Semesteroppgave

CrazyFlie triangulation network

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Contents

1 Introduction

[1] [2]

2 Background

3 System Description

3.1 Notation

A set $\mathcal{S}$ of points $\mathbf{x} \in \mathbb{R}^2$ is defined by it's boundary, $\delta\mathcal{S}$, and it's interior, $\mathrm{int}(\mathcal{S})$. The number of elements in a set $\mathcal{S}$ is denoted as $|\mathcal{S}|$.
The $Comb(\cdot)$ operator takes as arguments a set $\mathcal{S}$ and an integer n and returns all subset of $\mathcal{S}$ of lenght n:

$$Comb(\mathcal{S}, n) = \{\mathcal{A} : \mathcal{A} \subseteq \mathcal{S}, |\mathcal{A}| = n\} \tag{1}$$

3.2 Feasible space

As in [2], a *mission space*, Ω, is defined as a simple polygon [3]. Within the mission space there exists $N_o \geq 0$ obstacles, each one of which is defined as a simple polygon. The set of all obstacles, $\mathcal{O}$, is defined according to (2).

$$\mathcal{O} = \begin{cases} \{o_0 \dots o_{N_o - 1}\} & , N_o > 0 \\ \emptyset & , N_o = 0 \end{cases} \tag{2}$$

The obstacles in $\mathcal{O}$ constrains the movement of entities within the mission space, as it is not possible to penetrate the boundary of an obstacle. Due to this, once an entity is inside Ω, it is constrained to be positioned within Ω and outside $\mathrm{int}(o)\ \forall\ o \in \mathcal{O}$. From this we define the *feasible space*, $\mathcal{F}$, as all points where it is possible to place an entity:

$$\mathcal{F} = \{\mathbf{x} \in \mathbb{R}^2 : \mathbf{x} \in \Omega,\ \mathbf{x} \notin \mathrm{int}(o)\ \forall\ o \in \mathcal{O}\} = \Omega \setminus \bigcup_{o \in \mathcal{O}} \mathrm{int}(o) \tag{3}$$

3.3 Agent

An *agent*, denoted by an integer i, is described by its position $\mathbf{s}_i \in \mathbb{R}^2$ and it's maximum radius of communication, r_i. The probability of an agent i being able to communicate with another entity positioned at a point $\mathbf{x}$ is defined according to:

Find notation for $\mathbb{R}^5$ with non-negative values in last dimension

$$\hat{p} : (\mathbb{R}^2, \mathbb{R}^2, \mathbb{R}^+) \to [0, 1] \quad \hat{p}(\mathbf{s}_i, \mathbf{x}, r_i) = \begin{cases} p(\|\mathbf{s}_i - \mathbf{x}\|) & , \mathbf{x} \in V(\mathbf{s}_i, r_i) \\ 0 & , \mathbf{x} \in \mathbb{R}^2 \setminus V(\mathbf{s}_i, r_i) \end{cases} \tag{4}$$

Where $V(\mathbf{s}_i, r_i)$ denotes the *visible set* of agent i. Assuming line-of-sight (LoS) communication, meaning an agent cannot communicate with an entity if there is an obstacle or a mission space wall between them, the visible set of agent i is defined in (5).

$$V(\mathbf{s}_i, r_i) = \{\mathbf{x} \in \mathbb{R}^2 : \|\mathbf{s}_i - \mathbf{x}\| \leq r_i, \lambda\mathbf{x} + (1 - \lambda)\mathbf{s}_i \in \mathcal{F}\ \forall\ 0 \leq \lambda \leq 1\} \tag{5}$$

An example of the visible set for an agent is show in Figure 1.

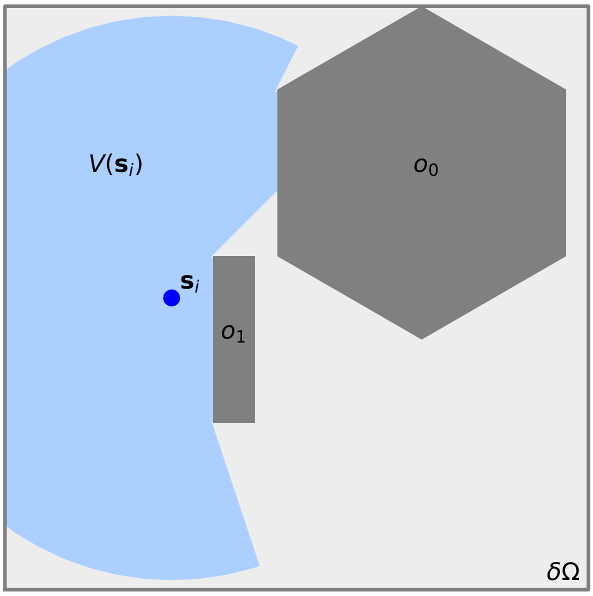

Figure 1: Visible set (light blue) for the agent placed at $\mathbf{s}_i$ in a rectangular mission space Ω with two obstacles ($\mathcal{O} = \{o_0, o_1\}$)

3.4 Swarm

A *swarm* consists if N discinct agents, each one denoted by an integer $i \in \mathcal{N} = \{0 \dots N-1\}$. The state of the swarm is described by the state of its participants , and is expressed as vector $\mathbf{S} \in \mathbb{R}^{2N}$ as shown in (6).

$$\mathbf{S} = \begin{bmatrix} \mathbf{s}_0 \\ \vdots \\ \mathbf{s}_{N-1} \end{bmatrix} \tag{6}$$

The maximum radii of communication of the swarm are represented as the vector $\mathbf{r} \in \mathbb{R}^N$ as shown in (7).

$$\mathbf{r} = \begin{bmatrix} r_0 & \dots & r_{N-1} \end{bmatrix}^T \tag{7}$$

Assuming that the distributions for all agents are independent lets us use (7) in [1] to express the probability of n members in the swarm being able to communicate with an entity at a point $\mathbf{x}$:

$$P(n, \mathbf{x}, \mathbf{S}, \mathbf{r}) = \sum_{A \in Comb(\mathcal{N}, n)} \left(\prod_{i \in \mathcal{A}} \hat{p}(\mathbf{s}_i, \mathbf{x}, r_i) \right) \left(\prod_{i \in \mathcal{N} \setminus \mathcal{A}} 1 - \hat{p}(\mathbf{s}_i, \mathbf{x}, r_i) \right) \tag{8}$$

4 Problem formulation

4.1 Trilateration

Given three agents with know positions $\mathbf{s}_i \in \mathbb{R}^2,\ i = 0, 1, 2$, the location of a beacon, denoted by $\mathbf{x}$, can be determined as follows:

1. The beacon pings agents at $\mathbf{s}_i,\ i = 0, 1, 2$ and starts three timers $t_i,\ i = 0, 1, 2$.
2. When the agents receive the ping, they instantly respond with a packet containing $\mathbf{s}_i$.
3. When receiving the packet from agent i, the beacon stops timer t_i and calculates the distance from itself to agent i: $d_i = \frac{1}{2}ct_i$, where c is the speed of light. The factor $\frac{1}{2}$ is due to the signal traveling two times the distance between the beacon and agent i (the ping travels from the beacon to the agent, and the packet sent by the agent travels back again).
4. Based on the distances $d_i,\ i = 0, 1, 2$ and the positions of the agents $\mathbf{s}_i,\ i = 0, 1, 2$ the beacon can determine its position by calculating the point where circles centered at $\mathbf{s}_i,\ i = 0, 1, 2$ with radii $d_i,\ i = 0, 1, 2$ intersect.

4.2 Coverage

5 Implementation

6 Simulation Results

7 Discussion

8 Future Work

References

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[3] E. W. Weisstein, "Simple polygon. From MathWorld–A Wolfram Web Resource," last visited on 10/11/2020. [Online]. Available: https://mathworld.wolfram.com/SimplePolygon.html