Zestaw 1

1.

$x R y \iff \log_2\left(\frac{x}{y}\right) \in \mathbb{Z}$

✓ • zwrotna $\quad \log_2\left(\frac{x}{x}\right) = \log_2(1) = 0 \in \mathbb{Z}$

✓ • symetryczna $\quad \log_2\left(\frac{x}{y}\right) = \log_2\left(\left(\frac{y}{x}\right)^{-1}\right) = -\log_2\left(\frac{y}{x}\right)$

$k \in \mathbb{Z} \iff -k \in \mathbb{Z}$

więc $\ x R y \Rightarrow y R x$

X • antysymetryczna $\quad (1R2 \wedge 2R1) \wedge 1 \neq 2$

X • spójna $\quad 1 \not R 3 \wedge 3 \not R 1 \wedge 1 \neq 3$

✓ • przechodnia $\quad x R y \wedge y R z \iff \log_2\left(\frac{x}{y}\right) \in \mathbb{Z} \wedge \log_2\left(\frac{y}{z}\right) \in \mathbb{Z}$

$\Rightarrow x = 2^m y \wedge y = 2^n z \qquad m, n \in \mathbb{Z}$

$\Rightarrow x = 2^{m+n} z$

$\Rightarrow \log_2\left(\frac{x}{z}\right) = \log_2\left(\frac{2^{m+n} z}{z}\right) = m+n \in \mathbb{Z}$

$\Rightarrow x R z$

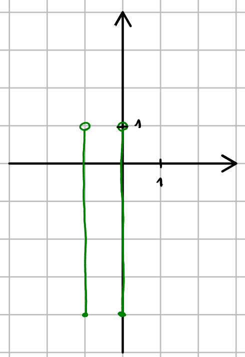
jest relacją równoważności

$$[1]_R = \{ n \in \mathbb{Z} : 2^n \} = \{ \ldots \tfrac{1}{4}, \tfrac{1}{2}, 1, 2, 4, \ldots \}$$

2.

$f : \mathbb{R}^2 \to \mathbb{R} \qquad f(x,y) = |3x + y|$

a)



$R_f \subset [0, +\infty)$

$f(0,0) = |3 \cdot 0 + 0| = 0$

$f(-1,1) = |-3+1| = 2$

$f(x_{min}, y_{min}) = f(-1,-4) = |-3-4| = 7$

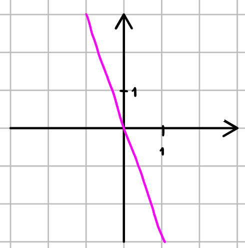
$f(\{0\} \times [-4,1)) = [0,4]$

$f(\{-1\} \times [-4,1)) = (2,7]$

$f(\{-1,0\} \times [-4,1)) = [0,7]$

b) $\ f^{-1}(\{0\}) = \{(x,y) \in \mathbb{R}^2 : |3x+y| = 0\} = \{(x,-3x) : x \in \mathbb{R}\}$

$|3x+y| = 0 \iff 3x + y = 0 \iff y = -3x$



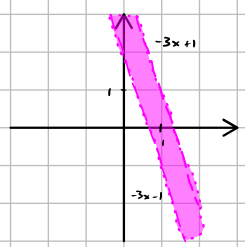
c) $\ f^{-1}((-\infty,1)) = f^{-1}([0,1))$

$f(x,y) = |3x+y| \geq 0$

$|3x+y| < 1$

$3x + y < 1 \ \wedge \ 3x + y > -1$

$y < -3x + 1 \ \wedge \ y > -3x - 1$



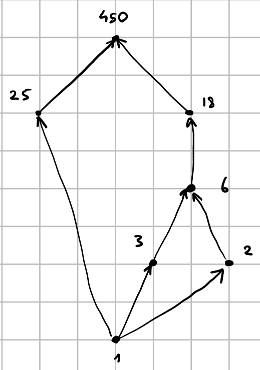
f jest surjekcją $\iff f(\mathbb{R}^2) = \mathbb{R}$

nieprawda, bo $\ f(\mathbb{R}^2) = [0, +\infty)$

nie jest surjekcją

3.

$(\{1, 2, 3, 6, 18, 25, 450\}, |)$



element największy: 450

element najmniejszy: 1

Łańcuch maksymalnej długości:

$\{1, 3, 6, 18, 450\}$

antyłańcuch maksymalnej długości:

$\{2, 3, 25\}$

Jest kratą bo dla każdej pary istnieje

kres górny - wspólna wielokrotność (450 lub mniejsza)

kres dolny - wspólny dzielnik (1 lub większy)

4.

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} -1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\frac{1}{2}\cdot w_3]{(-1)\cdot w_1,\ (-1)\cdot w_2} \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right] \xrightarrow{w_1+2w_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -1 & -2 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right] \xrightarrow[w_2+w_3]{w_1+2w_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -2 & 1 \\ 0 & 1 & 0 & 0 & -1 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -1 & -2 & 1 \\ 0 & -1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \qquad \begin{bmatrix} -1 & -2 & 1 \\ 0 & -1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}\begin{bmatrix} -1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5.

$$\begin{cases} x_1 + x_2 + x_4 + 2x_5 = 1 \\ -2x_1 - 2x_2 - x_3 + x_4 + 2x_5 = 1 \\ 4x_1 + 4x_2 + x_3 - x_4 = 1 \\ -4x_1 - 4x_2 - 2x_3 + x_4 + 3x_5 = 2 \end{cases} \qquad A = \begin{bmatrix} 1 & 1 & 0 & 1 & 2 \\ -2 & -2 & -1 & 1 & 2 \\ 4 & 4 & 1 & -1 & 0 \\ -4 & -4 & -2 & 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 2 & 1 \\ -2 & -2 & -1 & 1 & 2 & 1 \\ 4 & 4 & 1 & -1 & 0 & 1 \\ -4 & -4 & -2 & 1 & 3 & 2 \end{array}\right] \xrightarrow[w_4+w_3]{w_2+2w_1} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & 3 & 6 & 3 \\ 4 & 4 & 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 3 & 3 \end{array}\right] \xrightarrow{w_3-4w_1} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & 3 & 6 & 3 \\ 0 & 0 & 1 & -5 & -8 & -3 \\ 0 & 0 & -1 & 0 & 3 & 3 \end{array}\right] \xrightarrow[w_4+w_3]{w_2+w_3} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -2 & -2 & 0 \\ 0 & 0 & 1 & -5 & -8 & -3 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{-5} & \cancel{-5} & \cancel{0} \end{array}\right]$$

$w_4 = \frac{5}{2} w_2$

$$\xrightarrow{-\frac{1}{2}\cdot w_2} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -5 & -8 & -3 \end{array}\right] \xrightarrow[w_3+5w_2]{w_1-w_2} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -3 & -3 \end{array}\right]$$

$\text{rank}(A) = \text{rank}(A|B) = 3$ układ ma nieskończenie wiele rozwiązań zależnych od 2 parametrów

$$\begin{cases} x_1 + x_2 + x_5 = 1 \\ x_4 + x_5 = 0 \\ x_3 - 3x_5 = -3 \end{cases} \qquad \begin{cases} x_2 = 1 - x_1 - x_5 \\ x_4 = -x_5 \\ x_3 = -3 + 3x_5 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \\ 0 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ -1 \\ 3 \\ -1 \\ 1 \end{bmatrix} \qquad x_1, x_5 \in \mathbb{R}$$

Zestaw 2

1. $R \subset (0,+\infty)^2$ $\quad xRy \iff \log_{10}\left(\frac{y}{x}\right) \in \mathbb{Q}$

✓ • zwrotna $\quad \log_{10}\left(\frac{x}{x}\right) = \log_{10}(1) = 0 \in \mathbb{Q}$

✓ • symetryczna
$$xRy \iff \log_{10}\left(\frac{y}{x}\right) \in \mathbb{Q}$$
$$\iff -\log_{10}\left(\frac{y}{x}\right) \in \mathbb{Q}$$
$$\iff \log_{10}\left(\left(\frac{y}{x}\right)^{-1}\right) \in \mathbb{Q}$$
$$\iff \log_{10}\left(\frac{x}{y}\right) \in \mathbb{Q}$$
$$\iff yRx$$

× • antysymetryczna $\quad 10R100 \wedge 100R10 \wedge 10 \neq 100$

× • spójna $\quad 10R100 \wedge 100R10 \wedge 10 \neq 100$

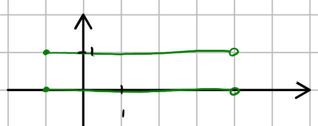
✓ • przechodnia
$$xRy \wedge yRz \iff \log_{10}\left(\frac{y}{x}\right) \in \mathbb{Q} \wedge \log_{10}\left(\frac{z}{y}\right) \in \mathbb{Q}$$
$$\iff y = 10^{q}x \wedge z = 10^{r}y \qquad q,r \in \mathbb{Q}$$
$$\implies z = 10^{r+q}x$$
$$\implies \log_{10}\left(\frac{z}{x}\right) = \log_{10}\left(\frac{10^{r+q}x}{x}\right) = r+q \in \mathbb{Q}$$
$$\implies xRz$$

jest relacją równoważności

$$[1]_R = \{10^x : x \in \mathbb{Q}\} = \{10, \tfrac{1}{10}, \sqrt[3]{1000}, \dots\}$$

2. $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = |x+3y|$

$f([-1,4) \times \{0,1\}) = [0,7)$



$f(-1,1) = |-1+3| = 2$

$f(-1,0) = |-1+0| = 1$

$f(4,1) \to |4+3| = 7$

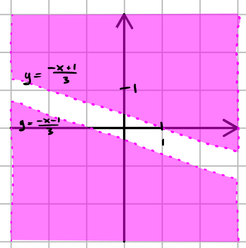
$f(4,0) \to |4+0| = 4$

$f(0,0) = |0+0| = 0$

$f^{-1}((1,+\infty)) = \{(x,y) \in \mathbb{R}^2 : |x+3y| > 1\}$

$x+3y > 1 \ \vee \ x+3y < -1$

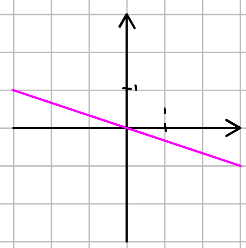
$y > -\frac{1}{3}x + \frac{1}{3} \ \vee \ y < -\frac{1}{3}x - \frac{1}{3}$



$f^{-1}(\{0\}) = \{(x,y) \in \mathbb{R}^2 : |x+3y| = 0\}$

$x+3y = 0$

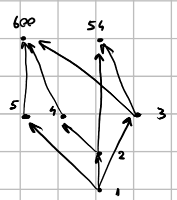
$y = -\frac{1}{3}x$



$f(-1,0) = |-1+0| = 1 = |1+0| = f(1,0)$

$(-1,0) \neq (1,0) \wedge f(-1,0) = f(1,0) \implies$ nie jest injekcją

3. $(\{1,2,3,4,5,54,600\}, |)$



elementy maksymalne: 54, 600

element najmniejszy: 1

łańcuch maksymalnej długości

$\{1,2,4,600\}$

antyłańcuch maksymalnej długości

$\{3,4,5\}$

zbiór nie jest kratą, bo nie istnieje

$\sup\{54,600\}$

4.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & -2 \end{bmatrix} \quad \left[\begin{array}{ccc|ccc} -1 & 0 & 0 & 1 & 0 & 0 \\ 2 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{w_2+2w_1} \left[\begin{array}{ccc|ccc} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 2 & 1 & 0 \\ 0 & 1 & -2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[w_3+w_2]{-1\cdot w_1} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 2 & 1 & 0 \\ 0 & 0 & -2 & 2 & 1 & 1 \end{array}\right] \xrightarrow[-\frac{1}{2}\cdot w_3]{-1\cdot w_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -2 & -1 & 0 \\ 0 & 0 & 1 & -1 & -\frac{1}{2} & -\frac{1}{2} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -1 & 0 & 0 \\ -2 & -1 & 0 \\ -1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \qquad \begin{bmatrix} -1 & 0 & 0 \\ -2 & -1 & 0 \\ -1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 2 & -1 & 0 \\ 0 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5.

$$\left[\begin{array}{ccccc|c} 3 & 3 & 1 & 0 & 0 & 0 \\ -2 & -2 & -1 & 1 & 2 & 1 \\ -5 & -5 & -2 & 1 & 2 & 1 \\ 2 & 2 & 0 & -1 & 1 & 2 \end{array}\right] \xrightarrow[w_2-w_3]{w_1-w_4} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & -1 & -2 \\ 3 & 3 & 1 & 0 & 0 & 0 \\ -5 & -5 & -2 & 1 & 2 & 1 \\ 2 & 2 & 0 & -1 & 1 & 2 \end{array}\right] \xrightarrow[\substack{w_3+5w_1 \\ w_4-2w_1}]{w_2-3w_1} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & -1 & -2 \\ 0 & 0 & -2 & -3 & 3 & 6 \\ 0 & 0 & 3 & 6 & -3 & -9 \\ \cancel{0} & \cancel{0} & \cancel{-2} & \cancel{-3} & \cancel{3} & \cancel{6} \end{array}\right] \xrightarrow{\frac{1}{3}w_3} \left[\begin{array}{ccccc|c} 1 & 1 & 1 & 1 & -1 & -2 \\ 0 & 0 & -2 & -3 & 3 & 6 \\ 0 & 0 & 1 & 2 & -1 & -3 \end{array}\right] \longrightarrow$$

$$\left[\begin{array}{cccccc} 1 & 1 & 1 & 1 & -1 & -2 \\ 0 & 0 & -2 & -3 & 3 & 6 \\ 0 & 0 & 1 & 2 & -1 & -3 \end{array}\right] \xrightarrow[w_2+2w_3]{w_1-w_3} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & -1 & -3 \end{array}\right] \xrightarrow[w_3-2w_2]{w_1+w_2} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -3 & -3 \end{array}\right]$$

$\mathrm{rank}(A) = \mathrm{rank}(A|B) = 3$ jest nieskończenie wiele rozwiązań zależnych od 2 parametrów

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -3 \\ 0 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ -1 \\ -1 \\ 3 \\ 1 \end{bmatrix} \qquad x_1, x_5 \in \mathbb{R}$$

Zestaw 3

1.

$$A = \begin{bmatrix} 0 & 2 & 1 & 0 & 1 \\ 0 & -1 & 3 & 2 & 0 \\ 1 & 2 & 0 & -1 & 3 \\ -1 & 4 & 0 & 3 & -3 \\ 1 & 3 & -1 & 0 & 2 \end{bmatrix} \xrightarrow[v_5 - v_3]{v_4 + v_3} \begin{bmatrix} 0 & 2 & 1 & 0 & 1 \\ 0 & -1 & 3 & 2 & 0 \\ 1 & 2 & 0 & -1 & 3 \\ 0 & 6 & 0 & 2 & 0 \\ 0 & 1 & -1 & 1 & -1 \end{bmatrix} \xrightarrow[v_3 - 2v_5,\ v_4 - 6v_5]{v_1 - 2v_5,\ v_2 + v_5} \begin{bmatrix} 0 & 0 & 3 & -2 & 3 \\ 0 & 0 & 2 & 3 & -1 \\ 1 & 0 & 2 & -3 & 5 \\ 0 & 0 & 6 & -4 & 6 \\ 0 & 1 & -1 & 1 & -1 \end{bmatrix} \xrightarrow[v_3 - v_2,\ v_4 - 3v_2]{v_1 - v_2}$$

$$\begin{bmatrix} 0 & 0 & 1 & -5 & 4 \\ 0 & 0 & 2 & 3 & -1 \\ 1 & 0 & 0 & -6 & 6 \\ 0 & 0 & 0 & -13 & 9 \\ 0 & 1 & -1 & 1 & -1 \end{bmatrix} \xrightarrow[v_5 + v_1]{v_2 - 2v_1} \begin{bmatrix} 0 & 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 13 & -9 \\ 1 & 0 & 0 & -6 & 6 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{-13} & \cancel{9} \\ 0 & 1 & 0 & -4 & 3 \end{bmatrix}$$

$$v_4 = -v_2$$

$$\begin{bmatrix} 0 & 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 13 & -9 \\ 1 & 0 & 0 & -6 & 6 \\ 0 & 1 & 0 & -4 & 3 \end{bmatrix} \xrightarrow{\frac{1}{13} v_2} \begin{bmatrix} 0 & 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 1 & -\frac{9}{13} \\ 1 & 0 & 0 & -6 & 6 \\ 0 & 1 & 0 & -4 & 3 \end{bmatrix} \xrightarrow[v_3 + 6v_2,\ v_4 + 4v_2]{v_1 + 5v_2} \begin{bmatrix} 0 & 0 & 1 & 0 & 4 - \frac{45}{13} \\ 0 & 0 & 0 & 1 & -\frac{9}{13} \\ 1 & 0 & 0 & 0 & 6 - \frac{54}{13} \\ 0 & 1 & 0 & 0 & 3 - \frac{36}{13} \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 6 - \frac{54}{13} \\ 0 & 1 & 0 & 0 & 3 - \frac{36}{13} \\ 0 & 0 & 1 & 0 & 4 - \frac{45}{13} \\ 0 & 0 & 0 & 1 & -\frac{9}{13} \end{bmatrix}$$

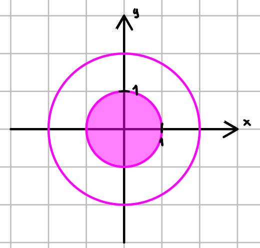
$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1 \neq 0$$

$\operatorname{rank} A = 4$

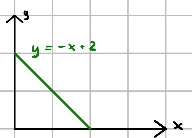
2.

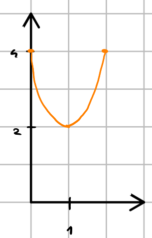
$f: \mathbb{R}^2 \to \mathbb{R} \qquad f(x,y) = \max\{1, x^2+y^2\}$

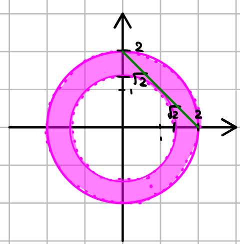
$$f^{-1}(\{0, 1, 4\}) = \emptyset \cup \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \le 1\} \cup \{(x,y) \in \mathbb{R}^2 : x^2+y^2 = 2^2\}$$



$$f^{-1}(f(\{(x,y) \in \mathbb{R} : x+y = 2 \land x \in [0,2]\})) = f^{-1}([2,4]) = \{(x,y) \in \mathbb{R}^2 : \sqrt{2}^2 \le x^2+y^2 \le 2^2\}$$



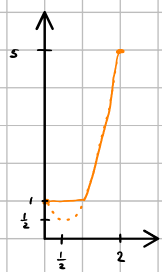




$$x^2+y^2 = x^2 + (-x+2)^2$$
$$= x^2 + x^2 - 4x + 4 = 2(x-1)^2 + 2$$

$$f(\{(x,y) \in \mathbb{R}^2 : x+y = 1 \land x \in [0,2]\}) = [1, 5]$$

$$x^2+y^2 = x^2 + (1-x)^2 = x^2 + 1 - 2x + x^2$$
$$= 2x^2 - 2x + 1 = 2\left(x - \tfrac{1}{2}\right)^2 + \tfrac{1}{2}$$



3.

$[-2, 2] \qquad x \sim y \iff \lfloor x^2 \rfloor = \lfloor y^2 \rfloor \qquad \lfloor \sqrt{2}^2 \rfloor = \lfloor 2 \rfloor = 2 \qquad \lfloor (\tfrac{1}{2})^2 \rfloor = \lfloor \tfrac{1}{4} \rfloor = 0$

$$[\sqrt{2}]_\sim = \{x \in [-2,2] : \lfloor x^2 \rfloor = 2\} = \{x \in [-2,2] : x^2 \in [2,3)\} = (-\sqrt{3}, -\sqrt{2}] \cup [\sqrt{2}, \sqrt{3})$$

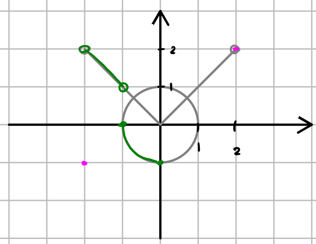
$$\left[\tfrac{1}{2}\right]_\sim = \{x \in [-2,2] : \lfloor x^2 \rfloor = 0\} = \{x \in [-2,2] : x^2 \in [0,1)\} = (-1, 1)$$

Liczba klas abstrakcji $\to 5 \qquad x \in [-2,2] \Rightarrow x^2 \in [0,4] \Rightarrow \lfloor x^2 \rfloor \in \{0,1,2,3,4\}$

4.

$(x_1, y_1) \leq_P (x_2, y_2) \iff x_1 \leq x_2 \wedge y_1 \leq y_2$

$A = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\} \cup \{(x,y) \in \mathbb{R}^2 : y = |x| \wedge x \in (-2,2)\}$



kres górny $(2,2)$
kres dolny $(-2,-1)$
nie ma elementu największego
nie ma elementów maksymalnych
nie ma elementu najmniejszego
elementy minimalne

5.

$$\left[\begin{array}{cccc|c} 1 & 0 & -4 & 8 & 1 \\ 0 & 2 & 1 & -4 & 4 \\ 2 & 1 & -3 & 5 & 4 \\ 3 & -2 & -1 & 4 & -1 \end{array}\right] \xrightarrow[\substack{w_3 - 2w_1 \\ w_4 - 3w_1}]{} \left[\begin{array}{cccc|c} 1 & 0 & -4 & 8 & 1 \\ 0 & 2 & 1 & -4 & 4 \\ 0 & 1 & 5 & -11 & 2 \\ 0 & -2 & 11 & -20 & -4 \end{array}\right] \xrightarrow[w_4 + 2w_3]{w_2 - 2w_3} \left[\begin{array}{ccccc} 1 & 0 & -4 & 8 & 1 \\ 0 & 0 & -9 & 18 & 0 \\ 0 & 1 & 5 & -11 & 2 \\ 0 & 0 & 21 & -42 & 0 \end{array}\right] \xrightarrow[\frac{1}{21} w_4]{-\frac{1}{9} w_2}$$

$$\left[\begin{array}{ccccc} 1 & 0 & -4 & 8 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 5 & -11 & 2 \\ \cancel{0} & \cancel{0} & \cancel{1} & \cancel{-2} & \cancel{0} \end{array}\right] \xrightarrow[w_3 - 5w_2]{w_1 + 4w_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & -1 & 2 \end{array}\right]$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & a \end{array}$$

$$\begin{cases} x = 1 \\ y = 2 + a \\ z = 2a \\ t = a \end{cases} \qquad \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + a \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} \qquad a \in \mathbb{R}$$