

$$z = x + yj$$

$$\operatorname{Re}\left(\frac{\bar{z}+j}{z-1}\right) \geq 1$$

$$\operatorname{Re}\left[\frac{x-yj+j}{x+yj-1}\right] = \operatorname{Re}\left[\frac{x+(1-y)j}{x-1+yj} \cdot \frac{(x-1)-yj}{(x-1)-yj}\right] = \operatorname{Re}\left[\frac{x(x-1)-xyj + (x-1)(1-y)j + (1-y)^2}{(x-1)^2 + y^2}\right]$$

$$= \frac{x(x-1) + y(1-y)}{(x-1)^2 + y^2} \geq 1$$

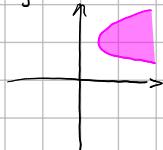
$$x(x-1) + y(1-y) \geq (x-1)^2 + y^2$$

$$x^2 - x + y - y^2 \geq x^2 - 2x + 1 + y^2$$

$$0 \geq 2y^2 - y + x + 1$$

$$x \leq -2y^2 + y - 1$$

$$x \geq 2y^2 - y + 1$$



2.

$$j^{24} = \bar{z} \cdot |z|^2$$

$$\varphi = -\frac{\pi}{12} + \frac{\pi}{3}k$$

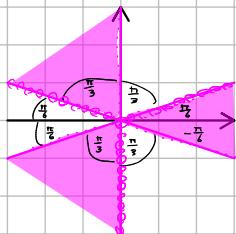
$$3. \quad 0 \leq \arg(jz^3) < \pi \quad z \neq 0$$

$$jz^3 = e^{\frac{\pi}{2}j} \cdot r^3 e^{3\varphi j} = r^3 e^{(3\varphi + \frac{\pi}{2})j}$$

$$0 + 2k\pi \leq 3\varphi + \frac{\pi}{2} < \pi + 2k\pi$$

$$-\frac{\pi}{2} + 2k\pi \leq 3\varphi < \frac{\pi}{2} + 2k\pi$$

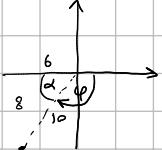
$$-\frac{\pi}{6} + \frac{2k\pi}{3} \leq \varphi < \frac{\pi}{6} + \frac{2k\pi}{3}$$



$$4. \quad -6 - 8j$$

$$\alpha = \arctan\left(\frac{8}{6}\right)$$

$$\arg(-6 - 8j) = \varphi = -\pi + \arctan\left(\frac{8}{6}\right)$$



5.

$$\sqrt[n]{1} = \left\{ e^{\frac{2k\pi j}{n}} : k \in \mathbb{Z} \wedge 0 \leq k < n \right\}$$

6.

$$z^4 = (jz+1)^4$$

$$\begin{aligned} z^4 - (jz+1)^4 &= (z^2 - (jz+1)^2)(z^2 + (jz+1)^2) \\ &= (z - (jz+1))(z + (jz+1))(z - j(jz+1))(z + j(jz+1)) \\ &= (z - jz + 1)(z + jz + 1)(z + 2 - jz)(z - z + j) \\ &= [z(1-j) + 1][z(1+j) + 1][2z - j] \cdot j \end{aligned}$$

$$z_0 = \frac{-1}{1-j} \quad z_1 = \frac{-1}{1+j} \quad z_2 = \frac{j}{2}$$