1

$$A \div B = (A \setminus B) \cup (B \setminus A)$$

a) $A \div B = \emptyset \overset{?}{\Leftrightarrow} A = B$

niech $A \div B = \emptyset$

$\Leftrightarrow (A \setminus B) \cup (B \setminus A) = \emptyset$

$\Leftrightarrow A \setminus B = \emptyset \wedge B \setminus A = \emptyset$

$\Leftrightarrow A \subseteq B \wedge B \subseteq A$

$\Leftrightarrow A = B$

b) $A \div \emptyset \overset{?}{=} A$

$A \div \emptyset = (A \setminus \emptyset) \cup (\emptyset \setminus A) = A \cup \emptyset = A$

c) $A \div B = (A \cup B) - (A \cap B)$

$(A \cup B) - (A \cap B) = \left[(A \setminus B) \cup (B \setminus A) \cup (A \cap B) \right] - (A \cap B) = (A \setminus B) \cup (B \setminus A) = A \div B$

2

$A = \{\emptyset, \mathbb{R}\}$ $\qquad B = \{0, \emptyset\}$ $\qquad C = \{\mathbb{R}, \mathbb{R}^2\}$

a) $B \setminus \mathbb{R} = \{\emptyset\}$

b) $(A \cup B) \setminus C = \{0, \emptyset, \mathbb{R}\} \setminus \{\mathbb{R}, \mathbb{R}^2\} = \{0, \emptyset\} = B$

c) $(A \div B) \cap \mathbb{R} = (\{\mathbb{R}\} \cup \{0\}) \cap \mathbb{R} = \{\mathbb{R}, 0\} \cap \mathbb{R} = \{0\}$

d) $2^B = \{\emptyset, \{0\}, \{\emptyset\}, \{0, \emptyset\}\}$

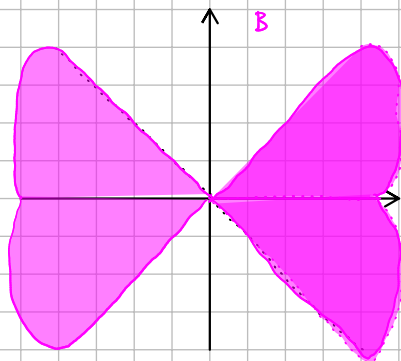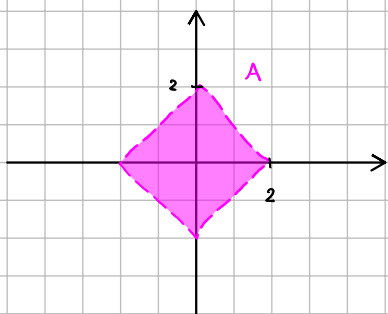e) $A \times B = \{(\emptyset, 0), (\emptyset, \emptyset), (\mathbb{R}, 0), (\mathbb{R}, \emptyset)\}$

3

$A = \{(x,y) \in \mathbb{R}^2 : |x| + |y| < 2\}$
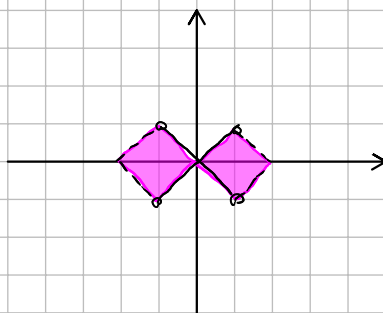
$B = \{(x, y) \in \mathbb{R}^2 : |y| \leq |x|\}$

$y \leq x$ dla $y \geq 0$ $x \geq 0$

$-y \leq x$ dla $y < 0$ $x \geq 0$

$-y \geq x$ $y \leq -x$ dla $y \geq 0$ $x < 0$

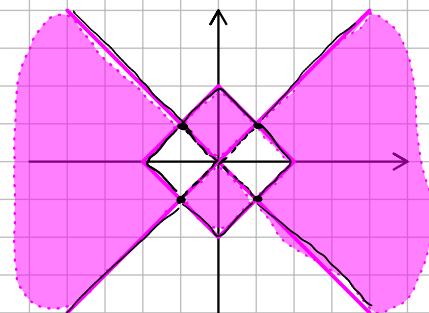$y \geq x$ $-y \leq -x$ dla $y < 0$ $x < 0$

A

B
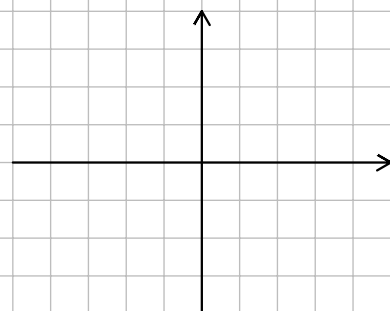
a) $A \cup B$
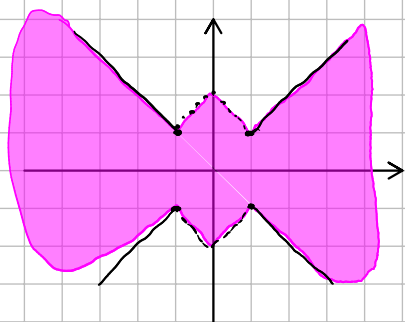
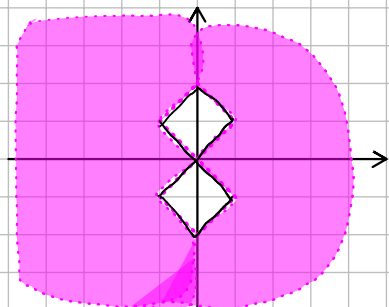b) $A \cap B$

c) $A \div B$

d) $\{(x,y) \in \mathbb{R}^2 : (x,y) \in A \Longrightarrow (x,y) \in B\}$

$\{(x,y) \in \mathbb{R}^2 : (x,y) \notin A \lor (x,y) \in B\}$

$= \mathbb{R}^2 - \{A \setminus B\}$

$$\sim(\sim(p \Rightarrow q)) \quad {\scriptstyle p \Rightarrow q}$$
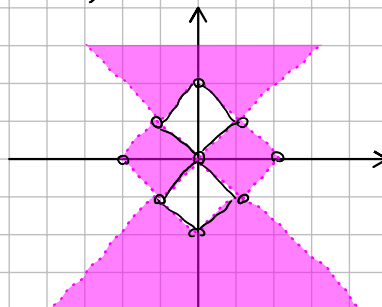$$\sim(p \land \sim q)$$
$$\sim p \lor q$$

e) $\{(x,y) \in \mathbb{R}^2 : (x,y) \in A \Longleftrightarrow (x,y) \in B\}$

$\{(x,y) \in \mathbb{R}^2 : ((x,y) \in A \land (x,y) \in B) \lor ((x,y) \notin A \land (x,y) \notin B)\}$

$\{(x,y) \in \mathbb{R}^2 : (x,y) \in A \cap B \lor (x,y) \notin A \cup B\}$

$\mathbb{R}^2 - (A \div B)$

4.

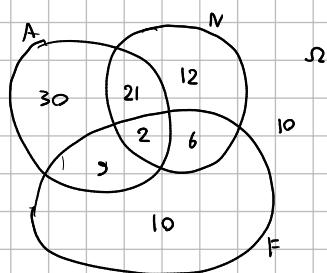$|A| = 62 \qquad |A \cap N| = 23 \qquad |A \cap N \cap F| = 2$

$|N| = 41 \qquad |A \cap F| = 11$

$|F| = 27 \qquad |N \cap F| = 8 \qquad |\Omega| = 100$

$|A \cup N \cup F| = 90$

$|\Omega \setminus (A \cup N \cup F)| = 10$

$|A \cup N \cup F| = |A| + |N| + |F| - |A \cap N| - |A \cap F| - |N \cap F| + |A \cap N \cap F|$

$62 + 41 + 27 - 23 - 11 - 8 + 2 = 90$

10 osób nie zna żadnego języka
12 zna tylko niemiecki

$x \notin N \Longrightarrow x \in A$

$x \in N \lor x \in A$

$\sim(x \in N \Rightarrow x \in A)$

$x \notin N \land x \notin A$

$x \in \Omega \setminus (A \cup N)$

mówią prawdą — $\{x \in \Omega : x \in N \lor x \in A\} = \{x \in \Omega : x \in N \cup A\}$

$|A \cup N| = |A| + |N| - |A \cap N| = 62 + 41 - 23 = 80$

$|\Omega| - |A \cup N| = 100 - 80 = 20$

skłamałoby 20 osób

5.

a) $A_t = \left\{ x \in \mathbb{R} : 1 + \dfrac{1}{t} \leq x \leq 4 + \dfrac{1}{t^2} \right\}$ $T = \mathbb{N}$

$A_1 = [2, 5]$ $\bigcup_{t \in T} A_t = (1, 5]$

$A_2 = [1\tfrac{1}{2}, 4\tfrac{1}{4}]$

$A_3 = [1\tfrac{1}{3}, 4\tfrac{1}{3}]$ $\bigcap_{t \in T} A_t = [2, 4]$

$\downarrow$

$(1, 4]$

b) $A_t = \left\{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 \geq t^2 \right\}$ gdzie $T = \mathbb{R}$

$\bigcup_{t \in T} A_t = \mathbb{R}^2$ $\bigcap_{t \in T} A_t = \emptyset$

c) $A_t = \left\{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 2 - \sin(t) \right\}$ gdzie $T = \mathbb{R}$

$A_{\frac{\pi}{2}} = \left\{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \right\} = \bigcap_{t \in T} A_t$ bo $-1 \leq \sin(t) \leq 1$

$A_{\frac{3\pi}{2}} = \left\{ (x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 3 \right\} = \bigcup_{t \in T} A_t$

d) $A_t = \left\{ x \in \mathbb{R} : x^2 + (2 - t^2)x - 2t^2 = 0 \right\}$ $T = \mathbb{R}$

$\Delta = (2 - t^2)^2 - 4(-2t^2) = 4 - 4t^2 + t^4 + 8t^2$

$\Delta = t^4 + 4t^2 + 4 = (t^2 + 2)^2$ $\forall_{t \in \mathbb{R}} \ \Delta > 0$

$x_1 = \dfrac{-(2 - t^2) - \sqrt{(t^2 + 2)^2}}{2} = \dfrac{t^2 - 2 - t^2 - 2}{2} = -2$

$x_2 = \dfrac{-(2 - t^2) + \sqrt{(t^2 + 2)^2}}{2} = \dfrac{t^2 - 2 + t^2 + 2}{2} = t^2$

$A_t = \left\{ -2, t^2 \right\}$

$\bigcap_{t \in T} A_t = \{-2\}$ $\bigcup_{t \in T} A_t = \{-2\} \cup [0, +\infty)$

e) $A_t = \left\{ x \in \mathbb{R} : 3 + (-1)^t - \dfrac{(-1)^t}{t} < x < 7 + (-1)^t - \dfrac{(-1)^t}{t} \right\}$ $T = \mathbb{N}$

$A_1 = (3 - 1 + 1, \ 7 + -1 + 1) = (3, 7)$ $A_2 = (3 + 1 - \tfrac{1}{2}, \ 7 + 1 - \tfrac{1}{2}) = (3\tfrac{1}{2}, 7\tfrac{1}{2})$

$A_3 = (3 - 1 + \tfrac{1}{3}, \ 7 - 1 + \tfrac{1}{3}) = (2\tfrac{1}{3}, 6\tfrac{1}{3})$ $A_4 = (3 + 1 - \tfrac{1}{4}, \ 7 + 1 - \tfrac{1}{4}) = (3\tfrac{3}{4}, 7\tfrac{3}{4})$

$A_5 = (3 - 1 + \tfrac{1}{5}, \ 7 - 1 + \tfrac{1}{5}) = (2\tfrac{1}{5}, 6\tfrac{1}{5})$ $2n \downarrow$
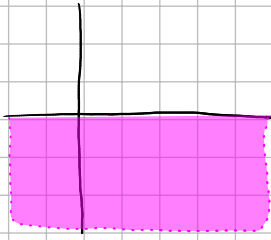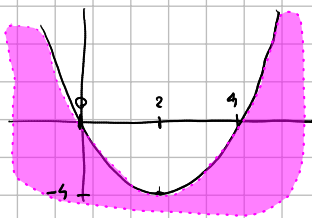
$2n+1 \downarrow \quad (2, 6]$ $[4, 8)$

$\bigcap_{t \in T} A_t = [4, 6]$ $\bigcup_{t \in T} A_t = (2, 8)$

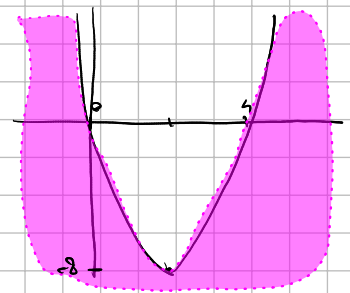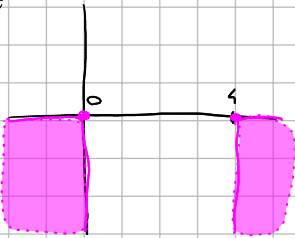f) $A_t = \{(x,y) \in \mathbb{R}^2 : y \leq tx(x-4)\}$, $T = \mathbb{N} \cup 0$
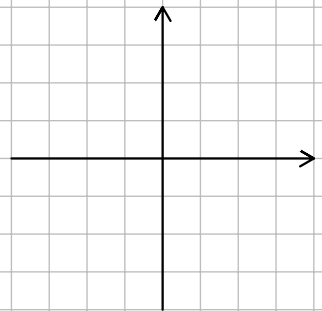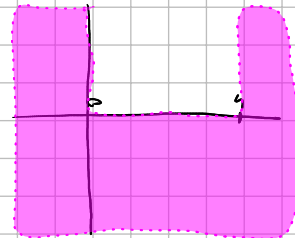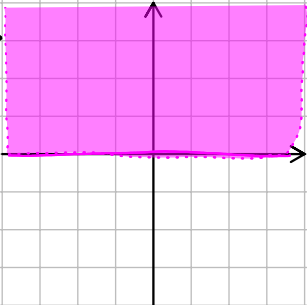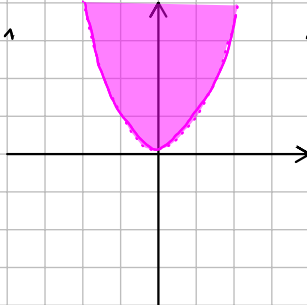
A_0 :

A_1 :

A_2 :

$\bigcap_{t \in T} A_t$

$\bigcup_{t \in T} A_t$
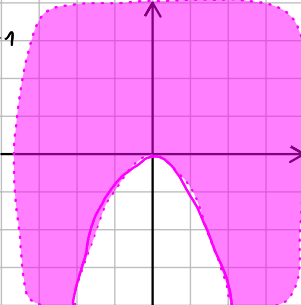
6.

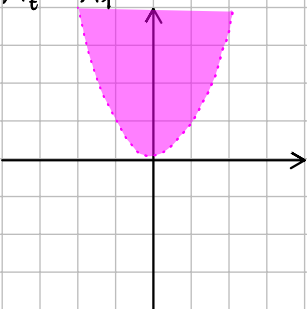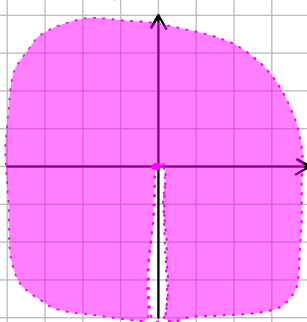$A_t = \{(x,y) \in \mathbb{R}^2 : y \geq tx^2\}$ $t \in \mathbb{R}$
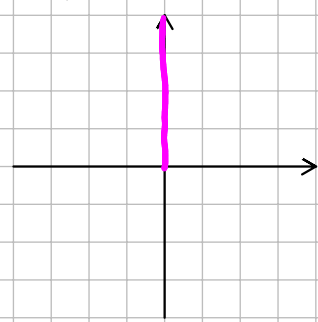
A_0

A_1

A_{-1}

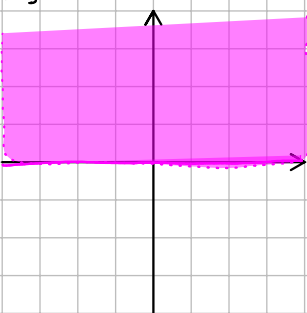a) $\bigcup_{t \in \mathbb{N}} A_t = A_1$

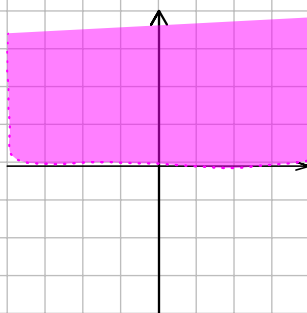b) $\bigcup_{t \in \mathbb{R}} A_t$

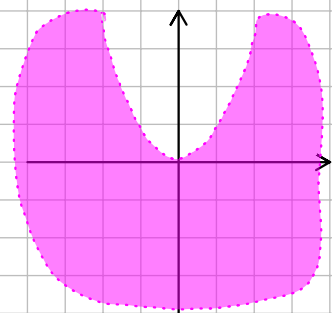e) $\bigcap_{t \in \mathbb{N}} A_t$

c) $\bigcup_{t \in [0,1]} A_t$
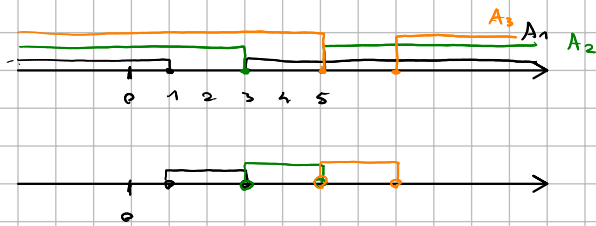
d) $\bigcup_{t \in (0,1)} A_t$

f) $\bigcup_{t \in [0,1]} A_t'$

7.

$$A_n = \mathbb{R} \setminus (2n-1, 2n+1) = \{x \in \mathbb{R} : |x - 2n| \geq 1\}, \qquad n \in \mathbb{N}$$

a) $\displaystyle\bigcup_{n=1}^{\infty} A_n = \mathbb{R}$

b) $\displaystyle\bigcap_{n=1}^{\infty} A_n = (-\infty, 1]$

c) $\displaystyle\bigcup_{n=1}^{\infty} A_n = (1, +\infty) - \{2n+1 : n \in \mathbb{N}\}$
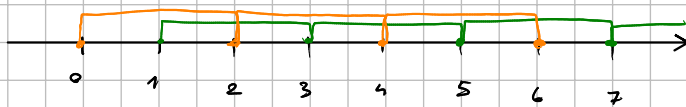
d) $\displaystyle\bigcap_{n=1}^{\infty} A_n = \emptyset$

8. $A_m = \{x \in \mathbb{R} : m - 1 - (-1)^m \leq x \leq m + 1 - (-1)^m\} \qquad m \in \mathbb{N}$

$A_1 = [1 - 1 + 1, 1 + 1 + 1] = [1, 3]$ $\qquad A_2 = [2 - 1 - 1, 2 + 1 - 1] = [0, 2]$

$A_3 = [3 - 1 + 1, 3 + 1 + 1] = [3, 5]$ $\qquad A_4 = [4 - 1 - 1, 4 + 1 - 1] = [2, 4]$

$A_5 = [5, 7]$ $\qquad\qquad\qquad\qquad A_6 = [4, 6]$

a) $\displaystyle\bigcap_{m=1}^{\infty} A_m = \emptyset$

b) $\displaystyle\bigcup_{m=1}^{\infty} A_m = [0, +\infty)$

c) $\displaystyle\bigcap_{m=1}^{\infty} (\mathbb{R} - A_m) = (-\infty, 0)$

d) $\displaystyle\bigcup_{m=1}^{\infty} (\mathbb{R} - A_m) = \mathbb{R}$

e) $\displaystyle\bigcup_{m \in 2\mathbb{N}} A_m = [0, +\infty)$