1.

$$\int e^{4x}\sin(2x)\,dx = \left|\begin{matrix} f = e^{4x} & g' = \sin(2x) \\ f' = 4e^{4x} & g = -\frac{1}{2}\cos(2x)\end{matrix}\right| = e^{4x}\cdot\frac{-1}{2}\cos(2x) - \int 4\cdot e^{4x}\cdot\frac{-1}{2}\cos(2x)\,dx =$$

$$= -\frac{1}{2}e^{4x}\cos(2x) + 2\int e^{4x}\cos(2x)\,dx = \left|\begin{matrix} f = e^{4x} & g' = \cos(2x) \\ f' = 4e^{4x} & g = \frac{1}{2}\sin(2x)\end{matrix}\right| = -\frac{1}{2}e^{4x}\cos(2x) + e^{4x}\sin(2x) - 4\int e^{4x}\sin(2x)\,dx$$

$$\int e^{4x}\sin(2x)\,dx = \frac{1}{2}e^{4x}\left(2\sin(2x) - \cos(2x)\right) - 4\int e^{4x}\sin(2x)\,dx$$

$$5\int e^{4x}\sin(2x)\,dx = \frac{1}{2}e^{4x}\left(2\sin(2x) - \cos(2x)\right) + C$$

$$\int e^{4x}\sin(2x)\,dx = \frac{1}{10}e^{4x}\left(2\sin(2x) - \cos(2x)\right) + C$$

2.

$$\int \frac{-2}{2+\sqrt{3x}}\,dx = \left|\begin{matrix} t = \sqrt{x} \\ dt = \frac{1}{2\sqrt{x}}dx \\ dx = 2t\,dt\end{matrix}\right| = \int \frac{-4t}{\sqrt{3}t+2}\,dt = \int \frac{-4t - \frac{8}{\sqrt{3}} + \frac{8}{\sqrt{3}}}{\sqrt{3}t+2}\,dt = \int \frac{-\frac{4}{\sqrt{3}}(\sqrt{3}t+2) + \frac{8}{\sqrt{3}}}{\sqrt{3}t+2}\,dt = \int -\frac{4}{\sqrt{3}} + \frac{\frac{8}{\sqrt{3}}}{\sqrt{3}t+2}\,dt$$

$$= -\int \frac{4\sqrt{3}}{3}\,dt + \frac{8}{\sqrt{3}}\cdot\frac{1}{\sqrt{3}}\int \frac{dt}{t+\frac{2}{\sqrt{3}}} = -\frac{4\sqrt{3}}{3}t + \frac{8}{3}\ln\left|t + \frac{2}{\sqrt{3}}\right| + C = -\frac{4\sqrt{3}}{3}\sqrt{x} + \frac{8}{3}\ln\left|\sqrt{x} + \frac{2\sqrt{3}}{3}\right| + C$$

$$= -\frac{4\sqrt{3}}{3}\sqrt{x} + \frac{8}{3}\ln\left|\sqrt{x} + \frac{2}{\sqrt{3}}\right| + \frac{8}{3}\ln|\sqrt{3}| + C = -\frac{4\sqrt{3}}{3}\sqrt{x} + \frac{8}{3}\ln|\sqrt{3x} + 2| + C$$

3.

$$\int \frac{4\,dx}{2+\sqrt{x}} = \left|\begin{matrix} t = \sqrt{x} \\ dt = \frac{1}{2\sqrt{x}}dx \\ dx = 2t\,dt\end{matrix}\right| = \int \frac{8t}{t+2}\,dt = \int \frac{8t+16-16}{t+2}\,dt = \int 8 - \frac{16}{t+2}\,dt = \int 8\,dt - 16\int\frac{dt}{t+2}$$

$$= 8t - 16\ln|t+2| + C = 8\sqrt{x} - 16\ln|\sqrt{x}+2| + C$$

4.

$$\int \arcsin^2(x)\,dx = \left|\begin{matrix} x = \sin(t) \quad t\in\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \\ t = \arcsin(x) \\ dx = \cos(t)\,dt\end{matrix}\right| = \int t^2\cos(t)\,dt = \left|\begin{matrix} f = t^2 & g' = \cos(t) \\ f' = 2t & g = \sin(t)\end{matrix}\right| = t^2\sin(t) - 2\int t\sin(t)\,dt$$

$$= \left|\begin{matrix} f = t & g' = \sin(t) \\ f' = 1 & g = -\cos(t)\end{matrix}\right| = t^2\sin(t) - 2\left[-t\cos(t) - \int -\cos(t)\,dt\right] = t^2\sin(t) + 2t\cos(t) - 2\int\cos(t)\,dt$$

$$= t^2\sin(t) + 2t\cos(t) - 2\sin(t) + C = x\arcsin^2(x) + 2\sqrt{1-x^2}\arcsin(x) - 2x + C$$

$$\cos(\arcsin(x)) = \sqrt{1-\sin^2(\arcsin(x))} = \sqrt{1-x^2}$$

5.

$$\int \sin^9(x)\,dx = \int (1-\cos^2(x))^4\sin(x)\,dx = \left|\begin{matrix} t = \cos(x) \\ dt = -\sin(x)dx\end{matrix}\right| = -\int (1-t^2)^4\,dt$$

$$= -\int (1-2t^2+t^4)^2\,dt = -\int 1 - 2t^2 + t^4 - 2t^2 + 4t^4 - 2t^6 + t^4 - 2t^6 + t^8\,dt = -\int 1 - 4t^2 + 6t^4 - 4t^6 + t^8\,dt$$

$$= -t + \frac{4}{3}t^3 - \frac{6}{5}t^5 + \frac{4}{7}t^7 - \frac{1}{9}t^9 + C$$

$$= -\cos(x) + \frac{4}{3}\cos^3(x) - \frac{6}{5}\cos^5(x) + \frac{4}{7}\cos^7(x) - \frac{1}{9}\cos^9(x) + C$$

$$\int \sin^3(x)\,dx = \int \sin^2(x)\sin(x)\,dx = \int (1-\cos^2(x))\sin(x)\,dx = \left|\begin{matrix} t = \cos(x) \\ dt = -\sin(x)dx\end{matrix}\right| = -\int 1-t^2\,dt = \int t^2 - 1\,dt = \frac{1}{3}t^3 - t + C = \frac{1}{3}\cos^3(x) - \cos(x) + C$$

6.

$$\int \sin^5(x)\,dx = \int (1-\cos^2(x))^2\sin(x)\,dx = \left|\begin{matrix} t = \cos(x) \\ dt = -\sin(x)dx\end{matrix}\right| = -\int (1-t^2)^2\,dt = -\int 1 - 2t^2 + t^4\,dt = \int -t^4 + 2t^2 - 1\,dt = -\frac{1}{5}\cos^5(x) + \frac{2}{3}\cos^3(t) - \cos(x) + C$$

7. $$\int \frac{dx}{\sin(6x)\cos^2(6x)} = \int \frac{\sin^2(6x)+\cos^2(6x)}{\sin(6x)\cos^2(6x)}\,dx = \int \frac{\sin(6x)}{\cos^2(6x)}\,dx + \int \frac{1}{\sin(6x)}\,dx$$

$$\int \frac{\sin(6x)}{\cos^2(6x)}\,dx = \left| \begin{aligned} t &= \cos(6x) \\ dt &= -6\sin(6x)\,dx \end{aligned} \right| = -\frac{1}{6}\int \frac{dt}{t^2} = -\frac{1}{6}\int t^{-2}\,dt = -\frac{1}{6}\cdot(-1)\,t^{-1} + C = \frac{1}{6\cos(6x)} + C$$

$$\int \frac{1}{\sin(6x)}\,dx = \int \frac{\sin(6x)}{\sin^2(6x)}\,dx = \int \frac{\sin(6x)}{1-\cos^2(6x)}\,dx = \left| \begin{aligned} t &= \cos(6x) \\ dt &= -6\sin(6x)\,dx \end{aligned} \right| = -\frac{1}{6}\int \frac{1}{1-t^2}\,dt =$$

$$\frac{1}{1-t^2} = \frac{-1}{t^2-1} = \frac{A}{t-1} + \frac{B}{t+1} = \frac{A(t+1)+B(t-1)}{t^2-1} = \frac{-\frac{1}{2}}{t-1} + \frac{\frac{1}{2}}{t+1}$$

$-1 = A(t+1) + B(t-1)$

$-1 = -2B \qquad B = \frac{1}{2}$

$-1 = 2A \qquad A = -\frac{1}{2}$

$$\int \frac{1}{\sin(6x)}\,dx = -\frac{1}{6}\left[\int \frac{-\frac{1}{2}}{t-1}\,dt + \int \frac{\frac{1}{2}}{t+1}\,dt\right] = -\frac{1}{6}\left[-\frac{1}{2}\ln|t-1| + \frac{1}{2}\ln|t+1|\right] + C = \frac{1}{12}\ln|\cos(6x)-1| - \frac{1}{12}\ln|\cos(6x)+1| + C$$

$$\int \frac{dx}{\sin(6x)\cos^2(6x)} = \frac{1}{6\cos(6x)} + \frac{1}{12}\ln|\cos(6x)-1| - \frac{1}{12}\ln|\cos(6x)+1| + C = -\frac{1}{12}\ln\left|\frac{\cos(6x)+1}{\cos(6x)-1}\right| + \frac{1}{6\cos(6x)} + C$$

8. $$\int \frac{7}{(x^2+9)^n}\,dx = 7\int \frac{dx}{(x^2+9)^n} = \left| \begin{aligned} t &= \tfrac{1}{3}x \\ 3t &= x \\ 3\,dt &= dx \end{aligned} \right| = 7\int \frac{3\,dt}{(9t^2+9)^n} = 21\int \frac{dt}{9^n(t^2+1)^n} = \frac{21}{9^n}\int \frac{dt}{(t^2+1)^n}$$

$$= \frac{21}{9^n}\left[\frac{1}{2(n-1)}\cdot\frac{t}{(1+t^2)^{n-1}} + \frac{2n-3}{2n-2}\int \frac{dt}{(1+t^2)^{n-1}}\right] = \frac{21\cdot\frac{1}{3}x}{9^n\cdot 2(n-1)\left(1+\frac{1}{9}x^2\right)^{n-1}} + \frac{21}{9^n}\cdot\frac{2n-3}{2n-2}\int \frac{\frac{1}{3}\,dx}{\left(1+\frac{1}{9}x^2\right)^{n-1}}$$

$$= \frac{7x}{18(n-1)(x^2+9)^{n-1}} + \frac{7}{18}\cdot\frac{2n-3}{n-1}\int \frac{dx}{(x^2+9)^{n-1}}$$

9. $$\int \frac{dx}{6\cos(x)+10} \quad \left| \begin{aligned} t &= \tan\left(\tfrac{x}{2}\right) \\ dt &= \frac{1}{\cos^2\left(\frac{x}{2}\right)}\cdot\frac{1}{2}\,dx \end{aligned} \qquad \begin{aligned} \frac{1}{\cos^2\left(\frac{x}{2}\right)} &= \frac{\cos^2\left(\frac{x}{2}\right)+\sin^2\left(\frac{x}{2}\right)}{\cos^2\left(\frac{x}{2}\right)} = 1+\tan^2\left(\tfrac{x}{2}\right) \\ dx &= \frac{2\,dt}{1+t^2} \end{aligned} \right| = \int \frac{\frac{2}{1+t^2}}{6\,\frac{1-t^2}{1+t^2}+10}\,dt = \int \frac{\frac{2}{1+t^2}}{\frac{6-6t^2+10+10t^2}{1+t^2}}\,dt = \int \frac{2}{4t^2+16}\,dt$$

$$\cos(x) = \frac{1-\tan^2\left(\frac{x}{2}\right)}{1+\tan^2\left(\frac{x}{2}\right)}$$

$$= \frac{1}{8}\int \frac{dt}{\frac{1}{4}t^2+1} = \frac{1}{8}\int \frac{dt}{\left(\frac{1}{2}t\right)^2+1} \left| \begin{aligned} u &= \tfrac{1}{2}t \\ du &= \tfrac{1}{2}\,dt \end{aligned} \right| = \frac{1}{4}\int \frac{du}{u^2+1} = \frac{1}{4}\arctan(u) + C = \frac{1}{4}\arctan\left(\tfrac{1}{2}t\right) + C =$$

$$= \frac{1}{4}\arctan\left(\frac{1}{2}\tan\left(\frac{x}{2}\right)\right) + C$$

10. $$\int \frac{9\,dx}{\sqrt[3]{4x}+\sqrt{4x}} = \left| \begin{aligned} t &= (4x)^{\frac{1}{6}} \\ dt &= \tfrac{1}{6}(4x)^{-\frac{5}{6}}\cdot 4\,dx \\ dx &= \tfrac{6}{4}\cdot(4x)^{\frac{5}{6}}\,dt = \tfrac{3}{2}t^5\,dt \end{aligned} \right| = \int \frac{9\cdot\frac{3}{2}t^5}{t^2+t^3}\,dt = \frac{27}{2}\int \frac{t^5}{t^2+t^3}\,dt = \frac{27}{2}\int \frac{t^3}{t+1}\,dt = \frac{27}{2}\int t^2-t+1-\frac{1}{t+1}\,dt$$

$$\frac{27}{2}\left[\int t^2-t+1\,dt - \int \frac{dt}{t+1}\right] = \frac{27}{2}\left[\frac{1}{3}t^3 - \frac{1}{2}t^2 + t - \ln|t+1|\right] + C$$

$$t^3 : (t+1) = t^2 - t + 1 \qquad \frac{t^3}{t+1} = t^2 - t + 1 - \frac{1}{t+1}$$

$$\begin{array}{l} t^3 \\ -(t^3+t^2) \\ \hline -t^2 \\ -(-t^2-t) \\ \hline t \\ -(t+1) \\ \hline -1 \end{array}$$

$$= \frac{9}{2}\sqrt{4x} - \frac{27}{4}\sqrt[3]{4x} + \frac{27}{2}\sqrt[6]{4x} - \frac{27}{2}\ln\left|\sqrt[6]{4x}+1\right| + C$$

11. $$\int \sqrt{\frac{x-1}{x-4}} \cdot \frac{dx}{(x-1)^2} = \int \frac{t}{\left(\frac{4t^2-1}{t^2-1}-1\right)^2} \cdot \frac{-6t}{(t^2-1)^2}\,dt = \int \frac{-6t^2\,dt}{\left(\frac{4t^2\cancel{-1}-t^2\cancel{+1}}{t^2-1}\right)^2 \cdot (t^2-1)^2} = \int \frac{-6t^2\,dt}{\frac{3t^4}{\cancel{(t^2-1)^2}}\cancel{(t^2-1)^2}} = \int \frac{-2\,dt}{3t^2} = -\frac{2}{3}\int t^{-2}\,dt$$

$$t^2 = \frac{x-1}{x-4}$$

$$t^2x - 4t^2 = x - 1$$

$$t^2x - x = 4t^2 - 1$$

$$x = \frac{4t^2-1}{t^2-1}$$

$$dx = \frac{8t(t^2-1) - (4t^2-1)\cdot 2t}{(t^2-1)^2}\,dt$$

$$dx = \frac{\cancel{8t^3} - 8t \cancel{-8t^3} + 2t}{(t^2-1)^2}\,dt$$

$$dx = \frac{-6t}{(t^2-1)^2}\,dt$$

$$= -\frac{2}{3}\cdot(-1)\,t^{-1} + C = \frac{2}{3}\sqrt{\frac{x-4}{x-1}} + C$$