Analysis 1

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October 2020

Theorem 1. *Prove that if $f : I \to \mathbb{R}$ is differentiable and its derivative is increasing then f is convex.*

Proof. Let $x < t < y$, where $t = x + \lambda(y - x)$ and $x, y \in I$, then by the mean value theorem there exists c, u such that $x < c < t < u < y$ with the following property

$$f^{'}(c) = \frac{f(t) - f(x)}{t - x}$$

$$f^{'}(u) = \frac{f(y) - f(t)}{y - t}$$

.

$$f^{'}(c) \leq f^{'}(u) \implies \frac{f(t) - f(x)}{t - x} \leq \frac{f(y) - f(t)}{y - t}$$

$$\frac{f(t) - f(x)}{\lambda y + (1 - \lambda)x - x} \leq \frac{f(y) - f(t)}{y - \lambda y - (1 - \lambda)x}$$

$$\frac{f(t) - f(x)}{\lambda(y - x)} \leq \frac{f(y) - f(t)}{(1 - \lambda)(y - x)}$$

$$(1 - \lambda)(f(t) - f(x)) \leq \lambda(f(y) - f(t))$$

$$f(t) \leq \lambda f(y) - (1 - \lambda)f(x)$$

□