1.

$$\lim_{n \to +\infty} \left(1 + \frac{\sqrt{n}}{n^2+1} \right)^{n(2\sqrt{n}+1)}$$

$$\frac{\sqrt{n}}{n^2+1} = \frac{1}{\frac{n^2+1}{\sqrt{n}}} \qquad \lim_{n \to \infty} \frac{n^2+1}{\sqrt{n}} = +\infty$$

$$n(2\sqrt{n}+1) = \frac{n^2+1}{\sqrt{n}} \cdot \frac{\sqrt{n}}{n^2+1} \cdot n(2\sqrt{n}+1) = \frac{n^2+1}{\sqrt{n}} \cdot \frac{2n^2 + n\sqrt{n}}{n^2+1}$$

$$\lim_{n \to \infty} \frac{2n^2 + \sqrt{n}}{n^2+1} = 2 \qquad \lim_{n \to \infty} \left(1 + \frac{1}{t} \right)^n = e$$

$$\lim_{n \to \infty} \left(1 + \frac{\sqrt{n}}{n^2+1} \right)^{n(2\sqrt{n}+1)} = \lim_{n \to \infty} \left(\left(1 + \frac{1}{\frac{n^2+1}{\sqrt{n}}} \right)^{\frac{n^2+1}{\sqrt{n}}} \right)^{\frac{2n^2+n\sqrt{n}}{n^2+1}} = e^2$$

2. **?**

$$\lim_{x \to -\infty} \left(\frac{1}{\pi} \operatorname{arccot}(x) \right)^x = e^{x \ln\left(\frac{1}{\pi} \operatorname{arccot}(x) \right)} = e$$

$$\cot(x) = \frac{\cos(x)}{\sin(x)}$$
$$x \to \pi \to \frac{-1}{0^+} = -\infty$$

$$\lim_{x \to -\infty} \operatorname{arccot}(x) = \pi \qquad \lim_{x \to -\infty} x \ln\left(\frac{1}{\pi} \operatorname{arccot}(x) \right) = \lim_{x \to -\infty} \frac{\ln\left(\frac{1}{\pi} \operatorname{arccot}(x) \right)}{\frac{1}{x}} \overset{\frac{0}{0}}{\underset{H}{=}} \lim_{x \to -\infty} \frac{\frac{1}{\frac{1}{\pi}\operatorname{arccot}(x)} \cdot \frac{-1}{x^2+1}}{-\frac{1}{x^2}} = \lim_{x \to -\infty} \frac{\pi x^2}{\operatorname{arccot}(x)(x^2+1)}$$

$$\overset{\frac{\infty}{\infty}}{\underset{H}{=}} \lim_{x \to -\infty} \frac{2\pi x}{\frac{-1}{x^2+1} \cdot (x^2+1) + \operatorname{arccot}(x) \cdot 2x} = \lim_{x \to -\infty} \frac{2\pi x}{-1 + 2x\operatorname{arccot}(x)} = \lim_{x \to -\infty} \frac{\pi}{-\frac{1}{2x} + \operatorname{arccot}(x)} = \frac{\pi}{0+\pi} = 1$$

3. $\int e^{x-x^2}(x^2-x)(2x-1)\,dx = \begin{vmatrix} t = x^2-x \\ dt = (2x-1)dx \end{vmatrix} = \int t e^{-t}\,dt = \begin{vmatrix} f=t & g'=e^{-t} \\ f'=1 & g=-e^{-t} \end{vmatrix} = -te^{-t} + \int e^{-t} = -te^{-t} - e^{-t} + C = -e^{-t}(t+1) + C$

$$= -e^{x-x^2}(x^2-x+1) + C$$

$$\int_0^\infty e^{x-x^2}(x^2-x)(2x-1)\,dx = \lim_{T \to +\infty} -e^{T-T^2}(T^2-T+1) + e^0 \cdot 0 = \lim_{T \to +\infty} \frac{-T^2+T-1}{e^{T^2-T}} \overset{\frac{\infty}{\infty}}{\underset{H}{=}} \lim_{T \to +\infty} \frac{-2T+1}{(2T-1)e^{T^2-T}} = \lim_{T \to +\infty} \frac{-1}{e^{T^2-T}} = 0$$

całka zbieżna

5.

$$xy' = xe^{\frac{y}{x}} + y$$
$$y' = e^{\frac{y}{x}} + \frac{y}{x} \qquad v = \frac{y}{x}$$
$$v'x + v = e^v + v$$
$$v'x = e^v$$
$$\frac{dv}{dx} = \frac{e^v}{x} \qquad \int \frac{dv}{e^v} = \int \frac{dx}{x}$$
$$-e^{-v} = \ln|x| + C$$
$$e^{-v} = -\ln|x| + C$$
$$-v = \ln(-\ln|x|) + C$$
$$y = -x\ln(-\ln|x|) + Cx$$

6. $\quad y'' - 2y' + y = \dfrac{e^x}{x^2} + 1$

$r^2 - 2r + 1 = (r-1)^2 = 0 \quad r_0 = 1$

$y(x) = C_1 e^x + C_2 x e^x \qquad \text{CORJ}$

$$\begin{bmatrix} e^x & x e^x \\ e^x & x e^x + e^x \end{bmatrix} \cdot \begin{bmatrix} C_1' \\ C_2' \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{e^x}{x^2} + 1 \end{bmatrix}$$

$C_1' = \dfrac{-(x e^x + e^x)\left(\frac{e^x}{x^2}+1\right)}{e^x(x e^x + e^x) - x e^x\, e^x} = \dfrac{-\left(\frac{e^{2x}}{x} + x e^x + \frac{e^{2x}}{x^2} + e^x\right)}{x e^{2x} + e^{2x} - x e^{2x}} = \dfrac{e^{2x}\left(\frac{-1}{x} - x e^{-x} - \frac{1}{x^2} - e^{-x}\right)}{e^{2x}} = -\dfrac{1}{x} - x e^{-x} - \dfrac{1}{x^2} - e^{-x}$

$C_2' = \dfrac{e^x\left(\frac{e^x}{x^2}+1\right)}{e^{2x}} = \dfrac{1}{x^2} + e^{-x}$

$\displaystyle\int e^{-x}\,dx = -e^{-x} \qquad \int \dfrac{1}{x}\,dx = \ln|x| \qquad \int \dfrac{1}{x^2}\,dx = -\dfrac{1}{x}$

$\displaystyle\int x e^{-x}\,dx = \begin{vmatrix} f = x & g' = e^{-x} \\ f' = 1 & g = -e^{-x} \end{vmatrix} = -x e^{-x} + \int e^{-x}\,dx = -x e^{-x} - e^{-x} = -e^{-x}(x+1)$

$C_1 = -\displaystyle\int \dfrac{1}{x}\,dx - \int x e^{-x}\,dx - \int \dfrac{1}{x^2}\,dx - \int e^{-x}\,dx = -\ln|x| + e^{-x}(x+1) + \dfrac{1}{x} + e^{-x} + C_1 = -\ln|x| + \dfrac{1}{x} + e^{-x}(x+2) + C_1$

$C_2 = \displaystyle\int \dfrac{1}{x^2}\,dx + \int e^{-x}\,dx = -\dfrac{1}{x} - e^{-x} + C_2$

$y(x) = \left(-\ln|x| + \dfrac{1}{x} + e^{-x}(x+2) + C_1\right)e^x + \left(-\dfrac{1}{x} - e^{-x} + C_2\right)x e^x$

$\qquad = -e^x \ln|x| + \dfrac{e^x}{x} + \cancel{x} + 2 + C_1 e^x - e^x - \cancel{x} + C_2 x e^x$

$\qquad = C_1 e^x + C_2 x e^x + \dfrac{e^x}{x} - e^x \ln|x| + 2 \qquad \text{CORN}$