Matematika pro fyziky 1: The X-Files 👽

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1 ? ? ?

$$\boldsymbol{p}, \boldsymbol{q} \in \mathbb{R}^2 \qquad H = \|\boldsymbol{p}\|^2 + \|\boldsymbol{q}\|^2$$

$$\dot{q}_j = \frac{\partial H}{\partial p_j} \qquad \dot{p}_j = -\frac{\partial H}{\partial q_j} - \zeta \left(\frac{\partial H}{\partial p_1} - \frac{\partial H}{\partial p_2} \right)$$

2 ! ! !

2.1 $\boldsymbol{p}, \boldsymbol{q} = ?$

$$\boldsymbol{v} = \begin{pmatrix} \boldsymbol{q} \\ \boldsymbol{p} \end{pmatrix} = \begin{pmatrix} q_1 \\ q_2 \\ p_1 \\ p_2 \end{pmatrix} \qquad \dot{\boldsymbol{v}} = \begin{pmatrix} -\frac{\partial H}{\partial q_1} - \zeta \left(\frac{\partial H}{\partial p_1} - \frac{\partial H}{\partial p_2} \right) \\ -\frac{\partial H}{\partial q_2} - \zeta \left(\frac{\partial H}{\partial p_1} - \frac{\partial H}{\partial p_2} \right) \\ \frac{\partial H}{\partial p_1} \\ \frac{\partial H}{\partial p_2} \end{pmatrix} = \underbrace{2 \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -\zeta & \zeta \\ 0 & 1 & -\zeta & \zeta \end{pmatrix}}_{M} \boldsymbol{v}$$

$$\boxed{\dot{\boldsymbol{v}} = M\boldsymbol{v} \quad \Longrightarrow \quad \boldsymbol{v} = \exp(tM)\boldsymbol{v_0}, \quad \boldsymbol{v_0} \in \mathbb{R}^2}$$

2.2 $\exp(tM) = ?$

$$\underbrace{2t \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & -\zeta & \zeta \\ 0 & 1 & -\zeta & \zeta \end{pmatrix}}_{tM} = \underbrace{\begin{pmatrix} tz & \frac{z}{2} - 1 & -tz & \frac{z}{2} + 1 \\ tz & \frac{z}{2} & -tz & \frac{z}{2} \\ -tz & 1 & -tz & 1 \\ -tz & 0 & -tz & 0 \end{pmatrix}}_{P^{-1}} \underbrace{\begin{pmatrix} -2t & 1 & 0 & 0 \\ 0 & -2t & 0 & 0 \\ 0 & 0 & 2t & 1 \\ 0 & 0 & 0 & 2t \end{pmatrix}}_{J} \underbrace{\begin{pmatrix} 0 & \frac{1}{2tz} & -\frac{1}{4t} & \frac{z-2}{4tz} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2tz} & \frac{1}{4t} & -\frac{z+2}{4tz} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}}_{P}$$