1.

$$\lim_{n\to+\infty}\left(1+\frac{\sqrt{n}}{n^2+1}\right)^{n(2\sqrt{n}+1)}$$

$$\frac{\sqrt{n}}{n^2+1}=\frac{1}{\frac{n^2+1}{\sqrt{n}}} \qquad \lim_{n\to\infty}\frac{n^2+1}{\sqrt{n}}=+\infty$$

$$n(2\sqrt{n}+1)=\frac{n^2+1}{\sqrt{n}}\cdot\frac{\sqrt{n}}{n^2+1}\cdot n(2\sqrt{n}+1)=\frac{n^2+1}{\sqrt{n}}\cdot\frac{2n^2+n\sqrt{n}}{n^2+1}$$

$$\lim_{n\to\infty}\frac{2n^2+\sqrt{n}}{n^2+1}=2 \qquad \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$$

$$\lim_{n\to\infty}\left(1+\frac{\sqrt{n}}{n^2+1}\right)^{n(2\sqrt{n}+1)}=\lim_{n\to\infty}\left(\left(1+\frac{1}{\frac{n^2+1}{\sqrt{n}}}\right)^{\frac{n^2+1}{\sqrt{n}}}\right)^{\frac{2n^2+n\sqrt{n}}{n^2+1}}=e^2$$

2. ?

$$\lim_{x\to-\infty}\left(\frac{1}{\pi}\operatorname{arccot}(x)\right)^x=e^{x\ln\left(\frac{1}{\pi}\operatorname{arccot}(x)\right)}=e$$

$$\cot(x)=\frac{\cos(x)}{\sin(x)}$$

$$x\to\pi^- \to \frac{-1}{0^+}=-\infty$$

$$\lim_{x\to-\infty}\operatorname{arccot}(x)=\pi \qquad \lim_{x\to-\infty}x\ln\left(\frac{1}{\pi}\operatorname{arccot}(x)\right)=\lim_{x\to-\infty}\frac{\ln\left(\frac{1}{\pi}\operatorname{arccot}(x)\right)}{\frac{1}{x}}\overset{\frac{0}{0}}{\underset{H}{=}}\lim_{x\to-\infty}\frac{\frac{1}{\frac{1}{\pi}\operatorname{arccot}(x)}\cdot\frac{-1}{x^2+1}}{-\frac{1}{x^2}}=\lim_{x\to-\infty}\frac{\pi x^2}{\operatorname{arccot}(x)(x^2+1)}$$

$$\overset{\frac{\infty}{\infty}}{\underset{H}{=}}\lim_{x\to-\infty}\frac{2\pi x}{\frac{-1}{x^2+1}\cdot(x^2+1)+\operatorname{arccot}(x)\cdot 2x}=\lim_{x\to-\infty}\frac{2\pi x}{-1+2x\operatorname{arccot}(x)}=\lim_{x\to-\infty}\frac{\pi}{-\frac{1}{2x}+\operatorname{arccot}(x)}=\frac{\pi}{0+\pi}=1$$

3. $$\int e^{x-x^2}(x^2-x)(2x-1)\,dx=\left|\begin{matrix}t=x^2-x\\ dt=(2x-1)dx\end{matrix}\right|=\int te^{-t}dt=\left|\begin{matrix}f=t & g'=e^{-t}\\ f'=1 & g=-e^{-t}\end{matrix}\right|=-te^{-t}+\int e^{-t}=-te^{-t}-e^{-t}+C=-e^{-t}(t+1)+C$$

$$-e^{x-x^2}(x^2-x+1)+C$$

$$\int_0^\infty e^{x-x^2}(x^2-x)(2x-1)\,dx=\lim_{T\to+\infty}-e^{T-T^2}(T^2-T+1)+e^0\cdot 0=\lim_{T\to+\infty}\frac{-T^2+T-1}{e^{T^2-T}}\overset{\frac{\infty}{\infty}}{\underset{H}{=}}\lim_{T\to+\infty}\frac{-2T+1}{(2T-1)e^{T^2-T}}=\lim_{T\to+\infty}\frac{-1}{e^{T^2-T}}=0$$

całka zbieżna

5.

$$xy'=xe^{\frac{y}{x}}+y$$

$$y'=e^{\frac{y}{x}}+\frac{y}{x} \qquad u=\frac{y}{x}$$

$$u'x+u=e^u+u$$

$$u'x=e^u$$

$$\frac{du}{dx}=\frac{e^u}{x} \qquad \int\frac{du}{e^u}=\int\frac{dx}{x}$$

$$-e^{-u}=\ln|x|+C$$

$$e^{-u}=-\ln|x|+C$$

$$-u=\ln(-\ln|x|)+C$$

$$y=-x\ln(-\ln|x|)+Cx$$

6. ?

$$y'' - 2y' + y = \frac{e^x}{x^2} + 1$$

$$r^2 - 2r + 1 = (r-1)^2 = 0 \quad r_0 = 1$$

$$y(x) = C_1 e^x + C_2 x e^x \quad \text{CORJ}$$

$$\begin{bmatrix} e^x & xe^x \\ e^x & xe^x + e^x \end{bmatrix} \cdot \begin{bmatrix} C_1' \\ C_2' \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{e^x}{x^2} + 1 \end{bmatrix}$$

$$C_1' = \frac{-(xe^x + e^x)\left(\frac{e^x}{x^2} + 1\right)}{e^x(xe^x + e^x) - xe^x e^x} = \frac{-\left(\frac{e^{2x}}{x} + xe^x + \frac{e^{2x}}{x^2} + e^x\right)}{xe^{2x} + e^{2x} - xe^{2x}} = \frac{e^{2x}\left(\frac{-1}{x} - xe^{-x} - \frac{1}{x^2} - e^{-x}\right)}{e^{2x}} = -\frac{1}{x} - xe^{-x} - \frac{1}{x^2} - e^{-x}$$

$$C_2' = \frac{e^x\left(\frac{e^x}{x^2} + 1\right)}{e^{2x}} = \frac{1}{x^2} + e^{-x}$$

$$\int e^{-x} dx = -e^{-x} \qquad \int \frac{1}{x} dx = \ln|x| \qquad \int \frac{1}{x^2} dx = -\frac{1}{x}$$

$$\int x e^{-x} dx = \left| \begin{matrix} f = x & g' = e^{-x} \\ f' = 1 & g = -e^{-x} \end{matrix} \right| = -xe^{-x} + \int e^{-x} dx = -xe^{-x} - e^{-x} = -e^{-x}(x+1)$$

$$C_1 = -\int \frac{1}{x} dx - \int x e^{-x} dx - \int \frac{1}{x^2} dx - \int e^{-x} dx = -\ln|x| + e^{-x}(x+1) + \frac{1}{x} + e^{-x} + C_1 = -\ln|x| + \frac{1}{x} + e^{-x}(x+2) + C_1$$

$$C_2 = \int \frac{1}{x^2} dx + \int e^{-x} dx = -\frac{1}{x} - e^{-x} + C_2$$

$$y(x) = \left(-\ln|x| + \frac{1}{x} + e^{-x}(x+2) + C_1\right)e^x + \left(-\frac{1}{x} - e^{-x} + C_2\right)xe^x$$

$$= -e^x \ln|x| + \frac{e^x}{x} + \cancel{x} + 2 + C_1 e^x - e^x - \cancel{x} + C_2 x e^x$$

$$= C_1 e^x + C_2 x e^x + \frac{e^x}{x} - e^x \ln|x| + 2 \qquad \text{CORN}$$