MATEMATIČKE FORMULE

ELEMENT

ALGEBARSKI IZRAZI

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

$$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$$

$$a^2 - b^2 = (a - b)(a + b)$$

$$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$$

$$a^4 - b^4 = (a - b)(a + b)(a^2 + b^2)$$

$$a^n - b^n = (a - b)(a^{n-1} + a^{n-2}b + a^{n-3}b^2 + \ldots + \\ + ab^{n-2} + b^{n-1}), \quad n \in \mathbf{N}$$

$$a^{2n+1} + b^{2n+1} = (a + b)(a^{2n} - a^{2n-1}b + a^{2n-2}b^2 - \ldots - \\ - ab^{2n-1} + b^{2n}), \quad n \in \mathbf{N}$$

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2ac + 2bc$$

POTENCIJE

$$a^m \cdot a^n = a^{m+n}$$

$$a^m : a^n = a^{m-n}$$

$$(ab)^n = a^n b^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(a^m)^n = a^{m \cdot n}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

KORIJENI

$$a^n = b \iff a = \sqrt[n]{b}$$

$$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

$$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$\sqrt[n]{a^m} = \sqrt[np]{a^{mp}}$$

$$\sqrt[n]{\sqrt[m]{a}} = \sqrt[nm]{a}$$

$$\sqrt[n]{a^m} = a^{\frac{m}{n}}$$

GRČKI ALFABET

A	α	alfa	I	ι	jota	P	ρ	ro
B	β	beta	K	κ	kapa	Σ	σ	sigma
Γ	γ	gama	Λ	λ	lambda	T	τ	tau
Δ	δ	delta	M	μ	mi	Υ	υ	ipsilon
E	ε	epsilon	N	ν	ni	Φ	ϕ	fi
Z	ζ	zeta	Ξ	ξ	ksi	X	χ	hi
H	η	eta	O	o	omikron	Ψ	ψ	psi
Θ	θ	theta	Π	π	pi	Ω	ω	omega

Velika slova identička latiničnim: $A, B, E, Z, H, I, K, M, N, O, P, T, X$ čitamo kao slova latinice. Malo slovo omikron ne razlikujemo od latiničnog o. Slova ϕ, θ i κ imaju i varijante φ, ϑ i $\varkappa$.

KOMPLEKSNI BROJEVI

$$z = x + yi, \quad x, y \in \mathbf{R}$$

$$i^2 = -1$$
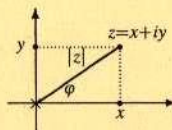
$$i^{4n} = 1, \quad i^{4n+1} = i,$$
$$i^{4n+2} = -1, \quad i^{4n+3} = -i$$

TRIGONOMETRIJSKI PRIKAZ

$$\varphi = \arg z$$

$$\cos\varphi = \frac{x}{|z|}, \quad \sin\varphi = \frac{y}{|z|}$$

$$z = |z|(\cos\varphi + i\sin\varphi)$$

ALGEBARSKE OPERACIJE

$$(a + bi) \pm (c + di) = (a \pm c) + (b \pm d)i$$

$$(a + bi)(c + di) = (ac - bd) + (ad + bc)i$$

$$\frac{a + bi}{c + di} = \frac{ac + bd + (bc - ad)i}{c^2 + d^2}$$

Konjugirani broj broju z
$$\overline{z} = x - yi$$
Apsolutna vrijednost broja z:
$$|z| = \sqrt{x^2 + y^2}$$

$$|z_1 + z_2| \leqslant |z_1| + |z_2|$$
$$|z_1 z_2| = |z_1| \cdot |z_2|$$
$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

MOIVREOVA FORMULA

$$z^n = |z|^n(\cos n\varphi + i\sin n\varphi),$$

$$\sqrt[n]{z} = \sqrt[n]{|z|}\left(\cos\frac{\varphi + 2k\pi}{n} + i\sin\frac{\varphi + 2k\pi}{n}\right),$$

$$k = 0, 1, \ldots, n - 1$$

FAKTORIJELE I BINOMNA FORMULA

$$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1)n, \quad 0! = 1,$$

$$\binom{n}{r} = \frac{n(n-1)\cdots(n-r+1)}{1 \cdot 2 \cdots r} = \frac{n!}{r!(n-r)!}$$

$$\binom{n}{r} = \binom{n}{n-r}, \quad \binom{n}{r} + \binom{n}{r+1} = \binom{n+1}{r+1}$$

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \\ + \binom{n}{n-1}ab^{n-1} + b^n$$
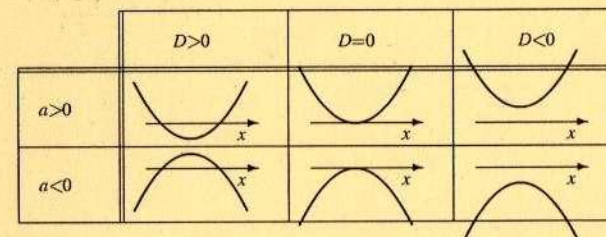
POLINOM DRUGOG STUPNJA

$$f : \mathbf{R} \to \mathbf{R}, \quad f(x) = ax^2 + bx + c, \quad a \neq 0, \ a, b, c \in \mathbf{R}$$

Kanonski oblik: $f(x) = a\left(x + \dfrac{b}{2a}\right)^2 + \dfrac{4ac - b^2}{4a}$.

Tjeme: $T\left(-\dfrac{b}{2a}, -\dfrac{b^2 - 4ac}{4a}\right)$. Os simetrije: $x = -\dfrac{b}{2a}$.

Diskriminanta: $D = b^2 - 4ac$.

	$D > 0$	$D = 0$	$D < 0$
$a > 0$			
$a < 0$			

KVADRATNA JEDNADŽBA

Opći oblik
$$ax^2 + bx + c = 0, \quad x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Normirani oblik
$$x^2 + px + q = 0, \quad x_{1,2} = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

Vièteove formule: $x_1 + x_2 = -\dfrac{b}{a} = -p, \quad x_1 x_2 = \dfrac{c}{a} = q$

Faktorizacija: $a(x - x_1)(x - x_2) = 0$ Vrsta rješenja:

$$\begin{cases} D > 0 : & \text{dva realna rješenja} \\ D = 0 : & \text{jedno dvostruko realno rješenje} \\ D < 0 : & \text{kompleksno konjugirana rješenja} \end{cases}$$

DIJELJENJE POLINOMA

$$P_n(x) = Q_{n-m}(x)P_m(x) + R(x)$$

stupanj ostatka R je $< m$

$$P_n(a) = 0 \iff$$
$$P_n(x) = (x - a)Q_{n-1}(x)$$

$P_n(a)$ je ostatak pri dijeljenju s $x - a$:
$$P_n(x) = (x - a)Q_{n-1}(x) + P_n(a)$$

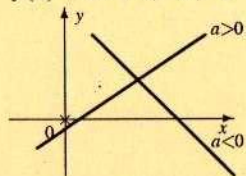## VIÈTEOVE FORMULE

$$P(x) = ax^3 + bx^2 + cx + d$$
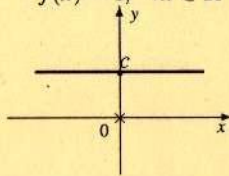$$= a(x - x_1)(x - x_2)(x - x_3)$$

$$x_1 + x_2 + x_3 = -\frac{b}{a}$$

$$x_1 x_2 + x_1 x_3 + x_2 x_3 = \frac{c}{a}$$

$$x_1 x_2 x_3 = -\frac{d}{a}$$

FUNKCIJE

Afina funkcija $f: \mathbf{R} \to \mathbf{R}$
$f(x) = ax + b, \quad a \neq 0$

Konstanta $f: \mathbf{R} \to \mathbf{R}$
$f(x) = c, \quad \forall x \in \mathbf{R}$
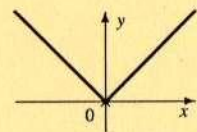

Apsolutna vrijednost realnog broja

$$| \; | : \mathbf{R} \to \mathbf{R}^+$$

$$|x| = \begin{cases} x, & x > 0 \\ 0, & x = 0 \\ -x, & x < 0 \end{cases}$$

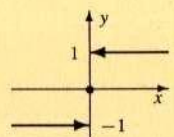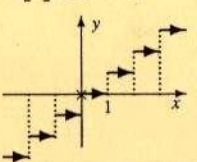$$|a + b| \leqslant |a| + |b|$$
$$|ab| = |a| \cdot |b|$$
$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$\text{sgn} : \mathbf{R} \to \{-1, 0, 1\}$

$\lfloor \; \rfloor : \mathbf{R} \to \mathbf{Z}$

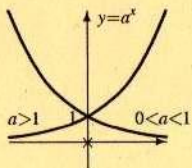$\{ \; \} : \mathbf{R} \to [0, 1\rangle$
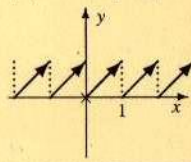
EKSPONENCIJALNA FUNKCIJA

$$f : \mathbf{R} \to \mathbf{R}^+$$
$$f(x) = a^x$$
$$a > 0, \quad a \neq 1$$

LOGARITAMSKA FUNKCIJA

$$f : \mathbf{R}^+ \to \mathbf{R}$$
$$f(x) = \log_a x, \quad a > 0, \quad a \neq 1$$
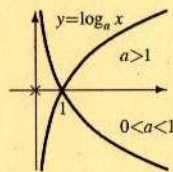
$$\log_a(xy) = \log_a x + \log_a y$$
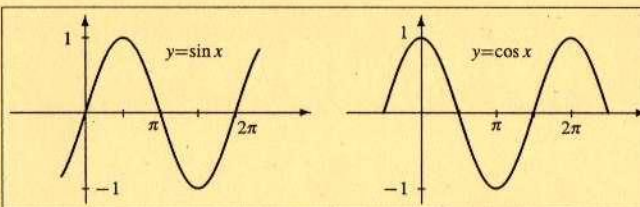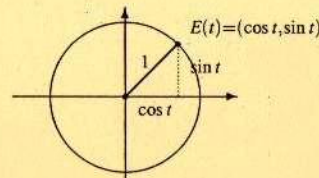$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$
$$\log_a x^n = n \log_a x$$
$$\log_a x = \frac{\log_b x}{\log_b a}$$

$$a^{\log_a x} = x$$
$$\log_a a^x = x$$

TRIGONOMETRIJSKE FUNKCIJE

$$\sin^2 t + \cos^2 t = 1$$
$$\operatorname{tg} t = \frac{\sin t}{\cos t}$$
$$\operatorname{tg} t \cdot \operatorname{ctg} t = 1$$

$E(t) = (\cos t, \sin t)$

$y = \sin x$

$y = \cos x$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
$\operatorname{tg} x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$-$	0	$-$	0
$\operatorname{ctg} x$	$-$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-$	0	$-$

SVOĐENJE NA PRVI KVADRANT

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$
$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\sin\left(\frac{\pi}{2} + x\right) = \cos x$$
$$\cos\left(\frac{\pi}{2} + x\right) = -\sin x$$

$$\sin(\pi - x) = \sin x$$
$$\cos(\pi - x) = -\cos x$$

$$\sin(\pi + x) = -\sin x$$
$$\cos(\pi + x) = -\cos x$$

$$\sin\left(\frac{3\pi}{2} - x\right) = -\cos x$$
$$\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$$

$$\sin(2\pi - x) = -\sin x$$
$$\cos(2\pi - x) = \cos x$$

SINUSOIDA

$$C \sin(\omega x + \varphi) = A \cos \omega x + B \sin \omega x$$
$$C = \sqrt{A^2 + B^2}, \quad \sin \varphi = \frac{A}{C}, \quad \cos \varphi = \frac{B}{C}$$

TRIGONOMETRIJSKI IDENTITETI

PERIODIČNOST

$$\sin(x + 2k\pi) = \sin x$$
$$\cos(x + 2k\pi) = \cos x$$
$$\operatorname{tg}(x + k\pi) = \operatorname{tg} x$$
$$\operatorname{ctg}(x + k\pi) = \operatorname{ctg} x$$

(NE)PARNOST

$$\sin(-x) = -\sin x$$
$$\cos(-x) = \cos x$$
$$\operatorname{tg}(-x) = -\operatorname{tg} x$$
$$\operatorname{ctg}(-x) = -\operatorname{ctg} x$$

FUNKCIJE DVOSTRUKOG ARGUMENTA

$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$\operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$$
$$\operatorname{ctg} 2x = \frac{\operatorname{ctg}^2 x - 1}{2 \operatorname{ctg} x}$$

FUNKCIJE POLOVINE ARGUMENTA

$$\sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$$
$$\cos^2 \frac{x}{2} = \frac{1 + \cos x}{2}$$
$$\operatorname{tg} \frac{x}{2} = \frac{1 - \cos x}{\sin x}$$
$$\operatorname{ctg} \frac{x}{2} = \frac{1 + \cos x}{\sin x}$$

FUNKCIJE ZBROJA I RAZLIKE

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$
$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$
$$\operatorname{tg}(x \pm y) = \frac{\operatorname{tg} x \pm \operatorname{tg} y}{1 \mp \operatorname{tg} x \operatorname{tg} y}$$
$$\operatorname{ctg}(x \pm y) = \frac{\operatorname{ctg} x \operatorname{ctg} y \mp 1}{\operatorname{ctg} y \pm \operatorname{ctg} x}$$

UNIVERZALNA ZAMJENA

$$t = \operatorname{tg} \frac{x}{2}, \quad \operatorname{tg} x = \frac{2t}{1 - t^2}$$
$$\sin x = \frac{2t}{1 + t^2}, \quad \cos x = \frac{1 - t^2}{1 + t^2}$$

FORMULE PRETVORBE

Zbroja u umnožak

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$
$$\sin x - \sin y = 2 \cos \frac{x+y}{2} \sin \frac{x-y}{2}$$
$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$
$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

Umnoška u zbroj

$$\sin x \cos y = \tfrac{1}{2}(\sin(x+y) + \sin(x-y))$$
$$\cos x \sin y = \tfrac{1}{2}(\sin(x+y) - \sin(x-y))$$
$$\cos x \cos y = \tfrac{1}{2}(\cos(x+y) + \cos(x-y))$$
$$\sin x \sin y = \tfrac{1}{2}(\cos(x-y) - \cos(x+y))$$

FUNKCIJE TROSTRUKOG ARGUMENTA

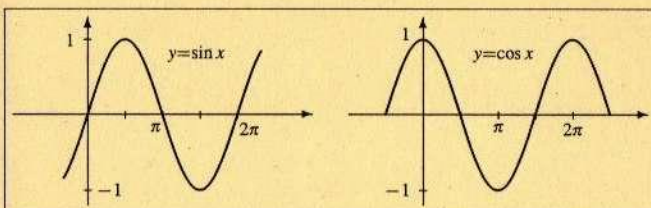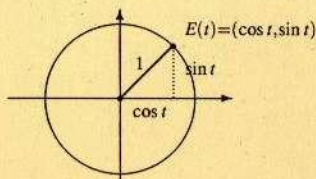$$\sin 3x = 3 \sin x - 4 \sin^3 x$$
$$\cos 3x = 4 \cos^3 x - 3 \cos x$$

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$$\sin^2 t + \cos^2 t = 1$$
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SINUSOIDA

$$C\sin(\omega x + \varphi) = A\cos\omega x + B\sin\omega x$$
$$C = \sqrt{A^2 + B^2}, \quad \sin\varphi = \frac{A}{C}, \quad \cos\varphi = \frac{B}{C}$$

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$$\cos x \cos y = \tfrac{1}{2}(\cos(x+y) + \cos(x-y))$$
$$\sin x \sin y = \tfrac{1}{2}(\cos(x-y) - \cos(x+y))$$

FUNKCIJE TROSTRUKOG ARGUMENTA

$$\sin 3x = 3\sin x - 4\sin^3 x$$
$$\cos 3x = 4\cos^3 x - 3\cos x$$

KOMBINATORIKA

Permutacije
$$P_n = n!$$

Permutacije s ponavljanjem
$$P_n^{r_1,\ldots,r_k} = \frac{n!}{r_1! \cdot \ldots \cdot r_k!}$$

Kombinacije
$$C_n^r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Kombinacije s ponavljanjem
$$\overline{C}_n^r = \binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$

ARITMETIČKI NIZ

$$a_{n+1} = a_n + d$$

Opći član
$$a_n = a_1 + (n-1)d$$

Suma niza
$$S_n = \frac{n}{2}(a_1 + a_n)$$

GEOMETRIJSKI NIZ

$$a_{n+1} = a_n q$$

Opći član
$$a_n = a_1 q^{n-1}$$

Suma niza
$$S_n = a_1 \frac{q^n - 1}{q - 1}$$

POSTOTNI RAČUN

$$P = Cp, \quad p = \frac{P}{C}, \quad C = \frac{P}{p}$$

P postotni iznos
p postotak
C osnovna vrijednost

SLOŽENI KAMATNI RAČUN

$$C_n = C_0 r^n, \quad r = 1 + p$$

C_0 glavnica
p postotak
n vrijeme u godinama

VJEROJATNOST

$$P(A) = \frac{M}{N}$$
$$P(A \cup B) = P(A) + P(B) - P(AB)$$
$$P(AB) = P(A)P(B \mid A)$$
$$P(A) = P(H_1)P(A|H_1) + \ldots + P(H_n)P(A|H_n)$$
$$P(H_k|A) = \frac{P(H_k)P(A|H_k)}{P(A)}$$

Ponavljanje pokusa:
$$p_k = P(X=k) = \binom{n}{k} p^k q^{n-k}$$

BESKONAČAN GEOMETRIJSKI RED

$$\lim_{n \to \infty} S_n = \frac{a_1}{1 - q},$$
$$|q| < 1$$

ARITMETIČKA SREDINA

$$A = \frac{x_1 + x_2 + \ldots + x_n}{n}$$

GEOMETRIJSKA SREDINA

$$G = \sqrt[n]{x_1 \cdot x_2 \cdot \ldots \cdot x_n}$$

HARMONIJSKA SREDINA

$$H = \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}}$$

$$H \leqslant G \leqslant A$$

JEDNOSTAVAN KAMATNI RAČUN

$$K = Cpv$$

K kamate
p postotak
C glavnica
v vrijeme u godinama

NEPREKIDNO UKAMAĆIVANJE

$$C_n = C_0 e^{pn}$$
$$e = 2{,}718\,281\ldots$$

PLANIMETRIJA

OBODNI I SREDIŠNJI KUT

$$\beta = 2\alpha$$

TETIVNI ČETVEROKUT

$$\alpha + \gamma = \beta + \delta$$
$$P^2 = (s-a)(s-b) \times$$
$$\times (s-c)(s-d)$$

TANGENCIJALNI ČETVEROKUT

$$a + c = b + d$$

JEDNAKOSTRANIČNI TROKUT

$$v = \frac{a\sqrt{3}}{2}, \quad r = \frac{a\sqrt{3}}{6}$$
$$R = \frac{a\sqrt{3}}{3}, \quad P = \frac{a^2\sqrt{3}}{4}$$

PRAVOKUTNI TROKUT

$$a^2 + b^2 = c^2$$
$$a^2 = pc, \quad b^2 = qc$$
$$v^2 = pq$$

KVADRAT

$$d = a\sqrt{2}$$
$$P = a^2$$

PRAVOKUTNIK

$$d = \sqrt{a^2 + b^2}$$
$$P = ab$$

PARALELOGRAM

$$e^2 + f^2 = 2(a^2 + b^2)$$
$$P = av = ab\sin\alpha$$
$$P = \frac{1}{2}ef\sin\varphi$$

TRAPEZ

$$P = sv$$
$$P = \frac{a+c}{2}v$$

KRUG

$$O = 2r\pi$$
$$P = r^2\pi$$

KRUŽNI VIJENAC

$$P = (R^2 - r^2)\pi$$
$$P_i = \frac{\pi\alpha}{360}(R^2 - r^2)$$
$$P_i = \frac{l_1 + l_2}{2}(R - r)$$

KRUŽNI ISJEČAK

$$l = \frac{r\pi\alpha}{180}$$
$$P_i = \frac{rl}{2} = \frac{r^2\pi\alpha}{360}$$

PRAVOKUTNI TROKUT
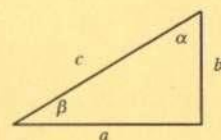
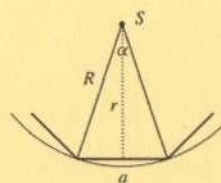$$\sin\alpha = \frac{a}{c} = \cos\beta$$
$$\cos\alpha = \frac{b}{c} = \sin\beta$$
$$\operatorname{tg}\alpha = \frac{a}{b} = \operatorname{ctg}\beta$$
$$\operatorname{ctg}\alpha = \frac{b}{a} = \operatorname{tg}\beta$$
$$P = \frac{1}{2}ab = \frac{1}{4}c^2\sin 2\alpha$$
$$= \frac{1}{2}a^2\operatorname{tg}\beta = \frac{1}{2}b^2\operatorname{tg}\alpha$$

PRAVILNI MNOGOKUT

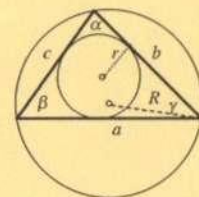Broj dijagonala: $\dfrac{n(n-3)}{2}$.

Zbroj unutarnjih kutova
$$(n-2) \cdot 180°$$
$$R = \frac{a}{2\sin\alpha/2}, \quad r = \frac{a}{2\operatorname{tg}\alpha/2}$$
$$P = \frac{n}{4}a^2\operatorname{ctg}\frac{\alpha}{2} = nr^2\operatorname{tg}\frac{\alpha}{2}$$
$$= \frac{n}{2}R^2\sin\alpha$$

TROKUT

$$\alpha + \beta + \gamma = 180°$$
$$a + b > c$$
$$P = \frac{1}{2}av_a = \frac{1}{2}bv_b = \frac{1}{2}cv_c$$
$$= rs, \quad (2s = a + b + c)$$
$$= \frac{abc}{4R}$$
$$= \sqrt{s(s-a)(s-b)(s-c)}$$

POUČAK O SINUSIMA

$$a : b : c = \sin\alpha : \sin\beta : \sin\gamma$$
$$a = 2R\sin\alpha$$
$$b = 2R\sin\beta$$
$$c = 2R\sin\gamma$$

POUČAK O KOSINUSIMA

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

POVRŠINA TROKUTA

$$P = \frac{1}{2}ab\sin\gamma$$
$$= \frac{1}{2}bc\sin\alpha = \frac{1}{2}ac\sin\beta$$
$$= \frac{a^2\sin\beta\sin\gamma}{2\sin\alpha}$$
$$= \frac{b^2\sin\alpha\sin\gamma}{2\sin\beta}$$
$$= \frac{c^2\sin\alpha\sin\beta}{2\sin\gamma}$$
$$= 2R^2\sin\alpha\sin\beta\sin\gamma$$

SIMETRALA I TEŽIŠNICA

$$s_\alpha = \frac{2}{b+c}\sqrt{bcs(s-a)}$$
$$4t_a^2 = 2b^2 + 2c^2 - a^2$$

STEREOMETRIJA

KOCKA

$$D = a\sqrt{3}$$
$$O = 6a^2$$
$$V = a^3$$

KVADAR

$$D^2 = a^2 + b^2 + c^2$$
$$O = 2(ab + ac + bc)$$
$$V = abc$$

PRIZMA

$$O = 2B + P$$
$$V = Bv$$

PIRAMIDA

$$O = B + P$$
$$V = \frac{1}{3}Bv$$

KRNJA PIRAMIDA

$$B : B_1 = h^2 : x^2$$
$$V = \frac{v}{3}(B + \sqrt{BB_1} + B_1)$$

VALJAK

$$P = 2r\pi v$$
$$O = 2r\pi(r + v)$$
$$V = r^2\pi v$$

STOŽAC

$$P = r\pi s$$
$$O = r\pi(r + s)$$
$$V = \frac{1}{3}r^2\pi v$$

KRNJI STOŽAC

$$P = \pi s(R + r)$$
$$O = \pi(R^2 + r^2 + (R+r)s)$$
$$V = \frac{\pi v}{3}(R^2 + r^2 + Rr)$$

KUGLA

$$O = 4R^2\pi$$
$$V = \frac{4}{3}R^3\pi$$

KAPICA I ODSJEČAK

$$P = 2R\pi v$$
$$V = \frac{1}{3}\pi v^2(3R - v)$$
$$V = \frac{1}{6}\pi v(3r^2 + v^2)$$

KUGLIN POJAS I SLOJ

$$O = 2R\pi v$$
$$V = \frac{\pi v}{6}(3r^2 + 3r_1^2 + v^2)$$

KUGLIN ISJEČAK

$$V = \frac{2}{3}R^2\pi v$$

ZBROJ I RAZLIKA VEKTORA

VEKTORSKI UMNOŽAK

$\vec{a} \times \vec{b}$ je okomit na $\vec{a}$ i $\vec{b}$, čini desni sustav i ima duljinu

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\varphi.$$

U komponentama:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} =$$

$$= (a_yb_z - a_zb_y)\vec{i} + (a_zb_x - a_xb_z)\vec{j} + (a_xb_y - a_yb_x)\vec{k}$$
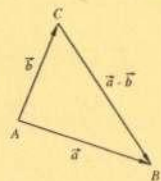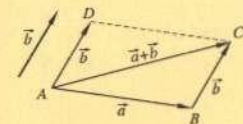
KOMPONENTE VEKTORA

$T_1(x_1,y_1), T_2(x_2,y_2), \vec{a} = \vec{T_1T_2}$

$\vec{a} = (x_2-x_1)\vec{i} + (y_2-y_1)\vec{j}$

$= a_x\vec{i} + a_y\vec{j}$

Jednakost vektora:

$\vec{a} = \vec{b} \iff a_x = b_x, \; a_y = b_y$

Duljina (norma, iznos) vektora:

$|\vec{a}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{a_x^2 + a_y^2}$

DETERMINANTE

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix}$$

$$- a_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$$= a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2$$

$$- a_1b_3c_2 - a_2b_1c_3 - a_3b_2c_1$$

SKALARNI UMNOŽAK

$\vec{a} = a_x\vec{i} + a_y\vec{j}, \; \vec{b} = b_x\vec{i} + b_y\vec{j},$

$\varphi = \angle(\vec{a}, \vec{b})$.

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}|\cos\varphi$

$= a_xb_x + a_yb_y$

Kut između vektora:

$\cos\varphi = \dfrac{a_xb_x + a_yb_y}{\sqrt{a_x^2+a_y^2}\sqrt{b_x^2+b_y^2}}$

Uvjet okomitosti:

$\vec{a} \perp \vec{b} \iff a_xb_x + a_yb_y = 0$

LINEARNI SUSTAVI

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

Rješenja:

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}, \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

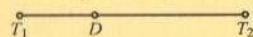## ANALITIČKA GEOMETRIJA RAVNINE

Udaljenost točaka T_1 i T_2:

$$|T_1T_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

Površina trokuta $\triangle T_1T_2T_3$:

$$P = \tfrac{1}{2}|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)|$$

Djelište dužine u omjeru λ,

$\vec{T_1D} = \lambda\,\vec{DT_2}$

$x_D = \dfrac{x_1 + \lambda x_2}{1+\lambda}$

$y_D = \dfrac{y_1 + \lambda y_2}{1+\lambda}$

Polovište dužine T_1T_2

$P\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

Težište trokuta $\triangle T_1T_2T_3$

$T\left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

PRAVAC

Eksplicitni oblik jednadžbe pravca

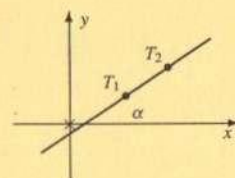$y = kx + l, \quad k = \text{tg}\,\alpha$

Implicitni oblik jednadžbe pravca

$Ax + By + C = 0$

$k = -\dfrac{A}{B}, \; l = -\dfrac{C}{B}$

Jednadžba pravca kroz točku T_1

$y - y_1 = k(x - x_1)$

Jednadžba pravca određenog točkama T_1 i T_2

$y - y_1 = \dfrac{y_2-y_1}{x_2-x_1}(x-x_1)$

Segmentni oblik jednadžbe pravca

$\dfrac{x}{m} + \dfrac{y}{n} = 1$

Udaljenost točke $T(x_0, y_0)$ od pravca $Ax + By + C = 0$

$d = \dfrac{|Ax_0 + By_0 + C|}{\sqrt{A^2+B^2}}$

Kut između pravaca p_1 i p_2

$\text{tg}\,\varphi = \left|\dfrac{k_2-k_1}{1+k_1k_2}\right|$

$\cos\varphi = \dfrac{|A_1A_2 + B_1B_2|}{\sqrt{A_1^2+B_1^2}\sqrt{A_2^2+B_2^2}}$

Uvjet paralelnosti pravaca

$k_1 = k_2$ ili $\dfrac{A_1}{A_2} = \dfrac{B_1}{B_2}$

Uvjet okomitosti pravaca

$k_1k_2 = -1$ ili $A_1A_2 + B_1B_2 = 0$

Simetrala kuta dvaju pravaca

$\dfrac{|A_1x + B_1y + C_1|}{\sqrt{A_1^2+B_1^2}} = \dfrac{|A_2x + B_2y + C_2|}{\sqrt{A_2^2+B_2^2}}$

KRIVULJE DRUGOG REDA

1. *Jednadžba krivulje*
2. *Jednadžba tangente u točki* (x_1, y_1)
3. *Uvjet dodira pravca* $y = kx + l$ *i krivulje*
4. *Koordinate dirališta*

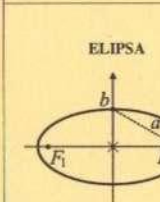### KRUŽNICA

1. $x^2 + y^2 = r^2$
2. $x_1x + y_1y = r^2$
3. $r^2(1+k^2) = l^2$
4. $\left(-\dfrac{kl}{1+k^2}, \dfrac{l}{1+k^2}\right)$
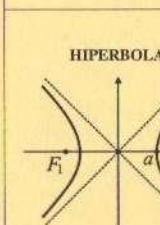
KRUŽNICA

1. $(x-p)^2 + (y-q)^2 = r^2$
2. $(x_1-p)(x-p) + (y_1-q)(y-q) = r^2$
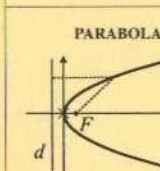3. $r^2(1+k^2) = (q - kp - l)^2$

ELIPSA

1. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$
2. $\dfrac{x_1x}{a^2} + \dfrac{y_1y}{b^2} = 1$
3. $a^2k^2 + b^2 = l^2$
4. $\left(-\dfrac{a^2k}{l}, \dfrac{b^2}{l}\right)$

linearni ekscentricitet $e^2 = a^2 - b^2$

numerički ekscentricitet $\varepsilon = e/a$

HIPERBOLA

1. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$
2. $\dfrac{x_1x}{a^2} - \dfrac{y_1y}{b^2} = 1$
3. $a^2k^2 - b^2 = l^2$
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linearni ekscentricitet $e^2 = a^2 + b^2$

numerički ekscentricitet $\varepsilon = e/a$

asimptote hiperbole $y = \pm\dfrac{b}{a}x$

PARABOLA

1. $y^2 = 2px$
2. $y_1y = p(x + x_1)$
3. $p = 2kl$
4. $\left(\dfrac{l}{k}, 2l\right)$
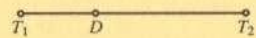
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Djelište dužine u omjeru λ,	Polovište dužine T_1T_2
$\overrightarrow{T_1D} = \lambda \overrightarrow{DT_2}$	$P\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

Težište trokuta $\triangle T_1T_2T_3$
$$T\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

$$x_D = \frac{x_1 + \lambda x_2}{1+\lambda}$$
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PRAVAC

Eksplicitni oblik jednadžbe pravca
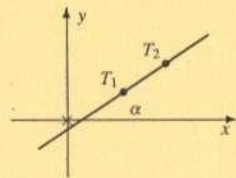$$y = kx + l, \quad k = \operatorname{tg}\alpha$$
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$$Ax + By + C = 0$$
$$k = -\frac{A}{B}, \quad l = -\frac{C}{B}$$
Jednadžba pravca kroz točku T_1
$$y - y_1 = k(x - x_1)$$

Jednadžba pravca određenog točkama T_1 i T_2
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Segmentni oblik jednadžbe pravca
$$\frac{x}{m} + \frac{y}{n} = 1$$
Udaljenost točke $T(x_0, y_0)$ od pravca $Ax + By + C = 0$
$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$
Kut između pravaca p_1 i p_2
$$\operatorname{tg}\varphi = \left|\frac{k_2 - k_1}{1 + k_1k_2}\right|$$
$$\cos\varphi = \frac{|A_1A_2 + B_1B_2|}{\sqrt{A_1^2 + B_1^2}\sqrt{A_2^2 + B_2^2}}$$
Uvjet paralelnosti pravaca
$$k_1 = k_2 \quad \text{ili} \quad \frac{A_1}{A_2} = \frac{B_1}{B_2}$$
Uvjet okomitosti pravaca
$$k_1k_2 = -1 \quad \text{ili} \quad A_1A_2 + B_1B_2 = 0$$
Simetrala kuta dvaju pravaca
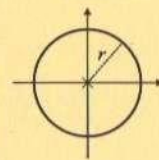$$\frac{|A_1x + B_1y + C_1|}{\sqrt{A_1^2 + B_1^2}} = \frac{|A_2x + B_2y + C_2|}{\sqrt{A_2^2 + B_2^2}}$$

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1. *Jednadžba krivulje*
2. *Jednadžba tangente u točki* (x_1, y_1)
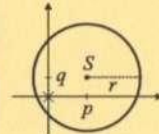3. *Uvjet dodira pravca* $y = kx + l$ *i krivulje*
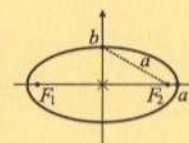4. *Koordinate diralište*

KRUŽNICA
1. $x^2 + y^2 = r^2$
2. $x_1x + y_1y = r^2$
3. $r^2(1 + k^2) = l^2$
4. $\left(-\dfrac{kl}{1+k^2}, \dfrac{l}{1+k^2}\right)$

KRUŽNICA
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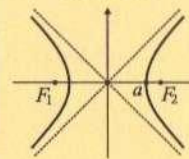2. $(x_1-p)(x-p) + (y_1-q)(y-q) = r^2$
3. $r^2(1+k^2) = (q - kp - l)^2$

ELIPSA
1. $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$
2. $\dfrac{x_1x}{a^2} + \dfrac{y_1y}{b^2} = 1$
3. $a^2k^2 + b^2 = l^2$
4. $\left(-\dfrac{a^2k}{l}, \dfrac{b^2}{l}\right)$

linearni ekscentricitet $e^2 = a^2 - b^2$
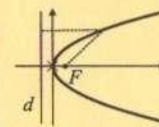numerički ekscentricitet $\varepsilon = e/a$

HIPERBOLA
1. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$
2. $\dfrac{x_1x}{a^2} - \dfrac{y_1y}{b^2} = 1$
3. $a^2k^2 - b^2 = l^2$
4. $\left(-\dfrac{a^2k}{l}, -\dfrac{b^2}{l}\right)$

linearni ekscentricitet $e^2 = a^2 + b^2$
numerički ekscentricitet $\varepsilon = e/a$
asimptote hiperbole $y = \pm\dfrac{b}{a}x$

PARABOLA
1. $y^2 = 2px$
2. $y_1y = p(x + x_1)$
3. $p = 2kl$
4. $\left(\dfrac{l}{k}, 2l\right)$

Derivacija zbroja
$$(u(x) \pm v(x))' = u'(x) \pm v'(x)$$
Derivacija umnoška
$$(u(x)\cdot v(x))' = u'(x)v(x) + u(x)v'(x)$$
Derivacija kvocijenta
$$\left(\frac{u(x)}{v(x)}\right)' = \frac{u'(x)v(x) - u(x)v'(x)}{v(x)^2}$$
Derivacija složene funkcije
$$(f(g(x))' = f'(g(x))\cdot g'(x)$$

INTEGRALI

Linearnost integrala
$$\int af(x)dx = a\int f(x)dx$$
$$\int(f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx$$
Parcijalna integracija
$$\int f(x)dg(x) = f(x)g(x) - \int g(x)df(x)$$
Newton-Leibnitzova formula
$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a), \qquad F'(x) = f(x)$$

DERIVACIJE	
$f(x)$	$f'(x)$
C	0
x	1
x^n	nx^{n-1}
$\dfrac{1}{x^n}$	$-\dfrac{n}{x^{n+1}}$
e^x	e^x
a^x	$a^x \ln a$
$\ln x$	$\dfrac{1}{x}$
$\log x$	$\dfrac{\log e}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\operatorname{tg} x$	$\dfrac{1}{\cos^2 x}$
$\operatorname{ctg} x$	$-\dfrac{1}{\sin^2 x}$

INTEGRALI	
$\int x^n dx$	$\dfrac{x^{n+1}}{n+1} + C$
$\int \dfrac{dx}{x}$	$\ln|x| + C$
$\int \dfrac{f'(x)}{f(x)}dx$	$\ln|f(x)| + C$
$\int e^x dx$	$e^x + C$
$\int a^x dx$	$\dfrac{a^x}{\ln a} + C$
$\int \sin x\, dx$	$-\cos x + C$
$\int \cos x\, dx$	$\sin x + C$
$\int \operatorname{tg} x\, dx$	$-\ln|\cos x| + C$
$\int \operatorname{ctg} x\, dx$	$\ln|\sin x| + C$
$\int \dfrac{dx}{\sin^2 x}$	$-\operatorname{ctg} x + C$
$\int \dfrac{dx}{\cos^2 x}$	$\operatorname{tg} x + C$