1.

$$A \div B = (A \setminus B) \cup (B \setminus A)$$

a) $A \div B = \emptyset \overset{?}{\iff} A = B$

niech $A \div B = \emptyset$

$\iff (A \setminus B) \cup (B \setminus A) = \emptyset$
$\iff A \setminus B = \emptyset \land B \setminus A = \emptyset$
$\iff A \subseteq B \land B \subseteq A$
$\iff A = B$

b) $A \div \emptyset \overset{?}{=} A$

$A \div \emptyset = (A \setminus \emptyset) \cup (\emptyset \setminus A) = A \cup \emptyset = A$

c) $A \div B = (A \cup B) - (A \cap B)$

$(A \cup B) - (A \cap B) = [(A \setminus B) \cup (B \setminus A) \cup (A \cap B)] - (A \cap B) = (A \setminus B) \cup (B \setminus A) = A \div B$

2.

$A = \{\emptyset, \mathbb{R}\}$ $\quad B = \{0, \emptyset\}$ $\quad C = \{\mathbb{R}, \mathbb{R}^2\}$

a) $B \setminus \mathbb{R} = \{\emptyset\}$

b) $(A \cup B) \setminus C = \{0, \emptyset, \mathbb{R}\} \setminus \{\mathbb{R}, \mathbb{R}^2\} = \{0, \emptyset\} = B$

c) $(A \div B) \cap \mathbb{R} = (\{\mathbb{R}\} \cup \{0\}) \cap \mathbb{R} = \{\mathbb{R}, 0\} \cap \mathbb{R} = \{0\}$

d) $2^B = \{\emptyset, \{0\}, \{\emptyset\}, \{0, \emptyset\}\}$

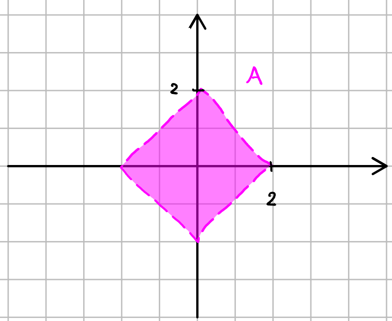
e) $A \times B = \{(\emptyset, 0), (\emptyset, \emptyset), (\mathbb{R}, 0), (\mathbb{R}, \emptyset)\}$

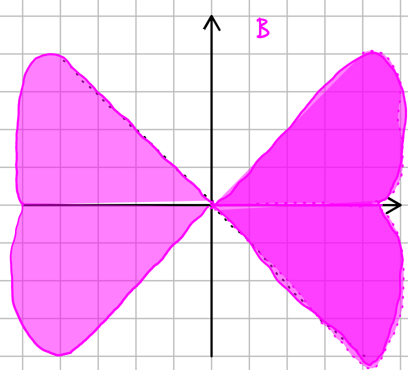
3.

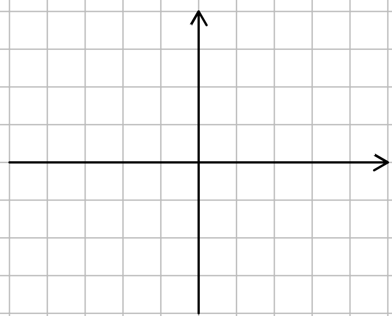
$A = \{(x,y) \in \mathbb{R}^2 : |x| + |y| < 2\}$

$B = \{(x,y) \in \mathbb{R}^2 : |y| \le |x|\}$

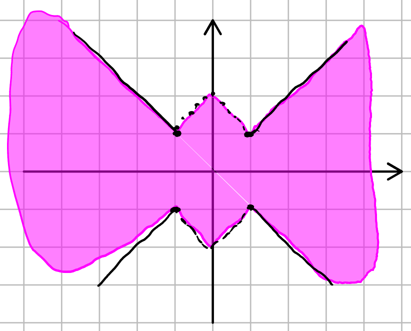
$y \le x$ dla $y \ge 0$ $x \ge 0$
$-y \le x$ dla $y < 0$ $x \ge 0$
$-y \ge x$ $\quad y \le -x$ dla $y \ge 0$ $x < 0$
$y \ge x$ $\quad -y \le -x$ dla $y < 0$ $x < 0$



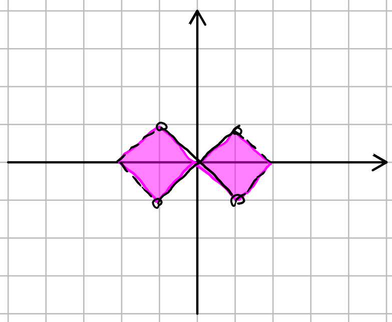




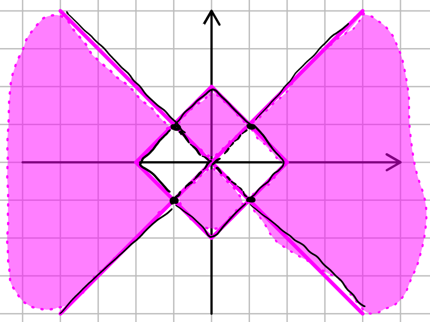
a) $A \cup B$



b) $A \cap B$



c) $A \div B$



d) $\{(x,y) \in \mathbb{R}^2 : (x,y) \in A \Rightarrow (x,y) \in B\}$

$\{(x,y) \in \mathbb{R}^2 : (x,y) \notin A \vee (x,y) \in B\}$

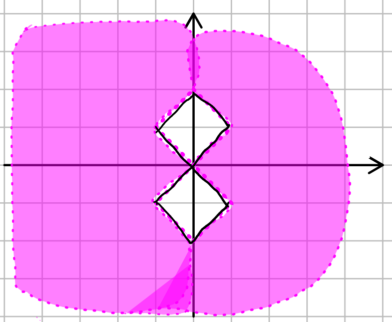
$= \mathbb{R}^2 - \{A \setminus B\}$

$p \Rightarrow q$

$\sim(\sim(p \Rightarrow q))$

$\sim(p \wedge \sim q)$

$\sim p \vee q$

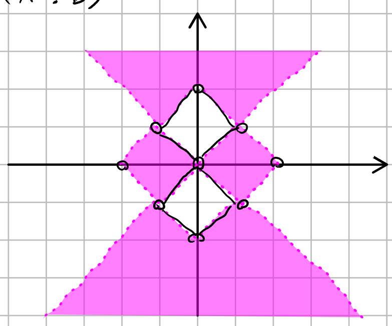


e) $\{(x,y) \in \mathbb{R}^2 : (x,y) \in A \Leftrightarrow (x,y) \in B\}$

$\{(x,y) \in \mathbb{R}^2 : ((x,y) \in A \wedge (x,y) \in B) \vee ((x,y) \notin A \wedge (x,y) \notin B)\}$

$\{(x,y) \in \mathbb{R}^2 : (x,y) \in A \cap B \vee (x,y) \notin A \cup B\}$

$\mathbb{R}^2 - (A \div B)$

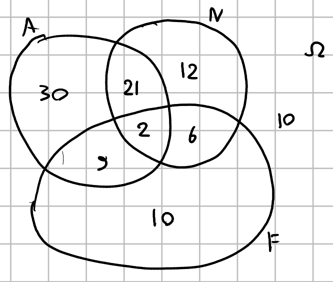


4.

$|A| = 62$ $\quad |A \cap N| = 23$ $\quad |A \cap N \cap F| = 2$

$|N| = 41$ $\quad |A \cap F| = 11$

$|F| = 27$ $\quad |N \cap F| = 8$ $\quad |\Omega| = 100$



$|A \cup N \cup F| = 90$

$|\Omega \setminus (A \cup N \cup F)| = 10$

$|A \cup N \cup F| = |A| + |N| + |F| - |A \cap N| - |A \cap F| - |N \cap F| + |A \cap N \cap F|$

$62 + 41 + 27 - 23 - 11 - 8 + 2 = 90$

10 osób nie zna żadnego języka

12 zna tylko niemiecki

$x \notin N \Rightarrow x \in A$

$x \in N \vee x \in A$

$\sim(x \notin N \Rightarrow x \in A)$

$x \notin N \wedge x \notin A$

$x \in \Omega \setminus (A \cup N)$

mówią prawdę – $\{x \in \Omega : x \in N \vee x \in A\} = \{x \in \Omega : x \in N \cup A\}$

$|A \cup N| = |A| + |N| - |A \cap N| = 62 + 41 - 23 = 80$

$|\Omega| - |A \cup N| = 100 - 80 = 20$

skłamało by 20 osób

5.

a) $A_t = \{x \in \mathbb{R} : 1 + \frac{1}{t} \leq x \leq 4 + \frac{1}{t^2}\}$ $\quad T = \mathbb{N}$

$A_1 = [2, 5]$ $\quad \bigcup_{t \in T} A_t = (1, 5]$

$A_2 = [1\frac{1}{2}, 4\frac{1}{4}]$

$A_3 = [1\frac{1}{3}, 4\frac{1}{9}]$ $\quad \bigcap_{t \in T} A_t = [2, 4]$

$\downarrow$

$(1, 4]$

b) $A_t = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \geqslant t^2\}$ gdzie $T = \mathbb{R}$

$\bigcup_{t \in T} A_t = \mathbb{R}^2$ $\quad \bigcap_{t \in T} A_t = \emptyset$

c) $A_t = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 2 - \sin(t)\}$ gdzie $T = \mathbb{R}$

$A_{\frac{\pi}{2}} = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\} = \bigcap_{t \in T} A_t$ $\quad$ bo $\quad -1 \leq \sin(t) \leq 1$

$A_{\frac{3\pi}{2}} = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 3\} = \bigcup_{t \in T} A_t$

d) $A_t = \{x \in \mathbb{R} : x^2 + (2 - t^2)x - 2t^2 = 0\}$ $\quad T = \mathbb{R}$

$\Delta = (2 - t^2)^2 - 4(-2t^2) = 4 - 4t^2 + t^4 + 8t^2$

$\Delta = t^4 + 4t^2 + 4 = (t^2 + 2)^2$ $\quad \forall_{t \in \mathbb{R}}\ \Delta > 0$

$$x_1 = \frac{-(2 - t^2) - \sqrt{(t^2+2)^2}}{2} = \frac{\cancel{t^2} - 2 - \cancel{t^2} - 2}{2} = -2$$

$$x_2 = \frac{-(2 - t^2) + \sqrt{(t^2+2)^2}}{2} = \frac{t^2 \cancel{-2} + t^2 \cancel{+2}}{2} = t^2$$

$A_t = \{-2, t^2\}$

$\bigcap_{t \in T} A_t = \{-2\}$ $\quad \bigcup_{t \in T} A_t = \{-2\} \cup [0, +\infty)$

e) $A_t = \{x \in \mathbb{R} : 3 + (-1)^t - \frac{(-1)^t}{t} < x < 7 + (-1)^t - \frac{(-1)^t}{t}\}$ $\quad T = \mathbb{N}$

$A_1 = (3 - 1 + 1, 7 + -1 + 1) = (3, 7)$ $\quad A_2 = (3 + 1 - \frac{1}{2}, 7 + 1 - \frac{1}{2}) = (3\frac{1}{2}, 7\frac{1}{2})$

$A_3 = (3 - 1 + \frac{1}{3}, 7 - 1 + \frac{1}{3}) = (2\frac{1}{3}, 6\frac{1}{3})$ $\quad A_4 = (3 + 1 - \frac{1}{4}, 7 + 1 - \frac{1}{4}) = (3\frac{3}{4}, 7\frac{3}{4})$

$A_5 = (3 - 1 + \frac{1}{5}, 7 - 1 + \frac{1}{5}) = (2\frac{1}{5}, 6\frac{1}{5})$ $\quad 2n \downarrow$

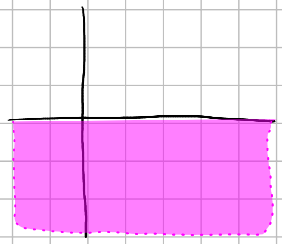
$2n+1 \downarrow$ $\quad [4, 8)$

$(2, 6]$

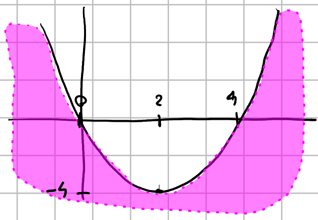
$\bigcap_{t \in T} A_t = [4, 6]$ $\quad \bigcup_{t \in T} A_t = (2, 8)$

f) $A_t = \{(x,y)\in\mathbb{R}^2 : y \le tx(x-4)\}$, $T = \mathbb{N}\cup 0$

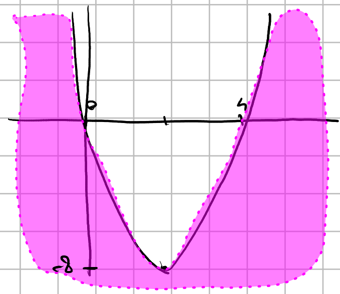
A_0 :



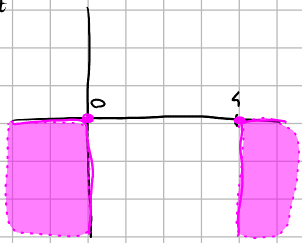
A_1 :



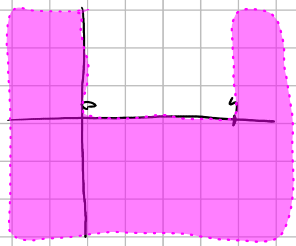
A_2 :

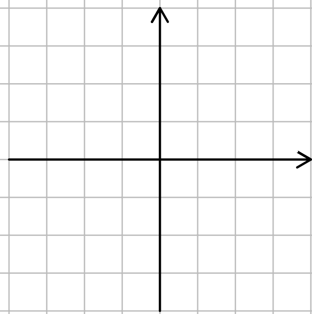


$\bigcap_{t\in T} A_t$



$\bigcup_{t\in T} A_t$

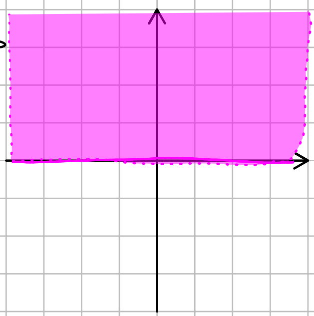




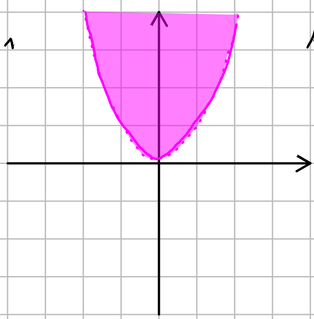
6.

$A_t = \{(x,y)\in\mathbb{R}^2 : y \ge tx^2\}$ $t\in\mathbb{R}$

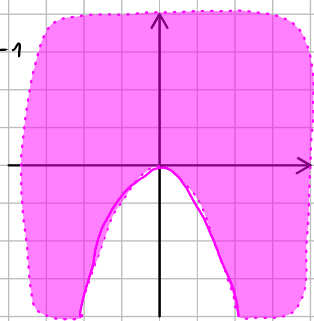
A_0



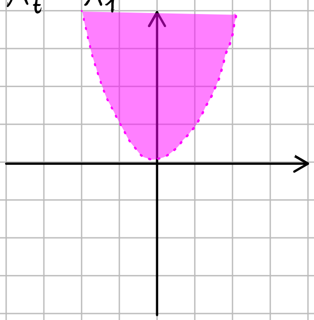
A_1



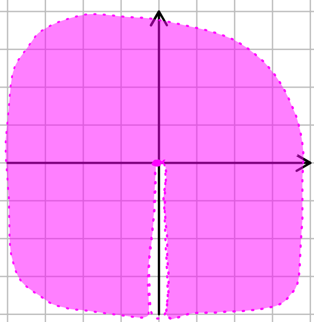
A_{-1}



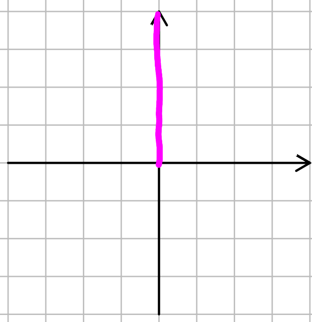
a) $\bigcup_{t\in\mathbb{N}} A_t = A_1$



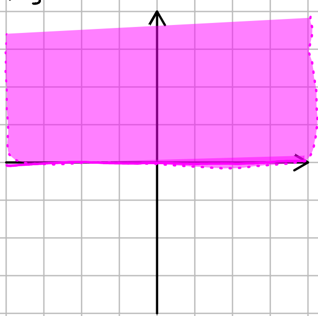
b) $\bigcup_{t\in\mathbb{R}} A_t$



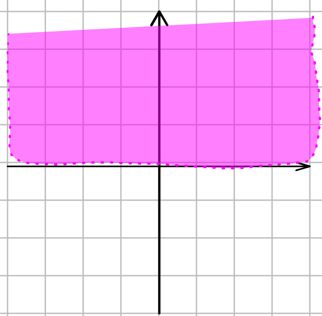
e) $\bigcap_{t\in\mathbb{N}} A_t$



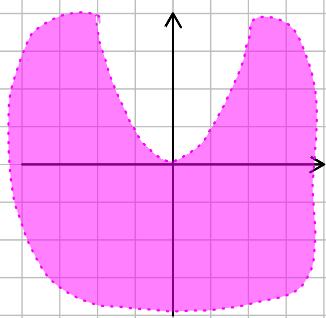
c) $\bigcup_{t\in[0,1]} At$



d) $\bigcup_{t\in(0,1)} A_t$

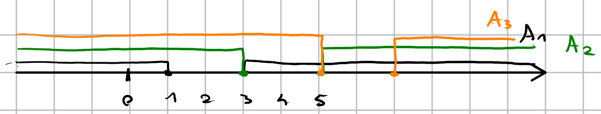


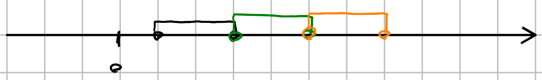
f) $\bigcup_{t\in[0,1]} A'_t$



7.

$A_n = \mathbb{R} \setminus (2n-1, 2n+1) = \{x \in \mathbb{R} : |x - 2n| \geq 1\}, \quad n \in \mathbb{N}$





a) $\bigcup_{n=1}^{\infty} A_n = \mathbb{R}$ $\qquad$ b) $\bigcap_{n=1}^{\infty} A_n = (-\infty, 1]$

c) $\bigcup_{n=1}^{\infty} A_n = (1, +\infty) - \{2n+1 : n \in \mathbb{N}\}$

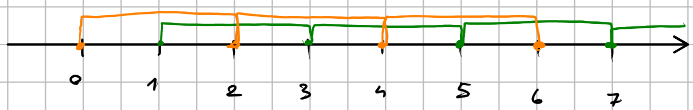
d) $\bigcap_{n=1}^{\infty} A_n = \emptyset$

8. $A_m = \{x \in \mathbb{R} : m - 1 - (-1)^m \leq x \leq m + 1 - (-1)^m\} \quad m \in \mathbb{N}$

$A_1 = [1 - 1 + 1, 1 + 1 + 1] = [1, 3]$ $\qquad$ $A_2 = [2 - 1 - 1, 2 + 1 - 1] = [0, 2]$

$A_3 = [3 - 1 + 1, 3 + 1 + 1] = [3, 5]$ $\qquad$ $A_4 = [4 - 1 - 1, 4 + 1 - 1] = [2, 4]$

$A_5 = [5, 7]$ $\qquad$ $A_6 = [4, 6]$



a) $\bigcap_{m=1}^{\infty} A_m = \emptyset$ $\qquad$ b) $\bigcup_{m=1}^{\infty} A_m = [0, +\infty)$ $\qquad$ c) $\bigcap_{m=1}^{\infty} (\mathbb{R} - A_m) = (-\infty, 0)$

d) $\bigcup_{m=1}^{\infty} (\mathbb{R} - A_m) = \mathbb{R}$ $\qquad$ e) $\bigcup_{m \in 2\mathbb{N}} A_m = [0, +\infty)$