1.

$$f(x) = \frac{3^{\frac{1}{x}} + 5}{5^{\frac{1}{x}} + 3}$$

$$f'_-(0) = \lim_{\Delta x \to 0^-} \frac{f(0+\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \to 0^-} \frac{\frac{3^{\frac{1}{\Delta x}} + 5}{5^{\frac{1}{\Delta x}} + 3} - \frac{3^0 + 5}{5^0 + 3}}{\Delta x}$$

$$\lim_{\Delta x \to 0^-} \frac{\frac{3^{\frac{1}{\Delta x}} + 5}{5^{\frac{1}{\Delta x}} + 3} - \frac{6}{4}}{\Delta x} = \left\{ \frac{\frac{5}{3} - \frac{3}{2}}{0^-} = \frac{\frac{1}{6}}{0^-} \right\} = -\infty \qquad \lim_{\Delta x \to 0^-} 3^{\frac{1}{\Delta x}} = \left\{ 3^{\frac{1}{0^-}} = 3^{-\infty} = 0 \right\}$$

nie istnieje skończona pochodna $f'_-(0)$ więc nie istnieje $f'(0)$

f nie jest ciągła w 0

2.

$$f(x) = \arcsin\left(\frac{1+x}{1-x}\right) \qquad \frac{d}{dt} \arcsin(t) = \frac{1}{\sqrt{1-t^2}}$$

$D_{\arcsin} = [-1, 1]$ $\quad 1 \notin D_f$

$$\frac{1+x}{1-x} \geq -1 \qquad\qquad \frac{1+x}{1-x} \leq 1$$

$$\frac{1+x}{1-x} + 1 \geq 0 \qquad\qquad \frac{1+x}{1-x} - 1 \leq 0$$

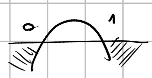
$$\frac{1+x+1-x}{1-x} \geq 0 \qquad\qquad \frac{1+x-1+x}{1-x} \leq 0$$

$$\frac{2}{1-x} \geq 0 \qquad\qquad \frac{2x}{1-x} \leq 0$$

$$2(1-x) \geq 0 \qquad\qquad 2x(1-x) \leq 0$$

$$1 - x \geq 0 \qquad\qquad -2x(x-1) \leq 0$$

$$1 \geq x$$



$x \in (-\infty, 0] \cup [1, +\infty)$

$D_f = (-\infty, 0]$

Nie może mieć asymptot pionowych

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \arcsin\left(\frac{1+x}{1-x}\right) = \lim_{x \to -\infty} \arcsin\left(\frac{x(1+\frac{1}{x})}{x(-1+\frac{1}{x})}\right)$$

$$= \lim_{x \to -\infty} \arcsin\left(\frac{1+\frac{1}{x}}{-1+\frac{1}{x}}\right) = \arcsin(-1) = -\frac{\pi}{2}$$

asymptota pozioma lewostronna $y = -\frac{\pi}{2}$

3.

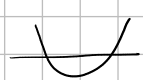
$$\lim_{x \to -\infty} \frac{\sqrt{2-x} - \sqrt{1-x}}{\sqrt{x^2+2x+3} + x} = \lim_{x \to -\infty} \frac{(\sqrt{2-x} - \sqrt{1-x})(\sqrt{2-x} + \sqrt{1-x})}{(\sqrt{x^2+2x+3} + x)(\sqrt{2-x} + \sqrt{1-x})}$$

$$\lim_{x \to -\infty} \frac{2-x-1+x}{\sqrt{(x^2+2x+3)(2-x)} + \sqrt{(x^2+2x+3)(1-x)} + \sqrt{x^2(2-x)} + \sqrt{x^2(1-x)}}$$

$$\lim_{x \to -\infty} \frac{1}{\sqrt{(x^2+2x+3)(2-x)} + \sqrt{(x^2+2x+3)(1-x)} + \sqrt{x^2(2-x)} + \sqrt{x^2(1-x)}} = \left\{ \frac{1}{+\infty + \infty + \infty + \infty} = \frac{1}{+\infty} \right\} = 0$$

$\downarrow$ $+\infty$ $\quad$ $\downarrow$ $+\infty$ $\quad$ $\downarrow$ $+\infty$ $\quad$ $\downarrow$ $+\infty$ $\quad$ $\downarrow$ $+\infty$ $\quad$ $\downarrow$ $+\infty$

$$x^2 + 3x + x + 3 = x(x+3) + (x+3) = (x+1)(x+3)$$



4.

$$f(x) = \begin{cases} \left(\frac{x^2+3x+2}{x+1}\right)^{\frac{3}{x+1}} & \text{dla} \quad x \neq -1 \wedge x > -2 \\ 3 & \text{dla} \quad x = -1 \end{cases}$$

$$x^2+3x+2 = x^2+2x+x+2 = (x+1)(x+2)$$

$$\lim_{x \to -1} \left(\frac{x^2+3x+2}{x+1}\right)^{\frac{3}{x+1}} = \lim_{x \to -1} \left[\frac{(x+1)(x+2)}{x+1}\right]^{\frac{3}{x+1}} = \lim_{x \to -1} (x+2)^{\frac{3}{x+1}}$$

$$\lim_{x \to -1} \left[\left(1+(x+1)\right)^{\frac{1}{x+1}}\right]^3 = e^3 \neq f(-1)$$

5.

$$f(x) = \begin{cases} \frac{\sin(x) - x}{x} & \text{dla} \quad x < 0 \\ a & \text{dla} \quad x = 0 \\ b + \frac{1-\cos(x)}{x^2} & \text{dla} \quad x > 0 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin(x) - x}{x} = \lim_{x \to 0^-} \frac{\sin(x)}{x} - 1 = 1 - 1 = 0$$

$$f(0) = \lim_{x \to 0^-} f(x) \iff a = 0$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \left[b + \frac{1-\cos(x)}{x^2}\right] = b + \lim_{x \to 0^+} \frac{1-\sqrt{\cos^2(x)}}{x^2} = b + \lim_{x \to 0^+} \left(\frac{\sin(x)}{x}\right)^2 \cdot \frac{1}{1+\sqrt{1-\sin^2(x)}}$$

$$= b + \frac{1}{2} \qquad b + \frac{1}{2} = 0 \iff b = -\frac{1}{2}$$

$$\lim_{x \to 0^+} \frac{1-\cos(x)}{x^2} = \frac{1-\sqrt{\cos^2(x)}}{x^2} = \frac{1-\sqrt{1-\sin^2(x)}}{x^2} \cdot \frac{1+\sqrt{1-\sin^2(x)}}{1+\sqrt{1-\sin^2(x)}}$$

$$= \frac{1-(1-\sin^2(x))}{x^2 \cdot (1+\cos(x))} = \frac{1-1+\sin^2(x)}{x^2 \cdot (1+\cos(x))} = \frac{\sin^2(x)}{x^2} \cdot \frac{1}{1+\cos(x)} \longrightarrow 1 \cdot \frac{1}{2} = \frac{1}{2}$$