9. DOMAĆA ZADAĆA IZ MATEMATIKE 1

1. $y = 3x^4 + 4x^3 - 12x^2 + 20$

$$y' = 12x^3 + 12x^2 - 24x = 0$$

$$x^3 + x^2 - 2x = 0$$

$$x(x^2 + x - 2) = 0$$

$$\mathbf{x_1 = 0}$$

$$\mathbf{y_1 = 20}$$

$$x^2 + x - 2 = 0$$

$$x_{2,3} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + 2} = -\frac{1}{2} \pm \sqrt{\frac{9}{4}} = -\frac{1}{2} \pm \frac{3}{2}$$

$$\mathbf{x_2 = -2}$$

$$\mathbf{y_2} = 48 - 32 - 48 + 20 = \mathbf{-12}$$

$$\mathbf{x_3 = 1}$$

$$\mathbf{y_3} = 3 + 4 - 12 + 20 = \mathbf{15}$$

2. $y^4 = 4x^4 + 6xy,\ A(1,2)$

$$4y^3y' = 16x^3 + 6y + 6xy'$$

$$y' = \frac{16x^3 + 6y}{4y^3 - 6x} = \frac{8x^3 + 3y}{2y^3 - 3x} = [x = 1, y = 2] = \frac{8 + 6}{16 - 3} = \frac{14}{13} = k_T$$

Jednadžba tangente: $y - y_0 = k_T(x - x_0)$

$$y - 2 = \frac{14}{13}(x - 1)$$

$$\boldsymbol{y = \frac{14}{13}x + \frac{12}{13}}$$

3. $xy^2 + x^4y^3 = 2,\ T(1,1)$

$$y^2 + 2xyy' + 4x^3y^3 + 3x^4y^2y' = 0$$

$$y'(2xy + 3x^4y^2) = -(y^2 + 4x^3y^3)$$

$$\boldsymbol{y'} = -\frac{y^2 + 4x^3y^3}{2xy + 3x^4y^2} = [x = 1, y = 1] = -\frac{1+4}{2+3} = \mathbf{-1}$$

$$y'' = -\frac{(y^2 + 4x^3y^3)'(2xy + 3x^4y^2) - (y^2 + 4x^3y^3)(2xy + 3x^4y^2)'}{(2xy + 3x^4y^2)^2}$$

$$(y^2 + 4x^3y^3)' = 2yy' + 12x^2y^3 + 12x^3y^2y' = -2$$

$$2xy + 3x^4y^2 = 5$$

$$y^2 + 4x^3y^3 = 5$$

$$(2xy + 3x^4y^2)' = 2y + 2xy' + 12x^3y^2 + 6x^4yy' = 6$$

$$\boldsymbol{y''} = -\frac{-2 \cdot 5 - 5 \cdot 6}{5^2} = -\frac{-10 - 30}{25} = \frac{40}{25} = \mathbf{\frac{8}{5}}$$

4. $x = acos^3t,\ y = asin^3t,\ t = \frac{\pi}{4}$

$$y' = \frac{\dot{y}}{\dot{x}} = \frac{3asin^2t \cdot cost}{-3acos^2t \cdot sint} = -\frac{sint}{cost} = -tgt$$

$$y'\left(t = \frac{\pi}{4}\right) = -tg\frac{\pi}{4} = -1 = k_T$$

$$x_0 = acos^3\frac{\pi}{4} = a\left(\frac{\sqrt{2}}{2}\right)^3 = \frac{2a\sqrt{2}}{8} = \frac{a\sqrt{2}}{4}$$

$$y_0 = asin^3\frac{\pi}{4} = a\left(\frac{\sqrt{2}}{2}\right)^3 = \frac{2a\sqrt{2}}{8} = \frac{a\sqrt{2}}{4}$$

Jednadžba tangente: $y - y_0 = k_T(x - x_0)$

$$y - \frac{a\sqrt{2}}{4} = -\left(x - \frac{a\sqrt{2}}{4}\right)$$

$$\boldsymbol{y = -x + \frac{a\sqrt{2}}{2}}$$

5. $x = t^2 + 1,\ \ y = 3t + e^t$

$$\boldsymbol{y'} = \frac{\dot{y}}{\dot{x}} = \boldsymbol{\frac{e^t + 3}{2t}}$$

$$\boldsymbol{y''} = \frac{d(y')}{dx} = \frac{\frac{d\left(\frac{\dot{y}}{\dot{x}}\right)}{dt}}{\frac{dx}{dt}} = \frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{\dot{x}^3} = \frac{2e^t - 2t(e^t + 3)}{8t^3} = \boldsymbol{\frac{e^t - te^t - 3t}{4t^3}}$$

$$y''' = \frac{d(y'')}{dx} = \frac{\frac{d\left(\frac{\dot{x}\ddot{y} - \ddot{x}\dot{y}}{\dot{x}^3}\right)}{dt}}{\frac{dx}{dt}} = \frac{(\dot{x}\ddot{y} - \ddot{x}\dot{y})'\dot{x}^3 - (\dot{x}\ddot{y} - \ddot{x}\dot{y})(\dot{x}^3)'}{\dot{x}^7}$$

$$y''' = \frac{(\ddot{x}\ddot{y} + \dot{x}\dddot{y} - \dddot{x}\dot{y} - \ddot{x}\ddot{y})\dot{x}^3 - (\dot{x}\ddot{y} - \ddot{x}\dot{y})(3\dot{x}^2\ddot{x})}{\dot{x}^7}$$

$$y''' = \frac{\dot{x}^4\dddot{y} - \dot{x}^3\dddot{x}\dot{y} - 3\dot{x}^3\ddot{x}\ddot{y} + 3\dot{x}^2\ddot{x}^2\dot{y}}{\dot{x}^7}$$

$$\boldsymbol{y''' = \frac{16t^4e^t - 48t^3e^t + 48t^2(e^t + 3)}{128t^7}}$$

6. $f(x) = sinx,\ I = \left[0, \frac{\pi}{2}\right]$

Langrageov teorem: $f(b) - f(a) = f'(c)(b - a)$

$$f\left(\frac{\pi}{2}\right) - f(0) = f'(c)\left(\frac{\pi}{2} - 0\right)$$

$$sin\frac{\pi}{2} - sin0 = \frac{\pi}{2}f'(c)$$

$$1 = \frac{\pi}{2}f'(c)$$

$$f'(c) = \frac{2}{\pi}$$

$$f'(x) = cosx \leftrightarrow f'(c) = cosc$$

$$cosc = \frac{2}{\pi}$$

$$\boldsymbol{c = arccos\frac{2}{\pi}}$$

7. $f(x) = arcsinx,\ I = [-1,1]$

Langrageov teorem: $f(b) - f(a) = f'(c)(b - a)$

$$f(1) - f(-1) = f'(c)(1 + 1)$$

$$arcsin1 - arcsin(-1) = 2f'(c)$$

$$\frac{\pi}{2} + \frac{\pi}{2} = 2f'(c)$$

$$f'(c) = \frac{\pi}{2}$$

$$f'(x) = \frac{1}{\sqrt{1 - x^2}} \leftrightarrow f'(c) = \frac{1}{\sqrt{1 - c^2}}$$

$$\frac{1}{\sqrt{1 - c^2}} = \frac{\pi}{2} \leftrightarrow \frac{1}{1 - c^2} = \frac{\pi^2}{4} \leftrightarrow 1 - c^2 = \frac{4}{\pi^2} \leftrightarrow c^2 = \frac{\pi^2 - 4}{\pi^2}$$

$$\boldsymbol{c = \pm\sqrt{\frac{\pi^2 - 4}{\pi^2}}}$$

8. $f(x) = chx,$ oko 0

Taylorova formula:

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

$c = 0$

$f(0) = ch0 = 1$

$f'(0) = sh0 = 0$

$f''(0) = ch0 = 1$

$f'''(0) = sh0 = 0$

…

Neparne derivacije = 0

Parne derivacije = 1 --> $2k$

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(2k)}(c)}{(2k)!}(x-c)^{2k} = \sum_{k=0}^{n} \frac{f^{(2k)}(0)}{(2k)!}(x-0)^{2k}$$

$$\boldsymbol{T_n(x) = \sum_{k=0}^{n} \frac{x^{2k}}{(2k)!} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^n}{(2n)!}}$$

9. $f(x) = shx,$ oko 0

Taylorova formula:

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(c)}{k!}(x-c)^k$$

$c = 0$

$f(0) = sh0 = 0$

$f'(0) = ch0 = 1$

$f''(0) = sh0 = 0$

$f'''(0) = ch0 = 1$

...

Neparne derivacije = 1 --> $2k + 1$

Parne derivacije = 0

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(2k+1)}(c)}{(2k+1)!}(x-c)^{2k+1} = \sum_{k=0}^{n} \frac{f^{(2k+1)}(0)}{(2k+1)!}(x-0)^{2k+1}$$

$$\boldsymbol{T_n(x) = \sum_{k=0}^{n} \frac{x^{2k+1}}{(2k+1)!} = x + \frac{x^3}{3!} + \frac{x^5}{5!} \dots + \frac{x^{2n+1}}{(2n+1)!}}$$

10. $f(x) = x^3 + 2x^2 - x + 1$, u $(x-1)$

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iz $(x-1) = (x-c) \leftrightarrow c = 1$

Polinom je trećeg stupnja --> računamo do treće derivacije.

$f(1) = 1 + 2 - 1 + 1 = 3$

$f'(1) = 3x^2 + 4x - 1 = 3 + 4 - 1 = 6$

$f''(1) = 6x + 4 = 6 + 4 = 10$

$f'''(1) = 6$

$$T_n(x) = \sum_{k=0}^{n} \frac{f^{(k)}(1)}{k!}(x-1)^k = 3 + 6(x-1) + \frac{10}{2!}(x-1)^2 + \frac{6}{3!}(x-1)^3$$

$$\boldsymbol{T_n(x) = 3 + 6(x-1) + 5(x-1)^2 + (x-1)^3}$$

11. 1) $\lim_{x\to0}\frac{2^x-2^{-x}}{x}$, 2) $\lim_{x\to\infty} x\cdot arcctg(2x)$

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1) $\lim_{x\to0}\frac{2^x-2^{-x}}{x}=\left(\frac{0}{0}\right)=\lim_{x\to0}\frac{2^x ln2+\frac{2^x ln2}{2^{2x}}}{1}=ln2+ln2=2ln2=\boldsymbol{ln4}$

2) $\lim_{x\to\infty} x\cdot arcctg(2x)=(\infty\cdot0)=\lim_{x\to\infty}\frac{arcctg(2x)}{\frac{1}{x}}=\left(\frac{0}{0}\right)=$

$$=\lim_{x\to\infty}\frac{-\frac{2}{4x^2+1}}{-\frac{1}{x^2}}=2\lim_{x\to\infty}\frac{x^2}{4x^2+1}=2\lim_{x\to\infty}\frac{1}{4+\frac{1}{x^2}}=2\cdot\frac{1}{4}=\boldsymbol{\frac{1}{2}}$$

12. 1) $\lim_{x\to 0}\left(\frac{sin3x}{3x}\right)^{\frac{1}{x}}$, 2) $\lim_{x\to 0}\sqrt{x}\cdot lnx$

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1) $\lim_{x\to 0}\left(\frac{sin3x}{3x}\right)^{\frac{1}{x}} = \left(\left(\frac{0}{0}\right)^{\infty}\right) = \lim_{x\to 0} e^{ln\left(\frac{sin3x}{3x}\right)^{\frac{1}{x}}} = \lim_{x\to 0} e^{\frac{1}{x}ln\left(\frac{sin3x}{3x}\right)}$

$$e^{\lim\limits_{x\to 0}\frac{1}{x}ln\left(\frac{sin3x}{3x}\right)} = e^{A}$$

$$A = \lim_{x\to 0}\frac{1}{x}ln\left(\frac{sin3x}{3x}\right) = \left(\frac{0}{0}\right) = \lim_{x\to 0}\left(\frac{1}{sin3x}\frac{3xcos3x - sin3x}{x}\right) = \left(\frac{0}{0}\right) =$$

$$= \lim_{x\to 0}\left(\frac{(3xcos3x - sin3x)'}{(xsin3x)'}\right) = \lim_{x\to 0}\left(\frac{-9xsin3x}{sin3x + 3xcos3x}\right) =$$

$$= \left(\frac{0}{0}\right) = \lim_{x\to 0}\left(\frac{-9sin3x - 27xcos3x}{3cos3x + 3cos3x - 3xsin3x}\right) = \frac{0}{6} = 0$$

$$\boldsymbol{\lim_{x\to 0}\left(\frac{sin3x}{3x}\right)^{\frac{1}{x}} = e^{0} = 1}$$

2) $\lim_{x\to 0}\sqrt{x}\cdot lnx = \lim_{x\to 0}\frac{lnx}{\frac{1}{\sqrt{x}}} = \left(\frac{\infty}{\infty}\right) = \lim_{x\to 0}\frac{\frac{1}{x}}{-\frac{\frac{1}{2\sqrt{x}}}{x}} = \lim_{x\to 0} -2\sqrt{x} = \mathbf{0}$

13. 1) $\lim_{x\to\frac{\pi}{2}}(cosx)^{\frac{\pi}{2}-x}$, 2) $\lim_{x\to 0}\left(\frac{tgx}{x}\right)^{\frac{1}{x}}$

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1) $\lim_{x\to\frac{\pi}{2}}(cosx)^{\frac{\pi}{2}-x} = \left|\begin{matrix}\frac{\pi}{2}-x=y\to x=\frac{\pi}{2}-y \\ y\to 0\end{matrix}\right| = \lim_{y\to 0}\left(cos\left(\frac{\pi}{2}-y\right)\right)^{y} =$

$$= \lim_{y\to 0}(siny)^{y} = (0^0) = \lim_{y\to 0} e^{ln(siny)^{y}} = \lim_{y\to 0} e^{yln(siny)} = e^{\lim\limits_{y\to 0} yln(siny)} = e^{A}$$

$$A = \lim_{y\to 0}\frac{ln(siny)}{\frac{1}{y}} = \left(\frac{\infty}{\infty}\right) = \lim_{y\to 0}\frac{\frac{cosy}{siny}}{-\frac{1}{y^2}} = -\lim_{y\to 0}\frac{y^2cosy}{siny} = \left(\frac{0}{0}\right) =$$

$$= -\lim_{y\to 0}\frac{2ycosy - y^2siny}{cosy} = -\frac{0}{1} = 0$$

$$\boldsymbol{\lim_{x\to\frac{\pi}{2}}(cosx)^{\frac{\pi}{2}-x} = e^{0} = 1}$$

2) $\lim_{x\to 0}\left(\frac{tgx}{x}\right)^{\frac{1}{x}} = \lim_{x\to 0} e^{\frac{1}{x}ln\frac{tgx}{x}} = e^{\lim_{x\to 0}\frac{1}{x}ln\frac{tgx}{x}} = e^{A}$

$$A = \lim_{x\to 0}\frac{1}{x}ln\frac{tgx}{x} = \left(\frac{0}{0}\right) = \lim_{x\to 0}\frac{x - sinx\cdot cosx}{xcos^2x\cdot tgx} = \lim_{x\to 0}\frac{x - sinx\cdot cosx}{xsinx\cdot cosx} = \left(\frac{0}{0}\right) =$$

$$= \lim_{x\to 0}\frac{2sin^2x}{x(cos^2x - sin^2x) + sinx\cdot cosx} = \left(\frac{0}{0}\right) =$$

$$= \lim_{x\to 0}\frac{2sin2x}{-2sin^2x + 2cos^2x - 2sin2x}$$

$$\boldsymbol{\lim_{x\to 0}\left(\frac{tgx}{x}\right)^{\frac{1}{x}} = 1}$$

14. 1) $\lim_{x\to-\infty}\frac{ln\left(ch(x+3)\right)}{x}$, 2) $\lim_{x\to0}\frac{ln\frac{sinx}{x}}{x^2}$

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1) $\lim_{x\to-\infty}\frac{ln\left(ch(x+3)\right)}{x} = \mathbf{0}$

- ne znam zašto, ali ovo je sto post točan odgovor, probao sam svašta raspisat al nisam dobio šta treba ☹, pa mi je Wolfram Alpha pomogla x]

2) $\lim_{x\to0}\frac{ln\frac{sinx}{x}}{x^2} = \left(\frac{0}{0}\right) = \lim_{x\to0}\frac{xcosx-sinx}{2x^2sinx} = \left(\frac{0}{0}\right) = \lim_{x\to0}\frac{-xsinx}{4xsinx+2x^2cosx} = \left(\frac{0}{0}\right)$

$$= \lim_{x\to0}\frac{-sinx - xcosx}{2(2sinx + 4xcosx - x^2sinx)} = \left(\frac{0}{0}\right) =$$

$$= \lim_{x\to0}\frac{-cosx - cosx + xsinx}{2(2cosx + 4cosx - 4xsinx + 2xcosx - x^2cosx)} = -\frac{2}{12} = \mathbf{-\frac{1}{6}}$$

15. $\lim_{x\to-\infty}\left(xe^{-\frac{1}{x^2}}-x\right)$

$$\lim_{x\to-\infty}\left(xe^{-\frac{1}{x^2}}-x\right)=\lim_{x\to\infty}\left(x-xe^{-\frac{1}{x^2}}\right)=\lim_{x\to\infty}\left(\frac{1-e^{-\frac{1}{x^2}}}{\frac{1}{x}}\right)=\left(\frac{0}{0}\right)=$$

$$=\lim_{x\to\infty}\left(\frac{-\frac{2}{x^3}e^{-\frac{1}{x^2}}}{-\frac{1}{x^2}}\right)=\lim_{x\to\infty}\left(\frac{2e^{-\frac{1}{x^2}}}{x}\right)=\frac{2}{\infty}=\mathbf{0}$$

16. $f(x) = \frac{-x^3-x^2+x+5}{(x+1)^2}$

Vertikalna asimptota: $(x+1)^2 = 0 \rightarrow x+1 = 0 \rightarrow x = -1$

$$\lim_{x\to -1^-} \frac{-x^3-x^2+x+5}{(x+1)^2} = \infty$$

$$\lim_{x\to -1^+} \frac{-x^3-x^2+x+5}{(x+1)^2} = \infty$$

Vertikalna asimptota je $x = -1$.

Horizontalna asimptota: ne postoji jer je stupanj polinoma u brojniku veći od stupnja polinoma u nazivniku.

Kosa asimptota: $k = \lim_{x\to\pm\infty} \frac{f(x)}{x}$, $l = \lim_{x\to\pm\infty}(f(x) - kx)$

$$k = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{-x^3-x^2+x+5}{x^3+2x^2+x} = -1$$

$$l = \lim_{x\to\pm\infty} (f(x) - kx) = \lim_{x\to\pm\infty} \left(\frac{-x^3-x^2+x+5}{(x+1)^2} + x\right) =$$

$$\lim_{x\to\pm\infty} \left(\frac{-x^3-x^2+x+5+x^3+2x^2+x}{x^2+2x+1}\right) = -1$$

Kosa asimptota je $y = -x - 1$.

17. $f(x) = e^{\frac{1}{x-2}}$

Vertikalna asimptota: $x - 2 = 0 \rightarrow x = 2$

$$\lim_{x \to 2^-} e^{\frac{1}{x-2}} = e^{-\infty} = 0$$

$$\lim_{x \to 2^+} e^{\frac{1}{x-2}} = e^{\infty} = \infty$$

Vertikalna asimptota je $x = 2$ (s desne strane).

Horizontalna asimptota:

$$\lim_{x \to \infty} e^{\frac{1}{x-2}} = 1$$

$$\lim_{x \to -\infty} e^{\frac{1}{x-2}} = 1$$

horizontalna asimptota je $y = 1$.

Kosa asimptota: $k = \lim_{x \to \pm\infty} \frac{f(x)}{x}$, $l = \lim_{x \to \pm\infty} (f(x) - kx)$

$$k = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{e^{\frac{1}{x-2}}}{x} = 0$$

Nema kose asimptote.

18. $f(x) = \frac{x}{ln^2 x}$

Vertikalna asimptota: $ln^2 x = 0 \rightarrow lnx = 0 \rightarrow x = 1$

$$\lim_{x \to 1^-} \frac{x}{ln^2 x} = \infty$$

$$\lim_{x \to 1^+} \frac{x}{ln^2 x} = \infty$$

Vertikalna asimptota je $x = 1$.

Horizontalna asimptota:

$$\lim_{x \to \pm\infty} \frac{x}{ln^2 x} = \left(\frac{\infty}{\infty}\right) = \lim_{x \to \infty} \frac{x}{2lnx} = \left(\frac{\infty}{\infty}\right) = \lim_{x \to \infty} \frac{x}{2} = \infty$$

Nema horizontalne asimptote.

Kosa asimptota: $k = \lim_{x \to \pm\infty} \frac{f(x)}{x}$, $l = \lim_{x \to \pm\infty} (f(x) - kx)$

$$k = \lim_{x \to \pm\infty} \frac{f(x)}{x} = \lim_{x \to \pm\infty} \frac{1}{ln^2 x} = 0$$

Nema kose asimptote.

19. $f(x) = arctg\,\frac{x^2}{x+1}$

Vertikalna asimptota: $x + 1 = 0 \rightarrow x = -1$

$$\lim_{x\to -1^-} arctg\frac{x^2}{x+1} = arctg(-\infty) = -\frac{\pi}{2}$$

$$\lim_{x\to -1^+} arctg\frac{x^2}{x+1} = arctg(\infty) = \frac{\pi}{2}$$

Nema vertikalnih asimptota.

Horizontalna asimptota:

$$\lim_{x\to\pm\infty} arctg\frac{x^2}{x+1} = arctg\lim_{x\to\pm\infty}\frac{x^2}{x+1} = arctg(\pm\infty) = \pm\frac{\pi}{2}$$

Horizontalne asimptote su $y = -\frac{\pi}{2}$ i $y = \frac{\pi}{2}$.

Kosa asimptota: $k = \lim_{x\to\pm\infty}\frac{f(x)}{x}$, $l = \lim_{x\to\pm\infty}(f(x) - kx)$

$$k = \lim_{x\to\pm\infty}\frac{f(x)}{x} = \lim_{x\to\pm\infty}\frac{arctg\frac{x^2}{x+1}}{x} = \left(\frac{\infty}{\infty}\right) = \lim_{x\to\pm\infty}\frac{x^2+2x}{\left(\frac{x^2}{x+1}\right)^2 + 1} = 0$$

Nema kose asimptote.

20. $f(x) = 2x - 1 - \sqrt{x^2 - x - 1}$

Nema vertikalnih asimptota.

Horizontalna asimptota:

$$\lim_{x\to\pm\infty}\left(\frac{4x^2 - 4x + 1 - x^2 + x + 1}{2x - 1 + \sqrt{x^2 - x - 1}}\right) = \infty$$

Nema horizontalnih asimptota.

Kosa asimptota: $k = \lim_{x\to\pm\infty}\frac{f(x)}{x}$, $l = \lim_{x\to\pm\infty}(f(x) - kx)$

$$k_1 = \lim_{x\to\infty}\frac{f(x)}{x} = \lim_{x\to\infty}\frac{2x - 1 - \sqrt{x^2 - x - 1}}{x} = 1$$

$$k_2 = \lim_{x\to-\infty}\frac{f(x)}{x} = \lim_{x\to\infty}\frac{-2x - 1 - \sqrt{x^2 - x - 1}}{-x} = 3$$

$$l_1 = \lim_{x\to\infty}(f(x) - k_1x) = \lim_{x\to\infty}\left(2x - 1 - \sqrt{x^2 - x - 1} - x\right) =$$

$$= \lim_{x\to\infty}\left(x - 1 - \sqrt{x^2 - x - 1}\right) = \lim_{x\to\infty}\frac{-x}{x - 1 + \sqrt{x^2 - x - 1}} = -\frac{1}{2}$$

$$l_2 = \lim_{x\to-\infty}(f(x) - k_2x) = \lim_{x\to-\infty}\left(2x - 1 - \sqrt{x^2 - x - 1} - 3x\right) =$$

$$= \lim_{x\to-\infty}\left(-x - 1 - \sqrt{x^2 - x - 1}\right) = -\lim_{x\to-\infty}\frac{3x}{x - 1 + \sqrt{x^2 - x - 1}} = -\frac{3}{2}$$

Kose asimptote su $y = x - \frac{1}{2}$ i $y = 3x - \frac{3}{2}$.