Machine Learning Exercises _chapter 9

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9.1 Show the cast ERm problem of linear regression with respect to the absolute value loss function, $l(h,(x,y)) = |h(x)-y|$

$$\min_{w} \sum_{i=1}^{m} |\langle w, x_i \rangle - y_i|$$

Solution: $|c|$ is the lower bound of a so we have: $|c| = \min a$ s.t. $c \leq a \wedge c \geq -a$

So $\min \sum_{i=1}^{m} |c_i|$: minimize $\sum_{i=1}^{m} a_i$ subject to $c_i \leq a_i$

$c_i \geq -a_i$

And the linear Program for absolute loss function regression is:

$$\min \sum_{i=1}^{m} a_i \quad \text{subject to} \quad \langle w, x_i \rangle - a_i \leq y_i$$

At last the obj is: $\sum_{i=1}^{m} a_i = [0\,0 \ldots 0 \ldots 1\,1 \ldots 1]\,[w_1 \ldots w_d\, a_1 \ldots a_d]^T$

9.3 we assume that $R = \max \|x_i\| = \max \|e_i\| = 1$ so we have

$$\forall i \in [m] \text{ s.t. } \langle w, x_i \rangle = \langle w, e_i \rangle = w_i \xrightarrow[w^* = [1, \dots, 1]]{\langle w, x_i \rangle = 1} \|w^*\|^2 = \langle w^*, w^* \rangle = \sum 1^2 = m$$

$T \leq (RB)^2$, we know this and $B \leq \sqrt{m}$ so we have:

$$T \leq (BR)^2 = m$$

We run the perceptron at step t $(w^{(0)} = [0, \dots, 0]^T)$:

$$w^{(t)} = w^{(t-1)} + e_t = \sum e_i \quad \text{so:}$$

$$\forall i \ \langle w^{(t)}, x_t \rangle = \langle w^{(t)}, e_t \rangle = 1 \longrightarrow i \leq t$$

hence at step m the algorithm stops.

$\square$

9.1 Show the cast ERm problem of linear regression with respect to the absalute value loss function, $\ell(h,(x,y)) = |h(x)-y|$

$$\min_{w} \sum_{i=1}^{m} |\langle w, x_i\rangle - y_i|$$

Solution: $|c|$ is the lower bound of a so we have: $|c| = \min a$ s.t. $c \leq a$ $\wedge$ $c \geq -a$

So $\min \sum_{i=1}^{m} |c_i|$: minimize $\sum_{i=1}^{m} a_i$ subject to $c_i \leq a_i$

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And the linear Program for absolute loss function regression is:

$$\min \sum_{i=1}^{m} a_i \quad \text{subject to} \quad \langle w, x_i\rangle - a_i \leq y_i$$

At last the obj is: $\sum_{i=1}^{m} a_i = [0\,0 \dots 0 \dots 1\,1 \dots 1]\,[w_1 - w_d\, a_1 \dots a_d]^T$

9.3 we assume that $R = \max \|x_i\| = \max \|e_i\| = 1$ so we have

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hence at step m the algorithm stops.

$\square$