1. KRATKA PROVJERA ZNANJA IZ MATEMATIKE 2

30.3.2016.

grupe 2, 4, 6

A

1. **(3 boda)**

   Paralelogram razapet vektorima $\vec{a}$ duljine 3 i $\vec{b}$ duljine 4 ima površinu 6. Odredite duljinu vektora $2\vec{a}+\vec{b}$ ako je $\angle(\vec{a},\vec{b})$ šiljasti.

2. **(4 boda)** Pravac p je presjek ravnina

$$\pi_1 \ldots x - z + 1 = 0,$$
$$\pi_2 \ldots \lambda x - y - z + 5 = 0.$$

   Odredite parametar λ tako da pravac p bude paralelan s ravninom $\pi_3 \ldots x + 3y - z = 0$.

3. **(3 boda)**

   Odredite i skicirajte prirodnu domenu funkcije

$$f(x,y) = \arccos \frac{y}{(x+1)^2}.$$

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grupe 2, 4, 6

B

1. **(3 boda)**

   Odredite površinu paralelograma razapetog vektorima $\vec{a}$ i $\vec{b}$ ako je $|\vec{a}| = 2$, $|\vec{b}| = 1$ i $|\vec{a}-\vec{b}| = \sqrt{5}$.

2. **(4 boda)**

   Odredite parametre λ i μ tako da ravnina $\pi_1 \ldots 3x + \lambda y + \mu z = 0$ bude okomita na presjek ravnina

$$\pi_2 \ldots 2x - y - z + 4 = 0,$$
$$\pi_3 \ldots x + z - 2 = 0.$$

3. **(3 boda)**

   Odredite i skicirajte prirodnu domenu funkcije

$$f(x,y) = \arcsin(x^2 + y^2 - 2y).$$

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grupe 1, 3, 5

A

1. **(4 boda)** Paralelepiped volumena $\sqrt{3}$ je razapet vektorima $\vec{a}$, $\vec{b}$ i $\vec{c}$, gdje je $|\vec{a}| = 2$, $|\vec{b}| = 1$, kut između $\vec{a}$ i $\vec{b}$ šiljasti, $|\vec{c}| = 1$ i $\vec{c}$ je okomit na $\vec{a}$ i $\vec{b}$. Odredite $|\vec{a} + \vec{b}|$.

2. **(3 boda)** Odredite parametar $\lambda \in \mathbb{R}$ za koji će pravac

$$p \ldots \frac{x}{4} = \frac{y+1}{-4} = \frac{z-3}{4}$$

biti paralelan s ravninama

$$\pi_1 \ldots 2x + \lambda y - z + 3 = 0$$
$$\pi_2 \ldots \lambda y + z - 2 = 0.$$

3. **(3 boda)** Odredite i skicirajte domenu funkcije

$$f(x,y) = \sqrt{\frac{y-x}{x+1}}.$$

1. KRATKA PROVJERA ZNANJA IZ MATEMATIKE 2

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grupe 1, 3, 5

B

1. **(3 boda)** Zadani su vektori $\vec{a}$ i $\vec{b}$ takvi da je $|\vec{a}| = 2$, $|\vec{b}| = 1$ i $|\vec{a} + \vec{b}| = \sqrt{7}$. Odredite volumen paralelepipeda razapetog s $\vec{a}$, $\vec{b}$ i $\vec{c} = \vec{a} \times \vec{b}$.

2. **(4 boda)** Odredite pravac p koji se nalazi u ravnini

$$\pi \ldots 2x - y + z + 4 = 0,$$

okomit je na pravac

$$q \ldots \frac{x-1}{2} = \frac{y+3}{3} = \frac{z-1}{1}$$

te prolazi sjecištem ravnine π i pravca q.

3. **(3 boda)** Odredite i skicirajte domenu funkcije

$$f(x,y) = \ln(xy) - \ln\left(y + 1 - x^2\right).$$

2, 4, 6 (A)

(1.) $P = |\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi$ (1)

$6 = 3 \cdot 4 \cdot \sin\varphi \Rightarrow \sin\varphi = \frac{1}{2} \Rightarrow \varphi = 30^\circ$

$|2\vec{a} + \vec{b}| = \sqrt{4\vec{a}^2 + \vec{b}^2 + 4\vec{a}\vec{b}} = \sqrt{4 \cdot 9 + 4^2 + 24\sqrt{3}} = 2\sqrt{13 + 6\sqrt{3}}$ (1)

$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi = 3 \cdot 4 \cdot \frac{\sqrt{3}}{2} = 6\sqrt{3}$ (1)

(2.) $\underset{(1)}{\vec{A}_p} = \vec{n}_1 \times \vec{n}_2 = (\vec{i} - \vec{k}) \times (\lambda\vec{i} - \vec{j} - \vec{k}) = -\vec{k} + \vec{j} - \lambda\vec{j} - \vec{i} =$

$= -\vec{i} + (-\lambda + 1)\vec{j} - \vec{k}$ (1)

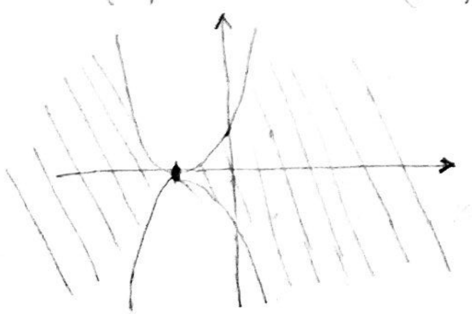
$p \parallel \Pi_3 \Rightarrow \vec{A}_p \perp \vec{n}_3 \Rightarrow \vec{A}_p \cdot \vec{n}_3 = 0$ (1)

$\Rightarrow (-1, -\lambda + 1, -1) \cdot (1, 3, -1) = 0 \Rightarrow 3(-\lambda + 1) = 0$

$\Rightarrow \boxed{\lambda = 1}$ (1)

(3.) $\underbrace{\frac{|y|}{(x+1)^2} \leq 1, \ x \neq -1}_{(1)}$ $\quad |y| \leq (x+1)^2$

$-(x+1)^2 \leq y \leq (x+1)^2$



(1)

$\mathcal{D}_f = \{(x, y) \mid |y| \leq (x+1)^2, \ x \neq -1\}$ (1)

2, 4, 6 (B)

(1.) $|\vec{a} - \vec{b}|^2 = 5 \Rightarrow \vec{a}^2 - 2\vec{a}\vec{b} + \vec{b}^2 = 5$

$\Rightarrow \underset{(1)}{5 - 2\vec{a} \cdot \vec{b} = 5} \Rightarrow \cos\angle(\vec{a}, \vec{b}) = 0$ (1)

$\Rightarrow \sin\angle(\vec{a}, \vec{b}) = 1$

$\Rightarrow P = |\vec{a} \times \vec{b}| = |\vec{a}| \cdot |\vec{b}| \cdot \sin\varphi = 2 \cdot 1 \cdot 1 = 2$ (1)

(2.) $p = \Pi_2 \cap \Pi_3$ $\quad \underset{(1)}{\vec{A}_p} = \vec{n}_2 \times \vec{n}_3 = (2\vec{i} - \vec{j} - \vec{k}) \times (\vec{i} + \vec{k})$

$= -2\vec{j} + \vec{k} - \vec{i} - \vec{j} =$

$= -\vec{i} - 3\vec{j} + \vec{k}$ (1)

$\Pi_1 \perp p \Rightarrow \underset{(1)}{\vec{A}_p \parallel \vec{n}_1} \Rightarrow \alpha \cdot (-1, -3, 1) = (3, \lambda, \mu)$

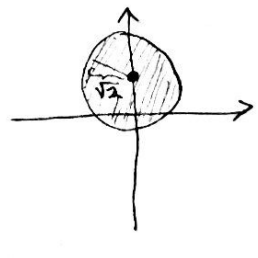
$\Rightarrow \alpha = -3 \Rightarrow \boxed{\lambda = 9}$ $\boxed{\mu = -3}$ (1)

(3.) $\underset{(1)}{|x^2 + y^2 - 2y| \leq 1} \Rightarrow -1 \leq x^2 + y^2 - 2y \leq 1 \ /+1$

$0 \leq x^2 + (y-1)^2 \leq 2$

$\mathcal{D}_f = \{(x, y) \mid x^2 + (y-1)^2 \leq (\sqrt{2})^2\}$ (1)

$K((0, 1), \sqrt{2})$



(1)

(A)

(1) $\sqrt{3} = |(\vec{a},\vec{b},\vec{c})| = |(\vec{a}\times\vec{b})\cdot\vec{c}| = |\vec{a}\times\vec{b}|\cdot|\vec{c}|\cdot\underbrace{|\cos\measuredangle(\vec{c},\vec{a}\times\vec{b})|}_{=1\ (1)}$

$= |\vec{a}||\vec{b}||\sin\measuredangle(\vec{a},\vec{b})|\cdot|\vec{c}| = 2|\sin\measuredangle(\vec{a},\vec{b})|$

$\Rightarrow |\sin\measuredangle(\vec{a},\vec{b})| = \frac{\sqrt{3}}{2} \overset{(1)}{\Rightarrow} \measuredangle(\vec{a},\vec{b}) = \frac{\pi}{3} \Rightarrow \cos\measuredangle(\vec{a},\vec{b}) = \frac{1}{2}$ (1)

$|\vec{a}+\vec{b}| = \sqrt{(\vec{a}+\vec{b})^2} = \sqrt{|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}} = \sqrt{5+2|\vec{a}||\vec{b}|\cos\measuredangle(\vec{a},\vec{b})}$

$= \underline{\underline{\sqrt{7}}}$ (1)

(2) $p \parallel \pi_1,\pi_2 \Rightarrow p \perp \vec{n}_1,\vec{n}_2 \Rightarrow \vec{s}_p = \alpha(\vec{n}_1\times\vec{n}_2)$, za neki α (1)

$\vec{n}_1\times\vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & \lambda & -1 \\ 0 & \lambda & 1 \end{vmatrix} = 2\lambda\vec{i} - 2\vec{j} + 2\lambda\vec{k}$ (1)

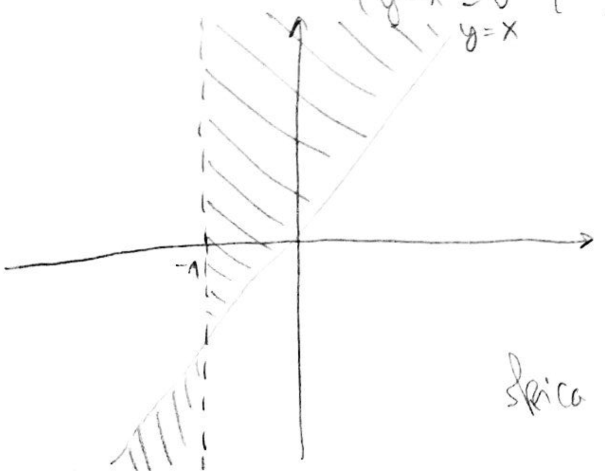
$(4,-4,4) = \alpha(2\lambda,-2,2\lambda) \Rightarrow \alpha = 2 \Rightarrow \underline{\underline{\lambda = 1}}$ (1)

(3) $x \neq -1$

$\frac{y-x}{x+1} \geq 0 \Rightarrow (y-x \geq 0\ \text{ i }\ x+1>0)$ (1)

ili

$(y-x \leq 0\ \text{ i }\ x+1<0)$ (1)



skica (1)

(B)

(1) $\sqrt{7} = |\vec{a}+\vec{b}| = \sqrt{|\vec{a}|^2+|\vec{b}|^2+2\vec{a}\cdot\vec{b}} = \sqrt{5+2|\vec{a}||\vec{b}|\cos\measuredangle(\vec{a},\vec{b})}$

$7 = 5+4\cos\measuredangle(\vec{a},\vec{b}) \Rightarrow \cos\measuredangle(\vec{a},\vec{b}) = \frac{1}{2}$ (1)

$V(\vec{a},\vec{b},\vec{a}\times\vec{b}) = |(\vec{a},\vec{b},\vec{a}\times\vec{b})| = |(\vec{a}\times\vec{b})^2|$

$= |\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2\sin^2\measuredangle(\vec{a},\vec{b})$ (1)

$= 4\cdot 1\cdot\left(\frac{\sqrt{3}}{2}\right)^2 = 3$ (1)

(2) $T_0 := \pi \cap g \qquad T\in g \Leftrightarrow \begin{aligned} x &= 2t+1 \\ y &= 3t-3 \\ z &= t+1 \end{aligned}$ (1)

$2(2t+1)-(3t-3)+(t+1)+4=0 \Rightarrow t=-5$

$\Rightarrow T_0 = (-9,-18,-4)$ (1)

$\vec{s}_p \perp \vec{n}_\pi, \vec{s}_g \Rightarrow \vec{s}_p = \vec{n}_\pi\times\vec{s}_g = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 2 & 3 & 1 \end{vmatrix} = -4\vec{i}+8\vec{k}$ (1)

$p \ldots \frac{x+9}{-4} = \frac{y+18}{0} = \frac{z+4}{8}$ (1)

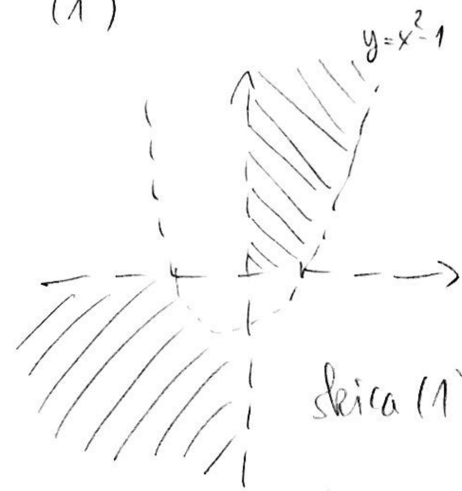
(3) $xy>0$ i $y>x^2-1$ (1)

$\Downarrow$

$x>0$ i $y>0$ (I kvadrant)

ili (1)

$x<0$ i $y<0$ (III kvadrant)



skica (1)