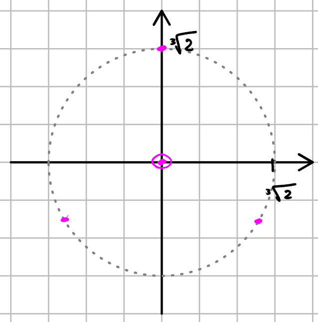
1.

$$z^5 (\sqrt{3}+j)^3 = z^2 (1+j)^8$$

a) dla $z=0$

$0=0$ ✓

b) dla $z \neq 0$



$$r^5 e^{5\varphi j} \cdot (2e^{\frac{\pi}{6}j})^3 = r^2 e^{2\varphi j} \cdot (\sqrt{2}\, e^{\frac{\pi}{4}j})^8$$

$$\frac{r^5 e^{5\varphi j}}{r^2 e^{2\varphi j}} = \frac{16\, e^{2\pi j}}{8\, e^{\frac{\pi}{2}j}} \qquad = e^{0j}$$

$$r^3 e^{3\varphi j} = 2e^{-\frac{\pi}{2}j}$$

$r^3 = 2$ $\qquad$ $3\varphi = -\frac{\pi}{2} + 2k\pi$

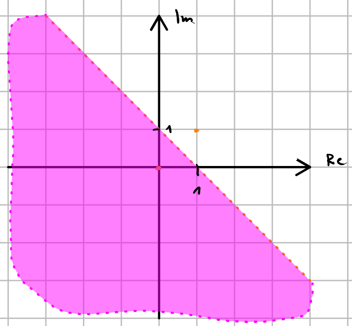
$r = \sqrt[3]{2}$ $\qquad$ $\varphi = \frac{-\frac{\pi}{2} + 2k\pi}{3}$

$$\left\{0,\ \sqrt[3]{2}\, e^{-\frac{\pi}{6}j},\ \sqrt[3]{2}\, e^{\frac{\pi}{2}j},\ \sqrt[3]{2}\, e^{-\frac{5\pi}{6}j}\right\}$$

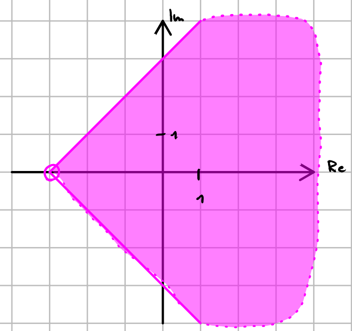
2.

$$\left\{ z \in \mathbb{C} :\ |z| < |z-1-j| \ \wedge\ \arg(z+3) \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \right\} = Z$$

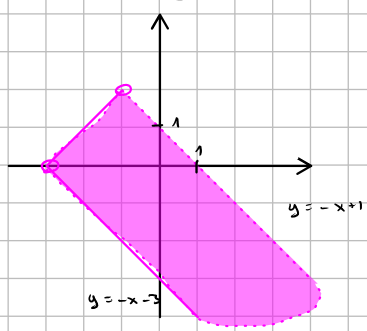
$|z| < |z-(1+j)|$



$\arg(z+3) \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$



Z



1) $51-53j$ $\qquad$ $-51+1=-50$

$-51-3=-54$

$-54 \leq -53 < -50$ ✓

$51-53j \in Z$

2) $67-66j$ $\qquad$ $-67+1=-66$

$-67-3=-70$

$-70 \leq -66 \cancel{<} -66$

$67-66j \notin Z$

3) $77+76j \notin Z$ na podstawie rysunku (I ćwiartka)

4) $88-85j$ $\qquad$ $-88+1=-87$

$-88-3=-91$

$-91 \leq -85 \cancel{<} -87$

$88-85j \notin Z$

3.

$$f(x) = \frac{-4x^3}{7(x+4)^2(x^4+16)} = \frac{-4x^3}{7(x+4)^2(x^2-2\sqrt{2}x+4)(x^2+2\sqrt{2}x+4)}$$

$$x^4+8x^2+16-8x^2 = (x^2+4)^2-8x^2 = (x^2-2\sqrt{2}x+4)(x^2+2\sqrt{2}x+4)$$

$\Delta = -8 < 0$ $\quad$ $\Delta = -8 < 0$

nad $\mathbb{R}$

$$f(x) = \frac{A}{x+4} + \frac{B}{(x+4)^2} + \frac{Cx+D}{x^2-2\sqrt{2}x+4} + \frac{Ex+F}{x^2+2\sqrt{2}x+4}$$

$$\sqrt[4]{-16} = \sqrt[4]{16e^{\pi j}} = \left\{ 2e^{\frac{\pi+2k\pi}{4}j} : k=0,1,2,3 \right\} = \left\{ 2e^{\frac{\pi}{4}j},\ 2e^{\frac{3\pi}{4}j},\ 2e^{-\frac{\pi}{4}j},\ 2e^{-\frac{3\pi}{4}j} \right\}$$

$$= \left\{ \sqrt{2}+\sqrt{2}j,\ -\sqrt{2}+\sqrt{2}j,\ \sqrt{2}-\sqrt{2}j,\ -\sqrt{2}-\sqrt{2}j \right\}$$

nad $\mathbb{C}$

$$f(x) = \frac{-4x^3}{7(x+4)^2(x-(\sqrt{2}+\sqrt{2}j))(x-(-\sqrt{2}+\sqrt{2}j))(x-(\sqrt{2}-\sqrt{2}j))(x-(-\sqrt{2}-\sqrt{2}j))}$$

$$= \frac{A}{x+4} + \frac{B}{(x+4)^2} + \frac{C}{x-(\sqrt{2}+\sqrt{2}j)} + \frac{D}{x-(-\sqrt{2}+\sqrt{2}j)} + \frac{E}{x-(\sqrt{2}-\sqrt{2}j)} + \frac{F}{x-(-\sqrt{2}-\sqrt{2}j)}$$

4.

$(z-j)^3 = (2-2j)^2$

$(z-j)^3 = (2\sqrt{2}e^{-\frac{\pi}{4}j})^2$

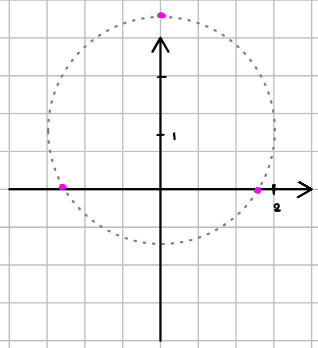
$(z-j)^3 = 8e^{-\frac{\pi}{2}j}$

$(z-j)^3 = (2e^{-\frac{\pi}{6}j})^3$

$$z-j \in \sqrt[3]{(2e^{-\frac{\pi}{6}j})^3} = \left\{ 2e^{\frac{-\frac{\pi}{2}+2k\pi}{3}j} ; k=0,1,2 \right\} = \left\{ 2e^{-\frac{\pi}{6}j},\ 2e^{\frac{\pi}{2}j},\ 2e^{-\frac{5\pi}{6}j} \right\}$$

$z-j \in \{\sqrt{3}-j,\ 2j,\ -\sqrt{3}-j\}$

$z \in \{\sqrt{3},\ 3j,\ -\sqrt{3}\}$



5. ?

$$\left\{ z \in \mathbb{C} : \operatorname{Im}(z(1-j)) < \operatorname{Re}(\overline{2-7j}) \wedge \arg\left(z - \frac{1}{j^5}\right) \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \right\} = Z$$

$$\operatorname{Im}(z(1-j)) = \operatorname{Im}(re^{\varphi j} \cdot \sqrt{2}e^{-\frac{\pi}{4}j}) = \operatorname{Im}(\sqrt{2}re^{(\varphi-\frac{\pi}{4})j})$$

$$= \operatorname{Im}(\sqrt{2}r(\cos(\varphi-\tfrac{\pi}{4}) + j\sin(\varphi-\tfrac{\pi}{4}))) = \sqrt{2}r\sin(\varphi-\tfrac{\pi}{4})$$

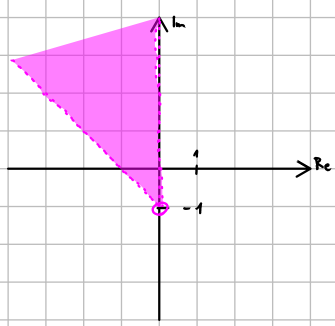
$$\operatorname{Re}(\overline{2-7j}) = \operatorname{Re}(2+7j) = 2$$

$\sqrt{2}r\sin(\varphi-\frac{\pi}{4}) < 2$

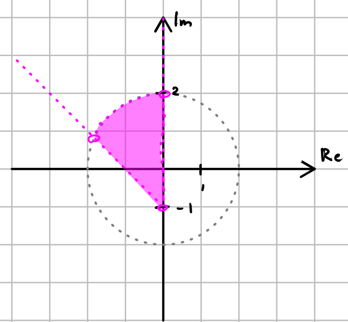
$r\sin(\varphi-\frac{\pi}{4}) < \sqrt{2}$ $\rightarrow$ dla każdego φ kiedy $r<\sqrt{2}$ bo $-1<\sin(\theta)<1$

$$\frac{1}{j^5} = \frac{1}{j^4 j} = \frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$$

$$\arg\left(z - \frac{1}{j^5}\right) = \arg(z+j)$$



$\arg(z+j) \in (\frac{\pi}{2}, \frac{3\pi}{4})$



Z

6. ?

$$f(x) = \frac{x^2+2x}{\underset{\Delta<0}{(x^2+4)}(x-2)}$$

nad $\mathbb{R}$

$$f(x) = \frac{x^2+2x}{(x^2+4)(x-2)} = \frac{Ax+B}{x^2+4} + \frac{C}{x-2} = \frac{Ax(x-2)+B(x-2)+C(x^2+4)}{(x^2+4)(x-2)}$$

$$x^2+2x = Ax(x-2)+B(x-2)+C(x^2+4)$$

$$x=2 \rightarrow 8 = 8C \Leftrightarrow C=1$$

$$x^2+2x = Ax^2-2Ax+Bx-2B+x^2+4$$
$$x^2+2x = (A+1)x^2+(B-2A)x+(4-2B)$$

$$\begin{cases} A+1=1 \\ B-2A=2 \\ 4-2B=0 \end{cases} \quad \begin{cases} A=0 \\ B=2 \\ B=2 \end{cases}$$

$$f(x) = \frac{2}{x^2+4} + \frac{1}{x-2}$$

nad $\mathbb{C}$

$$f(x) = \frac{x^2+2x}{(x^2+4)(x-2)} = \frac{x^2+2x}{(x+2j)(x-2j)(x-2)} = \frac{A}{x+2j} + \frac{B}{x-2j} + \frac{C}{x-2} = \frac{A(x-2j)(x-2)+B(x+2j)(x-2)+C(x+2j)(x-2j)}{(x+2j)(x-2j)(x-2)}$$

$$x^2+4 = (x+2j)(x-2j)$$

$$x^2+2x = A(x-2j)(x-2)+B(x+2j)(x-2)+C(x+2j)(x-2j)$$

$$x=-2j \rightarrow -4-4j = A(-4j)(-2-2j)$$
$$-4-4j = A(8+8j)$$
$$A = \frac{-4-4j}{8+8j}\cdot\frac{8-8j}{8-8j} = \frac{-32+32j-32j-32}{64+64} = \frac{-64-64j}{128} = -\frac{1}{2}-\frac{1}{2}j$$

$$x=2j \rightarrow -4+4j = B(4j)(-2+4j)$$
$$-1+j = B(-4-2j)$$
$$B = \frac{-1+j}{-4-2j}\cdot\frac{-4+2j}{-4+2j} = \frac{4-2j-4j-2}{16+4} = \frac{2-6j}{20} = \frac{1}{10}-\frac{3}{10}j$$

$$x=2 \rightarrow 8 = C(2+2j)(2-2j)$$
$$8 = C(4+4)$$
$$C=1$$

$$f(x) = \frac{-\frac{1}{2}-\frac{1}{2}j}{x+2j} + \frac{\frac{1}{10}-\frac{3}{10}j}{x-2j} + \frac{1}{x-2}$$

7

$\left[\exists_{n\in\mathbb{N}}\ \forall_{k\in\mathbb{N}}\ (n-k<1)\right] \Rightarrow \left[\forall_{n\in\mathbb{N}}\ \forall_{k\in\mathbb{N}}\ n\neq k\right]$

$n<k+1$ $\quad v(RHS)=0$ bo dla $n=k$ nieprawda że $n\neq k$

$v(LHS)=1$

prawda dla $n=1$

$[\sim(p\Rightarrow q)]\Leftrightarrow[p\wedge\sim q]$

$v(1\Rightarrow 0)=0$

$\sim\left[\left[\exists_{n\in\mathbb{N}}\ \forall_{k\in\mathbb{N}}\ (n-k<1)\right]\Rightarrow\left[\forall_{n\in\mathbb{N}}\ \forall_{k\in\mathbb{N}}\ n\neq k\right]\right]$

$\left[\exists_{n\in\mathbb{N}}\ \forall_{k\in\mathbb{N}}\ (n-k<1)\right]\wedge\sim\left[\forall_{n\in\mathbb{N}}\ \forall_{k\in\mathbb{N}}\ n\neq k\right]$

$\left[\exists_{n\in\mathbb{N}}\ \forall_{k\in\mathbb{N}}\ (n-k<1)\right]\wedge\left[\exists_{n\in\mathbb{N}}\ \exists_{k\in\mathbb{N}}\ n=k\right]$

8.

$\forall_{k\in\mathbb{N}}\ \exists_{n\in\mathbb{N}}\ (k=3n\Rightarrow n=3k)$ $\quad (p\Rightarrow q)\Leftrightarrow(\sim p\vee q)$

$\forall_{k\in\mathbb{N}}\ \exists_{n\in\mathbb{N}}\ (k\neq 3n\ \vee\ n=3k)$

prawda, bo dla każdej liczby naturalnej można znaleźć $3\cdot k$

$\sim\left[\forall_{k\in\mathbb{N}}\ \exists_{n\in\mathbb{N}}\ (k=3n\Rightarrow n=3k)\right]$ $\quad \sim(p\Rightarrow q)\Leftrightarrow p\wedge\sim q$

$\exists_{k\in\mathbb{N}}\ \forall_{n\in\mathbb{N}}\ (k=3n\ \wedge\ n\neq 3k)$

9.

$\forall_{x\in\mathbb{R}}\left[(x>5\Rightarrow x<2)\vee x>1\right]$

$\forall_{x\in\mathbb{R}}\left[(x\leq 5\ \vee\ x<2)\vee x>1\right]$

$\forall_{x\in\mathbb{R}}\left[x\in(-\infty,2)\vee x\in(1,+\infty)\right]$

$\forall_{x\in\mathbb{R}}\left[x\in(-\infty,2)\cup(1,+\infty)\right]$

$\forall_{x\in\mathbb{R}}\ x\in\mathbb{R}$ tautologia

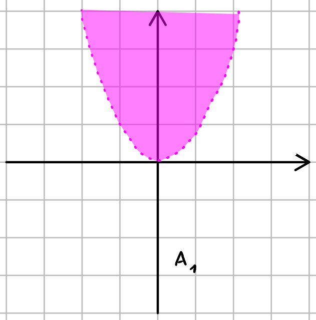
$\sim\left[\forall_{x\in\mathbb{R}}\left[(x>5\Rightarrow x<2)\vee x>1\right]\right]$

$\exists_{x\in\mathbb{R}}\left[\sim(x>5\Rightarrow x<2)\wedge\sim(x>1)\right]$

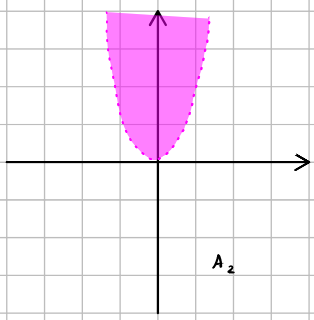
$\exists_{x\in\mathbb{R}}\left[(x>5\wedge x\geq 2)\wedge x\leq 1\right]$

10.

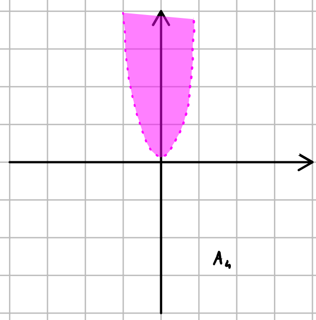
$A_n = \{ (x,y) \in \mathbb{R}^2 : y > nx^2 \}, \quad n \in \mathbb{N}$



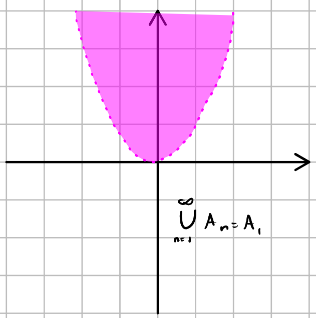
A_1



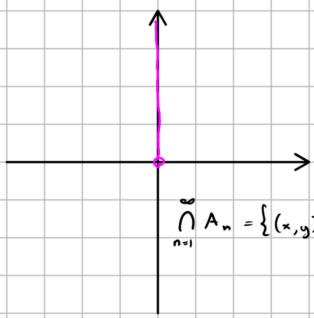
A_2



A_4



$\bigcup_{n=1}^{\infty} A_n = A_1$

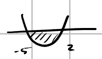


$\bigcap_{n=1}^{\infty} A_n = \{ (x,y) \in \mathbb{R} : x = 0 \wedge y > 0 \}$

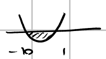
11.

$A_n = \{ x \in \mathbb{R} : (x - \frac{2}{n})(x + 5n) < 0 \}, \quad n \in \mathbb{N}$

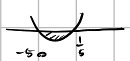
$A_1 : \quad (x-2)(x+5) < 0 \qquad (-5, 2)$



$A_2 : \quad (x-1)(x+10) < 0 \qquad (-10, 1)$

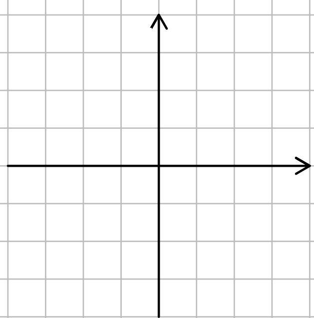


$A_{10} : \quad (x - \frac{1}{5})(x+50) < 0 \qquad (-50, \frac{1}{5})$



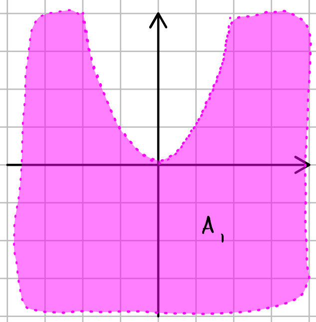
$\bigcup_{n=1}^{\infty} A_n = (-\infty, 2)$

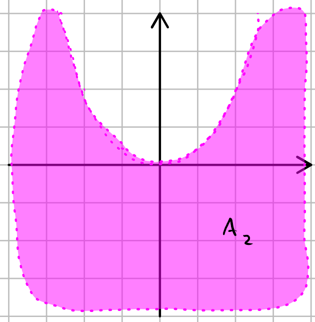
$\bigcap_{n=1}^{\infty} A_n = (-5, 0]$

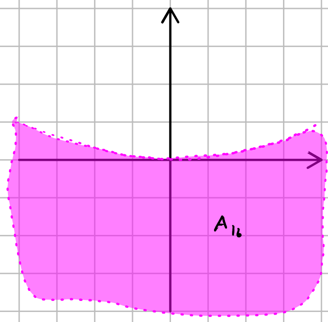


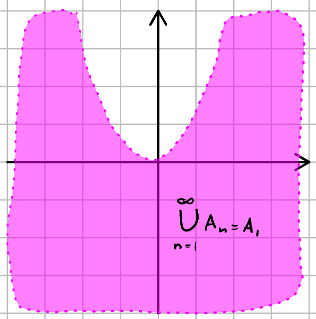
12.

$A_n = \left\{ (x,y) \in \mathbb{R}^2 : \; y < \frac{1}{n} x^2 \right\}, \quad n \in \mathbb{N}$

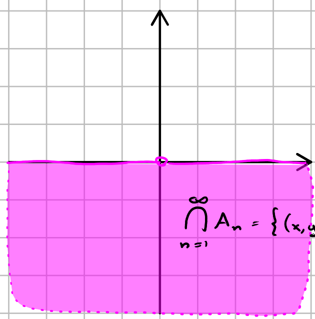








$\bigcup_{n=1}^{\infty} A_n = A_1$



$\bigcap_{n=1}^{\infty} A_n = \{ (x,y) \in \mathbb{R}^2 : \; y \leq 0 \} \setminus \{ (0,0) \}$