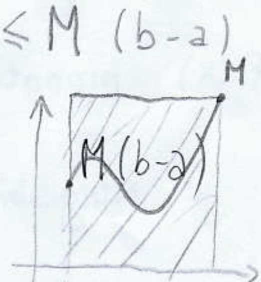
$$\sigma_\Delta = \sum_{i=1}^{n} f(\xi) \cdot \Delta x_i \quad , \xi \in \langle m_i, M_i \rangle \rightarrow \text{integralna suma}$$

$$s_\Delta \leq \sigma_\Delta \leq S_\Delta$$

očito je

$$m \cdot (b-a) \leq s_\Delta \leq \sigma_\Delta \leq S_\Delta \leq M\,(b-a)$$





neka je:

$$I_* = \sup_{\Delta} s_\Delta \quad \text{(donji Riemannov integral)}$$

$$I^* = \inf_{\Delta} S_\Delta \quad \text{(gornji Riemannov integral)}$$

<u>def.</u> Kažemo da je $f(x)$ integrabilna na $[a, b]$ ako je

$I_* = I^* = I$ i tada I nazivamo

ODREĐENI INTEGRAL od $f(x)$ na $[a,b]$

$$\int_a^b f(x)\,dx = \lim_{\substack{n \to \infty \\ \Delta x_i \to 0}} \sum_{i=1}^{n} f(\xi_i) \cdot \Delta x_i$$ – ako taj limes postoji i <u>konačan</u> je.

<u>Određeni i neodređeni integral nisu isti!</u>

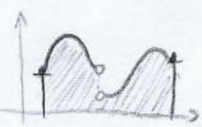
$$\int \frac{dx}{x} = \ln|x| + C \qquad \int_0^1 \frac{dx}{x} \text{ ne postoji!}$$

($\int \frac{dx}{x}$ ne postoji u nuli, ali $\int_0^1 \frac{dx}{x}$ uopće ne postoji)

<u>TM</u> Omeđena funkcija $f(x)$ na $[a,b]$ je integrabilna akko $\exists\, \varepsilon > 0$ postoji razdioba intervala Δ t.d $S_\Delta - s_\Delta < \varepsilon$.

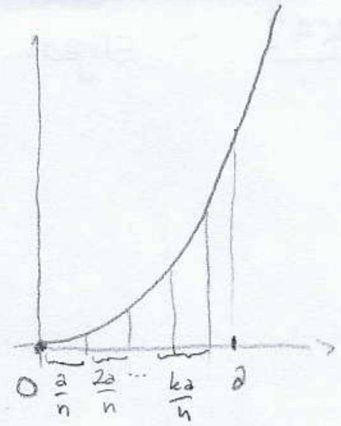
<u>TM</u> Ako je $f(x)$ neprekinuta na $[a,b]$ tada je integrabilna na $[a,b]$.

<u>TM</u> Ako je $f(x)$ omeđena na $[a,b]$ i ima konačno mnogo prekida, tada je integrabilna na $[a,b]$.



<u>TM</u> Ako je $f(x)$ monotona na $[a,b]$ i omeđena $(M;0)$, tada je integrabilna na $[a,b]$

ZAD Izračunaj $\int_0^a x^2\,dx$ po definiciji!



$\Delta x_i = \frac{a}{n}$

$$\int_0^a x^2\,dx = \sum_{k=1}^{n} f\left(\frac{ka}{n}\right)\cdot\frac{a}{n} =$$

$$= \lim_{n\to\infty}\sum_{k=1}^{n}\frac{k a^2}{n^2}\cdot\frac{a}{n} =$$

$$= a^3 \lim_{n\to\infty}\sum_{k=1}^{n}\frac{k^2}{n^3} = a^3\lim_{n\to\infty}\frac{\frac{1}{6}n(n+1)(2n+1)}{n^3} \sim \frac{\frac{2n^3}{6}}{\cancel{n^3}} = \frac{a^3}{3}$$

KOJA JE VEZA IZMEĐU ODREĐENOG I NEODREĐENOG INTEGRALA?

$$\int_0^a x^2\,dx = \frac{a^3}{3} \longleftrightarrow \int x^2\,dx = \frac{x^3}{3} + C$$

- očito je $\int_a^a f(x)\,dx = 0$

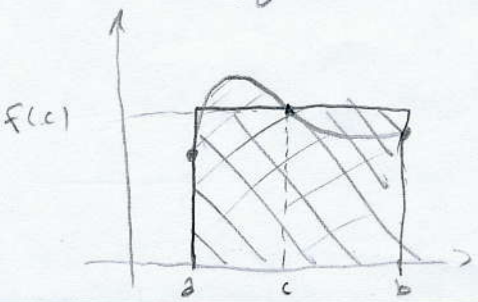
$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$$

TM - Teorem srednje vrijednosti integralnog računa

Neka je $f(x)$ neprekinuta na $[a,b]$. Tada postoji

$$c \in \langle a, b\rangle : \int_a^b f(x)\,dx = f(c)\cdot(b-a)$$



DOKAZ - Slijedi iz Riemannovih integrala

$$m\,(b-a) \leq \int_a^b f(x)dx \leq M\,(b-a) \qquad /:(b-a)>0$$

$$m \leq \frac{\int_a^b f(x)dx}{b-a} \leq M$$

- zbog neprekinutosti funkcije vrijedi

$$f(c) = \frac{\int_a^b f(x)dx}{b-a} \quad , \quad c \in [m, M]$$

TM Osnovni teorem diferencijalno - integralnog računa

Neka je $f(x)$ neprekinuta na $[a,b]$ i neka je $x \in [a,b]$.

Tada je $\phi(x) = \int_a^x f(t)dt$ diferencijabilna i vrijedi:

$$\phi'(x) = \frac{d}{dx}\int_a^x f(t)dt = f(x)$$

DOKAZ

$$\phi'(x) = \lim_{h\to 0} \frac{\phi(x+h)-\phi(x)}{h} = \lim_{h\to 0} \frac{\int_a^{x+h} f(t)dt - \int_a^x f(t)dt}{h} =$$

$$= \lim_{h\to 0} \frac{\int_x^{x+h} f(t)dt}{h} \underset{\exists c_h \in [x, x+h]}{\overset{TSVIR}{=}} \lim_{h\to 0} f(c_h) \overset{\text{zbog neprekinutosti limes ulazi u funkciju}}{=}$$

$$= f\left(\lim_{h\to 0} c_h\right) = \left| \begin{matrix} h=0 \\ c_h \in [x, x+0] \end{matrix} \right| = f(x)$$

TM - Newton - Leibnitzova formula

Neka je $f(x)$ neprekinuta na $[a, b]$ te neka je $F(x)$ neka njena primitivna funkcija.

Tada vrijedi :

$$\boxed{\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)}$$

DOKAZ - po osnovnom teoremu diferencijalno - integralnog računa

$\phi(x) = \int_a^b f(x)\,dx$ je neka primitivna funkcija od $f(x)$.

$F(x) = \phi(x) + C$ je neka druga primitivna funkcija.

Tada je $F(b) - F(a) = \phi(b) + C - \phi(a) - C = \phi(b) - \phi(a) =$

$$= \int_a^b f(x)\,dx - \int_a^a f(x)\,dx = \int_a^b f(x)\,dx .$$

11.2. Tehnike integriranja

A) Neposredno integriranje

Pr.

$$\int (2x^2 - 4e^x + 5)\,dx = \int 2x^2 dx - \int 4e^x dx + \int 5 dx =$$

$$= 2\int x^2 dx - 4\int e^x dx + 5\int dx =$$

$$= 2\cdot\frac{x^3}{3} - 4e^x + 5x + C \quad , C\in R$$

Zad.

$$\int \frac{(1+x)^2}{\sqrt{x}}\,dx = \int \frac{1+2x+x^2}{\sqrt{x}}\,dx = \int\frac{dx}{\sqrt{x}} + \int\frac{2x}{\sqrt{x}}dx + \int\frac{x^2}{\sqrt{x}}dx =$$

$$= \int x^{-\frac{1}{2}} dx + 2\int x^{\frac{1}{2}} dx + \int x^{\frac{3}{2}} dx =$$

$$= \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + 2\cdot\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$$

Zad.

$$\int \frac{x^2}{x^2+5}\,dx \quad \text{-->} \quad \int\frac{dx}{x^2+a^2} = \frac{1}{a}\cdot \operatorname{arctg}\frac{x}{a}$$

$$\int \frac{x^2+5-5}{x^2+5}\,dx = \int 1\,dx - 5\int\frac{dx}{x^2+5} = x - \frac{1}{\sqrt{5}}\cdot\operatorname{arctg}\frac{x}{\sqrt{5}} + C$$

Zad.

$$\int_0^{\frac{\pi}{4}} \operatorname{tg}^2 x\,dx = \int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{\cos^2 x}\,dx = \int_0^{\frac{\pi}{4}} \frac{1-\cos^2 x}{\cos^2 x}\,dx =$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos^2 x} - 1\right) dx = \operatorname{tg}x - x\,\Big|_0^{\frac{\pi}{4}} = 1 - \frac{\pi}{4}$$

Zad. $\int \frac{\sin x}{tg \frac{x}{2}} dx = \int \frac{\sin x}{\frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}} dx = \int \frac{2 \cancel{\sin \frac{x}{2}} \cos \frac{x}{2}}{\frac{\cancel{\sin \frac{x}{2}}}{\cos \frac{x}{2}}} dx =$

$\cos^2 x = \frac{1+\cos 2x}{2}$

- za sve integrale trigonom funkcija sa parnim potencijama

$\int 2\cos^2 \frac{x}{2} dx = 2\int \cos^2 \frac{x}{2} dx =$

$= 2 \cdot \frac{1}{2} \int (1+\cos x) dx = x + \sin x + C$

Pr. $\int \sin(3x+1) dx = ?$ $\int (x+\pi)^{69} dx = ?$

B) Metoda supstitucije

Pr. $\int \frac{dx}{3x+14} = \left| \begin{matrix} 3x+14 = t \ /' \\ 3dx = dt \\ dx = \frac{dt}{3} \end{matrix} \right| = \int \frac{\frac{dt}{3}}{t} = \frac{1}{3} \int \frac{dt}{t} = \frac{1}{3} \ln|t| + C$

$= \frac{1}{3} \ln|3x+14| + C$

Pr. $\int \frac{\ln^3 x}{x} dx = \left| \begin{matrix} \ln x = t \ /' \\ \frac{dx}{x} = dt \end{matrix} \right| = \int t^3 dt = \frac{t^4}{4} + C = \frac{\ln^4 x}{4} + C$

Pr. $\int e^{2x+5} dx = \left| \begin{matrix} t = 2x+5 \\ dt = 2dx \\ dx = \frac{dt}{2} \end{matrix} \right| = \int e^t \cdot \frac{dt}{2} = \frac{1}{2} \int e^t dt = \frac{1}{2} e^t + C = \frac{1}{2} e^{2x+5} + C$

$\int f(ax+b) dx = \frac{1}{a} \cdot F(ax+b)$ → samo za linearne argumente

$$\int x^2 \operatorname{ch}(x^3+1)\,dx = \left| \begin{matrix} t = x^3+1 \\ dt = 3x^2\,dx \end{matrix} \right| = \int \operatorname{ch} t \frac{dt}{3} = \frac{1}{3}\int \operatorname{ch} t\, dt = \frac{1}{3}\operatorname{sh} t + C =$$

$$= \frac{1}{3}\operatorname{sh}(x^3+1) + C$$

11. DZ.

(11.)

$$\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx = \left| \begin{matrix} \sin x = t \\ \cos x\, dx = dt \end{matrix} \right| = \int \frac{\cos^{\cancel{3}\,2} x}{\sqrt{t}} \cdot \frac{dt}{\cancel{\cos x}} = \int \frac{\cos^2 x}{\sqrt{t}}\,dt =$$

$$= \int \frac{1-\sin^2 x}{\sqrt{t}}\,dt = \int \frac{1-t^2}{\sqrt{t}}\,dt = \frac{t^{\frac{1}{2}}}{\frac{1}{2}} - \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + C =$$

$$= \frac{\sqrt{\sin x}}{\frac{1}{2}} - \frac{\sqrt{\sin^5 x}}{\frac{5}{2}} + C$$

$$\int \operatorname{tg} x\,dx = \int \frac{\sin x}{\cos x}\,dx = \left| \begin{matrix} \cos x = t \\ -\sin x\,dx = dt \end{matrix} \right| = \int -\frac{dt}{t} = -\int \frac{dt}{t} = -\ln|t| + C =$$

$$= -\ln|\cos x| + C$$

$$\int \frac{dx}{\sin x} \overset{\text{doskočica}}{=} \int \frac{\sin x}{\sin^2 x}\,dx = \int \frac{\sin x}{1-\cos^2 x}\,dx = \left| \begin{matrix} \cos x = t \\ -\sin x\,dx = dt \end{matrix} \right| =$$

$$\int \frac{-dt}{1-t^2} = \frac{1}{2}\ln\left|\frac{\cos x - 1}{\cos x + 1}\right| + C$$

Z1 - 09 - 4)

$$\int \frac{\operatorname{arctg} x + x}{x^2+1} = \int \frac{\operatorname{arctg} x}{x^2+1}dx + \int \frac{x}{x^2+1}dx =$$

$$\left| \begin{array}{l} \operatorname{arctg} x = v \\ \frac{1}{x^2+1}dx = dv \end{array} \right| \quad \left| \begin{array}{l} x^2+1 = t \\ 2x dx = dt \\ x dx = \frac{dt}{2} \end{array} \right|$$

$$= \int v\, dv + \frac{1}{2}\int \frac{dt}{t} = \frac{v^2}{2} + \frac{1}{2}\ln|t| + C =$$

$$= \frac{\operatorname{arctg}^2 x}{2} + \frac{1}{2}\ln|x^2+1| + C$$

Z1 - 08 - 4)

$$\int \frac{x^9}{x^{10}-2x^5+5}dx = \left| \begin{array}{l} t = x^5 \\ dt = 5x^4 dx \\ dx = \frac{dt}{5x^4} \end{array} \right| = \int \frac{x^{\cancel{9}5}}{t^2-2t+5} \cdot \frac{dt}{5\cancel{x^4}} =$$

$$= \int \frac{t}{t^2-2t+5}\cdot\frac{dt}{5} = \frac{1}{5}\int \frac{t}{\underbrace{t^2-2t+5}_{\text{na puni kvadrat}}}dt = \frac{1}{5}\int \frac{t}{(t-1)^2+4}dt =$$

$$= \left| \begin{array}{l} t-1 = u \\ dt = du \end{array} \right| = \frac{1}{5}\int \frac{u+1}{u^2+4}du = \frac{1}{5}\int \frac{u}{u^2+4}du + \frac{1}{5}\int \frac{1}{u^2+4}du =$$

$$= \frac{1}{5}\cdot\frac{1}{2}\cdot\ln|u^2+4| + \frac{1}{5}\cdot\frac{1}{2}\cdot\operatorname{arctg}\frac{u}{2} + C =$$

$$= \frac{1}{10}\ln|(x^5-1)^2+4| + \frac{1}{10}\operatorname{arctg}\frac{x^5-1}{2} + C$$

$\boxed{11. DZ.}$ 7.

$$\int_0^{\sqrt{3}} x^5 \sqrt{x^2+1}\, dx = \left| \begin{array}{l} x^2+1=t \\ 2x\,dx = dt \end{array} \right| = \frac{1}{2} \int_1^4 (t-1)^2 \sqrt{t}\, dt =$$

$$= \frac{1}{2} \int_1^4 \left(t^{\frac{5}{2}} - 2t^{\frac{3}{2}} + t^{\frac{1}{2}}\right) dt = \frac{1}{2} \cdot \frac{t^{\frac{7}{2}}}{\frac{7}{2}} - \frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{1}{2} \cdot \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \Bigg|_1^4 =$$

$$= \frac{848}{105}$$

$\boxed{11. ZZV.}$ 6.

$$\int_{\frac{1}{2}}^{\frac{5}{4}} \frac{x\, dx}{\sqrt{\underbrace{2+x-x^2}_{\text{puni kvadrat}}}} = \int_{\frac{1}{2}}^{\frac{5}{4}} \frac{x\, dx}{\sqrt{\frac{9}{4} - (x-\frac{1}{2})^2}} = \left| \begin{array}{l} x - \frac{1}{2} = t \\ dx = dt \end{array} \right| =$$

$$2 - \left(x^2 - x + \frac{1}{4} - \frac{1}{4}\right)$$

$$2 - \left(x - \frac{1}{2}\right)^2 + \frac{1}{4}$$

$$= \int_0^{\frac{3}{4}} \frac{t + \frac{1}{2}}{\sqrt{\frac{9}{4} - t^2}}\, dt = \int_0^{\frac{3}{4}} \frac{t}{\sqrt{\frac{9}{4} - t^2}}\, dt + \frac{1}{2} \int_0^{\frac{3}{4}} \underbrace{\frac{dt}{\sqrt{\frac{9}{4} - t^2}}}_{\arcsin \frac{x}{a} = \frac{t}{\frac{3}{2}}} =$$

$$\frac{9}{4} - t^2 = u$$

$$-2t\,dt = du$$

$$= -\frac{1}{2} \int_{\frac{9}{4}}^{\frac{27}{16}} \frac{du}{\sqrt{u}} + \frac{1}{2} \int_0^{\frac{3}{4}} \frac{dt}{\sqrt{\frac{9}{4} - t^2}} = -\sqrt{u} \Bigg|_{\frac{9}{4}}^{\frac{27}{16}} + \frac{1}{2} \arcsin \frac{2t}{3} \Bigg|_0^{\frac{3}{4}} =$$

$$= \frac{3}{2} - \frac{3\sqrt{3}}{4} + \frac{\pi}{12}$$

C) PARCIJALNA INTEGRACIJA

MOTIVACIJA : $\int xe^x\,dx = ?$ $\quad \int x\sin x\,dx = ?$

$$\boxed{\int f(x)\,g'(x)\,dx = f(x)\cdot g(x) - \int f'(x)\cdot g(x)\,dx}$$

DOKAZ:

$$\left(f(x)\cdot g(x) - \int f'(x)g(x)\,dx\right)' =$$

$$= \cancel{f'(x)\cdot g(x)} + f(x)\cdot g'(x) - \cancel{f'(x)g(x)} =$$

$$= f(x)\cdot g'(x) \xrightarrow{\text{kraće}} \int u\,dv = uv - \int v\,du$$

Pr.

$$\int xe^x\,dx = \begin{vmatrix} u = x & dv = e^x \\ du = dx & v = e^x \end{vmatrix} = \overset{u\cdot v}{xe^x} - \int \overset{v\,\cdot\,du}{e^x dx} =$$

$$= xe^x - e^x + C = e^x(x-1) + C$$

Z1-08-3) $\int (x^2 \ln x)\,dx = \begin{vmatrix} u = \ln x & dv = x^2 \\ du = \frac{1}{x} & v = \frac{x^3}{3} \end{vmatrix} = \frac{x^3}{3}\cdot \ln x - \int \frac{x^{\cancel{3}\,2}}{3}\cdot\frac{1}{x}\,dx =$

nešto što se lako integrira

nešto što se lako derivira, a teško integrira

$$= \frac{x^3}{3}\ln x - \frac{1}{3}\cdot\frac{x^3}{3} + C$$

u ⟵ ○ ⟶ dv

L	I	A	T	E
logaritam	inverz	algebra	trigonometrija	eksponencijalna
(ln)	(arc, ar)	(x^n)	$\cos x$	e^x, a^x
		$(\cos^n x)$	$\operatorname{tg} x$	
			$\operatorname{ch} x$	

(Pr.) $\int \arcsin x \cdot dx = \begin{vmatrix} u=\arcsin x & dv = dx \\ du = \frac{1}{\sqrt{1-x^2}}dx & v = x \end{vmatrix} =$

$x \cdot \arcsin x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx = \begin{vmatrix} 1-x^2=t \\ -2x\,dx=dt \end{vmatrix} =$

$= x\arcsin x + \frac{1}{2}\int \frac{dt}{\sqrt{t}} = x\arcsin x + \sqrt{1-x^2} + C$

21-08-

$\int e^{-x}\sin x\, dx = \begin{vmatrix} u=e^{-x} & v' = \sin x\, dx \\ du = -e^{-x}dx & v = -\cos x \end{vmatrix} = e^{-x}\cdot(-\cos x) - \int \cos x \cdot e^{-x} dx =$

$= \begin{vmatrix} u=e^{-x} & dv = \cos x\, dx \\ du = -e^{-x}dx & v = \sin x \end{vmatrix} = -e^{-x}\cos x - \left(e^{-x}\sin x + \int \sin x \cdot e^{-x} dx\right)$

- ciklički integral $\longrightarrow$ eksp. trig

$I = \int e^{-x}\sin x\, dx$

$I = -e^{-x}\cos x - e^{-x}\sin x - I$

$2I = -e^{-x}(\cos x + \sin x)$

$I = \frac{-e^{-x}(\cos x + \sin x)}{2} + C$ //