

Customer Search and Product Returns

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Abstract

Online retailers are challenged by frequent product returns with \$428 Billion in merchandise returned to US retailers in 2020 (National Retail Foundation 2021). Previous research has focused on linking customers' purchase and return decisions. However, online retailers have access to the information which precedes the purchase decision – customer search. We demonstrate that customer search information provides important insights about product returns. Using data from a large European apparel retailer, we propose and estimate a joint model of customer search, purchase, and return decisions. We then provide theory and data indicating that using search filters, viewing multiple colors of a product, spending more time, and purchasing the last item searched are negatively associated with the probability of a return. Finally, we use the proposed model to optimize the online assortment as well as the product display order on the retailer's website.

Keywords: product returns, consumer search, sequential search, online retailing

1. Introduction

Online retailers are challenged by frequent product returns. Product returns often significantly decrease the profits of the firm by reducing the revenue (the firm must refund the returned product) and increasing the cost (backward logistics, dry cleaning, etc.)¹. Return costs are so high that major online retailers such as Amazon and Walmart have begun to allow customers to keep the item because it is sometimes less costly to refund the purchase price than bear the return costs (Wall Street Journal 2021). Zara announced it will be charging online shoppers for returns unless the items are returned to the physical store (BBC 2022). Managing product returns is critical to retailers' profitability. For example, L.L. Bean spent \$50 million per year on returns costs, amounting to about 30% of the retailer's annual profits (Abbey et al., 2018).

Product returns are typically studied in the "purchase/post-purchase" framework where researchers assume the product purchase event as the starting point of the customer journey. With this assumption, the authors have established that product characteristics jointly affect the probability of purchase and return because the option to return a product has value to the customer and impacts purchase decisions. From a managerial perspective, research suggests that changes in the return policy (for example, towards a more lenient policy) would impact customers' purchase behavior.

We build on that research. Online retailing allows us to track the customer journey from the moment the customer starts looking for a product (pre-purchase), to the moment he or she decides on whether to keep or return the product (post-purchase). We seek to determine whether observing customer's search (the "pre-purchase" events) helps us understand mechanisms by which search and returns are related and, hence, provide insights about product return management.

¹ According to the Wall Street Journal (2021), online returns can cost \$10 to \$20 per returned item, excluding freight.

Using an empirical-theoretical framework, we develop a rational model of customer search in the presence of a return option. Using browsing sessions linked to data on purchasing and returning items at a major European apparel online retailer, we estimate our model and demonstrate the importance of modeling search and returns jointly. The data and analysis suggest strategies by which a retailer can maximize its profits.

A formal empirically-grounded model helps researchers and managers better understand the relationship between search and product returns. A deeper understanding of the mechanisms by which search and returns are related could identify new opportunities such as targeted special offers based on observed search. Improved return-probability prediction might provide insights with which to manage backward logistics. To motivate the value of including prepurchase search when managing returns, consider Nelly and Wendy who both purchased the same pair of jeans. Nelly kept the jeans while Wendy returned them for a full refund. From the purchase data, these customers are indistinguishable, however, search data might reveal that Nelly used pre-search filters, searched many options, and spent considerable time reviewing the webpage — all of which are related to lower return probabilities. Wendy, on the other hand, purchased the least-expensive jeans from the front page without using filters, reviewing other options, nor spending much time searching. Other insights include observations that customers are less likely to return products if they browse many colors of the same item and/or purchase the last-clicked product.

Knowing these relationships, the firm might develop policies to encourage Wendy to search more such as better pre-search tools, incentives to evaluate more colors, incentives to search longer, and better website layouts that make it easier to search many items. To test creative policies, the firm can use our model and estimated parameters for policy simulations — we demonstrate example policy simulations such as reducing search costs. Other policy simulations test modifications to online assortment. Then, recognizing that any theoretical model is incomplete and full causation is difficult to establish, the firm can test its policies with A/B experiments. Such policy development and testing are particularly important in countries (e.g., Western Europe) with strong consumer protection that dictates return policies but not low-cost changes to website design.

2. Related Literature

This paper connects two fields of research: Product returns and customer search. The first field of research has studied product returns in both theoretical and empirical frameworks. Theoretically, authors demonstrate that the option to return products serves as a risk-reducing instrument to experience the product (Che, 1996; also studied empirically in Petersen and Kumar 2015) or as a signal of item quality (Moorthy & Srinivasan, 1995). The applied stream of research focuses on the optimization of return policies by firms. Authors recognize the tradeoff between higher demand and higher return rates when firms use lenient policies and suggest that the optimal return policy must be balanced (e.g., Davis et al. 1998; Bower and Maxham-III 2012; Abbey et al. 2018) because overly strict return policies lead to a decrease in purchases (Bechwati and Siegal 2005). Janakiraman et al. (2016) provide an extensive review of the effect of return policy leniency on purchases and returns.

Anderson et al. (2009) propose a structural model where the option to return is embedded in a customer purchase decision—the customer learns private information only after purchasing the product. Other empirical studies demonstrate that a variety of factors affect the probability of product returns including price, discounts, marketing instruments (e.g., free shipping), or the truthfulness of product reviews. (e.g., Petersen and Kumar 2009, 2010; Sahoo et al. 2018; Shehu et al. 2020; El Kihal and Shehu 2022). Empirical studies have focused on concrete instruments, such as visualization systems and online product forums. These instruments decrease product uncertainty in the match of the product to the customer and, therefore, product returns (Hong and Pavlou 2014). Recent research has used machine learning to accurately predict returns and identify product-related features to be considered when selecting and designing fashion items for the retailer's website (Cui et al. 2020 and Dzyabura et al. 2022).

The second field of research is customer search. Customer search is an established and mature field of research both empirically and theoretically. The literature typically follows either sequential (Weitzman 1979) or simultaneous (Stigler 1961) approaches. In both approaches, the customer knows the distribution of the rewards and searches to fully resolve uncertainty. Most of the literature focuses on sequential search buttressed by Bronnenberg,

Kim, and Mela (2016) who report strong evidence to support sequential search. Recent papers allow for flexible preference heterogeneity (Morozov et al. 2021), add learning (e.g., Ke et al. 2016; Branco et al. 2012; Dzyabura and Hauser 2019), multiple attributes (Kim et al. 2010), intermediaries (Dukes and Liu 2015), or search duration (Ursu et al. 2020) and search fatigue (Ursu, Zhang, and Honka 2022).

The availability of click-stream data has enabled researchers to empirically study customer search behavior (e.g., Bronnenberg et al 2016; Chen and Yao 2017; Ursu, Wang, and Chintagunta 2020) and provide detailed insights on search-to-purchase customer behavior. For example, Bronnenberg et al. (2016) examine consumer search behavior for cameras and show that early search is highly predictive of consumer purchase and that the first-time discovery of the purchased alternative happens towards the end of the search. Chen and Yao (2017) show that refinement tools significantly impact consumer behavior and the market structure. Ursu, Wang, and Chintagunta (2020) study search duration and quantify consumer preferences and search costs, to provide insights about how much information to provide on their platforms. We build on these insights to show that such information also affects returns.

Previous literature on search and on product returns, both theoretical and empirical, focuses on a portion of the customer journey. The product-returns literature focuses on the purchase-to-returns portion and the search literature on the search-to-purchase portion. Our research expands the focus by considering both the pre-purchase and post-purchase customer journey in online retailing – search-to-purchase-to-returns.

In doing so, we link the two fields of research, contribute to both, and investigate new phenomena. First, we embed the “pre-purchase” events in a traditional “purchase/post-purchase” framework studied in returns literature. We seek to improve the knowledge on why product returns happen and how the retailer could manage them. Second, our model extends the existing search models by accounting for returns. (Our analyses generalize directly to cancellations in the travel industry, such as for hotels, Airbnbs, airlines, cruise ships, and resorts.) Our goal is to demonstrate that the option to return impacts the way customers search for the product. By modeling the search-to-purchase-to-return customer journey, we gain insight on how to manage the entire process more profitably. Not only do search patterns

help predict returns, but nudging search can influence net purchases after accounting for returns. Finally, we seek to explore managerially relevant applications of the model unachievable without jointly modeling search, purchase, and returns.

3. Data

We sought and obtained online-channel individual-level data from a large apparel retailer in Western Europe. We focus on the online channel because (1) most returns are through the online channel (our retailer has in total 53% of sales being returned—typical for the European apparel industry) and (2) the online channel is an ideal situation in which to observe search, sales, and returns for each customer. Ultimately, insights from online shopping should be relevant to offline shopping, but we leave that to future research. We preprocessed data by removing noise and outliers (for example, extremely short/long sessions). In this paper, we focus on orders which had at most one product purchased. This focus isolates the impact of search on product returns by excluding situations when the customer purchases several colors or variations of a product with an intention to keep only one. Our model-free evidence suggests that qualitative behavior of customers does not change for situations in which multiple products are purchased. We leave the extension to multiple-item orders for future research. A detailed description of data pre-processing can be found in Appendix A.1.

The retailer sells medium-price fashion products for women, men, and children. (The retailer is mostly for adult apparel, which compromises 95% of the purchases.) As is typical for Europe, the retailer has a generous return policy, where items can be returned for free, for a full refund, within 60 days after the purchase with or without providing a reason.

The data include both mobile and desktop usage and consist of three main components:

1. **Search** records the sequence of actions made by the customer during the browsing session. We observe products listed on the webpage for the customer and the set of products considered (clicked), as well as the sequence of clicks. We also observe all actions (e.g., clicking on a product, sorting by price) and the timing between the different actions, allowing us to observe how much time a customer spends on a specific product page.

2. **Purchases** includes the products purchased (if any) by the customer during browsing sessions. These data include product characteristics such as price, category, fabric, size, brand, color, and product image. We also observe the products' base price and can infer whether it was sold with a discount.
3. **Returns** contains information on whether the purchased product was kept or returned by the customer. It also includes the date and stated reason for the return.

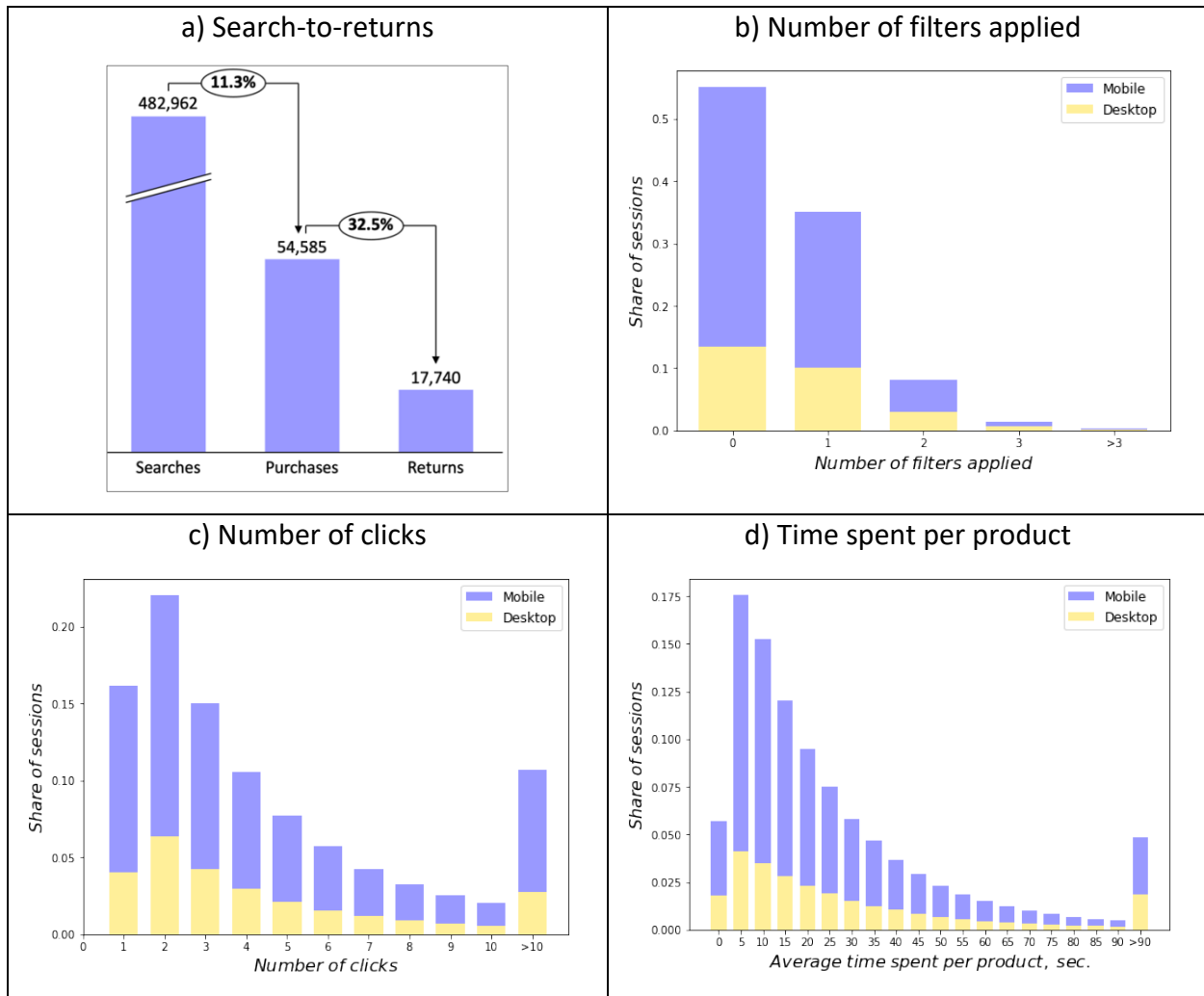
All three components of the data are matched by a unique identifier. For each session, we observe the complete customer journey from opening the website to deciding whether to keep a fashion item. This allows us to build and estimate the model which combines customer search, purchase, and return decisions. Our observation period is between 1 October 2019 and 15 May 2020. Over this period, in our final dataset we observe 482,962 search sessions, of which 54,585 (11.3%) result in a purchase. In 32.5% of the cases, customers decide to return the product purchased. As anticipated, the return rate for the single-item subsample is lower than the return rate for the multiple-item subsample, but the qualitative implications remain intact. Figure 1 provides summary statistics for our data.

Search Descriptives. Customers can access the retailer's website through desktop or a mobile device (73% access through a mobile device in our data). On the website, the customer observes a product list, which displays an image of the product, its price, and category, and small front view picture of the product. Customers can click on products or use refinement tools to select a more specific product list. The website offers two types of refinement tools: filtering and sorting. Customers can filter by brand, color, products on sale, new products, or product size. They can sort the product list by price (ascending or descending), new products, or top sellers. When the customer clicks on a specific product, further information is revealed on the product page, such as more (and higher quality) product images, and detailed product description. Amongst the 482,962 search sessions median customers search lengths is 263 seconds before purchasing or leaving the retailer's website. On average, they browse 5 product pages spending 38 seconds per page (Figure 1). 45% of customers use at least one refinement search tool (Figure 1).

Purchase Descriptives. Out of the 482,962 search sessions, 54,585 ended with a purchase. Customers choose among 16 product categories, as predefined by the retailer (e.g., jeans, blouses, dresses, coats, shoes). The most popular purchased product categories are “jackets and coats” (27%) and jeans (16%). (Figure A.1 in the Appendix A.1)

Return Descriptives. In our subsample 33% of purchased products are returned. Dresses and jumpsuits have the highest probability of return (46% and 47% respectively) and t-shirts have the lowest return rate (11%). Figure A.1 in the Appendix A.1 shows the distribution of return rate by category.

Figure 1. Summary statistics of the data



4. Model

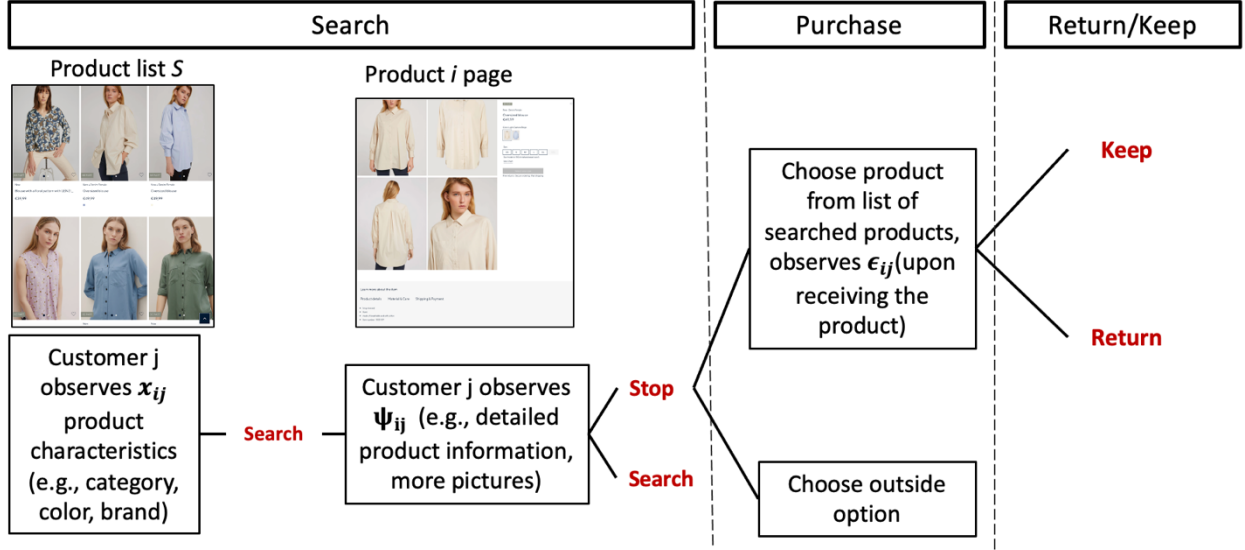
Following the literature cited in §2, we model search as sequential. Consider a customer, i , who is searching for a product on the retailer's website. When visiting the website, the customer observes products on the website from which to choose (or click). The total number of products available to customer i is S_i . Each product view from this list contains information about the product (for example, product category, price, etc.). After evaluating these products, the customer may choose to click on a product to reveal additional information. For instance, a click on a product page could reveal a detailed image of the product or additional information such as the fabric. We capture this by assuming that the customer observes characteristics of the product, x_{ij} , before customer i clicks on product j , and receives a signal ψ_{ij} after clicking on the product. This signal is a noisy estimate of how much the customer would like the product when it arrives home. We assume that the customer pays search costs to reveal the signal ψ_{ij} (mental costs, mouse navigation, or time costs).

After evaluating a clicked product, the customer has two options: continue the search by clicking on the next product or stop the search. In case of stopping the search, the customer purchases one of the previously clicked products (or chooses the outside option). In case the customer decided to purchase a product, the customer must wait until the product arrives at home. At home, the customer receives additional information about the purchased product and decides on whether to keep it or return it to the retailer. We assume that the product inspection at home reveals a true customer's individual preference shock ϵ_{ij} for the product, which captures all the product information revealed to the customer upon physical product evaluation (for example, the customer understands the physical fit of the product, its expected use, or simply how close it is to the customer's fashion taste). Returns are costly. The customer pays return costs R_i (despite the "free" returns policy return, return costs include printing the return label, bringing the package to the postal office, mental costs, and time spent returning the item).

In §4.1 we specify the utility of the customer and the effect of the returns option. We elaborate search costs in §4.2. Finally, we explain the optimal search rules under the availability

of returns in §4.3. Figure 2 illustrates the timing of the customer search-purchase-return journey as well as all the available information in each stage.

Figure 2. Customer actions and observed information in the pre-purchase, purchase, and post-purchase stages



4.1. Utility and Returns

For ease of notation, we number products such that j is the index of a sequence in which customer i searches ($j = 1$ implies the first clicked product). The customer utility of purchasing the product j from the website could take one of three possible forms in Equation (1). Search costs are paid prior to the realization of this utility.

$$(1) \quad u_{ij} = \begin{cases} \mu^u(x_{ij}) + \epsilon_{ij} & \text{if customer purchases product } j \neq 0 \text{ and keeps it} \\ -R_i & \text{if customer purchases product } j \neq 0 \text{ and returns it} \\ 0 & \text{if customer chooses the outside option } j = 0 \end{cases}$$

where x_{ij} is a vector of product characteristics (e.g., category, color, brand, etc.); $\epsilon_{ij} \sim \mathcal{N}(0, \sigma_\epsilon)$ is the individual preference shock; and R_i return costs (e.g., shipping fee, travel time to the postal office, etc.). Note that because j is an index of a sequence, x_{ij} takes on different values for different customers. Without loss of generality, we set the utility of the outside option to zero and specify $\mu^u(x_{ij})$ to have a nonzero intercept.

As outlined in Figure 2, x_{ij} represents information readily available on the website before starting the search and ψ_{ij} represents information revealed after the customer clicks on the product page (additional information about the product like reviews, high-quality pictures,

etc.); ϵ_{ij} is revealed after the customer makes a purchase decision (for example, at home the customer tries the received product).

To understand how the return option impacts the search, remember that the customer observes ϵ_{ij} only after receiving the product, therefore the customer must make a purchase decision without knowing ϵ_{ij} , but anticipating it will be revealed. After clicking on the product, the customer observes ψ_{ij} which is a noisy signal of the individual preference shock ϵ_{ij} . That is:

$$(2) \quad \psi_{ij} = \eta_{ij} + \epsilon_{ij}$$

where $\eta_{ij} \sim \mathcal{N}(0, \sigma_{\eta_{ij}})$ is the noise of the signal. We assume that η_{ij} and ϵ_{ij} are independently distributed and that the customer knows the distribution of ϵ_{ij} and η_{ij} before search as well as all model parameters.

Because the customer knows the distribution of $\psi_{ij}|\epsilon_{ij}$, it is feasible to compute for each clicked product the expected utility given the signal ψ_{ij} (see Appendix A.2 for details):

$$(3) \quad \begin{aligned} v_{ij} &= \mathbb{E}_{\epsilon_{ij}}[(\mu_{ij}^u + \epsilon_{ij}) \cdot \mathcal{I}(\mu_{ij}^u + \epsilon_{ij} \geq -R_i) + (-R_i) \cdot \mathcal{I}(\mu_{ij}^u + \epsilon_{ij} < -R_i) | \psi_{ij}] \\ &= \sigma_{v_{ij}} \cdot T\left(\frac{\mu_{ij}^u + R_i}{\sigma_{v_{ij}}} + \frac{\psi_{ij} \cdot \sigma_{v_{ij}}}{\sigma_{\eta_{ij}}^2}\right) - R_i \\ \sigma_{v_{ij}} &= \sqrt{\frac{\sigma_{\epsilon_{ij}}^2 \cdot \sigma_{\eta_{ij}}^2}{\sigma_{\epsilon_{ij}}^2 + \sigma_{\eta_{ij}}^2}} \end{aligned}$$

where $T(x) = x \cdot \Phi(x) + \varphi(x)$; $\Phi(\cdot)$ and $\varphi(\cdot)$ are standard normal cdf and pdf respectively and $\mathcal{I}(\text{condition}) = 1$ if condition is TRUE.

Equation (3) demonstrates how the return option impacts customer search—because v_{ij} depends on the random variable ψ_{ij} that is unobservable before search (or click), the return option changes the distribution of the reward from search. It is easy to show that $T(x) \geq x$ and thus $v_{ij} \geq \mu_{ij}^u + \frac{\sigma_{\epsilon_{ij}}^2}{\sigma_{\epsilon_{ij}}^2 + \sigma_{\eta_{ij}}^2} \psi_{ij}$. This implies that for any product characteristics, the option to return improves the customer's expected search utility. Intuitively, by the principle of optimality, having the option, but not the obligation, to return a product is weakly better than a strategy of “always keep the product.”

4.2. Search Costs

Each additional search (or click) requires the customer to pay a search cost. For example, mental or physical effort of reviewing the additional information, or clicking/moving mouse. We denote the search costs, c_{ij} , and model its relationship to the search environment by:

$$(4) \quad \log c_{ij} = \mu^c(z_{ij}) + \xi_{ij} = z'_{ij}\beta^c + \xi_{ij} = \mu^c_{ij} + \xi_{ij}$$

where z_{ij} is a vector of characteristics affecting the search costs (position on page, device, page number, age, etc.), β^c are levels of costs coefficients, and $\xi_{ij} \sim \mathcal{N}(0, \sigma_{\xi_{ij}})$ is an individual product-level shock on search costs that follows a normal distribution.

We assume that the customer observes both z_{ij} and ξ_{ij} before the search (or before clicking on the product). In §5.2 we explain that the assumption of heterogeneous search costs is essential for model parameter estimation.

4.3. Optimal Search Strategy

Because the return option changes the distribution of the reward, standard decision rules (Weitzman 1979) must be updated. We summarize these decision rules below and provide derivations in Appendix A.3:

1. **Selection rule.** If the customer is going to make a new search (or click), the click would be the option with the highest reservation utility ω_{ij} derived from the system of Equations (5).

$$(5) \quad \begin{aligned} c_{ij} &= \sigma_{v_{ij}} \cdot \int_{\theta}^{\infty} T\left(\frac{\mu^u_{ij} + R_i}{\sigma_{v_{ij}}} + \frac{\psi_{ij} \cdot \sigma_{v_{ij}}}{\sigma_{\eta_{ij}}^2}\right) - T\left(\frac{\mu^u_{ij} + R_i}{\sigma_{v_{ij}}} + \frac{\theta \cdot \sigma_{v_{ij}}}{\sigma_{\eta_{ij}}^2}\right) \cdot dF(\psi_{ij}) \\ \omega_{ij} &= \sigma_{v_{ij}} \cdot T\left(\frac{\mu^u_{ij} + R_i}{\sigma_{v_{ij}}} + \frac{\theta \cdot \sigma_{v_{ij}}}{\sigma_{\eta_{ij}}^2}\right) - R_i \end{aligned}$$

Notice that the second equation is a 1-to-1 mapping $\theta \rightarrow \omega_{ij}$ and thus, to find ω_{ij} , we need only solve the first equation for θ .

2. **Stopping rule.** The customer stops the search when the customer's maximal expected utility of searched options exceeds the maximal reservation utilities of unsearched options.

3. **Purchase rule.** The customer purchases either the option from the set of searched ones, which yields the highest expected utility, or the outside option.

Equation (5) illustrates the difference between the standard search model and the model with a return option. Specifically, by changing the reward distribution, the return option also changes the reservation utilities for the customer. Thus, the return option could change the order in which the customer searches products and could change the stopping rule. The option to return also changes the purchase rule, as in this case the customer would compare $v_{ij}(\psi_{ij})$ rather than $\mu^u(x_{ij}) + \epsilon_{ij}$ which depends on the parameters of the return option. It is helpful to compare Equation (5) with a classic case of “no returns allowed”. When $R \rightarrow +\infty$ and $\sigma_{v_{ij}} \rightarrow 0$, Equation (5) converges to the standard equation for reservation utilities.

5. Empirical Specification and Estimation Strategy

In this section, we describe in detail the estimation strategy and empirical specification.

5.1. Utility

We assume that the utility of customer i 's purchase of product j is in Equation (6) a linear function of product characteristics. To focus on search and returns and not overparameterize the model, we assume that customers have identical preference vectors (homogeneous preferences for product characteristics):

$$(6) \quad \mu^u(x_{ij}) + \epsilon_{ij} = x'_{ij}\beta^u + \epsilon_{ij},$$

where x_{ij} is a vector of product characteristics; β^u is a vector of the customer's sensitivity to product attributes; and $\epsilon_{ij} \sim \mathcal{N}(0, \sigma_{\epsilon_{ij}})$ follows i.i.d. standard normal distribution. Without loss of generality, because utility is unique only to a positive linear transformation, we set $\sigma_{\epsilon_{ij}} = \sigma_{\epsilon} = 1$.

Recall that the customer receives a noisy signal ψ_{ij} before purchase. In Equation (7), we assume that the noisy signal is normally distributed $\eta_{ij} \sim \mathcal{N}(0, \sigma_{\eta_{ij}})$, and that this signal depends upon the product and session characteristics. Specifically,

$$(7) \quad \log \sigma_{\eta_{ij}} = y'_{ij} \beta^\eta$$

where y_{ij} is a vector of product/session characteristics which could affect the quality of the signal; β^ν is a vector of weights for the strength of signal quality.

From Equations (5-7), it follows that the expected purchase utility under the return option is a function of model parameters β^u, β^ν, R (return costs) observable variables x_{ij}, y_{ij} and the unobservable shock ψ_{ij} . In notation, we write this as:

$$(8) \quad v_{ij} = v_{ij}(\beta^u, \beta^\nu, R, x_{ij}, y_{ij}, \psi_{ij})$$

5.2. Search Costs

To capture heterogeneity in search costs, we specify search costs as is Equation (4). For identification, we set $\sigma_{\xi_{ij}} = \sigma_\xi = 1$. The random cost assumption is important because without it, the likelihood function could be equal to zero for some customers. To see why the model would collapse, consider the case when the costs are not random. Constant costs imply that the reservation utilities in Equation (5) would be deterministic. For a given list of products, all products would have the same fixed reservation utilities for every consumer. Therefore, given the optimal search rules, all customers viewing this list would click on products in the same order which contradicts our data. Mathematically, the observed data would make infeasible the set of constraints regarding the search order (Equation (11) left part). Without randomness in costs, there is no combination of values of model parameters such that there is a non-zero probability of observing data where the search-purchase varies among consumers.

5.3. Likelihood

Let S_i denote the number of products presented to the customer (for example, the number of products the customer sees on the main page of the website). From this set of products, the customer searches (clicks) on C_i products according to optimal search rules in §4.3. Recall that the index j represents the order in which the customer searches for the product (e.g., $j = 2$ denotes the second searched product). We assume that for $j > C_i$ the order of products does not matter.

The customer acts as if the customer's actions were described by knowing all parameters of the model, as well as observing the shocks $\epsilon_{ij}, \psi_{ij}, \xi_{ij}$, which are not observable by the researcher. Because our data enable us to observe all other information that is available to the customer, we use the optimal search rules from §4.3 to write constraints on model parameters.

Consider a customer who searched for C_i products; purchased a product with index b and decided to keep it. In Equations (9-12), the following constraints on the customer parameters are implied where $\mathcal{I}(\text{condition}) = 1$ if the condition is satisfied.

Return. The customer keeps a purchased product b if the product utility is greater than the negative return costs R :

$$(9) \quad \mathcal{I}(x'_{ib}\beta^u + \epsilon_{ib} \geq -R)$$

Purchase. The customer purchases the product if its expected utility from Equation (5) is greater than the expected utility of all other products in the consideration set. ($j = 0$ is the outside option).

$$(10) \quad \mathcal{I}\left(v_{ib} \geq \max_{s=0..C_i} v_{is}\right)$$

Search continuation. After searching option j , the customer continues the search if the best option on hand, $\max_{s=0..j} v_{is}$, is worse than the value of searching the unsearched options.

The customer would choose the option with maximal reservation utility ω_{ij} :

$$(11) \quad \forall j < C_i \quad \mathcal{I}\left(\omega_{ij+1} \geq \max_{s=j+2, \dots, S_i} \omega_{is}\right) \cdot \mathcal{I}\left(\max_{s=0..j} v_{is} \leq \max_{s=j+1, \dots, S_i} \omega_{is}\right)$$

Search stopping. The customer stops searching when the maximal expected utility of searched options is higher than the value of searching the remaining options:

$$(12) \quad \mathcal{I}\left(\omega_{iC_i} \geq \max_{s=C_i+1, \dots, S_i} \omega_{is}\right) \cdot \mathcal{I}\left(\max_{s=0..C_i} v_{is} \geq \max_{s=C_i+1, \dots, S_i} \omega_{is}\right)$$

Equations (9-12) define the set of constraints that must be satisfied to observe the given search sequence. Multiplication of these equations produces a binary variable W_i which takes 1 if and only if all constraints are satisfied. The case when the customer decides to return a

product (or chooses the outside option) closely follows derivations in Equations (9-12). In Appendix A.4, we show that the variable W_i can be rewritten in a more compact way²:

$$\begin{aligned}
 W_i &= W_i(\beta^u, \beta^c, \beta^\eta, R, x_i, y_i, z_i, \epsilon_i, \psi_i, \xi_i) = \\
 &= \left[\prod_{j=1 \dots C_i-1} \mathcal{I}(\omega_{ij} \geq \omega_{i,j+1}) \right] \cdot \mathcal{I}\left(\omega_{iC_i} \geq \max_{j=C_i+1, \dots, S_i} \omega_{ij}\right) \cdot \\
 (13) \quad &\cdot \left[\prod_{j=0 \dots C_i-1} \mathcal{I}(v_{ij} \leq \min\{\omega_{iC_i}, v_{ib}\}) \right] \cdot \mathcal{I}(v_{iC_i} \leq v_{ib}) \cdot \mathcal{I}\left(v_{ib} \geq \max_{j=L+1, \dots, S_i} \omega_{ij}\right) \\
 &\cdot \mathcal{I}(x'_{ib}\beta^u + \epsilon_{ib} \geq -R)
 \end{aligned}$$

Because the researcher does not observe $\epsilon_i, \psi_i, \xi_i$, we obtain the probability of observing the given search sequence of customer i by integrating out these variables:

$$(14) \quad P_i = P_i(\beta^u, \beta^c, \beta^\eta, R, x_i, y_i, z_i) = \iiint W_i \cdot dF(\epsilon_i, \psi_i, \xi_i)$$

Given this probability for each observation in a sample, we compute the log-likelihood function:

$$(15) \quad LL(\beta^u, \beta^c, \beta^\eta, R) = \sum_{i=1 \dots N} \log P_i(\beta^u, \beta^c, \beta^\eta, R, x_i, y_i, z_i)$$

5.4. Estimation

Variables x_i, y_i, z_i are observable $\forall i$, thus, if computations were feasible, we would find the estimate of the parameter vector $\beta = (\beta^u, \beta^c, \beta^\eta, R)$ by maximizing the log-likelihood function in Equation (15). Unfortunately, this maximization is not feasible for two reasons.

First, the reservation utilities ω_{ij} from Equation (5) are not computed directly because they are defined through implicit functions. Without further simplification and approximation, we estimate that solving for the maximum-likelihood estimates would require more than 1,000

² We drop the product index for compactness. For example, x_i should be read as a set of variables for all products in the search set $\{x_{ij} : j \in \{0 \dots S_i\}\}$

years with today's computers. To make computation feasible, we approximate the function $\omega_{ij}(\mu_{ij} + R, \sigma_{\eta_{ij}}, c_{ij})$ with interpolation techniques. Details are given in Appendix A.3.

Second, when integrating over unobserved shocks $\epsilon_i, \psi_i, \xi_i$, there is no known closed-form solution for this integral. We must approximate the integral. We considered the following approximations.

Accept-reject simulator (e.g., as in Chen and Yao 2017). An accept-reject simulator replaces the true probability P_i with a simulated probability \hat{P}_i . In this approach, for given parameter estimates, we simulate B random draws of shocks from corresponding distributions and calculate the share of draws in which $W_i = 1$ (all constraints in Equations (13) are satisfied). The parameter vector corresponding to the largest share of draws is the maximum-likelihood estimator. The challenge with this approach is that the nature of search data makes P_i close to zero and thus requires large values of B with a correspondingly substantial increase in computation time. Compounding the computational limit is the fact that this approach produces a non-smooth objective function which requires the use of non-gradient optimization methods (e.g., Nelder-Mead method). Such methods are substantially slower. In §6 we demonstrate that this approach is not feasible for our data because this approach produces highly imprecise estimates.

Accept-reject simulator with smoothing (e.g., as in Honka and Chintagunta 2019; Ursu 2018). To make estimation feasible, we consider replacing the sharp constraints, such as $\mathcal{I}(a < b)$ with a continuous function of differences $b - a$. This approach punishes large violations of the constraint but allows small differences. While this approach is often feasible, it does not work well for the search-purchase-return model. First, most of constraints of the form $\mathcal{I}(a < b)$ have arguments a and b bounded from below (for example, v_{ij} is bounded below by $-R$ because $T(x) \rightarrow 0$ if $x \rightarrow \infty$). Because of these bounds, the difference $b - a$ does not translate well into a probability that the constraint was violated. Second, returns are represented by a single constraint and we observe returns only for the searches that ended with a purchase (approximately, 10% of the sample). This implies that the “returns constraints” constitute a small proportion of all the constraints in the model. Thus, violation of the “return constraints” would have a much lower impact on the final objective function and the optimal

solution to Equation (15) would be obtained by fitting the “non-return constraints” for while violating the “return constraints.” Because of this imbalance, smoothing of all constraints poses threats to the estimation quality.

Partly closed form integration. The third approach to feasibility recognizes that some, but not all variables, in the constraints can be integrated out, at least theoretically. For example, the return constraint $\mathcal{I}(x'_{ib}\beta^u + \epsilon_{ib} \geq -R)$ could be replaced with probability $1 - \Phi((x'_{ib}\beta^u + R_i) \cdot q_{ib}^v + p_{ib}^v \cdot \psi_{ib})$ by integrating out the shock $\epsilon_{ib}|\psi_{ib}$. In Appendix A.5, we prove that we can rewrite the binary variable W_i as:

$$(16) \quad \widetilde{W}_i = \mathcal{I}(\text{condition 1}) \cdot \mathbb{P}[\text{condition 2}]$$

- *Condition 1* – constraints which cannot be integrated out and thus represented by binary variable
- *Condition 2* – constraints which could be integrated out and represented by continuous number from 0 to 1.

In Appendix A.5, we demonstrate that *Condition 1* has only one sharp constraint that cannot be integrated out. Hence, the requirement on the number of draws for the sharp constrain is significantly smaller than it would be if all constraints were simulated.

To mitigate the concern that the optimal solution might ignore the returns data due to imbalance of the number of constraints, we implement a two-stage approach:

1. In the first stage, we estimate only the purchase-return model.

$$(17) \quad W_i^{FS} = W_i^{FS}(\beta^u, \beta^\eta, R, x_i, y_i, \epsilon_i, \psi_i) = \left[\prod_{j=0 \dots C_i} \mathcal{I}(v_{ij} \leq v_{ib}) \right] \cdot \mathcal{I}(x'_{ib}\beta^u + \epsilon_{ib} \geq -R)$$

2. In the second stage, we fix the parameters related to product returns (R and β^η) and estimate all other parameters by simulating the variable from Equation (17).

The two-stage approach is an approximation that works well with partly-closed-form integration, but it is an approximation. To examine the implications of the approximation, we

create synthetic data with known parameters and apply the three alternative methods. In §6, we demonstrate that the two-stage approach with partly-closed-form integration achieves the highest accuracy and recovers the values of parameters on simulated data quite well. See Appendix A.6.

6. Synthetic Data to Examine the Properties of the Alternative Estimators

We use synthetic data to examine the feasibility of our model estimation, its properties, and identification. Synthetic data analyses demonstrate that the likelihood in Equation (17) enables us to recover the true parameters of the model. We also compare the estimated parameters to the classical model of customer search (no option to return the product).

We simulated 10,000 synthetic customers according to the search model described in §4. So that the synthetic data are relevant, we chose parameters that closely mimic the real data. Specifically, the synthetic data have the same marginal probabilities of purchase and return as the real data. We assume that the retailer has two products (for example, two product categories) and provide the customer with the list of 20 products on the webpage.

To capture the essence of the model and make all estimation methods feasible, in synthetic data:

1. The utility of the customer depends on the category and constant term. A negative intercept implies that the customer has a positive outside option.
2. The quality of the signal depends on the category.
3. The logarithm of search cost is a function of the product position on the page and a constant term.
4. The return cost is constant.

Our simulation has in total six parameters to be estimated. In Appendix A.6, we describe in detail the parameters of the synthetic data. For models with returns and without returns, we use the zero vector as a starting. The results of the estimation with the two-stage approach are shown in Table 1.

Table 1. Results of model estimation on simulated data

Variable	True parameter values	Estimates of the model with returns	Estimates of the model without returns
Utility: constant	-1.4	-1.347	-1.568
Utility: category dummy	-0.3	-0.280	-0.099
Noise β^η : category	-0.5	-0.443	
Search costs: constant	-4	-3.944	-3.360
Search costs: page rank	0.3	0.312	0.312
Return costs: constant	-1.5	-1.427	

Table 1 demonstrates that the search-purchase-return model, and the model without returns, can be estimated using the two-stage approach and that the true parameters can be recovered well for the full model. We expect precision to improve with a larger sample of synthetic customers because we observe precision improving from 5,000 to 10,000 synthetic customers. We tried alternative approaches discussed in §5.4 and found that the estimates were substantially worse than the ones provide by the focal method both in terms of accuracy and the required computation resource/time. See Appendix A.7.

Table 1 also demonstrates what is lost when returns are not modeled explicitly. Not modeling returns (a) overestimates the utility of a product, (b) underestimates the absolute value of a category dummy, and (c) slightly underestimates search costs. Naturally, when returns are not modeled, there is no estimate of return costs. These differences are intuitive. Returns have an option value. If they are not modeled, this option value is absorbed in the base utility. Also, when returns are not modeled, it appears to the researcher that search is less costly because the return option makes it more attractive to search. Lastly, underestimation of the absolute value of the category dummy is due to the particular parameters of our simulation. Specifically, category decreases the expected utility directly (-0.3 is less than zero) but also increases the utility by providing better signal quality (-0.5 is less than zero). As a result, the customer would purchase this product more frequently because the customer would expect to keep it. As a result, the no-return model would overestimate (taking into account the sign) the utility of this product.

7. Estimation Results

Having demonstrated that we can recover known parameters from synthetic data, we proceed to the actual data. Table 2 reports the estimation results for our model. The first column reports the sensitivity to attributes of the mean utility β^u from Equation (6), the second reports sensitivity of the variance of the signal β^η from Equation (7), and the estimates of search costs β^c from Equation (4). The return costs are at the end of the table.

Table 2 suggests variation in mean product utilities and quality of the signals across categories. For example, “Dresses” has a substantially higher variance of the signal than “Polo Shirts.” This difference is not surprising (based on fashion experts) because the choice of a dress is more nuanced than choosing a polo shirt. Customers need more information to make a correct purchase decision. Consistently with Dzyabura et al. (2022), colorful items are harder to evaluate than black and blue items, resulting in a higher return rate online. Interestingly, apparel products made from natural fabrics provide a better signal of the customer preference shock. Likely, synthetic fabrics look attractive based on the online images, but, for some customers, look less attractive when the customer inspects the product at home. Natural fabrics, on the other hand, look more consistently attractive online and in the home.

Table 2 suggests that the product position on the website affects the search costs substantially. Specifically, a product in the middle of the list has 8.8% higher search costs compared with a product at the beginning of the list. The result is qualitatively consistent with the results in Ursu (2018), where the position of the product on the website affects the search costs in a randomized setting. This implies, for a given set of product characteristics, customers are more likely to click on the first displayed item. Interestingly, mobile device users have lower search costs implying that either it is easier for customers to navigate the website through a mobile device or that those customers are more experienced with search. This is an interesting finding that retailers could use to implement different policies for different versions of the website (desktop vs. mobile).

Table 2. Results of model estimation on real data

Variable	Mean Utility	Signal Variance
Utility constant	-1.447	
Relative Price	-0.003	
Blouses	-0.191	-0.213
Pants	-0.046	0.012
Jackets and coats	0.014	0.105
Jeans	-0.054	-0.029
Dresses	0.080	0.242
Shoes	-0.166	-0.185
Polo Shirts	-0.376	-0.814
Tops	-0.319	-0.494
Knit	-0.223	-0.319
Sweat	-0.259	-0.416
Shorts	-0.293	-0.339
Skirts	-0.006	0.119
Blazer	-0.078	-0.026
Classic shirts	-0.272	-0.484
Brand: Denim	-0.009	0.019
Brand: Menplus	-0.025	0.012
Proportion of natural fabric	-0.080	-0.165
Color: Blue	-0.031	-0.057
Color: Gray	-0.020	-0.039
Color: White	0.047	0.123
Color: Green	0.019	-0.009
Color: Yellow	0.028	0.003
Color: Red	0.059	0.098
Search performed on a desktop (not mobile)	0.072	0.005
Search Costs Parameters		
Fixed Mean Search Costs	-4.092	
Variable Position Search Costs (log)	0.303	
Variable Page Search Costs	0.100	
Search performed on a desktop (not mobile)	0.231	
Return Costs Parameters		
Return Costs (log)	-1.3472	

8. Model Implications

8.1. Insights Obtained from the Estimated Search-Purchase-Return Model and Comparisons with Data

The analytical model and the parameter estimates provide qualitative insights about the interrelation between search and returns. These relationships are valuable, but we must not over-interpret them. Some relationships are valuable in the sense that if we observe a type of search behavior, the product is less likely to be returned. Although the data-based results are not necessarily causal, the theoretical model suggests potential policies that we can evaluate with policy simulations recognizing that the firm needs A/B tests to confirm the superiority of a proposed policy.

For example, consider the following insight: if the last-clicked product is purchased, then it is less likely to be returned. This is a description of an empirical relationship in the data. The last click does not cause a lower return, but rather, if we observe that the last-clicked product is purchased, then we observe a lower probability of return, likely because the consumer found a hidden-gem product that matched the customer's tastes. On the other hand, if the customer searches a variety of colors, the customer is less likely to return the product. A partial explanation of this data-based observation is likely due to idiosyncratic customer characteristics, but part of the explanation is due to empirical search costs. Our model suggests that if search costs are reduced, the customer will search more colors and find the best color match. We leave to future research, experiments that establish causality. However, even without causality, the insights are useful because they indicate how search data are indicators of future returns.

Table 3 summarizes the insights. For each insight, we provide intuition from the theoretical search-purchase-return model. We then cite raw (model-free) evidence from the data to support the insight.

Table 3. Summary of model insights about customer search and product returns

Model Insight	Intuition	Data Support
The last clicked product is purchased → lower return probability	Customer finds a perfect match product	Correlational evidence
More clicks on products → higher return probability	Customer struggles to find a good product	Figure 3a
Many colors of product clicked → lower return probability	Customer likes the product style and searches for best color fit	Figure 3b
Usage of pre-search tool → lower return probability	Customer has a higher preference for a particular attribute and can products with that attribute quickly	Figure 3c
Long product views → lower return probability	Customer extracts a lot of information about the product	Figure 3d
Product with low share of returns → more likely to be clicked	Customer prefers exploring “safer” (low-returned) options	Figure 3e

Customers who purchase the last clicked product are less likely to return it. Intuitively, this means that the customer found a product that matched well the customer’s preferences (for example, a “dream dress”). The customer does not want to continue the search as it would only drain valuable time. Having a dream purchase results in lower probability of return. We now provide formal motivation.

Recall that C_i is the index of the last product clicked and imagine customer i purchased this product. Thus, from Equation (13), we write down all constraints which involve the utility of a purchased product:

$$(18) \quad \left[\prod_{j=0 \dots C_i-1} \mathcal{I}(v_{ij} \leq \min\{\omega_{iC_i}, v_{iC_i}\}) \right] \cdot \mathcal{I}\left(v_{iC_i} \geq \max_{j=L+1, \dots, S_i} \omega_{ij}\right) \cdot \mathcal{I}(x'_{iC_i} \beta^u + \epsilon_{iC_i} \geq -R)$$

All constraints in Equation (18) bound the value of v_{iC_i} from below. Thus, the value of v_{iC_i} could be quite large due to a large positive signal of product fit ψ_{iC_i} . In this case, the value of ϵ_{iC_i} is likely to be large as well due to the structure of the signal, hence, the customer would not return the product. Practically, this implies that the information about whether the last clicked product was purchased allows the firm to identify customers who received a very good

signal. When we examine the data, we observe that customer who purchased the last searched item are 8 percentage points less likely to return the product.

Customers who make more clicks prior to purchase are more likely to return the product. Intuitively, the customer who clicks on many products is hesitating between different options and does not have a strong preference for any of them. For example, imagine a very unlucky customer who cannot find an item he or she likes, but ends up purchasing the item with a utility just above the outside option.

More formally, recall the simplified equivalent constraints explaining the search behavior of the customer in Equation (13). After dropping the less relevant constraints, we get:

$$(19) \quad \left[\prod_{j=1 \dots C_i-1} \mathcal{I}(\omega_{ij} \geq \omega_{ij+1}) \right] \cdot \left[\prod_{j=0 \dots C_i-1} \mathcal{I}(v_{ij} \leq \min\{\omega_{iC_i}, v_{ib}\}) \right]$$

The right set of constraints implies that all the clicked options' expected purchase utilities (except the last one) are bounded from above by at least ω_{iC_i} . At the same time the left set of constraints implies that the ω_{ij} is a decreasing function of option order click j . Therefore, a customer, who searched longer (or made more clicks), would likely have a lower upper bound ω_{iC_i} and hence a lower purchase utility with consequent higher return probability.

The retailer observes this information online and may implement policies which would reduce the need to search additional options, say by a recommendation system based on observed search costs. The retailer could also reduce the return rate by showing additional random products for the customer at zero search costs. We explore this policy in §7.2.

Data are consistent with this theoretical result. In Figure 3a, we observe a strong positive correlation between the number of clicks and probability of return.

Customers who browse many colors of the purchased product are less likely to return the product. Intuitively, the customer already likes the style of the product, say a dress, and is now searching for the best color match.

Although our model assumes homogeneous customers, we still can model this scenario by assuming homogeneity in preferences for all characteristics other than color, but heterogeneity in color preference. Formally, we assume other characteristics are identical (e.g.,

shape, style, etc.) and we can write the product utility as sum of non-color and color sub-utilities $u^{final} = u^{nc} + u^c$. The fact that customer clicked on different colors of the product implies that the expected utility of these different colors was higher than other products. Because the non-color-related characteristics are identical, it is likely that u^{nc} is high and the customer tried to maximize u^c .

From the data, we find that customers who looked at many colors have a lower probability of returning the purchased item. Figure 3b demonstrates that customers who looks at one additional color are 1 percentage point less likely to return the product.

Although our data and model suggest a relationship that may or may not be causal, the theoretical model suggests that the retailer may enhance purchase and reduce returns by reducing the search costs of alternative colors of a chosen product (increase u^c and u^{final}). For example, subject to experimental A/B tests, the retailer might suggest alternative colors to the customer at the time of search or at checkout.

Customers who apply pre-search tools have a lower probability of return. Intuitively, consider two customers (A and B) looking for a dress. Customer A does not have particular preferences while B wants a black dress made of a natural fabric. Customer B applies pre-search filter to narrow search while Customer A is content to search from the default webpage. Suppose both customers buy the same dress. Customer B is more likely to find the correct match to Customer B's preference and is thus less likely to return the product. Customer A is less likely to find the best match.

More formally, by using pre-search filters, the customer changes the distribution from which to sample the products (for example, browse only products made of natural fabrics). Typically, the application of pre-search filters requires paying an additional search cost beyond those estimated in Table 2 (for example, navigating through the menu, reading, clicking). This implies that customer faces a tradeoff: sample from a better distribution by paying search costs or sample from the default distribution for free. Our data mix variation among customers with retailer policies. However, the theoretical model suggests that reducing search costs through pre-search filters increases purchases and reduces returns.

Model-free evidence in Figure 3c illustrates these findings. Specifically, we observe that customers who apply one additional pre-search filter are 4 percentage points less likely to return purchased products.

Customers who spend substantial time reviewing the product page are less likely to return the product. Intuitively, consider the scenario when a customer opened the product page and started to explore information about products. The customer reads the descriptions, looks at pictures, inspects at the patterns, etc. If the customer searches for a long time, then it is likely that the customer obtained a substantial amount of information about the product—enough to be confident that the product will match customer’s needs. Thus, we expect customers who spend on average more time reviewing the products are less likely to return the product.

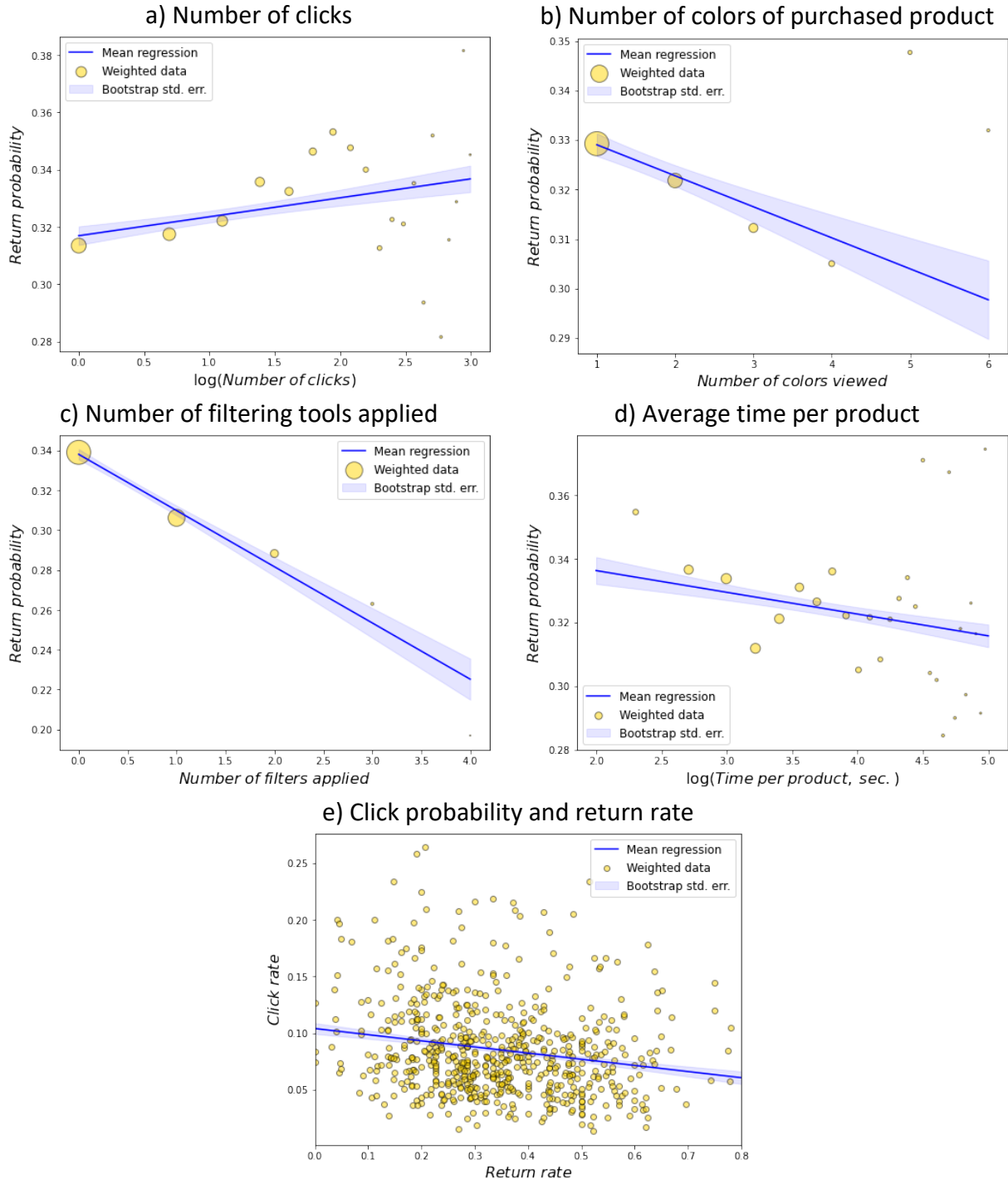
Mathematically, by consuming each unit of information, the customer gets a signal ψ_Δ about the variance of noise quality σ_{η_Δ} for the cost of c_Δ . The customer could choose how many of these signals to obtain before he or she makes a purchase decision. For example, each signal ψ_Δ might be an additional picture or an additional line of item description. If the customer receives T signals, then the final signal would be $\psi = \sum_t \frac{\psi_{\Delta t}}{T} = \sum_t \frac{\eta_{\Delta t}}{T} + \epsilon \equiv \eta + \epsilon$. Because $\eta \sim \mathcal{N}\left(0, \frac{\sigma_{\eta_\Delta}}{T}\right)$, the variance of the noise of the final signal is a decreasing function of time spent. Thus, more time spent leads to higher overall quality of the signal and thus lower return probability. Our data support these findings where Figure 3d demonstrates the decreasing pattern.

Products with a lower return rate are more likely to be clicked. Intuitively, customers have high return costs when effort and time are considered. Customers may prefer to minimize these costs by including safer options in their searches.

Formally, the reservation utility is an increasing function of the quality of the signal and thus the product with a better signal quality would have a higher reservation utility and thus more likely to be clicked according to the optimal search rules. However, products with better signals have a lower probability of being returned.

From the data in Figure 3d, we find a negative correlation between the return rate of the item and click probability where an item which is clicked 5 percentage points more has a 10 percentage point lower probability of being returned.

Figure 3. Informativeness of search on product returns



8.2. Policy Simulations: Implications for Retailer Actions

Our data are observations of customers searching in the empirical retail environment. Although the retailer has indicated that it has not designed its website to influence returns, we cannot rule out that it has done so. Our data are not policy experiments. However, once we fit the search-purchase-return model, we can assume that the parameters are close to those which would be obtained if we could have taken the retailer's website design decisions into account. (We provide these cautions to highlight that any policy simulations are subject to future empirical tests.)

With these caveats, we use the analytical model to simulate what would happen if the search environment were changed by the retailer ('what if' simulations). We present these recommendations as hypotheses consistent with the data and analytic model, recognizing that we have not fully modeled, nor do we have the data to explore, endogenous retailer decisions.

Table 4 summarizes the results of the policy simulations. In each policy simulation, we change the retailer's website policy and observe the resulting (simulated) customer behavior.

Table 4. Summary of policy simulation results

Policy	Main Findings
Improving the website design to decrease search costs	Increase in purchase rate and decrease in return rate
Modifying the effort and time to return a product in the online channel	Increase in purchase rate and increase in return rate
Changing customers' search set by changing the online assortment	Retailer may improve the return rate and profit by removing some products from the website
Changing the product ranking	Order of displaying product impacts the return rate

Improving the website design to decrease search costs. The retailer may invest in modifying the website to decrease customer search costs. We consider a hypothetical scenario where the retailer can modify the website to reduce fixed search costs by 5%. Policy simulations predict that lower search costs would benefit both for the customer and the retailer. Specifically, a 5% decrease in search costs results in 9.1% increase in purchase probability and an increase of 1.9% in customer surplus.

Moreover, our model provides additional insights on the positive effect of decreasing the search costs. It shows that decreasing the search costs would lead to a decrease in the return rate via allowing the customer to choose a better option. To test this, we assume that the retailer could make the search costs for 1 random product equal to 0 (for example, through a recommendation system). This implies that the customer can always make an additional cost-free click on the product shown. Adding this one additional searched product leads to a 3.8% decrease in the return rate.

Modifying the time and effort to return a product in the online channel. In many countries, product returns are mandated by law and customers have the right to return any product within a specified deadline. However, even in strictly regulated environments, the retailer can make the returns process harder or easier for customers. For example, the retailer may make customers print the label themselves, complete a complicated return form, or require customers to return to a bricks-and-mortar store (as recently implemented by major retailer. See BBC 2022).

In the returns literature, it is well documented that the return option (and how lenient or strict it is) affects both the purchase and the return decision of the customer. Our model supports and extends these insights. Specifically, our model allows us to distinguish between two reasons for a change in purchase probability. One reason is the improved expected utility of the product as well documented in Simester et al. (2008). A second reason is that decreased return costs improve the reservation utilities of products and encourage the customer to click on more products. Specifically, a 10% decrease in return costs leads to an 7.8% increase in the number of clicks which translates into a 6.3% increased purchase probability. In our data, the model shows that 37.4% of the total purchase probability increase is explained by the second reason – greater search.

Changing consumers' search set by changing the online assortment. Typically, the retailer has full control of its online assortment. Even if the retailer wishes to retain an omnichannel positioning, it may make some products harder to access by placing them at the end of the product list. If this were done, only customers with a strong preference for the

product would be able to find it (for example, by using pre-search filters). We can simulate the policy of excluding a small percentage of products from the online assortment.

As an illustration, we assume the retailer could exclude 20 products from its online assortment. We choose 20 because it is a negligible change in the assortment (0.15% of all products) and not expected to cause major structural changes in the website. We divided all products into 690 equally sized groups based on the predicted return rate for the products. We estimate the change in purchase/return rate by simulating the elimination of one of these groups from the online assortment. For each of these groups we obtained a pair of numbers – change in purchase rate and change in the return rate (in comparison to the baseline of all products). We position all such product-group deletions in Figure 4a. The findings are qualitatively similar for other number of excluded products.

The results demonstrate that most products pose a trade-off to the retailer by increasing/decreasing both purchase and return probabilities (upper-right and lower-left quadrants) – for these product groups, each retailer can evaluate the profit impact by using its return processing costs and product markup. There are also many product groups with a uniformly negative impact (lower-right quadrant). These product groups should be retained by the retailer. Interestingly, there are product groups with net improvement (upper-left quadrant) which have a positive impact both on return and purchase probabilities. The retailer should consider excluding these product groups from customers' search set, either by making them hard to search or by removing them from the online assortment.

Figure 4a is illustrative of the concept of assortment management. There are $\sim 10^{332}$ potential product groups comprising 1% of the products and 10^{1186} comprising 5% or fewer of the products. Selecting the optimal assortment is beyond the scope of this paper. The optimization problem is finite and, thus, an optimum exists, but approximations might prove necessary.

Changing the product ranking. From Table 2 it follows that the customer is more likely to click on products displayed at the top of the website because the search costs are an increasing function of product position. This implies that changing the order in which the retailer displays products on the website could substantially impact customer's search behavior

and, by implication, returns. One way to encourage customers to click on a higher-utility products is a greedy algorithm: rank the products based on their mean utility. In this case, the products which are liked by customers are displayed at the top of the website, allowing customers to browse these more easily and thus improve the customer surplus (Ursu, 2018). However, such a greedy product ranking might have a negative effect on product returns.

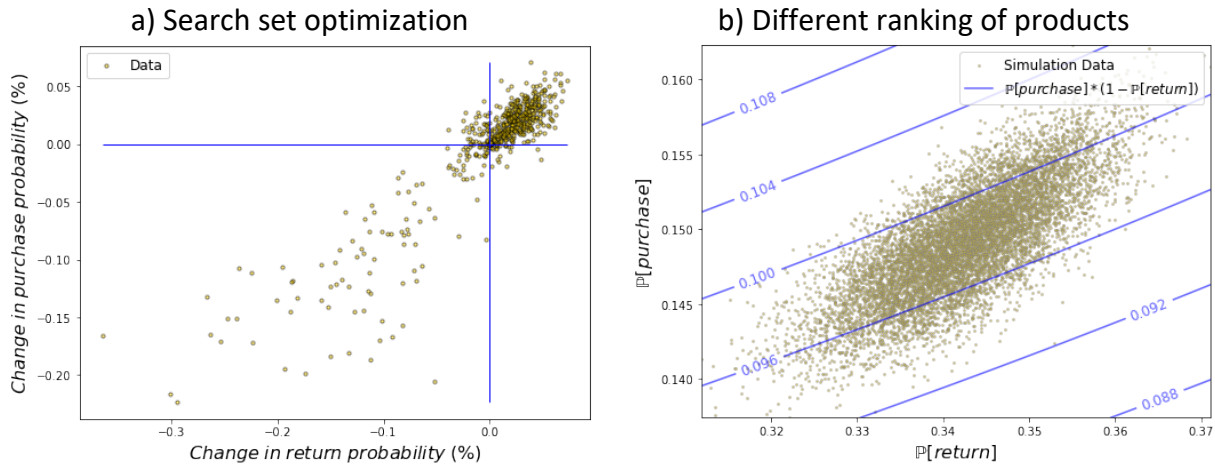
We use the estimates of our model from Table 2 to rank products according to their estimated mean product utility without considering returns. Our policy simulation suggests that, although the purchase probability increases substantially (by 11.7%), the customer surplus increases only by 0.4%. However, ranking on mean product utility increases the return rate by impressive 9.6% and may negate the profit improvement.

We can likely do better by taking returns into account. Unfortunately, reservation utilities (considering returns) are nonlinear functions of parameters negating a simple approach. Brute force is also not feasible because the number of rankings is factorial in the number of products. Our retailer has 48 products per page on its website, resulting in $48! \sim 10^{61}$ possible combinations. Exhaustive enumeration is infeasible with current computational power.

To illustrate the potential improvement that is possible with alternative rankings, we randomly selected 48 products and plotted in Figure 4b the purchase and return probabilities for 10,000 different rankings of these products. Figure 4b illustrates the potential improvement. For this set of 48 products, we observe that the $\mathbb{P}[\text{return}]$ ranges within $[31\%, 37\%]$ and $\mathbb{P}[\text{purchase}]$ within $[13.5\%, 16.5\%]$. Without knowing the retailers profit margins and return costs, we cannot choose the optimal ranking from this set. As a further illustration we consider a simple metric, $\mathbb{P}[\text{purchase}] \cdot (1 - \mathbb{P}[\text{return}])$, which maximizes sales net of returns. We plot $\mathbb{P}[\text{purchase}] \cdot (1 - \mathbb{P}[\text{return}]) = \text{const}$ in the Figure 4b for different value of the *const*. In Appendix A.7, a greedy algorithm based on this metric and applied to 100 random product sets increases this metric by 6.4% relative to a random ranking. Optimization, which is beyond the scope of this paper, would increase the metric even more. Based on these results, we expect that, for any metric the retailer uses to balance sales and returns, we can improve profits, as evaluated with policy simulations, with a greedy algorithm

based on that metric. This improvement is easily achievable and could be optimized. Furthermore, the revised ranking is relatively easy to evaluate with A/B testing vs. ranking by utility alone.

Figure 4. Policy simulation illustrations



9. Conclusions and Future Research

Managing product returns is highly relevant but also challenging. Online retailers are facing high return rates that are associated with high return costs. Improving how a retailer can manage product returns has a direct and considerable impact on the firm's bottom line. To the best of our knowledge, retailers and researchers have not yet investigated the complete search-to-purchase-to-returns customer journey to generate insights and suggest strategies by which a retailer can maximize profits. Our goal was to establish that observing customer search helps revealing the mechanisms by which, search and returns are related and provide insights that would help retailers develop easy-to-implement, profit improving strategies that would not be obtained otherwise.

Using an empirical-theoretical framework, we developed a rational model of customer search in the presence of a return option. We obtained data from a major European apparel online retailer to estimate the model and demonstrate the importance of modeling search and returns jointly. The data and analysis suggest strategies by which a retailer can maximize its profits.

Our model provides insights on how search and returns are related. More specifically, the model shows that purchasing the last clicked product, browsing multiple product colors,

searching more, and using pre-search tools are linked to a lower return probability. The retailer's data provide supportive evidence for these insights.

In policy simulations, we investigate strategies that can help the retailer better manage product returns. Some of these strategies such as improving some aspects of the website design are relatively low-cost in comparison to changes in the return policy. Moreover, in countries with strong customer protection legislation, it is impossible to change the return policy but, generally, companies are not limited in the way they design their websites. We show that reducing the search costs through a more efficient website design decreases the return rate, while increasing the purchase rate. Our policy simulations also show that rank-ordering products on the website based on their mean utility increases purchase probabilities, but also return probabilities. Retailers are well-advised to investigate the sizes of effects individually and assess whether, based on their prices and profit margins, this type of rank-ordering might be suboptimal. Finally, our policy simulations also reveal that removing products from customers' search set, via excluding them from the online assortment, might also be a promising Changing customers' consideration set by changing the online assortment strategy.

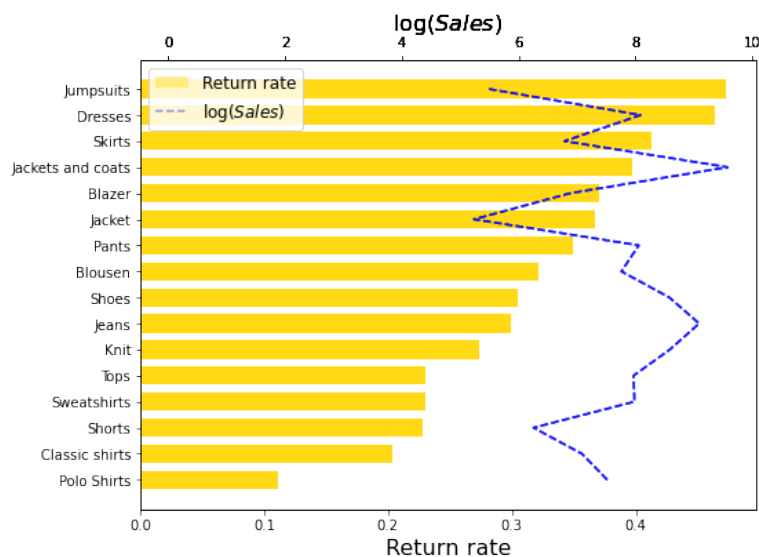
Appendix

Appendix A.1. Data preprocessing and additional summary statistics

In our paper, we preprocessed our data to obtain better estimates of the model parameters. We took the following steps in our preprocessing:

1. Remove non-fashion products (e.g. linen, towels) and kid's apparel. These products constitute a small proportion of our data and are not the focus of our retailer.
2. Remove sessions without product page views. This could happen if the customer comes to the website from the third-party website and lands directly on the product page. These sessions do not represent the true search process and we are not able to recover the set of products from which the customer was choosing.
3. Remove sessions that do not have any clicked products after a page view and sessions which have clicked products before a page view. This implies that we keep only sessions with the clean search process: the customer views the product page and selects the product to click. The alternative could happen if the customer found a product through an alternative means (from a third-party website) and in this case, it is impossible to infer the set of products from which he or she was choosing.
4. Remove 0.5% of longest/shortest sessions based on time spent. This includes accident clicks and long-tail outliers, for example, if a customer forgot to close the website.

Figure A.1. Sales and return rate by category



Appendix A.2. Derivation of expected purchase utility

Without loss of generality, we drop all indices and subscripts in this section to preserve readability. Remember $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon)$, $\eta \sim \mathcal{N}(0, \sigma_\eta)$ where ϵ and η are independent. Thus, ϵ and η have a joint normal distribution with diagonal variance-covariance matrix. From the properties of joint normal distribution, it follows that variable $\psi = \eta + \epsilon$ and ϵ also have a joint normal distribution. Hence, one can find the conditional distribution $\epsilon|\psi \sim \mathcal{N}\left(\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\eta^2} \psi; \frac{\sigma_\epsilon^2 \cdot \sigma_\eta^2}{\sigma_\epsilon^2 + \sigma_\eta^2}\right)$

In §4.1 we wanted to find the purchase expected utility as

$$\begin{aligned} v &= \mathbb{E}_\epsilon[(\mu^u + \epsilon) \cdot \mathcal{I}(\mu^u + \epsilon \geq -R) + (-R) \cdot \mathcal{I}(\mu^u + \epsilon < -R)|\psi] = \\ (A1) \quad &= \mathbb{E}_\epsilon[(\mu^u + \epsilon + R) \cdot \mathcal{I}(\mu^u + \epsilon + R \geq 0)|\psi] - R = \\ &= \mathbb{E}_\xi[\xi \cdot \mathcal{I}(\xi \geq 0)|\psi] - R \end{aligned}$$

where $\xi = \mu^u + \epsilon + R$. Because $\epsilon|\psi$ is normally distributed then $\xi|\psi$ is also normally distributed and thus we can compute the expectation above as:

$$\begin{aligned} v &= \frac{1}{\sqrt{2\pi} \cdot \sigma_\xi} \int_0^{+\infty} t \cdot e^{-\frac{(t-\mu_\xi)^2}{2 \cdot \sigma_\xi^2}} dt - R = \\ (A2) \quad &= \frac{1}{\sqrt{2\pi} \cdot \sigma_\xi} \int_0^{+\infty} (t - \mu_\xi) \cdot e^{-\frac{(t-\mu_\xi)^2}{2 \cdot \sigma_\xi^2}} dt + \frac{\mu_\xi}{\sqrt{2\pi} \cdot \sigma_\xi} \int_0^{+\infty} e^{-\frac{(t-\mu_\xi)^2}{2 \cdot \sigma_\xi^2}} dt - R = \\ &= \sigma_\xi \cdot \varphi\left(\frac{\mu_\xi}{\sigma_\xi}\right) + \mu_\xi \cdot \Phi\left(\frac{\mu_\xi}{\sigma_\xi}\right) - R = \sigma_\xi \cdot \left(\varphi\left(\frac{\mu_\xi}{\sigma_\xi}\right) + \frac{\mu_\xi}{\sigma_\xi} \cdot \Phi\left(\frac{\mu_\xi}{\sigma_\xi}\right)\right) - R = \sigma_\xi \cdot T\left(\frac{\mu_\xi}{\sigma_\xi}\right) - R \end{aligned}$$

where $\mu_\xi = \mu^u + R + \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\eta^2} \psi$; $\sigma_\xi = \sqrt{\frac{\sigma_\epsilon^2 \cdot \sigma_\eta^2}{\sigma_\epsilon^2 + \sigma_\eta^2}}$ and $T(x) = x \cdot \Phi(x) + \varphi(x)$

Appendix A.3. Derivation of reservation utilities for model with product returns.

In the original paper, Weitzman (1979) demonstrated that the reservation utility z for a product could be found from Equation (A3) where we drop the individual (i) and product (j) indices for compactness:

$$(A3) \quad c = \int_z^\infty (u - z) \cdot dF(u)$$

We demonstrated in §4.1 that the return option changes the distribution of the reward and thus, in our case, we need to find the distribution of the expected purchase utility from Equation (A2). Notice that the randomness comes from the signal of customer preference ψ while all other parameters are fixed and known to the customer.

$$\begin{aligned}
F(u) &= \mathbb{P}[v(\psi) \leq u] = \mathbb{P}\left[\sigma_\xi \cdot T\left(\frac{\mu_\xi}{\sigma_\xi}\right) - R \leq u\right] = \mathbb{P}\left[T\left(\frac{\mu_\xi}{\sigma_\xi}\right) \leq \frac{u+R}{\sigma_\xi}\right] = \\
&= \mathbb{P}\left[T\left(\frac{\mu_\xi}{\sigma_\xi}\right) \leq \frac{u+R}{\sigma_\xi}\right] = \mathbb{P}\left[\frac{\mu_\xi}{\sigma_\xi} \leq T^{-1}\left(\frac{u+R}{\sigma_\xi}\right)\right] = \mathbb{P}\left[\mu_\xi \leq \sigma_\xi \cdot T^{-1}\left(\frac{u+R}{\sigma_\xi}\right)\right] = \\
&= \mathbb{P}\left[\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\eta^2} \psi \leq \sigma_\xi \cdot T^{-1}\left(\frac{u+R}{\sigma_\xi}\right) - \mu^u - R\right] = \\
(A4) \quad &= \mathbb{P}\left[\frac{\psi}{\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}} \leq \frac{\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}}{\sigma_\epsilon^2} \left(\sigma_\xi \cdot T^{-1}\left(\frac{u+R}{\sigma_\xi}\right) - \mu^u - R\right)\right] = \\
&= \Phi\left[\frac{\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}}{\sigma_\epsilon^2} \left(\sigma_\xi \cdot T^{-1}\left(\frac{u+R}{\sigma_\xi}\right) - \mu^u - R\right)\right]
\end{aligned}$$

Next, we plug-in the distribution from Equation (A4) into Equation (A3) and obtain:

$$\begin{aligned}
c &= \int_{\frac{\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}}{\sigma_\epsilon^2} \left(\sigma_\xi \cdot T^{-1}\left(\frac{z+R}{\sigma_\xi}\right) - \mu^u - R\right)}^{\infty} \sigma_\xi \cdot T\left(\frac{\mu^u + R + \frac{\sigma_\epsilon^2}{\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}} \cdot t}{\sigma_\xi}\right) - R - z) \cdot d\Phi(t) = \\
(A5) \quad &= \sigma_\xi \int_{\theta}^{\infty} T\left(\frac{\mu^u + R + \frac{\sigma_\epsilon^2}{\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}} t}{\sigma_\xi}\right) - T\left(\frac{\mu^u + R + \frac{\sigma_\epsilon^2}{\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}} \theta}{\sigma_\xi}\right) d\Phi(t)
\end{aligned}$$

where we used the substitution $z = T\left(\frac{\mu^u + R + \frac{\sigma_\epsilon^2}{\sqrt{\sigma_\epsilon^2 + \sigma_\eta^2}} \theta}{\sigma_\xi}\right) - R$

Approximating the solution to the equation

In the paper, we made an identifying assumption that $\sigma_\epsilon = 1$. Thus, from Equation (A5) it could be seen that the reservation utility is a function of three parameters: $z^* = f(\mu^u + R, \sigma_\eta, c) = f(x_1, x_2, x_3)$. During the optimization algorithm – finding this function for each customer-product combination is not feasible as it involves many integration steps.

To circumvent the computational burden, we used trilinear interpolation technique. Specifically, for three-dimensional variable (x_1, x_2, x_3) , we constructed a grid of values and computed the exact reservation utilities for each element of the grid. Notice that in this case, the space of possible values of (x_1, x_2, x_3) is divided into 3-dimensional cubes. For each of these cubes, we know the exact values of reservation utilities in eight vertices. For any vector within the cube, we approximate the reservation utility function $f(x_1, x_2, x_3)$ as:

$$(A6) \quad \begin{aligned} f_{true}(x_1, x_2, x_3) &\simeq f_{approx}(x_1, x_2, x_3) = \\ &= \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_1 x_2 + \alpha_5 x_2 x_3 + \alpha_6 x_1 x_3 + \alpha_7 x_1 x_2 x_3 \end{aligned}$$

Where we require $f_{approx}(x_1, x_2, x_3) = f_{true}(x_1, x_2, x_3)$ at the grid (or cube vertices) points. Because f_{approx} has eight parameters and eight constraints, the linear system has a unique solution for each cell.

Appendix A.4. Deriving the equivalent set of constraints on model parameters

After combining Equation (9-12), we can compute the variable W_i from Equation (13). For compactness and without loss of generality, we drop customer related index i :

$$(A7) \quad \begin{aligned} W = & \left[\prod_{j=0}^{C-1} \mathcal{J} \left(\omega_{j+1} \geq \max_{s=j+2, \dots, S} \omega_s \right) \cdot \mathcal{J} \left(\max_{s=0 \dots j} v_s \leq \max_{s=j+1, \dots, S} \omega_s \right) \right] \cdot \mathcal{J} \left(\omega_C \geq \max_{s=C+1, \dots, S} \omega_s \right) \\ & \cdot \mathcal{J} \left(\max_{s=0 \dots C} v_s \geq \max_{s=C+1, \dots, S} \omega_s \right) \cdot \mathcal{J} \left(v_b \geq \max_{s=0 \dots C} v_s \right) \cdot \mathcal{J} (x'_b \beta^u + \epsilon_b \geq -R) \end{aligned}$$

Consider the first part of the equation:

$$(A8) \quad \left[\prod_{j=0}^{C-1} \mathcal{J} \left(\omega_{j+1} \geq \max_{s=j+2, \dots, S} \omega_s \right) \right] \cdot \mathcal{J} \left(\omega_C \geq \max_{s=C+1, \dots, S} \omega_s \right) =$$

$$\begin{aligned}
&= \left[\prod_{j=0}^{C-1} \left[\prod_{s=j+2}^S \mathcal{I}(\omega_{j+1} \geq \omega_s) \right] \right] \cdot \mathcal{I} \left(\omega_C \geq \max_{s=C+1, \dots, S} \omega_s \right) = \\
&= \left[\prod_{j=1}^{C-1} \mathcal{I}(\omega_j \geq \omega_{j+1}) \right] \cdot \mathcal{I} \left(\omega_C \geq \max_{j=C+1, \dots, S} \omega_j \right)
\end{aligned}$$

Notice that Equation (A7) is a necessary condition for $W = 1$. Thus, we can assume that these inequalities hold in further derivations. Specifically, it follows that $\max_{s=j+1, \dots, S} \omega_s = \omega_{j+1}$ and we can rewrite the part of the equation as:

$$\begin{aligned}
(A9) \quad \prod_{j=0}^{C-1} \mathcal{I} \left(\max_{s=0 \dots j} v_s \leq \max_{s=j+1, \dots, S} \omega_s \right) &= \prod_{j=0}^{C-1} \mathcal{I} \left(\max_{s=0 \dots j} v_s \leq \omega_{j+1} \right) = \prod_{j=0}^{C-1} \left[\prod_{s=0}^j \mathcal{I}(v_s \leq \omega_{j+1}) \right] \\
&= \prod_{j=0}^{C-1} \left[\prod_{s=0}^j \mathcal{I}(v_s \leq \omega_{j+1}) \right] = \prod_{j=0}^{C-1} \mathcal{I}(v_j \leq \omega_C)
\end{aligned}$$

Similarly, we find that:

$$\begin{aligned}
(A10) \quad \mathcal{I} \left(\max_{s=0 \dots C} v_s \geq \max_{s=C+1, \dots, S} \omega_s \right) \cdot \mathcal{I} \left(v_b \geq \max_{s=0 \dots C} v_s \right) &= \\
&= \mathcal{I} \left(v_b \geq \max_{s=C+1, \dots, S} \omega_s \right) \cdot \prod_{j=0}^C \mathcal{I}(v_j \leq v_b)
\end{aligned}$$

Finally, combining all Equations (A8-A10) and adding the returns inequality, we find the equivalent simplified form of the inequality constraints:

$$\begin{aligned}
(A11) \quad W &= \left[\prod_{j=1}^{C-1} \mathcal{I}(\omega_j \geq \omega_{j+1}) \right] \cdot \mathcal{I} \left(\omega_C \geq \max_{j=C+1, \dots, S} \omega_j \right) \cdot \left[\prod_{j=0}^{C-1} \mathcal{I}(v_j \leq \omega_C) \right] \\
&\cdot \mathcal{I} \left(v_b \geq \max_{s=C+1, \dots, S} \omega_s \right) \cdot \left[\prod_{j=0}^C \mathcal{I}(v_j \leq v_b) \right] \cdot \mathcal{I}(x'_b \beta^u + \epsilon_b \geq -R) = \\
&= \left[\prod_{j=1}^{C-1} \mathcal{I}(\omega_j \geq \omega_{j+1}) \right] \cdot \mathcal{I} \left(\omega_C \geq \max_{j=C+1, \dots, S} \omega_j \right) \cdot
\end{aligned}$$

$$\begin{aligned}
& \cdot \left[\prod_{j=0}^{C-1} \mathcal{I}(v_j \leq \min\{\omega_C, v_b\}) \right] \cdot \mathcal{I}(v_C \leq v_b) \cdot \mathcal{I}\left(v_b \geq \max_{j=C+1, \dots, S} \omega_j\right) \\
& \cdot \mathcal{I}(x'_b \beta^u + \epsilon_b \geq -R)
\end{aligned}$$

Appendix A.5. Derivation of semi-closed form likelihood

As in the previous sections, we drop the customer related index i for compactness.

Recall the set of constraints in simplified form from Equation (13)

$$\begin{aligned}
W &= \left[\prod_{j=1}^{C-1} \mathcal{I}(\omega_j \geq \omega_{j+1}) \right] \cdot \mathcal{I}\left(\omega_C \geq \max_{j=C+1, \dots, S} \omega_j\right) \cdot \\
(A12) \quad & \cdot \left[\prod_{j=0}^{C-1} \mathcal{I}(v_j \leq \min\{\omega_C, v_b\}) \right] \cdot \mathcal{I}(v_C \leq v_b) \cdot \mathcal{I}\left(v_b \geq \max_{j=C+1, \dots, S} \omega_j\right) \\
& \cdot \mathcal{I}(x'_b \beta^u + \epsilon_b \geq -R)
\end{aligned}$$

Where v_j is a function of unobservable shock ψ_j ; ω_j is a function of unobserved shock ξ_j and unobserved shock ϵ_b . To compute the likelihood, we need to integrate out all these unobserved shocks. In Equation (A13) we use the fact that all ψ_j and ξ_j are independent by assumption, while ϵ_b and ψ_b are dependent.

$$\begin{aligned}
(A13) \quad & \iiint W \cdot dF(\epsilon_b, \psi_b, \xi_b) = \iiint W \cdot \prod_{j=1, j \neq b}^C dF_{\psi_j}(\psi_j) \cdot \prod_{j=1}^S dF_{\xi_j}(\xi_j) \cdot dF_{\psi_b, \epsilon_b}(\psi_b, \epsilon_b) = \\
& = \iiint W \cdot \prod_{j=1, j \neq b}^C dF_{\psi_j}(\psi_j) \cdot \prod_{j=1}^S dF_{\xi_j}(\xi_j) \cdot dF_{\epsilon_b | \psi_b}(\epsilon_b | \psi_b) \cdot dF_{\psi_b}(\psi_b)
\end{aligned}$$

The distribution $F_{\epsilon_b | \psi_b}(\epsilon_b | \psi_b)$ is known from Appendix A.3. and thus we can integrate out the variable ϵ_b as only one constraint depends on it.

$$(A14) \quad \int \mathcal{I}(x'_b \beta^u + \epsilon_b \geq -R) \cdot F_{\epsilon_b | \psi_b}(\epsilon_b | \psi_b) = 1 - \Phi \left[-\frac{R + x'_b \beta^u + \frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_\eta^2} \psi_b}{\sqrt{\frac{\sigma_\epsilon^2 \cdot \sigma_\eta^2}{\sigma_\epsilon^2 + \sigma_\eta^2}}} \right]$$

$$= RP_b(\psi_b)$$

Next, we notice that $\{\xi_j: j = C + 1 \dots S\}$ appear only in two constraints which could be simplified to

$$\begin{aligned}
 & \iiint \mathcal{I}\left(\omega_C \geq \max_{j=C+1, \dots, S} \omega_j\right) \cdot \mathcal{I}\left(v_b \geq \max_{j=C+1, \dots, S} \omega_j\right) \prod_{j=C+1}^S dF_{\xi_j}(\xi_j) \\
 &= \iiint \mathcal{I}\left(\min\{v_b, \omega_C\} \geq \max_{j=C+1, \dots, S} \omega_j\right) \prod_{j=C+1}^S dF_{\xi_j}(\xi_j) = \\
 (A15) \quad &= \iiint \prod_{j=C+1}^S \mathcal{I}(\min\{v_b, \omega_C\} \geq \omega_j) \prod_{j=C+1}^S dF_{\xi_j}(\xi_j) \\
 &= \prod_{j=C+1}^S \int \mathcal{I}(\min\{v_b, \omega_C\} \geq \omega_j) dF_{\xi_j}(\xi_j)
 \end{aligned}$$

Notice that $\omega_j(\xi_j)$ is an invertible function for each j (Equation (5) implies that this function would be different depending on product characteristics and costs). Thus, we can compute:

$$\begin{aligned}
 & \prod_{j=C+1}^S \int \mathcal{I}(\min\{v_b, \omega_C\} \geq \omega_j(\xi_j)) dF_{\xi_j}(\xi_j) \\
 (A16) \quad &= \prod_{j=C+1}^S \int \mathcal{I}(\omega_j^{-1}(\min\{v_b, \omega_C\}) \geq \xi_j) dF_{\xi_j}(\xi_j) = \prod_{j=C+1}^S F_{\xi_j}[\omega_j^{-1}(\min\{v_b, \omega_C\})]
 \end{aligned}$$

Next, we modify constraints related to purchase decision. However, in this case we consider three separate cases: choosing the outside option, choosing the last searched option, all else:

$$(A17) \quad \left[\prod_{j=0}^{C-1} \mathcal{I}(v_j \leq \min\{\omega_C, v_b\}) \right] \cdot \mathcal{I}(v_C \leq v_b) =$$

$$\left\{ \begin{array}{ll} \left[\prod_{j=1}^{C-1} \mathcal{I}(v_j \leq \min\{\omega_c, v_0\}) \right] \cdot \mathcal{I}(v_c \leq v_0) \cdot \mathcal{I}(v_0 \leq \omega_c), & \text{if } b = 0 \\ \left[\prod_{j=1}^{C-1} \mathcal{I}(v_j \leq \min\{\omega_c, v_c\}) \right] \cdot \mathcal{I}(v_0 \leq \min\{\omega_c, v_c\}), & \text{if } b = C \\ \left[\prod_{j=1, j \neq b}^{C-1} \mathcal{I}(v_j \leq \min\{\omega_c, v_b\}) \right] \mathcal{I}(v_c \leq v_b \leq \omega_c) \cdot \mathcal{I}(v_0 \leq \min\{\omega_c, v_b\}) & \end{array} \right.$$

Notice that all other inequalities except those in Equation (A17) do not depend on the unobserved shocks $\{\psi_j: j = 1 \dots C, j \neq b\}$ and thus could be integrated out. Because $v_j(\psi_j)$ is an invertible function for each j (Equation (3) implies that this function would be different depending on product characteristics and costs). Consider integration of different cases from Equation (A17) and remember that in case the customer chose the non-outside option, we should keep the result from Equation (A14) as return probability $RP_b(\psi_b)$ depends on ψ_b .

Choice of outside option or $b = 0$

$$\begin{aligned} & \iiint \left[\prod_{j=1}^{C-1} \mathcal{I}(v_j(\psi_j) \leq \min\{\omega_c, v_0\}) \right] \cdot \mathcal{I}(v_c \leq v_0) \cdot \mathcal{I}(v_0 \leq \omega_c) \prod_{j=1}^C dF_{\psi_j}(\psi_j) \\ &= \iiint \left[\prod_{j=1}^{C-1} \mathcal{I}(\psi_j \leq v_j^{-1}(\min\{\omega_c, v_0\})) \right] \cdot \mathcal{I}(\psi_c \leq v_c^{-1}(v_0)) \\ (A18) \quad & \cdot \mathcal{I}(v_0 \leq \omega_c) \prod_{j=1}^C dF_{\psi_j}(\psi_j) = \\ &= \mathcal{I}(v_0 \leq \omega_c) \cdot \left[\prod_{j=1}^{C-1} F_{\psi_j}(v_j^{-1}(\min\{\omega_c, v_0\})) \right] \cdot F_{\psi_c}(v_c^{-1}(v_0)) \end{aligned}$$

Choice of last clicked option or $b = C$

$$\begin{aligned}
& \iiint RP_C(\psi_C) \cdot \left[\prod_{j=1}^{C-1} \mathcal{I}(v_j \leq \min\{\omega_C, v_C\}) \right] \cdot \mathcal{I}(v_0 \leq \min\{\omega_C, v_C\}) \prod_{j=1}^C dF_{\psi_j}(\psi_j) \\
&= \iiint RP_C(\psi_C) \cdot \left[\prod_{j=1}^{C-1} \mathcal{I}(\psi_j \leq v_j^{-1}(\min\{\omega_C, v_0\})) \right] \cdot \mathcal{I}(v_0 \leq v_C) \cdot \\
& \quad \cdot \mathcal{I}(v_0 \leq \omega_C) \prod_{j=1}^C dF_{\psi_j}(\psi_j) = \\
&= \mathcal{I}(v_0 \leq \omega_C) \cdot \int RP_C(\psi_C) \cdot \left[\prod_{j=1}^{C-1} F_{\psi_j}(v_j^{-1}(\min\{\omega_C, v_C\})) \right] \cdot \mathcal{I}(v_0 \leq v_C) dF_{\psi_C}(\psi_C) \\
&= \mathcal{I}(v_0 \leq \omega_C) \cdot \int_{v_C^{-1}(v_0)}^{+\infty} RP_C(\psi_C) \cdot \prod_{j=1}^{C-1} F_{\psi_j}(v_j^{-1}(\min\{\omega_C, v_C\})) dF_{\psi_C}(\psi_C)
\end{aligned}
\tag{A19}$$

Choice of other options or $0 < b < C$

$$\begin{aligned}
& \iiint RP_b(\psi_b) \cdot \left[\prod_{j=1, j \neq b}^{C-1} \mathcal{I}(v_j \leq \min\{\omega_C, v_b\}) \right] \cdot \mathcal{I}(v_C \leq v_b \leq \omega_C) \cdot \\
& \quad \cdot \mathcal{I}(v_0 \leq \min\{\omega_C, v_b\}) \prod_{j=1}^C dF_{\psi_j}(\psi_j) = \\
&= \iiint RP_b(\psi_b) \cdot \left[\prod_{j=1, j \neq b}^{C-1} \mathcal{I}(\psi_j \leq v_j^{-1}(\min\{\omega_C, v_b\})) \right] \cdot \mathcal{I}(\psi_C \leq v_C^{-1}(v_b)) \cdot \\
& \quad \cdot \mathcal{I}(v_0 \leq v_b \leq \omega_C) \cdot \mathcal{I}(v_0 \leq \omega_C) \prod_{j=1}^C dF_{\psi_j}(\psi_j) = \\
&= \mathcal{I}(v_0 \leq \omega_C) \cdot \int_{v_b^{-1}(v_0)}^{v_b^{-1}(\omega_C)} RP_b(\psi_b) \cdot \left[\prod_{j=1, j \neq b}^{C-1} F_{\psi_j}(v_j^{-1}(\min\{\omega_C, v_b\})) \right] \cdot \\
& \quad \cdot F_{\psi_C}(v_C^{-1}(v_b)) dF_{\psi_b}(\psi_b)
\end{aligned}
\tag{A20}$$

Notice that after combining Equations (A18-A20) and recalling Equations (A14) and (A16), we can rewrite the original integral in Equation (A13) as:

$$(A21) \quad \iiint W \cdot dF(\epsilon_i, \psi_i, \xi_i) = \iiint \mathcal{I}(v_0 \leq \omega_c) \prod_{j=1}^{C-1} \mathcal{I}(\omega_j \geq \omega_{j+1}) \int_{\underline{\psi}_b}^{\bar{\psi}_b} B(\xi_c, \psi_b) dF_{\psi_b}(\psi_b) \prod_{j=1}^C dF_{\xi_j}(\xi_j)$$

where $B(\cdot, \cdot)$ is a function which depends only on two unobserved shocks: ξ_c through ω_c and ψ_b through v_b .

Next, notice that only $\omega_c(\xi_c)$ depends on ξ_c , therefore, Equation (A21) could be rewritten as:

$$(A22) \quad \iiint W \cdot dF(\epsilon_i, \psi_i, \xi_i) = \int_{-\infty}^{+\infty} \int_{\underline{\psi}_b}^{\bar{\psi}_b} [\mathcal{I}(v_0 \leq \omega_c) \cdot B(\xi_c, \psi_b) \cdot D(\xi_c)] dF_{\psi_b}(\psi_b) dF_{\xi_c}(\xi_c)$$

$$D(\xi_c) = \iiint \prod_{j=1}^{C-1} \mathcal{I}(\omega_j \geq \omega_{j+1}) \cdot \prod_{j=1}^{C-1} dF_{\xi_j}(\xi_j)$$

Notice that in Equation (A22), we need to simulate only $C + 1$ random shocks in comparison with $S + C + 1$ in Equation (A12). Because typically $S \gg C$ (number of viewed products is much higher than number of clicks), this is already a large improvement. However, $\prod_{j=1}^{C-1} \mathcal{I}(\omega_j \geq \omega_{j+1})$ could have quite many sharp constraints in longer sessions, which may result in a higher chance of having zero-valued integral approximation in Equation (A22). Also, as discussed in the paper, the reservation utility $\omega_j \rightarrow +\infty$ if search costs $c_j \rightarrow \bar{c}_j$, where \bar{c}_j is an upper bound on costs and could be found from Equation (A5) by making $\theta \rightarrow -\infty$. This implies that we can consider only values of the parameters which keeps the search costs for clicked products lower than their corresponding upper bounds.

We notice that $\prod_{j=1}^{C-1} \mathcal{I}(\omega_j \geq \omega_{j+1})$ has a chain-like structure. Thus, we can sample random shocks iteratively. Let's assume we sampled some value of ξ_C^g with ξ_C^g being a realization of this random variable (thus, $\omega_C^g = \omega_C(\xi_C^g)$ also sampled). In this case, we may sample ξ_{C-1} in a way that $\omega_{C-1} \geq \omega_C^g$ (or $\xi_{C-1} \leq \min\{\omega_{C-1}^{-1}(\omega_C(\xi_C^g)), \bar{c}_{C-1}\}$ for random shock itself), however, we should adjust for probability of such event $F_{\xi_{C-1}}(\min\{\omega_{C-1}^{-1}(\omega_C(\xi_C^g)), \bar{c}_{C-1}\})$. Notice that after generating ξ_{C-1}^g , we can repeat this procedure for ξ_{C-2} and so on.

After recursively applying of the procedure discussed in the previous paragraph for each j , we obtain the random sample $(\xi_1^g, \dots, \xi_C^g)$ such that $\prod_{j=1}^{C-1} \mathcal{I}(\omega_j \geq \omega_{j+1}) = 1$ but the probability needs to be adjusted by:

$$(A23) \quad \prod_{j=1}^{C-1} F_{\xi_j}(\min\{\omega_j^{-1}(\omega_{j+1}(\xi_{j+1}^g)), \bar{c}_j\})$$

Therefore, by using the recursive shock generation we can eliminate $\prod_{j=1}^{C-1} \mathcal{I}(\omega_j \geq \omega_{j+1})$ from Equation (A21). Finally, notice that we can eliminate $\mathcal{I}(v_0 \leq \omega_C)$ from Equation (A21) by sampling ξ_C from distribution such that $v_0 \leq \omega_C$ holds and adjust the probability by $F_{\xi_C}(\min\{\omega_C^{-1}(v_0), \bar{c}_C\})$.

At the end, we summarize the procedure which we used to compute the objective function as follows:

1. Set the probability $W^g \leftarrow 1$
2. Generate random shock ξ_C^g from truncated $F_{\xi_C | \xi_C \leq \min\{\omega_C^{-1}(v_0), \bar{c}_C\}}$ and set $W^g \leftarrow W^g \cdot F_{\xi_C}(\min\{\omega_C^{-1}(v_0), \bar{c}_C\})$
3. If customer chose the non-outside option – generate random shock ψ_b^g from truncated $F_{\psi_b | \psi_b \geq \underline{\psi}_b}$ and set $W^g \leftarrow W^g \cdot (1 - F_{\psi_b}(\underline{\psi}_b))$, where $\underline{\psi}_b$ could be found from Equations (A19) and (A20)
4. Compute $B(\xi_C^g, \psi_b^g)$ from Equations (A14-A20) and set $W^g \leftarrow W^g \cdot B(\xi_C^g, \psi_b^g)$

5. Generate recursively random shocks $(\xi_1^g, \dots, \xi_{C-1}^g)$ discussed in this subsection. Set
$$W^g \leftarrow W^g \cdot \prod_{j=1}^{C-1} F_{\xi_j} \left(\min \left\{ \omega_j^{-1} \left(\omega_{j+1}(\xi_{j+1}^g) \right), \bar{c}_j \right\} \right)$$
6. If customer did not purchase the last option clicked, then set $W^g \leftarrow W^g \cdot \mathcal{I}[v_b(\psi_b^g) \leq \omega_c(\xi_c^g)]$
7. Repeat steps (1-6) G times with different random seed. The estimate of likelihood for one individual would be $\hat{P} = \sum_g W^g / G$

Appendix A.6. Additional analysis on synthetic data

In §6 we discussed the estimation on simulated data. In this subsection, we compare our method with the methods that are widely used in literature discussed in §5.4. The results of the simulations are in Table A.1.

Table A.1. Comparison with alternative estimation methods

Variable	True parameter values	Our approach	Simulated maximum likelihood	Smoothed simulated maximum likelihood	Maximizing the true likelihood
Utility: constant	-1.4	-1.347	-0.285	-2.246	-1.267
Utility: category dummy	-0.3	-0.280	0.204	1.988	-0.269
Noise variance β^η : category	-0.5	-0.443	-0.093	2.365	-0.379
Search cost: constant	-4	-3.944	-0.155	-8.666	-3.841
Search cost: page rank	0.3	0.312	0.047	0.261	0.311
Return cost: constant	-1.5	-1.427	-0.556	1.302	-1.091
Computation time, min	0	5	173	72	11
Relative memory usage	0	1	500	50	1

Table A.1 demonstrates that our method provides the best accuracy and takes the least time to converge. The main efficiency gains come from the fact that our approach allows approximating the likelihood well with a very small number of random shocks (~20) while the SML method fails to give a correct estimation even with 10,000 random shocks. The smoothed SML method provides an even worse quality for the reasons discussed in the main paper.

Next, we compare the two-stage approach in our paper with the direct maximization of the likelihood in Equation A.1. First, we observe that the true likelihood maximization recovers

parameters better than all other methods and the estimates are comparable with the true parameters. This supports the identifiability of the model. Second, we see that the parameters related to returns (3rd and 6th rows) have the worst accuracy and are shrunk towards zero. This observation supports our justification that the returns data is underweighted in the full maximization. Notice that this argument is even stronger on the real data which have longer sessions than in simulated data – this implies that the share of return constraints is even smaller. Finally, we note that by increasing the sample size, the true likelihood maximization improves the accuracy.

Appendix A.7. Description of the greedy algorithm for ranking optimization.

Let's denote Ω a list of K ordered products. Assume this list ordered randomly and without loss of generality $\Omega_1 = \{x_1, x_2, \dots, x_K\}$, where x_i characteristics of the i^{th} product in a list. Next, assume $\mathcal{V}(\cdot)$ is a function which takes as input an ordered list of products Ω and returns the number representing the “performance” of Ω . For example, in our paper $\mathcal{V}(\cdot)$ would be a procedure which simulates the behavior of customers who face some ordered list of products Ω on the website. In §8.2 the returning value of this function is the probability that the customer purchases a product and keeps it, however, any other value function is possible.

The algorithm which was used in §8.2 could be summarized as follows:

1. For each $k \in \{1 \dots K\}$ repeat:
 - a. For each $j \in \{k \dots K\}$ repeat:
 - i. Construct Ω_{jk} by switch positions of product j and k in a list Ω_k
 - ii. Compute and store $v_{jk} = \mathcal{V}(\Omega_{jk})$
 - b. Choose the index j^* which maximizes v_{jk}
 - c. Store $\Omega_k = \Omega_{j^*k}$
2. Return Ω_K as the greedy-optimal ranking

Intuitively, we iteratively (one-by-one) optimize each position of the list while taking into account that products chosen as best on previous iterations remain on the same spot. It is straightforward to show that this algorithm requires $\sim K^2$ evaluations of function $\mathcal{V}(\cdot)$ which could be very slow to compute. Thus, it provides a feasible approximation to the complex problem of finding the best ranking.

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