4. školska zadaća iz matematike 2 za grupe 1.06 i 1.08
10.06.2009.

Grupa B

1. (3b) Naći opće rješenje diferencijalne jednadžbe

$$y' = \frac{2x - y + 1}{2x - y}.$$

2. (4b) Naći ortogonalne trajektorije familije kružnica čije je središte na osi ordinata, a prolaze ishodištem.

3. (3b) Naći opće rješenje diferencijalne jednadžbe

$$\left(\frac{x^2}{y} - 1\right)dx + \left(3y - \frac{x}{y}\right)dy = 0.$$

1. $\dfrac{dy}{dx} = \dfrac{2x - y + 1}{2x - y}.$

$$2x - y + 1 = 0$$
$$2x - y = 0$$

$$2x - y = z \Rightarrow y = 2x - z \Rightarrow \frac{dy}{dx} = 2 - \frac{dz}{dx}$$

$$\frac{z+1}{z} = 2 - \frac{dz}{dx} \Rightarrow \frac{z-1}{z} = \frac{dz}{dx} \Rightarrow \frac{zdz}{z-1} = dx$$

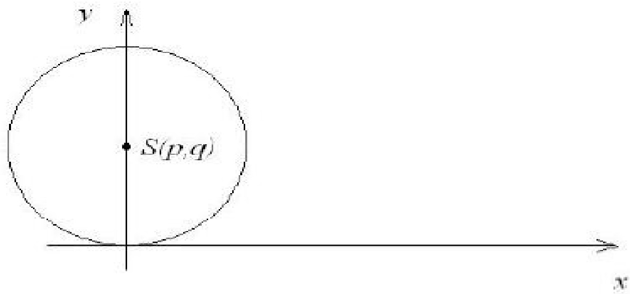
$$\int \frac{zdz}{z-1} = \int dx$$

$$z - 1 = u \Rightarrow z = u + 1 \Rightarrow dz = du$$

$$\int \frac{(u+1)du}{u} = x + C \Rightarrow u + \ln|u| = x + C \Rightarrow z - 1 + \ln|z-1| = x + C$$

$$2x - y - 1 + \ln|2x - y - 1| = x + C \Rightarrow x - y + \ln|2x - y - 1| = C$$

2.



$(x-p)^2+(y-q)^2=r^2$

uvrstimo toč ishodišta i koordinate središta

$(0-0)^2+(0-q)^2=r^2$

$q=\pm r \Rightarrow q=r=C$

$x^2+(y-C)^2=C^2 \Rightarrow$ *deriviramo po x da se riješimo konstante*

$2x+2(y-C)y'=0 \Rightarrow C=y+\frac{x}{y'} \Rightarrow$ *vratimo C u poč. jednadžbu*

$$x^2+\left(y-y+\frac{x}{y'}\right)^2=\left(y+\frac{x}{y'}\right)^2 \Rightarrow x^2+\left(\frac{x}{y'}\right)^2=y^2+2\frac{xy}{y'}+\left(\frac{x}{y'}\right)^2$$

$$x^2y'=y^2y'+2xy \Rightarrow y'=\frac{2xy}{x^2-y^2} \Rightarrow y_1'=\frac{2xy}{x^2-y^2}$$

uvjet ortogonalnosti $\Rightarrow y_2'=-\frac{1}{y_1'}=-\frac{1}{\frac{2xy}{x^2-y^2}}=-\frac{x^2-y^2}{2xy}$

$$\frac{dy}{dx}=-\frac{x^2-y^2}{2xy}=-\frac{1-z^2}{2z}=\frac{z^2-1}{2z}$$

$$\frac{dz}{dx}x=\frac{z^2-1}{2z}-z=\frac{-z^2-1}{2z}=-\frac{z^2+1}{2z} \Rightarrow \frac{2zdz}{z^2+1}=-\frac{dx}{x}$$

$$\int\frac{2zdz}{z^2+1}=\int-\frac{dx}{x} \qquad z^2+1=u \Rightarrow 2zdz=du$$

$$\int\frac{du}{u}=\ln\frac{C}{x} \Rightarrow \ln u=\ln\frac{C}{x} \Rightarrow \ln\left(z^2+1\right)=\ln\frac{C}{x} \Rightarrow z^2+1=\frac{C}{x}$$

$$\frac{y^2}{x^2}+1=\frac{C}{x} \Rightarrow y^2+x^2=Cx$$

3. $\left(\dfrac{x^2}{y}-1\right)dx+\left(3y-\dfrac{x}{y}\right)dy=0.$

$$P_y' = -\frac{x^2}{y^2}; \quad Q_x' = -\frac{1}{y}$$

$-$ *nije egzaktna* $\Rightarrow$ *Eulerov multiplikator*

$$\frac{1}{P}\left(P_y' - Q'\right) = \frac{1}{\dfrac{x^2}{y}-1}\left(\frac{1}{y}-\frac{x^2}{y^2}\right) = \frac{y}{x^2-y}\cdot\frac{y-x^2}{y^2} = -\frac{y}{y^2} = -\frac{1}{y}$$

$$\mu = \mu(y)$$

$$\ln\mu(y) = -\int\frac{1}{P}\left(P_y' - Q'\right)dy = -\int -\frac{dy}{y} = \int\frac{dy}{y} = \ln y$$

$\mu(y) = y \Rightarrow$ *pomnožimo početnu jednadžbu sa* y

$$\left(x^2 - y\right)dx + \left(3y^2 - x\right)dy = 0$$

$$u(x,y) = \int_{x_0=0}^{x}\left(x^2 - y\right)dx + \int_{y_0=0}^{y}\left(3y^2 - x_0\right)dy = \int_0^x x^2dx - y\int_0^x dx + 3\int_0^x y^2dy = \frac{x^3}{3} - xy + y^3$$

$$\frac{x^3}{3} - xy + y^3 = C \Rightarrow x^3 - 3xy + 3y^3 = C$$