1.

$S_x = \{-2, -1, 0, 1, 3\}$

$$p_x(x) = \begin{cases} \dfrac{x}{a} & x \in \{-2, -1, 0\} \\ \dfrac{x}{a^2} & x \in \{1, 3\} \\ 0 \end{cases}$$

$a \in \mathbb{R}$

$$\sum_{x_i \in S_x} p_x(x_i) = 1$$

x	-2	-1	0	1	3
p_x	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{16}$	$\frac{3}{16}$

$$\dfrac{-2}{a} + \dfrac{-1}{a} + \dfrac{0}{a} + \dfrac{1}{a^2} + \dfrac{3}{a^2} = 1$$

$$F_x(t) = \begin{cases} 0 & t < -2 \\ \frac{1}{2} & -2 \leq t < -1 \\ \frac{3}{4} & -1 \leq t < 1 \\ \frac{13}{16} & 1 \leq t < 3 \\ 1 & t \geq 3 \end{cases}$$

$-3a + 4 = a^2$

$a^2 + 3a - 4 = 0$

$a^2 + 4a - a - 4 = 0$

$(a+4)(a-1) = 0$

$a = 1 \lor a = -4$

$p_x : S_x \to [0, 1] \implies a = -4$

2.

$$F_x(x) \begin{cases} 0 & x < 1 \\ \frac{1}{8} & 1 \le x < 2 \\ \frac{1}{2} & 2 \le x < 3 \\ \frac{3}{4} & 3 \le x < 4 \\ 1 & x \ge 4 \end{cases}$$

$$P(X = a) = F_x(a) - \lim_{t \to a^-} F_x(t)$$

$$ozn \qquad F_x(a) - F_x(a^-)$$

$$p_x(x) = P(X = x) = F_X(x) - F_X(x^-)$$

$$S_x = \{1, 2, 3, 4\}$$

$$p_x(x) = \begin{cases} \frac{1}{8} & x = 1 \\ \frac{3}{8} & x \in \{2, 3\} \\ \frac{1}{4} & x = 4 \\ 0 & w. p.p. \end{cases}$$

$$x^2 - 5x + 6 = x^2 - 3x - 2x + 6$$
$$x(x - 3) - 2(x - 3)$$
$$(x - 3)(x - 2)$$

$$P(X^2 - X = 0) = P(X = 0 \lor X = 1) = P(X = 0) + P(X = 1) = p_x(0) + p_x(1) = 0 + \frac{1}{8} = \frac{1}{8}$$

$$P(X^2 - 5X + 6 \le 0) = P(2 \le X \le 3) = F_x(3) - \lim_{t \to 2^-} F_x(t) = \frac{3}{4} - \frac{1}{8} = \frac{5}{8}$$

3.

$\Omega = \{0, 1, 2, 3\}$

$P(\{\omega\}) = \frac{1}{4}$

$X(\omega) = \sin\left(\frac{\pi}{2}\omega\right)$ $S_X = \{-1, 0, 1\}$

$Y(\omega) = \cos\left(\frac{\pi}{2}\omega\right)$ $S_Y = \{-1, 0, 1\}$

a)

$$p_X(x) = \begin{cases} \frac{1}{4} & dla \ x \in \{-1, 1\} \\ \frac{1}{2} & dla \ x = 0 \end{cases}$$

$$p_Y(y) = \begin{cases} \frac{1}{4} & dla \ y \in \{-1, 1\} \\ \frac{1}{2} & dla \ y = 0 \end{cases}$$

Mają takie same rozkłady

ω	0	1	2	3
X	0	1	0	-1
Y	1	0	-1	0

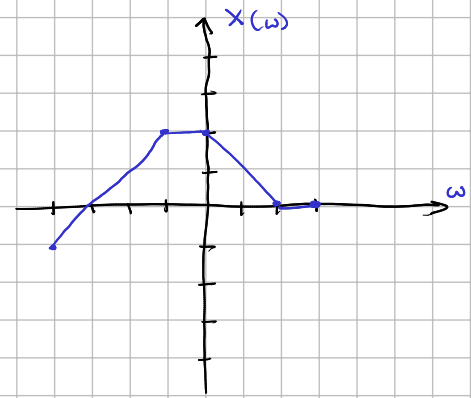**b)** $P(\{\omega \in \Omega : X(\omega) = Y(\omega)\}) = P(\emptyset) = 0$

4.

$$\Omega = [-4, 3]$$

$$X(\omega) = \begin{cases} \omega + 3 & -4 \leq \omega \leq -1 \\ 2 & -1 < \omega \leq 0 \\ 2 - \omega & 0 < \omega \leq 2 \\ 0 & 2 < \omega \leq 3 \end{cases}$$
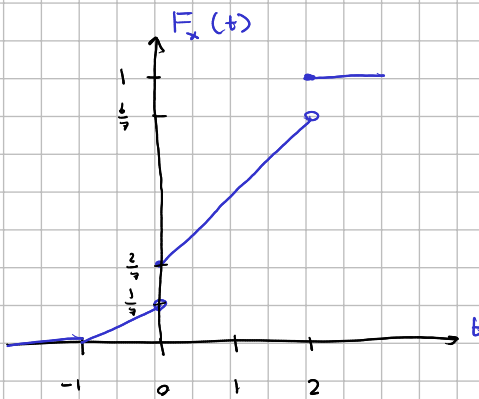
P — prawdopodobieństwo geometryczne

$$X^{-1}((-\infty, t]) = \begin{cases} \emptyset & t < -1 \\ [-4, t-3] & -1 \leq t < 0 \\ [-4, t-3] \cup [2-t, 3] & 0 \leq t < 2 \\ [-4, 3] & t \geq 2 \end{cases}$$

$$F_x(t) = P\left(X^{-1}((-\infty, t])\right) = \begin{cases} 0 & t < -1 \\ \dfrac{t+1}{7} & -1 \leq t < 0 \\ \dfrac{2t+2}{7} & 0 \leq t < 2 \\ 1 & t \geq 2 \end{cases}$$

$$\frac{|[-4, t-3]|}{|[-4, 3]|} = \frac{t-3-(-4)}{7} = \frac{t+1}{7}$$

$$\frac{|[-4, t-3] \cup [2-t, 3]|}{7} = \frac{t+1 + (3-(2-t))}{7} = \frac{2t+2}{7}$$

Rozkład nie jest ani ciągły ani dyskretny

$$S_x = [-1, 2]$$

5.

10 monet
- 2 z dwoma orłami N' inne zadanie
- 8 normalnych N

X — liczba rzutów

a) rzucamy do pierwszego orła

$$p_X(1) = P(O|N')P(N') + P(O|N)P(N) = 1 \cdot \frac{2}{10} + \frac{1}{2} \cdot \frac{8}{10} = \frac{6}{10}$$

$$p_X(2) = \left(0 \cdot \frac{2}{10} + \frac{1}{2} \cdot \frac{8}{10}\right) \cdot \left(1 \cdot \frac{2}{10} + \frac{1}{2} \cdot \frac{8}{10}\right) = \frac{4}{10} \cdot \frac{6}{10}$$

$$p_X(3) = \left(0 \cdot \frac{2}{10} + \frac{1}{2} \cdot \frac{8}{10}\right)^2 \cdot \left(1 \cdot \frac{2}{10} + \frac{1}{2} \cdot \frac{8}{10}\right)$$

$$p_X(x) = \left[P(O'|N')P(N') + P(O'|N)P(N)\right]^{x-1} \cdot \left[P(O|N')P(N') + P(O|N)P(N)\right] = \left(\frac{2}{5}\right)^{x-1} \cdot \frac{3}{5}$$

b) rzucamy do pierwszego orła ale max 3 razy

$$p_X(3) = \left(\frac{2}{5}\right)^{3-1} \cdot \frac{3}{5} + \left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)^2 \left(\frac{3}{5} + \frac{2}{5}\right)$$
                2 razy reszka i raz (orzeł v reszka)

$$p_X(x) = \begin{cases} \left(\frac{2}{5}\right)^{x-1} \cdot \frac{3}{5} & \text{dla} \quad x \in \{1,2\} \\[2mm] \left(\frac{2}{5}\right)^2 & \text{dla} \quad x = 3 \\[2mm] 0 & \text{w p.p.} \end{cases}$$

c) rzucamy do wypadnięcia drugiego orła

$$p_X(x) = \binom{x-1}{1} \left(\frac{2}{5}\right)^{x-2} \left(\frac{3}{5}\right)^2 \qquad x \geqslant 2$$

           → 2 orły

       → $x-2$ reszka

    → za którym z pierwszych $x-1$ razy będzie 1szy orzeł

6.

Dystrybuanta spełnia warunki

- $F: \mathbb{R} \rightarrow [0, 1]$

- $\lim\limits_{t \to -\infty} F(t) = 0 \quad \wedge \quad \lim\limits_{t \to +\infty} F(t) = 1$

- F jest niemalejąca

- F jest co najmniej prawostronnie ciągła

a) $\quad F(t) = \arctan(t)$ ✗

$\quad \lim\limits_{t \to \infty} \arctan(t) = \dfrac{\pi}{2}$

b) $\quad F(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-2t} & t > 0 \end{cases}$ ✗

$\quad$ niedokreślona dla $t = 0$

c) $\quad F(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2} & t = 0 \\ 1 & t > 0 \end{cases}$ ✗

$\quad \lim\limits_{t \to 0^+} F(t) = 1 \neq F(0) = \dfrac{1}{2}$

$\quad$ nie jest prawostronnie ciągła

d) $\quad F(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2}\left(t + \frac{1}{2}\right) & 0 \leq t < 1 \\ 1 & t > 1 \end{cases}$ ✗

$\quad$ niedokreślona dla $t = 1$

7.

Rzucamy kostką, aż suma oczek przekroczy 6

X – liczba rzutów kostką

$F_x(1) = p_x(1) = 0$

$F_x(2) = p_x(2) + p_x(1) = \frac{21}{36} + 0 = \frac{21}{36}$

$F_x(7) = 1$ bo $X = 7$ jest zdarzeniem pewnym   (?)