Hi, I'm Dr. Abhishek Thakur

I'm on a mission to help 100,000 students to achieve technical skills, to earn passive income using digital coaching.

Digital Marketer, Entrepreneur & A Mentor

Thanks & Regard

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* $\boxed{\text{Computing}}$ *

Antecedent $\longrightarrow$ Consequent

$$(X) \longrightarrow \boxed{Y = f(X)} \longrightarrow Y$$

f : formal method | Algorithm | mapping function

— Precise Solution

— Unambiguous & Accurate

— Mathematical Model

* Hard Computing *

* Precise Results
* Control Actions → Unambiguous
  → Formally defined
* Numerical Problem
* Searching & Sorting
* Computational Geometry Problem

* Soft Computing *

* Imprecision
* Dynamic
* Uncertainty
* low Solution Cost
* Do not require Mathematical Model

FUZZY LOGIC ⇒ Neural Network ⇒ Genetic Computing

Hard Computing	Soft Computing
① Precision	① Imprecision
② Based on Binary Logic	② Based on Fuzzy Logic
③ No Approximation	③ Approximation
④ Exact Input Data	④ Noisy Data
⑤ Strictly Sequential	⑤ Allow parallel Computing
⑥ Certainty	⑥ Uncertainty

* FUZZY LOGIC *

Mathematical Language

Fuzzy logic deals with Fuzzy Set

Relational logic

Boolean Logic

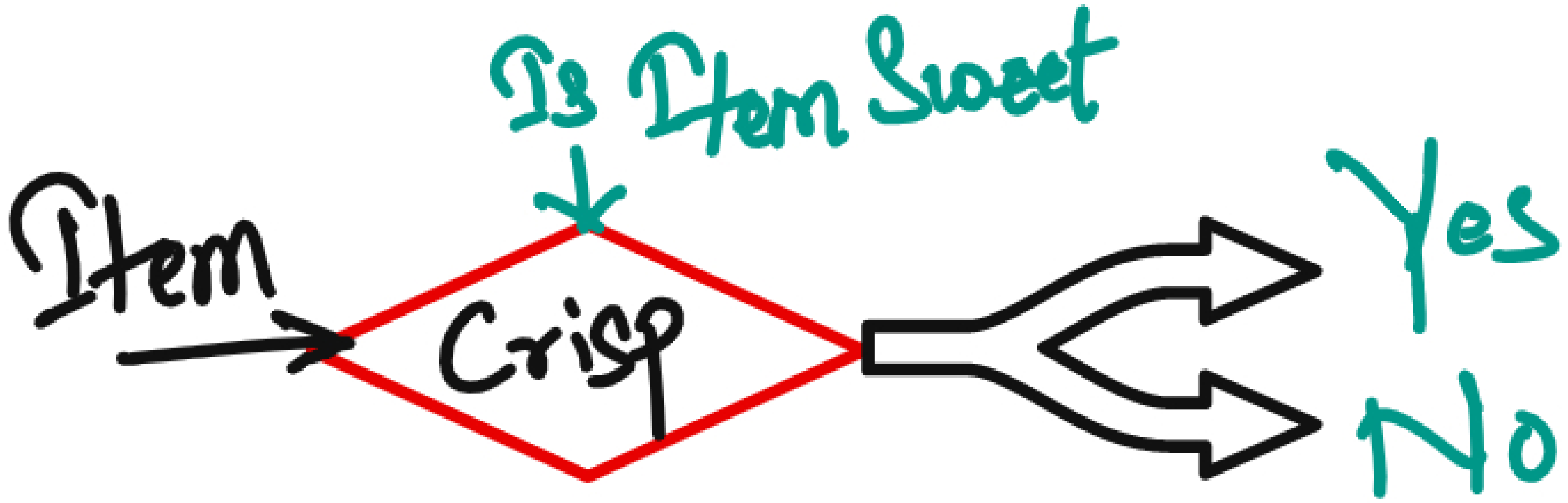
Predicate logic

It deals with Fuzzy Algebra

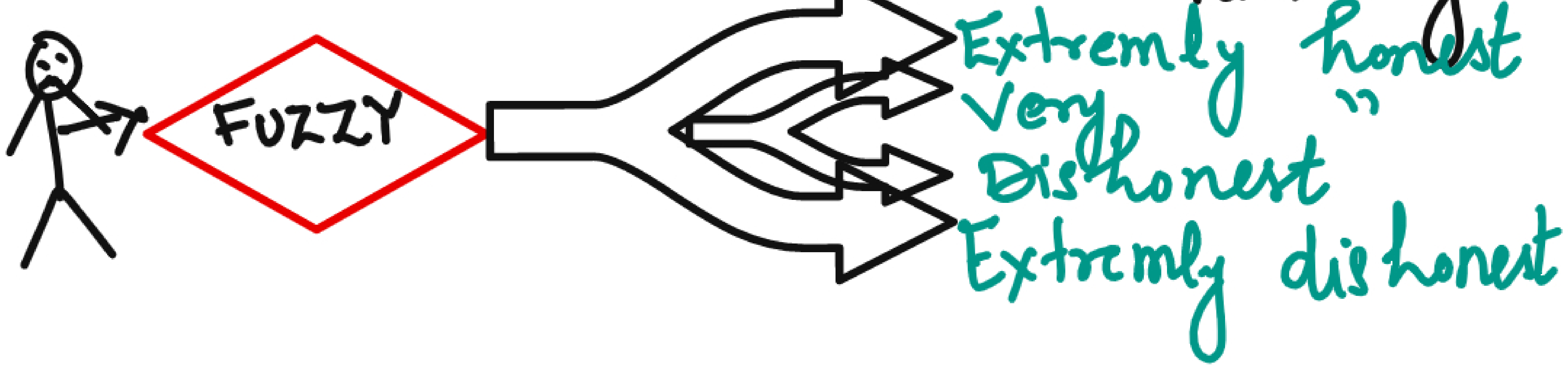
Crisp Logic Vs Fuzzy Logic

Crisp O/P
- Yes or No
- True or False

Fuzzy O/P
- May be
- May Not be
- Absolutely
- Partially



Is Person Honest

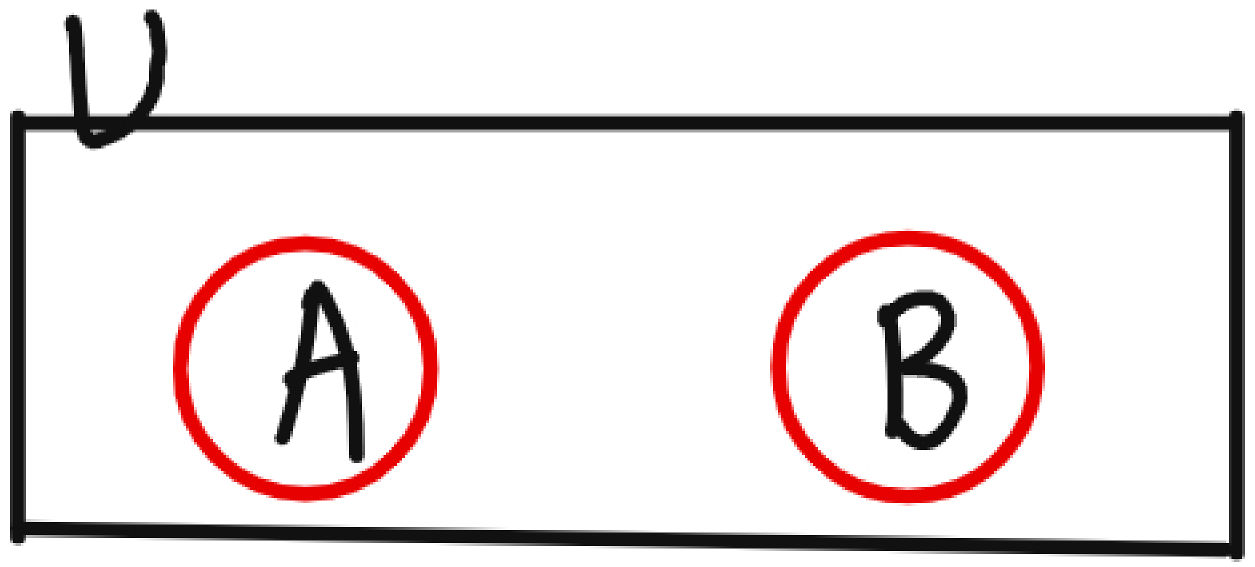


* Crisp Set *

U : All Students

A : In 10th Class

B : In 12th Class



FUZZY SET

U: All Students

G: Good Students $G = \{G, \mu(G)\}$

S: Bad Students $\mu(\,)$ degree of Goodness

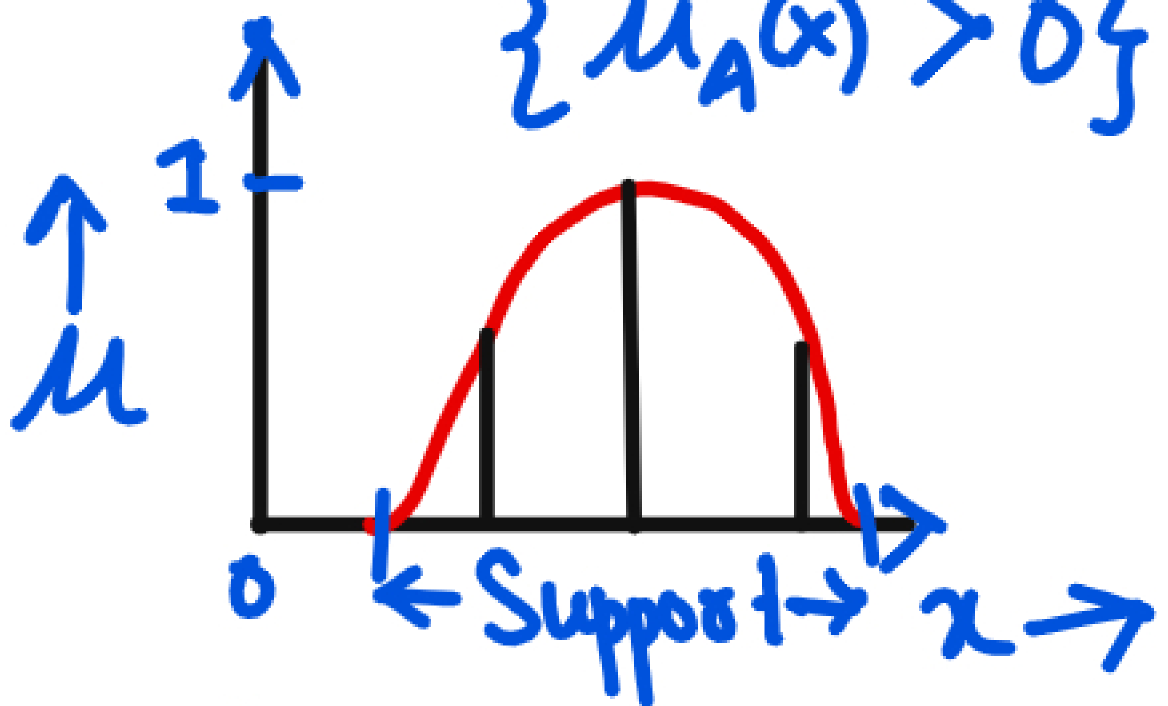
$$G = \{(A, 0.9), (B, 0.7), (C, 0.1), (D, 0.3)\}$$

$$B = \{(A, 0.1), (B, 0.3), (C, 0.9), (D, 0.7)\}$$

* Membership Function *

(I) Support :-

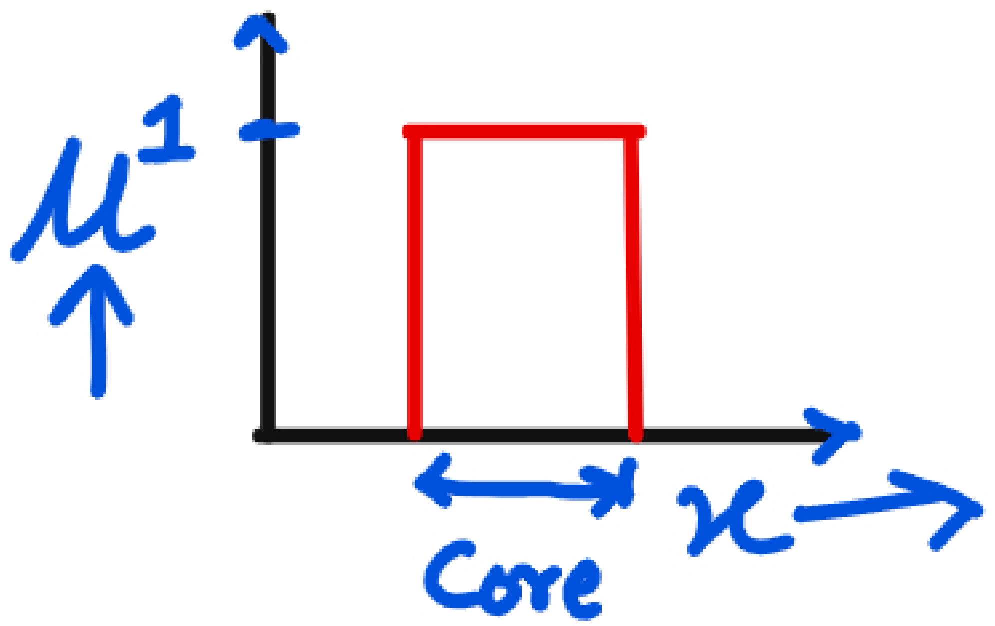
set of all points such that $\{\mu_A(x) > 0\}$



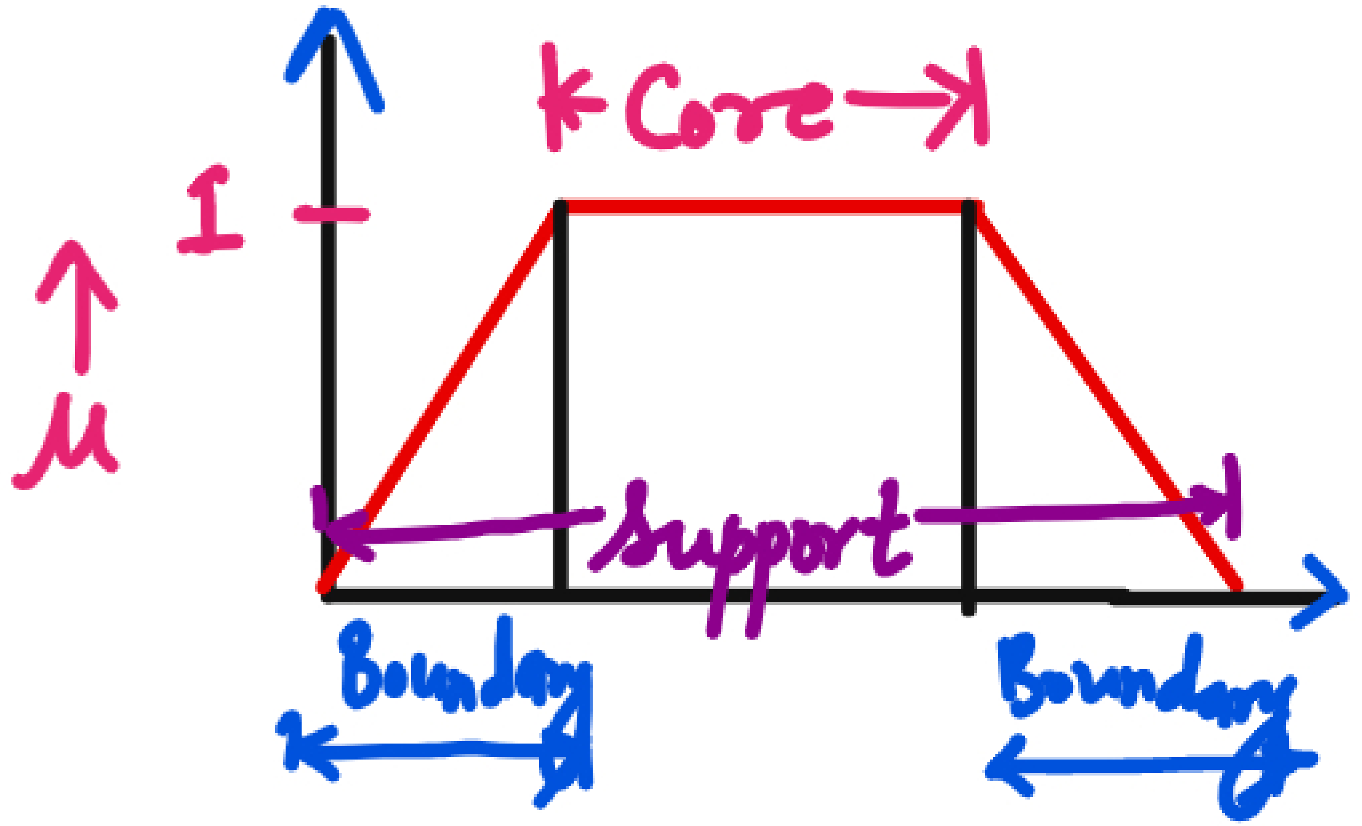
$$\text{Support}(A) = \{x \mid \mu_A(x) > 0\}$$

(II) Core :- $\mu_A(x) = 1$

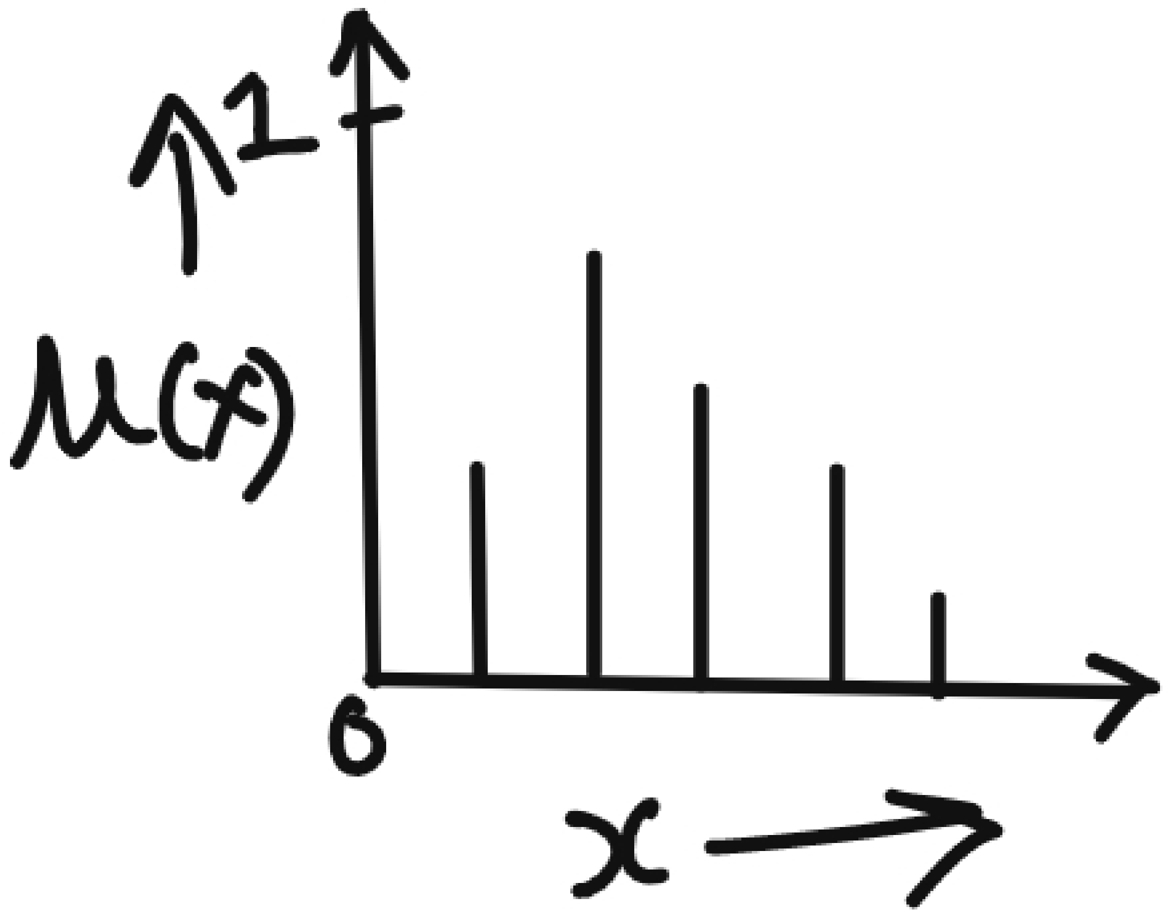
$$\text{Core}(A) = \{x : \mu_A(x) = 1\}$$



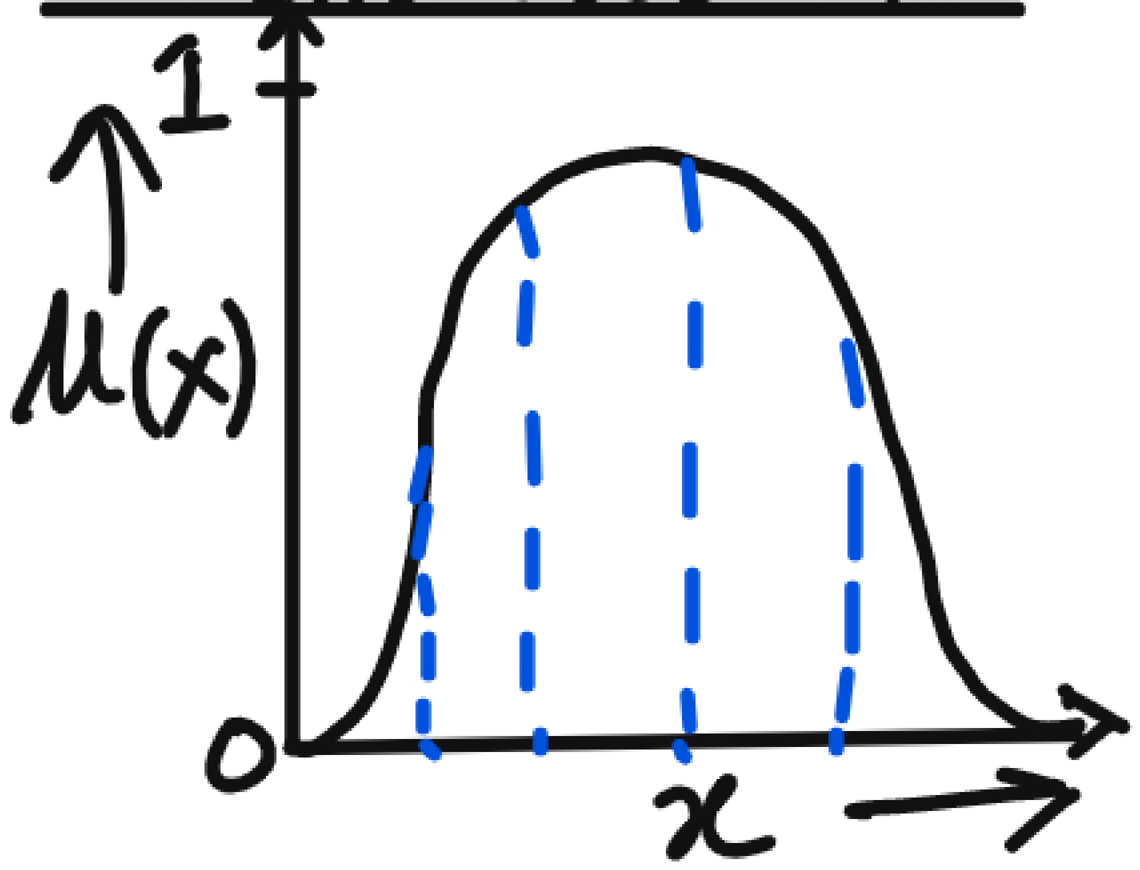
(III) Boundary $(1 > \mu_A(x) > 0)$



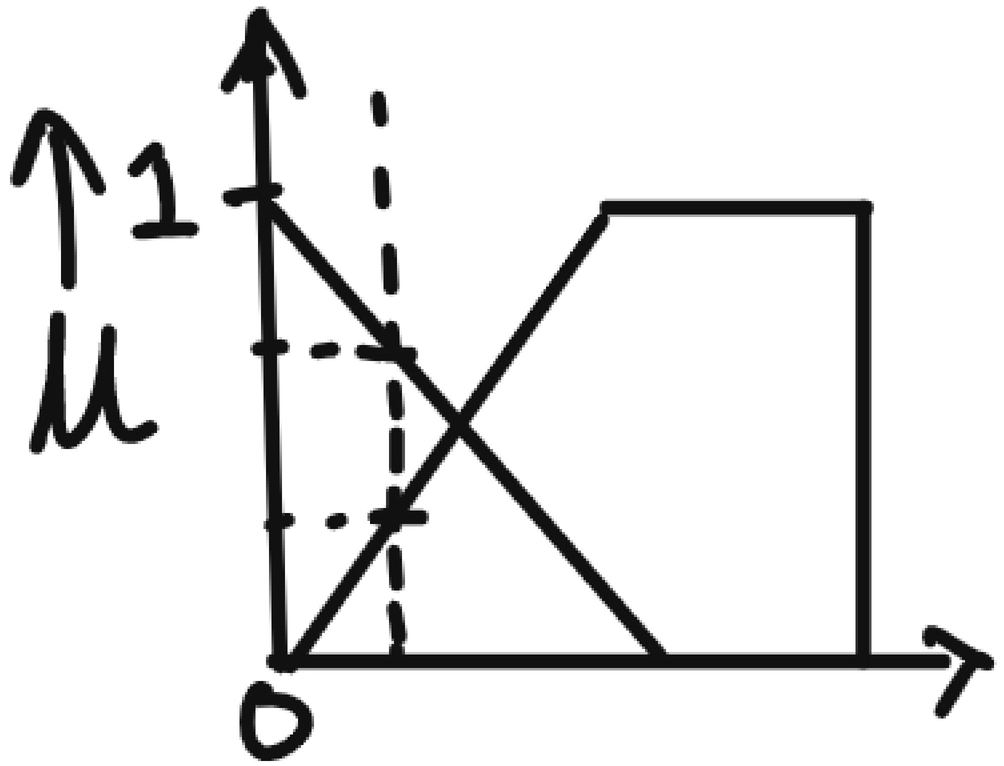
Discrete MF



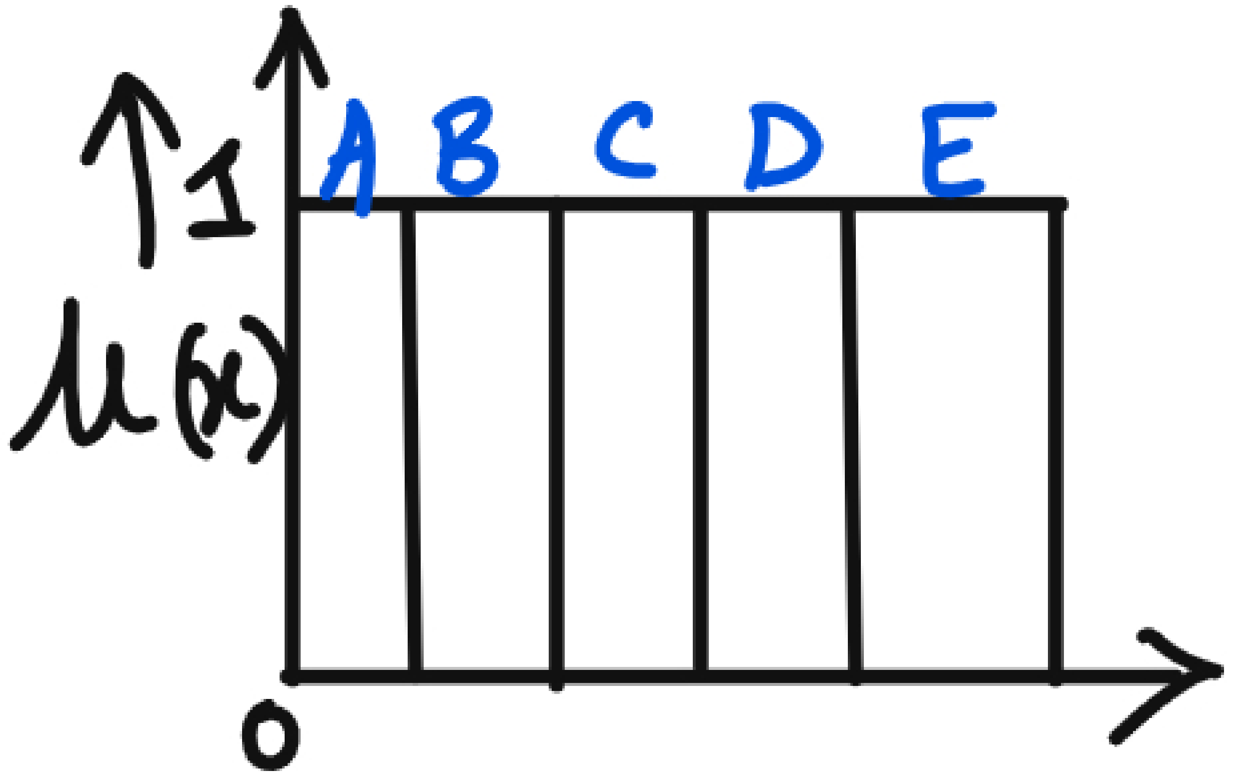
Continuous MF



Fuzzy Sets



Crisp Set



* Fuzzy Set Operation *

① **Union :-** $(A \cup B)$

$$\mu_{(A \cup B)}(x) = \max\left(\mu_A(x), \mu_B(x)\right)$$

② **Intersection :-** $(A \cap B)$

$$\mu_{(A \cap B)}(x) = \min\left(\mu_A(x), \mu_B(x)\right)$$

Example :- $A = \{(x_1, 0.6), (x_2, 0.7), (x_3, 0.4)\}$

$B = \{(x_1, 0.3), (x_2, 0.2), (x_3, 0.5)\}$

$(A \cup B) = \{(x_1, \ ), (x_2, \ ), (x_3, \ )\}$

$(A \cap B) = \{(x_1, \ ), (x_2, \ ), (x_3, \ )\}$

③ Complement: A^c

$$\mu_{(A^c)}(x) = 1 - \mu_A(x)$$

Example :-

$A = \{(x_1, 0.6), (x_2, 0.7), (x_3, 0.4)\}$

$A^c = \{(x_1, \quad), (x_2, \quad), (x_3, \quad)\}$

$B = \{(x_1, 0.3), (x_2, 0.2), (x_3, 0.5)\}$

$B^c = \{(x_1, \quad), (x_2, \quad), (x_3, \quad)\}$

④ Vector Product : (A.B)

$$\mu_{A.B}(x) = \mu_A(x) \cdot \mu_B(x)$$

Example :

$A = \{(x_1, 0.6), (x_2, 0.7), (x_3, 0.4)\}$

$B = \{(x_1, 0.3), (x_2, 0.2), (x_3, 0.5)\}$

$(A.B) = \{(x_1, 0.18), (x_2, \quad), (x_3, \quad)\}$

⑤ Cartesion Product : $(A \times B)$

$$\mu_{A \times B}(x, y) = \min(\mu_A(x), \mu_B(y))$$

Example:

$$A(x) = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5)\}$$

$$B(y) = \{(y_1, 0.8), (y_2, 0.6), (y_3, 0.3)\}$$

$$(A \times B) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.2 & 0.2 & \\ 0.3 & & \\ & & \end{bmatrix} \end{array}$$

⑥ Scalar Product : $(\alpha \times A)$

$$\mu_{\alpha A}(x) = \alpha \times \mu_A(x)$$

⑦ Equlity : $(A = B)$

$$\mu_A(x) = \mu_B(x)$$

⑧ Power : (A^α)

$$\mu_{A^\alpha}(x) = (\mu_A(x))^\alpha$$

⑨ Sum: $(A+B)$

$$\mu_{A+B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x)$$

⑩ Difference: $(A-B)$

$$\mu_{A-B}(x) = \mu_{A \cap B^c}(x)$$

⑪ Disjunctive Sum $(A \oplus B)$

$$\mu_{(A \oplus B)}(x) = (A^c \cap B) \cup (A \cap B^c)$$

Fuzzy Set Properties

① Commutative: $A \cap B = B \cap A$

$A \cup B = B \cup A$

② Associative: $A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

③ Distributive: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

④ Idempontence:

$A \cup A = A$; $A \cap A = \phi$

$A \cup \phi = A$; $A \cap \phi = \phi$

⑤ Transitive:

If $A \subseteq B$; $B \subseteq C$ then $A \subseteq C$

⑥ Demorgan's Law:

$(A \cap B)^c = A^c \cup B^c$

$(A \cup B)^c = A^c \cap B^c$

Crisp Relation

$A = \{1, 2, 3\}$ $B = \{4, 7, 8\}$

$A \times B = \{(1,4)(1,7)(1,8)(2,4)(2,7)(2,8)(3,4)(3,7)(3,8)\}$

$R = \{(1,4)(2,7)(3,8)\}$

$$R = \begin{array}{c|ccc} & 4 & 7 & 8 \\ \hline 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \end{array}$$

$R_2 = \{(a,b) / a > b, (a,b) \in A \times B\}$

$R_2 = ?$

$R_1 = \{(a, b) / a < b, (a,b) \in A \times B\}$

$R_1 = \{(1,4)(1,7)(1,8)(2,4)(2,7)(2,8)(3,7)(3,8)(3,4)\}$

Operation on Crisp Relation

① Union : $(A \cup B)$ ② Intersection : $(A \cap B)$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A \cup B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad A \cap B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Fuzzy Relation & Operation

$$A = \{(x_1, 0.6)(x_2, 0.2)(x_3, 0.3)$$

$$B = \{(y_1, 0.7)(y_2, 0.3)(y_3, 0.4)$$

$$\mu_R(x,y) = \mu_{A \times B}(x,y)$$

$$= \min[\mu_A(x), \mu_B(y)]$$

$$R = A \times B = \begin{array}{c c} & \begin{array}{ccc} Y_1 & Y_2 & Y_3 \end{array} \\ \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array} & \begin{bmatrix} 0.6 & 0.3 & 0.4 \\ 0.2 & - & - \\ 0.3 & - & - \end{bmatrix} \end{array}$$

FUZZY IF THEN Rule

- Fuzzy Implication
- Fuzzy Rule
- Fuzzy Conditional Statement

If x is A then y is B

- x is A: Premise / antecedent
- y is B: Conclusion / Consequence

Fuzzy Rule: R denoted as $R: A \rightarrow B$

Eg:- If Temp is High then Pressure is low

$$T_{HIGH} = \{(25, 0.1), (30, 0.2), (35, 0.3), (40, 0.6)\}$$

$$P_{LOW} = \{(2, 0.3), (5, 0.5)(6, 0.4)\}$$

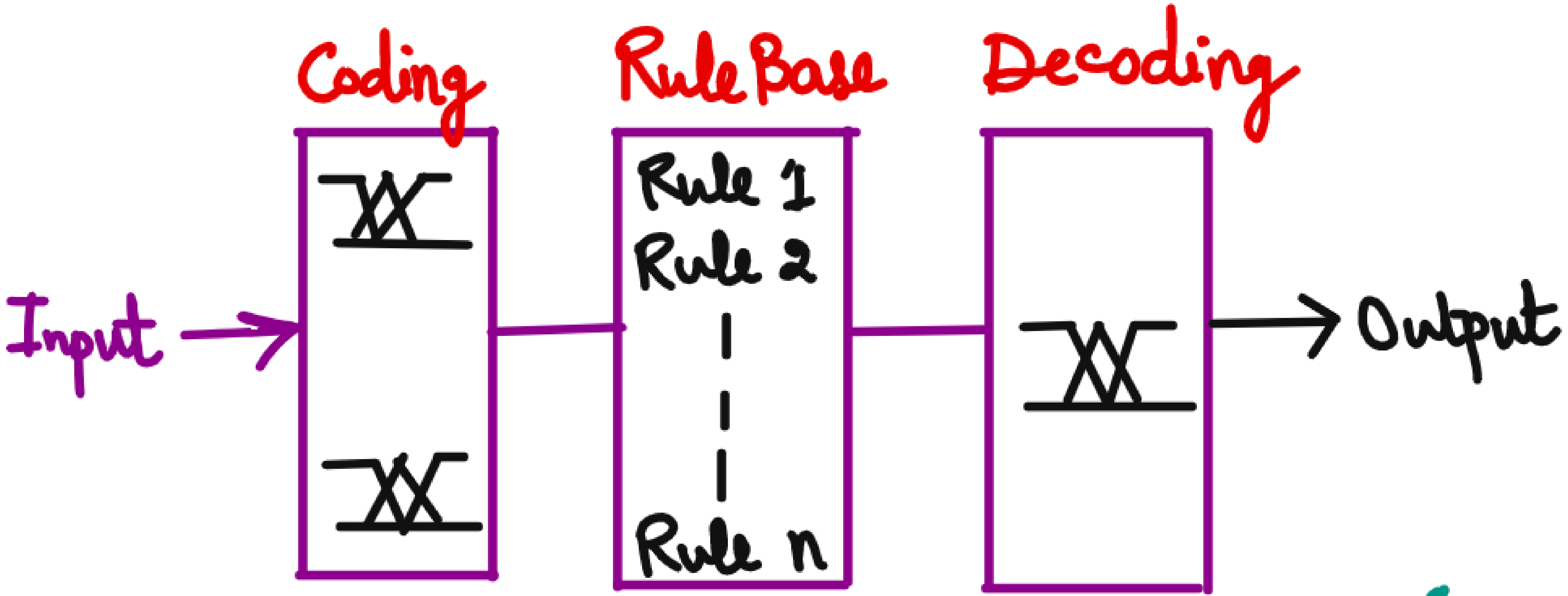
If Temp is High then Pressure is Low

$$R : T_{HIGH} \to P_{LOW}$$

$R : A \to B$ (Cartesion Product)

$$A \times B$$
$$\downarrow$$
$$\min(\mu_A, \mu_B)$$

$$R = \begin{array}{c} \\ 25 \\ 30 \\ 35 \\ 40 \end{array} \begin{array}{c} \begin{array}{ccc} 2 & 5 & 6 \end{array} \\ \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.2 & 0.2 & 0.2 \\ 0.3 & 0.5 & \text{—} \\ 0.3 & 0.5 & \text{—} \end{bmatrix} \end{array}$$



Fuzzification Inference Defuzzification

Example:- Inverted Pendulum

Input ① Error (difference)
② Angular Velocity

Output :- Current (mA)

Defuzzification

Fuzzy to Crisp conversion

Crisp Input $\rightarrow$ Fuzzy System $\rightarrow$ Crisp Output

1. Lambda Cut Method
2. Maxima methods
3. Weighted Sum Method
4. Centroid Methods

Lamda-cut method

Fuzzy Set A $\longrightarrow$ Crisp Set A_λ $(0 \le \lambda \le 1)$

$$A_\lambda = \{x \mid \mu_A(x) \geq \lambda\}$$

Example:

$A = \{(x_1, 0.2), (x_2, 0.4), (x_3, 0.6)\}$

$\lambda = 0.3$

$A_{0.3} = \{(x_1, 0), (x_2, 1), (x_3, 1)\} = \{x_2, x_3\}$

$B = \{(y_1, 0.5), (y_2, 0.4), (y_3, 0.7)\}$

$\lambda = 0.7$

$B_{0.7} = \{(y_1, 0), (y_2, 0), (y_3, 1)\} = \{y_3\}$

Lamda-cut method (Fuzzy Relation)

$$R = \begin{bmatrix} 0.2 & 1 \\ 0.3 & 0.6 \end{bmatrix} \quad \lambda = 1, 0.5, 0.1, 0$$

$$R_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad R_{0.5} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$$

Perform Cartesian Product

$$R_{0.1} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad R_0 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

R - Fuzzy Relation

R_λ - Crisp Relation

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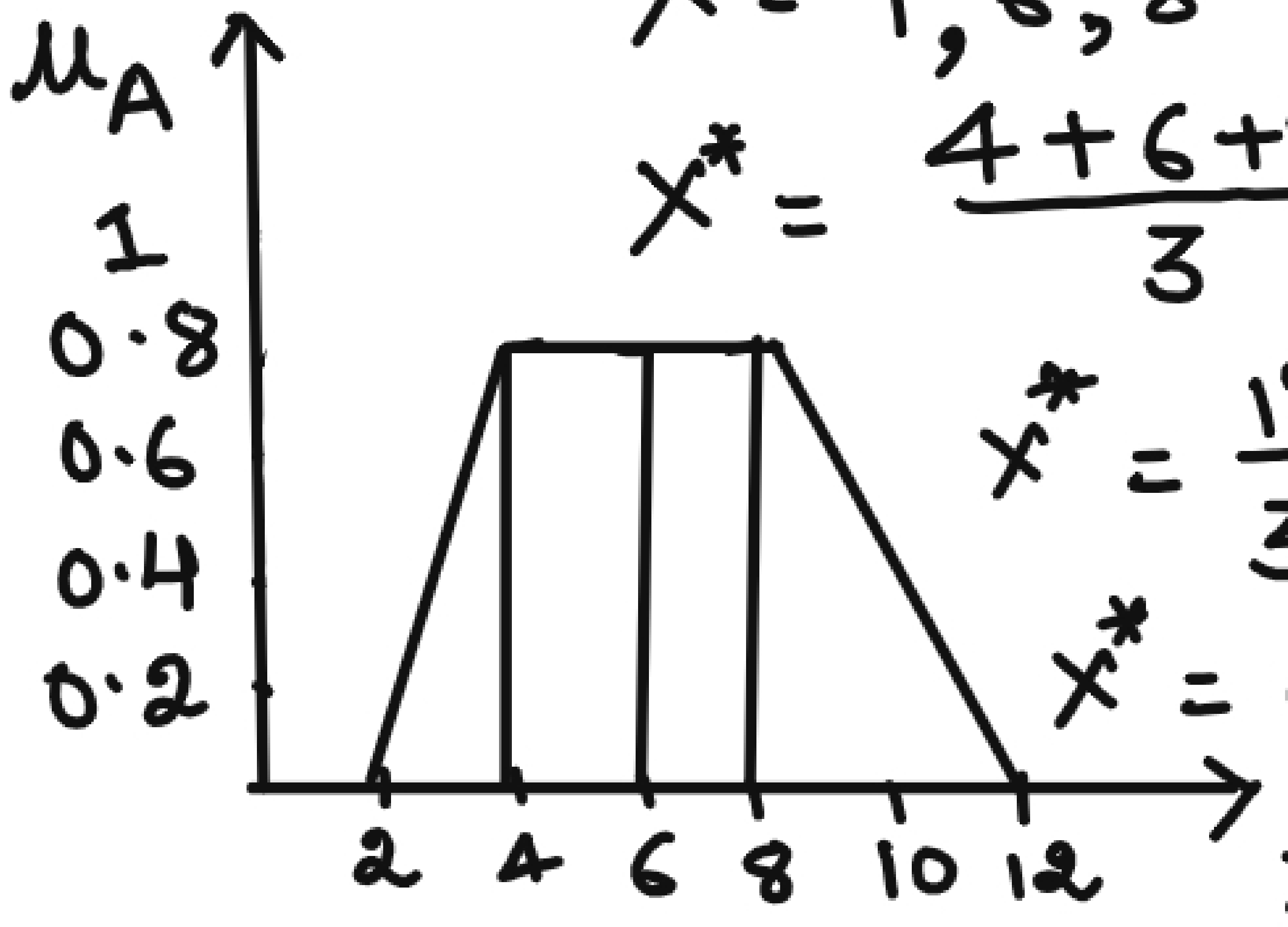
Maxima Method

1) First of Maxima (FOM) ($x^* = 4$)
2) Last of Maxima (LOM) ($x^* = 8$)
3) Mean of Maxima (MOM) ($x^* = 6$)

$$x^* = \frac{\sum x_i \in M \, X_i}{|M|}$$



$X = 4, 6, 8$

$$x^* = \frac{4+6+8}{3}$$

$$x^* = \frac{18}{3}$$

$$x^* = 6$$

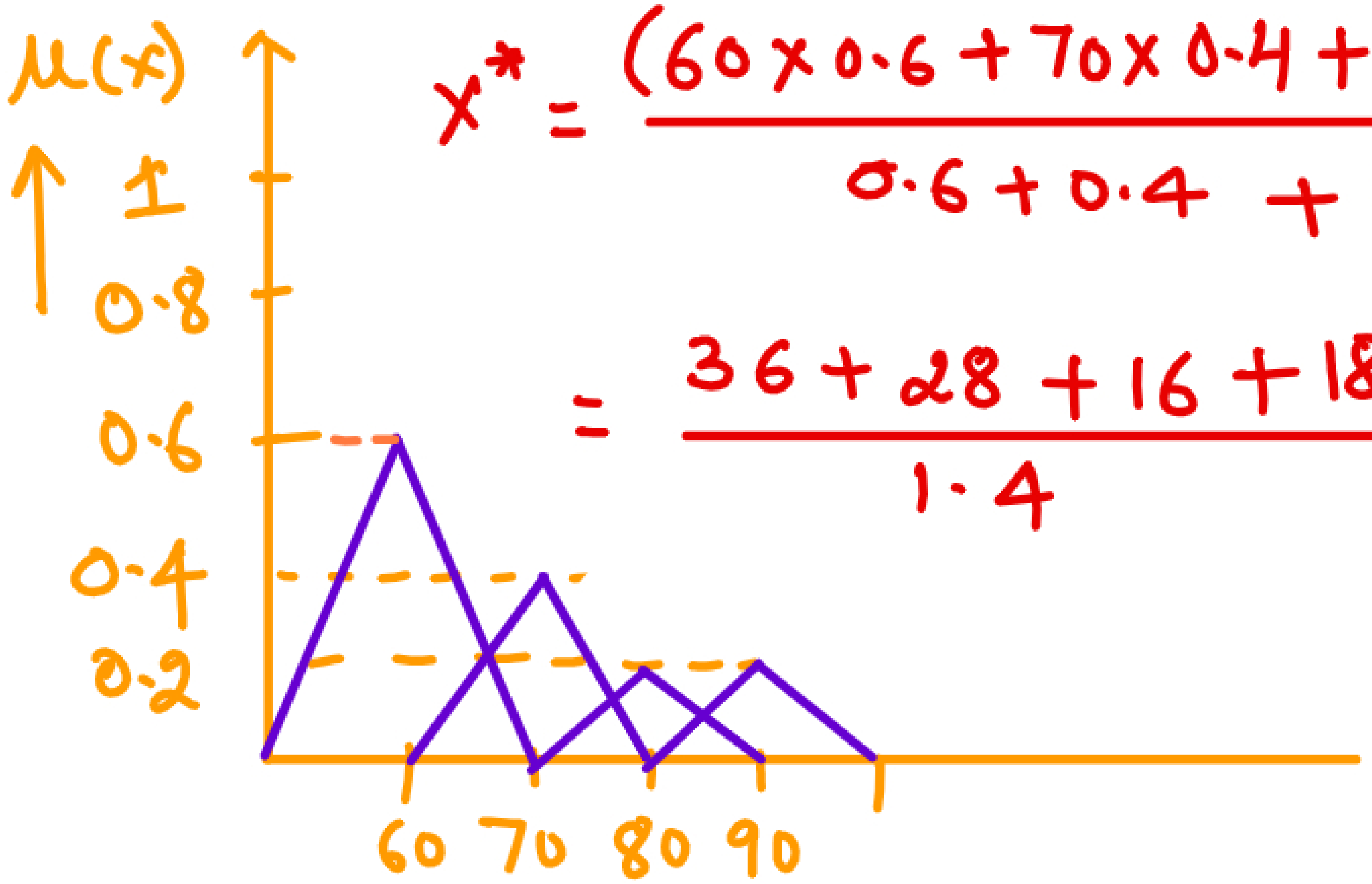
$M = \{ x \mid \mu_A(x) = \text{height of fuzzy set} \}$

$|M|$ = Cardinality of set M

x^* = Crisp Value

Weighted Average Method

$$X^* = \frac{\sum \mu(x) \cdot x}{\sum \mu(x)}$$



$$x^* = \frac{(60 \times 0.6 + 70 \times 0.4 + 80 \times 0.2 + 90 \times 0.2)}{0.6 + 0.4 + 0.2 + 0.2}$$

$$= \frac{36 + 28 + 16 + 18}{1.4} = 70$$

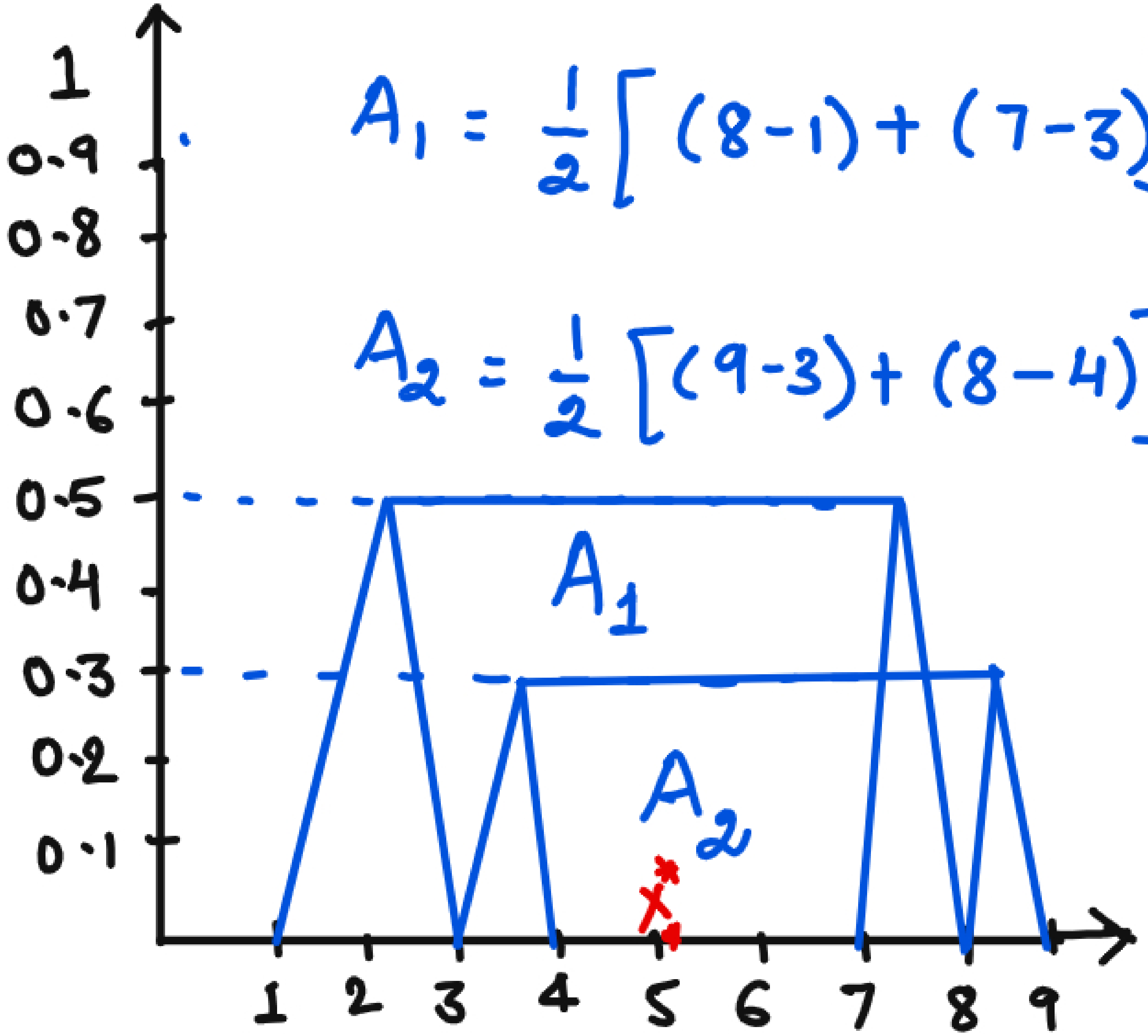
36
28
16
18
2
70 $\cancel{980}$
$\cancel{14}$
2
14
7
98

Center of Sum (COS)

$$x^* = \frac{\sum A\, x_c}{\sum A}$$

$$A_1 = \frac{1}{2}\left[(8-1)+(7-3)\right] \times 0.5 = \frac{1}{2}\left[7+4\right] \times 0.5 = \frac{5.5}{2} = 2.75$$

$$A_2 = \frac{1}{2}\left[(9-3)+(8-4)\right] \times 0.3 = \frac{1}{2}\left[6+4\right] \times 0.3 = \frac{3}{2} = 1.5$$



Fuzzy Set C_1 and C_2

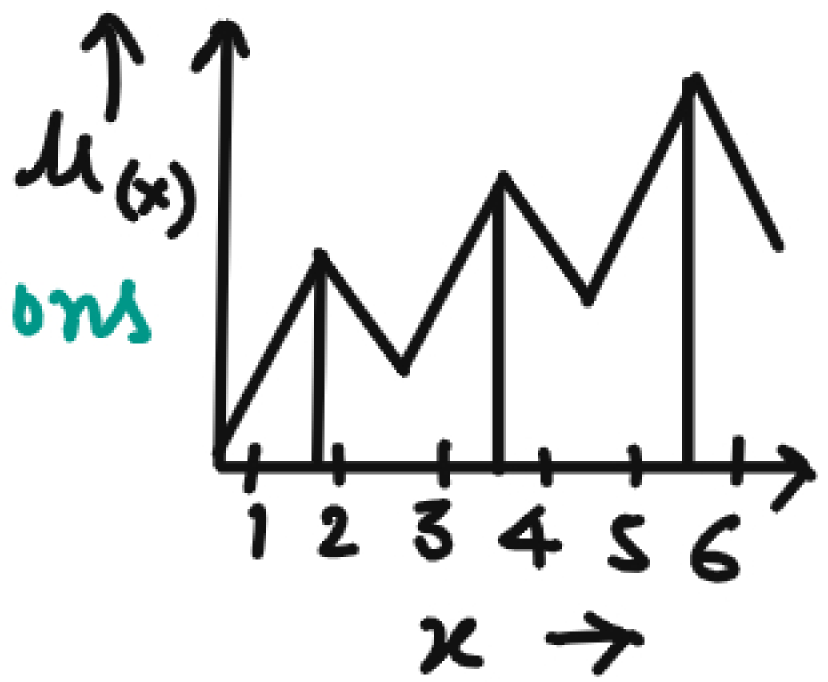
$$x^* = \frac{A_1 x_{c_1} + A_2 x_{c_2}}{A_1 + A_2}$$

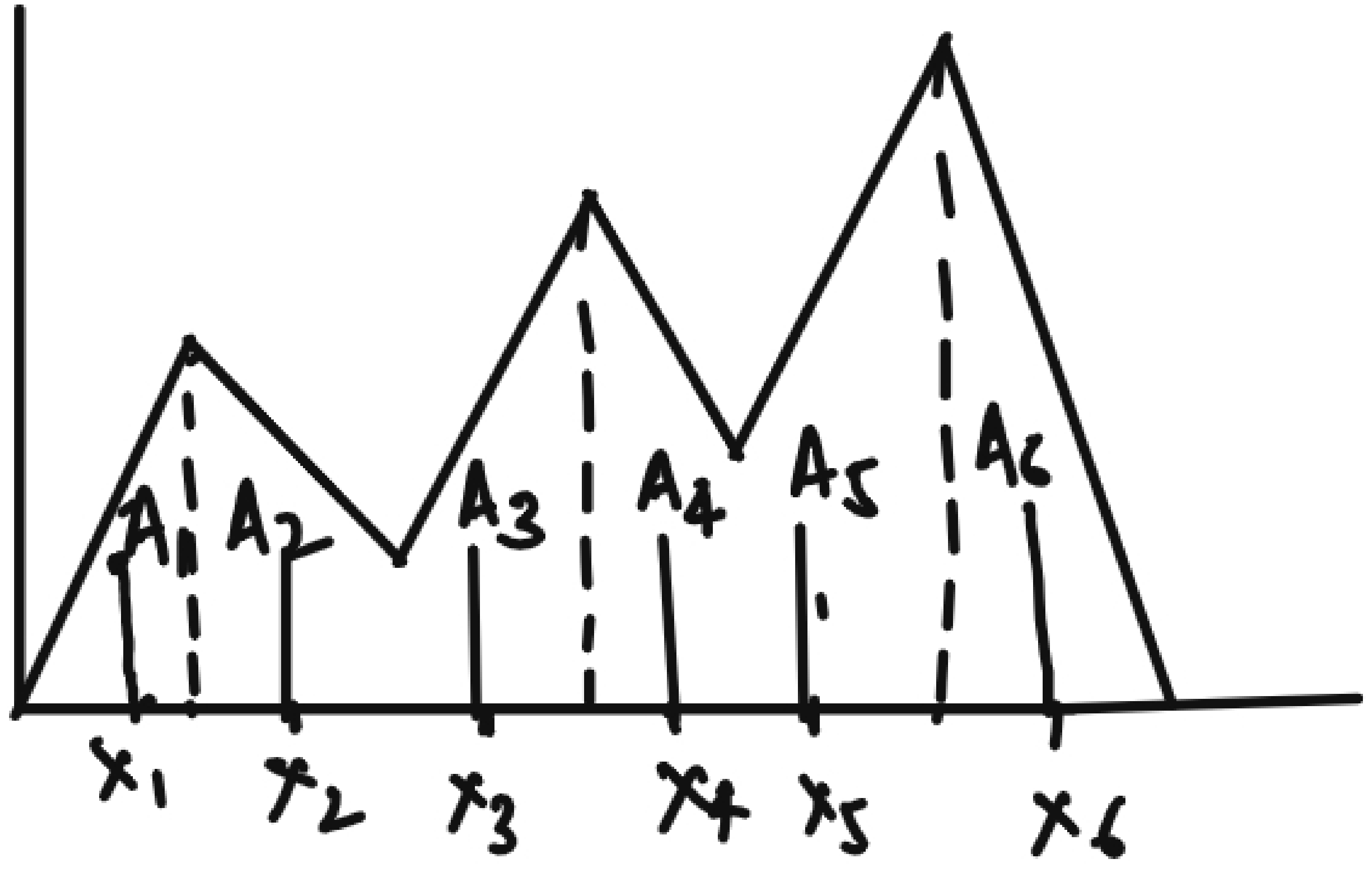
$$= \frac{2.75 \times 5 + 1.5 \times 6}{2.75 + 1.5} = \frac{13.75 + 9}{4.25}$$

$$= \frac{22.75}{4.25} = 5.35$$

Centroid Method

Center of Gravity (COG) :- Divide into smaller portions





$$X^* = \frac{\sum_{i=1}^{n} X_i A_i}{\sum_{i=1}^{n} A_i}$$

X_i - Center of Gravity

X^* - Crisp fuzzy set

Truth Values & Tables in Fuzzy Logic

X is A

⇒ London is in UK
Subject — Predicate

- Truth table defines logic functions of 2 propositions

- Truth value of proposition in fuzzy logic are in range [0, 1]

eg :- P : Ram is Boy

$T(P) = 0.8$

1> Conjunction ($\wedge$) : X AND Y. $\rightarrow T_{(X \text{ AND } Y)} = T(X) \wedge T(Y) = \min[T(X), T(Y)]$

2> Disjunction ($\vee$) : X OR Y $\rightarrow T_{(X \text{ OR } Y)} = T(X) \vee T(Y) = \max[T(X), T(Y)]$

3> Implication ($\rightarrow$) : If Then Y $\rightarrow T(\text{NOT } X) = 1 - T(X)$

4> Bidirectional ($\leftrightarrow$) : X If F Y $\rightarrow T(X \rightarrow Y) = \max[1 - T(X), \min[T(X), T(Y)]]$

Fuzzy Proposition

P: Ram is Boy

$T(P) = 0.0$

$T(P) = 0.2$

$T(P) = 0.8$

$T(P) = 1.0$

Q: Ram is Intelligent

$T(Q) = 0.6$

⇒ Ram is not Intelligent

$T(\bar{Q}) = 1 - T(Q) = 1 - 0.6 = 0.4$

⇒ Ram is Boy and so is intelligent

$T(P \wedge Q) = \min(T(P) - T(Q))$

$= \min(0.8, 0.6)$

$= 0.6$

- Fuzzy Predicates: Shridhar is tall
  (tall, Short, qick)
- Fuzzy Predicates modifier:
  (very, fairly, moderately, rather Slightly)

- Fuzzy quntifiers: (Most, several, Many)

- Fuzzy qualifiers:
  - Based on Truth (X is t)
  - Based on Probability (X is λ)
  - Based on Possibility (X is π)

Crisp Proposition deals with 0 and 1 OR True and False only

Decomposition of Rules

1> Multiple Conjuctive antecedents

If x is $A_1, A_2, A_3, ------ A_n$ Then Y is B_m

$$A_m = A_1 \cap A_2 \cap A_3 \cap ------ \cap A_n$$

$$\mu_{A_m}(x) = \min[\mu_{A_1}(x), \mu_{A_2}(x) ----- \mu_{A_n}(x)]$$

$\Rightarrow$ If A_m Then B_m

2> Multiple disjunctive antecedent

3> Conditional statements

⇒ If A_1 then B_1 else B_2

⇒ If A_1 then B_1

⇒ If NOT A_1 then B_2

4> Nested If then Rules

If A_1 then [If A_1 then [If A_2 then B_1]

⇒ If A_1 AND A_2 Then B_1

Aggregation of Fuzzy Rules

1) Conjunctive System of Rules

- Rules to be jointly Satisfied
- Using AND
- Using Intersection

$$y = y_1 \text{ AND } y_2 \text{ AND } y_3 \text{ ---- AND } y_n$$

$$y = y_1 \cap y_2 \cap y_3 \text{ ------ } \cap y_n$$

$$\mu_y = \min[\mu_{y_1}(y), \mu_{y_2}(y), \mu_{y_3}(y), \text{----} \mu_{y_n}(y)]$$

3) Disjunctive System of Rules

- The satisfaction of at least one Rule
- OR is used

$$y = y_1 \text{ or } y_2 \text{ or } y_3 \text{ ---... or } y_n$$

$$y = y_1 \cup y_2 \cup y_3 \text{ ----- } \cup y_n$$

$$\mu_y(y) = \max[\mu_{y_1}(y), \mu_{y_2}(y), \mu_{y_3}(y) \text{ ---- } \mu_{y_n}(y)$$

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Takagi Segeno Kang Fuzzy Rule Base System (TSK-FRBS)

Sugeno et al. suggested new model based on rules whose antecedent is composed of "Lingistic variables" and the Consequent (output) is represented by a "function" of the input variables.

Ex :- $x_i \in X$ input variable, Y : output variable

If x_1 is $\hat{A}_1$ and x_2 is $\hat{A}_2$ and .... and x_n is $\hat{A}_n$

then $y = p_0 + p_1 x_1 + \cdots + p_n x_n$

Consequent expression constitutes a linear combination of the Variables involved in the antecedent.

$P = \{p_0, p_1, \ldots, p_n\}$ are parameters

A_i : fuzzy set or a linguistic label that points to a particular member of a fuzzy partition of a linguistic Variable

$$\text{Output} = \frac{\sum_{i=1}^{m} h_i Y_i}{\sum_{i=1}^{m} h_i}$$

where m : number of rules present in the knowledge base

y_i : output of each rule $1 \le i \le m$

$h_i : T(A_{i1}(x_1), \ldots \ldots A_{in}(x_n))$

The matching degree between the antecedent part of the ith rule and the current input to the system.

$x_0 = (x_1, \cdots, x_n)$

TSK System do not need defuzzification being there output real numbers

It divides the input space in several fuzzy subspaces and defines a linear input output relationship in each one of the subspace.

Adv :- The system present a set of compact system equation that allows the parameters p_i to be estimated by mean of classical regression methods which facilitates the design process.

Dis: The structure of the rule consequents : difficult to be understood by human experts.

Singleton fuzzy rule system

Here, the rule consequent taken a single real valued number.

Eg: If x_1 is $\hat{A}_1$ and x_2 is $\hat{A}_2$ - - - - and x_n is $\hat{A}_n$ then y is y_0

Fuzzy Rule based Classifier

A fuzzy rule based classifier is an automatic classification system that uses fuzzy rules or knowledge representation tool.

Eg: If x_1 is A_1 and x_2 is A_2 and ... and x_n is A_n

Then Y is C_j (class label)

Here, Y is categorical variable, C is a class label

We use majority voting for the final result.

Example of Takagi and Sugeno's Approach

Rule: fuzzy antecedent then functional Consequent

i.e. Input (fuzzy) $\rightarrow$ Output (functional)

eg: If (x_1 is $\hat{A}_{i1}$) and (x_2 is $\hat{A}_{i2}$) and ....

.. and (x_{in} is $\hat{A}_{in}$) then

$$y_i = a_{i0} + a_{i1}x_1 + a_{i2}x_2 + \cdots a_{in}x_n$$

where $a_{i0}, a_{i1}, a_{i2} \cdots, a_{in}$ are coefficient

The weights of ith rule is

$$w_i = \mu_i \hat{A}_1 (x_1) \times \mu_1 \hat{A}_2 (x_2) \times \cdots \times \mu_i \hat{A}_n (x_n)$$

and

$$y = \frac{\sum_{i=1}^{k} w_i y_i}{\sum_{i=1}^{k} w_i}$$

Where k is the total of number rules

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Eg :- Let two Inputs : Input I and Input 2 be there and the corresponding linguistic states are

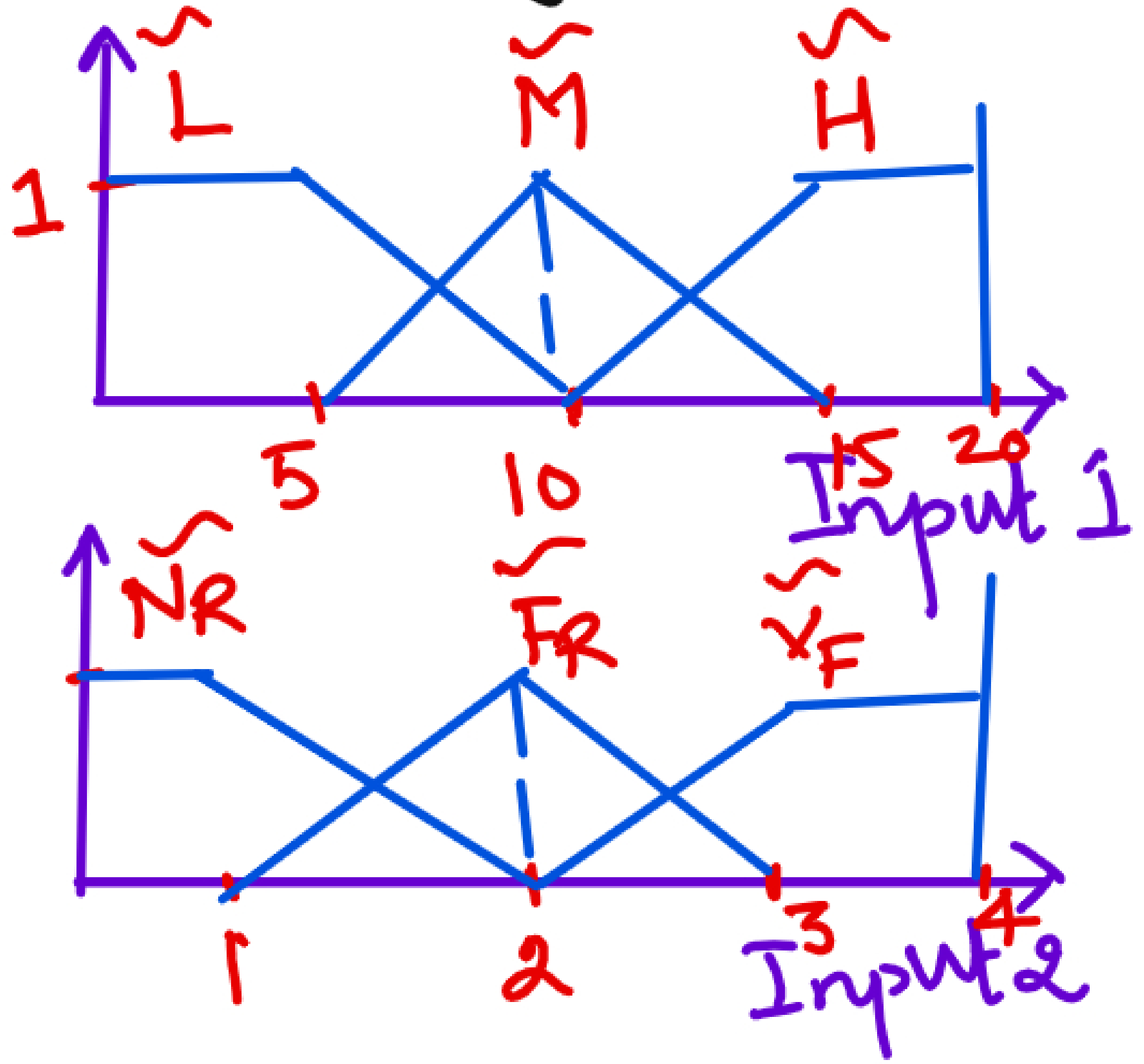
for Input 1 : Low ($\tilde{L}$), Medium ($\tilde{M}$), Heigh ($\tilde{H}$)

for Input 2 : Near ($\tilde{N}_R$), Far ($\tilde{F}_R$), Very far ($\tilde{V}_F$)

Total rules $3 \times 3 = 9$

Input 1 (rows), Input 2 (columns)

	$\tilde{N}_R$	$\tilde{F}_R$	$\tilde{V}_F$
$\tilde{L}$	y_1	y_2	y_3
$\tilde{M}$	y_4	y_5	y_6
$\tilde{H}$	y_7	y_8	y_9



The output of the i^{th} rule $\quad 1 \leq j, k \leq 3$

$$y_i = f(\text{Input}_1, \text{Input}_2)$$

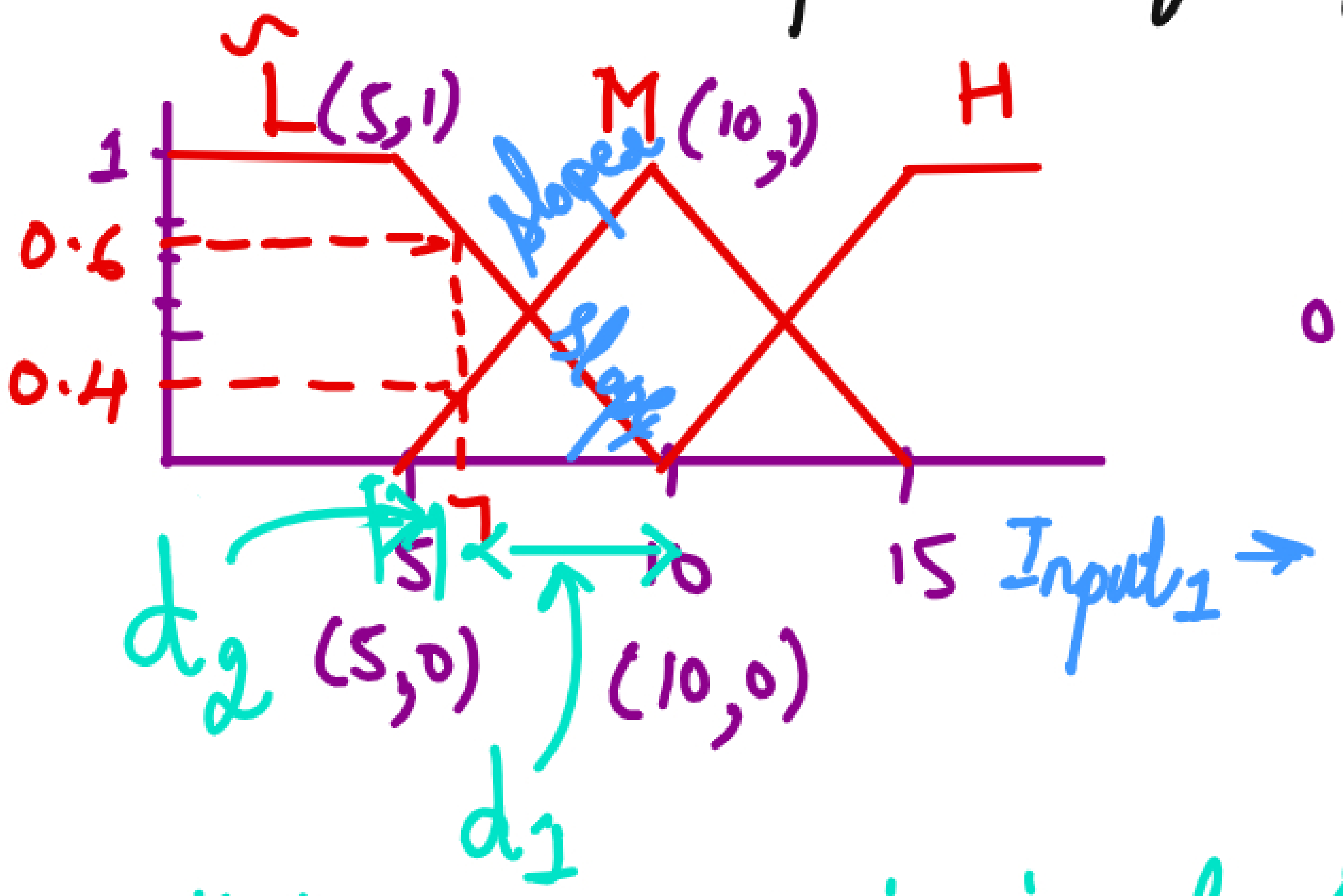
$$= a_{ij}\ \text{Input}_1 + b_{ik}\ \text{Input}_2$$

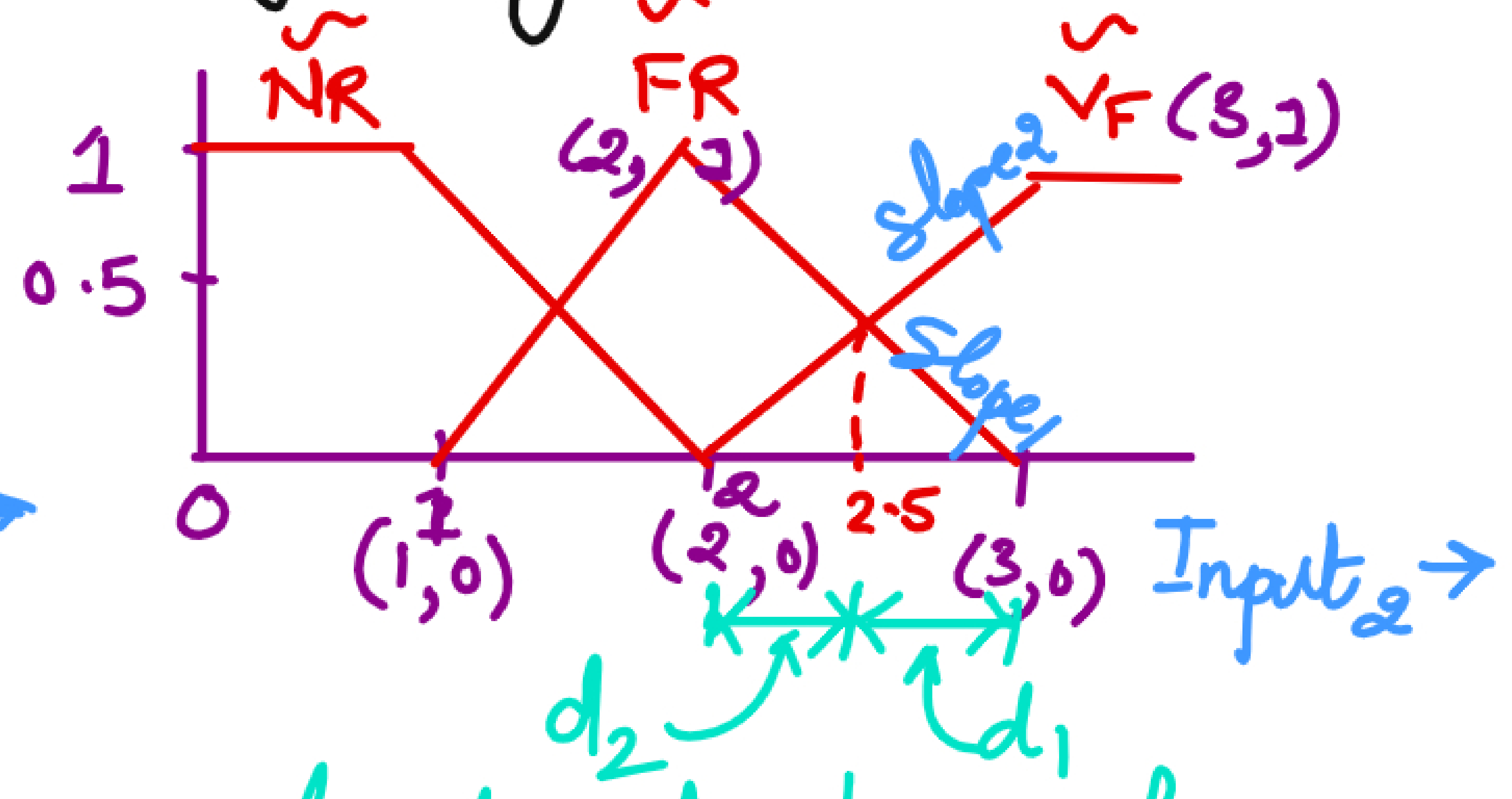
$\text{Input}_1 = \hat{L}, \hat{M}$, and $\hat{H}$ respectively

Coefficient of Input_1

$b_{i1} = 1$, $b_{i2} = 2$, $b_{i3} = 3$ for the inputs of (Input_2) $\widehat{NR}$, $\widehat{FR}$ and $\widehat{VF}$ respectively.

If $Input_1 = 7$ and $Input\ 2 = 2.5$ then what will be the output of fuzzy logic control





Using the principal similarity of triangle

$$Slope\ 1 = \left|\frac{1-0}{5-10}\right| = \frac{1}{5}.$$

$$\mu_{\tilde{L}}(Input_1) = \frac{1}{5} \times 3 = .6$$

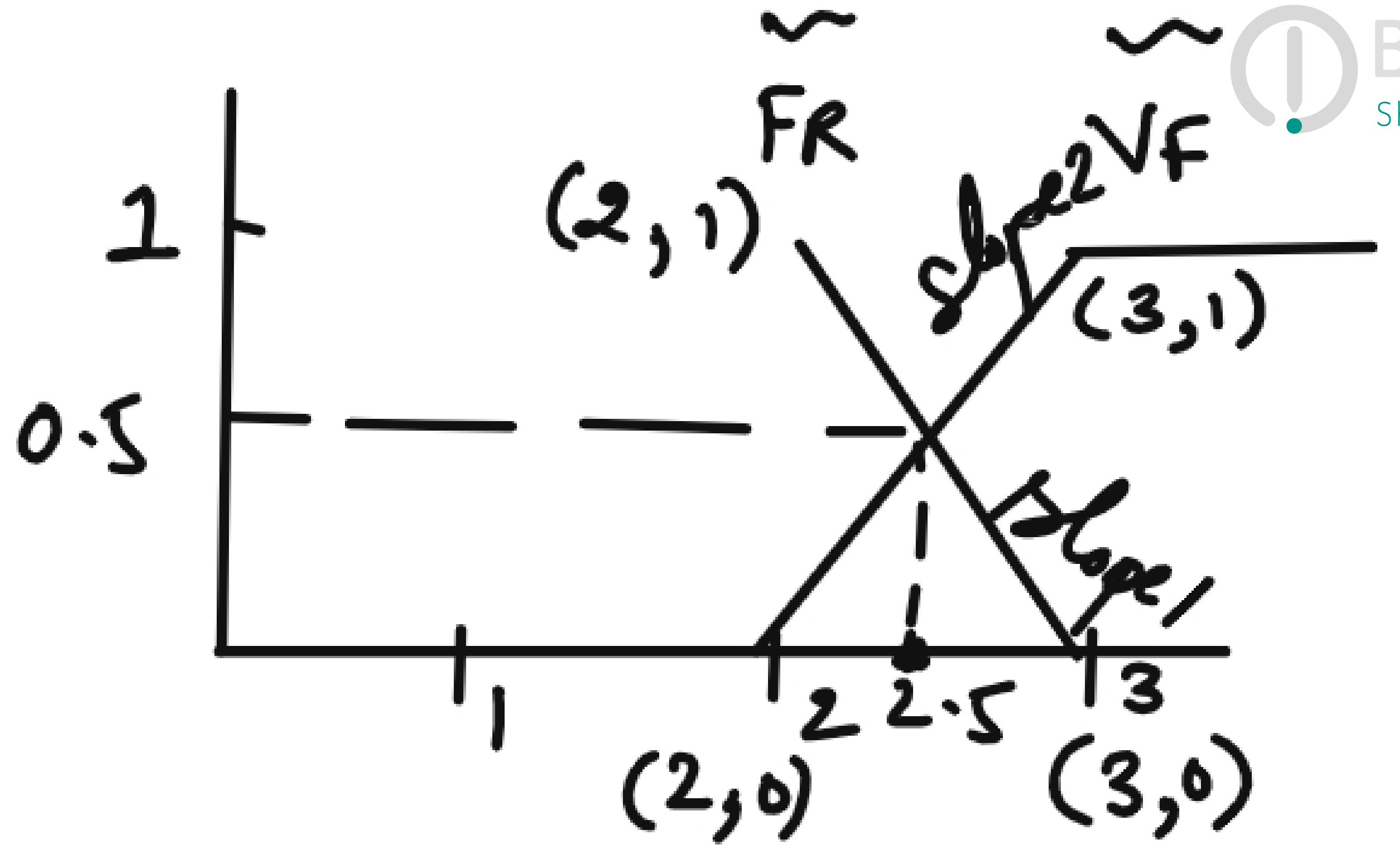
$$d_1 = 10 - 7 = 3$$

$$\mu(x) = d \times slope$$

$$\text{Slope } 2 : \quad 7-5 = 2$$

$$\text{Slope}2 : \left|\frac{1-0}{10-5}\right| = \frac{1}{5}$$

$$\mu_{\tilde{M}}(\text{Input}_1) = \frac{1}{5} \times 2 = 0.4$$



$$\text{Input}_2 = 2.5$$

$$\text{Slope}1 = \left|\frac{1-0}{2-3}\right| = \frac{1}{1} = 1$$

$$d_1 = 3 - 2.5 = 0.5$$

$$\mu_{\tilde{FR}}(\text{Input}_2) = 1 \times 0.5 = 0.5$$

$$\text{Slope}2 = \left|\frac{1-0}{3-2}\right| = \frac{1}{1} = 1$$

$$d_2 = 2.5 - 2 = 0.5$$

$$\mu_{\tilde{VF}}(\text{Input}2) = 1 \times 0.5 = 0.5$$

$Input_2 = 2.5$

		(0) NR	(0.5) FR	(0.5) VF
0.6	$\tilde{L}$	✗ R_1	✓ R_2	✓ R_3
0.4	$\tilde{M}$	✗ R_4	✓ R_5	✓ R_6
0	$\tilde{H}$	✗ R_7	✗ R_8	✗ R_9

$Input_1 = 7$

R_2: $Input_1$ is $\tilde{L}$ and $Input_2$ is $\tilde{FR}$

R_3: $Input_1$ is $\tilde{L}$ and $Input_2$ is $\tilde{VF}$

R_5: $Input_1$ is $\tilde{M}$ and $Input_2$ is $\tilde{FR}$

R_6: $Input_1$ is $\tilde{M}$ and $Input_2$ is $\tilde{VF}$

Weights:

$R_2 : W_2 = \mu_{\tilde{L}}(Input_1) \times \mu_{\tilde{FR}}(Input_2) = 0.6 \times 0.5 = 0.3$

$R_3 : W_3 = \mu_{\tilde{L}}(Input_1) \times \mu_{\tilde{VF}}(Input_2) = 0.6 \times 0.5 = 0.3$

$R_5 : W_5 = \mu_{\tilde{M}}(Input_1) \times \mu_{\tilde{FR}}(Input_2) = 0.4 \times 0.5 = 0.2$

$R_6 : W_6 = \mu_{\tilde{M}}(Input_1) \times \mu_{\tilde{VF}}(Input_2) = 0.4 \times 0.5 = 0.2$

$\tilde{L} = a_{i_1} = 1$ $\quad\tilde{NR} = b_{i_1} = 1$

$\tilde{M} = a_{i_2} = 2$ $\quad\tilde{FR} = b_{i_2} = 2$

$\tilde{H} = a_{i_3} = 3$ $\quad\tilde{VF} = b_{i_3} = 3$

R_{21} $\quad R_3$ $\quad R_{51}$ $\quad R_6$

$Input_1 = 7$

$Input_2 = 2.5$

Then y

$$y_2 = a_{21}\, Input_1 + b_{22} \cdot Input_2 = 1 \times 7 + 2 \times 2.5 = 12$$

$$y_3 = a_{31}\, Input_1 + b_{33}\, Input_2 = 1 \times 7 + 3 \times 2.5 = 14.5$$

$$y_5 = a_{52}\, Input_1 + b_{52}\, Input_2 = 2 \times 7 + 2 \times 2.5 = 19$$

$$y_6 = a_{62}\, Input_1 + b_{63}\, Input_2 = 2 \times 7 + 3 \times 2.5 = 21.5$$

The crisp output

$$Y = \frac{w_2 y_2 + w_3 y_3 + w_5 y_5 + w_6 y_6}{w_2 + w_3 + w_5 + w_6}$$

$$= \frac{0.3 \times 12 + .3 \times 14.5 + .2 \times 19 + .2 \times 21.5}{.3 + .3 + .2 + .2}$$

$$= \frac{16.05}{1}$$

$$= 16.05$$

Mamdani Fuzzy model Sum with solved Example

Design a controller to determine the Wash Time of a domastic machine. Assume the input is dirt & grease on cloths. Use three descriptors for input variables and five descriptor for output variables. Derive the set of rules for controller action and defuzzification. The design should be supported by figure wherever possible. Show that if the cloths are solid to a larger degree the wash time will be more and vice versa.

Steps to Solve

Step 1: Identify input and output variables and decide descriptor for the same.

Step 2:- Define membership functions for each of input and output variables.

Step 3:- Form a rule base

Step 4:- Rule Evaluation

Step 5:- Defuzzification

Step 1:- Identify Input and Output variable and decide descriptor.

* Here inputs are 'dirt' and 'grease'
Assume there %age.

* Output is 'wash time' measured in minutes

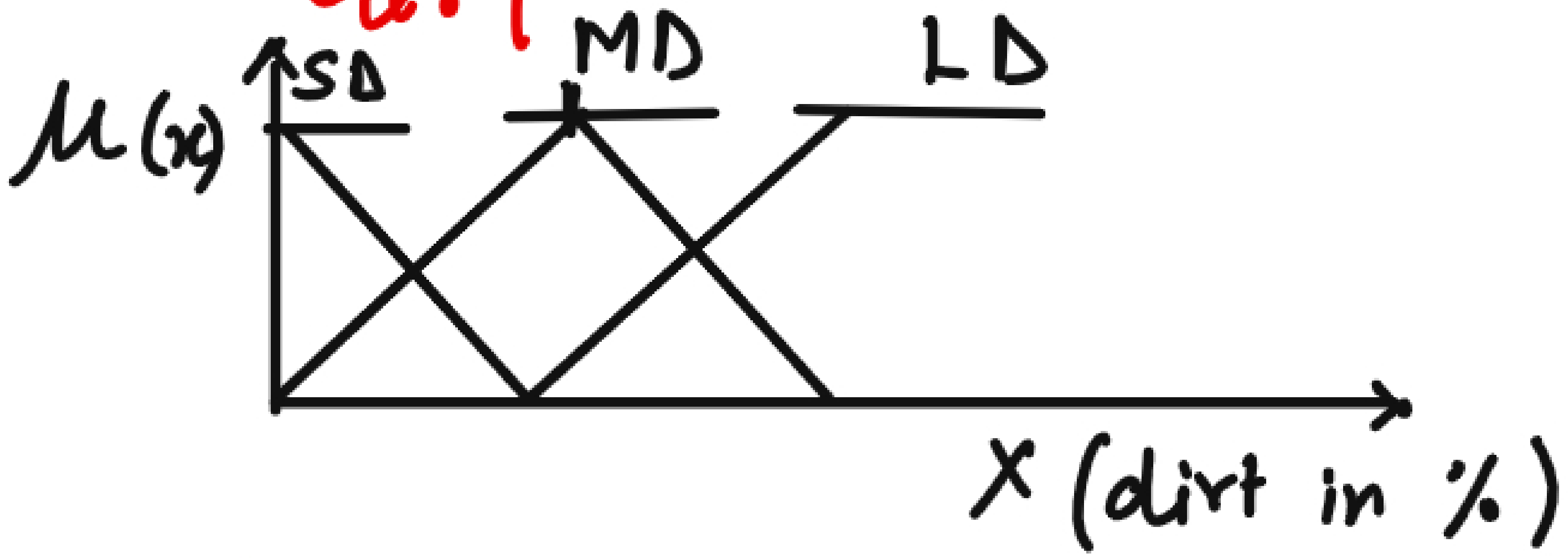
Descriptor for Input Variable

DIRT	GREASE	Wash Time
SD - Small Dirt	NG - No Grease	VS - Very Short
MD - Medium Dirt	MG - Medium Grease	S - Short
LD - Large Dirt	LG - Large Grease	M - Medium
		L - Large
		VL - Very Large

Step2 :- Define Membership function for each of the input and output variable.

We use triangular MF

① Membership function for dirt

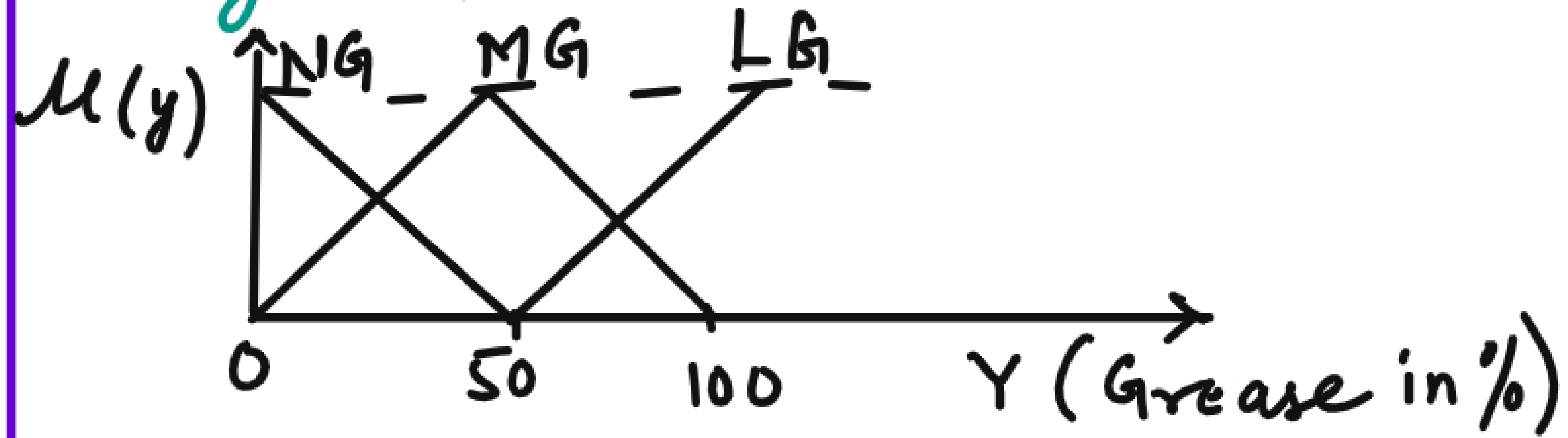


$$\mu_{SD}(x) = \frac{50-x}{50}, \quad 0 \le x \le 50$$

$$\mu_{MD}(x) = \begin{cases} \frac{x}{50}, & 0 \le x \le 50 \\ \frac{100-x}{50}, & 50 \le x \le 100 \end{cases}$$

$$\mu_{LD}(x) = \frac{x-50}{50}, \quad 50 \le x \le 100$$

② Membership function for grease

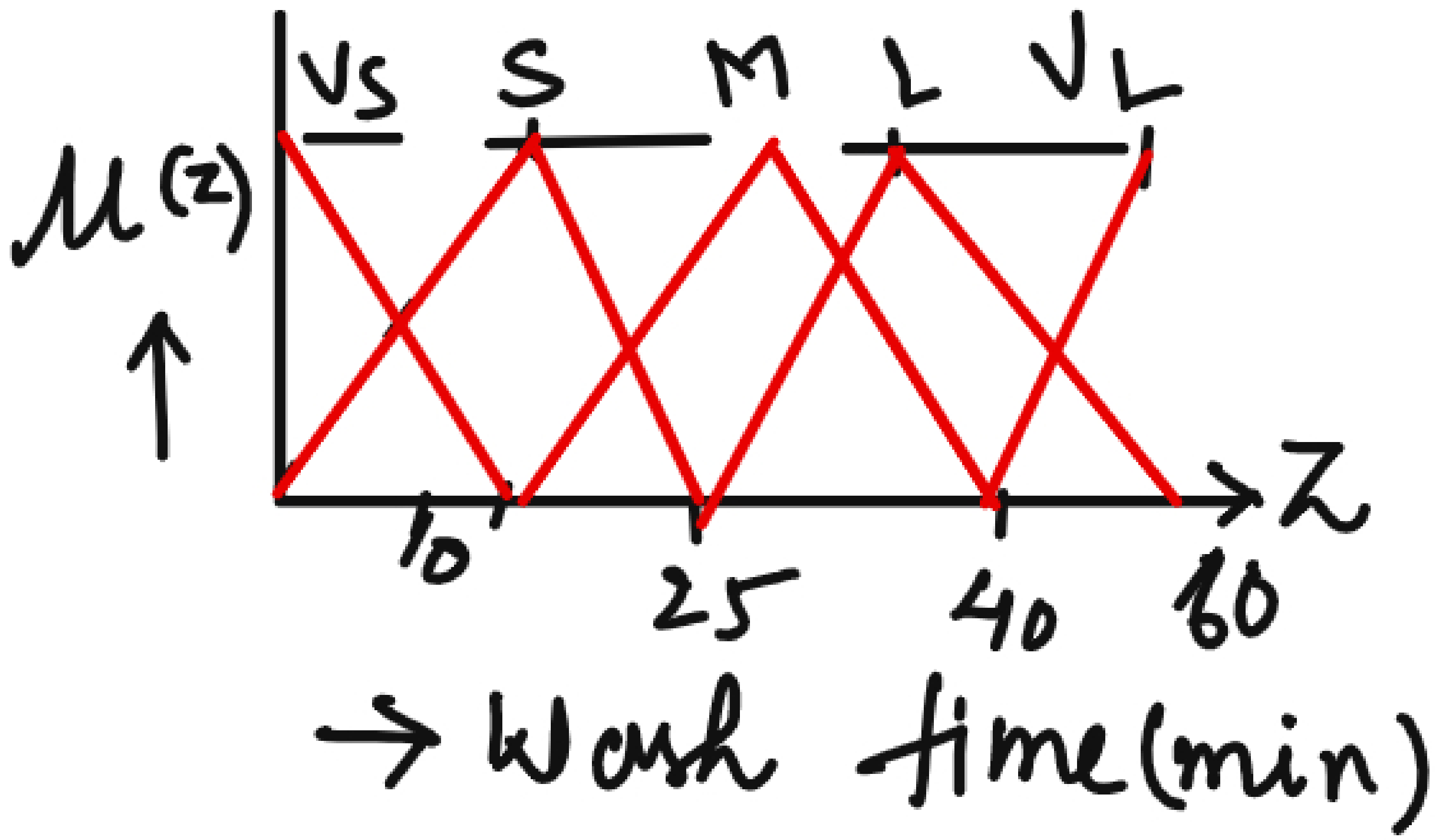


$$\mu_{NG}(Y) = \frac{50-Y}{50}, \quad 0 \le y \le 50$$

$$\mu_{MG}(Y) = \begin{cases} \frac{y}{50}, & 0 \le y \le 50 \\ \frac{100-y}{50}, & 50 \le y \le 100 \end{cases}$$

$$\mu_{LG}(Y) = \frac{y-50}{50}, \quad 50 \le y \le 100$$

3) Membership function for Wash Time



$$\mu_{VS}(z) = \frac{10-z}{10}, \quad 0 \le z \le 10$$

$$\mu_S(z) = \begin{cases} \frac{z}{10}, & 0 \le z \le 10 \\ \frac{25-z}{15}, & 10 \le z \le 25 \end{cases}$$

$$\mu_M(z) = \begin{cases} \frac{z-10}{15}, & 10 \le z \le 25 \\ \frac{40-z}{15}, & 25 \le z \le 40 \end{cases}$$

$$\mu_L(z) = \begin{cases} \frac{z-25}{15}, & 25 \le z \le 40 \\ \frac{60-z}{15}, & 40 \le z \le 60 \end{cases}$$

$$\mu_{VL}(z) = \frac{z-40}{20}, \quad 40 \le z \le 60$$

Step 3: Form a Rule Base

$x \backslash y$	NG	MG	LG
SD	VS	M	L
MD	S	M	L
LD	M	L	VL

Evaluate $\mu_{MD}(x)$ and $\mu_{LD}(x)$ for $x = 60$, we get

$$\mu_{MD}(60) = \frac{100-60}{50} = \frac{40}{50} = \frac{4}{5}$$

$$\mu_{LD}(60) = \frac{60-50}{50} = \frac{10}{50} = \frac{1}{5}$$

Step 4: Rule Evaluation

Assume Dirt = 60%

Grease = 70%

$\mu_{MD}(x) = \frac{100-x}{50}$ | $\mu_{LD}(x) = \frac{x-50}{50}$

Similarly Grease = 70% Maps 2 MFs

$\mu_{MG}(y) = \frac{100-y}{50}$ | $\mu_{LG}(y) = \frac{y-50}{50}$

Evaluate $\mu_{MG}(y)$ & $\mu_{LG}(y)$ for $y = 70$ we get

$$\mu_{MG}(70) = \frac{100-70}{50} = \frac{30}{50} = \frac{3}{5}$$

$$\mu_{LG}(70) = \frac{70-50}{50} = \frac{20}{50} = \frac{2}{5}$$

The four equation leads to 4 rules we need to evaluate

1) Dirt is Medium and Greace is Medium

2) Dirt is Medium and Greace is Large

3) Dirt is Large and Greace is Medium

4) Dirt is Large and Greace is Large

Dirt \ Greace	M	L
M		
L		

Since the antecedent is connected by the operator by and operator we use min operator to evaluate strength of each rule

Strength of Rule 1:- $D_M\ G_M$

$$S_1 = \min\left(\mu_{MD}(60), \mu_{MG}(70)\right) = \min\left(4/5, 3/5\right) = \frac{3}{5}$$

Strength of Rule 2: $D_M\ G_L$

$$S_2 = \min\left(\mu_{MD}(60), \mu_{LG}(70)\right) = \min\left(4/5, 2/5\right) = \frac{2}{5}$$

Strength of Rule 3: $D_L\ G_M$

$$S_3 = \min\left(\mu_{LD}(60), \mu_{MG}(70)\right) = \min\left(\frac{1}{5}, \frac{3}{5}\right) = \frac{1}{5}$$

Strength of Rule 4: $D_L\ G_L$

$$S_4 = \min\left(\mu_{LD}(60), \mu_{LG}(70)\right) = \min\left(\frac{1}{5}, \frac{2}{5}\right) = \frac{1}{5}$$

D \ G		$\mu_{MG}(70)$	$\mu_{LG}(70)$
	×	×	×
$\mu_{MD}(60)$	×	3/5	2/5
$\mu_{LD}(60)$	×	1/5	1/5

Max MF

×	×	×
×	M	L
×	L	VL

Step 5 :- Defuzzification

Since we use 'Mean of Max' defuzzification technique

Maximum Strength = $\max(S_1, S_2, S_3, S_4) = \max\left(\frac{3}{5}, \frac{2}{5}, \frac{1}{5}, \frac{1}{5}\right)$

$= 3/5$ (This Correspond to Rule I)

Rule I: Dirt is Medium and Grease is Medium has maximum Strength (3/5)

To find out the final defuzzification value, we now take the average (mean) of $\mu_M(z)$

$$\mu_M(z) = \frac{z-10}{15} \quad \text{and} \quad \mu_M(z) = \frac{40-z}{15}$$

$$\frac{3}{5} = \frac{z-10}{15} \qquad \frac{3}{5} = \frac{40-z}{15}$$

$$\frac{3}{\cancel{5}} \times \cancel{15}^{3} = z-10 \qquad \frac{3}{\cancel{5}} \times \cancel{15}^{3} = 40-z$$

$$z = 9+10 \qquad z = 40-9$$

$$z = 19 \qquad z = 31$$

$$z^* = \frac{19+31}{2} = \frac{50}{2} = 25 \text{ min}$$

$$\boxed{z^* = 25 \text{ min}}$$