

$$1b \quad x y' = \tan(y) \quad y\left(\frac{1}{2}\right) = \frac{5\pi}{6}$$

$$\frac{dy}{dx} = \frac{\tan(y)}{x} \rightarrow \int \frac{dy}{\tan(y)} = \int \frac{dx}{x}$$

$$\int \frac{dy}{\tan(y)} = \int \frac{\cos(y)}{\sin(y)} dy = \left| \begin{matrix} t = \sin(y) \\ dt = \cos(y) dy \end{matrix} \right| = \int \frac{dt}{t}$$

$$\ln |\sin(y)| = \ln |x| + C$$

$$\sin(y) = Cx$$

$$y \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$$

$$\sin\left(\frac{5\pi}{6}\right) = C \cdot \frac{1}{2}$$

$$y - \pi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$\frac{1}{2} = \frac{1}{2} C \rightarrow C = 1$$

$$y = \pi - \arcsin(x)$$

$$\sin(y) = x$$

2a

$$y' - \frac{2x}{1+x^2} y = 1+x^2$$

$$\frac{dy}{dx} - \frac{2x}{1+x^2} y = 1+x^2 \rightarrow \int \frac{dy}{y} = \int \frac{2x}{x^2+1} dx$$

$$\int \frac{2x}{x^2+1} dx = \left| \begin{matrix} t = x^2+1 \\ dt = 2x dx \end{matrix} \right| = \int \frac{dt}{t} = \ln |x^2+1|$$

$$\ln |y| = \ln |x^2+1| + C$$

$$y = C(x^2+1)$$

$$y' = C'(x^2+1) + 2xC$$

$$C'(x^2+1) + 2xC - \frac{2x}{x^2+1} \cdot C(x^2+1) = x^2+1$$

$$C'(x^2+1) - 2xC - 2xC = x^2+1$$

$$C' = 1 \quad C = \int 1 dx = x + C$$

$$y(x) = (x+C)(x^2+1)$$

2c

$$y' + 4y = 5 \sin(3x)$$

$$1^\circ \quad y' + 4y = 0$$

$$\frac{dy}{dx} = -4y \quad \int \frac{dy}{y} = \int -4 dx$$

$$\ln |y| = -4x + C$$

$$y = e^{-4x+C} = C e^{-4x} \quad \text{COR}$$

$$2^\circ \quad 0 \cdot 3x + 4 = 5 \rightarrow y_1 = A \cos(3x) + B \sin(3x)$$

$$y_1' = -3A \sin(3x) + 3B \cos(3x)$$

$$-3A \sin(3x) + 3B \cos(3x) + 4A \cos(3x) + 4B \sin(3x) = 5 \sin(3x)$$

$$\begin{cases} 4A + 3B = 0 & B = -\frac{4}{3}A \end{cases}$$

$$\begin{cases} -3A + 4B = 5 & -3A - \frac{16}{3}A = 5 \end{cases}$$

$$-\frac{25}{3}A = 5 \quad A = -\frac{3}{5} \quad B = \frac{4}{5}$$

$$y_1 = -\frac{3}{5} \cos(3x) + \frac{4}{5} \sin(3x) \quad C \in \mathbb{R}$$

$$y(x) = -\frac{3}{5} \cos(3x) + \frac{4}{5} \sin(3x) + C e^{-4x} \quad \text{COR}$$

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$$y' - \frac{xy}{2(x^2-1)} = \frac{x}{2y} \quad y(2) = \sqrt{3}$$

$$y' + p(x)y = q(x) \quad y' \rightarrow v = y^2$$

$$y = \sqrt{v}$$

$$y' = \frac{1}{2\sqrt{v}} v'$$

$$\frac{v'}{2\sqrt{v}} - \frac{x\sqrt{v}}{2(x^2-1)} = \frac{x}{2\sqrt{v}} \quad | \cdot 2\sqrt{v}$$

$$\frac{dv}{dx} - \frac{x}{x^2-1} v = -x$$

$$1^\circ \quad \frac{dv}{dx} = \frac{x}{x^2-1} v \rightarrow \int \frac{dv}{v} = \int \frac{x}{x^2-1} dx$$

$$\ln|v| = \frac{1}{2} \ln|x^2-1| + C$$

$$v = C\sqrt{x^2-1}$$

$$v' = C' \sqrt{x^2-1} + \frac{C}{2\sqrt{x^2-1}} \cdot 2x = C' \sqrt{x^2-1} + \frac{Cx}{\sqrt{x^2-1}}$$

$$C' \sqrt{x^2-1} + \frac{Cx}{\sqrt{x^2-1}} = \frac{x}{x^2-1} \cdot C\sqrt{x^2-1} = x$$

$$C' = \frac{x}{\sqrt{x^2-1}} \quad C = \int \frac{x}{\sqrt{x^2-1}} dx = \left| \begin{matrix} t=x^2-1 \\ dt=2x dx \end{matrix} \right| = \int \frac{1}{2t} dt = \frac{1}{2} \ln t + C = \sqrt{x^2-1} + C$$

$$y^2 = v(x) = (\sqrt{x^2-1} + C) \sqrt{x^2-1} = C\sqrt{x^2-1} + x^2 - 1$$

$$y(2) = \sqrt{3}$$

$$3 = C\sqrt{3} + 3 \rightarrow C=0$$

$$y = \sqrt{x^2-1}$$

4a

$$y'' - 4y' = 8x \quad y(0) = 1 \quad y'(0) = -1$$

$$1^\circ \quad y'' - 4y' = 0$$

$$r^2 - 4r = 0 \quad r(r-4) = 0$$

$$r_1 = 0 \quad r_2 = 4$$

$$y_0 = C_1 e^{0x} + C_2 e^{4x} = C_1 + C_2 e^{4x} \quad \text{COR}$$

$$2^\circ \quad y'' - 4y' = 8x$$

0 + 0 ist 1. Leedingterm gegeben

$$y_1 = x(Ax + B) = Ax^2 + Bx$$

$$y_1' = 2Ax + B$$

$$y_1'' = 2A$$

$$3^\circ \quad 2A - 4(2Ax + B) = 8x$$

$$2A - 8Ax - 4B = 8x$$

$$\begin{cases} 8 = -8A & A = -1 \\ 0 = 2A - 4B & B = -\frac{1}{2} \end{cases}$$

$$y_1 = -x^2 - \frac{1}{2}x \quad \text{CSRN}$$

$$4^\circ \quad y = y_1 + y_0 = C_1 + C_2 e^{4x} - x^2 - \frac{1}{2}x \quad \text{COR}$$

$$5^\circ \quad y(0) = 1 \quad y'(0) = -1 \quad y' = 4C_2 e^{4x} - 2x - \frac{1}{2}$$

$$\begin{cases} 1 = C_1 + C_2 & C_2 = -\frac{1}{8} \\ -1 = 4C_2 - \frac{1}{2} & C_1 = \frac{3}{8} \end{cases}$$

$$\text{CSRN} \quad y = \frac{3}{8} - \frac{1}{8} e^{4x} - x^2 - \frac{1}{2}x$$

4b

$$y'' - 2y' + y = 4\sin^2\left(\frac{x}{2}\right) \quad y(0) = 2 \quad y'(0) = 1$$

$$1^\circ \quad y'' - 2y' + y = 0$$

$$r^2 - 2r + 1 = (r-1)^2 = 0$$

$$r_0 = 1 \rightarrow y_0 = C_1 e^x + C_2 x e^x$$

$$2^\circ \quad \cos(2x) = 1 - 2\sin^2(x)$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$4\sin^2\left(\frac{x}{2}\right) = 4 \cdot \frac{1}{2}(1 - \cos(x)) = 2 - 2\cos(x)$$

$\alpha + \beta j = 0 + j$  mit jetzt partiell integrieren

$$y_1 = A \cos(x) + B \sin(x) + C$$

$$y_1' = B \cos(x) - A \sin(x)$$

$$y_1'' = -A \cos(x) - B \sin(x)$$

$$3^\circ \quad -A \cos(x) - B \sin(x) - 2B \cos(x) + 2A \sin(x) + A \cos(x) + B \sin(x) + C = 2 - 2\cos(x)$$

$$\begin{cases} 0 = 2A & A = 0 \\ -2 = -2B & B = 1 \\ 2 = C & C = 2 \end{cases}$$

$$y_1 = \sin(x) + 2 \quad \text{CSRN}$$

$$4^\circ \quad y = y_0 + y_1 = C_1 e^x + C_2 x e^x + \sin(x) + 2 \quad \text{CORN}$$

$$5^\circ \quad y' = C_1 e^x + C_2 x e^x + C_2 e^x + \cos(x)$$

$$y(0) = 2 = C_1 \cdot 2 \rightarrow C_1 = 0$$

$$y'(0) = 1 = C_1 + C_2 \cdot 1 \rightarrow C_2 = 0$$

$$y(x) = \sin(x) + 2 \quad \text{CSRN}$$

5a

$$y'' + y = \tan(x)$$

$$1^\circ \quad y'' + y = 0$$

$$r^2 + 1 = 0$$

$$r_1 = j \quad r_2 = -j$$

$$\alpha = 0 \quad \beta = 1$$

$$y_0 = C_1 e^0 \cos(x) + C_2 e^0 \sin(x) = C_1 \cos(x) + C_2 \sin(x)$$

$$2^\circ \quad \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \tan(x) \end{bmatrix}$$

$$C_1' = \frac{-\sin(x) \tan(x)}{\cos^2(x) + \sin^2(x)} = \frac{-\sin^2(x)}{\cos(x)}$$

$$C_2' = \frac{\cos(x) \tan(x)}{1} = \sin(x)$$

$$C_2 = \int \sin(x) dx = -\cos(x) + C_2$$

$$\frac{1}{1-t^2} = \frac{A}{1-t} + \frac{B}{1+t} = \frac{A(1+t) + B(1-t)}{1-t^2}$$

$$t=1 \rightarrow \begin{cases} 1 = 2A \\ 1 = 2B \end{cases} \quad \begin{matrix} A = \frac{1}{2} \\ B = \frac{1}{2} \end{matrix}$$

$$\int \frac{dt}{1-t^2} = \frac{1}{2} \int \frac{dt}{1-t} + \frac{1}{2} \int \frac{dt}{1+t} = -\frac{\ln|1-t|}{2} + \frac{\ln|1+t|}{2} = \frac{1}{2} \ln \left| \frac{1+\sin(x)}{1-\sin(x)} \right|$$

$$C_1 = - \int \frac{\sin^2(x)}{\cos(x)} dx = - \int \frac{\sin^2(x)}{\cos^2(x)} \cos(x) dx = - \int \frac{\sin^2(x)}{1-\sin^2(x)} \cos(x) dx = \left| \begin{matrix} t = \sin(x) \\ dt = \cos(x) dx \end{matrix} \right| = - \int \frac{t^2}{1-t^2} dt = \int \frac{1-t^2-1}{1-t^2} dt = \int 1 - \frac{1}{1-t^2} dt$$

$$= \int 1 dt - \int \frac{dt}{1-t^2} = \sin(x) - \frac{1}{2} \ln \left| \frac{1+\sin(x)}{1-\sin(x)} \right| + C_1$$

$$y(x) = \left( \sin(x) - \frac{1}{2} \ln \left| \frac{1+\sin(x)}{1-\sin(x)} \right| + C_1 \right) \cos(x) + (-\cos(x) + C_2) \sin(x)$$

$$y(x) = C_1 \cos(x) + C_2 \sin(x) - \frac{1}{2} \cos(x) \ln \left| \frac{1+\sin(x)}{1-\sin(x)} \right| + \cancel{\sin(x) \cos(x)} - \cancel{\sin(x) \cos(x)}$$

$$5b \quad y'' - 2y' + y = \frac{e^x}{x^2+1}$$

$$1^\circ \quad y'' - 2y' + y = 0$$

$$r^2 - 2r + 1 = (r-1)^2 = 0$$

$$r_0 = 1 \rightarrow y_0 = C_1 e^x + C_2 x e^x$$

$$2^\circ \quad \begin{bmatrix} e^x & x e^x \\ e^x & x e^x + e^x \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{e^x}{x^2+1} \end{bmatrix}$$

$$W = e^x (x e^x + e^x) - x e^x \cdot e^x = x e^{2x} + e^{2x} - x e^{2x} = e^{2x}$$

$$C_1' = \frac{-x e^x \cdot \frac{e^x}{x^2+1}}{e^{2x}} = \frac{-x}{x^2+1}$$

$$C_2' = \frac{e^x \cdot \frac{e^x}{x^2+1}}{e^{2x}} = \frac{1}{x^2+1}$$

$$C_1 = \int \frac{-x}{x^2+1} dx = -\frac{1}{2} \int \frac{2x}{x^2+1} dx = \left| \begin{matrix} t = x^2+1 \\ dt = 2x dx \end{matrix} \right| = -\frac{1}{2} \int \frac{dt}{t} = -\frac{1}{2} \ln|x^2+1| + C_1$$

$$C_2 = \int \frac{dx}{x^2+1} = \arctan(x) + C_2$$

$$3^\circ \quad y(x) = \left( -\frac{1}{2} \ln|x^2+1| + C_1 \right) e^x + (\arctan(x) + C_2) x e^x$$

$$= C_1 e^x + C_2 x e^x + x \arctan(x) e^x - \frac{1}{2} \ln|x^2+1| e^x$$

$$y'' - 3y' + 2y = 3e^x$$

$$1^\circ \quad y'' - 3y' + 2y = 0$$

$$r^2 - 3r + 2 = r^2 - 2r - r + 2 = (r-2)(r-1) = 0$$

$$\rightarrow y_0 = C_1 e^x + C_2 e^{2x}$$

$$2^\circ \quad \alpha=1 \quad \beta=0 \quad 1 \rightarrow 1\text{-te Ordnung partielle}$$

$$y_1 = A x e^x$$

$$y_1' = A x e^x + A e^x$$

$$y_1'' = A x e^x + 2A e^x$$

$$3^\circ \quad \cancel{A x e^x} + 2A e^x - 3\cancel{A x e^x} - 3A e^x + 2\cancel{A x e^x} = 3e^x$$

$$3 = -A$$

$$A = -3 \rightarrow y_1 = -3x e^x$$

$$4^\circ \quad y = y_0 + y_1 = C_1 e^x + C_2 e^{2x} - 3x e^x$$

$$\bullet \quad x y' = y \ln\left(\frac{y^2}{x^2}\right)$$

$$y' = \frac{y}{x} \ln\left(\left(\frac{y}{x}\right)^2\right)$$

$$v(x) = \frac{y}{x}$$

$$y' = \frac{d}{dx} v x = v' x + v$$

$$v' x + v = v \ln(v^2) = 2v \ln(v)$$

$$v' x = 2v \ln(v) - v$$

$$\frac{dv}{dx} = \frac{2v \ln(v) - v}{x} \quad \int \frac{dv}{2v \ln(v) - v} = \int \frac{dx}{x}$$

$$\int \frac{dv}{2v \ln(v) - v} = \int \frac{1}{2 \ln(v) - 1} \cdot \frac{dv}{v} = \left| \frac{t = 2 \ln(v) - 1}{dt = 2 \cdot \frac{1}{v} dv} \right| = \frac{1}{2} \int \frac{dt}{t} = \frac{1}{2} \ln|t| + C = \frac{1}{2} \ln|2 \ln(v) - 1| + C$$

$$\frac{1}{2} \ln|2 \ln(v) - 1| = \ln|x| + C$$

$$\sqrt{2 \ln(v) - 1} = C x$$

$$2 \ln(v) - 1 = C x^2 \quad C \geq 0$$

$$\ln(v) = \frac{C x^2 + 1}{2}$$

$$v = e^{\frac{C x^2 + 1}{2}} \quad y = x e^{\frac{C x^2 + 1}{2}}$$

$$\bullet \quad y'' - 4y' + 4y = \frac{e^{2x}}{x^2} + 2$$

$$1^\circ \quad y'' - 4y' + 4y = 0$$

$$r^2 - 4r + 4 = (r-2)^2 = 0$$

$$r_0 = 2 \rightarrow y_0 = C_1 e^{2x} + C_2 x e^{2x}$$

$$2^\circ \quad \begin{bmatrix} e^{2x} & x e^{2x} \\ 2e^{2x} & e^{2x} + 2x e^{2x} \end{bmatrix} \cdot \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{e^{2x}}{x^2} + 2 \end{bmatrix}$$

$$W = e^{2x} (e^{2x} + 2x e^{2x}) - 2e^{2x} \cdot x e^{2x} = e^{4x} + 2x e^{4x} - 2x e^{4x} = e^{4x}$$

$$C_1' = \frac{-x e^{2x} \cdot \left(\frac{e^{2x}}{x^2} + 2\right)}{e^{4x}} = \frac{\frac{-x e^{4x}}{x^2} - 2x e^{2x}}{e^{4x}} = -\frac{1}{x} - 2x e^{-2x}$$

$$C_2' = \frac{e^{2x} \left(\frac{e^{2x}}{x^2} + 2\right)}{e^{4x}} = \frac{1}{x^2} + 2e^{-2x}$$

$$\int \frac{1}{x} dx = \ln|x| + C \quad \int \frac{1}{x^2} dx = -\frac{1}{x} + C$$

$$\int e^{-2x} dx = -\frac{1}{2} e^{-2x} + C \quad \int x e^{-2x} dx = \left| \begin{array}{l} t=x \\ t'=1 \end{array} \right| \begin{array}{l} y=e^{-2x} \\ y=-\frac{1}{2} e^{-2x} \end{array} = -\frac{1}{2} x e^{-2x} + \frac{1}{2} \int e^{-2x} dx = -\frac{1}{2} x e^{-2x} - \frac{1}{4} e^{-2x} + C$$

$$C_1 = -\int \frac{1}{x} dx - 2 \int x e^{-2x} dx = -\ln|x| + x e^{-2x} + \frac{1}{2} e^{-2x} + C_1$$

$$C_2 = \int \frac{1}{x^2} dx + 2 \int e^{-2x} dx = -\frac{1}{x} - e^{-2x} + C_2$$

$$y = \left(-\ln|x| + x e^{-2x} + \frac{1}{2} e^{-2x} + C_1\right) e^{2x} + \left(-\frac{1}{x} - e^{-2x} + C_2\right) x e^{2x}$$

$$= C_1 e^{2x} + C_2 x e^{2x} + x + \frac{1}{2} - e^{2x} \ln|x| - e^{2x} - x$$

$$= C_1 e^{2x} + C_2 x e^{2x} + \frac{1}{2} - e^{2x} \ln|x| - e^{2x}$$

$$= C_1 e^{2x} + C_2 x e^{2x} + \frac{1}{2} - e^{2x} \ln|x|$$

$$\bullet \quad x y' = x e^{\frac{y}{x}} + y$$

$$y' = e^{\frac{y}{x}} + \frac{y}{x}$$

$$v = \frac{y}{x} \quad y' = v'x + v$$

$$v'x + v = e^v + v$$

$$v'x = e^v$$

$$\frac{dv}{dx} = \frac{e^v}{x} \quad \int \frac{dv}{e^v} = \int \frac{dx}{x}$$

$$\int \frac{dv}{e^v} = \int e^{-v} dv = -e^{-v} + C$$

$$-e^{-v} = \ln|x| + C$$

$$e^{-v} = \ln\left|\frac{1}{x}\right| + C$$

$$\ln|e^{-v}| = \ln\left|\ln\left|\frac{1}{x}\right|\right| + C$$

$$-v = \ln\left|\ln\left|\frac{1}{x}\right|\right|$$

$$y = -x \ln\left|\ln\left|\frac{1}{x}\right|\right|$$