4)

$$y'^2 + 2x y' + 2y = 0$$

$$y = -x y' - y'^2 \qquad y' = p(x)$$

$$y = -xp - p^2 \quad / \frac{d}{dx}$$

$$p = -p - x p' - 2pp'$$

$$2p = p'(-x - 2p)$$

$$p' = \frac{-2p}{x+2p} \qquad x = x(p) \qquad p' = \frac{dp}{dx} = \frac{1}{\frac{dx}{dp}} = \frac{1}{x'}$$

$$x' = -\frac{x+2p}{2p} = -\frac{1}{2}\frac{x}{p} - 1$$

$$x' + \frac{1}{2}\frac{1}{p}x = -1 \qquad \text{Lineer}$$

$$x' + \frac{1}{2}\frac{1}{p}x = 0 \qquad \frac{dx}{x} = -\frac{1}{2}\frac{dp}{p}$$

$$\ln x = \ln \frac{c}{\sqrt{p}} \Rightarrow x = \frac{c(p)}{\sqrt{p}}$$

$$\frac{c'}{\sqrt{p}} - \frac{1}{2}\frac{c}{p^{\frac{3}{2}}} + \frac{1}{2}\frac{c}{p^{\frac{3}{2}}} = -1$$

$$c' = -\sqrt{p} \qquad c(p) = -\frac{2}{3}\sqrt{p}p^2 = -\frac{2}{3}p\sqrt{p} + C$$

$$x = -\frac{2}{3}p + \frac{c}{\sqrt{p}} = \frac{c}{\sqrt{p}} - \frac{2}{3}p$$

$$y = -xp - p^2 = \frac{2}{3}p^2 - c\sqrt{p} + p^2 = -\frac{1}{3}p^2 - c\sqrt{p}$$

$$\boxed{x = \frac{c}{\sqrt{p}} - \frac{2}{3}p} \qquad \boxed{y = -\frac{1}{3}p^2 - c\sqrt{p}}$$

Zad 9

1)

(1) $y = 2xy' + \sin y'$ $y' = p(x)$

$y = 2xp + \sin p$

$p = 2p + 2xp' + p'\cos p$

$-p = p'(2x + \cos p)$

$p' = -\frac{p}{2x + \cos p}$ $x = x(p)$

$x' = -2\frac{x}{p} - \frac{\cos p}{p}$

$x' + \frac{2}{p}x = -\frac{\cos p}{p}$ Linearna

(2) $x' + \frac{2}{p}x = 0$

$\frac{dx}{x} = -2\frac{dp}{p}$

$x = \frac{c}{p^2}$

$\frac{c'}{p^2} - 2\frac{c}{p^3} + 2\frac{c}{p^3} = -\frac{\cos p}{p}$

$c' = -p\cos p$

$C(p) = -\int p\cos p + C$

$\int p\cos p = \left| \begin{matrix} u = p & du = dp \\ dv = \cos p\, dp & v = \sin p \end{matrix} \right|$

$= p\cdot\sin p + \cos p$

$C(p) = -p\sin p - \cos p + C$

(3) $x = -\frac{\sin p}{p} - \frac{\cos p}{p^2} + \frac{c}{p^2}$

$y = 2xp + \sin p = -2\sin p + \sin p - \frac{2\cos p}{p} + \frac{2c}{p}$

2.) $e^x = \frac{y^2 + y'^2}{2y'}$ $x = \ln\left(\frac{y^2 + p^2}{2p}\right)$ $y' = p = p(y)$ $\frac{dy}{dx} = p$ $dy = p \cdot dx$

$dx = \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial p}dp$

$f'_y = \frac{2p}{y^2+p^2} \cdot \frac{2y}{2p} = \frac{2y}{y^2+p^2}$ $f'_p = \frac{2p}{y^2+p^2} \cdot \left(-\frac{y^2}{2p^2} + \frac{1}{2}\right)$

$dy = p\left[\frac{2y}{y^2+p^2}dx + \frac{2p}{y^2+p^2}\left(-\frac{y^2}{2p^2} + \frac{1}{2}\right)dp\right] \cdot y^2+p^2$

$dx(y^2+p^2) = p\left[2py\,dx + \left(-\frac{y^2}{p} + p\right)dp\right]$

$dx(y^2 - 2py + p^2) = (-y^2 + p^2)\,dp$ $/dx$

$(p-y)^2 = p'(p^2 - y^2)$ $\Rightarrow p = y$ $\frac{dy}{y} = dx \Rightarrow y = c\cdot e^x$

$(p - y) = p'(p + y)$

$$(p-y) = p'(p+y) \Rightarrow p' = \frac{p-y}{p+1} = \frac{y\left(\frac{p}{y}-1\right)}{y\left(\frac{p}{y}+1\right)}$$ Hungarian

$$t = \frac{p}{y} \quad p' = t + yt' \quad t + yt' = \frac{t-1}{t+1}$$

$$yt' = \frac{t-1-t-t^2}{t+1} = -\frac{t^2+1}{t+1}$$

$$\int \frac{t+1}{t^2+1}\,dt = -\int \frac{dy}{y} = -\ln y + C$$

$$\frac{1}{2}\int \frac{d(t^2+1)}{t^2+1} + \int \frac{dt}{t^2+1} = \frac{1}{2}\ln(t^2+1) + \arctan t$$

$$\ln\sqrt{\frac{p^2}{y^2}+1} + \ln y + \arctan\frac{p}{y} = C$$

$$\boxed{\ln\sqrt{p^2+y^2} + \arctan\frac{p}{y} = C}$$

3)

$$x = y'\sqrt{y'^2+1}$$

$$\boxed{x = p\sqrt{p^2+1}}$$

$$y' = p(x) \qquad \frac{dy}{dx} = p \qquad dy = dx \cdot p$$

$$dx = f'_y\,dy + f'_p\,dp \Rightarrow f'_y = 0$$

$$f'_p = \sqrt{p^2+1} + \frac{p}{2\sqrt{p^2+1}}\,2p = \frac{p^2+1+p^2}{\sqrt{p^2+1}} = \frac{2p^2+1}{\sqrt{p^2+1}}$$

$$dy = p\cdot dx \Rightarrow \int dy = \int \frac{p(2p^2+1)}{\sqrt{p^2+1}}\,dp$$

$$\int \frac{2p^3+p}{\sqrt{p^2+1}} = (Ap^2+Bp+C)\sqrt{p^2+1} + \lambda\int \frac{1}{\sqrt{p^2+1}} \quad \frac{d}{dp}$$

$$\frac{2p^3+p}{\sqrt{p^2+1}} = (2Ap+B)\sqrt{p^2+1} + \frac{(Ap^2+Bp+C)}{\sqrt{p^2+1}}\cdot p + \frac{\lambda}{\sqrt{p^2+1}} \qquad \Big/\sqrt{p^2+1}$$

$$2p^3+p = 2Ap^3 + Bp^2 + 2Ap + B + Ap^3 + Bp^2 + Cp + \lambda$$

$$\boxed{A = \frac{2}{3}} \qquad \boxed{B = 0} \qquad 2A + C = 1 \qquad B + \lambda = 0 \quad \boxed{\lambda = 0}$$

$$C = 1 - \frac{4}{3} = \boxed{-\frac{1}{3}}$$

$$\boxed{y = \left(\frac{2}{3}p^2 - \frac{1}{3}\right)\sqrt{1+p^2}}$$

ЮЗ

2) 4) $y'(x-\ln y')=1$ $\quad y'=p(y) \quad \frac{dx}{dy}=p \quad dx=p\,dy$

$$x=\ln p+\frac{1}{p}$$

$dx=\int p\,dp+\int dx$

$=0$

$dx=\left(\frac{1}{p}-\frac{1}{p^2}\right)dp$

$dx=p\,dy=\left(1-\frac{1}{p}\right)dp \Rightarrow \int dy=\int\left(1-\frac{1}{p}\right)dp$

$$y=p-\ln p+C$$