

$$z = x + jy$$

$$\operatorname{Re}\left(\frac{\bar{z} + j}{z - 1}\right) \geq 1$$

$$\operatorname{Re}\left[\frac{x - jy + j}{x + jy - 1}\right] = \operatorname{Re}\left[\frac{x + (1-j)j}{x - 1 + jy} \cdot \frac{(x-1) - jy}{(x-1) - jy}\right] = \operatorname{Re}\left[\frac{x(x-1) - xyj + (x-1)(-y)j + (1-j)y}{(x-1)^2 + y^2}\right]$$

$$= \frac{x(x-1) + y(1-y)}{(x-1)^2 + y^2} \geq 1$$

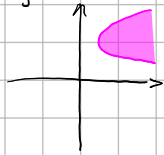
$$x(x-1) + y(1-y) \geq (x-1)^2 + y^2$$

$$x^2 - x + y - y^2 \geq x^2 - 2x + 1 + y^2$$

$$0 \geq 2y^2 - y + x + 1$$

$$x \leq -2y^2 - y - 1$$

$$x \geq 2y^2 - y + 1$$



2.

$$jz^4 = \bar{z} \cdot |z^4|$$

$$\varphi = -\frac{\pi}{12} + \frac{\pi}{3}k$$

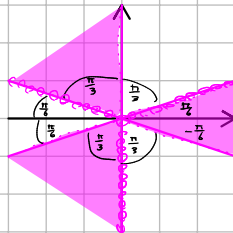
3. $0 \leq \arg(jz^3) < \pi \quad z \neq 0$

$$jz^3 = e^{\frac{\pi}{2}j} r^3 e^{3\varphi j} = r^3 e^{(3\varphi + \frac{\pi}{2})j}$$

$$0 + 2k\pi \leq 3\varphi + \frac{\pi}{2} < \pi + 2k\pi$$

$$-\frac{\pi}{2} + 2k\pi \leq 3\varphi < \frac{\pi}{2} + 2k\pi$$

$$-\frac{\pi}{6} + \frac{2k\pi}{3} \leq \varphi < \frac{\pi}{6} + \frac{2k\pi}{3}$$

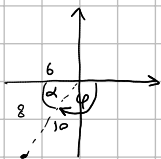


4.

$$-6 - 8j$$

$$\alpha = \arctan\left(\frac{4}{3}\right)$$

$$\arg(-6 - 8j) = \varphi = -\pi + \arctan\left(\frac{4}{3}\right)$$



5.

$$\sqrt[n]{1} = \left\{ e^{\frac{2k\pi}{n}j} : k \in \mathbb{Z} \wedge 0 \leq k < n \right\}$$

6.

$$z^4 = (jz + 1)^4$$

$$z^4 - (jz + 1)^4 = (z^2 - (jz + 1)^2)(z^2 + (jz + 1)^2)$$

$$= (z - (jz + 1))(z + (jz + 1))(z - j(jz + 1))(z + j(jz + 1))$$

$$= (z - jz + 1)(z + jz + 1)(z + 2 - j)(z - 2 + j)$$

$$= [2(1-j) + 1][z(1+j) + 1][2z - j] \cdot j$$

$$z_0 = \frac{-1}{1-j}$$

$$z_1 = \frac{-1}{1+j}$$

$$z_2 = \frac{j}{2}$$