

Zestaw 0

$$1. \quad u = a + bi \quad z = c + di$$

Alternatywny sposób

$$a) |u+z|^2 = |u|^2 + |z|^2 + 2\operatorname{Re}(uz)$$

$$|z|^2 = z \bar{z} \quad 2\operatorname{Re}(u) = u + \bar{u}$$

$$\text{LHS} = |a+bi+c+di|^2 = (\sqrt{(a+c)^2 + (b+d)^2})^2 = a^2 + b^2 + c^2 + d^2 + 2ac + 2bd$$

$$\begin{aligned} \text{RHS} &= |a+bi|^2 + |c+di|^2 + 2\operatorname{Re}((a+bi)(c+di)) \\ &= (\sqrt{a^2+b^2})^2 + (\sqrt{c^2+d^2})^2 + 2\operatorname{Re}(ac-ad+ci+bd) \\ &= a^2 + b^2 + c^2 + d^2 + 2ac - 2bd = \text{LHS} \end{aligned}$$

$$b) |u-z|^2 = |u|^2 + |z|^2 - 2\operatorname{Re}(uz)$$

$$\begin{aligned} |u+(-z)|^2 &= |u|^2 + |-z|^2 + 2\operatorname{Re}(u \cdot \bar{z}) \\ &= |u|^2 + |z|^2 - 2\operatorname{Re}(uz) \end{aligned}$$

$$c) |u+z|^2 + |u-z|^2 = |u|^2 + |z|^2 + 2\operatorname{Re}(uz) + |u|^2 + |z|^2 - 2\operatorname{Re}(uz) = 2|u|^2 + 2|z|^2$$

tożsamej rozumielego boku



$$\begin{aligned} b^2 &= \frac{1}{2}e^2 + \frac{1}{2}f^2 + 2 - \frac{1}{2}ef \cos(\alpha) \\ b^2 &= \frac{1}{2}e^2 + \frac{1}{2}f^2 + \frac{1}{2}ef \cos(\alpha) \\ a^2 &= \frac{1}{2}e^2 + \frac{1}{2}f^2 + \frac{1}{2}ef \cos(\pi - \alpha) \\ a^2 &= \frac{1}{2}e^2 + \frac{1}{2}f^2 - \frac{1}{2}ef \cos(\alpha) \end{aligned}$$

$$\begin{aligned} a^2 + b^2 &= \frac{1}{2}e^2 + \frac{1}{2}f^2 \\ e^2 + f^2 &= 2a^2 + 2b^2 \\ |u+z|^2 + |u-z|^2 &= 2|u|^2 + 2|z|^2 \end{aligned}$$

2.



$$c^2 = a^2 + b^2 - 2ab \cos(\alpha)$$

$$\begin{aligned} \cos(\alpha) &= \frac{a^2 + b^2 - c^2}{2ab} = \frac{|\vec{a}|^2 + |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{a}| \cdot |\vec{b}|} = \frac{|\vec{a}|^2 + |\vec{b}|^2 - |\vec{c}|^2 - |\vec{b}|^2 + 2\operatorname{Re}(c\bar{b})}{2|\vec{a}| \cdot |\vec{b}|} \\ &= \frac{2\operatorname{Re}((x_A + y_A i)(x_B - y_B i))}{|\vec{a}| |\vec{b}|} = \frac{x_A x_B + y_A y_B}{|\vec{a}| \cdot |\vec{b}|} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} \end{aligned}$$

$$\vec{c} = \vec{b} - \vec{a} = \begin{bmatrix} x_B - x_A \\ y_B - y_A \end{bmatrix}$$

$$|\vec{c}|^2 = |\vec{b} - \vec{a}|^2 = x_A^2 + x_B^2 + y_A^2 + y_B^2 - 2x_A x_B - 2y_A y_B$$

$$\cos(\alpha) = \frac{\operatorname{Re}(uz)}{|uz|}$$

$$|uz| = |u||z| = |u||z| = |uz|$$

$$\sin(\alpha) = \sqrt{\frac{|\operatorname{Im}(uz)|^2}{|uz|^2}} = \sqrt{\frac{|\operatorname{Im}(uz)|^2}{|uz|^2}}$$

$$S = \frac{1}{2}ab \sin(\alpha)$$

$$\sin(\alpha) = \sqrt{1 - \cos^2(\alpha)} = \sqrt{\frac{(x_A^2 + y_A^2)(x_B^2 + y_B^2) - (x_A x_B + y_A y_B)^2}{(x_A^2 + y_A^2)(x_B^2 + y_B^2)}}$$



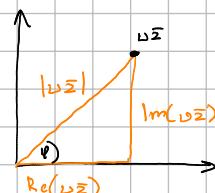
$$= \frac{(x_A y_B - x_B y_A)^2}{\sqrt{x_A^2 + y_A^2} \cdot \sqrt{x_B^2 + y_B^2}} = \frac{|x_A y_B - x_B y_A|}{\sqrt{x_A^2 + y_A^2} \cdot \sqrt{x_B^2 + y_B^2}}$$

$$3. \quad \sin(\alpha) = \frac{|x_A y_B - x_B y_A|}{\sqrt{x_A^2 + y_A^2} \cdot \sqrt{x_B^2 + y_B^2}}$$

$$S = \frac{1}{2}ab \sin(\alpha) = \frac{1}{2}\sqrt{x_A^2 + y_A^2} \sqrt{x_B^2 + y_B^2} \cdot \frac{|x_A y_B - x_B y_A|}{\sqrt{x_A^2 + y_A^2} \cdot \sqrt{x_B^2 + y_B^2}} = \frac{1}{2}|x_A y_B - x_B y_A|$$

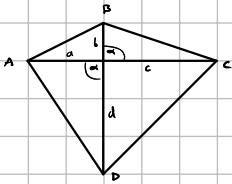
$$S = \frac{1}{2}ab \sin(\alpha) = \frac{1}{2}|u||z| \cdot \frac{|\operatorname{Im}(uz)|}{|uz|} = \frac{1}{2}|\operatorname{Im}((x_A + y_A i)(x_B - y_B i))| = \frac{1}{2}|-x_A y_B + x_B y_A| = \frac{1}{2}|x_A y_B - x_B y_A|$$

$$S = \frac{1}{2} \begin{vmatrix} x_A & x_B \\ y_A & y_B \end{vmatrix}$$



4.

$$|AB|^2 + |CD|^2 = |AD|^2 + |BC|^2 \Leftrightarrow AC \perp BD$$



$$\begin{aligned} |AB|^2 &= a^2 + b^2 + 2ab \cos(\alpha) & |AD|^2 &= a^2 + d^2 - 2ad \cos(\alpha) \\ |CD|^2 &= c^2 + d^2 + 2cd \cos(\alpha) & |BC|^2 &= b^2 + c^2 - 2bc \cos(\alpha) \\ a^2 + b^2 + c^2 + d^2 + 2\cos(\alpha)(ab + cd) &= a^2 + b^2 + c^2 + d^2 - 2\cos(\alpha)(ad + bc) \\ \cos(\alpha)(ab + cd) &= -\cos(\alpha)(ad + bc) \end{aligned}$$

$$1^\circ \cos(\alpha) = 0 :$$

$$0 = 0 \quad \checkmark$$

$$2^\circ \cos(\alpha) \neq 0 :$$

$$\underbrace{ab + cd}_{> 0} = -\underbrace{(ad + bc)}_{< 0}$$

$$\cos(\alpha) = 0 \Leftrightarrow AC \perp BD$$

sprawdzosc

$$|\alpha + d|^2 + |\beta + c|^2 = |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 + 2\operatorname{Re}(\alpha\bar{\delta}) + 2\operatorname{Re}(\beta\bar{\gamma})$$

$$|\alpha + \beta|^2 + |\gamma + \delta|^2 = |\alpha|^2 + |\beta|^2 + |\gamma|^2 + |\delta|^2 + 2\operatorname{Re}(\alpha\bar{\beta}) + 2\operatorname{Re}(\gamma\bar{\delta})$$

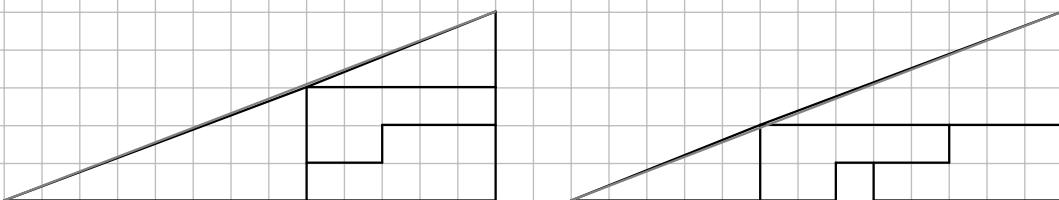
5.

$$(0,0) \ (3,3) \ (13,5)$$

$$S = \frac{1}{2} |x_1y_2 - x_2y_1|$$

$$S = \frac{1}{2} |8 \cdot 5 - 13 \cdot 3| = \frac{1}{2} - \text{wzmacnica pola trójkąta i oznacza wychodzącego na trójkąt}$$

stąd bierze się ilość w zasadzie brakującej kredytu



6.

$$D = W + a \quad \text{obrot o } 90^\circ \odot$$

$$T_1 = D + a \cdot (-j)$$

$$x = \frac{T_1 + T_2}{2} = \frac{W + a - aj + W + b + bj}{2}$$

$$S = W + b \quad \text{obrot o } 90^\circ \odot$$

$$T_2 = S + b \cdot j$$

$$= \frac{W + (D - W) - (D - W)j + W + (S - W) + (S - W)j}{2}$$

$$= \frac{D - Dj + Wj + S + Sj - Wj}{2} = \frac{D(1-j) + S(1+j)}{2}$$

