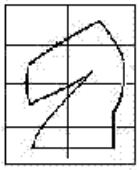
Steven H. Cullinane Generating the octad generator. Expository note. April 28, 1985.

0	1
x	x+1
x^2	x^2+1
x^2+x	x^2+x+1

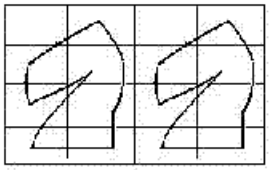
GF(8)
(mod x^3-x-1)



A Singer 7-cycle
S_1 on GF(8)

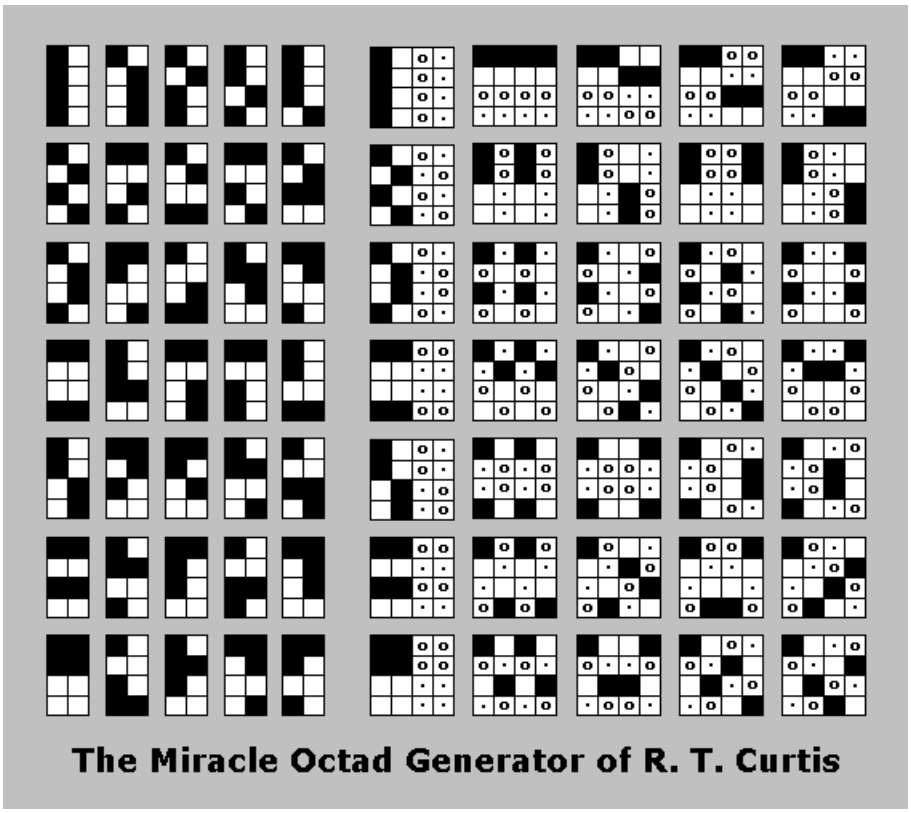
0000	0001	1000	1001
0010	0011	1010	1011
0100	0101	1100	1101
0110	0111	1110	1111

The linear 4-space
L over GF(2)



A linear map S_2 on L
(= 2 copies of S_1)

S_1 and S_2 acting on row 1 below yield the Miracle Octad Generator [3]:



Apart from its use in studying the 759 octads of a Steiner system S(5,8,24) -- and hence the Mathieu group M_{24} -- the Curtis MOG nicely illustrates a natural correspondence C (Conwell [2], p. 72) between

(a) the 35 partitions of an 8-set H (such as GF(8) above, or Conwell's 8 "heptads") into two 4-sets, and
(b) the 35 partitions of L into four parallel affine planes.

Two of the H-partitions have a common refinement into 2-sets iff the same is true of the corresponding L-partitions. (Cameron [1], p. 60).

Note that C is particularly natural in row 1, and that partitions 2-5 in each row have similar structures.

1. Cameron, P. J., Parallelisms of Complete Designs, Camb. U. Pr. 1976.
2. Conwell, G. M., The 3-space PG(3,2) and its group, Ann. of Math. 11 (1910) 60-76.
3. Curtis, R. T., A new combinatorial approach to M_{24}, Math. Proc. Camb. Phil. Soc. 79 (1976) 25-42.

For an image of the original 1985 typed note, click here.