Analysis 1

Mike Desgrottes

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Theorem 1. *Suppose that* $\alpha \in End(v)$ *and* $V = W_1 \bigoplus .. \bigoplus W_k$ *such that each subspace is* α*-invariant. Prove that there exists a basis* B *for* V *such that the representation matrix of* α *with resprct to* B *has the form* $A = [A_{ij}]$ *are zero matrices for all* $i \neq j$.

Proof. Let B_i be a basis for W_i, then the matrix representation of α with respect to B_i is contained in W_i because W_i is α-invariant. $\bigcup_{i=1}^{k} B_i$ is a basis for V. Then for every $v \in B_i$, then $\alpha(v) \notin B_j$ for $j \neq i$. Hence $A = diag(A_{11}, ..., A_{kk})$. □

Theorem 2. *Let* A *be a matrix such that its characteristic polynomial is completely reducible. Prove that* A *is nilpotent if and only if* $spec(\alpha) = \{0\}$.

Proof. Since the characteristic polynomial of A is completely reducible, then A is similar to a diagonal matrix. Hence, the diagonal matrix is also nilpotent. Let $B = diag(\lambda_1, ..., \lambda_n)$ is the diagonal matrix, then $B^k = diag(\lambda_1^k, ..., \lambda_n^k) = 0 \implies \lambda_i = 0$ for all i. So, A being nilpotent implies $spec(A) = \{0\}$.

Suppose that the spectrum of A is trivial, then A is similar to the zero matrix. Hence $A = P^T B P = 0$ which is nilpotent. □

Theorem 3. *Prove that if* J *is a* m *x* m *Jordan block with zero maind diagonal, then* $det(tI - J) = t^m$ *is the minimal polynomial of* J.

Theorem 4. *let* $p(t) = t^3 - 6t^2 + 11t - 6$. *Describe the Jordan Canonical forms of the 3 x 3 matrices A that statisfies* $p(A) = 0$ *if such matrices exist.*