

$$2700. \text{ a) } \frac{2l}{\pi} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin \frac{n\pi x}{l}}{n}; \quad \text{ b) } \frac{l}{2} - \frac{4l}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos \frac{(2n-1)\pi x}{l}}{(2n-1)^2}.$$

$$2701. \text{ a) } \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{2}, \text{ gdje je } b_{2k+1} = \frac{8}{\pi} \left[\frac{\pi^2}{2k+1} - \frac{4}{(2k+1)^3} \right], \quad b_{2k} = -\frac{4\pi}{k};$$

$$\text{ b) } \frac{4\pi^2}{3} - 16 \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\cos \frac{n\pi x}{2}}{n^2}.$$

$$2702. \text{ a) } \frac{8}{\pi^2} \sum_{n=0}^{\infty} (-1)^n \frac{\sin \frac{(2n+1)\pi x}{2}}{(2n+1)^2}; \quad \text{ b) } \frac{1}{2} - \frac{4}{\pi^2} \sum_{n=0}^{\infty} \frac{\cos (2n+1)\pi x}{(2n+1)^2}.$$

$$2703. \frac{2}{3} - \frac{9}{2\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos \frac{2n\pi x}{3} + \frac{1}{2\pi^2} \sum_{n=1}^{\infty} \frac{\cos 2n\pi x}{n^2}.$$

GLAVA IX

2704. Da.

2705. Ne.

2706. Da.

2707. Da.

2708. Da.

2709. a) Da; b) ne.

2710. Da.

2714. $y - xy' = 0$.2715. $xy' - 2y = 0$.2716. $y - 2xy' = 0$.2717. $x \, dx + y \, dy = 0$.2718. $y' = y$.2719. $3y^2 - x^2 = 2xyy'$.2720. $xyy' (xy^2 + 1) = 1$.2721. $y = xy' \ln \frac{x}{y}$.2722. $2xy'' + y' = 0$.2723. $y'' - y' - 2y = 0$.2724. $y'' + 4y = 0$.2725. $y''' - 2y'' + y' = 0$.2726. $y'' = 0$.2727. $y''' = 0$.2728. $(1 + y'^2) y''' - 3y' y'^2 = 0$.2729. $y^2 - x^2 = 25$.2730. $y = xe^{2x}$.2731. $y = -\cos x$.2732. $y = \frac{1}{6} (-5e^{-x} + 9e^x - 4e^{2x})$.2738. 2,593 (točna je vrijednost $y = e$).2739. 4,780 [točno, $y = 3(e - 1)$].2740. 0,946 (točna je vrijednost $y = 1$).2741. 1,826 (točna je vrijednost $y = \sqrt{3}$).2742. $\operatorname{ctg}^2 y = \operatorname{tg}^2 x + C$.2743. $x = \frac{Cy}{\sqrt{1+y^2}}; y = 0$.2744. $x^2 + y^2 = \ln Cx^2$.2745. $y = a + \frac{Cx}{1+ax}$.2746. $\operatorname{tg} y = C(1 - e^x)^3; x = 0$.2747. $y = C \sin x$.2748. $2e^{\frac{y^2}{2}} = \sqrt{e}(1 + e^x)$.2749. $1 + y^2 = \frac{2}{1-x^2}$.2750. $y = 1$.2751. $\arctg(x+y) = x + C$.2752. $8x + 2y + 1 = 2 \operatorname{tg}(4x + C)$.2753. $x + 2y + 3 \ln |2x + 3y - 7| = C$.2754. $5x + 10y + C = 3 \ln |10x - 5y + 6|$.2755. $\rho = \frac{C}{1 - \cos \rho}$ ili $y^2 = 2Cx + C^2$.

$$2756. \ln \rho = \frac{1}{2 \cos^2 \varphi} - \ln |\cos \varphi| + C \text{ ili } \ln |x| - \frac{y^2}{2x^2} = C.$$

$$2757. \text{Pravac } y = Cx \text{ ili hiperbola } y = \frac{C}{x}. \text{ Uputa. Duljina tangente jednaka je } \sqrt{y^2 + \left(\frac{y}{y'}\right)^2}.$$

$$2758. y^2 - x^2 = C. \quad 2759. y = Ce^{\frac{x}{a}}. \quad 2760. y^2 = 2px.$$

$$2761. y = ax^2. \text{ Uputa. Prema uvjetima je } \frac{\int_0^x xy \, dx}{\int_0^x y \, dx} = \frac{3}{4} x. \text{ Derivirajući dvaput po } x \text{ dobivamo di-}$$

ferencijalnu jednadžbu.

$$2762. y^2 = \frac{1}{3} x.$$

$$2763. y = \sqrt{4 - x^2} + 2 \ln \frac{2 - \sqrt{4 - x^2}}{|x|}.$$

$$2764. \text{Pramen pravaca } y = kx.$$

$$2765. \text{Porodicu sličnih elipsa } 2x^2 + y^2 = C^2.$$

$$2766. \text{Porodicu hiperbola } x^2 - y^2 = C.$$

$$2767. \text{Porodicu kružnica } x^2 + (y - b)^2 = b^2.$$

$$2768. y = x \ln \left| \frac{C}{x} \right|.$$

$$2769. y = \frac{C}{x} - \frac{x}{2}. \quad 2770. x = Ce^{\frac{x}{y}}.$$

$$2771. (x - C)^2 - y^2 = C^2; (x - 2)^2 - y^2 = 4; y = \pm x.$$

$$2772. \sqrt{\frac{x}{y}} + \ln |y| = C.$$

$$2773. y = \frac{C}{2} x^2 - \frac{1}{2C}; x = 0.$$

$$2774. (x^2 + y^2)^3 (x + y)^2 = C.$$

$$2775. y = x \sqrt{1 - \frac{3}{8} x}.$$

$$2776. (x + y - 1)^3 = C(x - y + 3).$$

$$2777. 3x + y + 2 \ln |x + y - 1| = C.$$

$$2778. \ln |4x + 8y + 5| + 8y - 4x = C.$$

$$2779. x^2 = 1 - 2y.$$

2780. Rotacioni paraboloid. *Rješenje.* Na temelju simetrije traženo zrcalo je rotaciona ploha. Ishodište koordinatnog sustava postavlja se u izvor svjetla; os x je smjer pramena zraka. Ako tangenta u po volji odabranoj točki $M(x, y)$ krivulje presjeka tražene površine i ravnine xOy tvori s osi x kut φ , a odsječak, koji spaja ishodište koordinatnog sustava s točkom $M(x, y)$, kut α , onda je $\operatorname{tg} \alpha = \operatorname{tg} 2\varphi = \frac{2 \operatorname{tg} \varphi}{1 - \operatorname{tg}^2 \varphi}$. Ali $\operatorname{tg} \alpha = \frac{y}{x}$, $\operatorname{tg} \varphi = y'$. Tražena diferencijalna jednadžba je $y - yy'^2 = 2xy'$ i njeno je rješenje $y^2 = 2Cx + C^2$. Ravninski presjek je parabola. Tražena je površina rotacioni paraboloid.

$$2781. (x - y)^2 - Cy = 0.$$

$$2782. x^2 = C(2y + C).$$

$$2783. (2y^2 - x^2)^3 = Cx^2. \text{ Uputa. Upotrijebite da je površina jednaka } \int_a^x y \, dx.$$

$$2784. y = Cx - x \ln |x|.$$

$$2785. y = Cx + x^2.$$

$$2786. y = \frac{1}{6} x^4 + \frac{C}{x^2}.$$

$$2787. x \sqrt{1 + y^2} + \cos y = C. \text{ Uputa. Jednadžba je linearna s obzirom na } x \text{ i } \frac{dx}{dy}.$$

$$2788. x = Cy^2 - \frac{1}{y}.$$

$$2789. y = \frac{e^x}{x} + \frac{ab - ea}{x}.$$

$$2790. y = \frac{1}{2} (x \sqrt{1 - x^2} + \arcsin x) \sqrt{\frac{1 + x}{1 - x}}. \quad 2791. y = \frac{x}{\cos x}.$$

$$2792. y = (x^2 + Cx) = 1.$$

$$2794. x^2 = \frac{1}{y + Cy^2}.$$

$$2797. xy = Cy^2 + a^2.$$

$$2799. x = y \ln \frac{y}{a}.$$

$$2801. x^2 + y^2 - Cy + a^2 = 0.$$

$$2803. \frac{x^3}{3} + xy^2 + x^2 = C.$$

$$2805. x^2 + y^2 - 2 \operatorname{arctg} \frac{y}{x} = C.$$

$$2807. \frac{x^2}{2} + ye^{\frac{x}{y}} = 2.$$

$$2809. \frac{x}{y} + \frac{x^2}{2} = C.$$

$$2811. (x \sin y + y \cos y - \sin y) e^x = C.$$

$$2812. (x^2 C^2 + 1 - 2Cy)(x^2 + C^2 - 2Cy) = 0; \text{ singularni integral je } x^2 - y^2 = 0.$$

$$2813. \text{Opći integral je } (y + C)^2 = x^3; \text{ singularnog integrala nema.}$$

$$2814. \text{Opći integral je } \left(\frac{x^2}{2} - y + C \right) \left(x - \frac{y^2}{2} + C \right) = 0; \text{ singularnog integrala nema.}$$

$$2815. \text{Opći integral je } y^2 + C^2 = 2Cx; \text{ singularni je integral } x^2 - y^2 = 0.$$

$$2816. y = \frac{1}{2} \cos x \pm \frac{\sqrt{3}}{2} \sin x.$$

$$2818. \begin{cases} x = e^p + pe^p + C, \\ y = p^2 e^p. \end{cases}$$

Singularno rješenje je $y = 0$.

$$2820. 4y = x^2 + p^2, \ln |p - x| = C + \frac{x}{p - x}.$$

$$2821. \ln \sqrt{p^2 + y^2} + \operatorname{arctg} \frac{p}{y} = C, \quad x = \ln \frac{y^2 + p^2}{2p}. \text{ Singularno rješenje je } y = e^x.$$

$$2822. y = C + \frac{x^2}{C}; y = \pm 2x.$$

$$2824. \begin{cases} x = Ce^{-p} - 2p + 2, \\ y = C(1 + p)e^{-p} - p^2 + 2. \end{cases}$$

$$2793. y = x \ln \frac{C}{x}.$$

$$2795. y^3(3 + Ce^{\cos x}) = x.$$

$$2798. y^2 + x + ay = 0.$$

$$2800. \frac{a}{x} + \frac{b}{y} = 1.$$

$$2802. \frac{x^2}{2} + xy + y^2 = C.$$

$$2804. \frac{x^4}{4} - \frac{3}{2} x^2 y^2 + 2x + \frac{y^3}{3} = C.$$

$$2806. x^2 - y^2 = Cy^3.$$

$$2808. \ln |x| - \frac{y^2}{x} = C.$$

$$2810. \frac{1}{y} \ln x + \frac{1}{2} y^2 = C.$$

$$2817. \begin{cases} x = \sin p + \ln p, \\ y = p \sin p + \cos p + p + C. \end{cases}$$

$$2819. \begin{cases} x = 2p - \frac{2}{p} + C, \\ y = p^2 + 2 \ln p. \end{cases}$$

$$2823. \begin{cases} x = \ln |p| - \arcsin |p| + C, \\ y = |p| + \sqrt{1 - p^2}. \end{cases}$$

$$2825. \begin{cases} x = \frac{1}{3} (Cp^{-\frac{1}{2}} - p), \\ y = \frac{1}{6} (2Cp^{\frac{1}{2}} + p^3). \end{cases}$$

Uputa. Diferencijalna jednadžba iz koje određujemo x kao funkciju od p je homogena.

$$2826. y = Cx + C^2; \quad y' = -\frac{x^2}{4}.$$

$$2828. y = Cx + \sqrt{1+C^2}; \quad x^2 + y^2 = 1.$$

$$2830. xy = C.$$

$$2832. \text{Astroida } x^{\frac{2}{3}} + y^{\frac{2}{3}} = l^{\frac{2}{3}}.$$

2833. a) Homogena; $y = xu$; b) linearna s obzirom na x ; $x = uv$; c) linearna s obzirom na y ; $y = u \cdot v$; d) Bernoullijeva jednađba; $y = uv$; e) sa separiranim varijablama; f) Clairautova jednađba; svedite na oblik $y = xy' \pm \sqrt{y'^2}$; g) Langrangeova jednađba; derivirajte po x ; h) Bernoullijeva jednađba; $y = uv$; i) svedite na jednađbu sa separiranim varijablama; $u = x + y$; j) Lagrangeova jednađba; derivirajte po x ; k) Bernoullijeva jednađba s obzirom na x ; $x = uv$; l) jednađba s totalnim diferencijalima; m) linearna; $y = uv$; n) Bernoullijeva jednađba; $y = u \cdot v$.

$$2834. \text{a) } \sin \frac{y}{x} = -\ln |x| + C; \quad \text{b) } x = y \cdot e^{Cy+1}. \quad 2835. x^2 + y^4 = Cy^2.$$

$$2836. y = \frac{x}{x^2 + C}.$$

$$2837. xy \left(C - \frac{1}{2} \ln^2 x \right) = 1.$$

$$2838. y = Cx + C \ln C; \quad \text{singularno rješenje je } y = -e^{-(x+1)}.$$

$$2839. y = Cx + \sqrt{-aC}; \quad \text{singularno rješenje je } y = \frac{a}{4x}.$$

$$2840. 3y + \ln \frac{|x^3 - 1|}{(y+1)^6} = C.$$

$$2841. \frac{1}{2} e^{2x} - ey - \arctg y - \frac{1}{2} \ln(1+y^2) = C.$$

$$2842. y = x^2(1 + Ce^x).$$

$$2843. x = y^2(C - e^{-y}).$$

$$2844. y = Ce^{-\sin x} + \sin x - 1.$$

$$2845. y = ax + C\sqrt{1-x^2}.$$

$$2846. y = \frac{x}{x+1}(x + \ln |x| + C).$$

$$2847. x = Ce^{\sin y} - 2a(1 + \sin y).$$

$$2848. \frac{x^2}{2} + 3x + y + \ln[(x-3)^{10}|y-1|^3] = C. \quad 2849. 2 \arctg \frac{y-1}{2x} = \ln Cx.$$

$$2850. x^2 = 1 - \frac{2}{y} + Ce^{-\frac{2}{y}}.$$

$$2851. x^3 = Ce^y - y - 2.$$

$$2852. \sqrt{\frac{y}{x}} + \ln |x| = C.$$

$$2853. y = x \arcsin(Cx).$$

$$2854. y^2 = Ce^{-2x} + \frac{2}{5} \sin x + \frac{4}{5} \cos x.$$

$$2855. xy = C(y-1).$$

$$2856. x = Ce^y - \frac{1}{2}(\sin y + \cos y).$$

$$2857. py = C(p-1).$$

$$2858. x^4 = Ce^4y - y^3 - \frac{3}{4}y^2 - \frac{3}{8}y - \frac{3}{32}.$$

$$2859. (xy+C)(x^2y+C) = 0.$$

$$2860. \sqrt{x^2+y^2} - \frac{x}{y} = C.$$

$$2861. xe^y - y^2 = C.$$

$$2862. \begin{cases} x = \frac{C}{p^2} - \frac{\sqrt{1+p^2}}{2p} + \frac{1}{2p^2} \ln(p + \sqrt{1+p^2}), \\ y = 2px + \sqrt{1+p^2}. \end{cases}$$

2863. $y = xe^{Cx}$.

2864. $2e^x - y^4 = Cy^2$.

2865. $\ln|y+2| + 2 \arctg \frac{y+2}{x-3} = C$.

2866. $y^2 + Ce^{-\frac{y^2}{2}} + \frac{1}{x} - 2 = 0$.

2867. $x^2 y = Ce^{\frac{y}{a}}$. 2868. $x + \frac{x}{y} = C$.

2869. $y = \frac{C - x^4}{4(x^2 - 1)^{3/2}}$.

2870. $y = C \sin x - a$.

2871. $y = \frac{a^2 \ln(x + \sqrt{a^2 + x^2}) + C}{x + \sqrt{a^2 + x^2}}$.

2872. $(y - Cx)(y^2 - x^2 + C) = 0$.

2873. $y = Cx + \frac{1}{C^2}$, $y = \frac{3}{2} \sqrt[3]{2x^2}$.

2874. $x^3 + x^2 y - y^2 x - y^3 = C$.

2875. $p^3 + 4y^2 = Cy^3$. 2876. $y = x - 1$.

2877. $y = x$. 2878. $y = 2$.

2879. $y = 0$.

2880. $y = \frac{1}{2}(\sin x + \cos x)$.

2881. $y = \frac{1}{4}(2x^2 + 2x + 1)$.

2882. $y = e^{-x} + 2x - 2$.

2883. a) $y = x$; b) $y = Cx$, gdje je C po volji; točka $(0, 0)$ je singularna točka diferencijalne jednadžbe.

2884. a) $y^2 = x$; b) $y^2 = 2px$; $(0, 0)$ je singularna točka.

2885. a) $(x - C)^2 + y^2 = C^2$; b) nema rješenja; c) $x^2 + y^2 = x$; $(0, 0)$ je singularna točka.

2886. $y = e^{\frac{x}{y}}$. 2887. $y = (\sqrt{2a} \pm \sqrt{x})^2$. 2888. $y^2 = 1 - e^{-x}$.

2889. $r = Ce^{a\varphi}$. *Uputa.* Predite na polarne koordinate.

2890. $3y^2 - 2x = 0$.

2891. $r = k\varphi$. 2892. $x^2 + (y - b)^2 = b^2$. 2893. $y^2 + 16x = 0$.

2894. Hiperbole $y^2 - x^2 = C$ ili kružnice $x^2 + y^2 = C^2$.

2895. $y = \frac{1}{2}(e^x + e^{-x})$. *Uputa.* Upotrijebite da je površina $\int_0^x y \, dx$, a dužina luka $\int_0^x \sqrt{1 + y'^2} \, dx$.

2896. $x = \frac{a^2}{y} + Cy$. 2897. $y^3 = 4C(C + a - x)$.

2898. *Uputa.* Koristite se time da je rezultanta sile teže i centrifugalne sile normalna na površinu. Uzmemo li os vrtnje za os y i označimo li sa ω kutnu brzinu vrtnje, dobit ćemo za ravninski presjek kroz os tražene plohe diferencijalnu jednadžbu $g \frac{dy}{dx} = \omega^2 x$:

2899. $p = e^{-0,000167h}$. *Uputa.* Tlak na svakom nivou vertikalnog zračnog stupca možemo smatrati da je uvjetovan samo tlakom gornjih slojeva. Upotrijebite Boyle-Mariotteov zakon prema kojemu je gustoća proporcionalna tlaku. Tražena diferencijalna jednadžba je $dp = -k\rho \, dh$.

2900. $s = \frac{1}{2}klw$. *Uputa.* Jednadžba je $ds = kw \cdot \frac{l-x}{l} \, dx$.

2901. $s = \left(p + \frac{1}{2}w\right)kl$.

2902. $T = a + (T_0 - a)e^{-kt}$.

2903. Kroz jedan sat.

2904. $\omega = 100 \left(\frac{3}{5}\right)^t \text{ o/min.}$

2905. Za 100 godina raspadne se 4,2% početne količine Q_0 . *Uputa.* Jednadžba je $\frac{dQ}{dt} = kQ$;

$$Q = Q_0 \left(\frac{1}{2}\right)^{\frac{t}{1600}}.$$

2906. $t \approx 35,2$ s. *Uputa.* Jednadžba je $\pi (h^2 - 2h) dh = \pi \left(\frac{1}{10}\right)^2 v dt$.

2907. $\frac{1}{1024}$. *Uputa.* Jednadžba je $dQ = -kQ dh$; $Q = Q_0 \left(\frac{1}{2}\right)^{\frac{h}{3}}$

2908. $v \rightarrow \sqrt{\frac{gm}{k}}$ kada $t \rightarrow \infty$ (k je koeficijent proporcionalnosti). *Uputa.* Jednadžba je

$$m \frac{dv}{dt} = mg - kv^2; \quad v = \sqrt{\frac{gm}{k}} \tanh \left(t \sqrt{\frac{gk}{m}} \right).$$

2909. 18,1 kg. *Uputa.* Jednadžba je $\frac{dx}{dt} = k \left(\frac{1}{3} - \frac{x}{300} \right)$.

2910. $i = \frac{E}{R^2 + L^2 \omega^2} [(R \sin \omega t - L \omega \cos \omega t) + L \omega e^{-\frac{R}{L}t}]$. *Uputa.* Jednadžba je $Ri + L \frac{di}{dt} = E \sin \omega t$.

2911. $y = x \ln |x| + C_1 x + C_2$.

2912. $1 + C_1 y^2 = \left(C_2 + \frac{C_1 x}{\sqrt{2}} \right)^2$.

2913. $y = \ln |e^{2x} + C_1| - x + C_2$.

2914. $y = C_1 + C_2 \ln |x|$.

2915. $y = C_1 e^{C_2 x}$.

2916. $y = \pm \sqrt{C_1 x + C_2}$.

2917. $y = (1 + C_1^2) \ln |x + C_1| - C_1 x + C_2$.

2918. $(x - C_1) = a \ln \left| \sin \frac{y - C_2}{a} \right|$ za $a \neq 0$;
 $y = C$ za $a = 0$.

2919. $y = \frac{1}{2} (\ln |x|)^2 + C_1 \ln |x| + C_2$.

2920. $x = \frac{1}{C_1} \ln \left| \frac{y}{y + C_1} \right| + C_2$; $y = C$.

2921. $y = C_1 e^{C_2 x} + \frac{1}{C_2}$.

2922. $y = \pm \frac{1}{2} \left[x \sqrt{C_1^2 - x^2} + C_1^2 \arcsin \frac{x}{C_1} \right] + C_2$.

2923. $y = (C_1 e^x + 1) x + C_2$.

2924. $y = (C_1 x - C_1^2) e^{\frac{x}{C_1} + 1} + C_2$; $y = \frac{e}{2} x^2 + C$ (singularno rješenje).

2925. $y = C_1 x (x - C_1) + C_2$; $y = \frac{x^3}{3} + C$ (singularno rješenje).

2926. $y = \frac{x^3}{12} + \frac{x^2}{2} + C_1 x \ln |x| + C_2 x + C_3$.

2927. $y = \sin (C_1 + x) + C_2 x + C_3$.

2928. $y = x^3 + 3x$.

2929. $y = \frac{1}{2} (x^2 + 1)$.

2930. $y = x + 1$.

2931. $y = Cx^2$.

2932. $y = C_1 \frac{1 + C_2 e^x}{1 - C_2 e^x}$; $y = C$.

2933. $x = C_1 + \ln \left| \frac{y - C_2}{y + C_2} \right|$.

2934. $x = C_1 - \frac{1}{C_1} \ln \left| \frac{y}{y + C_2} \right|$.

2935. $x = C_1 y^2 + y \ln y + C_2$.

2936. $2y^2 - 4x^2 = 1$.

2937. $y = x + 1$.

2938. $y = \frac{x^2 - 1}{2(e^2 - 1)} - \frac{e^2 - 1}{4} \ln |x|$ ili $y = \frac{1 - x^2}{2(e^2 + 1)} + \frac{e^2 + 1}{4} \ln |x|$.

2939. $y = \frac{1}{2} x^2$

2940. $y = \frac{1}{2} x^2$.

2941. $y = 2e^x$.

2942. $x = -\frac{3}{2} (y + 2)^{2/3}$.

2943. $y = e^x$.

2944. $y^2 = \frac{e}{e - 1} + \frac{e^{-x}}{1 - e}$.

2945. $y = \frac{2\sqrt{2}}{3} x^{3/2} - \frac{8}{3}$.

2946. $y = \frac{3e^{3x}}{2 + e^{3x}}$. 2947. $y = \sec^2 x$. 2948. $y = \sin x + 1$. 2949. $y = \frac{x^2}{4} - \frac{1}{2}$.
2950. $x = -\frac{1}{2}e^{-y^2}$. 2951. Rješenja nema. 2952. $y = e^x$. 2953. $y = 2 \ln |x| - \frac{2}{x}$.
2954. $y = \frac{(x+C_1+1)^2}{2} + \frac{4}{3}C_1(x+1)^{\frac{3}{2}} + C_2$. Singularno rješenje je $y = C$.
2955. $y = C_1 \frac{x^2}{2} + (C_1 - C_1^2)x + C_2$. Singularno rješenje je $y = \frac{(x+1)^3}{12} + C$.
2956. $y = \frac{1}{12}(C_1 + x)^4 + C_2 x + C_3$.
2957. $y = C_1 + C_2 e^{C_1 x}$; $y = 1 - e^x$; $y = -1 + e^{-x}$; singularno rješenje je $y = \frac{4}{C-x}$.
2958. Kružnice. 2959. $(x - C_1)^2 - C_2 y^2 + k C_2^2 = 0$.
2960. Lančanica $y = a \operatorname{ch} \frac{x-x_0}{2}$. Kružnica $(x-x_0)^2 + y^2 = a^2$.
2961. Parabola $(x-x_0)^2 = 2ay - a^2$. Cikloida $x-x_0 = a(t - \sin t)$, $y = a(1 - \cos t)$.
2962. $e^{ay} + C_2 = \sec(ax + C_1)$. 2963. Parabola.
2964. $y = \frac{C_1}{2} \frac{H}{q} e^{\frac{q}{H}x} + \frac{1}{2C_1} \frac{H}{q} e^{-\frac{q}{H}x} + C_2$ ili $y = a \operatorname{ch} \frac{x+C}{a} + C_2$, gdje je H konstantno horizon-
- talno naprezanje, a $\frac{H}{q} = a$. Uputa. Diferencijalna jednačba je $\frac{d^2 y}{dx^2} = \frac{q}{H} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$.
2965. Jednačba gibanja je $\frac{d^2 s}{dt^2} = g(\sin \alpha - \mu \cos \alpha)$. Zakon gibanja je $s = \frac{gt^2}{2}(\sin \alpha - \mu \cos \alpha)$.
2966. $s = \frac{m}{k} \ln \operatorname{ch} \left(t \sqrt{g \frac{k}{m}} \right)$. Uputa. Jednačba gibanja je $m \frac{d^2 s}{dt^2} = mg - k \left(\frac{ds}{dt} \right)^2$.
2967. Za 6,45 s. Uputa. Jednačba gibanja je $\frac{300}{g} \frac{d^2 x}{dt^2} = -10v$.
2968. a) Ne; b) da; c) da; d) da; e) ne; f) ne; g) ne; h) da.
2969. a) $y'' + y = 0$; b) $y'' - 2y' + y = 0$; c) $x^2 y'' - 2xy' + 2y = 0$; d) $y''' - 3y'' + 4y' - 2y = 0$.
2970. $y = 3x - 5x^2 + 2x^3$.
2971. $y = \frac{1}{x}(C_1 \sin x + C_2 \cos x)$. Uputa. Primijenite supstituciju $y = y_1 u$.
2972. $y = C_1 x + C_2 \ln x$. 2973. $y = A + Bx^2 + x^3$.
2974. $y = \frac{x^2}{3} + Ax + \frac{B}{x}$. Uputa. Partikularna rješenja homogene jednačbe su $y_1 = x$, $y_2 = \frac{1}{x}$.
- Metodom varijacija konstanti dobivamo $C_1 = \frac{x}{2} + A$; $C_2 = -\frac{x^3}{6} + B$.
2975. $y = A + B \sin x + C \cos x + \ln |\sec x + \operatorname{tg} x| + \sin x \ln |\cos x| - x \cos x$.
2976. $y = C_1 e^{2x} + C_2 e^{3x}$. 2977. $y = C_1 e^{-3x} + C_2 e^{3x}$.

- 2978.** $y = C_1 + C_2 e^x$. **2979.** $y = C_1 \cos x + C_2 \sin x$.
2980. $y = e^x (C_1 \cos x + C_2 \sin x)$. **2981.** $y = e^{-2x} (C_1 \cos 3x + C_2 \sin 3x)$.
2982. $y = (C_1 + C_2 x) e^{-x}$. **2983.** $y = e^{2x} (C_1 e^{x\sqrt{2}} + C_2 e^{-x\sqrt{2}})$.
2984. Kada je $k > 0$, $y = C_1 e^{x\sqrt{k}} + C_2 e^{-x\sqrt{k}}$; kada je $k < 0$, $y = C_1 \cos \sqrt{-k}x + C_2 \sin \sqrt{-k}x$.
2985. $y = e^{-\frac{x}{2}} \left(C_1 e^{\frac{\sqrt{5}}{2}x} + C_2 e^{-\frac{\sqrt{5}}{2}x} \right)$. **2986.** $y = e^{\frac{x}{6}} \left(C_1 \cos \frac{\sqrt{11}}{6}x + C_2 \sin \frac{\sqrt{11}}{6}x \right)$.
2987. $y = 4e^x + e^{4x}$. **2988.** $y = e^{-x}$. **2989.** $y = \sin 2x$. **2990.** $y = 1$.
2991. $y = a \operatorname{ch} \frac{x}{a}$. **2992.** $y = 0$. **2993.** $y = C \sin \pi x$.
2994. a) $xe^{2x}(Ax^2+Bx+C)$; b) $A \cos 2x + B \sin 2x$; c) $A \cos 2x + B \sin 2x + Cx^2 e^{2x}$; d) $e^x(A \cos x + B \sin x)$; e) $e^x(Ax^2+Bx+C) + xe^{2x}(Dx+E)$; f) $xe^x[(Ax^2+Bx+C) \cos 2x + (Dx^2+Ex+F) \sin 2x]$.
2995. $y = (C_1 + C_2 x) e^{2x} + \frac{1}{8} (2x^2 + 4x + 3)$. **2996.** $y = e^{\frac{x}{2}} \left(C_1 \cos \frac{x\sqrt{3}}{2} + C_2 \sin \frac{x\sqrt{3}}{2} \right) + x^3 + 3x^2$.
2997. $y = (C_1 + C_2 x) e^{-x} + \frac{1}{9} e^{2x}$. **2998.** $y = C_1 e^x + C_2 e^{7x} + 2$.
2999. $y = C_1 e^x + C_2 e^{-x} + \frac{1}{2} xe^x$. **3000.** $y = C_1 \cos x + C_2 \sin x + \frac{1}{2} x \sin x$.
3001. $y = C_1 e^x + C_2 e^{-2x} - \frac{2}{5} (3 \sin 2x + \cos 2x)$. **3002.** $y = C_1 e^{2x} + C_2 e^{-3x} + x \left(\frac{x}{10} - \frac{1}{25} \right) e^{2x}$.
3003. $y = (C_1 + C_2 x) e^x + \frac{1}{2} \cos x + \frac{x^2}{4} e^x - \frac{1}{8} e^{-x}$.
3004. $y = C_1 + C_2 e^{-x} + \frac{1}{2}x + \frac{1}{20} (2 \cos 2x - \sin 2x)$.
3005. $y = e^x (C_1 \cos 2x + C_2 \sin 2x) + \frac{x}{4} e^x \sin 2x$. **3006.** $y = \cos 2x + \frac{1}{3} (\sin x + \sin 2x)$.
3007. 1) $x = C_1 \cos \omega t + C_2 \sin \omega t + \frac{A}{\omega^2 - p^2} \sin pt$; 2) $x = C_1 \cos \omega t + C_2 \sin \omega t - \frac{A}{2\omega} t \cos \omega t$.
3008. $y = C_1 e^{3x} + C_2 e^{4x} - xe^{4x}$. **3009.** $y = C_1 + C_2 e^{2x} + \frac{x}{4} - \frac{x^2}{4} - \frac{x^3}{6}$.
3010. $y = e^x (C_1 + C_2 x + x^2)$. **3011.** $y = C_1 + C_2 e^{2x} + \frac{1}{2} xe^{2x} - \frac{5}{2} x$.
3012. $y = C_1 e^{-2x} + C_2 e^{4x} - \frac{1}{9} e^x + \frac{1}{5} (3 \cos 2x + \sin 2x)$.
3013. $y = C_1 + C_2 e^{-x} + e^x + \frac{5}{2} x^2 - 5x$. **3014.** $y = C_1 + C_2 e^x - 3xe^x - x - x^2$.
3015. $y = \left(C_1 + C_2 x + \frac{1}{2} x^2 \right) e^{-x} + \frac{1}{4} e^x$.
3016. $y = (C_1 \cos 3x + C_2 \sin 3x) e^x + \frac{1}{37} (\sin 3x + 6 \cos 3x) + \frac{e^x}{9}$.
3017. $y = (C_1 + C_2 x + x^2) e^{2x} + \frac{x+1}{8}$.

$$3018. y = C_1 + C_2 e^{3x} - \frac{1}{10} (\cos x + 3 \sin x) - \frac{x^2}{6} - \frac{x}{9}.$$

$$3019. y = \frac{1}{8} e^{2x}(4x+1) - \frac{x^3}{6} - \frac{x^2}{4} + \frac{x}{4}.$$

$$3020. y = C_1 e^x + C_2 e^{-x} - x \sin x - \cos x.$$

$$3021. y = C_1 e^{-2x} + C_2 e^{2x} - \frac{e^{2x}}{20} (\sin 2x + 2 \cos 2x).$$

$$3022. y = C_1 \cos 2x + C_2 \sin 2x - \frac{x}{4} (3 \sin 2x + 2 \cos 2x) + \frac{1}{4}.$$

$$3023. y = e^x (C_1 \cos x + C_2 \sin x - 2x \cos x). \quad 3024. y = C_1 e^x + C_2 e^{-x} + \frac{1}{4} (x^2 - x) e^x.$$

$$3025. y = C_1 \cos 3x + C_2 \sin 3x + \frac{1}{4} x \sin x - \frac{1}{16} \cos x + \frac{1}{54} (3x-1) e^{3x}.$$

$$3026. y = C_1 e^{3x} + C_2 e^{-x} + \frac{1}{9} (2-3x) + \frac{1}{16} (2x^2-x) e^{3x}.$$

$$3027. y = C_1 + C_2 e^{2x} - 2xe^x - \frac{3}{4}x - \frac{3}{4}x^2. \quad 3028. y = \left(C_1 + C_2 x + \frac{x^3}{6} \right) e^{2x}.$$

$$3029. y = C_1 e^{-3x} + C_2 e^x - \frac{1}{8} (2x^2+x) e^{-3x} + \frac{1}{16} (2x^2+3x) e^x.$$

$$3030. y = C_1 \cos x + C_2 \sin x + \frac{x}{4} \cos x + \frac{x^2}{4} \sin x - \frac{x}{8} \cos 3x + \frac{3}{32} \sin 3x. \text{ Uputa. Produkt kosinusa pretvorite u sumu kosinusa.}$$

$$3031. y = C_1 e^{-x\sqrt{2}} + C_2 e^{x\sqrt{2}} + xe^x \sin x + e^x \cos x.$$

$$3032. y = C_1 \cos x + C_2 \sin x + \cos x \ln \left| \operatorname{ctg} \left(\frac{x}{2} + \frac{\pi}{4} \right) \right|.$$

$$3033. y = C_1 \cos x + C_2 \sin x + \sin x \cdot \ln \left| \operatorname{tg} \frac{x}{2} \right|. \quad 3034. y = (C_1 + C_2 x) e^x + xe^x \ln |x|.$$

$$3035. y = (C_1 + C_2 x) e^{-x} + xe^{-x} \ln |x|.$$

$$3036. y = C_1 \cos x + C_2 \sin x + x \sin x + \cos x \ln |\cos x|.$$

$$3037. y = C_1 \cos x + C_2 \sin x - x \cos x + \sin x \ln |\sin x|.$$

$$3038. \text{ a) } y = C_1 e^x + C_2 e^{-x} + (e^x + e^{-x}) \operatorname{arctg} e^x; \quad \text{ b) } y = C_1 e^{x\sqrt{2}} + C_2 e^{-x\sqrt{2}} + e^{x^2}.$$

$$3040. \text{ Jednadžba gibanja je } \frac{2}{g} \left(\frac{d^2 x}{dt^2} \right) = 2 - k(x+2) \quad (k=1); \quad T = 2\pi \sqrt{\frac{2}{g}} \text{ s.}$$

$$3041. x = \frac{2g \sin 30t - 60 \sqrt{g} \sin \sqrt{g} t}{g - 900} \text{ cm. Uputa. Ako } x \text{ računamo od položaja u kojem teret}$$

miruje, onda je $\frac{4}{g} x'' = 4 - k(x_0 + x - y - l)$, gdje je x_0 udaljenost točke mirovanja tereta od početne tačke ovješnja opruge, l je duljina opruge u stanju mirovanja; prema tome je $k(x_0 - l) = 4$ i $\frac{4}{g} \frac{d^2 x}{dt^2} = -k(x - y)$, gdje je $k = 4$, $g = 981 \text{ cm/s}^2$.

$$3042. m \frac{d^2 x}{dt^2} = k(b-x) - k(b+x); \quad x = c \cos \left(t \sqrt{\frac{2k}{m}} \right)$$

$$3043. 6 \frac{d^2 s}{dt^2} = gs; \quad t = \sqrt{\frac{6}{g}} \ln(6 + \sqrt{35}).$$

3044. a) $r = \frac{a}{2} (e^{\omega t} + e^{-\omega t})$; b) $r = \frac{v_0}{2\omega} (e^{\omega t} - e^{-\omega t})$. *Uputa.* Diferencijalna jednačba gibanja je $\frac{d^2 r}{dt^2} = \omega^2 r$.

3045. $y = C_1 + C_2 e^x + C_3 e^{12x}$.

3046. $y = C_1 + C_2 e^{-x} + C_3 e^x$.

3047. $y = C_1 e^{-x} + e^{\frac{x}{2}} \left(C_2 \cos \frac{\sqrt{3}}{2} x + C_3 \sin \frac{\sqrt{3}}{2} x \right)$.

3048. $y = C_1 + C_2 x + C_3 e^{x\sqrt{2}} + C_4 e^{-x\sqrt{2}}$.

3049. $y = e^x (C_1 + C_2 x + C_3 x^2)$.

3050. $y = e^x (C_1 \cos x + C_2 \sin x) + e^{-x} (C_3 \cos x + C_4 \sin x)$.

3051. $y = (C_1 + C_2 x) \cos 2x + (C_3 + C_4 x) \sin 2x$.

3052. $y = C_1 + C_2 e^{-x} + e^{\frac{x}{2}} \left(C_3 \cos \frac{\sqrt{3}}{2} x + C_4 \sin \frac{\sqrt{3}}{2} x \right)$.

3053. $y = (C_1 + C_2 x) e^{-x} + (C_3 + C_4 x) e^x$.

3054. $y = C_1 e^{ax} + C_2 e^{-ax} + C_3 \cos ax + C_4 \sin ax$.

3055. $y = (C_1 + C_2 x) e^{\sqrt{3}x} + (C_3 + C_4 x) e^{-\sqrt{3}x}$.

3056. $y = C_1 + C_2 x + C_3 \cos ax + C_4 \sin ax$.

3057. $y = C_1 + C_2 x + (C_3 + C_4 x) e^{-x}$.

3058. $y = (C_1 + C_2 x) \cos x + (C_3 + C_4 x) \sin x$.

3059. $y = e^{-x} (C_1 + C_2 x + \dots + C_n x^{n-1})$.

3060. $y = C_1 + C_2 x + \left(C_3 + C_4 x + \frac{x^2}{2} \right) e^x$.

3061. $y = C_1 + C_2 x + 12x^2 + 3x^3 + \frac{1}{2} x^4 + \frac{1}{20} x^5 + (C_3 + C_4 x) e^x$.

3062. $y = C_1 e^x + e^{-\frac{x}{2}} \left(C_2 \cos \frac{\sqrt{3}}{2} x + C_3 \sin \frac{\sqrt{3}}{2} x \right) - x^3 - 5$.

3063. $y = C_1 + C_2 x + C_3 x^2 + C_4 e^{-x} + \frac{1}{1088} (4 \cos 4x - \sin 4x)$.

3064. $y = C_1 e^{-x} + C_2 + C_3 x + \frac{3}{2} x^2 - \frac{1}{3} x^3 + \frac{1}{12} x^4 + e^x \left(\frac{3}{2} x - \frac{15}{4} \right)$.

3065. $y = C_1 e^{-x} + C_2 \cos x + C_3 \sin x + e^x \left(\frac{x}{4} - \frac{3}{8} \right)$.

3066. $y = C_1 + C_2 \cos x + C_3 \sin x + \sec x \ln |\cos x| - \operatorname{tg} x \sin x + x \sin x$.

3067. $y = e^{-x} + e^{-\frac{x}{2}} \left(\cos \frac{\sqrt{3}}{2} x + \frac{1}{\sqrt{3}} \sin \frac{\sqrt{3}}{2} x \right) + x - 2$.

3068. $y = (C_1 + C_2 \ln x) \frac{1}{x}$.

3069. $y = C_1 x^3 + \frac{C_2}{x}$.

3070. $y = C_1 \cos (2 \ln x) + C_2 \sin (2 \ln x)$.

3071. $y = C_1 x + C_2 x^2 + C_3 x^3$.

3072. $y = C_1 + C_2 (3x+2)^{-\frac{4}{3}}$.

3073. $y = C_1 x^2 + \frac{C_2}{x}$.

3074. $y = C_1 \cos (\ln x) + C_2 \sin (\ln x)$.

3075. $y = C_1 x^3 + C_2 x^2 + \frac{1}{2} x$.

3076. $y = (x+1)^2 [C_1 + C_2 \ln (x+1)] + (x+1)^3$.

3077. $y = x (\ln x + \ln^2 x)$.

3078. $y = C_1 \cos x + C_2 \sin x, z = C_2 \cos x - C_1 \sin x$.

$$3079. y = e^{-x} (C_1 \cos x + C_2 \sin x), \quad z = \frac{1}{5} e^{-x} [(C_2 - 2C_1) \cos x - (C_1 + 2C_2) \sin x].$$

$$3080. y = (C_1 - C_2 - C_1 x) e^{-2x}, \quad z = (C_1 x + C_2) e^{-2x}.$$

$$3081. x = C_1 e^t + e^{-\frac{t}{2}} \left(C_2 \cos \frac{\sqrt{3}}{2} t + C_3 \sin \frac{\sqrt{3}}{2} t \right),$$

$$y = C_1 e^t + e^{-\frac{t}{2}} \left(\frac{C_3 \sqrt{3} - C_2}{2} \cos \frac{\sqrt{3}}{2} t - \frac{C_2 \sqrt{3} + C_3}{2} \sin \frac{\sqrt{3}}{2} t \right),$$

$$z = C_1 e^t + e^{-\frac{t}{2}} \left(\frac{-C_3 \sqrt{3} - C_2}{2} \cos \frac{\sqrt{3}}{2} t + \frac{C_2 \sqrt{3} - C_3}{2} \sin \frac{\sqrt{3}}{2} t \right).$$

$$3082. x = C_1 e^{-t} + C_2 e^{2t}, \quad y = C_3 e^{-t} + C_2 e^{2t}, \quad z = -(C_1 + C_3) e^{-t} + C_2 e^{2t}.$$

$$3083. y = C_1 + C_2 e^{2x} - \frac{1}{4} (x^2 + x), \quad z = C_2 e^{2x} - C_1 + \frac{1}{4} (x^2 - x - 1).$$

$$3084. y = C_1 + C_2 x + 2 \sin x, \quad z = -2C_1 - C_2 (2x + 1) - 3 \sin x - 2 \cos x.$$

$$3085. y = (C_2 - 2C_1 - 2C_2 x) e^{-x} - 6x + 14, \quad z = (C_1 + C_2 x) e^{-x} + 5x - 9; \quad C_1 = 9, \quad C_2 = 4,$$

$$y = 14(1 - e^{-x}) - 2x(3 + 4e^{-x}), \quad z = -9(1 - e^{-x}) + x(5 + 4e^{-x}).$$

$$3086. x = 10 e^{2t} - 8e^{3t} - e^t + 6t - 1; \quad y = -20e^{2t} + 8e^{3t} + 3e^t + 12t + 10.$$

$$3087. y = \frac{2C_1}{(C_2 - x)^2}, \quad z = \frac{C_1}{C_2 - x}.$$

$$3088. \text{ a) } \frac{(x^2 + y^2)y}{x^2} = C_1, \quad \frac{z}{y} = C_2; \text{ b) } \ln \sqrt{x^2 + y^2} = \operatorname{arctg} \frac{y}{x} + C_1, \quad \frac{z}{\sqrt{x^2 + y^2}} = C_2. \text{ c) } \textit{Uputa.}$$

Integriranjem homogene jednadžbe $\frac{dx}{x-y} = \frac{dy}{x+y}$, nalazimo prvi integral $\ln \sqrt{x^2 + y^2} = \operatorname{arctg} \frac{y}{x} +$

$+ C_1$. Nadalje upotrebom svojstava izvedenih razmjera, imamo $\frac{dz}{z} = \frac{x dx}{x(x-y)} = \frac{y dy}{y(x+y)} =$

$= \frac{x dx + y dy}{x^2 + y^2}$. Odatle je $\ln z = \frac{1}{2} \ln (x^2 + y^2) + \ln C_2$ i prema tome, $\frac{z}{\sqrt{x^2 + y^2}} = C_2$; d) $x + y +$

$+ z = 0, x^2 + y^2 + z^2 = 6$. *Uputa.* Primjenom svojstava izvedenih razmjera imamo: $\frac{dx}{y-z} =$

$= \frac{dy}{z-x} = \frac{dz}{x-y} = \frac{dx + dy + dz}{0}$; odatle je $dx + dy + dz = 0$ i prema tome $x + y + z = C_1$. Analogno

je $\frac{x dx}{x(y-z)} = \frac{y dy}{y(z-x)} = \frac{z dz}{z(x-y)} = \frac{x dx + y dy + z dz}{0}$; $x dx + y dy + z dz = 0$ i $x^2 + y^2 + z^2 = C_2$.

Na taj način integralne krivulje su kružnice $x + y + z = C_1, x^2 + y^2 + z^2 = C_2$. Iz početnih uvjeta $x = 1, y = 1, z = -2$ dobit ćemo $C_1 = 0, C_2 = 6$.

$$3089. y = C_1 x^2 + \frac{C_2}{x} - \frac{x^2}{18} (3 \ln^2 x - 2 \ln x), \quad Z = 1 - 2C_1 x + \frac{C_2}{x^2} + \frac{x}{9} (3 \ln^2 x + \ln x - 1).$$

$$3090. y = C_1 e^{x\sqrt{2}} + C_2 e^{-x\sqrt{2}} + C_3 \cos x + C_4 \sin x + e^x - 2x,$$

$$Z = -C_1 e^{x\sqrt{2}} - C_2 e^{-x\sqrt{2}} - \frac{C_3}{4} \cos x - \frac{C_4}{4} \sin x - \frac{1}{2} e^x + x.$$

- 3091.** $x = \frac{v_0 m \cos \alpha}{k} \left(1 - e^{-\frac{k}{m} t} \right)$, $y = \frac{m}{k^2} (k v_0 \sin \alpha + mg) \left(1 - e^{-\frac{k}{m} t} \right) - \frac{mgt}{k}$. *Rješenje.*
 $m \frac{dv_x}{dt} = -k v_x$; $m \frac{dv_y}{dt} = -k v_y - mg$ pri početnim uvjetima: $x_0 = y_0 = 0$, $v_{x0} = v_0 \cos \alpha$,
 $v_{y0} = v_0 \sin \alpha$ pri $t = 0$. Integriranjem dobijemo: $v_x = v_0 \cos \alpha e^{-\frac{k}{m} t}$, $k v_y + mg =$
 $= (k v_0 \sin \alpha + mg) e^{-\frac{k}{m} t}$.
- 3092.** $x = \alpha \cos \frac{k}{\sqrt{m}} t$, $y = \frac{v_0 \sqrt{m}}{k} \sin \frac{k}{\sqrt{m}} t$, $\frac{x^2}{a^2} + \frac{k^2 y^2}{m v_0^2} = 1$. *Uputa.* Diferencijalne jednadžbe gibanja su: $m \frac{d^2 x}{dt^2} = -k^2 x$; $m \frac{d^2 y}{dt^2} = -k^2 y$. **3093.** $y = -2 - 2x - x^2$.
- 3094.** $y = \left(y_0 + \frac{1}{4} \right) e^{2(x-1)} - \frac{1}{2} x + \frac{1}{4}$.
- 3095.** $y = \frac{1}{2} + \frac{1}{4} x + \frac{1}{8} x^2 + \frac{1}{16} x^3 + \frac{9}{32} x^4 + \frac{21}{320} x^5 + \dots$
- 3096.** $y = \frac{1}{3} x^3 - \frac{1}{7 \cdot 9} x^7 + \frac{2}{7 \cdot 11 \cdot 27} x^{11} - \dots$
- 3097.** $y = x + \frac{x^2}{1 \cdot 2} + \frac{x^3}{2 \cdot 3} + \frac{x^4}{3 \cdot 4} + \dots$; red konvergira za $-1 \leq x \leq 1$.
- 3098.** $y = x - \frac{x^2}{(1!)^2 \cdot 2} + \frac{x^3}{(2!)^2 \cdot 3} - \frac{x^4}{(3!)^2 \cdot 4} + \dots$; red konvergira za $-\infty < x < +\infty$. *Uputa.*
 Koristite se metodom neodređenih koeficijenata.
- 3099.** $y = 1 - \frac{1}{3!} x^3 + \frac{1 \cdot 4}{6!} x^6 - \frac{1 \cdot 4 \cdot 7}{9!} x^9 + \dots$; red konvergira za $-\infty < x < +\infty$.
- 3100.** $y = \frac{\sin x}{x}$. *Uputa.* Koristite se metodom neodređenih koeficijenata.
- 3101.** $y = 1 - \frac{x^2}{2^2} + \frac{x^4}{2^2 \cdot 4^2} - \frac{x^6}{2^2 \cdot 4^2 \cdot 6^2} + \dots$; red konvergira za $|x| < +\infty$. *Uputa.* Koristite se metodom neodređenih koeficijenata.
- 3102.** $x = a \left(1 - \frac{1}{2!} t^2 + \frac{2}{4!} t^4 - \frac{9}{6!} t^6 + \frac{55}{8!} t^8 - \dots \right)$.
- 3103.** $u = A \cos \frac{a\pi t}{l} \sin \frac{\pi x}{l}$. *Uputa.* Koristite se uvjetima: $u(0, t) = 0$; $u(l, t) = 0$, $u(x, 0) = A \sin \frac{\pi x}{l}$, $\frac{\partial u(x, 0)}{\partial t} = 0$.
- 3104.** $u = \frac{2l}{\pi^2 a} \sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} \sin \frac{(2k+1)\pi a t}{l} \sin \frac{(2k+1)\pi x}{l}$. *Uputa.* Koristite se uvjetima:
 $u(0, t) = 0$, $u(l, t) = 0$, $u(x, 0) = 0$, $\frac{\partial u(x, 0)}{\partial t} = 1$.
- 3105.** $u = \frac{8h}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \sin \frac{n\pi a t}{2} \cos \frac{n\pi a t}{l} \sin \frac{n\pi x}{l}$. *Uputa.* Koristite se ovim uvjetima:
 $\frac{\partial u(x, 0)}{\partial t} = 0$, $u(0, t) = 0$, $u(l, t) = 0$, $u(x, 0) = \begin{cases} \frac{2hx}{l} \text{ za } 0 < x \leq \frac{l}{2}, \\ 2h \left(1 - \frac{x}{l} \right) \text{ za } \frac{l}{2} < x < l. \end{cases}$

3106. $u = \sum_{n=0}^{\infty} A_n \cos \frac{(2n+1)a\pi t}{2l} \sin \frac{(2n+1)\pi x}{2l}$, gdje su koeficijenti

$$A_n = \frac{2}{l} \int_0^l \frac{x}{l} \sin \frac{(2n+1)\pi x}{2l} dx = \frac{8(-1)^n}{(2n+1)^2 \pi^2}. \text{ Uputa. Koristite se ovim uvjetima:}$$

$$u(0, t) = 0, \frac{\partial u(l, t)}{\partial x} = 0, u(x, 0) = \frac{x}{l}, \frac{\partial u(x, 0)}{\partial t} = 0.$$

3107. $u = \frac{400}{\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^3} (1 - \cos n\pi) \sin \frac{n\pi x}{100} \cdot e^{-\frac{a^2 n^2 \pi^2 t}{100^3}}$. Uputa. Koristite se ovim uvjetima:

$$u(0, t) = 0, u(100, t) = 0, u(x, 0) = 0,01 x (100 - x).$$

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3108. a) $\leq 1''$; $\leq 0,0023\%$; b) $\leq 1\text{mm}$; $\leq 0,26\%$; c) ≤ 1 d) $\leq 0,0016\%$.

3109. a) $\leq 0,05$; $\leq 0,021\%$; b) $\leq 0,0005$; $\leq 1,45\%$; c) $\leq 0,005$; $\leq 0,16\%$.

3110. a) 2 znamenke; $48 \cdot 10^3$ ili $49 \cdot 10^3$, jer je broj uključen između 47877 i 48845; b) 2 znamenke; 15; c) 1 znamenka; $6 \cdot 10^2$. Praktički rezultat treba pisati u obliku $(5,9 \pm 0,1) \cdot 10^2$.

3111. a) 29,5; b) $1,6 \cdot 10^2$; c) 43,2.

3112. a) 84,2; b) 18,5 ili $18,47 \pm 0,01$; c) rezultat izračunavanja nema točnih znamenaka jer je razlika jednaka stotinki pri mogućoj vrijednosti apsolutne pogreške od jedne stotinke.

3113. $1,8 \pm 0,3 \text{ cm}^2$. Uputa. Koristite se formulom za prirast površine kvadrata.

3114. a) $30,0 \pm 0,2$; b) $43,7 \pm 0,1$; c) $0,3 \pm 0,1$.

3115. $19,9 \pm 0,1 \text{ m}^2$.

3116. a) $1,1295 \pm 0,0002$; b) $0,120 \pm 0,006$; c) kvocijent može varirati između 48 i 62. Prema tome u kvocijentu nije moguće smatrati pouzdanim niti jednu decimalnu znamenku.

3117. 0,480. Posljednja brojka može varirati za 1.

3118. a) 0,1729; b) $277 \cdot 10^3$; c) 2.

3119. $(2,05 \pm 0,01) \cdot 10^3 \text{ cm}^2$.

3120. a) 1,648; b) $4,025 \pm 0,001$; c) $9,006 \pm 0,003$.

3121. $4,01 \cdot 10^3 \text{ cm}^2$. Apsolutna pogreška iznosi 6,5 cm. Relativna pogreška iznosi 0,16%.

3122. Kateta je jednaka $13,8 \pm 0,2 \text{ cm}$; $\sin \alpha = 0,44 \pm 0,01$; $\alpha = 26^\circ 15' \pm 35'$.

3123. $2,7 \pm 0,1$.

3124. 0,27 A.

3125. Duljinu njihala treba izmjeriti s točnošću do 0,3 cm; brojeve π i q uzmite s tri znamenke (prema principu istih utjecaja).

3126. Polumjere i izvodnicu izmjerite s relativnom pogreškom 1/300. Broj π uzmite s tri znamenke (prema principu istih utjecaja).

3127. Veličinu l izmjerite s točnošću 0,2% a s s točnošću od 0,7% (prema principu istih utjecaja).