17.1.

$$\sum_{n=1}^{\infty} 3^{-(n-2)} = \sum_{n=1}^{\infty} 9\left(\frac{1}{3}\right)^n = \sum_{n=0}^{\infty} 3\left(\frac{1}{3}\right)^n$$

$$S = \frac{a}{1-q} = \frac{3}{1-\frac{1}{3}} = \frac{9}{2}$$

17.2.

$$\sum_{n=1}^{\infty} \frac{2^{n-1}}{(e-1)^{2n}} = \sum_{n=1}^{\infty} \frac{1}{2}\frac{2^n}{[(e-1)^2]^n} = \sum_{n=0}^{\infty} \frac{1}{2} * \frac{2}{(e-1)^2}\left[\frac{2}{(e-1)^2}\right]^n$$

$$S = \frac{a}{1-q} = \frac{\frac{1}{(e-1)^2}}{1-\frac{2}{(e-1)^2}} = \frac{1}{(e-1)^2-2} = \frac{1}{e^2-2e-1}$$

17.3.

$$\sum_{n=1}^{\infty} \frac{1}{n^2-\frac{1}{4}} = \sum_{n=1}^{\infty} \frac{1}{n^2-\left(\frac{1}{2}\right)^2} = \sum_{n=1}^{\infty} \frac{1}{\left(n-\frac{1}{2}\right)\left(n+\frac{1}{2}\right)} = \sum_{n=1}^{\infty} \frac{1}{n-\frac{1}{2}} - \frac{1}{n+\frac{1}{2}}$$

$$S_n = \left(2-\frac{2}{3}\right) + \left(\frac{2}{3}-\frac{2}{5}\right) + \cdots + \left(\frac{1}{n-\frac{1}{2}} - \frac{1}{n+\frac{1}{2}}\right) = 2 - \frac{1}{n+\frac{1}{2}}$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} 2 - \frac{1}{n+\frac{1}{2}} = 2$$

17.4.

$$\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \sum_{n=2}^{\infty} \frac{1}{n^2-1^2} = \sum_{n=2}^{\infty} \frac{1}{(n-1)(n+1)} = \sum_{n=2}^{\infty} \frac{1}{2}\left(\frac{1}{n-1} - \frac{1}{n+1}\right)$$

$$S_n = \frac{1}{2}\left[\left(1-\frac{1}{3}\right) + \left(\frac{1}{2}-\frac{1}{4}\right) + \left(\frac{1}{3}-\frac{1}{5}\right) + \cdots + \left(\frac{1}{n-2}-\frac{1}{n}\right) + \left(\frac{1}{n-1}-\frac{1}{n+1}\right)\right]$$

$$= \frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n}-\frac{1}{n+1}\right)$$

$$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{1}{2}\left(1+\frac{1}{2}-\frac{1}{n}-\frac{1}{n+1}\right) = \frac{3}{4}$$

17.5.

$$\sum_{n=1}^{\infty}\frac{1}{n(n+1)(n+2)}=\sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{1}{n}-\frac{2}{n+1}+\frac{1}{n+2}\right)=\sum_{n=1}^{\infty}\frac{1}{2}\left(\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+1}+\frac{1}{n+2}\right)$$

$$S_n=\frac{1}{2}\left[\left(1-\frac{1}{2}-\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{3}-\frac{1}{3}-\frac{1}{4}\right)+\cdots+\left(\frac{1}{n}-\frac{1}{n+1}-\frac{1}{n+1}+\frac{1}{n+2}\right)\right]$$

$$=\frac{1}{2}\left(1-\frac{1}{2}-\frac{1}{n+1}+\frac{1}{n+2}\right)$$

$$S=\lim_{n\to\infty}S_n=\lim_{n\to\infty}\frac{1}{2}\left(1-\frac{1}{2}-\frac{1}{n+1}+\frac{1}{n+2}\right)=\frac{1}{4}$$

18.1.

$$\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^p}$$

$$\int_2^{\infty}\frac{dn}{n(\ln n)^p}=\left|\begin{matrix}\ln n=t\\ dn=n*dt\end{matrix}\right|=\int_{\ln 2}^{\infty}\frac{dt}{t^p}=\frac{t^{1-p}}{1-p}\Bigg|_{\ln 2}^{\infty}=\frac{\infty^{1-p}}{1-p}-\frac{(\ln 2)^{1-p}}{1-p}=|p>1|=\frac{(\ln 2)^{1-p}}{p-1}$$

18.2.

$$\sum_{n=1}^{\infty}\frac{1}{n\ln n\ln\ln n}$$

$$\int_1^{\infty}\frac{dn}{n\ln n\ln\ln n}=\left|\begin{matrix}\ln n=t\\ dn=n*dt\end{matrix}\right|=\int_{\ln 1}^{\infty}\frac{dt}{t\ln t}=\left|\begin{matrix}\ln t=u\\ dt=n*du\end{matrix}\right|=\int_{\ln\ln 1}^{\infty}\frac{du}{u}=\ln|u|\,\Big|_{\ln\ln 1}^{\infty}=\infty$$

19.1.

$$\sum_{n=1}^{\infty}\frac{n-1}{(n+1)2^n}$$

$$b_n=\frac{1}{2^n}$$

$$\lim_{n\to\infty}\frac{a_n}{b_n}=\lim_{n\to\infty}\frac{\frac{n-1}{(n+1)2^n}}{\frac{1}{2^n}}=1$$

$$\sum_{n=1}^{\infty}b_n=\sum_{n=1}^{\infty}\frac{1}{2^n}=\sum_{n=1}^{\infty}\left(\frac{1}{2}\right)^n\rightarrow \mathit{konvergira}\ za\ |q|<1$$

19.2.

$$\sum_{n=1}^{\infty} \frac{(n+2)(n+3)}{n(n+1)^3}$$

$$b_n = \frac{1}{n^2}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{(n+2)(n+3)}{n(n+1)^3}}{\frac{1}{n^2}} = 1$$

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^2} \rightarrow \mathit{konvergira}\ za\ r > 1$$

19.3.

$$\sum_{n=1}^{\infty} \frac{2n-1}{n^2+2n}$$

$$b_n = \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{2n-1}{n^2+2n}}{\frac{1}{n}} = 2$$

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n} \rightarrow \mathit{divergira}\ za\ r \leq 1$$

19.4.

$$\sum_{n=1}^{\infty} \frac{n+2}{n^2\sqrt{n+1}}$$

$$b_n = \frac{1}{\sqrt{n^3}}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+2}{n^2\sqrt{n+1}}}{\frac{1}{\sqrt{n^3}}} = 1$$

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}} \rightarrow \mathit{konvergira}\ za\ r > 1$$

19.5.

$$\sum_{n=1}^{\infty} \frac{(n+1)^n}{n^{2n}} = \sum_{n=1}^{\infty} \left(\frac{n+1}{n^2}\right)^n$$

$$b_n = \frac{1}{n^n}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\left(\frac{n+1}{n^2}\right)^n}{\frac{1}{n^n}} = \lim_{n\to\infty} \left(\frac{n(n+1)}{n^2}\right)^n = \lim_{n\to\infty} \left[\left(1+\frac{1}{n^2}\right)^{n^2}\right]^{\frac{1}{n}} = \lim_{n\to\infty} e^{\frac{1}{n}} = 1$$

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{n^n} = \sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n \rightarrow \textit{konvergira za } |q| < 1$$

Napomena: $a_1 = 1$, nakon njega je $|q| < 1$

19.6.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1}}{n^2+1}$$

$$b_n = \frac{1}{\sqrt{n^3}}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{\sqrt{n+1}}{n^2+1}}{\frac{1}{\sqrt{n^3}}} = \frac{\sqrt{n^3(n+1)}}{\sqrt{(n^2+1)^2}} = 1$$

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}} \rightarrow \textit{konvergira za } r > 1$$

20.1.

$$\sum_{n=2}^{\infty} \frac{2n-1}{n^3-n}$$

$$b_n = \frac{1}{n^2}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{2n-1}{n^3-n}}{\frac{1}{n^2}} = 2$$

$$\sum_{n=2}^{\infty} b_n = \sum_{n=2}^{\infty} \frac{1}{n^2} \rightarrow \textit{konvergira za } r > 1$$

20.2.

$$\sum_{n=1}^{\infty} \sin\frac{1}{n}$$

$$b_n = \frac{1}{n}$$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\sin\frac{1}{n}}{\frac{1}{n}} = \lim_{1/n\to 0}\frac{\sin\frac{1}{n}}{\frac{1}{n}} = 1$$

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty}\frac{1}{n} \to \mathit{divergira}\ za\ r < 1$$

20.3.

$$\sum_{n=1}^{\infty}\left(1-\cos\frac{1}{n}\right) = \left|\sin^2\frac{x}{2} = \frac{1-\cos x}{2}\right| = \sum_{n=1}^{\infty} 2\sin^2\frac{1}{2n}$$

$$b_n = \left(\frac{1}{2n}\right)^2$$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{2\sin^2\frac{1}{2n}}{\left(\frac{1}{2n}\right)^2} = 2 * \lim_{1/2n\to 0}\left(\frac{\sin\frac{1}{2n}}{\frac{1}{2n}}\right)^2 = 2$$

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty}\frac{1}{4n^2} \to \mathit{konvergira}\ za\ r > 1$$

20.4.

$$\sum_{n=1}^{\infty}\frac{1+3^n}{n^2+3^n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{1+3^n}{n^2+3^n} = \lim_{n\to\infty}\frac{\frac{1}{3^n}+1}{\frac{n^2}{3^n}+1} = 1 \to \text{divergira}$$

Napomena: 3^n raste brže od n^2

20.5.

$$\sum_{n=1}^{\infty} \frac{n+2^n}{1+3^n}$$

$$b_n = \left(\frac{2}{3}\right)^n$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n+2^n}{1+3^n}}{\frac{2^n}{3^n}} = \lim_{n\to\infty} \frac{3^n(n+2^n)}{2^n(1+3^n)} = \lim_{n\to\infty} \frac{n+2^n}{\left(\frac{2}{3}\right)^n + 2^n} = \lim_{n\to\infty} \frac{\frac{n}{2^n}+1}{\frac{1}{3^n}+1} = 1$$

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n \to \mathit{konvergira}\ za\ |q| < 1$$

20.6.

$$\sum_{n=1}^{\infty} \frac{n!}{(n+2)!-1}$$

$$b_n = \frac{1}{n^2}$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n!}{(n+2)!-1}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2 n!}{(n+2)(n+1)n!-1} = \lim_{n\to\infty} \frac{n^2}{(n+2)(n+1)-\frac{1}{n!}} = 1$$

$$\sum_{n=1}^{\infty} b_n = \sum_{n=1}^{\infty} \left(\frac{1}{3}\right)^n \to \mathit{konvergira}\ za\ |q| < 1$$

21.1.

$$\sum_{n=1}^{\infty} \frac{1\cdot 2 \cdots (2n-1)}{2\cdot 5 \cdots (3n-1)}$$

$$q = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{1\cdot 2 \cdots (2n-1)(2n)(2n+1)}{2\cdot 5 \cdots (3n-1)(3n)(3n+1)(3n+2)}}{\frac{1\cdot 2 \cdots (2n-1)}{2\cdot 5 \cdots (3n-1)}}$$

$$= \lim_{n\to\infty} \frac{(2n)(2n+1)}{(3n)(3n+1)(3n+2)} = 0 \to \mathit{konvergira}\ za\ q < 1$$

21.2.

$$\sum_{n=1}^{\infty} n!\, e^{n}$$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} n!\, e^{n} = \infty \;\rightarrow$ divergira

21.3.

$$\sum_{n=1}^{\infty} \frac{2\cdot 4 \cdots 2n}{4\cdot 7 \cdots (3n+1)}$$

$$q = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\frac{2\cdot 4\cdots 2n\cdot(2n+1)(2n+2)}{4\cdot 7\cdots(3n+1)\cdot(3n+2)\cdot(3n+3)\cdot(3n+4)}}{\frac{2\cdot 4\cdots 2n}{4\cdot 7\cdots(3n+1)}} =$$

$$= \lim_{n\to\infty} \frac{(2n+1)(2n+2)}{(3n+2)(3n+3)(3n+4)} = 0 \rightarrow \mathit{konvergira}\ za\ q < 1$$

21.4.

$$\sum_{n=1}^{\infty} n^2 \left(\frac{2}{3}\right)^n$$

$$q = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^2\left(\frac{2}{3}\right)^{n+1}}{n^2\left(\frac{2}{3}\right)^{n}} = \lim_{n\to\infty} \frac{2}{3}\frac{n^2+2n+1}{n^2} = \frac{2}{3} \rightarrow \mathit{konvergira}\ za\ q < 1$$

22.1.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{1+n^4}$$

(1) $\quad \dfrac{1}{1+n^4} > \dfrac{1}{1+(n+1)^4} \rightarrow niz\ (a_n)\ je\ \ padajući$

(2) $\quad \lim\limits_{n\to\infty} \frac{1}{1+n^4} = 0$

$$\sum_{n=1}^{\infty} \frac{1}{1+n^4} \sim \sum_{n=1}^{\infty} \frac{1}{n^4}$$

konvergira apsolutno

22.2.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{2^n - n}$$

(1) $\quad \frac{1}{2^n - n} > \frac{1}{2 * 2^n - n + 1} \rightarrow niz\ (a_n)\ je\ \ padajući$

(2) $\quad \lim_{n\to\infty} \frac{1}{2^n - n} = 0$

$$\sum_{n=1}^{\infty} \frac{1}{2^n - n} \sim \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$$

konvergira apsolutno

22.3.

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(\ln n)}$$

(1) $\quad \frac{1}{\ln(\ln n)} > \frac{1}{\ln(\ln n + 1)} \rightarrow niz\ (a_n)\ je\ \ padajući$

(2) $\quad \lim_{n\to\infty} \frac{1}{\ln(\ln n)} = 0$

$$\frac{1}{\ln(\ln n)} > \frac{1}{n}, n \geq 3 \rightarrow divergira$$

konvergira uvjetno

22.4.

$$\sum_{n=1}^{\infty} \frac{\sin n}{n^2}$$

$$\sum_{n=1}^{\infty} \left|\frac{\sin n}{n^2}\right| \leq \sum_{n=1}^{\infty} \frac{1}{n^2} \rightarrow konvergira\ apsolutno$$

22.5.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n+1}{n}$$

$$\sum_{n=1}^{\infty} \frac{n+1}{n} \rightarrow \lim_{n\to\infty} \frac{n+1}{n} = 1 \rightarrow divergira$$

22.6.

$$\sum_{n=1}^{\infty} \frac{n\cos n\pi}{n^2+1} = \sum_{n=1}^{\infty} \frac{n(-1)^n}{n^2+1}$$

(1) $\quad \dfrac{n}{n^2+1} > \dfrac{n+1}{(\mathrm{n}+1)^2+1} \rightarrow niz\ (a_n)\ je\ \ padajući$

(2) $\quad \lim\limits_{n\to\infty} \dfrac{n}{n^2+1} = 0$

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1} \sim \sum_{n=1}^{\infty} \frac{1}{n} \rightarrow divergira$$

konvergira uvjetno

23.1.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}n}{n^2+1}$$

(1) $\quad \dfrac{n}{n^2+1} > \dfrac{n+1}{(\mathrm{n}+1)^2+1} \rightarrow niz\ (a_n)\ je\ \ padajući$

(2) $\quad \lim\limits_{n\to\infty} \dfrac{n}{n^2+1} = 0$

$$\sum_{n=1}^{\infty} \frac{n}{n^2+1} \sim \sum_{n=1}^{\infty} \frac{1}{n} \rightarrow divergira$$

konvergira uvjetno

23.2.

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}\sqrt{n}}{n+1}$$

(1) $\quad \dfrac{\sqrt{n}}{\mathrm{n}+1} > \dfrac{\sqrt{n+1}}{\mathrm{n}+2} \rightarrow niz\ (a_n)\ je\ \ padajući$

(2) $\quad \lim\limits_{n\to\infty} \dfrac{\sqrt{n}}{\mathrm{n}+1} = 0$

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+1} \sim \sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}} \rightarrow konvergira$$

konvergira apsolutno

23.3.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n^2}$$

$$(1) \qquad \frac{\ln n}{n^2} > \frac{\ln(n+1)}{(n+1)^2} \rightarrow niz\ (a_n)\ je\ \ padajući\ (n \geq 2)$$

$$(2) \qquad \lim_{n\to\infty} \frac{\ln n}{n^2} = \lim_{n\to\infty} \frac{\frac{1}{n}}{2n} = 0$$

$$\frac{\ln n}{n^2} > \frac{1}{n^2}, n \geq 3 \rightarrow konvergira$$

konvergira apsolutno

23.4.

$$\sum_{n=1}^{\infty} (-1)^n \left(1 - n \ln \frac{n+1}{n}\right)$$

$$(1) \qquad \frac{\ln n}{n^2} > \frac{\ln(n+1)}{(n+1)^2} \rightarrow niz\ (a_n)\ je\ \ padajući\ (n \geq 2)$$

$$(2) \qquad \lim_{n\to\infty} \left(1 - n \ln \frac{n+1}{n}\right) = \left|t = \frac{1}{n}\right| = 1 - \lim_{t\to 0} \frac{\ln(1+t)}{t} = 1 - \lim_{t\to 0} \frac{\frac{1}{1+t}}{1} = 0$$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{1 - n \ln \frac{n+1}{n}}{\frac{1}{n}} = \left|t = \frac{1}{n}\right| = \lim_{t\to 0} \frac{1 - \frac{\ln(1+t)}{t}}{t} = \lim_{t\to 0} \frac{t - \ln(1+t)}{t^2} = \lim_{t\to 0} \frac{1 - \frac{1}{1+t}}{2t}$$

$$= \lim_{t\to 0} \frac{1}{2+2t} = \frac{1}{2}$$

$$1 - n \ln \frac{n+1}{n} \sim \frac{1}{n} \rightarrow divergira$$

konvergira uvjetno