11. DOMAĆA ZADAĆA – 1. dio

1.

(a)

$$\int \frac{2x+1}{x^4}dx = 2\int \frac{1}{x^3}dx + \int \frac{1}{x^4}dx = \frac{2x^{-2}}{-2} + \frac{x^{-3}}{-3} + C = -\frac{1}{3x^3} - \frac{1}{x^2} + C$$

(b)

$$\int \left(1 - \frac{1}{x^2}\right)\sqrt{x}dx = \int \sqrt{x}dx - \int \frac{\sqrt{x}}{x^2}dx = \int x^{\frac{1}{2}}dx - \int x^{-\frac{3}{2}}dx = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + C = \frac{2}{3}\sqrt{x^3} + \frac{2}{\sqrt{x}} + C$$

2.

(a)

$$\int_0^1 x(2x+1)^5 dx = \begin{bmatrix} 2x+1=t & x=\dfrac{t-1}{2} \\ 2dx=dt \to dx=\dfrac{dt}{2} & t_D=1, t_G=3 \end{bmatrix} = \int_1^3 \frac{t-1}{2}t^5\frac{dt}{2} =$$

$$= \frac{1}{4}\int_1^3 (t-1)t^5 dt = \frac{1}{4}\int_1^3 (t^6 - t^5)dt = \frac{1}{4}\left(\frac{t^7}{7} - \frac{t^6}{6}\right)\Big|_1^3 = \frac{1}{4}\left(\frac{3^7}{7} - \frac{3^6}{6} - \left(\frac{1}{7} - \frac{1}{6}\right)\right) = \frac{2005}{42}$$

(b)

$$\int_0^1 x\sqrt{3x+1}dx = \begin{bmatrix} 3x+1=t & x=\dfrac{t-1}{3} \\ 3dx=dt \to dx=\dfrac{dt}{3} & t_D=1, t_G=4 \end{bmatrix} = \int_1^4 \frac{t-1}{3}\sqrt{t}\frac{dt}{3} =$$

$$= \frac{1}{9}\int_1^4 (t\sqrt{t} - \sqrt{t})dt = \frac{1}{9}\int_1^4 \left(t^{\frac{3}{2}} - t^{\frac{1}{2}}\right)dt = \frac{1}{9}\left(\frac{t^{\frac{5}{2}}}{\frac{5}{2}} - \frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)\Big|_1^4 = \frac{1}{9}\left(\frac{2}{5}\left(4^{\frac{5}{2}} - 1\right) - \frac{2}{3}\left(4^{\frac{3}{2}} - 1\right)\right) = \frac{116}{135}$$

3.

(a)

$$\int_0^{\sqrt{3}} x\sqrt{4-x^2}dx = \begin{bmatrix} 4-x^2=t \\ -2xdx=dt \to xdx=-\frac{dt}{2} \\ t_D=4, t_G=1 \end{bmatrix} = -\frac{1}{2}\int_4^1 \sqrt{t}dt = -\frac{1}{2}\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\Big|_4^1 =$$

$$= -\frac{1}{3}\left(1-4^{\frac{3}{2}}\right) = \frac{7}{3}$$

(b)

$$\int_0^{\sqrt{3}} \frac{xdx}{\sqrt{x^2+1}} = \begin{bmatrix} x^2+1=t \\ 2xdx=dt \to xdx=\frac{dt}{2} \\ t_D=1, t_G=4 \end{bmatrix} = \frac{1}{2}\int_1^4 \frac{dt}{\sqrt{t}} = \frac{1}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}}\Big|_1^4 = \left(\sqrt{2}-1\right) = 1$$

4.

(a)

$$\int \frac{e^{2x}}{\sqrt{e^x+1}}dx = \int \frac{e^x \cdot e^x}{\sqrt{e^x+1}}dx = \begin{bmatrix} e^x+1=t \to e^x=t-1 \\ e^x dx = dt \end{bmatrix} =$$

$$= \int \frac{(t-1)dt}{\sqrt{t}} = \int \frac{tdt}{\sqrt{t}} - \int \frac{dt}{\sqrt{t}} = \int t^{\frac{1}{2}}dt - \int t^{-\frac{1}{2}}dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C = \frac{2}{3}\sqrt{t^3} - 2\sqrt{t} + C =$$

$$= \frac{2}{3}\sqrt{(e^x+1)^3} - 2\sqrt{e^x+1} + C = \frac{2}{3}(e^x-2)\sqrt{e^x+1} + C$$

(b)

$$\int \frac{\ln^3 x}{x}dx = \begin{bmatrix} \ln x = t \\ \frac{dx}{x} = dt \end{bmatrix} = \int t^3 dt = \frac{t^4}{4} + C = \frac{\ln^4 x}{4} + C$$

5.

(a)

$$\int \operatorname{ctg}^2 x\,dx = \begin{bmatrix} \sin^2 x + \cos^2 x = 1 \;/:\sin^2 x \\ \operatorname{ctg}^2 x = \dfrac{1}{\sin^2 x} - 1 \end{bmatrix} = \int \left(\frac{1}{\sin^2 x} - 1\right) dx =$$

$$= \int \frac{dx}{\sin^2 x} - \int dx = -\operatorname{ctg} x - x + C$$

(b)

$$\int \operatorname{th}(2x)\,dx = \int \frac{\operatorname{sh}(2x)}{\operatorname{ch}(2x)} dx = \begin{bmatrix} \operatorname{ch}(2x) = t \\ 2\operatorname{sh}(2x)\,dx = dt \to \operatorname{sh}(2x)\,dx = \dfrac{dt}{2} \end{bmatrix} = \frac{1}{2}\int \frac{dt}{t} =$$

$$= \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln|\operatorname{ch}(2x)| + C$$

6.

$$\int_1^e (x^2 - 2x + 3)\ln x\,dx = \int_1^e x^2 \ln x\,dx - 2\int_1^e x\ln x\,dx + 3\int_1^e \ln x\,dx$$

$$\int_1^e x^2 \ln x\,dx = \begin{bmatrix} u = \ln x & dv = x^2 dx \\ du = \dfrac{dx}{x} & v = \dfrac{x^3}{3} \end{bmatrix} = \frac{x^3 \ln x}{3}\Big|_1^e - \frac{1}{3}\int_1^e x^2 dx = \frac{e^3}{3} - \frac{x^3}{9}\Big|_1^e = \frac{2e^3 + 1}{9}$$

$$-2\int_1^e x\ln x\,dx = \begin{bmatrix} u = \ln x & dv = x dx \\ du = \dfrac{dx}{x} & v = \dfrac{x^2}{2} \end{bmatrix} = -x^2 \ln x\Big|_1^e - \int_1^e x dx = -e^2 + \frac{x^2}{2}\Big|_1^e = -\frac{e^2}{2} - \frac{1}{2}$$

$$3\int_1^e \ln x\,dx = \begin{bmatrix} u = \ln x & dv = dx \\ du = \dfrac{dx}{x} & v = x \end{bmatrix} = 3x\ln x\Big|_1^e - 3\int_1^e dx = 3e - 3x\Big|_1^e = 3$$

$$\int_1^e (x^2 - 2x + 3)\ln x\,dx = \frac{2e^3 + 1}{9} - \frac{e^2}{2} - \frac{1}{2} + 3 = \frac{2}{9}e^3 - \frac{1}{2}e^2 + \frac{47}{18}$$

7.

$$\int_0^{\sqrt{3}} x^5\sqrt{x^2+1}dx = \int_0^{\sqrt{3}} x\cdot x^4\sqrt{x^2+1}dx = \begin{bmatrix} x^2+1=t \\ 2xdx = dt \to xdx = \dfrac{dt}{2} \\ x^2 = t-1 \to x^4 = (t-1)^2 \end{bmatrix} =$$

$$= \int_1^4 (t-1)^2\sqrt{t}\frac{dt}{2} = \frac{1}{2}\int_1^4 \left(t^2\sqrt{t} - 2t\sqrt{t} + \sqrt{t}\right)dt = \frac{1}{2}\left(\frac{t^{\frac{7}{2}}}{\frac{7}{2}} - 2\frac{t^{\frac{5}{2}}}{\frac{5}{2}} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)\Big|_1^4 =$$

$$= \frac{1}{2}\left(\frac{2}{7}(128-1) - \frac{4}{5}(32-1) + \frac{2}{3}(8-1)\right) = \frac{848}{105}$$

8.

$$\int_0^{16} \frac{dx}{\left(1+\sqrt[4]{x}\right)^2} = \begin{bmatrix} 1+\sqrt[4]{x} = t \to \sqrt[4]{x} = t-1 \to \sqrt[4]{x^3} = (t-1)^3 \\ \dfrac{dx}{4\sqrt[4]{x^3}} = dt \to dx = 4\sqrt[4]{x^3}dt \to dx = 4(t-1)^3dt \\ t_D = 1, t_G = 3 \end{bmatrix} =$$

$$= \int_1^3 \frac{4(t-1)^3dt}{t^2} = 4\int_1^3 \left(t - 3 + \frac{3}{t} - \frac{1}{t^2}\right)dt = 4\left(\frac{t^2}{2} - 3t + 3\ln|t| + \frac{1}{t}\right)\Big|_1^3 =$$

$$= 4\left(\frac{9}{2} - \frac{1}{2} - 9 + 3 + 3\ln 3 + \frac{1}{3} - 1\right) = 12\ln 3 - \frac{32}{3}$$

9.

$$\int_0^1 \frac{x^2dx}{x^6+1} = \begin{bmatrix} x^3 = t \\ 3x^2dx = dt \to x^2dx = \dfrac{dt}{3} \end{bmatrix} = \frac{1}{3}\int_0^1 \frac{dt}{t^2+1} = \frac{1}{3}\operatorname{arctg} t\,\Big|_0^1 = \frac{1}{3}\cdot\frac{\pi}{4} = \frac{\pi}{12}$$

10.

$$\int_0^1 \ln(2x+1)\,dx = \begin{bmatrix} 2x+1 = t \\ 2dx = dt \to dx = \dfrac{dt}{2} \end{bmatrix} = \frac{1}{2}\int_1^3 \ln t\,dt = \begin{bmatrix} u = \ln t & dv = dt \\ du = \dfrac{dt}{t} & v = t \end{bmatrix} =$$

$$= \frac{1}{2}\left(t\ln t\,\Big|_1^3 - \int_1^3 dt\right) = \frac{1}{2}(3\ln 3 - 2) = \frac{3}{2}\ln 3 - 1$$