1.

$$f(x) = x\arctan\left(\frac{x}{x-1}\right) \qquad D_f = \mathbb{R}\setminus\{1\}$$

$$\lim_{x\to 1^-} f(x) = \left| 1\cdot \arctan\left(\frac{1}{0^-}\right) \to \arctan(-\infty)\right| = -\frac{\pi}{2}$$

$$\lim_{x\to 1^+} f(x) = \left| 1\cdot \arctan\left(\frac{1}{0^+}\right) \to \arctan(+\infty)\right| = \frac{\pi}{2}$$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \arctan\left(\frac{x}{x-1}\right) = \arctan(1) = \frac{\pi}{4}$$

$$\lim_{x\to+\infty}\left[f(x) - \frac{\pi}{4}x\right] = \lim_{x\to+\infty} x\left[\arctan\left(\frac{x}{x-1}\right) - \frac{\pi}{4}\right] = |+\infty\cdot 0| = \lim_{x\to+\infty} \frac{\arctan\left(\frac{x}{x-1}\right) - \frac{\pi}{4}}{\frac{1}{x}} \overset{\frac{0}{0}}{\underset{H}{=}} \lim_{x\to+\infty} \frac{\frac{1}{1+\frac{x^2}{x^2-2x+1}}\cdot\frac{x-1-x}{(x-1)^2}}{-\frac{1}{x^2}}$$

$$= \lim_{x\to+\infty} \frac{\frac{(x-1)^2}{2x^2-2x+1}\cdot\frac{-1}{(x-1)^2}}{-\frac{1}{x^2}} = \lim_{x\to+\infty} \frac{x^2}{2x^2-2x+1} = \frac{1}{2}$$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \arctan\left(\frac{x}{x-1}\right) = \frac{\pi}{4}$$

$$\lim_{x\to-\infty}\left[f(x) - \frac{\pi}{4}x\right] = \lim_{x\to-\infty} \frac{\arctan\left(\frac{x}{x-1}\right) - \frac{\pi}{4}}{\frac{1}{x}} \overset{\frac{0}{0}}{\underset{H}{=}} \lim_{x\to-\infty} \frac{x^2}{2x^2-2x+1} = \frac{1}{2}$$

brak asymptot pionowych i obustronna asymptota ukośna $\frac{\pi}{4}x + \frac{1}{2}$

2.

$$\lim_{x\to+\infty} \frac{\int_1^{x^2}\left[\frac{2}{t} - \ln\left(\frac{1+t}{t}\right)\right]dt}{x^2} \overset{\frac{?}{\infty}}{\underset{H}{=}} \lim_{x\to+\infty} \frac{\left[\frac{2}{x} - \ln\left(\frac{x+1}{x}\right)\right]\cdot 2x}{2x} = \lim_{x\to+\infty}\left[\frac{2}{x} - \ln\left(\frac{x+1}{x}\right)\right] = 0 - \ln(1) = 0$$

3.

$$f(x) = \frac{1}{\sqrt{x(x+1)(x+2)}} \qquad x\in[2,3] \qquad |V| = \pi\int_a^b f^2(x)\,dx$$

$$\frac{1}{x(x+1)(x+2)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x+2} = \frac{A(x+1)(x+2) + Bx(x+2) + Cx(x+1)}{x(x+1)(x+2)}$$

$x=0 \to 1 = 2A \quad A = \frac{1}{2}$

$x=-1 \to 1 = -B \quad B = -1$

$x=-2 \to 1 = 2C \quad C = \frac{1}{2}$

$$\int \frac{dx}{x(x+1)(x+2)} = \frac{1}{2}\int\frac{dx}{x} - \int\frac{dx}{x+1} + \frac{1}{2}\int\frac{dx}{x+2} = \frac{1}{2}\ln|x| - \ln|x+1| + \frac{1}{2}\ln|x+2| + C$$

$$|V| = \pi\int_2^3 \frac{dx}{x(x+1)(x+2)} = \pi\left[\frac{1}{2}\ln(3) - \ln(4) + \frac{1}{2}\ln(5) - \frac{1}{2}\ln(2) + \ln(3) - \frac{1}{2}\ln(4)\right]$$

$$= \pi\left[\frac{1}{2}\ln(5) - \frac{3}{2}\ln(4) + \frac{3}{2}\ln(3) - \frac{1}{2}\ln(2)\right]$$

$$= \frac{\pi}{2}\left[\ln(5) + 3\ln(3) - 3\ln(4) - \ln(2)\right] = \frac{\pi}{2}\ln\left(\frac{135}{128}\right)$$

4.

$$f(x,y) = \begin{cases} \dfrac{x^3 - y + 1}{\sqrt{x^2 + (y-1)^2}} & (x,y)\neq(0,1) \\ 0 & (x,y) = (0,1) \end{cases}$$

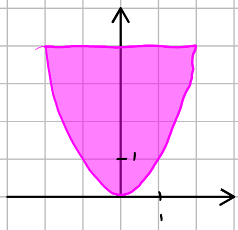
$$\frac{\partial f}{\partial x}(0,1) = \lim_{\Delta x\to 0}\frac{f(\Delta x,1) - f(0,1)}{\Delta x} = \lim_{\Delta x\to 0}\frac{\frac{\Delta x^3 - 1 + 1}{\sqrt{\Delta x^2 + (1-1)^2}} - 0}{\Delta x} = \lim_{\Delta x\to 0}\frac{\Delta x^2}{|\Delta x|} = \lim_{\Delta x\to 0}\frac{|\Delta x|^{\cancel{2}}}{\cancel{|\Delta x|}} = 0$$

$$\frac{\partial f}{\partial y}(0,1) = \lim_{\Delta y\to 0}\frac{f(0,1+\Delta y) - f(0,1)}{\Delta y} = \lim_{\Delta y\to 0}\frac{\frac{-1-\Delta y+1}{\sqrt{(1+\Delta y-1)^2}} - 0}{\Delta y} = \lim_{\Delta y\to 0}\frac{-1}{|\Delta y|} = \lim_{\Delta y\to 0} -\mathrm{sgn}(\Delta y)$$ nie istnieje bo granice jednostronne są różne

5.

$f(x,y) = (x-1)(y-1)$

$\overline{D} = \{(x,y) \in \mathbb{R}^2 : -2 \leq x \leq 2 \ \wedge \ x^2 \leq y \leq 4\}$



Int D:

$f_x = y - 1 \quad \to y = 1$

$f_y = x - 1 \quad \to x = 1$

$(1,1) \in \overline{D}$

$f_{xx} = 0 \quad f_{xy} = 1$

$f_{yx} = 1 \quad f_{yy} = 0$

$W(1,1) = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1 < 0$ nie ma ekstremum w $(1,1)$

∂D:

1° $y = 4$

$f(x,y) = (x-1)\cdot(4-1) = 3x - 3$

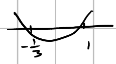
$\max\{3x-3 : x \in [-2,2]\} = 3$

$\min\{3x-3 : x \in [-2,2]\} = -9$

2° $y = x^2$

$f(x,y) = (x-1)(x^2-1) = x^3 - x^2 - x + 1 = g(x)$

$\frac{dg}{dx} = 3x^2 - 2x - 1 = 3x^2 - 3x + x - 1 = (3x+1)(x-1)$



$g(-\frac{1}{3}) = -\frac{1}{27} - \frac{1}{9} + \frac{1}{3} + 1 = \frac{-1-3+9+27}{27} = \frac{32}{27}$ max

$g(1) = 1 - 1 - 1 + 1 = 0$ min

wartość największa 3

wartość najmniejsza -9

6. ...