

1.

$$F(x) = \begin{cases} 0 & x < -1 \\ 1-x^2 & x \in [-1, 0) \\ 1 & x \geq 0 \end{cases}$$

$$f(x) = \frac{d}{dx} F(x) = \begin{cases} -2x & x \in [-1, 0) \\ 0 & \text{u. a.ä.} \end{cases}$$

$$EX = \int_{-1}^0 x(-2x) dx = -2 \int_{-1}^0 x^2 dx = -\frac{2}{3} \cdot x^3 \Big|_{-1}^0 = -\frac{2}{3} (0 - (-1)^3) = -\frac{2}{3}$$

$$EX^2 = \int_{-1}^0 x^2(-2x) dx = -2 \int_{-1}^0 x^3 dx = -\frac{1}{2} \cdot x^4 \Big|_{-1}^0 = -\frac{1}{2} (0 - (-1)^4) = \frac{1}{2}$$

$$VX = \frac{1}{2} - \left(-\frac{2}{3}\right)^2 = \frac{2}{18} - \frac{4}{18} = -\frac{2}{18}$$

$$Y = \sum_{k=1}^{100} X_k \approx N(100 EX, 100 VX) = N(-1200, 100)$$

$$\begin{aligned} P(|Y| \leq 1220) &= P(-1220 \leq Y \leq 1220) = F(1220) - F(-1220) \\ &= \Phi\left(\frac{1220 + 1200}{10}\right) - \Phi\left(\frac{-1220 + 1200}{10}\right) \\ &= \Phi(2.2) - \Phi(-2) \approx 1 - (1 - \Phi(2)) = \Phi(2) \\ &\approx 0.9772 \end{aligned}$$

2.

$$X_k \sim P(\lambda=3.5) \quad \text{Länge u. 1 Tag}$$

$$a) \quad Y = \sum_{k=1}^{52} X_k \approx N(52 EX, 52 VX) = N(182, 182)$$

$$\begin{aligned} P(Y > 200) &= 1 - P(Y \leq 200) = 1 - F(200) = 1 - \Phi\left(\frac{200 - 182}{\sqrt{182}}\right) \\ &= 1 - \Phi(1.33) \approx 0.0918 \end{aligned}$$

$$b) \quad Y = \sum_{k=1}^{52} X_k \sim P\left(\sum_{k=1}^{52} 3.5\right) = P(182)$$

$$\begin{aligned} P(Y > 200) &= 1 - P(Y \leq 200) = 1 - \sum_{k=0}^{200} P(Y=k) \\ &= 1 - \sum_{k=0}^{200} e^{-182} \cdot \frac{182^k}{k!} \approx 0.0867 \end{aligned}$$

3.

$$Z = 10\,000 \cdot 30 - X \cdot 2500 \quad \text{Zysk}$$

$$X \sim B(n=10000, p=0.006) \quad \text{Körbe unbeschädigt}$$

$$\begin{aligned} P(Z > 125\,000) &= P(300\,000 - 2500X > 125\,000) \\ &= P(X < 70) = \sum_{k=0}^{69} \binom{10000}{k} (0.006)^k (1-0.006)^{10000-k} \\ &\approx 0.29 \end{aligned}$$

$$X \approx N(np, np(1-p)) = N(60, 59.64)$$

$$P(X=60) = F(60) = \Phi\left(\frac{60-60}{\sqrt{59.64}}\right) = \Phi(0) \approx 0.3750$$

4.

$$\begin{aligned} X &\sim B(n=200, p=0.02) \quad \text{Körbe bTigeln} \\ &\approx N(4, 3.92) \end{aligned}$$

$$P(X \leq M) = 0.75 \Rightarrow \Phi\left(\frac{M-4}{\sqrt{3.92}}\right) = \Phi\left(\frac{2}{2}\right) \Rightarrow M = 5.31$$

M gewinnt bei 5 oder 6

5.

$$L_1 \sim U([0.07, 0.08]) \sim \text{stare assigne } \frac{L}{\text{km}}$$

$$L_2 \sim U([0.06, 0.07]) \sim \text{nave assigne } \frac{L}{\text{km}}$$

$$X_1 = \sum_{i=1}^{600} L_1 \approx N(600 E L_1, 600 V L_1) = N(45, 600 V L_1)$$

$$X_2 = \sum_{i=1}^D L_2 \approx N(D E L_2, D V L_2) = N(0.065 D, D V L_2)$$

$$P(X_1 \leq 45) = \Phi\left(\frac{45-45}{\sqrt{600 V L_1}}\right) = \Phi(0)$$

$$P(X_2 \leq 45) = P(X_1 \leq 45) \Rightarrow \Phi\left(\frac{45-0.065 D}{\sqrt{D V L_2}}\right) = \Phi(0) \Rightarrow 45 = 0.065 D \Rightarrow D \approx 692$$