Question 1.
What is the complexity of Insertion sort for an input scenario where two numbers inside every consecutive pair of a sorted array are switched. For example, A = [7, 6, 9, 8, 11, 10, 12, 11, …, a_j, a_{j-1}, …, a_n, a_{n-1}].

Question 2.
Every patient must get an appointment beforehand to go to the Akdeniz University hospital on a particular day. Appointed patients for each day are given distinct patient numbers starting from 1. Hence, the first appointed patient gets the patient number 1, the second appointed patient gets 2 and so on. Unfortunately, the patients come to their appointments in a totally random order. Also, some patients do not come to their appointments. Assume that m patients got patient numbers on a particular day and n<m of them came to their appointment. These patients' patient numbers are stored in an n-dimensional array A ordered with respect to their arrival times (i.e. not their patient numbers).
Give an O(n) average-time complexity algorithm to find the patient with the lowest patient number who did not show up for his/her appointment on that day. For example, let m=20 and n=11 with A=[16,13,9,5,4,13,1,6,7,8,2].
Then, the algorithm must return 3.
Hint: Divide-and-conquer.

Question 3.
Let f(n) ≥ 0, g(n) ≥ 0 be two functions, and a > 0 be a constant. State whether the statements below are true or not. For each statement that you think is true, show that it is true with a formal argument based on the definitions of asymptotic notations. For false statements, give a counter-example.

(a) f(2n)=Θ(f(n)).

False. Counter-example: Choose $f(n)=2^n$

(b) If $f(n)=O(n^a)$, then $f(2n)=O(n^a)$.

True. Formal argument: By $f(n) = O(n^a)$, there exists a c such that $f(n)<=cn^a$ for all $n>=n_0$. Let's define a new variable z=2n. Since z=2n>n, $f(z=2n)<=cz^a$ for all $z>=2n_0$.

(c) If f(n) = O(g(n)), log(f(n)) ≥ 0 and log(g(n)) ≥ 0, then log(f(n)) = O(log(g(n)))

False. Counter-example: Choose f(n)=2, g(n)=1.

(d) If f(n)≥1, then f(n)+a =O(f(n)).

True. Formal argument: We need to find positive constants c and n_0 such that f(n)+a<=cf(n) for all $n>=n_0$. Choose c=a+1. Then, we need to show if a<=af(n) is really true. Yes, due to the given condition f(n)≥1.