Analysis 1

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Theorem 1. *Prove that if f is differentiable and its derivative is increasing then f is convex.*

Proof. Let $x < t < y$, where $t = x + \lambda(y - x)$, then by the mean value theorem there exists c, u such that $x < c < t < u < y$ with the following property

$$f^{'}(c) = \frac{f(t) - f(x)}{t - x}$$

$$f^{'}(u) = \frac{f(y) - f(t)}{y - t}$$

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$f^{'}(c) \leq f^{'}(u) \implies \frac{f(t)-f(x)}{t-x} \leq \frac{f(y)-f(t)}{y-t} \implies f(t) \leq \frac{t-x}{y-t}(f(y) - f(t)) + f(x) \leq \frac{t-x}{y-t}(f(y) - f(x)) + f(x)$ $\square$