Exercises

1. $h(x) = \begin{cases} y_i & \text{if } \exists i \in [m] \text{ s.t. } x_i = x \\ 0 & \text{otherwise.} \end{cases}$

$$S = \{(x_i, f(x_i, f(x_i))\}_{i=1}^{m}$$

we define the P_s function as follows:

$$P_s(x) = \sum_{i=1}^{m} (f(x_i) - 1) \times (x - x_i)^2$$

now we have:

$$\forall i \text{ s.t. } f(x_i) = 1 \Longrightarrow P_s(x) = 0$$

otherwise: $\quad P_s(x) < 0$

Exercise 2.

If we assume that:

$$S = \{(x_i, f(x_i))\}_{i=1}^{m}$$

now we have:

$$\mathop{E}_{S|_x \sim D^m}[L_S(h)] = \mathop{E}_{S|_x \sim D^m}\left[\frac{1}{m}\sum_{i=1}^{m} \mathbb{1}_{[h(x_i) \neq f(x_i)]}\right] =$$

$$= \frac{1}{m}\sum_{i=1}^{m} E[\mathbb{1}_{[h(x_i) \neq f(x_i)]}]$$

$$= \frac{1}{m}\sum_{i=1}^{m} P([h(x_i) \neq f(x_i)]) \text{(by i.i.d.)}$$

$$= \frac{1}{m} \times m \times L_{D,f}(h) = L_{D,f}(h)$$

(By definition)

The proof is complete.

Exercise 3.

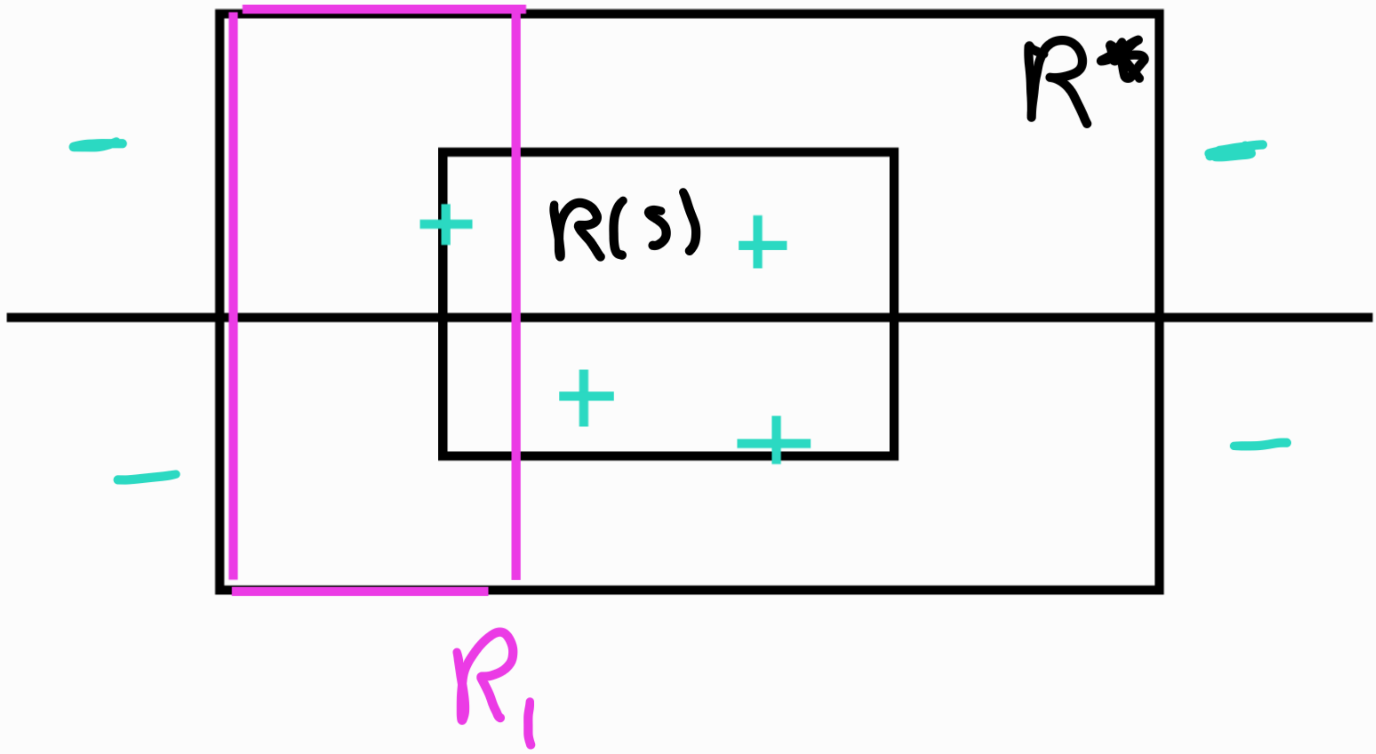
A)we have to show that A is an ERM:

$$ERM_H(S) \in \underset{h \in H}{\arg\min}\, L_S(h)$$

We assume realizability of h and A by the definition give us lable 1 if the object is in the smallest rectangle and lable 0 if the object is not in the rectangle then all the good objects is in the rectangle and bad ones not in there. Therefore A is an ERM.

B)We use hint: Assume that $R^* = (a_1^*, b_1^*, a_2^*, b_2^*)$ is the rectangle that generates the lables,and let f be the corresponding hypothesis.

$\forall S \;\; R(S) \subseteq R^*$



We assume that S contains posetive examples

in all of the rectangles R1,R2,R3,R4 and know that :

$$L_{(D,f)}(R(S)) = D(R^* - R(S))$$

We define the R1, R2,R3,R4 as in the hint.
Then we show that if we have:

$$K = \{S : L_{(D,f)}(A(S)) > \varepsilon\}$$

Then:

$$D(K) \leqslant D\Big(\bigcup_{i=1}^{4} \overbrace{\{S|_x : S|_x \cap R_i = \phi\}}^{S'_i}\Big)_x$$

$$\leqslant \sum_{i=1}^{4} D(S'_i) \quad \text{(union bound)}$$

For show that our sentence is true, we should proof that:

$$D^m(K) \leqslant \delta$$

$$\forall i \in [4] \quad D^m(S'_i) \leqslant \delta/4$$

Now we have:

$$D^m(S_i') = \left(1 - \frac{\varepsilon}{4}\right)^m \leq e^{\left(-m\frac{\varepsilon}{4}\right)}$$

Therefor:

$$D^m(K) \leq \sum_{i=1}^{4} D^m(S_i') \leq 4e^{\left(-\frac{m\varepsilon}{4}\right)}$$

(i.i.d.)

Size of our training set:

$$\boxed{4\log\left(\frac{4}{\delta}\right)/\varepsilon}$$

C) Assume that: $\begin{cases} a_1 \leqslant b_1 \\ a_2 \leqslant b_2 \\ \vdots \\ a_d \leqslant b_d \end{cases}$

Then we define the h:

$$h_{(a_1,b_1,\dots,a_d,b_d)}(x_1,\dots,x_d) = \begin{cases} 1 & \text{if } \forall i \in [d]\ a_i \leqslant x_i \leqslant b_i \\ 0 & \text{otherwise.} \end{cases}$$

And the class of rectangles in R^d will be:

$$H^d = \{ h_{(a_1,b_1,a_2,b_2,\dots,a_d,b_d)} : \forall i \in [d], a_i \leqslant b_i \}$$

This algorithm is an ERM.

Our trauning set's size : $\dfrac{2d \log(2d/\delta)}{\varepsilon}$

D)In this problem the target is to find m and we proof that the max size of our training set is:

$$M = \boxed{2d\log(2d/\delta)/\varepsilon}$$

Our runtime is: $O(md)$