RJEŠENJA PRVE DOMAĆE ZADAĆE!

1. ZADATAK

$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n-1)}$$

Rastavljanje:

$$\frac{1}{(2n+1)(2n-1)} = \frac{A}{2n+1} + \frac{B}{2n-1}$$

$$1 = A * 2n + A + B * 2n - B$$

$$2n * A + 2n * B = 0 / : 2n$$

$$A = -B$$

$$A - B = 1$$

$$A = \frac{1}{2} \ B = \frac{1}{2}$$

$$a_n = \frac{1}{(2n+1)(2n-1)} = \frac{1}{2} * \frac{1}{2n-1} - \frac{1}{2} * \frac{1}{2n+1}$$

$$n = 1 \quad a_1 = \frac{1}{2} * 1 - \frac{1}{2} * \frac{1}{3} = \frac{1}{2} - \frac{1}{6}$$

$$n = 2 \quad a_2 = \frac{1}{2} * 1 - \frac{1}{2} * \frac{1}{5} = \frac{1}{6} - \frac{1}{5}$$

$$S_n = a_1 + a_2 + a_3 + a_4 + ... + a_n = (\frac{1}{2} - \frac{1}{6}) + \frac{1}{6} - \frac{1}{5} + ... + \frac{1}{2} * \frac{1}{2n-1} - \frac{1}{2} * \frac{1}{2n+1} = \frac{1}{2} - \frac{1}{2} * \frac{1}{2n+1} =$$

$$= \frac{1}{2} * (1 - \frac{1}{2n+1}) = \frac{1}{2} * \frac{2n}{2n+1} = \frac{n}{2n+1}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n}{2n+1} = \frac{1}{2}$$

2. ZADATAK

$$\sum_{n=0}^{\infty} \frac{ch(n)}{10^n}$$

$$n = \infty \quad a_n = \frac{ch(n)}{10^n} = \frac{1}{2} * \frac{e^n}{10^n} + \frac{1}{2} * \frac{1}{3^n * 10^n}$$

$$n = 0 \quad a_1 = \frac{1}{2} + \frac{1}{2}$$

$$n = 1 \quad a_2 = \frac{1}{2} * \frac{e}{10} + \frac{1}{2} * \frac{1}{10 * e}$$

$$S_n = a_1 + a_2 + a_3 + a_4 + ... + a_n = (\frac{1}{2}((1+1) + (\frac{e}{10} + \frac{1}{10e}) + ... + (\frac{e^n}{10^n} + \frac{1}{3^n * 10^n})) =$$

$$S_n = \frac{1}{2}((1 + (\frac{e}{10} + \frac{e^2}{10^2} + ... + \frac{e^n}{10^n}) + (1 + \frac{1}{10e} + \frac{1}{100 * e^2} + ... + \frac{1}{3^n * 10^n})) =$$

$$q_1 = \frac{e}{10} \; q_2 = \frac{1}{10e}$$

$$S_n = \frac{1}{2} * (\frac{a_1}{1 - q_1} + \frac{a_1}{1 - q_2})$$

$$S_n = \frac{1}{2} * (\frac{1}{1 - \frac{e}{10}} + \frac{a_1}{1 - \frac{1}{10e}})$$

$$S_n = \frac{1}{2} * (\frac{10}{10 - e} + \frac{10e}{10e - 1})$$

3. ZADATAK

$$\sum_{n=0}^{\infty} \frac{2^n - 3^{n+2}}{6^{n+1}} = \sum_{n=1}^{\infty} \frac{2^n}{6^{n+1}} - \sum_{n=1}^{\infty} \frac{3^{n+2}}{6^{n+1}} = \frac{1}{6} * \sum_{n=1}^{\infty} \frac{1}{3^n} - \frac{3}{2} \sum_{n=1}^{\infty} \frac{1}{2^n}$$

$$S_n = \frac{1}{6} * \frac{\frac{1}{3}}{\frac{2}{3}} - \frac{3}{2} * \frac{\frac{1}{2}}{\frac{1}{2}} = \frac{1}{12} - \frac{3}{2} = \frac{1}{12} - \frac{18}{12} = -\frac{17}{18}$$

4. ZADATAK

$$\sum_{n=0}^{\infty} \frac{2^n + 3^n}{(a+1)^n} = \sum_{n=0}^{\infty} (\frac{2}{a+1})^n + \sum_{n=0}^{\infty} (\frac{3}{a+1})^n$$

$$S_n = (\frac{1}{1 - \frac{2}{a+1}} + \frac{1}{1 - \frac{3}{a+1}}) = \frac{a+1}{a-1} + \frac{a+1}{a-2} = \frac{a^2 - 2a + a - 2 + a^2 - a + a - 1}{(a-1)(a-2)} = \frac{2a^2 - a - 3}{(a-1)(a-2)}$$

Uvjet:

$$\mid \frac{2}{a+1} \mid < 1 \; i \; \mid \frac{3}{a+1} \mid < 1$$

$$a > 1 \; , \; a < -3 \; za \; \mid \frac{2}{a+1} \mid < 1$$

$$a > 2 \; , \; a < -4 \; za \; \mid \frac{3}{a+1} \mid < 1$$

$Konačno \ rješenje \ je : \; a \in \langle -\infty, -4 \rangle \cup \langle 2, +\infty \rangle$

4. ZADATAK

$$\sum_{n=0}^{\infty} \cos \frac{1}{\sqrt{n+1}}$$

$Nužni \ uvjet \ da \ bi \ red \ konvergirao : \; \lim_{n \to \infty} \cos \frac{1}{\sqrt{n+1}} = 0$

$$\lim_{n \to \infty} \cos \frac{1}{\sqrt{n+1}} = \lim_{n \to \infty} (\cos(\lim_{n \to \infty} \frac{1}{\sqrt{n+1}})) = \lim_{n \to \infty} \cos 0 = 1 \Rightarrow \; red \ divergira$$

6. ZADATAK

$$\sum_{n=1}^{\infty} \frac{3^n * n!}{n^n} \Rightarrow \; D'Alembertov \; kriterij \; za \; divergentnost$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} \geq 0$$

$$\lim_{n\to\infty} \frac{\frac{3^n*3*n!*(n+1)}{(n+1)^n*(n+1)}}{\frac{3^n*n!}{n^n}} \geq 1$$

$$\lim_{n\to\infty} \frac{3 * n^n}{(n+1)^n} \geq 0 \;\; \Rightarrow \frac{3}{e} \geq 1 \Rightarrow divergira$$

7. ZADATAK

$$\sum_{n=1}^{\infty} \frac{2^n * (n!)^2}{(2n)!} \Rightarrow \; D'Alembertov \; kriterij \; za \; divergentnost$$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} \geq 0$$

$$\lim_{n\to\infty} \frac{\frac{2^n*2*(n!)^2*(n+1)^2}{(2n)!*(2n+1)*2*(n+1)}}{\frac{2^n*(n!)^2}{(2n)!}} \geq 1$$

$$\lim_{n\to\infty} \frac{n+1}{2n+1} \geq 1 \;\; \Rightarrow \frac{1}{2} \geq 1 \Rightarrow konvergira$$

8. ZADATAK

$$\sum_{n=1}^{\infty} n * \tan \frac{\pi}{2^(n+1)} \Rightarrow \; D'Alembertov \; kriterij \; za \; divergentnost$$

$$\lim_{n\to\infty} \frac{(n+1) * \tan(\frac{\pi}{2^{n+2}})}{n * \tan \frac{\pi}{2^{n+1}}} \geq 1$$

$$\lim_{n\to\infty} \frac{n+1}{n} * \lim_{n\to\infty} \frac{\tan(\frac{\pi}{2^{n+2}})}{\tan \frac{\pi}{2^{n+1}}} = 1 * \lim_{n\to\infty} \frac{\tan(\frac{\pi}{4} * 2^{-n})}{\tan(\frac{\pi}{2} * 2^{-n})} = \frac{\frac{\pi}{4} * 2^{-n}}{\frac{\pi}{2} * 2^{-n}} = \frac{1}{2} \Rightarrow \frac{1}{2} \geq 1 \Rightarrow konvergira$$

9. ZADATAK

$$\sum_{n=1}^{\infty} (\frac{2n^2+1}{3n^2-1})^n \Rightarrow \; Cauchyjev \; kriterij$$

$$q = \lim_{n\to\infty} \sqrt[n]{(\frac{2n^2+1}{3n^2-1})^n} = \lim_{n\to\infty} \frac{2n^2+1}{3n^2-1} = \frac{2}{3} \qquad \frac{2}{3} < 1 \Rightarrow konvergira$$

10. ZADATAK

$$\sum_{n=1}^{\infty} \frac{1}{3^n} * (\frac{n}{n+1})^{n^2} \Rightarrow \; Cauchyjev \; kriterij$$

$$q=\lim_{n\to\infty}\sqrt[n]{3^{-n}*(\frac{n}{n+1})^{n2}}=\lim_{n\to\infty}\sqrt[n]{3^{-n}}*\lim_{n\to\infty}\sqrt[n]{(\frac{n}{n+1})^{n2}}=\frac{1}{3}*\lim_{n\to\infty}(\frac{n}{n+1})^{\frac{n^2}{n}}=\frac{1}{3}*\lim_{n\to\infty}(\frac{n}{n+1})^{n}=$$

$$\frac{1}{3}*\lim_{n\to\infty}(\frac{1}{1+\frac{1}{n}})^{n}=\frac{1}{3e}\quad\frac{1}{3e}<1\quad\Rightarrow konvergira$$

11. ZADATAK

$$\sum_{n=1}^{\infty}2^{n}*\sin(\frac{2}{3^{n}}-\frac{1}{3^{n+1}})\Rightarrow\ D'Alembertov\ kriterij$$

$$q=\lim_{n\to\infty}\frac{2^{n}*2*\sin(\frac{2}{3^{n}*3}-\frac{1}{3^{n}*9})}{2*n*\sin(\frac{2}{3^{n}}-\frac{1}{3^{n}*3})}=\lim_{n\to\infty}\frac{2*\sin(\frac{1}{9}*\frac{5}{3^{n}})}{\sin(\frac{1}{3}*\frac{5}{3^{n}})}=\frac{2*\frac{1}{9}*\frac{5}{3^{n}}}{\frac{1}{3^{n}}*\frac{5}{3^{n}}}=\frac{2}{3}$$

$$\frac{2}{3}<1\quad\Rightarrow konvergira$$

12. ZADATAK

$$\sum_{n=1}^{\infty}\frac{\sqrt[3]{n^{4}+n^{2}+1}}{\sqrt[4]{n^{6}+n^{3}+1}}\Rightarrow\ D'Alembertov\ kriterij$$

$$\lim_{n\to\infty}\frac{\frac{\sqrt[3]{(n+1)^{4}+(n+1)^{2}+1}}{\sqrt[4]{(n+1)^{6}+(n+1)^{3}+1}}}{\frac{\sqrt[3]{n^{4}+n^{2}+1}}{\sqrt[4]{n^{6}+n^{3}+1}}}=\lim_{n\to\infty}\frac{\sqrt[12]{((n+1)^{4}+(n+1)^{2}+1)^{4}+(n^{6}+n^{3}+1)^{3}}}{\sqrt[12]{((n+1)^{6}+(n+1)^{3}+1)^{3}}+(n^{4}+n^{2}+1)^{4}}$$

Pogledati najveće potencije i uočiti da se skrate pa je $q=1\ \Rightarrow konvergira$

13. ZADATAK

$$\sum_{n=1}^{\infty}\frac{(-1)^{n}}{n(n-1)(n-2)}=\frac{-1}{6}+\frac{1}{24}+...+\frac{(-1)^{n}}{n(n-1)(n-2)}\ \Rightarrow Alternirani\ Leibnitzov\ red$$

1) $\frac{1}{6}>\frac{1}{24}>...>\frac{(1)^{n}}{n(n-1)(n-2)}$

2) $\lim_{n\to\infty}\frac{1}{n(n-1)(n-2)}=\lim_{n\to\infty}\frac{1}{n}*\lim_{n\to\infty}\frac{1}{(n+1)(n+2)}=0\ \Rightarrow konvergira$

14. ZADATAK

$$\sum_{n=1}^{\infty}\frac{\cos(n^{2}+n+1)}{n^{2}+n+1}\ \Rightarrow poredbeni\ kriterij$$

Usporedimo s : $\sum_{n=1}^{\infty}\frac{2}{n^{2}+n+1}$ *najednostavnije je i veći je od zadanog.*

$$\lim_{n\to\infty}\frac{2}{n^{2}+n+1}=0\ <1\ \Rightarrow konvergira,\ \check{s}to\ zna\check{c}i\ da\ i\ po\check{c}etni\ niz\ konvergira$$

15. ZADATAK

$$\sum_{n=1}^{\infty}(-1)^n * \tan\frac{1}{\sqrt[3]{n}} \quad \Rightarrow Alternirajući\ red$$

$$1)\quad \sum_{n=1}^{\infty}(-1)^n * \tan\frac{1}{\sqrt[3]{n}} = -0.017 + 0.013 - 0.012 + 0.010 - ... + (-1)^n * \tan\frac{1}{\sqrt[3]{n}} \ \Rightarrow padajući\ red$$

$$2)\quad \lim_{n\to\infty}\tan\frac{1}{\sqrt[3]{n}} = \lim_{n\to\infty}\frac{1}{\sqrt[3]{n}} = 0 \ \Rightarrow konvergira$$

$Apsolutna\ konvergencija??$

16. ZADATAK

$$\sum_{n=1}^{\infty}(-1)^{n+1}(\sqrt{n^2+1}-n)$$

$$1)\quad \sum_{n=1}^{\infty}(-1)^{n+1}(\sqrt{n^2+1}-n) = 0.414 - 0.236 + 0.162 + -... + (-1)^{n+1}(\sqrt{n^2+1}-n) \ \Rightarrow padajući\ red$$

$Dalje??$

17. ZADATAK

$$\sum_{n=1}^{\infty}\frac{\sqrt{n^3+n^2}-\sqrt{n^3+1}}{n^a}$$

18. ZADATAK

$$\sum_{n=1}^{\infty}\sqrt{n+a}-\sqrt[4]{n^2+n+b}$$

$$\sum_{n=1}^{\infty}\sqrt{n+a}-\sqrt[4]{n^2+n+b} * \frac{\sqrt{n+a}+\sqrt[4]{n^2+n+b}}{\sqrt{n+a}+\sqrt[4]{n^2+n+b}} = \sum_{n=1}^{\infty}\frac{n+a-\sqrt{n^2+n+b}}{\sqrt{n+a}+\sqrt[4]{n^2+n+b}} * \frac{n+a+\sqrt{n^2+n+b}}{n+a+\sqrt{n^2+n+b}} =$$

$$\sum_{n=1}^{\infty}\frac{n^2+2na+a^2-n^2-n-b}{(\sqrt{n+a}+\sqrt[4]{n^2+n+b})(n+a+\sqrt{n^2+n+b})} \approx \sum_{n=1}^{\infty}\frac{n(2a-1)}{4n^{\frac{3}{2}}} \ \Rightarrow Usporedba\ s\ harm\ redom\ \frac{1}{n^r}$$

$$1)\quad r=\frac{1}{2} \Rightarrow \ divergira\ (za\ tu\ vrijednost);\ da\ bi\ konvergirao\ trebalo\ bi\ biti\ a=\frac{1}{2}\ (nazivnik\ je\ onda\ 0).$$

$$Provjera:\ za\ a=\frac{1}{2}.\ Usporedba\ s\ \frac{1}{n^{\frac{3}{2}}},\quad r=\frac{3}{2}\ >\ 1 \quad \Rightarrow\ konvergira.$$

19. ZADATAK

$$\sum_{n=1}^{\infty}\frac{\ln(1+e^{-n})}{n}, \quad b_n = \frac{e^{-n}}{n}$$

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \frac{\ln(1+e^{-n})}{e^{-n}} = 1 \quad \Rightarrow Niz\ \sum a_n\ konvergira$$

$$Provjerimo\ konvergenciju\ s\ \sum_{n=1}^{\infty}\frac{1}{n*e^n} \ \Rightarrow\ Cauchy$$

$$q = \lim_{n\to\infty} \frac{1}{\sqrt[n]{n} * e} = \frac{1}{e} < 1 \;\Rightarrow\; konvergira$$

20. ZADATAK

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}} \ln\left(\frac{n+1}{n-1}\right)$$