Vektori

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Tomislav Šikić ima princip predavanja na način da uz predavanje intenzivno koristi knjižice, a i Ljubo je u zaostatku pa je dodatno ubrzao. Zato su poglavlja površno obrađena!

<u>Vektorski produkt (umožak)</u> Knjižica 3, 18. str.

Vrijedi:

$\left|\vec{a} \times \vec{b}\right| = |\vec{a}| \cdot |\vec{b}| \cdot |\sin\alpha|$ → U knjižici se za vektor koristi $\boldsymbol{a}$ umjesto $\vec{a}$

$\vec{a} \times \vec{b}$ je okomit na vektor $\vec{a}$ i na $\vec{b}$

Trojka $(\vec{a}, \vec{b}, \vec{a} \times \vec{b})$ čini desni sustav. → Pravilo desne ruke

Svojstva:

$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

$(\lambda\vec{a}) \times \vec{b} = \lambda(\vec{a}) \times \vec{b}$

$(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$ $\vec{a}$ i $\vec{b}$ su kolinearni → $\vec{a} \times \vec{b} = \vec{0}$

<u>Vektorski umnožak u koordinatnom sustavu</u> Knjižica 3, 20. str.

$\vec{\imath} \times \vec{\imath} = 0 \qquad \vec{\jmath} \times \vec{\jmath} = 0 \qquad \vec{k} \times \vec{k} = 0$

$\vec{\imath} \times \vec{\jmath} = \vec{k} \qquad \vec{\jmath} \times \vec{k} = \vec{\imath} \qquad \vec{k} \times \vec{\imath} = \vec{\jmath}$

$\vec{\jmath} \times \vec{\imath} = -\vec{k} \qquad \vec{k} \times \vec{\jmath} = -\vec{\imath} \qquad \vec{\imath} \times \vec{k} = -\vec{\jmath}$

$\vec{a} \times \vec{b} = (a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k}) \times (b_x\vec{\imath} + b_y\vec{\jmath} + b_z\vec{k})$

$= a_xb_x\vec{\imath} \times \vec{\imath} + a_xb_y\vec{\imath} \times \vec{\jmath} + a_xb_z\vec{\imath} \times \vec{k} + a_yb_x\vec{\jmath} \times \vec{\imath} + a_yb_y\vec{\jmath} \times \vec{\jmath} + a_yb_z\vec{\jmath} \times \vec{k} + a_zb_x\vec{k} \times \vec{\imath} + a_zb_y\vec{k} \times \vec{\jmath} + a_zb_z\vec{k} \times \vec{k}$

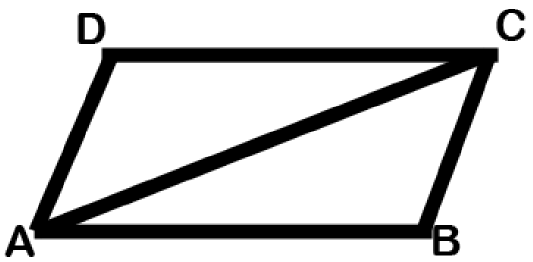
$$= (a_yb_z - a_zb_y)\vec{\imath} + (a_zb_x - a_xb_z)\vec{\jmath} + (a_xb_y - a_yb_x)\vec{k}$$

Računanje vektorskog umnoška   Knjižica 3, 21. str.

$$\vec{a} = a_x\vec{\imath} + a_y\vec{\jmath} + a_z\vec{k} \qquad \vec{b} = b_x\vec{\imath} + b_y\vec{\jmath} + b_z\vec{k}$$

$$\vec{a} + \vec{b} = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix}\vec{\imath} - \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix}\vec{\jmath} + \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}\vec{k} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

PRIMJER 14



$$P_\Delta = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AD}\right| \qquad P_\Delta = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{BC}\right|$$

$$\left|\overrightarrow{AB} \times \overrightarrow{BC}\right| = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ -1 & 2 & -4 \\ 6 & 2 & -4 \end{vmatrix} \qquad P_\Delta = \frac{b \cdot v_b}{2} \qquad v_b = \frac{2 \cdot 7\sqrt{5}}{|\overrightarrow{AC}|}$$

$$\overrightarrow{AB} = (0 - 1, 0 - -2, -4) = (-1, 2, -4)$$

$$\overrightarrow{BC} = (6, 2, -4)$$

Mješoviti umnožak   Knjižica 3, 22. str.

$$\begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} c_x & c_y & c_z \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}$$

Rastav vektora po bazi   Knjižica 3, 24. str.

ZADATAK

$$\vec{a} = (3, 1, 4) \qquad D = \begin{vmatrix} 3 & 1 & 4 \\ -2 & 0 & 6 \\ 3 & 6 & 5 \end{vmatrix} = 0 + 18 - 48 - 0 - 108 + 10 \neq 0$$

$$\vec{b} = (-2, 0, 6)$$

$$\vec{c} = (3, 6, 5)$$

$$\alpha, \beta, \gamma = ?$$

$\vec{d} = (10,10,10)$ → Možemo iskazati $\vec{d}$ pomoću $\vec{a}$, $\vec{b}$, i $\vec{c}$ jer im je $D \neq 0$

Ljubo bolestan

$\vec{d} = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$

$(10,10,10) = \alpha(3,1,4) + \beta(-2,0,6) + \gamma(3,6,5)$

$(10,10,10) = (3\alpha - 2\beta + 3\gamma, \alpha + 6\gamma, 4\alpha + 6\beta + 5\gamma)$

$3\alpha - 2\beta + 3\gamma = 10$

$\alpha + 6\gamma = 10$

$4\alpha + 6\beta + 5\gamma = 10$

$A = \begin{bmatrix} 3 & -2 & 3 \\ 1 & 0 & 6 \\ 4 & 6 & 5 \end{bmatrix} \qquad D = \begin{vmatrix} 3 & 1 & 4 \\ -2 & 0 & 6 \\ 3 & 6 & 5 \end{vmatrix}$ → D smo imali na početku!

<u>Rastav vektora u ortogonalnoj bazi</u> Knjižica 3, 25. str.

$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k} \ /\cdot\vec{i} /\cdot\vec{j} /\cdot\vec{k}$

$\vec{a}\vec{i} = a_x\vec{i}\cdot\vec{i} + 0 + 0$

$\vec{a}\vec{j} = 0 + a_y\vec{j}\cdot\vec{j} + 0$

$\vec{a}\vec{k} = 0 + 0 + a_z\vec{k}\cdot\vec{k}$

$a_x = \vec{a}\vec{i} \qquad a_y = \vec{a}\vec{j} \qquad a_z = \vec{a}\vec{k}$

Pravac i ravnina

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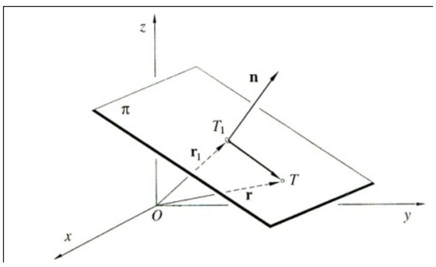
Sve se odvija u R^3 → Trodimenzionalni vektorski prostor



$\vec{n} \cdot \overrightarrow{T_1T} = (A, B, C) \cdot (X - X_1, Y - Y_1, Z - Z_1)$

$\boxed{A(X - X_1) + B(Y - Y_1) + C(Z - Z_1) = 0}$

PRIMJER 1

3)

$T(1,-2,0)$

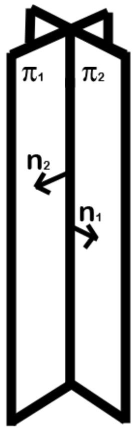
$\overrightarrow{TS}, S(0,-1,1)$ $\vec{n}$ → Normala

$\overrightarrow{TS} = \vec{n} = (-1,1,1)$

$-1(x-1) + 1(Y+2) + 1(Z-0) = 0$

PRIMJER 2

1)



π=?

$\overrightarrow{n_1} \in \pi$

$\overrightarrow{n_2} \in \pi$ $\vec{n} = \overrightarrow{n_1} \times \overrightarrow{n_2}$

$$\vec{n} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix} \rightarrow \begin{vmatrix} x-1 & y-2 & z+1 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 0$$

2)

$T(2,1,3)$

$S(1,0,-1)$

$\overrightarrow{ST} = (1,1,4)$ $\overrightarrow{n_1} = (3,-2,1)$

$$\vec{n} = \overrightarrow{n_1} \times \overrightarrow{ST} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 1 \\ 1 & 1 & 4 \end{vmatrix} \rightarrow \begin{vmatrix} x-2 & y-1 & z-3 \\ 3 & -2 & -1 \\ 1 & 1 & 4 \end{vmatrix} = 0$$

- Ako ima grešaka (matematičkih ili gramatičkih, kako koga smeta :D) ili nešto nedostaje (moguće da nije sve zapisano) ili imate neku ideju, javite mi na PM ili direktno mailom na Telefunken@fer2.net