$$\int_a^b f(x)\,dx = F(b) - F(a)$$

$$\int f(x)\,dx = F(x)$$

$$\int_a^b f(g(t))\,g'(t)\,dt = \left| \begin{array}{l} s = g(t) \\ ds = g'(t)\,dt \end{array} \right| = \int_{g(a)}^{g(b)} f(s)\,ds$$

$$\int_{(\frac{\pi}{2})^2}^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = \left| \begin{array}{l} t = \sqrt{x} \\ dt = \frac{dx}{2\sqrt{x}} \\ dx = 2t\,dt \end{array} \right| = \int_{\frac{\pi}{2}}^{\pi} \frac{\sin(t)}{t} \cdot 2t\,dt = 2\int_{\frac{\pi}{2}}^{\pi} \sin(t)\,dt = -2\cos(t)\Big|_{\frac{\pi}{2}}^{\pi} = -2(-1-0) = 2$$

1.

a) $$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_0^a f(x)\,dx = \int_0^a f(-x)\,dx + \int_0^a f(x)\,dx = \int_0^a [f(x) + f(-x)]\,dx$$

$$\left| \begin{array}{l} t = -x \\ dt = -dx \end{array} \right| = -\int_a^0 f(-t)\,dt = \int_0^a f(-t)\,dt$$

b) $f(x) = f(-x)$

$$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_0^a f(x)\,dx = \int_0^a f(-x)\,dx + \int_0^a f(x)\,dx = 2\int_0^a f(x)\,dx$$

$$\begin{array}{l} t = -x \\ dt = -dx \end{array} \quad -\int_a^0 f(-t)\,dt = \int_0^a f(-t)\,dt$$

c) $$\int_{-a}^{a} f(x)\,dx = \int_{-a}^{0} f(x)\,dx + \int_0^a f(x)\,dx = \int_0^a f(-x)\,dx + \int_0^a f(x)\,dx = \int_0^a [f(x) - f(x)]\,dx = \int_0^a 0\,dx = 0$$

$$\begin{array}{l} t = -x \\ dt = -dx \end{array} = -\int_a^0 f(-t)\,dt = \int_0^a f(-t)\,dt$$