

1.

$$a) \sum_{n=1}^{\infty} \frac{2^n}{n+1} x^n$$

$$\lambda = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{2^{n+1}}{n+2}}{\frac{2^n}{n+1}} \right| = \lim_{n \rightarrow \infty} \frac{2^{n+1}}{n+2} \cdot \frac{n+1}{2^n} = \lim_{n \rightarrow \infty} \frac{2n+2}{n+2} = 2 \rightarrow R = \frac{1}{2}$$

$$\text{dla } x = \frac{1}{2} \quad \sum_{n=1}^{\infty} \frac{2^n}{n+1} \cdot \left(\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n+1} = \sum_{n=2}^{\infty} \frac{1}{n} \text{ rozbiegły (szereg harmoniczny)}$$

$$\text{dla } x = -\frac{1}{2} \quad \sum_{n=1}^{\infty} \frac{2^n}{n+1} \cdot \left(-\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n+1}$$

$$\lim_{n \rightarrow \infty} \frac{1}{n+1} = 0 \Rightarrow \text{szereg zbieżny (kryterium Leibnizsa)}$$

 przedział zbieżności $[-\frac{1}{2}, \frac{1}{2})$

$$b) \sum_{n=1}^{\infty} \frac{\ln(n)}{n} x^n$$

$$\lambda = \lim_{n \rightarrow \infty} \left| \frac{\ln(n+1)}{n+1} \right| = \lim_{n \rightarrow \infty} \frac{\ln(n+1)}{\ln(n)} \cdot \frac{n}{n+1} = 1 \cdot 1 = 1 \rightarrow R = 1$$

$$\lim_{x \rightarrow \infty} \frac{\ln(x+1)}{\ln(x)} \stackrel{\infty}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x+1}}{\frac{1}{x}} = \lim_{x \rightarrow \infty} \frac{x}{x+1} = 1$$

$$\text{dla } x = 1 \quad \sum_{n=1}^{\infty} \frac{\ln(n)}{n} \cdot 1^n = \sum_{n=1}^{\infty} \frac{\ln(n)}{n}$$

$$\int \frac{\ln(x)}{x} dx = \left| \frac{t = \ln(x)}{dt = \frac{1}{x} dx} \right| = \int t dt = \frac{1}{2} \ln^2(x) + C$$

$$\int_1^{\infty} \frac{\ln(x)}{x} dx = \lim_{T \rightarrow \infty} \frac{1}{2} \ln^2(T) - \frac{1}{2} \ln(1) = +\infty \text{ rozbiegły} \rightarrow \text{szereg rozbiegły}$$

$$\text{dla } x = -1 \quad \sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n} \quad \lim_{n \rightarrow \infty} \frac{\ln(n)}{n} = 0 \quad \text{szereg zbieżny (k. Leibnizsa)}$$

$$\lim_{x \rightarrow \infty} \frac{\ln(x)}{x} \stackrel{0}{=} \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = 0$$

 przedział zbieżności $[-1, 1)$

$$c) \sum_{n=1}^{\infty} \frac{n(2n+1)}{6^n} x^{2n} \text{ wyznaczam granicę w zależności od } x$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)(2n+3)}{6^{n+1}} \cdot \frac{x^{2n+2}}{x^{2n}} \right| = \lim_{n \rightarrow \infty} \left| \frac{2n^2+5n+3}{2n^2+n} \cdot \frac{x^{2n+2}}{x^{2n}} \cdot \frac{6^n}{6^{n+1}} \right| = \left| 1 \cdot x^2 \cdot \frac{1}{6} \right| = \frac{|x|^2}{6} = \frac{|x|^2}{6}$$

$$\text{zbieżność dla } 0 \leq \frac{|x|^2}{6} < 1 \quad |x|^2 < 6 \quad x \in (-\sqrt{6}, \sqrt{6})$$

$$\text{dla } x = \sqrt{6} \quad \sum_{n=1}^{\infty} \frac{n(n+1)}{6^n} \cdot 6^n = \sum_{n=1}^{\infty} n(n+1) \text{ rozbiegły}$$

$$\text{dla } x = -\sqrt{6} \quad \sum_{n=1}^{\infty} \frac{n(n+1)}{6^n} (-6)^n = \sum_{n=1}^{\infty} (-1)^n n(n+1) \quad \lim_{n \rightarrow \infty} n(n+1) = +\infty \text{ rozbiegły}$$

 przedział zbieżności $(-\sqrt{6}, \sqrt{6})$

$$d) \sum_{n=0}^{\infty} \frac{(6-2x)^n}{3^n \cdot 2^n}$$

$$\frac{(6-2x)^n}{3^n \cdot 2^n} = \frac{[-2(x-3)]^n}{3^n \cdot 2^n} = (-1)^n \frac{2^n}{3^n \cdot 2^n} (x-3)^n \quad x_0=3$$

symetrycznie względem środka
 $\lim_{n \rightarrow \infty} n \sqrt[n]{\left| \frac{(6-2x)^n}{3^n \cdot 2^n} \right|} = \lim_{n \rightarrow \infty} n \sqrt[n]{\frac{2^n}{3^n \cdot 2^n}} = \frac{2}{3} \rightarrow R = \frac{3}{2} \rightarrow \text{przedział } (3 - \frac{3}{2}, 3 + \frac{3}{2}) = (\frac{3}{2}, \frac{9}{2})$

$$\frac{2^n}{3^n \cdot 3^n} \leq \frac{2^n}{3^n \cdot 2^n} \leq \frac{2^n}{3^n} \quad \lim_{n \rightarrow \infty} \sqrt[n]{\left(\frac{2}{3} \right)^n} = \frac{2}{3} \quad \lim_{n \rightarrow \infty} n \sqrt[n]{\frac{1}{3} \cdot \left(\frac{2}{3} \right)^n} = \frac{2}{3}$$

dla $x = \frac{3}{2}$ $\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n \cdot 2^n} \cdot \left(-\frac{3}{2}\right)^n = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{2^n}{3^n \cdot 2^n} \cdot (-1)^n \cdot \frac{3^n}{2^n} = \sum_{n=0}^{\infty} \frac{3^n}{3^n \cdot 2^n}$

$$\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} \frac{3^n}{3^n \cdot 2^n} = \lim_{n \rightarrow \infty} \frac{3^n}{3^n (1 + (\frac{1}{2}))^n} = \lim_{n \rightarrow \infty} \frac{1}{1 + (\frac{1}{2})^n} = 1 \neq 0 \quad \text{rozbicie, bo nie spełnia warunku koniecznego}$$

dla $x = \frac{9}{2}$ $\sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n \cdot 2^n} \left(\frac{9}{2}\right)^n = \sum_{n=0}^{\infty} (-1)^n \frac{3^n}{3^n \cdot 2^n}$

$$\lim_{n \rightarrow \infty} \frac{3^n}{3^n \cdot 2^n} = \lim_{n \rightarrow \infty} \frac{1}{1 + (\frac{1}{2})^n} = 1 \neq 0 \quad \text{rozbicie} \Rightarrow \text{konvergenca Leibnizowa}$$

przedział zbieżności $(\frac{3}{2}, \frac{9}{2})$

$$2. \frac{x^2}{2} + \frac{x^6}{6} + \dots + \frac{x^{4n+2}}{4n+2} + \dots = \sum_{n=0}^{\infty} \frac{x^{4n+2}}{4n+2}$$

$$\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{x^{4n+6}}{4n+6}}{\frac{x^{4n+2}}{4n+2}} \right| = \lim_{n \rightarrow \infty} \left| x^4 \cdot \frac{4n+2}{4n+6} \right| = |x|^4$$

$$0 \leq |x|^4 < 1 \Leftrightarrow x \in (-1, 1)$$

dla $x = 1$ $\sum_{n=0}^{\infty} \frac{1}{4n+2}$ rozbicie (harmoniczny)

dla $x = -1$ $\sum_{n=0}^{\infty} \frac{1}{4n+2}$ rozbicie

$$R > 0 \quad x \in (-1, 1)$$

$$S(x) = \sum_{n=0}^{\infty} \frac{x^{4n+2}}{4n+2} \quad S'(x) = \sum_{n=0}^{\infty} \frac{d}{dx} \frac{x^{4n+2}}{4n+2} = \sum_{n=0}^{\infty} \frac{1}{4n+2} (4n+2) x^{4n+1} = \sum_{n=0}^{\infty} x^{4n+1}$$

$$S'(x) = \sum_{n=0}^{\infty} x \cdot x^{4n} = x \sum_{n=0}^{\infty} (x^4)^n = x \cdot \frac{1}{1-x^4} = \frac{x}{1-x^4}$$

sąsiedztwo geometryczne

$$S(x) = \int S'(x) dx = \int \frac{x}{1-x^4} dx = - \int \frac{x}{x^4-1} dx$$

$$\frac{x}{x^4-1} = \frac{x}{(x^2+1)(x+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1} + \frac{D}{x-1} = \frac{A(x^2-1) + B(x^2-1) + C(x^2+1)(x-1) + D(x^2+1)(x+1)}{x^4-1}$$

$$x = Ax^3 - Ax + Bx^2 - B + Cx^3 - Cx^2 + Cx - C + Dx^3 + Dx^2 + Dx + D$$

$$x = (A+C+D)x^3 + (B-C+D)x^2 + (-A+C+D)x + (-B-C+D)$$

$$\begin{array}{c} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ -1 & 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & 1 & 0 \end{array} \right] \xrightarrow{U_3+U_1} \left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & -2 & 2 & 0 \end{array} \right] \xrightarrow{U_2+U_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 2 & \frac{1}{2} \\ 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 4 & 1 \end{array} \right] \xrightarrow{\frac{1}{2}U_3} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 2 & \frac{1}{2} \\ 0 & 0 & 1 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & \frac{1}{4} \end{array} \right] \xrightarrow{U_2-2U_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} \end{array} \right] \end{array}$$

$$\frac{x}{x^4-1} = \frac{-\frac{1}{4}x}{x^2+1} + \frac{\frac{1}{4}}{x+1} + \frac{\frac{1}{4}}{x-1} \quad \left| \begin{array}{l} t = x^2+1 \\ dt = 2x dx \end{array} \right. \quad -\frac{1}{4} \int \frac{dt}{t}$$

$$\int \frac{x}{x^4-1} dx = -\frac{1}{2} \int \frac{x}{x^2+1} dx + \frac{1}{4} \int \frac{dx}{x+1} + \frac{1}{4} \int \frac{dx}{x-1} = -\frac{1}{4} \ln|x^2+1| + \frac{1}{4} \ln|x+1| + \frac{1}{4} \ln|x-1| + C$$

$$S(x) = -\int \frac{x}{x^2-1} dx = \frac{1}{4} \left[\ln|x^2+1| - \ln|x+1| - \ln|x-1| \right] = \frac{1}{4} \ln \left| \frac{x^2+1}{x^2-1} \right|$$

$$S(0) = 0 \rightarrow \frac{1}{4} \ln|1+1| + C = 0 \rightarrow C = 0$$

3.

$$a) f(x) = x^4 \cdot e^{-2x} \quad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad \text{zulässig } \cup \mathbb{R}$$

$$f(x) = x^4 \cdot \sum_{n=0}^{\infty} \frac{(-2x)^n}{n!} = \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{n!} x^{n+4}$$

$$b) f(x) = \frac{1}{1+\alpha^2 x^2} = \frac{1}{1-(-\alpha^2 x^2)} \quad \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \text{zulässig } \text{ dla } |x| < 1 \\ \alpha > 0$$

$$f(x) = \sum_{n=0}^{\infty} (-\alpha^2 x^2)^n = \sum_{n=0}^{\infty} (-1)^n (\alpha x)^{2n}$$

$$\text{zulässig } \text{ dla } |(\alpha x)^2| < 1 \Leftrightarrow \alpha^2 |x|^2 < 1 \Leftrightarrow |x|^2 < \frac{1}{\alpha^2} \Leftrightarrow x \in (-\frac{1}{\alpha}, \frac{1}{\alpha})$$

$$c) f(x) = 2 \sin(x) \sin(3x) = 2 \cdot \frac{1}{2} [\cos(-2x) - \cos(4x)] = \cos(2x) - \cos(4x)$$

$$\cos(2x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} \quad \text{zulässig } \cup \mathbb{R}$$

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n}}{(2n)!} - \sum_{n=0}^{\infty} (-1)^n \frac{(4x)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} (4^n - 16^n)$$

$$4. \quad f(x) = \frac{1}{x} = x^{-1} \quad x_0 = 3$$

$$f(x) = x^{-1} \quad f^{(n)}(x) = (-1)^n n! x^{-n-1}$$

$$f' = -x^{-2}$$

$$f'' = 2x^{-3}$$

$$f''' = -6x^{-4}$$

$$f^{(4)} = 24x^{-5}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x-x_0)^n = \sum_{n=0}^{\infty} \frac{(-1)^n n! \cdot 3^{-n-1}}{n!} (x-3)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} (x-3)^n$$

$$\lambda = \lim_{n \rightarrow \infty} \sqrt[n]{\left| \frac{(-1)^n}{3^{n+1}} \right|} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{1}{3^n} \cdot \frac{1}{3}} = \frac{1}{3} \rightarrow R = 3 \rightarrow \text{präzise } (3-3, 3+3) = (0, 6)$$

$$\text{dla } x = 0$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} (-3)^n = \sum_{n=0}^{\infty} \frac{1}{3} \cdot \frac{(-1)^n \cdot 3^n}{3^n} = \frac{1}{3} \sum_{n=0}^{\infty} 1 \quad \text{rechlässig}$$

$$\text{dla } x = 6$$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} \cdot 3^n = \frac{1}{3} \sum_{n=0}^{\infty} (-1)^n \quad \text{rechlässig}$$

$$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} (x-3)^n \quad \text{dla } x \in (0, 6)$$

5. ?

$$f(x) = \frac{1-x}{1+x} = \frac{1+x-2x}{1+x} = 1-2x \cdot \frac{1}{1-(-x)}$$

$$f(x) = 1-2x \sum_{n=0}^{\infty} (-x)^n = 1-2x \sum_{n=0}^{\infty} (-1)^n x^n = 1-2 \sum_{n=0}^{\infty} (-1)^n x^{n+1}$$

$$6. f(x) = \begin{cases} 6 & \text{dla } 0 < x < 2 \\ 3x & \text{dla } 2 < x < 4 \end{cases}$$

1. uzupełniam punkt nieciągłości

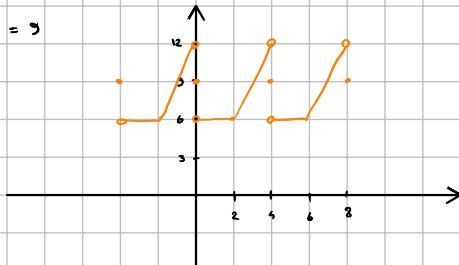
$$f(2) = \frac{1}{2} [\lim_{x \rightarrow 2^+} f(x) + \lim_{x \rightarrow 2^-} f(x)] = \frac{1}{2} [6 + 6] = 6$$

2. uzupełniam krańce przedziału

$$f(0) = f(4) = \frac{1}{2} [\lim_{x \rightarrow 0^+} f(x) + \lim_{x \rightarrow 4^-} f(x)] = \frac{1}{2} [6 + 12] = 9$$

3. rozwiążmy na przediale $[0, 4]$

$$\text{okres } \varphi = 2\pi - 4 \rightarrow \pi = 2$$



$$a_0 = \frac{1}{2} \int_0^4 f(x) dx = \frac{1}{2} \int_0^2 6 dx + \frac{1}{2} \int_2^4 3x dx = \frac{1}{2} \cdot 12 + \frac{1}{2} \cdot 18 = 15$$

$$a_n = \frac{1}{2} \int_0^4 f(x) \cos\left(\frac{n\pi x}{2}\right) dx = \frac{1}{2} \int_0^2 6 \cos\left(\frac{n\pi}{2}x\right) dx + \frac{1}{2} \int_2^4 3x \cos\left(\frac{n\pi}{2}x\right) dx$$

$$\int \cos\left(\frac{n\pi}{2}x\right) dx = \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}x\right) + C$$

$$\int x \cos\left(\frac{n\pi}{2}x\right) dx = \left| \begin{array}{l} f = x, g' = \cos\left(\frac{n\pi}{2}x\right) \\ f' = 1, g = \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}x\right) \end{array} \right| = \frac{2}{n\pi} x \sin\left(\frac{n\pi}{2}x\right) - \frac{2}{n\pi} \int \sin\left(\frac{n\pi}{2}x\right) dx = \frac{2}{n\pi} x \sin\left(\frac{n\pi}{2}x\right) + \left(\frac{2}{n\pi}\right)^2 \cos\left(\frac{n\pi}{2}x\right) + C$$

$$a_n = \frac{6}{n\pi} \cdot \left. \sin\left(\frac{n\pi}{2}x\right) \right|_0^2 + \frac{3}{n\pi} \cdot \left. x \sin\left(\frac{n\pi}{2}x\right) \right|_2^4 + \frac{6}{n^2\pi^2} \cdot \left. \cos\left(\frac{n\pi}{2}x\right) \right|_2^4$$

$$a_n = \frac{6}{n^2\pi^2} \cdot [\cos(2\pi n) - \cos(\pi n)] = \frac{6}{n^2\pi^2} \left[1 - \cos(\pi n) \right] = \begin{cases} \frac{12}{n^2\pi^2} & \text{dla } n = 2k+1 \\ -\frac{6}{n^2\pi^2} (1 - (-1)^n) & \text{dla } n = 2k \end{cases}$$

$$b_n = \frac{1}{2} \int_0^4 f(x) \sin\left(\frac{n\pi x}{2}\right) dx = \frac{1}{2} \int_0^2 6 \sin\left(\frac{n\pi}{2}x\right) dx + \frac{1}{2} \int_2^4 3x \sin\left(\frac{n\pi}{2}x\right) dx$$

$$\int \sin\left(\frac{n\pi}{2}x\right) dx = -\frac{2}{n\pi} \cos\left(\frac{n\pi}{2}x\right) + C$$

$$\int x \sin\left(\frac{n\pi}{2}x\right) dx = \left| \begin{array}{l} f = x, g' = \sin\left(\frac{n\pi}{2}x\right) \\ f' = 1, g = -\frac{2}{n\pi} \cos\left(\frac{n\pi}{2}x\right) \end{array} \right| = -\frac{2}{n\pi} x \cos\left(\frac{n\pi}{2}x\right) + \frac{2}{n\pi} \int \cos\left(\frac{n\pi}{2}x\right) dx = -\frac{2}{n\pi} x \cos\left(\frac{n\pi}{2}x\right) + \frac{4}{n^2\pi^2} \sin\left(\frac{n\pi}{2}x\right) + C$$

$$b_n = -\frac{6}{n\pi} \cdot \left. \cos\left(\frac{n\pi}{2}x\right) \right|_0^2 - \frac{3}{n\pi} \cdot \left. x \cos\left(\frac{n\pi}{2}x\right) \right|_2^4 + \frac{6}{n^2\pi^2} \cdot \left. \sin\left(\frac{n\pi}{2}x\right) \right|_2^4$$

$$b_n = -\frac{6}{n\pi} \cdot [\cos(\pi n) - 1] - \frac{3}{n\pi} \cdot [4\cos(2\pi n) - 2\cos(\pi n)] + 0$$

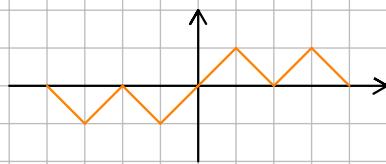
$$b_n = \begin{cases} -\frac{6}{n\pi} & \text{dla } n = 2k \\ \frac{12}{n\pi} - \frac{12}{n^2\pi^2} = -\frac{6}{n\pi} & \text{dla } n = 2k+1 \end{cases}$$

$$f(x) = \frac{15}{2} + \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos\left(\frac{2n-1}{2}\pi x\right) - \frac{6}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi}{2}x\right)$$

tylko nieparzyste wyże zamienia $n \rightarrow 2n-1$

a) szereg sinusowy

$$f(x) = \begin{cases} x & \text{dla } 0 \leq x \leq 1 \\ 2-x & \text{dla } 1 \leq x \leq 2 \end{cases}$$

rozwińując na $[0, 2]$ $L=2$ 

$$b_n = \int_0^2 f(x) \sin\left(\frac{n\pi}{2}x\right) dx = \int_0^1 x \sin\left(\frac{n\pi}{2}x\right) dx + \int_1^2 (2-x) \sin\left(\frac{n\pi}{2}x\right) dx = \int_0^1 x \sin\left(\frac{n\pi}{2}x\right) dx + 2 \int_1^2 \sin\left(\frac{n\pi}{2}x\right) dx - \int_1^2 x \sin\left(\frac{n\pi}{2}x\right) dx$$

$$\int \sin\left(\frac{n\pi}{2}x\right) dx = -\frac{2}{n\pi} \cos\left(\frac{n\pi}{2}x\right) + C$$

$$\int x \sin\left(\frac{n\pi}{2}x\right) dx = \left| \begin{array}{l} f=x \quad g'=\sin\left(\frac{n\pi}{2}x\right) \\ f'=1 \quad g=-\frac{2}{n\pi} \cos\left(\frac{n\pi}{2}x\right) \end{array} \right| = -\frac{2x}{n\pi} \cos\left(\frac{n\pi}{2}x\right) + \frac{2}{n\pi} \int \cos\left(\frac{n\pi}{2}x\right) dx = -\frac{2x}{n\pi} \cos\left(\frac{n\pi}{2}x\right) + \frac{4}{n^2\pi^2} \sin\left(\frac{n\pi}{2}x\right) + C$$

$$b_n = -\frac{2}{n\pi} \cdot x \cos\left(\frac{n\pi}{2}x\right) \Big|_0^1 + \frac{4}{n^2\pi^2} \sin\left(\frac{n\pi}{2}x\right) \Big|_0^1 - \frac{4}{n\pi} \cdot \cos\left(\frac{n\pi}{2}x\right) \Big|_1^2 + \frac{2}{n\pi} \cdot x \cos\left(\frac{n\pi}{2}x\right) \Big|_1^2 - \frac{4}{n^2\pi^2} \sin\left(\frac{n\pi}{2}x\right) \Big|_1^2$$

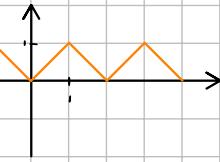
$$= -\frac{2}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \frac{4}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) - \frac{4}{n\pi} \cos(n\pi) + \frac{4}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \frac{4}{n\pi} \cos(n\pi) - \frac{2}{n\pi} \cos\left(\frac{n\pi}{2}\right) + \frac{4}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{8}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) = \begin{cases} 0 & \text{dla } n=2k \\ \frac{8}{n^2\pi^2} & \text{dla } n=4k-3 \quad (4k+1) \\ -\frac{8}{n^2\pi^2} & \text{dla } n=4k-1 \quad (4k+3) \end{cases} = \begin{cases} 0 & \text{dla } n=2k \\ (-1)^{k+1} \frac{8}{n^2\pi^2} & \text{dla } n=2k+1 \end{cases}$$

$$f(x) = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^2} \sin\left(\frac{2n-1}{2}\pi x\right) \quad (\text{tylko nieparzyste wyrazy})$$

b) szereg cosinusowy

$$f(x) = \begin{cases} x & \text{dla } 0 \leq x \leq 1 \\ 2-x & \text{dla } 1 \leq x \leq 2 \end{cases}$$

rozwińając na $[0, 2]$ $L=2$ 

$$a_0 = \int_0^2 f(x) dx = 1$$

$$a_n = \int_0^2 f(x) \cos\left(\frac{n\pi}{2}x\right) dx = \int_0^1 x \cos\left(\frac{n\pi}{2}x\right) dx + 2 \int_1^2 \cos\left(\frac{n\pi}{2}x\right) dx - \int_1^2 x \cos\left(\frac{n\pi}{2}x\right) dx$$

$$\int \cos\left(\frac{n\pi}{2}x\right) dx = \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}x\right) + C$$

$$\int x \cos\left(\frac{n\pi}{2}x\right) dx = \left| \begin{array}{l} f=x \quad g'=\cos\left(\frac{n\pi}{2}x\right) \\ f'=1 \quad g=\frac{2}{n\pi} \sin\left(\frac{n\pi}{2}x\right) \end{array} \right| = \frac{2x}{n\pi} \sin\left(\frac{n\pi}{2}x\right) - \frac{2}{n\pi} \int \sin\left(\frac{n\pi}{2}x\right) dx = \frac{2}{n\pi} x \sin\left(\frac{n\pi}{2}x\right) + \frac{4}{n^2\pi^2} \cos\left(\frac{n\pi}{2}x\right) + C$$

$$a_n = \frac{2}{n\pi} \sin\left(\frac{n\pi}{2}\right) - 0 + \frac{4}{n^2\pi^2} \cos\left(\frac{n\pi}{2}\right) - \frac{4}{n\pi} + \frac{2}{n\pi} \sin(n\pi) - \frac{4}{n\pi} \sin(n\pi) + \frac{2}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) - \frac{4}{n^2\pi^2} \cos(n\pi) + \frac{4}{n^2\pi^2} \cos\left(\frac{n\pi}{2}\right)$$

$$a_n = -\frac{4}{n^2\pi^2} (1 + \cos(n\pi)) = \begin{cases} -\frac{8}{n^2\pi^2} & \text{dla } n=2k \\ 0 & \text{dla } n=2k+1 \end{cases}$$

$$f(x) = \frac{1}{2} + \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n)^2} \cos\left(\frac{2n\pi}{2}x\right) = \frac{1}{2} - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{\cos(n\pi x)}{n^2}$$