$$y' = \frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{x'}$$

$$\boxed{y' = \frac{1}{x'}}$$

$$xy - y^2 x' = \pm 2a^2 \quad / : (-y^2)$$

$$\boxed{x' - \underbrace{\frac{1}{y}}_{f(x)} x = \underbrace{\pm \frac{2a^2}{y^2}}_{g(x)}} \quad \text{linearna}$$

$x' = \frac{dx}{dy}$

$C = C(y)$

$\oplus \quad x = Cy + \frac{a^2}{y}$

opće rješenje

$\ominus \quad x = Cy - \frac{a^2}{y}$

30.05.2011.

12. Dif. jed. višeg reda

opći oblik: $(*)$ $F(x, y, y', \dots, y^{(n)}) = 0$ n-tog reda

opći integral od $(*)$: $\Phi(x, y, C_1, \dots, C_n) = 0$

* INTEGRIRANJE SNIŽAVANJEM REDA JED. *

1. Dif. jed. oblika $y^{(n)} = f(x)$

- rješava se uzastopnim integriranjem

$$y^{(n)} = \frac{dy^{(n-1)}}{dx} = f(x) \Rightarrow dy^{(n-1)} = f(x)dx \quad / \int$$

$$y^{(n-1)} = \int f(x)dx + C_1$$

$$y^{(n-2)} = \int\left(\int f(x)dx + C_1\right)dx + C_2$$

itd

① Naći rj. Cauchyevog problema 3. reda

$y''' = 2x$

$y(0) = -1$

$y'(0) = 0$

$y''(0) = 1$

$y''' = \frac{dy''}{dx} = 2x$

$dy'' = 2x\,dx \; / \int$

$y'' = x^2 + C_1$

iz $y''(0) = 1 \Rightarrow 1 = 0^2 + C_1$

$C_1 = 1 \Rightarrow y'' = x^2 + 1$

$\frac{dy'}{dx} = x^2 + 1$

$dy' = (x^2+1)\,dx \; / \int$

$y' = \frac{x^3}{3} + x + C_2$

iz $y'(0) = 0 \Rightarrow 0 = \frac{0^3}{3} + 0 + C_2$

$C_2 = 0 \Rightarrow y' = \frac{x^3}{3} + x$

$\frac{dy}{dx} = \frac{x^3}{3} + x$

$dy = \left(\frac{x^3}{3} + x\right) dx \; / \int$

$y = \frac{x^4}{12} + \frac{x^2}{2} + C_3$

iz $y(0) = -1 \Rightarrow -1 = \frac{0^4}{12} + \frac{0^2}{2} + C_3$

$C_3 = -1 \Rightarrow \boxed{y = \frac{x^4}{12} + \frac{x^2}{2} - 1}$

2. Dif. jed. oblika $F(x, y^{(n)}) = 0$

a) Ako se jed. $F(x, y^{(n)}) = 0$ može riješiti po $y^{(n)}$, postupak se svodi na 1., s time da može biti više jed.

Opći integral je onda:

$$[y - f_1(x, C_1, \dots, C_n)]\,[y - f_2(x, C_1, \dots, C_n)] \dots = 0$$

b) Ako se jed. $F(x, y^{(n)}) = 0$ ne može riješiti po $y^{(n)}$, rješenje tražimo u parametarskom obliku:

$$\begin{cases} x = f(t) \\ y^{(n)} = \varphi(t) \end{cases}$$

za pogodno odabrane funkc. f i φ tako da poč. jed. bude zadovoljena.

$$y^{(n)} = \frac{dy^{(n-1)}}{dx}$$

$$dy^{(n-1)} = y^{(n)}\,dx = \underbrace{\varphi(t)}_{y^{(n)}} \cdot \underbrace{f'(t)\,dt}_{dx} \quad / \int$$

$$y^{(n-1)} = \int \varphi(t)\, f'(t)\,dt + C_1 = \varphi_1(t, C_1)$$

$$\begin{cases} x = f(t) \\ y^{(n-1)} = \varphi_1(t, C_1) \end{cases}$$

itd. n-puta

$\Rightarrow$ dobije se opće rj. u parametarskom obliku

$$\begin{cases} x = f(t) \\ y = \varphi_n(t, C_1, \dots, C_n) \end{cases}$$

① Naći opće rj. dif. jed.

$$x - y'' e^{y''} = 0$$

PARAMETRIZACIJA JED. $y'' = t \Rightarrow x - te^t = 0$

$$\begin{cases} x = te^t \\ y'' = t \end{cases}$$

$y'' = t \to \dfrac{dy'}{dx} = t$

$dy' = t\,dx = t\,(1 \cdot e^t + te^t)\,dt$

$\hookrightarrow$ iz $x = te^t$

$$y' = \int \underbrace{e^t (t^2 + t)\,dt}_{dv}, \quad \underbrace{(t^2+t)}_{u} = \ldots = (t^2 - t + 1)e^t + C_1$$

$$\begin{cases} x = te^t \\ y' = (t^2 - t + 1)e^t + C_1 \end{cases}$$

$y' = \dfrac{dy}{dx}$

$$dy = y'dx = \underbrace{\left[(t^2 - t + 1)e^t + C_1\right]}_{y'} \underbrace{(e^t + te^t)\,dt}_{dx} \quad / \int$$

$$y = \int \Big[e^{2t} \underbrace{(t^2 - t + 1)(t + 1)}_{t^3 + 1} + C_1 e^t (t+1) \Big]\,dt$$

***

$$y = \frac{1}{8}(4t^3 - 6t^2 + 6t + 1)e^{2t} + C_1 te^t + C_2$$

opće rj. u par. obliku:

$$\begin{cases} x = te^t \\ y = \frac{1}{8}(4t^3 - 6t^2 + 6t + 1)e^{2t} + C_1 te^t + C_2 \end{cases}$$

3. Dif. jed. oblika $F(x, y^{(k)}, \ldots, y^{(n)}) = 0$

$\Rightarrow$ nema $y, y', \ldots, y^{(k-1)}$

SUPST. $y^{(k)} = z$ $\quad$ z - nova zav. var.

$$y^{(k+1)} = \frac{d}{dx} y^{(k)} = \frac{dz}{dx} = z'$$

$\Rightarrow$ ovom supst. snižava se red dif. jed. za k

$$F(x, z, \ldots, z^{(n-k)}) = 0$$

(1) Naći opće rj. dif. jed.

$y'' + (y')^2 + 1 = 0$ $\quad$ - nema y

SUPST $y' = z$

$$y'' = \frac{dy'}{dx} = \frac{dz}{dx} = z'$$

$$z' + z^2 + 1 = 0$$

$$\frac{dz}{z^2+1} + dx = 0 \quad / \int$$

$$\operatorname{arctg} z = -x + C_1$$

$$z = \operatorname{tg}(-x + C_1)$$

$$y' = \operatorname{tg}(-x + C_1)$$

$$dy = \operatorname{tg}(-x + C_1)\, dx \quad / \int$$

$$y = \int \operatorname{tg}(-x + C_1)\, dx + C_2$$

opće rj. $y = \ln|\cos(-x + C_1)| + C_2$

01.06.2011.

4. Dif. jed. oblika $F(y, y', \ldots, y^{(n)}) = 0$

- nema 'x'
- red dif. jed. snižava se za 1 supst. $\boxed{\begin{matrix} y' = p \\ p = p(y) \end{matrix}}$

$$y'' = \frac{dy'}{dx} = \frac{dp}{dx}\frac{dy}{dy} = \frac{dy}{dx}\frac{dp}{dy} = p \cdot p'$$

$$y''' = \frac{dy''}{dx} = \frac{d}{dx}(pp')\frac{dy}{dy} = \frac{dy}{dx}\frac{d}{dy}(pp') = p\left[(p')^2 + pp''\right]$$

- poč. jed. prelazi u jed.:

$$F(y, y', \ldots, y^{(n)}) = F(y, p, pp', p(p')^2 + p^2p'', \ldots) =$$
$$= F(y, p, \ldots, p^{(n-1)}) = 0$$

1 Naći opće rj.

$$yy'' + (y')^2 = 0 \rightarrow \text{nema 'x'}$$

SUPST. $y' = p$, $p = p(y)$

$y'' = pp'$

$$y \cdot pp' + p^2 = 0$$
$$p(yp' + p) = 0 \quad /: p \neq 0$$
$$yp' + p = 0$$
$$y\frac{dp}{dy} + p = 0$$
$$\frac{dp}{p} + \frac{dy}{y} = 0 \,/ \int$$
$$\ln|p| + \ln|y| = \ln C$$
$$|py| = C_{\pm}$$

$py = C_1$

$y'y = C_1$

$\frac{dy}{dx} y = C_1$

$y\,dy = C_1\,dx \quad / \int$

$\frac{y^2}{2} = C_1 x + \frac{C_2}{2} \quad / \cdot 2$

$y^2 = K_1 x + K_2$ opći integral

$y = \pm \sqrt{K_1 x + K_2}$ OPĆE RJ.

$\Rightarrow$ slučaj $p = 0$

$y' = \frac{dy}{dx} = 0 \Rightarrow y = C_1$ SINGULARNO

** 5. Dif. jed. oblika $F(x, y, y', \ldots, y^{(n)}) = 0$ homogene u varijablama $y, y', \ldots, y^{(n)}$

$\Rightarrow$ to znači: $F(x, ty, ty', \ldots, ty^{(n)}) = t^{\alpha} F(x, y, y', \ldots, y^{(n)})$

- postiže se snižavanje reda zadane jed. za 1

supst.: $\boxed{y = e^{\int z\,dx}}$, z - nova zav. var.

$y' = e^{\int z\,dx} \cdot z = z \cdot y$

$y'' = \frac{d}{dx}(z \cdot y) = z'y + zy' = z'y + z^2 y = y(z' + z^2)$

$y''' = \ldots = y(z'' + 3zz' + z^3)$

① Naći opće rj.:

**

$$x^2 y y'' = (y - xy')^2 \rightarrow \text{dif. jed. homogena u } y, y', y''$$

$\alpha = 2$

SUPST. $y = e^{\int z dx}$, $z = z(x)$ nova zav. var

$$y' = e^{\int z dx} z$$

$$y'' = e^{\int z dx} z \cdot z + e^{\int z dx} z' = e^{\int z dx}(z^2 + z')$$

$$x^2 \cdot e^{\int z dx} \cdot e^{\int z dx}(z^2 + z') = (e^{\int z dx} - x e^{\int z dx} \cdot z)^2$$

$$x^2(z^2 + z') e^{2\int z dx} = e^{2\int z dx}(1 - xz)^2 \quad / : e^{2\int z dx}$$

$$x^2(z^2 + z') = (1 - xz)^2$$

$$\cancel{x^2 z^2} + x^2 z' = 1 - 2xz + \cancel{x^2 z^2} \quad / : x^2$$

$$z' + \frac{2z}{x} = \frac{1}{x^2} \qquad \text{LINEARNA}$$

*** (varijacija konst.)

$$z = \frac{C_1}{x^2} + \frac{1}{x}$$

$$y = e^{\int z dx} = e^{\int (\frac{C_1}{x^2} + \frac{1}{x}) dx} = e^{-\frac{C_1}{x} + \ln|x| + C_2}$$

$$y = e^{C_2} e^{\ln|x|} \cdot e^{-\frac{C_1}{x}}$$

$$y = K_2 |x| e^{-\frac{K_1}{x}} \qquad \text{opće rj.}$$

6. Dif. jed. za koje vrijedi: $F(x, y, \ldots, y^{(n)}) = \frac{d}{dx}\left[\Phi(x, y, \ldots, y^{(n-1)})\right] = 0$

- ako je gornji uvjet ispunjen dif. jed. možemo direktno integrirati tj. pomnožimo s dx i dobijemo:

$$\int d\Phi(x, y, \ldots, y^{(n-1)}) = \int 0 \, dx = C_1$$

$$\Phi(x, y, \ldots, y^{(n-1)}) = C_1$$

pa je dakle red poč. jed. snižen za 1

① Naći opće rj.

$$y'' + xy' + y = 0 \iff \frac{d}{dx}(y' + xy) = 0$$

$$d(y' + xy) = 0 \quad / \int$$

$$\int d(y' + xy) = C_1$$

$$y' + xy = C_1 \qquad \text{LINEARNA}$$

***

$$y = e^{-\frac{x^2}{2}}\left[C_1 \int e^{\frac{x^2}{2}} dx + C_2\right] \quad \text{opće rj.}$$

② $yy'' = 2(y')^2 \; /: y, y' \iff \frac{y''}{y'} = 2\frac{y'}{y} \iff \frac{d}{dx}(\ln(y')) = 2\frac{d}{dx}(\ln y)$

$$(\ln y)' = \frac{1}{y} y'$$

$$\iff \frac{d}{dx}\left[\ln(y') - 2\ln y\right] = 0 \quad / \cdot dx, \int$$

$$\int d\left[\ln(y') - 2\ln y\right] = C_1$$

$$\ln(y') - 2\ln y = C_1$$

$$\ln \frac{y'}{y^2} = C_1$$

$$\frac{y'}{y^2} = K_1 \implies -\frac{1}{y} = K_1 x + K_2$$

$$y = \frac{1}{C_1 x + C_2} \quad \text{opće rj.}$$

③ Riješite Cauchyev problem:

$$y'' = 2y^3 \quad \to \text{nema 'x'}$$

$$\begin{cases} y(0) = 1 \\ y'(0) = 1 \end{cases} \qquad \text{SUPST. } y' = p, \; p = p(y)$$

$$y'' = \frac{dy'}{dx} = \frac{dp}{dx}\frac{dy}{dy} = \frac{dy}{dx}\frac{dp}{dy} = p \cdot \frac{dp}{dy}$$

$$p \cdot \frac{dp}{dy} = 2y^3$$

$\Rightarrow$

$pdp = 2y^3dy \quad / \int$

$\frac{p^2}{2} = \cancel{2}\frac{y^4}{\cancel{4}} + \frac{C_1}{2} \quad / \cdot 2$

$p^2 = y^4 + C_1$

$\Rightarrow$ uzimamo u obzir poč. uvjet: $x=0,\ y=1,\ y'=1$

$$1^2 = 1^4 + C_1 \Rightarrow C_1 = 0$$

$$p^2 = y^4 \quad / \sqrt{\ }$$

$$p = \pm y^2$$

$$\frac{dy}{dx} = \pm y^2$$

$$\frac{dy}{y^2} = \pm dx \quad / \int$$

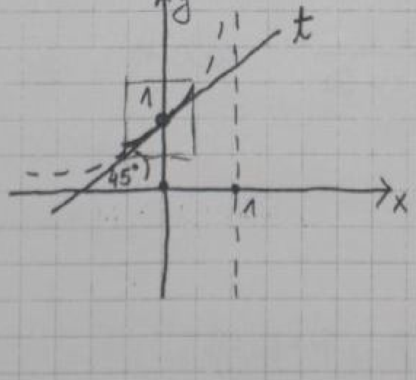
$$-\frac{1}{y} = \pm x + C_2$$

$\Rightarrow$ poč. uvjet: $-\frac{1}{1} = \pm 0 + C_2 \Rightarrow C_2 = -1$

$$-\frac{1}{y} + 1 = \pm x \quad /^2$$

$$x^2 = \left(\frac{y-1}{y}\right)^2$$

1) $y = \frac{1}{1-x}$ $\Rightarrow$ jedno od rješenja



* LINEARNE DIF. JEDNADŽBE *

Jed. oblika: (*) $A_n(x)y^{(n)} + A_{n-1}(x)y^{(n-1)} + \dots + A_1(x)y' + A_0(x)y = f(x)$
zove se LINEARNA DIF. JED. n-tog REDA

FUNKC. SMETNJE

Ako je funk. smetnje $f(x) \equiv 0$ kažemo da je homogena.
Ako su funk. $A_n, A_{n-1}, \dots, A_1, A_0$ konstante jed. (*) je LINEARNA DIF. JED. n-tog REDA S KONST. KOEFICIJENTIMA.

* LINEARNA DIF. JED. 2. REDA *

Jed. oblika: (*) $y'' + p(x)y' + q(x)y = 0$ zove se HOMOGENA LINEARNA DIF. JED. DRUGOG REDA S NEKONST. KOEF.

** STAVAK 1.

A. Ako je y_1 rješenje od (*) onda je rj. također i funk. Cy_1, za bilo koju vrijed. od C

B. Ako su y_1 i y_2 dva rj. od (*), onda je i funkc. $y_1 + y_2$ također rj. od (*)

DOKAZ A.

$$y_1'' + p(x)y_1' + q(x)y_1 \equiv 0$$
$$(Cy_1)'' + p(x)(Cy_1)' + q(x)(C_1y_1) =$$
$$= C[y_1'' + p(x)y_1' + q(x)y_1] \equiv 0 \text{ prema pretp.}$$

Q.E.D

DOKAZ B.

$\Rightarrow$ 2 pretp. : $\begin{cases} y_1'' + p(x) y_1' + q(x) y_1 \equiv 0 \\ y_2'' + p(x) y_2' + q(x) y_2 \equiv 0 \end{cases}$

$$\Rightarrow (y_1 + y_2)'' + p(x)(y_1 + y_2)' + q(x)(y_1 + y_2) =$$
$$= (y_1'' + y_2'') + p(x)(y_1' + y_2') + q(x)(y_1 + y_2) =$$
$$= y_1'' + p(x) y_1' + q(x) y_1 + y_2'' + p(x) y_2' + q(x) y_2 \equiv 0 + 0 \equiv 0$$

Q.E.D.

STAVAK 2.

Ako je $y_1(x)$ bilo koje rj. hom. lin. dif. jed. (*), onda je svako drugo rj. oblika:

$$\boxed{y = C_1 y_1(x) + C_2 y_2(x)} \quad !!$$

gdje je:

$$y_2(x) = y_1(x) \cdot \int \left[\frac{1}{y_1^2(x)} \cdot e^{-\int p(x)dx} \right] dx$$

$\Rightarrow$ DOKAZ STR. 78, 79 - NE TREBA ZNATI !

03

(1) Naći opće rj. dif. jed. $xy'' + 2y' + xy = 0$ znajući da je $y_1 = \frac{\sin x}{x}$ jedno njezino rj.

$$y = \underbrace{C_1 y_1(x)}_{\text{poznato}} + \underbrace{C_2 y_2(x)}_{\text{iz S2}}$$

PROVJERA:

$$x\left(\frac{\sin x}{x}\right)'' + 2\left(\frac{\sin x}{x}\right)' + x\,\frac{\sin x}{x} = 0$$

$$y_2(x) = y_1(x)\int\left(\frac{1}{[y_1(x)]^2}\, e^{-\int p(x)\,dx}\right)dx$$

$$y'' + p(x)y' + q(x)y = 0$$

$$y'' + \underbrace{\frac{2}{x}}_{p(x)}y' + \underbrace{1}_{q(x)}\cdot y = 0$$

$$e^{-\int p(x)dx} = e^{-\int \frac{2}{x}dx} = e^{-2\int\frac{dx}{x}} = e^{\ln(x^{-2})} = \frac{1}{x^2}$$

$$y_2(x) = y_1(x)\int\left(\frac{1}{[y_1(x)]^2}\, e^{-\int p(x)dx}\right)dx =$$

$$= \frac{\sin x}{x}\int\left(\frac{x}{\sin x}\right)^2 \frac{1}{x^2}\,dx = \frac{\sin x}{x}\int\frac{dx}{\sin x} = \frac{\sin x}{x}(-\mathrm{ctg}x) =$$

$$= -\frac{\cancel{\sin x}}{x}\,\frac{\cos x}{\cancel{\sin x}} = -\frac{\cos x}{x}$$

OPĆE RJ.

$$y = C_1 y_1(x) + C_2 y_2(x) = C_1\frac{\sin x}{x} + C_2\left(-\frac{\cos x}{x}\right)$$

$$y = \frac{1}{x}\left[K_1 \sin x + K_2\cos x\right]$$

$$\left.\begin{aligned} y_1(x) &= \frac{\sin x}{x} \\ y_2(x) &= -\frac{\cos x}{x}\end{aligned}\right\}\ \text{BAZA OPĆEG RJ.}$$

* LINEARNE HOMOGENE DIF. JED. 2. REDA S KONST. KOEF. *

opći oblik: $\boxed{y'' + a_1y' + a_0y = 0}$ (1)

$\Rightarrow$ potražimo rj. jed. (1) u obliku $y = e^{rx}$

$$\left.\begin{aligned} y' &= re^{rx} \\ y'' &= r^2e^{rx}\end{aligned}\right\}\ u\ (1) \Rightarrow r^2e^{rx} + a_1re^{rx} + a_0e^{rx} = 0$$

$$e^{rx}(r^2 + a_1r + a_0) = 0 \quad /:e^{rx}$$

ALGEBARSKA JED. u var. r – ~~KARAKTERISTIČNA~~ JED. PRIDRUŽENA DIF. JED. (1) $\boxed{r^2 + a_1r + a_0 = 0}$ (2)

$\Rightarrow$

$u\ (1) \Rightarrow \left.\begin{array}{l} y \rightsquigarrow 1 \\ y' \rightsquigarrow r \\ y'' \rightsquigarrow r^2 \end{array}\right\} \Rightarrow (2)$

Neka su r_1, r_2 rješenja karakter. jed. (2). S obzirom na diskriminantu kvadratne jed. imamo 3 mogućnosti:

1) r_1, r_2 REALNI I RAZLIČITI

$\left.\begin{array}{l} y_1 = e^{r_1 x} \\ y_2 = e^{r_2 x} \end{array}\right\}$ LINEARNO NEZAVISNA RJ. OD (1)

OPĆE RJ. $y = C_1 y_1(x) + C_2 y_2(x) = C_1 e^{r_1 x} + C_2 e^{r_2 x}$

2) $r_1 = r_2 = r = -\frac{a_1}{2}$ $(D=0)$

$y_1 = e^{r_1 x} = e^{rx}$ → jedno rj.

⇒ koristeći S2 nalazimo 2. lin. nez. rj. $y_2(x)$

$$y_2(x) = y_1(x) \int \left(\frac{1}{y_1^2(x)} e^{-\int p(x)dx} \right) dx =$$

$$= e^{rx} \int - \frac{1}{e^{2rx}} e^{\underbrace{-\int a_1 dx}_{e^{-a_1 x}}} dx =$$

$$= e^{rx} \int e^{\overbrace{(a_1 - 2r)}^{=0} x} dx = x e^{rx}$$

$\left.\begin{array}{l} y_1(x) = e^{rx} \\ y_2(x) = x e^{rx} \end{array}\right\}$ BAZA RJ.

OPĆE RJ. $y = C_1 e^{rx} + C_2 x e^{rx} = e^{rx}(C_1 + x C_2)$

3) r_1, r_2 KONJUGIRANO KOMPLEKSNI

$r_1 = \alpha + i\beta$, $r_2 = \alpha - i\beta$

$\Rightarrow$ korištenjem čuvene Eulerove formule:

$$e^{ix} = \cos x + i \sin x$$
$$e^{-ix} = \cos x - i \sin x$$

možemo opće rj. zapisati u obliku:

$$y = C_1 e^{r_1 x} + C_2 e^{r_2 x} = C_1 e^{(\alpha + i\beta)x} + C_2 e^{(\alpha - i\beta)x} =$$
$$= C_1 e^{\alpha x} \cdot e^{i\beta x} + C_2 e^{\alpha x} e^{-i\beta x} =$$
$$= C_1 e^{\alpha x}\left[\cos(\beta x) + i \sin(\beta x)\right] +$$
$$+ C_2 e^{\alpha x}\left[\cos(\beta x) - i \sin(\beta x)\right] =$$
$$= e^{\alpha x}\left[(C_1 + C_2)\cos(\beta x) + i(C_1 - C_2)\sin(\beta x)\right] =$$
$$= e^{\alpha x}\left[K_1 \cos(\beta x) + K_2 \sin(\beta x)\right] \text{ OPĆE RJ.}$$

Dakle imamo ovaj stavak:

** STAVAK 3.

Homogena lin. dif. jed. 2. reda s konst. koef.

$$y'' + a_1 y' + a_0 y = 0$$

ima za bazu rješenja sljedeće funkcije, ovisno o korijenima r_1, r_2 pripadne karakteristične jed.

$$r^2 + a_1 r + a_0 = 0$$

1) r_1, r_2 realni i različiti

$$\left.\begin{aligned} y_1 &= e^{r_1 x} \\ y_2 &= e^{r_2 x} \end{aligned}\right\}$$

2) $r_1 = r_2 = r$

$$\left.\begin{aligned} y_1 &= e^{rx} \\ y_2 &= x e^{rx} \end{aligned}\right\}$$

$\Rightarrow$

3) $r_1 = \alpha + i\beta$, $r_2 = \alpha - i\beta$

$$y_1 = e^{\alpha x} \cos(\beta x)$$

$$y_2 = e^{\alpha x} \sin(\beta x)$$

Opće rj. u ova tri slučaja je:

$$y = C_1 y_1 + C_2 y_2$$

Naći opće rj. dif. jed.

① $y'' + y' - 2y = 0$

KARAKT. JED. $y \rightsquigarrow 1$, $y' \rightsquigarrow r$, $y'' \rightsquigarrow r^2$

$$f(r) \equiv r^2 + r - 2 = 0 \Rightarrow r_{1,2} = \frac{-1 \pm 3}{2}$$

$$r_1 = -2$$

$$r_2 = 1$$

opće rj. $y = C_1 e^{-2x} + C_2 e^{x}$

② $y'' + 5y' = 0$

karakt. jed: $r^2 + 5r = 0$

$$r(r+5) = 0$$

$$r_1 = -5$$

$$r_2 = 0$$

$$y = C_1 e^{-5x} + C_2 e^{0x} = C_1 e^{-5x} + C_2$$

③ $y'' + 4y' + 4y = 0$

$$r^2 + 4r + 4 = 0$$

$$(r+2)^2 = 0$$

$$r_{1,2} = -2$$

$$y = C_1 e^{-2x} + C_2 x e^{-2x} = e^{-2x}(C_1 + C_2 x)$$

(4) $y'' - 2y' + 5y = 0$

$$r^2 - 2r + 5 = 0$$

$$r_{1,2} = \frac{2 \pm \sqrt{-16}}{2} = \frac{2 \pm 4i}{2}$$

$$r_1 = 1 + 2i$$

$$r_2 = 1 - 2i$$

$$y = e^{1x}\left[C_1 \cos(2x) + C_2 \sin(2x)\right]$$

(5) $y'' + 4y = 0$

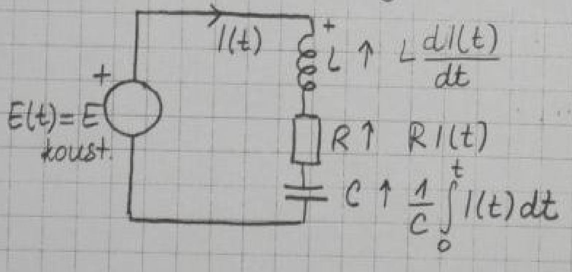
$$r^2 + 4 = 0$$

$$r_{1,2} = \pm 2i \begin{cases} 0 + 2i & \alpha = 0 \\ 0 - 2i & \beta = 2 \end{cases}$$

$$y = e^{0}\left[C_1 \cos(2x) + C_2 \sin(2x)\right] = C_1 \cos(2x) + C_2 \sin(2x)$$

* PRIMJENA DIF. JED. *

⇒ SERIJSKI RLC krug



$$E(t) = E = L\frac{dI(t)}{dt} + RI(t) + \frac{1}{C}\int_0^t I(t)dt \quad / \frac{d}{dt}$$

$$LI''(t) + RI'(t) + \frac{1}{C} I(t) = 0$$

$$I''(t) + \underbrace{\frac{R}{L}}_{a_1} I'(t) + \underbrace{\frac{1}{LC}}_{a_0} I(t) = 0$$

KRITIČNI SLUČAJ: $\Delta = \left(\frac{R}{L}\right)^2 - 4\frac{1}{LC} = 0$

$$\frac{R^2}{L^2} - \frac{4}{LC} = 0 \quad / L^2C$$

$$R^2C = 4L$$

⇒ PROČITATI IZ KNJIŽICE

$$R^2 = \frac{4L}{C}$$