Algorithm Analysis Fall 2022

MIDTERM

Due Date and Instructions

Please submit a video explaining your solutions till Tuesday, November 15, 2022.

Problem 1 [30 pts]

Using your favorite programming or scripting language, implement INSERTION-SORT and MERGE-SORT. Then, using identical platforms for the two algorithms, answer the following questions:

(a) Consider instance sizes $n = 2^4, 2^8, 2^{12}, 2^{16}, 2^{20}$. Generate 10 random instances (such that either the numbers in the array are randomly generated, or the ordering of the numbers is randomized). Plot the runtime of each algorithm as a function of n (you need to use error bars to visualize the distribution of runtimes for different random instances). At what value of n does MERGE-SORT become more efficient than INSERTION-SORT? Which algorithm has a more variable runtime for different instances that have the same size?

(b) For each value of n considered above, generate an instance that represents the worst case for INSERTION-SORT. Repeat the same analysis (in this case, you do not need error bars since you have a single instance and both algorithms are deterministic, but running the algorithms multiple times and reporting averages can be more reliable to account for other factors). What is the critical value of n in this case?

Problem 2 [30 pts]

The YouTube video at http://www.youtube.com/watch?v=ywWBy6J5gz8 shows Hungarian Küküllőmenti legényes folk dancers implementing a version of QUICKSORT. Use this video to answer the following questions:

(a) Write down the pseudo-code for the PARTITION procedure implemented by the folk dancers.

(b) Prove the correctness of your algorithm using the method of loop invariants and analyze its running time.

Problem 3 [15 pts]

Considering functions $f(n) \geq 0$, $g(n) \geq 0$, and constant $c > 0$, indicate whether each of the following statements is true. Prove the statements that are true by providing a formal argument that is based on the definition of asymptotic notation. For statements that are false, provide a counter-example to prove that they are false.

(a) $\max\{f(n), g(n)\} = \Theta(f(n) + g(n))$.

(b) $f(n) + c = O(f(n))$.

(c) If $f(n) \geq 1$, then $f(n) + c = O(f(n))$.

Problem 4 [15 pts]

Let $0 < \epsilon < 1 < a < b$ be constants. Solve the following recurrences using Master Method, noting the case that applies.

(a) [5 pts] $T(n) = bT(n/a) + \Theta(n)$.

(b) [5 pts] $T(n) = a^2T(n/a) + \Theta(n^2)$.

(c) [5 pts] $T(n) = T(\epsilon n) + n^\epsilon$.

Problem 5 [10 pts]

Prove the solution to the following recurrence using Substitution Method:

For any constant $k > 0$, if $T(n) = \Theta(n) + \sum_{i=1}^{k} T(n/2^i)$, then $T(n) = O(n)$.