1.

$X \sim P(\lambda_1 = 5) \Rightarrow EX = 5$ (5 zgłoszeń / 1h)

a)

T – czas między zgłoszeniami

$ET = \frac{1h}{5} = \frac{1}{\lambda_2} \Rightarrow T \sim exp(\lambda_2 = 5)$

$\lambda_2 = \lambda_1 \cdot 1$

b) $P(T \leq \frac{1}{4}) = F_T(\frac{1}{4}) = (1 - e^{-5 \cdot \frac{1}{4}}) \cdot \mathbb{1}_{[0,\infty)}(\frac{1}{4}) = 1 - e^{-\frac{5}{4}} \approx 71\%$

c) $P(T > 1) = 1 - P(T \leq 1) = 1 - F(1) = 1 - (1 - e^{-5}) = e^{-5} < 1\%$

2.

$X \sim N(2,4)$

a) $P(|X+1| \leq 5) = P(X+1 \leq 5 \wedge X+1 \geq -5)$

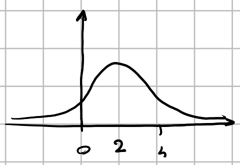
$= P(X \leq 4 \wedge X \geq -6) = P(-6 \leq X \leq 4) = F(4) - F(-6)$

$= \phi\left(\frac{4-2}{2}\right) - \phi\left(\frac{-6-2}{2}\right) = \phi(1) - \phi(-4)$

$= \phi(1) - (1 - \phi(4)) = \phi(1) + \phi(4) - 1$

$\simeq 0.8413$

b)



$P(X > 6) < \frac{1}{2}$

$P(X < 0) + P(X < 4) = 1$

$P(-4 < X < 8) > P(10 < X < 17)$

3.

$X \sim N(4000, \sigma^2)$

a) $P(X \leq 2400) = 15.87\% = F(2400) = \phi\left(\frac{2400-4000}{\sigma}\right)$

$0.1587 = \phi\left(-\frac{1600}{\sigma}\right) = 1 - \phi\left(\frac{1600}{\sigma}\right)$

$\phi\left(\frac{1600}{\sigma}\right) = .8413 \Rightarrow \frac{1600}{\sigma} \approx 1 \Rightarrow \sigma = 1600$

b) $P(4000 \leq X \leq 5600) = F(5600) - F(4000)$

$= \phi(1) - \phi(0) = 0.8413 - 0.5 = 0.3413$

4.

$X \sim N(10, 4)$

$0.98 = P(X \leq x) = F(x) = \Phi\left(\frac{x-10}{2}\right)$

$\frac{x-10}{2} = 2.05 \Rightarrow x = 14,1$

5.

$X \sim N(21, 9)$

$P(X < x_1) = P(X \geq x_2) = P(x_1 \leq X < x_2) = \frac{1}{3}$

$\frac{1}{3} = P(X < x_1) = F(x_1) = \phi\left(\frac{x_1 - 21}{3}\right) = 1 - \phi\left(\frac{21 - x_1}{3}\right)$

$\phi\left(\frac{21 - x_1}{3}\right) = \frac{2}{3} \Rightarrow \frac{21 - x_1}{3} \simeq 0.44 \Longrightarrow x_1 \simeq 19.68$

$\frac{1}{3} = P(X \geq x_2) = 1 - P(X < x_2) = 1 - F(x_2)$

$\frac{2}{3} = \phi\left(\frac{x_2 - 21}{3}\right) \Rightarrow 0.44 = \frac{x_2 - 21}{3} \Rightarrow x_2 = 22.32$

6.

X	0	1	2	3	4	5	6	7	8	9
X^2	0	1	4	9	16	25	36	49	64	81
$Y = X^2 \% 10$	0	1	4	9	6	5	6	9	4	1

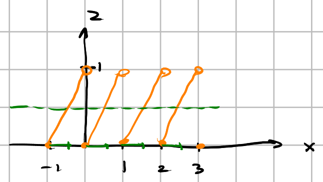
$S_y = \{0, 1, 4, 5, 6, 9\}$

y	0	1	4	5	6	9
$P(Y=y)$	.1	.2	.2	.1	.2	.2

7.

$X \sim U([-1,3])$

$Z = X - \lfloor X \rfloor$



$S_Z = [0,1]$

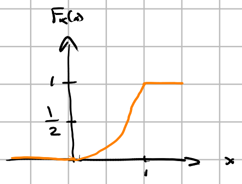
$$F_Z(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{4}\left[(-1+t)-(-1) + t-0 + (1+t)-1 + (2+t)-2\right] = t & t \in [0,1) \\ 1 & t \geq 1 \end{cases}$$

$Z \sim U([0,1]) \implies f_Z(z) = \mathbb{1}_{[0,1]}(z)$

8.

$$f_X(x) = 2x \, \mathbb{1}_{(0,1)}(x)$$

$$F_X(x) = \begin{cases} 0 & x \le 0 \\ x^2 & x \in (0,1) \\ 1 & x \ge 1 \end{cases}$$



$$Y = \begin{cases} \frac{1}{2} & 0 < X < \frac{1}{2} \\ 1 - X & \frac{1}{2} \le X < 1 \end{cases} \qquad S_Y = (0, \tfrac{1}{2})$$

$$F_Y(t) = P(Y \le t) = \begin{cases} 0 & t < 0 \\ & \\ 1 & t \ge \frac{1}{2} \end{cases}$$