(1) $\underbrace{(3x+6y^2)}_{P}dx + \underbrace{\left(6xy+\frac{4y^3}{x}\right)}_{Q}dy = 0$ EGZAKTNA ?

$P'_y = 12y$

$Q'_x = 6y + 4y^3 \cdot \frac{-1}{x^2} = 6y - \frac{4y^3}{x^2}$ } $P'_y \neq Q'_x$

EULEROV MULTIPLIK.

$\boxed{\ln \mu = \int \frac{1}{Q}(P'_y - Q'_x)\,dx}$

$P'_y - Q'_x = 6y + \frac{4y^3}{x^2}$

$\frac{1}{Q}\cdot(P'_y - Q'_x) = \frac{1}{6xy+\frac{4y^3}{x}} \cdot \frac{6y+\frac{4y^3}{x^2}}{1} =$

$= \frac{x}{\cancel{6x^2y+4y^3}} \cdot \frac{\cancel{6x^2y+4y^3}}{x^2} = \frac{1}{x}$

$\ln \mu = \int \frac{1}{x}\,dx = \ln|x| \quad / e^{\wedge}$

$\boxed{\mu = x}$ - pomnožimo jed. s μ kako bi dobili egzaktnu

$\underbrace{(3x^2+6xy^2)}_{P}dx + \underbrace{(6x^2y+4y^3)}_{Q}dy = 0$

- ako hoćete možete provjeriti da li je egzaktno ali nije obavezno ($P'_y = Q'_x$)

$u(x,y) = \int_0^x (3x^2+6xy^2)\,dx + \int_0^y (6x_0^2y+4y^3)\,dy$ $\boxed{x_0=0}$

$= \frac{3x^3}{3} + \frac{6y^2x^2}{2} + \frac{4y^4}{4}$

$= \boxed{x^3 + 3x^2y^2 + y^4 = C}$ RJEŠENJE

② $y' + \frac{y}{x} = x^8$

$y' + p(x) \cdot y = q(x) \Rightarrow$ LDJ 1. reda

(1-8, str. 2 PDF)

1. korak $\Rightarrow$ rješavamo homogenu

$$y' + \frac{y}{x} = 0$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$$\frac{1}{y} dy = \frac{-1}{x} dx \quad / \int$$

$$\int \frac{1}{y} dy = -\int \frac{1}{x} dx + C$$

$$\ln|y| = -\ln|x| + C$$

$$\ln|y| = \ln\left|\frac{C}{x}\right| \quad / e^{\wedge}$$

$$\boxed{y_H = \frac{C}{x}}$$

$$y = \frac{\frac{x^{10}}{10} + C}{x} = \boxed{\frac{x^9}{10} + \frac{C}{x} = y}$$

konačno rješenje

2. korak $\Rightarrow$ metodom varijacije konst. određujemo C

$$\left.\begin{aligned} y &= \frac{C(x)}{x} \\ y' &= \left(\frac{C(x)}{x}\right)' = \frac{C'(x) \cdot x - C(x)}{x^2} \end{aligned}\right\}$$ uvrštavamo u zadanu jednadžbu

$$\frac{C'(x)x - C(x)}{x^2} + \frac{\frac{C(x)}{x}}{\frac{x}{1}} = x^8$$

$$\frac{C'(x)x - C(x)}{x^2} + \frac{C(x)}{x^2} = x^8$$

$$\frac{C'(x)\cancel{x} \cancel{- C(x) + C(x)}}{x^{\cancel{2}}} = x^8 \quad / \cdot x$$ ($C(x)$ se uvijek mora pokratiti)

$$C'(x) = x^9 \quad / \int$$

$$\boxed{C(x) = \frac{x^{10}}{10} + C}$$ – uvrštavamo u homogenu

(3) krivulja ≠ pravac = ?

tangenta ⇒ trokut $P = 4$

$$\hookrightarrow \frac{x}{a} + \frac{y}{b} = 1 \Big/ \frac{d}{dx} \qquad P_{\Delta} = \frac{|a \cdot b|}{2}$$

(3)

$$\frac{1}{a} + \frac{y'}{b} = 0 \qquad |a \cdot b| = 8$$

(1) $\boxed{y' = \frac{-b}{a}}$ $\qquad \boxed{b = \pm\frac{8}{a}}$ $\qquad b = \pm\frac{8}{\sqrt{\frac{-8}{y'}}}$

(2) $y' = \frac{-8}{a^2}$

$$\frac{y}{b} = 1 - \frac{x}{a} \Big/ \cdot b \qquad a^2 = \frac{-8}{y'}$$

$$y = b - \frac{xb}{a} \Bigg| \begin{matrix}\text{uvrstimo} \\ a \cdot b\end{matrix} \Bigg| \qquad \boxed{a = \sqrt{\frac{-8}{y'}}}$$

$$y = \frac{8}{\sqrt{\frac{-8}{y'}}} + \frac{8x}{\frac{-8}{y'}}$$

$\psi(y')$

$$y = xy' + 8 \cdot \frac{1}{\sqrt{\frac{-8}{y'}}}$$ CLAIRAUTOVA DIF. JED. (2.13 PDF, str. 5)

$$y = xy' + \psi(y')$$

$$y = xp + 8\sqrt{\frac{-p}{8}} \Big/ \frac{d}{dx}$$

− zamjena $y' = p$ +

$$y' = p = p + xp' + \left(8\sqrt{\frac{-p}{8}}\right)'$$

$$\frac{1}{\sqrt{\frac{-8}{y'}}} = \sqrt{\frac{-y'}{8}}$$

$$\cancel{p = p} + xp' - \frac{p'}{2\sqrt{\frac{-p}{8}}}$$

$$8\left(\sqrt{\frac{-p}{8}}\right)' = 8 \cdot \frac{1}{2\sqrt{\frac{-p}{8}}} \cdot \left(\frac{-1}{8}\right) p' = \frac{-p'}{2\sqrt{\frac{-p}{8}}}$$

$$p'\left(x - \frac{1}{2\sqrt{\frac{-p}{8}}}\right) = 0$$

(1) $p' = 0$ ⇒ opće rješenje koje daje familiju krivulja i to nam ne treba

(2) $x - \frac{1}{2\sqrt{\frac{-p}{8}}} = 0$ ⇒ singularno ⇒ TO NAM TREBA ⇒

(2) $x = \frac{1}{2\sqrt{\frac{-p}{8}}} \Big/ ^2$

$x^2 = \frac{1}{\cancel{4}}\cdot\frac{1}{\frac{-p}{\cancel{8}\,2}}$

$x^2 = \frac{-2}{p}$

$p = \frac{-2}{x^2} \quad (p = y')$

$\frac{dy}{dx} = \frac{-2}{x^2}$

$dy = -2\cdot\frac{1}{x^2}\,dx \Big/ \int$

$y = -2\int \frac{1}{x^2}\,dx$

$y = -2\cdot\frac{x^{-1}}{-1},$

$\boxed{y = \frac{2}{x}}$

- posto je $b = \pm\frac{8}{a}$ onda kada uvrstimo $b = \frac{-8}{a}$ dobivamo rješenje $y = \frac{-2}{x}$ pa je konačno r.:

$\boxed{y = \pm\frac{2}{x}}$

④ $y=(y')^2-3xy'+3x^2$

PARAMETARSKI OBLIK DIF. JED.

$y=f(x,y')$

(2.5, str. 3 PDF)

(*) $y=p^2-3xp+3x^2 \quad \Big/ \frac{d}{dx}$

$y'=2pp'-3p-3xp'+6x$ — zamjena $y'=p,\ p=p(x)$

$p=2pp'-3p-3xp'+6x$

$2pp'\underbrace{-3p-p}_{-4p}-3xp'+6x=0$

$p'(2p-3x)-2(2p-3x)=0$

$(2p-3x)(p'-2)=0$

① $2p-3x=0 \Rightarrow$ daje nam singularno rješenje

$\boxed{p=\frac{3}{2}x}$ - uvrštavamo u (*)

$y=\frac{9}{4}x^2-\frac{9x^2}{2}+3x^2$ SINGULARNO

$y=\frac{9x^2-18x^2+12x^2}{4}=\boxed{\frac{3}{4}x^2=y}$

② $p'-2=0 \Rightarrow$ opće rješenje, zato gdje je p' je opće

$p'=2$

$\frac{dp}{dx}=2$

$dp=2dx \quad /\int$

$\int dp=2\int dx+C$

$\boxed{p=2x+C}$ - uvrstimo u (*)

$\boxed{y=(2x+C)^2-3x(2x+C)+3x^2}$

OPĆE RJEŠENJE

str. 122

DIF. JED. VIŠEG REDA
SNIŽAVAMO RED
ZAMJENOM $y'=p$, $p=p(y)$
3.4, str. 7 PDF

(5) $2(y')^2 = (y-1)y''$

$2p^2 = (y-1)pp'$

$2p^2 = (y-1)p\frac{dp}{dy}$

$\boxed{y' = p} = \frac{dy}{dx}$

$y'' = p' = \frac{dp}{dx}\cdot\frac{dy}{dy} = \frac{dp}{dy}\cdot\frac{dy}{dx}$

$y'' = p'\cdot p = \boxed{p'p = y''}$

$\frac{2}{y-1}\,dy = \frac{\cancel{p}}{p^2}\,dp$

$\frac{2}{y-1}\,dy = \frac{1}{p}\,dp \;/\int$

$2\int\frac{1}{y-1}\,dy = \int\frac{1}{p}\,dp + C \quad \left|\begin{array}{l} y-1=u \\ dy=du\end{array}\right|$

$2\int\frac{1}{u}\,du = \int\frac{1}{p}\,dp + C$

$2\ln|y-1| = \ln|p| + C$

$\ln(y-1)^2 = \ln(pC) \;/ e^{\wedge}$

$(y-1)^2 = pC$

$\boxed{p = \frac{(y-1)^2}{C}} = y'$

$\frac{dy}{dx} = \frac{(y-1)^2}{C}$

$\frac{1}{(y-1)^2}\,dy = \frac{1}{C}\,dx \;/\int$

$\int\frac{1}{(y-1)^2}\,dy = \frac{1}{C}\int dx + C_2 \quad \left|\begin{array}{l} y-1=u \\ dy=du\end{array}\right|$

$\int\frac{1}{u^2}\,du = \frac{1}{C}\int dx + C_2$

$\frac{u^{-1}}{-1} = \frac{1}{C}x + C_2$

$\frac{-1}{y-1} = \frac{1}{C}x + C_2 \;/\cdot C$

$C_2\cdot C$

$\frac{-C}{y-1} = x + C_3 \;/\cdot(y-1)$

$-C = (x+C_3)(y-1)$

da prilagodimo službenim rješenjima

$-C = C_2,\; C_3 = \epsilon$

$\boxed{(x+\epsilon)(y-1) = C_2}$

⑥ $y'' + y = \dfrac{1}{(\sin x)^3}$, $y\left(\frac{\pi}{2}\right) = 0$, $y'\left(\frac{\pi}{2}\right) = 1$

→ funkcija smetnje $f(x)$

① rješavamo homogen jed.

$$y'' + y = 0$$

$$r^2 + 1 = 0$$

$$\boxed{r = \pm i} = \alpha \pm \beta i \Rightarrow \boxed{\alpha = 0}, \boxed{\beta = 1}$$

rješenje je oblika $y_H = e^{\alpha x}(C_1 \cos \beta x + C_2 \sin \beta x)$ jer su rješenja konjugirano - kompleksna

$$\boxed{y_H = C_1 \cos x + C_2 \sin x}$$

② metodom varijacije konstanti određujemo C_1 i C_2

$$C_1'(x) \cos x + C_2'(x) \sin x = 0 \qquad /: \cos x$$

$$-C_1'(x) \sin x + C_2'(x) \cos x = \frac{1}{(\sin x)^3} \qquad /: \sin x$$

$$\left.\begin{aligned} C_1'(x) + C_2'(x) \frac{\sin x}{\cos x} &= 0 \\ -C_1'(x) + C_2'(x) \frac{\cos x}{\sin x} &= \frac{1}{(\sin x)^4} \end{aligned}\right\} +$$

$$C_2'(x) \left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} \right) = \frac{1}{(\sin x)^4}$$

$$C_2'(x) \left(\frac{\sin^2 x + \cos^2 x}{\cos x \sin x} \right) = \frac{1}{(\sin x)^{\cancel{4}\,3}} \qquad / \cdot \cos x \, \cancel{\sin x}$$

$$C_1'(x) + \frac{\cancel{\cos x}}{(\sin x)^{\cancel{3}\,2}} \cdot \frac{\cancel{\sin x}}{\cancel{\cos x}} = 0 \qquad \boxed{C_2'(x) = \frac{\cos x}{(\sin x)^3}}$$

$$\boxed{C_1'(x) = \frac{-1}{(\sin x)^2}}$$

- potrebno dobiti $C_1(x)$ i $C_2(x)$ integracijom $C_1'(x)$ i $C_2'(x)$

$\Rightarrow$

$$C_1'(x) = \frac{-1}{\sin^2 x} \quad / \int$$

$$C_1(x) = -\int \frac{1}{\sin^2 x}\,dx + C = \left| \text{TABLIČNI INTEGRAL} \right| = -(-\operatorname{ctg} x) + C$$

$$\boxed{C_1(x) = \operatorname{ctg} x + C}$$

$$C_2'(x) = \frac{\cos x}{\sin^3 x} \quad / \int$$

$$C_2(x) = \int \frac{\cos x}{\sin^3 x}\,dx + C = \left| \begin{matrix} \sin x = u \\ \cos x\,dx = du \\ dx = \frac{du}{\cos x} \end{matrix} \right| = \int \frac{\cancel{\cos x}}{u^3} \cdot \frac{du}{\cancel{\cos x}} + C$$

$$= \int \frac{1}{u^3}\,du + C = \int u^{-3}\,du + C = \frac{u^{-2}}{-2} + C$$

$$= \frac{-1}{2u^2} + C = \frac{-1}{2\sin^2 x} + C$$

$$\boxed{C_2(x) = \frac{-1}{2\sin^2 x} + C}$$

$C_1(x)$ i $C_2(x)$ uvrštavamo u y_H

$$\Rightarrow \boxed{y = (\operatorname{ctg} x + C_1)\cos x + \left(\frac{-1}{2\sin^2 x} + C_2\right)\sin x}$$

- uvrštavamo uvjete da bi izračunali C_1 i C_2

① $y = 0,\ x = \frac{\pi}{2}$; $\cos\frac{\pi}{2} = 0$, $\sin\frac{\pi}{2} = 1$

$$0 = \underbrace{(\operatorname{ctg}\tfrac{\pi}{2} + C_1)\cos\tfrac{\pi}{2}}_{0} + \left(\frac{-1}{2\sin^2\frac{\pi}{2}} + C_2\right)\sin\frac{\pi}{2}$$

$$0 = \frac{-1}{2} + C_2 \Rightarrow \boxed{C_2 = \frac{1}{2}}$$

② $y' = 1,\ x = \frac{\pi}{2}$

$$y' = \underbrace{(\operatorname{ctg} x + C_1)'\cos x}_{0} + (\operatorname{ctg} x + C_1)(\cos x)' + \left(\frac{-1}{2\sin^2 x} + C_2\right)'\sin x$$

$$+ \underbrace{\left(\frac{-1}{2\sin^2 x} + C_2\right)(\sin x)'}_{0}$$

$(\sin x)' = \cos x = 0$

$\Longrightarrow$

str. $\boxed{25}$

$$y' = C_1(\cos x)' + \left(\frac{-1}{2\sin^2 x} + C_2\right)' \sin x$$

$$= -C_1 \sin x + \left(\frac{-1}{2}\left[\sin^{(-2)} x\right]' + C_2'\right) \sin x$$

$$= -C_1 \sin x + \left(\underbrace{\frac{-1}{2} \cdot (-2) \cdot \sin^{-3} x \cdot \overset{0}{\overset{\shortparallel}{\cos x}}}_{0} + \overset{0}{\overset{\shortparallel}{C_2'}}\right) \sin x$$

$$y' = -C_1 \sin x$$

$$1 = -C_1 \cdot 1$$

$$\boxed{C_1 = -1}$$

$$y = (\operatorname{ctg} x - 1)\cos x + \left(\frac{-1}{2\sin^2 x} + \frac{1}{2}\right) \sin x$$

$$= \frac{\cos^2 x}{\sin x} - \cos x - \frac{1}{2\sin x} + \frac{1}{2}\sin x$$

$$\boxed{y = -\cos x + \frac{1}{2}\sin x - \frac{1}{2\sin x} + \frac{\cos^2 x}{\sin x}}$$