PRIMJERI DIF. JDBI

a) jednadžba separiranih varijabla

$f(y)\,dy = g(x)\,dx$

1) $yy' - x = x^3$ prebacimo varijable na suprotne strane

$yy' = x^3 + x$

$\frac{y\,dy}{dx} = x^3 + x$ Tražimo oblik $f(y)dy = g(x)dx$

$y\,dy = (x^3 + x)\,dx \;/ \int$ integriramo

$\frac{y^2}{2} = \frac{x^4}{4} + \frac{x^2}{2} + C \;/ \cdot 2$

$y^2 = \frac{x^4}{2} + x^2 + 2C = \frac{x^4}{2} + x^2 + C$

2) $y' \operatorname{tg} x = y$

$\int \frac{dx}{\operatorname{tg} x} = \int \frac{\cos x}{\sin x}\,dx = \left| \begin{array}{l} t = \sin x \\ dt = \cos x\,dx \end{array} \right.$

$= \int \frac{dt}{t} = \ln|t| + \ln C = \ln|\sin x \cdot c|$

$\frac{y'}{y} = \frac{1}{\operatorname{tg} x}$

$\frac{dy}{y\,dx} = \frac{1}{\operatorname{tg} x}$

$\frac{dy}{y} = \frac{dx}{\operatorname{tg} x} \;/ \int$

$\ln|y| = \int \frac{dx}{\operatorname{tg} x}$ supstitucija

$\ln|y| = \ln|\sin x \cdot c|$ zbog c možemo maknut apsolutno

$\underline{y = \sin x \cdot c}$

b) homogene jednadžbe prvog reda

$M(tx,ty)=t^{\alpha}M(x,y)$ $\rightarrow$ rješava se supstitucijom

$y'=f\left(\frac{y}{x}\right)$

1) $y'=-\frac{x+y}{x}$

$y'=-\frac{x}{x}-\frac{y}{x}$

$y'=-1-\frac{y}{x}$ supstitucija $z=\frac{y}{x}$; $z=\frac{y}{x}\Big|'$

$$dz=\frac{dy\cdot x-dx\cdot y}{x^2}$$

$$x^2dz=dy\cdot x-dx\cdot y$$

$$dy\cdot x=x^2dz+dx\cdot y \;/:dx$$

$$\frac{dy}{dx}\cdot x=x^2\frac{dz}{dx}+y \;/:x$$

$$\frac{dy}{dx}=x\frac{dz}{dx}+\frac{y}{x}$$

$$y'=xz'+z$$

$xz'+z=-1-z$

$xz'=-1-2z$

$\frac{dz}{1+2z}=-\frac{dx}{x}$

$\ln|1+2z|=\ln\left|\frac{c}{x}\right|$

$1+2z=\frac{c}{x}$

$2z=\frac{c}{x}-1$

$z=\frac{c}{2x}-\frac{1}{2}$

$z=\frac{c}{x}-\frac{1}{2}$

$\frac{y}{x}=\frac{c}{x}-\frac{1}{2}$

$\boxed{y=c-\frac{x}{2}}$

c) Svođenje na homogene jednadžbe

$$y' = \frac{1-3x-3y}{1+x+y}$$

$1-3x-3y=0$

$1+x+y=0$

nema rj.

$z = x+y \; /'$ supstitucija

$dz = dx + dy$

$dy = dz - dx \; /:dx$

$y' = z' - 1$

$$\frac{1+z}{2(1-z)} = \frac{(z-1)+2}{-2(z-1)}$$

$$z' - 1 = \frac{1-3z}{1+z}$$

$$z' = \frac{1-3z+1+z}{1+z}$$

$$z' = \frac{2-2z}{1+z}$$

$$\frac{1+z}{2(1-z)}dz = dx \; / \int$$

$-z + 2\ln|z-1| = 2x + C$

$C = 3x + y + 2\ln|x+y-1|$

d) $y' + p(x)y = g(x)$

$y' + p(x)y = 0$

$y = y_h + y_p$

1) $\frac{dy}{dx} - \frac{y}{x} = x$

pripadna homogena jdba:

$\frac{dy}{dx} - \frac{y}{x} = 0$

$\frac{dy}{dx} = \frac{y}{x}$ integriramo

$\ln|y| = \ln|x| + \ln|C|$

$y_h = xC$ varijacija konstanta

$y = x \cdot C(x) \quad /'$

Vraćamo u početnu

$y' = x' \cdot C(x) + xC'(x)$

$C(x) + x\,C'(x) - C(x) = x$

$C'(x) = 1 \quad / \int$

$C(x) = x + C_1$

$y = x(x + C_1) = x^2 + xC_1$

2) $\frac{dy}{dx} + \frac{2y}{x} = x^3$

Pripadna homogena

$y' + \frac{2y}{x} = 0$

$y' = -\frac{2y}{x}$

$-\frac{1}{2} \ln|y| = \ln|xc|$

$\frac{1}{\sqrt{y}} = xc$

$y = \frac{c}{x^2}$

$\left.\begin{array}{l} y = \frac{c(x)}{x^2} \quad /' \\ y' = \frac{c'(x) \cdot x^2 - c(x) \cdot 2x}{x^4} \end{array}\right\}$ vraćamo u početnu

$\frac{c'(x) \cdot x^2 - \cancel{c(x) \cdot 2x}}{x^4} + \frac{2c(x)}{\cancel{x^3}} = x^3$

$c'(x) = x^5 \quad / \int$

$c(x) = \frac{x^6}{6} + c_1$

$y = \frac{c_1}{x^2} + \frac{x^4}{6}$

e) bernulijeva jednadžba

$$y' + P(x)y = Q(x)y^n \qquad n \neq 0 \quad n \neq 1 \qquad z = y^{1-n}$$

$$y' = \frac{4}{x}y + x\sqrt{y}$$

$n = \frac{1}{2}$

$$z = y^{\frac{1}{2}}$$

$$z' = -\frac{y'}{2y^{\frac{1}{2}}}$$

$$y' = -2z' \cdot z$$

$$-2z'z = \frac{4}{x}z^2 + xz \Big/ : z$$

$$-2z' = \frac{4}{x}z + x \Big/ : -2$$

$$z' + \frac{2z}{x} = -\frac{x}{2}$$

pripadna homogena

$$z' + \frac{2z}{x} = 0$$

$$z' = \frac{-2z}{x}$$

$$-\frac{1}{2}\ln|z| = \ln|x \cdot c| \Big/ \cdot (-2)$$

$$z = \frac{c}{x^2}$$

$$\left.\begin{aligned} z &= \frac{c(x)}{x^2} \\ z' &= \frac{c'(x)x^2 - 2xc(x)}{x^4} \end{aligned}\right\} \text{Vraćamo}$$

$$\frac{c'(x)\cancel{x^2} - \cancel{2xc(x)}}{x^{\cancel{4}2}} + \frac{\cancel{2c(x)}}{\cancel{x^3}} = -\frac{x}{2}$$

$$c'(x) = -\frac{x^3}{2} \Big/ \int$$

$$c(x) = -\frac{x^4}{8} + C_1 \qquad y = \left(-\frac{x^2}{8} + \frac{C_1}{x^2}\right)^2$$

$$z = \frac{-x^2}{8} + \frac{C_1}{x^2}$$

4) EGZAKTNE JEDNADŽBE

$$P(x,y)dx + Q(x,y)dy = 0$$

$$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \quad \text{onda} \quad U(x,y) = \int_{x_0}^{x} P(x,y)dx + \int_{y_0}^{y} Q(x_0,y)dy$$

$x_0, y_0 \rightarrow$ biramo iz domene

ako uvjet nije zadovoljen onda eulerov multiplikator

1) $(x+y)dx + (x+2y)dy = 0$

$P = x+y$

$Q = x+2y$

$P_y' = 1$

$Q_x' = 1$

$$U(x,y) = \int_0^x P(x,y)\,dx + \int_0^y Q(0,y)\,dy = \frac{x^2}{2} + xy + y = C$$

2) $(x+y^2)dx - 2xy\,dy = 0$

$P = x+y^2$

$Q = -2xy$

$P_y' = 2y$

$Q_x' = -2y$

$$\ln M(x) = \int \frac{1}{-2xy}(4y)dx = \int \frac{-2}{x}dx$$

$\ln M(x) = -2\ln|x|$

$M(x) = \frac{1}{x^2}$

$P = \frac{1}{x} + \left(\frac{y}{x}\right)^2$

$Q = -2\frac{y}{x}$

$$U(x,y) = \int_0^x P(x,0)\,dx + \int_0^y Q(x,y)\,dy = \ln|x| - \frac{y^2}{x} = C$$

7) $P = y + \ln x$

$Q = -x$

$P'_y = 1$

$Q'_x = -1$

$$\ln \mu(x) = \int \frac{1}{-x} (2) \, dx$$

$$\ln \mu(x) = -2 \int \frac{1}{x} \, dx$$

$$\mu(x) = \frac{1}{x^2}$$

$$P = \frac{y}{x^2} + \frac{\ln x}{x^2}$$

$$Q = -\frac{1}{x}$$

$$U(x,y) = \int_1^x \frac{\ln x}{x^2} \, dx + \int_0^y -\frac{1}{x} \, dy$$

$$= -\frac{y}{x} + \left(-\frac{\ln x}{x} - \frac{1}{x} \right) \Bigg|_1^x$$

$$= -\frac{y}{x} - \frac{\ln x}{x} - \frac{1}{x}$$