

1.

$$a) f: \mathbb{R} \rightarrow [-1, 1] \quad f(x) = \sin(x) \\ A = [0, 4\pi] \quad B = [2\pi, 4\pi] \quad A \setminus B = [0, 2\pi)$$

$$f(A \setminus B) = f([0, 2\pi)) = [-1, 1]$$

$$f(A) = f([0, 4\pi]) = [-1, 1]$$

$$f(B) = f([2\pi, 4\pi]) = [-1, 1]$$

$$f(A) \setminus f(B) = [-1, 1] \setminus [-1, 1] = \emptyset$$

$$[-1, 1] \not\subseteq \emptyset$$

$$f(A \setminus B) \not\subseteq f(A) \setminus f(B)$$

$$b) f: \mathbb{R} \rightarrow [-1, 1] \quad f(x) = \sin(x) \quad A = [0, 2\pi]$$

$$f([0, 2\pi]) = [-1, 1]$$

$$f^{-1}([-1, 1]) = \mathbb{R}$$

$$\mathbb{R} \not\subseteq [-1, 1]$$

$$f^{-1}(f([0, 2\pi])) \not\subseteq A$$

$$c) f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \sin(x)$$

$$C = [-1, 2]$$

$$f^{-1}([-1, 2]) = \mathbb{R}$$

$$f(\mathbb{R}) = [-1, 1]$$

$$[-1, 2] \not\subseteq [-1, 1]$$

$$C \not\subseteq f(f^{-1}(C))$$

2.

$$a) f: \mathbb{R} \setminus \{1\} \rightarrow \mathbb{R} \quad f(x) = \frac{2x+3}{x-1}$$

$$f(x) = \frac{2x-2+5}{x-1} = \frac{5}{x-1} + 2$$

$$R_f = \mathbb{R} \setminus \{2\}$$

$$y = \frac{2x+3}{x-1}$$

$$yx - y = 2x + 3$$

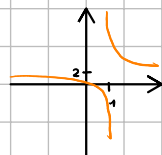
$$yx - 2x = y + 3$$

$$x(y-2) = y+3$$

$$x = \frac{y+3}{y-2}$$

$$f^{-1}: \mathbb{R} \setminus \{2\} \rightarrow \mathbb{R} \setminus \{1\}$$

$$f^{-1}(x) = \frac{x+3}{x-2}$$



$$b) f: [3\pi, 4\pi] \rightarrow \mathbb{R} \quad f(x) = e^{1-2\cos(x)}$$

$$x \in [3\pi, 4\pi] \Rightarrow \cos(x) \in [-1, 1] \Rightarrow 1-2\cos(x) \in [-1, 3] \Rightarrow f(x) \in [e^{-1}, e^3]$$

$$R_f = [e^{-1}, e^3]$$

$$y = e^{1-2\cos(x)}$$

$$\ln(y) = 1-2\cos(x)$$

$$\cos(x) = \frac{1-\ln(y)}{2}$$

$$x = \arccos\left(\frac{1-\ln(y)}{2}\right) + 3\pi$$

$$\arccos: [-1, 1] \rightarrow [0, \pi]$$

$$\arccos(x) + 3\pi \in [3\pi, 4\pi]$$

$$f^{-1}: [e^{-1}, e^3] \rightarrow [3\pi, 4\pi]$$

$$f^{-1}(x) = \arccos\left(\frac{1-\ln(x)}{2}\right) + 3\pi$$

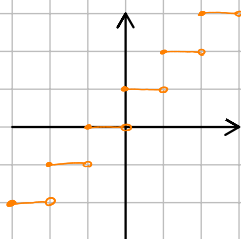
3.

a) $f: \mathbb{R} \rightarrow \mathbb{Z} \quad f(x) = \lfloor x \rfloor + 1$

nie jest iniekcyjna

nie jest surjekcyjna $f(1.5) = f(1.51)$

surjekcyjna $\forall n \in \mathbb{Z} \quad f(n-1) = n$



$$f^{-1}(\{0\}) = \{x \in \mathbb{R} : \lfloor x \rfloor + 1 = 0\} = [-1, 0)$$

$$f^{-1}(\mathbb{Z} \setminus \mathbb{N}) = \{x \in \mathbb{R} : \lfloor x \rfloor + 1 \in \mathbb{Z} \setminus \mathbb{N}\} = \{x \in \mathbb{R} : \lfloor x \rfloor < 0\} = (-\infty, 0)$$

$$f(0) = 1 \notin \mathbb{Z} \setminus \mathbb{N}$$

$$f(-0.01) = 0 \in \mathbb{Z} \setminus \mathbb{N}$$

$$f((0, +\infty)) = \mathbb{N}$$

$$f(0) = 1 \quad f(0.001) = 1 \quad f(0.999) = 1$$

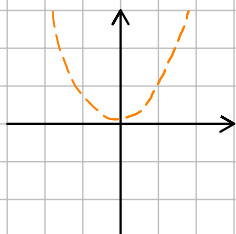
$$f(1) = 2 \quad f(1.001) = 2 \quad f(1.999) = 2$$

$$f^{-1}(f([-n, n])) = f^{-1}(\{-3, -2, -1, 0, 1, 2, 3, 4\}) = [-4, 4)$$

b) $f: \mathbb{Q} \rightarrow \mathbb{Q} \quad f(x) = x^2 + 1$

nie jest iniekcyjna $f(-1) = f(1)$

nie jest surjekcyjna $\sim \exists x \in \mathbb{Q} \quad f(x) = 0 \in \mathbb{Q}$



$$f^{-1}\left(\left\{\frac{13}{9}\right\}\right) = \left\{\frac{2}{3}, -\frac{2}{3}\right\}$$

$$f^{-1}\left(\left[\frac{10}{9}, 2\right] \cap \mathbb{Q}\right) = \left([-1, -\frac{1}{3}] \cup [\frac{1}{3}, 1]\right) \cap \mathbb{Q}$$

$$x^2 + 1 \leq 2 \quad \wedge \quad x^2 + 1 \geq \frac{10}{9}$$

$$x^2 - 1 \leq 0 \quad \wedge \quad x^2 - \frac{1}{9} \geq 0$$

$$(x-1)(x+1) \leq 0 \quad \wedge \quad (x-\frac{1}{3})(x+\frac{1}{3}) \geq 0$$

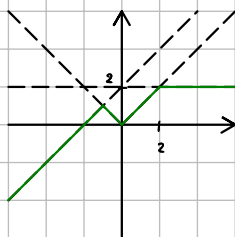


$$x^2 + 1 = \frac{3}{2} \Leftrightarrow x^2 - \frac{1}{2} = 0 \Leftrightarrow (x - \frac{\sqrt{2}}{2})(x + \frac{\sqrt{2}}{2}) = 0 \Leftrightarrow x \in \left[-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right]$$

$$[1, 2] \cap \mathbb{Q} \not\subseteq f(\mathbb{Q}) \quad \text{bo} \quad \sim \exists x \in \mathbb{Q} \quad f(x) = \frac{3}{2} \quad \wedge \quad \frac{3}{2} \in [1, 2] \cap \mathbb{Q}$$

c) $f: \mathbb{R} \rightarrow \mathbb{R} \quad f(x) = \min\{|x|, 2, x+2\}$

$$f(x) = \begin{cases} x+2 & \text{dla } x \in (-\infty, -1] \\ |x| & \text{dla } x \in (-1, 2] \\ 2 & \text{dla } x \in (2, +\infty) \end{cases}$$



nie jest iniekcyjna $f(2) = f(3)$

nie jest surjekcyjna $\sim \exists x \in \mathbb{R} \quad f(x) = 3$

$$f((-1, 3]) = [0, 2]$$

$$f^{-1}(\{-1, 1, 2\}) = \{-3, -1\} \cup [2, +\infty)$$

$$f^{-1}(f(\{0, 1, 5\})) = f^{-1}(\{0, 1, 2\}) = \{-2, -1, 0, 1\} \cup [2, +\infty)$$

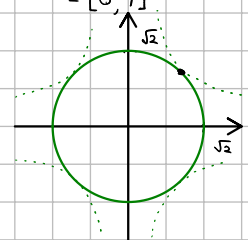
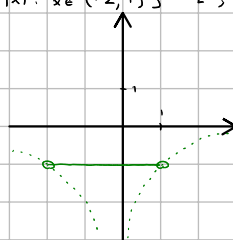
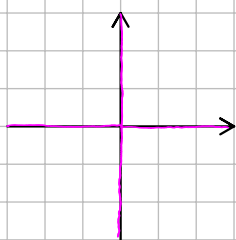
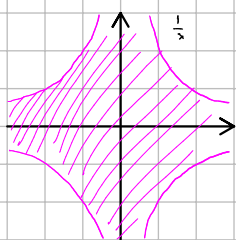
d) $f: \mathbb{R}^2 \rightarrow \mathbb{R} \quad f((x,y)) = |xy|$

$f^{-1}((-5, 2]) = f^{-1}([0, 2])$

$f^{-1}(\{0\})$

$f((-2, 1) \times \{-1\})$
 $= \{ |x| : x \in (-2, 1) \} = [0, 2)$

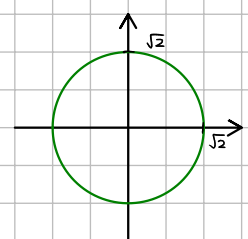
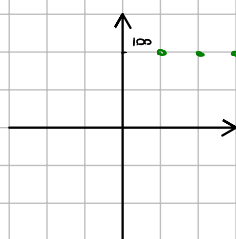
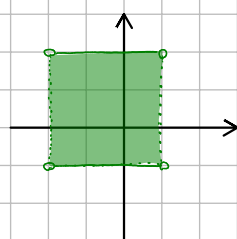
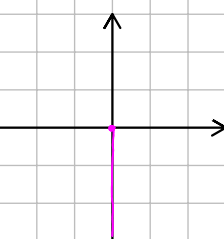
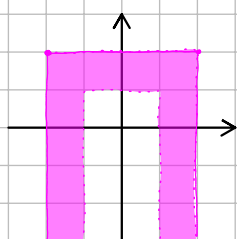
$f(\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 2\})$
 $= [0, 1]$



nie jest iniekcyjna
 nie jest suriekcyjna
 $f((1, 1)) = f((-1, 1))$
 $R_f = [0, +\infty) \neq \mathbb{R}$

min $f((0, \sqrt{2})) = 0$
 max $f((\sqrt{2} \cos(\frac{\pi}{4}), \sqrt{2} \sin(\frac{\pi}{4})))$
 $= f((1, 1)) = 1$

e) $f: \mathbb{R}^2 \rightarrow \mathbb{R} \quad f((x,y)) = \max\{|x|, |y|\}$



$f^{-1}((1, 2])$

$f^{-1}(\{0\})$

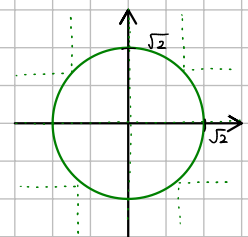
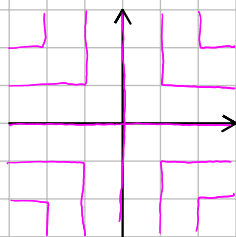
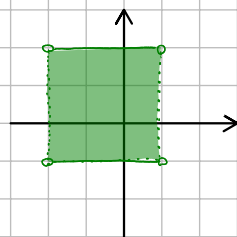
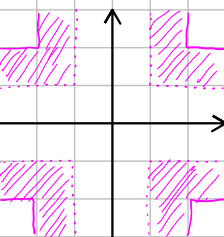
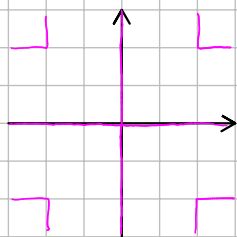
$f((-2, 1) \times [-1, 2])$
 $= [0, 2]$

$f(\mathbb{N} \times \{100\})$
 $= \{\max\{n, 100\} : n \in \mathbb{N}\}$
 $= \{\max\{n, 100\} : n \in \mathbb{N}\}$
 $= \{100, 101, \dots\}$

$f(\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 2\})$
 $= [0, \sqrt{2}]$
 $f((0, -\sqrt{2})) = 0$
 $f((0, \sqrt{2})) = \sqrt{2}$

nie jest iniekcyjna bo $f((-2, 1)) = f(2, 1) = 2$
 nie jest suriekcyjna bo $f(\mathbb{R}^2) = [0, +\infty) \neq \mathbb{R}$

f) $f: \mathbb{R}^2 \rightarrow \mathbb{R} \quad f((x,y)) = \min\{|x|, |y|\}$



$f^{-1}(\{-2, 0, 2\})$

$f^{-1}((1, 2])$

$f((-2, 1) \times [-1, 2])$
 $= [0, 2]$

$f(f^{-1}(\{-1, 0, 1, 2\}))$
 $= \{-1, 0, 1, 2\}$

$f(\{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 2\})$
 $= [0, \sqrt{2}]$

nie jest iniekcyjna
 nie jest suriekcyjna
 $f((-1, 0)) = f((1, 0)) = 0$
 $R_f = [0, +\infty)$