Semesteroppgave

CrazyFlie trilateration network

Magnus Berdal

Contents

1 Introduction

INCLUDE :

- WHAT IS THE SOCIETAL NEED?
- " " " TECHNOLOGICAL NEED?
- WHY WAS THIS STUFF NOT DONE BEFORE?
- HOW DO YOU INTEND TO CONTRIBUTE?

2 Background

- BE SURE TO CONNECT IT WITH THE TECHNOLOGICAL ADVANCES THAT MADE DRONES A USABLE TECHNOLOGY

- NOTATION: MAYBE MAKE A SECTION "NOTATION" SO THAT READERS CAN USE IT AS A REFERENCE (THUS PROMOTE 3.1 TO A SECTION AND MAKE IT INCLUDE ALL THE SYMBOLS)

3 System Description

3.1 Notation

A set $\mathcal{S}$ of points $\mathbf{x} \in \mathbb{R}^2$ is defined by it's boundary, $\delta\mathcal{S}$, and it's interior, $\text{int}(\mathcal{S})$. The number of elements in a set $\mathcal{S}$ is denoted as $|\mathcal{S}|$.
The $Comb(\cdot)$ operator takes as arguments a set $\mathcal{S}$ and an integer n and returns all subset of $\mathcal{S}$ of lenght n:

$$Comb(\mathcal{S}, n) = \{\mathcal{A} : \mathcal{A} \subseteq \mathcal{S}, |\mathcal{A}| = n\} \tag{1}$$

3.2 Feasible space

As in [1], a *mission space*, Ω, is defined as a simple polygon [2]. Within the mission space there exists $N_o \geq 0$ obstacles, each one of which is defined as a simple polygon. The set of all obstacles, $\mathcal{O}$, is defined according to (2).

$$\mathcal{O} = \begin{cases} \{o_0 \dots o_{N_o-1}\} & , N_o > 0 \\ \emptyset & , N_o = 0 \end{cases} \tag{2}$$

The obstacles in $\mathcal{O}$ constrains the movement of entities within the mission space, as it is not possible to penetrate the boundary of an obstacle. Due to this, once an entity is inside Ω, it is constrained to be positioned within Ω and outside $\text{int}(o)\ \forall\ o \in \mathcal{O}$. From this we define the *feasible space*, $\mathcal{F}$, as all points where it is possible to place an entity:

$$\mathcal{F} = \{\mathbf{y} \in \mathbb{R}^2 : \mathbf{y} \in \Omega,\ \mathbf{y} \notin \text{int}(o)\ \forall\ o \in \mathcal{O}\} = \Omega \setminus \bigcup_{o \in \mathcal{O}} \text{int}(o) \tag{3}$$

3.3 Agent

INDEX

An *agent*, denoted by ~~an integer~~ a, is defined by its position $\mathbf{x}_a \in \mathbb{R}^2$ and its maximum range of communication r_a. From this we define the communication disk of agent a:

$$D_a = \{\mathbf{y} \in \mathbb{R}^2 : \|\mathbf{x}_a - \mathbf{y}\| \leq r_a\} \tag{4}$$

Assuming line-of-sight (LoS) communication, meaning an agent cannot communicate with an entity if there is an obstacle or a mission space wall between them, we define the *visible set* of agent a:

$$V(\mathbf{x}_a, r_a) = \{\mathbf{y} \in \mathbb{R}^2 : \mathbf{y} \in D_a, \lambda\mathbf{y} + (1-\lambda)\mathbf{x}_a \in \mathcal{F}\ \forall\ 0 \leq \lambda \leq 1\} \tag{5}$$

The counterpart to the visible set, called the invisible set of agent a, is simply defined as:

$$V_c(\mathbf{x}_a, r_a) = \mathcal{F} \setminus V(\mathbf{x}_a, r_a) \tag{6}$$

An example of the visible set for an agent is show in Figure 1.

The probability of an agent a being able to communicate with another entity positioned at a point $\mathbf{y}$, from now on called the local probability of agent a, is defined according to:

Find notation for $\mathbb{R}^5$ with non-negative values in last dimension

$$\hat{p} : (\mathbb{R}^2, \mathbb{R}^2, \mathbb{R}^+) \to [0, 1] \quad \hat{p}(\mathbf{x}_a, r_a, \mathbf{y}) = \begin{cases} p(\|\mathbf{x}_a - \mathbf{y}\|) > 0 & , \mathbf{y} \in V(\mathbf{x}_a, r_a) \\ 0 & , \mathbf{y} \in V_c(\mathbf{x}_a, r_a) \end{cases} \tag{7}$$

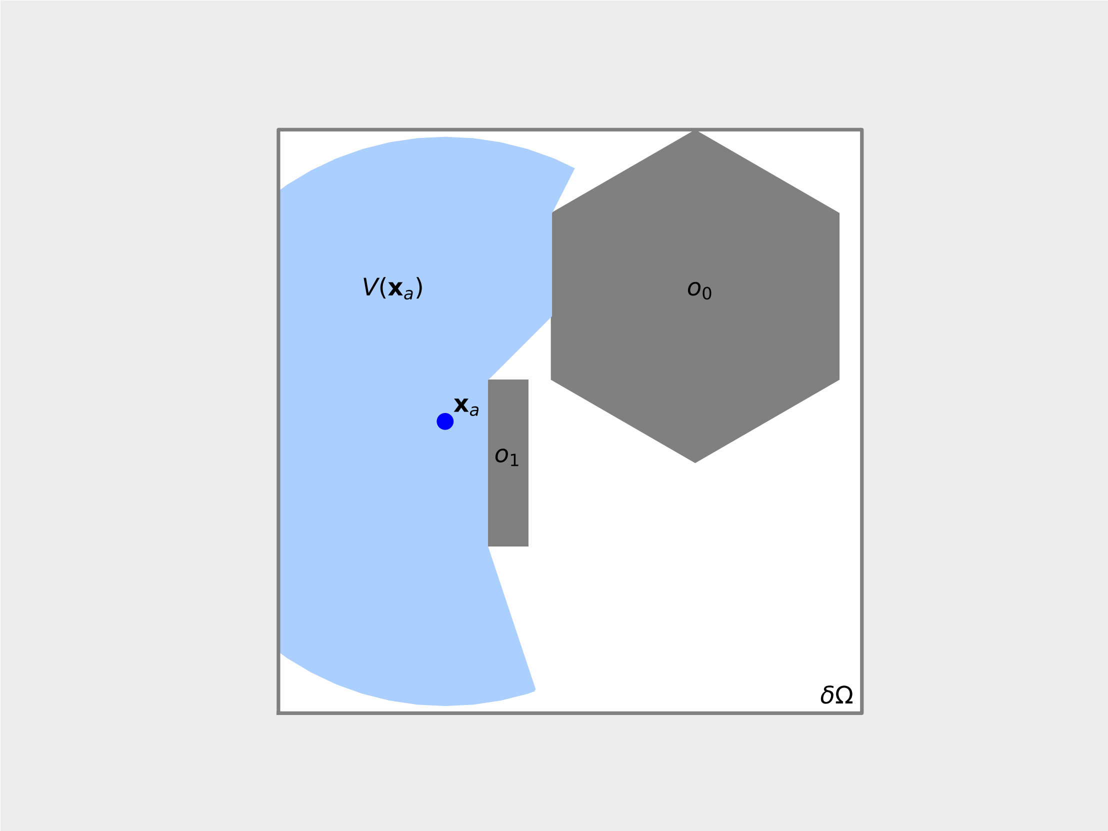

Figure 1: Visible set (light blue) for the agent placed at $\mathbf{x}_a$ in a rectangular mission space Ω with two obstacles ($\mathcal{O} = \{o_0, o_1\}$)

Update figure: $\mathbf{s}_i$ should be $\mathbf{x}_a$

3.4 Swarm

A *swarm*, $\mathcal{S}$ of size N is a set of agents $\{a_0 \dots a_{N-1}\}$. The state of the swarm is described by the state of its participants , and is expressed as vector $\mathbf{x}_\mathcal{S} \in \mathbb{R}^{2N}$ as shown in (8).

$$\mathbf{x}_\mathcal{S} = \begin{bmatrix} \mathbf{x}_{a_0} \\ \vdots \\ \mathbf{x}_{a_{N-1}} \end{bmatrix} \tag{8}$$

The maximum radii of communication of the swarm are represented as the vector $\mathbf{r} \in \mathbb{R}^N$ as shown in (9).

$$\mathbf{r}_\mathcal{S} = \begin{bmatrix} r_{a_0} & \dots & r_{a_{N-1}} \end{bmatrix}^T \tag{9}$$

Assuming that the distributions for all agents in the swarm are independent lets us use (7) in [3] to express the probability of n members in the swarm, $\mathcal{S}$, being able to communicate with an entity at a point $\mathbf{y}$:

$$\Phi_{\mathcal{S}}^{n}(\mathbf{y}) = Pr(N_{com}(\mathcal{S},\mathbf{y}) = n) = \sum_{A \in Comb(\mathcal{S},n)} \prod_{a \in \mathcal{A}} \hat{p}(\mathbf{x}_a, \mathbf{y}, r_a) \prod_{a \in \mathcal{S} \setminus \mathcal{A}} \big(1 - \hat{p}(\mathbf{x}_a, \mathbf{y}, r_a)\big) \tag{10}$$

Later we will use the probability of *at least* n members in a swarm beign able to communicate with an entity placed at $\mathbf{y}$, which is defined as:

$$\begin{aligned} \Phi_{\mathcal{S}}^{n^+}(\mathbf{y}) = Pr(N_{com}(\mathcal{S},\mathbf{y}) \geq n) = 1 - \sum_{i=0}^{n-1} Pr(N_{com}(\mathcal{S},\mathbf{y}) = i) \\ = 1 - \sum_{i=0}^{n-1} \Phi_{\mathcal{S}}^{i}(\mathbf{y}) \end{aligned} \tag{11}$$

4 Problem formulation

4.1 Multilateration

Multilateration is the process of determining the positions of unknown points in space by measurements of distances from known points [4]. In order to perform this task in two-dimensional space, at least three known points are needed.

Given $n \geq 3$ agents located at nonlinear positions $\mathbf{x}_a \in \mathbb{R}^2$, $0 \leq a < n$ the location of an entity, denoted by $\mathbf{y} \in \mathbb{R}^2$, can be determined as follows:

1. The entity broadcasts signal and starts a timer at t_0.
2. Agents at $\mathbf{x}_a$ receives broadcasted signal and immediately responds with a packet containing $\mathbf{x}_a$.
3. When receiving the packet from agent a, the entity stores the time of reception in a variable $t_{1,a}$.
4. When at least 3 agents have responded, the entity calculates the distance from itself to agent a: $d_a = \frac{1}{2}c(t_{1,a} - t_0)$, where c is the speed of light. The factor $\frac{1}{2}$ is due to the signal traveling two times the distance between the entity and agent a (the ping travels from the entity to the agent, and the packet sent by the agent travels back again).
5. Based on the distances, d_a, and the positions of the agents, $\mathbf{x}_a$, the entity can determine its position by calculating the point where circles centered at $\mathbf{x}_a$ with radii d_a intersect.

As the pings sent by the entity that is to be located travel at the speed of light, sufficient spread of the agents is necessary to ensure accurate locating. This is due to the resolution of the internal clock of the entity setting a bound on accuracy of time measurements, and thus distance measurements. Increasing the distance between agents causes the distance traveled by the pings to differ more, and thus decreasing the probability of mutliple pings returning to the entity within the same clock cycle. Figure 2 shows how the position of an entity can be determined from the known positions of 3 agents.

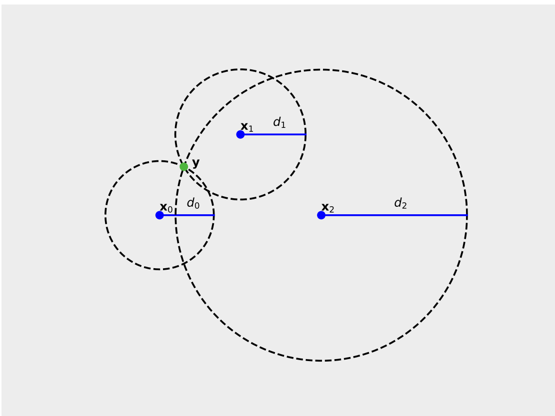

Figure 2: Position, $\mathbf{y}$, of entity determined by trilateration using known positions of $n = 3$ agents.

mention how spread of agents is benefitial

4.2 Coverage

As the goal of the swarm is to set up a network of agents in order to deliver precise positional data to entities entering the mission space, it is clear from the previous section that three or more agents are needed to perform the task. Hence a point $\mathbf{y} \in \mathcal{F}$ is said to be *covered* iff. it is within communication range of at least three agents.

4.3 Objective function derivation

The objective function presented here is inspired by [1], but differs in that for the purpose of multilateration, it is required that at least three agents must be within range of a point in order for the point to be covered.
We assume we have a total of N agents in a swarm $\mathcal{N}$ at our disposal. Furthermore it is assumed that all agents have homogenous local probability with respect to r_a:

$$\begin{aligned} r_a &= r \; \forall \; a \in \mathcal{N} \\ \tilde{V}(\mathbf{x}_a) &= V(\mathbf{x}_a, r) \\ \tilde{V}_c(\mathbf{x}_a) &= V_c(\mathbf{x}_a, r) \\ \tilde{p}(\mathbf{x}_a, \mathbf{y}) &= \hat{p}(\mathbf{x}_a, \mathbf{y}, r) \end{aligned} \tag{12}$$

The probability of a point $\mathbf{y}$ being covered by $\mathcal{N}$ can now be expressed as:

$$\Phi_{\mathcal{N}}^{3^+}(\mathbf{y}) = 1 - \Phi_{\mathcal{N}}^{0}(\mathbf{y}) - \Phi_{\mathcal{N}}^{1}(\mathbf{y}) - \Phi_{\mathcal{N}}^{2}(\mathbf{y}) \tag{13}$$

In order to formulate a distributed optimization algorithm, we rewrite the coverage supplied by $\mathcal{N}$ with focus on a single drone a. We partition the swarm, $\mathcal{N}$, into two disjoint sets: $\{a\}$ and $\mathcal{N} \setminus \{a\}$. Using this we can rewrite (13) as:

$$\begin{aligned} \Phi_{\mathcal{N}}^{3^+}(\mathbf{y}) = 1 & \\ & - \big(1 - \tilde{p}(\mathbf{x}_a, \mathbf{y})\big) \prod_{k \in \mathcal{N} \setminus \{a\}} \big(1 - \tilde{p}(\mathbf{x}_k, \mathbf{y})\big) \\ & - \tilde{p}(\mathbf{x}_a, \mathbf{y}) \prod_{k \in \mathcal{N} \setminus \{a\}} \big(1 - \tilde{p}(\mathbf{x}_k, \mathbf{y})\big) \\ & - \big(1 - \tilde{p}(\mathbf{x}_a, \mathbf{y})\big) \sum_{j \in \mathcal{N} \setminus \{a\}} \tilde{p}(\mathbf{x}_j, \mathbf{y}) \prod_{k \in \mathcal{N} \setminus \{a\} \setminus \{j\}} \big(1 - \tilde{p}(\mathbf{x}_k, \mathbf{y})\big) \\ & - \tilde{p}(\mathbf{x}_a, \mathbf{y}) \sum_{j \in \mathcal{N} \setminus \{a\}} \tilde{p}(\mathbf{x}_j, \mathbf{y}) \prod_{k \in \mathcal{N} \setminus \{a\} \setminus \{j\}} \big(1 - \tilde{p}(\mathbf{x}_k, \mathbf{y})\big) \\ & - \big(1 - \tilde{p}(\mathbf{x}_a, \mathbf{y})\big) \sum_{\mathcal{A} \in Comb(\mathcal{N} \setminus \{a\}, 2)} \prod_{j \in \mathcal{A}} \tilde{p}(\mathbf{x}_j, \mathbf{y}) \prod_{k \in \mathcal{N} \setminus \{a\} \setminus \mathcal{A}} \big(1 - \tilde{p}(\mathbf{x}_k, \mathbf{y})\big) \\ & = 1 \\ & - \prod_{k \in \mathcal{N} \setminus \{a\}} \big(1 - \tilde{p}(\mathbf{x}_k, \mathbf{y})\big) \\ & - \sum_{j \in \mathcal{N} \setminus \{a\}} \tilde{p}(\mathbf{x}_j, \mathbf{y}) \prod_{k \in \mathcal{N} \setminus \{a\} \setminus \{j\}} \big(1 - \tilde{p}(\mathbf{x}_k, \mathbf{y})\big) \\ & - \big(1 - \tilde{p}(\mathbf{x}_a, \mathbf{y})\big) \sum_{\mathcal{A} \in Comb(\mathcal{N} \setminus \{a\}, 2)} \prod_{j \in \mathcal{A}} \tilde{p}(\mathbf{x}_j, \mathbf{y}) \prod_{k \in \mathcal{N} \setminus \{a\} \setminus \mathcal{A}} \big(1 - \tilde{p}(\mathbf{x}_k, \mathbf{y})\big) \end{aligned} \tag{14}$$

Applying (10) to (14) yields:

$$\begin{aligned}\Phi_{\mathcal{N}}^{3^+}(\mathbf{y}) &= 1 - \Phi_{\mathcal{N}\setminus\{a\}}^{0}(\mathbf{y}) - \Phi_{\mathcal{N}\setminus\{a\}}^{1}(\mathbf{y}) - \Phi_{\mathcal{N}\setminus\{a\}}^{2}(\mathbf{y})\big(1 - \tilde{p}(\mathbf{x}_a, \mathbf{y})\big) \\ &= \Phi_{\mathcal{N}\setminus\{a\}}^{3^+}(\mathbf{y}) + \Phi_{\mathcal{N}\setminus\{a\}}^{2}(\mathbf{y})\tilde{p}(\mathbf{x}_a, \mathbf{y})\end{aligned} \tag{15}$$

Rewriting (15) yields:

$$\begin{aligned}\Phi_{\mathcal{N}}^{3^+}(\mathbf{y}) &= \Phi_{\mathcal{N}\setminus\{a\}}^{3^+}(\mathbf{y}) - \tilde{p}(\mathbf{x}_a, \mathbf{y})\Phi_{\mathcal{N}\setminus\{a\}}^{3^+}(\mathbf{y}) + \tilde{p}(\mathbf{x}_a, \mathbf{y})\Phi_{\mathcal{N}\setminus\{a\}}^{2^+}(\mathbf{y}) \\ &= (1 - \tilde{p}(\mathbf{x}_a, \mathbf{y}))\Phi_{\mathcal{N}\setminus\{a\}}^{3^+}(\mathbf{y}) + \tilde{p}(\mathbf{x}_a, \mathbf{y})\Phi_{\mathcal{N}\setminus\{a\}}^{2^+}(\mathbf{y})\end{aligned} \tag{16}$$

It is clear that the probability of the point $\mathbf{y}$ being covered can be seen on as an interpolation between two probability measures with the local probability of agent a as the interpolation variable. Low local probability of agent a means that the probability of coverage supplied the swarm as a whole depends more on the coverage supplied by the swarm excluding agent a. In the extreme case where the local probability of agent a is zero, the probability of covering $\mathbf{y}$ depends only on the coverage supplied by the swarm excluding agent a.

Higher local probability of agent a means that the contribution of a towards covering the point $\mathbf{y}$ is greater, thus less weight is put on the the probability of the swarm excluding agent a covering the point. Instead more weight is put on the probability of at least *two* other agents being able to communicate with an entity at $\mathbf{y}$. This is due to the fact that the probability of agent a being able to communicate with said entity is higher, and we need only two or more other agents to be able to communicate with the entity at $\mathbf{y}$ to make the total number of agents covering $\mathbf{y}$ three or more.

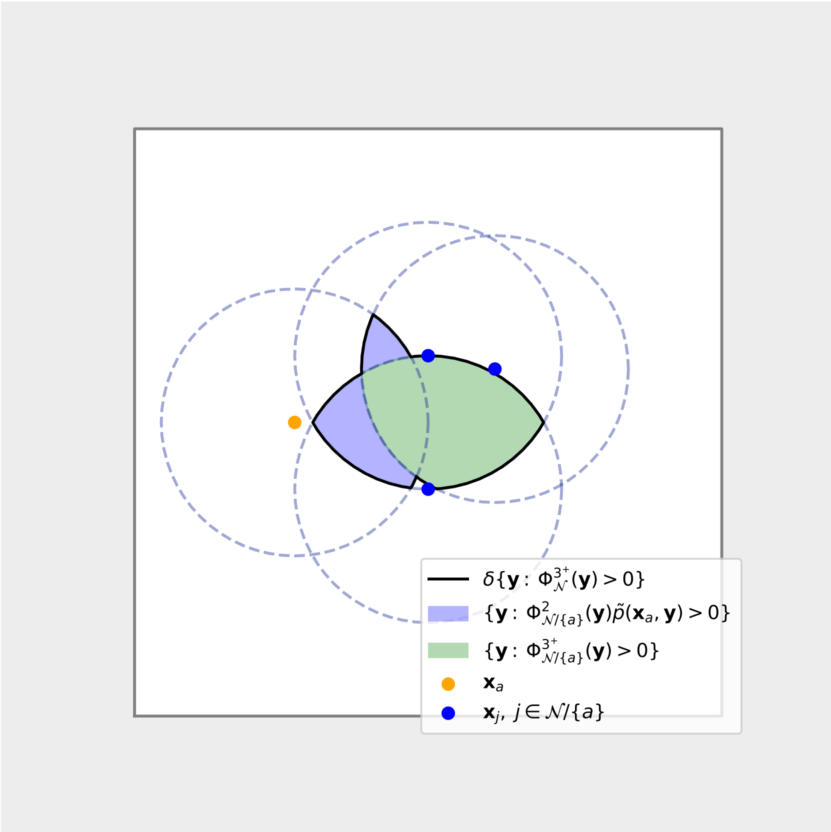

Figure 3: Non-zero regions of integrads in (17). Note that perturbing the orange circle (position of agent a) does not affect the green region, as it is defined only by the intersections of the disks surrounding the blue points (other agents in the swarm).

As the goal of the swarm is to cover as large a portion of $\mathcal{F}$ as possible, we define the global objective function as:

$$\begin{aligned} H(\mathbf{x}_\mathcal{N}) &= \int_\mathcal{F} \Phi^{3+}_\mathcal{N}(\mathbf{y})d\mathbf{y} = \int_\mathcal{F} \Phi^{3+}_{\mathcal{N}\setminus\{a\}}(\mathbf{y}) + \Phi^{2}_{\mathcal{N}\setminus\{a\}}(\mathbf{y})\tilde{p}(\mathbf{x}_a, \mathbf{y})d\mathbf{y} \\ &= \int_\mathcal{F} \Phi^{3+}_{\mathcal{N}\setminus\{a\}}(\mathbf{y})d\mathbf{y} + \int_\mathcal{F} \Phi^{2}_{\mathcal{N}\setminus\{a\}}(\mathbf{y})\tilde{p}(\mathbf{x}_a, \mathbf{y})d\mathbf{y} \end{aligned} \tag{17}$$

We note that the first term in (17) is independent of the position of a in both its domain and integrand. This independence is visualized in Figure 3. Thus we can rewrite the objective function as:

$$H(\mathbf{x}_\mathcal{N}) = \tilde{H}(\mathbf{x}_{\mathcal{N}\setminus\{a\}}) + H_a(\mathbf{x}_\mathcal{N}) \tag{18}$$

Where the *local* objective of agent a is defined as:

$$H_a(\mathbf{x}_\mathcal{N}) = \int_\mathcal{F} \Phi^{2}_{\mathcal{N}\setminus\{a\}}(\mathbf{y})\tilde{p}(\mathbf{x}_a, \mathbf{y})d\mathbf{y} \tag{19}$$

As in [1] we note that from the viewpoint of agent a, the swarm can be partitioned into three disjoint sets: $\{a\}$, $\mathcal{B}_a$ and $\mathcal{C}_a$. The latter sets are defined as:

$$\mathcal{B}_a = \{j \in \mathcal{N} \setminus \{a\} : \|\mathbf{x}_a - \mathbf{x}_j\| \leq r\} \tag{20a}$$

$$\mathcal{C}_a = \{j \in \mathcal{N} \setminus \{a\} : \|\mathbf{x}_a - \mathbf{x}_j\| > r\} \tag{20b}$$

The set $\mathcal{B}_a$, from now on called the neighbours of a, contains all agents in the swarm, $\mathcal{N}$, whose communication disks form a non-empty intersection with that of a. $\mathcal{C}_a$ contains all agents whose communication disks do not intersect with that of a.

Applying (20) to (19) yields:

$$\begin{aligned} H_a(\mathbf{x}_{\mathcal{N}}) &= \int_{\mathcal{F}} \Phi^2_{\mathcal{N}\setminus\{a\}}(\mathbf{y})\tilde{p}(\mathbf{x}_a, \mathbf{y})d\mathbf{y} \\ &= \int_{\mathcal{F}} \Big(\Phi^2_{\mathcal{B}_a}(\mathbf{y}) + \Phi^2_{\mathcal{C}_a}(\mathbf{y}) + \Phi^1_{\mathcal{B}_a}(\mathbf{y})\Phi^1_{\mathcal{C}_a}(\mathbf{y})\Big)\tilde{p}(\mathbf{x}_a, \mathbf{y})d\mathbf{y} \end{aligned} \tag{21}$$

Partitioning the domain of integration into the visible set and invisible set of agent a, and noting that $\tilde{p}(\mathbf{x}_j, \mathbf{y}) = 0 \ \forall \ j \in \mathcal{C}_a, \ \mathbf{y} \in \tilde{V}(\mathbf{x}_a)$ such that $\Phi^n_{\mathcal{C}_a}(\mathbf{y}) = 0 \ \forall \ n \in \mathbb{Z}^+, \ \mathbf{y} \in \tilde{V}(\mathbf{x}_a)$, and $\tilde{p}(\mathbf{x}_a, \mathbf{y}) = 0 \ \forall \ \mathbf{y} \in \tilde{V}_c(\mathbf{x}_a)$ yields:

$$\begin{aligned} H_a(\mathbf{x}_{\mathcal{N}}) &= \int_{\tilde{V}(\mathbf{x}_a)} \Big(\Phi^2_{\mathcal{B}_a}(\mathbf{y}) + \Phi^2_{\mathcal{C}_a}(\mathbf{y}) + \Phi^1_{\mathcal{B}_a}(\mathbf{y})\Phi^1_{\mathcal{C}_a}(\mathbf{y})\Big)\tilde{p}(\mathbf{x}_a, \mathbf{y})d\mathbf{y} \\ &+ \int_{\tilde{V}_c(\mathbf{x}_a)} \Big(\Phi^2_{\mathcal{B}_a}(\mathbf{y}) + \Phi^2_{\mathcal{C}_a}(\mathbf{y}) + \Phi^1_{\mathcal{B}_a}(\mathbf{y})\Phi^1_{\mathcal{C}_a}(\mathbf{y})\Big)\tilde{p}(\mathbf{x}_a, \mathbf{y})d\mathbf{y} \\ &= \int_{\tilde{V}(\mathbf{x}_a)} \Phi^2_{\mathcal{B}_a}(\mathbf{y})p(\|\mathbf{x}_a - \mathbf{y}\|)d\mathbf{y} \end{aligned} \tag{22}$$

Using (22) the global objective function can be written as:

$$H(\mathbf{x}_{\mathcal{N}}) = \int_{\tilde{V}(\mathbf{x}_a)} \Phi^2_{\mathcal{B}_a}(\mathbf{y})p(\|\mathbf{x}_a - \mathbf{y}\|)d\mathbf{y} + \int_{\mathcal{F}} \Phi^{3+}_{\mathcal{N}\setminus\{a\}}(\mathbf{y})d\mathbf{y} \tag{23}$$

where the first term is the local objective function of agent a which depends only on the position of a (in both domain and integrand) and the positions of the neighbours of a. The second term does not depend on the position of a.

4.4 Optimization problem formulation

An optimal coverage has to fulfill three demands: The largest possible portion of $\mathcal{F}$ must be the within range of at least three agents, agents must be positioned within the feasible space, and neighbours cannot be located at linear positions. Thus for any agent a, we want it to position itself such that it makes the largest possible contribution towards the global objective. Furthermore it is desirable that agents do not cluster together as the accuracy of trilateration depends on the spread of the agents. Furthermore the position, $\mathbf{y}$, of an entity cannot be uniquely determined from the known position of three agents if they are positioned along a straight line. Thus we need to enforce that the positions of an agent and its two closest neighbours are nonlinear. Using (22) we formulate the overall local objective for an agent a as:

$$
\begin{aligned}
&\max_{\mathbf{x}_a} \int_{\tilde{V}(\mathbf{x}_a)} \Phi^2_{\mathcal{B}_a}(\mathbf{y}) p(\|\mathbf{x}_a - \mathbf{y}\|) d\mathbf{y} - k_1 \sum_{j \in \mathcal{B}_a} e^{1 - k_2 \|\mathbf{x}_a - \mathbf{x}_j\|} \\
&\text{s.t. } \mathbf{x}_a \in \mathcal{F} \\
&\quad (\mathbf{x}_i - \mathbf{x}_a) \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} (\mathbf{x}_j - \mathbf{x}_a)^T \neq 0 \\
&\quad i = \arg\min_i \|\mathbf{x}_a - \mathbf{x}_i\| \quad \text{s.t } i \in \mathcal{B}_a \\
&\quad i = \arg\min_j \|\mathbf{x}_a - \mathbf{x}_j\| \quad \text{s.t } i \in \mathcal{B}_a \setminus \{i\}
\end{aligned} \tag{24}
$$

5 Implementation

5.1 Local probability

We assume that the agents have perfect communication capabilities within their maximum range. Thus we set

$$\tilde{p}(\mathbf{x}_a, \mathbf{y}) = \begin{cases} 1, & \mathbf{y} \in \tilde{V}(\mathbf{x}_a) \\ 0, & \mathbf{y} \in \tilde{V}_c(\mathbf{x}_a) \end{cases} = 1_{\left\{\mathbf{y} \in \tilde{V}(\mathbf{x}_a)\right\}} \tag{25}$$

where $1_{\{\cdot\}}$ is the indicator function, which is simply equal to one if the clause in the subscript is true and zero otherwise. This implies that the global objective function (17) is simply the area of all intersctions of three or more visible sets.

5.2 Computing the local objective function

We partition the neighbours of a into two sets:

$$\mathcal{B}_{a\tilde{V}}(\mathbf{y}) = \{j \in \mathcal{B}_a : \mathbf{y} \in \tilde{V}(\mathbf{x}_j)\} \tag{26a}$$

$$\mathcal{B}_{a\tilde{V}_c}(\mathbf{y}) = \{j \in \mathcal{B}_a : \mathbf{y} \in \tilde{V}_c(\mathbf{x}_j)\} \tag{26b}$$

Now the local objective function can be written as:

$$\begin{aligned} \tilde{H}_a(\mathbf{x}_{\{a\}\cup\mathcal{B}_a}) &= \int_{V(\mathbf{x}_a)} \Phi^2_{\mathcal{B}_a}(\mathbf{y}) 1_{\{\mathbf{y}\in\tilde{V}(\mathbf{x}_a)\}} d\mathbf{y} = \int_{V(\mathbf{x}_a)} \Phi^2_{\mathcal{B}_a}(\mathbf{y}) d\mathbf{y} \\ &= \int_{V(\mathbf{x}_a)} \sum_{n=0}^{2} \Phi^n_{\mathcal{B}_{a\tilde{V}}}(\mathbf{y}) \Phi^{2-n}_{\mathcal{B}_{a\tilde{V}_c}}(\mathbf{y}) d\mathbf{y} = \int_{V(\mathbf{x}_a)} \sum_{n=0}^{2} 1_{\{|B_{a\tilde{V}}(\mathbf{y})|=n\}} 1_{\{2-n=0\}} d\mathbf{y} \\ &= \int_{V(\mathbf{x}_a)} 1_{\{|B_{a\tilde{V}}(\mathbf{y})|=2\}} d\mathbf{y} \end{aligned} \tag{27}$$

Thus the value of the local objective function is simply the area where the visible set of a overlaps with those of excatly two neighbouring agents.

6 Simulation Results

MAKE IT "BY ANSWERED QUESTION".

IN OTHER WORDS, HAVE SOMETHING LIKE

6.1 WHAT IS THE EFFECT OF A ON B?

BLA BLA

6.2 WHAT HAPPENS IF XXX?

BLA BLA

6.3 WHAT IS THE CAUSE OF YYY?

⋮

IN THIS WAY IT WILL BE MUCH MORE INTERESTING!

7 Discussion

8 Future Work

References

[1] X. Sun, C. G. Cassandras, and K. Gokbayrak, "Escaping local optima in a class of multi-agent distributed optimization problems: A boosting function approach," 2014.

[2] E. W. Weisstein, "Simple polygon. From MathWorld–A Wolfram Web Resource," last visited on 10/11/2020. [Online]. Available: https://mathworld.wolfram.com/SimplePolygon.html

[3] Y. H. Wang, "On the number of successes in independent trials," *Statistica Sinica*, vol. 3, no. 2, pp. 295–312, 1993. [Online]. Available: http://www.jstor.org/stable/24304959

[4] C. Wolff, "Multilateration," last visited on 13.11.2020. [Online]. Available: https://www.radartutorial.eu/02.basics/rp52.en.html