Zestaw zadań 1

1.

a) $\sim(\exists x\in \mathbb{R}\ \forall y\in\mathbb{R}\ \ x \geq y)$ zdanie logiczne (prawda)

b) $\phi(p):\ \forall n\in\mathbb{N}\ (n\neq 1 \wedge n\neq p) \Rightarrow p \bmod n \neq 0$? $p\in\mathbb{N} \wedge p>1 \wedge \sim[\exists x,y\in\mathbb{N}\ \ y,x>1 \ \wedge\ yx=p]$

c) $a \bmod 8 = 0 \wedge b \bmod 8 = 0 \ \wedge \ \sim(\exists n\in\mathbb{R}\ \ n>8 \wedge a \bmod n = 0 \wedge b \bmod n = 0)$

d) $\sim(\forall n\in\mathbb{N}\ \ 2n+1 \equiv 0 \bmod 3)$

e) $\forall n\in\mathbb{N}\ \exists r\in\mathbb{R}\ \ n = r^2$

f) $k\in\mathbb{N} \wedge (k \not\equiv 0 \bmod 7 \vee k\equiv 0 \bmod 3)$

g) $\exists x\in\mathbb{R}_-\ \forall y\in\mathbb{R}_-\setminus\{x\}\ \ x>y$

2.

a) tak a') nie istnieje największa liczba naturalna

b) nie, nie istnieje mniejsza liczba naturalna od 1

b') tak, $y=1$

c) nie, dla $y=x+1$ nie istnieje takie z

3. $x\in\mathbb{R}$

a) $\varphi(x):\ x<e \Rightarrow x>\pi$

$x<e \Rightarrow x>\pi$

$\Leftrightarrow \sim(x<e) \vee x>\pi$

$x\geq e \vee x>\pi$

$\{x\in\mathbb{R}:\ x\geq e \vee x>\pi\}$

$[e,+\infty)$

b) $\varphi(x):\ x<e \Leftrightarrow x\leq\pi$

$\{x\in\mathbb{R}:\ (x<e \wedge x\leq\pi) \vee (x\geq e \wedge x>\pi)\}$

$(-\infty,e)\cup(\pi,+\infty)$

c) $\varphi(x):\ \exists y\in\mathbb{R}\ \ x<\sin(y)$ $-1<\sin(y)<1$

$\{x\in\mathbb{R}: \varphi(x)\} = (-\infty, 1)$

d) $\varphi(x):\ \forall y\in\mathbb{R}\ \ x<y^2+\pi$

$0\leq y^2$

$\pi\leq y^2+\pi$

$\{x\in\mathbb{R}: \varphi(x)\} = (-\infty,\pi)$

e) $\varphi(x):\ x>e \Rightarrow (\forall y\in\mathbb{R}\ \ x<y^2+\pi)$

$\{x\in\mathbb{R}:\ (x>e \wedge \forall y\in\mathbb{R}\ x<y^2+\pi) \vee (x\leq e)\} = (-\infty,e] \cup (e,\pi) = (-\infty,\pi)$

f) $\varphi(x):\ (\exists y\in\mathbb{R}\ x<\sin(y)) \Leftrightarrow x>e$

?

~~$\{x\in\mathbb{R}: \varphi(x)\} = [(-\infty,1)\cap(e,+\infty)] \cup [[1,+\infty)\cap(-\infty,e]] = \emptyset \cup [1,e] = [1,e]$~~

4.

...

5.

a) $\exists x \in R \, [x < \pi \Rightarrow \sin(x) > \pi]$

zdanie prawdziwe, dla $x \in [\pi, +\infty)$ jest $0 \Rightarrow 0$

$\sim(\exists x \in \mathbb{R} \, [x < \pi \Rightarrow \sin(x) > \pi])$

$\forall x \in R \sim [x < \pi \Rightarrow \sin(x) > \pi]$

$\forall x \in R \, [x < \pi \land \sin(x) \leq \pi]$

b) $(\exists x \in \mathbb{R} \; x < \pi) \Rightarrow (\exists x \in \mathbb{R} \; \sin(x) > \pi)$

zdanie fałszywe $1 \Rightarrow 0$

$\sim[(\exists x \in R \; x < \pi) \Rightarrow (\exists x \in R \; \sin(x) > \pi)]$

$(\exists x \in R \; x < \pi) \land \sim(\exists x \in R \; \sin(x) > \pi)$

$(\exists x \in R \; x < \pi) \land (\forall x \in R \; \sin(x) \leq \pi)$

c) $\forall x, y \in \mathbb{R} \, [x < y \Rightarrow \exists q \in Q \, (x < q < y)]$

zdanie prawdziwe

$\sim[\forall x, y \in R \, [x < y \Rightarrow \exists q \in Q \, (x < q < y)]]$

$\exists x, y \in R \sim [x < y \Rightarrow \exists q \in Q \, (x < q < y)]$

$\exists x, y \in R \; x < y \land \sim[\exists q \in Q \, (x < q \land q < y)]$

$\exists x, y \in R \; x < y \land \forall q \in Q \, (x \geq q \lor q \geq y)$

1

c) $\phi(a,b): \quad a,b \in \mathbb{N} \wedge \exists c \in \mathbb{N} \ a = 8c \wedge \exists d \in \mathbb{N} \ b = 8d \wedge \sim\left(\exists e \in \mathbb{N}_{2} \ \exists f \in \mathbb{N} \ \exists g \in \mathbb{N} \ ef = a \wedge eg = b\right)$

d) $\sim\left(\forall x \in \mathbb{N} \ \exists n \in \mathbb{N} \left(x = 2n - 1 \Longrightarrow \exists a \in \mathbb{N} \ x = 3a\right)\right)$

f) $\phi(k): \quad \sim(\exists n \in \mathbb{N} \ k = 7n) \vee \exists m \in \mathbb{N} \ k = 3m$