

$$\int_a^b f(x) dx = F(b) - F(a)$$

$$\int f(x) dx = F(x)$$

$$\int_a^b f(g(t)) g'(t) dt = \left| \begin{array}{l} s = g(t) \\ ds = g'(t) dt \end{array} \right| = \int_{g(a)}^{g(b)} f(s) ds$$

$$\int_{(\frac{\pi}{2})^2}^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}} dx = \left| \begin{array}{l} t = \sqrt{x} \\ dt = \frac{dx}{2\sqrt{x}} \\ dx = 2t dt \end{array} \right| = \int_{\frac{\pi}{2}}^{\pi} \frac{\sin(t)}{t} \cdot 2t dt = 2 \int_{\frac{\pi}{2}}^{\pi} \sin(t) dt = -2 \cos(t) \Big|_{\frac{\pi}{2}}^{\pi} = -2(-1 - 0) = 2$$

1.

$$\begin{aligned} \text{a) } \int_{-a}^a f(x) dx &= \int_{-a}^0 f(x) dx + \int_0^a f(x) dx = \int_0^a f(-x) dx + \int_0^a f(x) dx = \int_0^a [f(x) + f(-x)] dx \\ &\quad \left| \begin{array}{l} t = -x \\ dt = -dx \end{array} \right| = - \int_0^a f(-t) dt = \int_0^a f(-t) dt \end{aligned}$$

$$\text{b) } f(x) = f(-x)$$

$$\begin{aligned} \int_{-a}^a f(x) dx &= \int_{-a}^0 f(x) dx + \int_0^a f(x) dx = \int_0^a f(-x) dx + \int_0^a f(x) dx = 2 \int_0^a f(x) dx \\ &\quad \left| \begin{array}{l} t = -x \\ dt = -dx \end{array} \right| = - \int_0^a f(-t) dt = \int_0^a f(t) dt \end{aligned}$$

$$\begin{aligned} \text{c) } \int_{-a}^a f(x) dx &= \int_{-a}^0 f(x) dx + \int_0^a f(x) dx = \int_0^a f(-x) dx + \int_0^a f(x) dx = \int_0^a [f(x) - f(x)] dx = \int_0^a 0 dx = 0 \\ &\quad \left| \begin{array}{l} t = -x \\ dt = -dx \end{array} \right| = - \int_0^a f(-t) dt = \int_0^a f(t) dt \end{aligned}$$