1.

a) $\sum\limits_{n=1}^{\infty} \frac{2^n}{n+1} x^n$

$\lambda = \lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \left| \frac{\frac{2^{n+1}}{n+2}}{\frac{2^n}{n+1}} \right| = \lim\limits_{n\to\infty} \frac{2^{n+1}}{n+2} \cdot \frac{n+1}{2^n} = \lim\limits_{n\to\infty} \frac{2n+2}{n+2} = 2 \rightarrow R = \frac{1}{2}$

dla $x=\frac{1}{2}$ $\sum\limits_{n=1}^{\infty} \frac{2^n}{n+1} \cdot \left(\frac{1}{2}\right)^n = \sum\limits_{n=1}^{\infty} \frac{1}{n+1} = \sum\limits_{n=2}^{\infty} \frac{1}{n}$ rozbieżny (szereg harmoniczny)

dla $x=-\frac{1}{2}$ $\sum\limits_{n=1}^{\infty} \frac{2^n}{n+1} \cdot \left(-\frac{1}{2}\right)^n = \sum\limits_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n+1}$

$\lim\limits_{n\to\infty} \frac{1}{n+1} = 0 \Rightarrow$ szereg zbieżny (kryterium Leibniza)

przedział zbieżności $\left[-\frac{1}{2}, \frac{1}{2}\right)$

b) $\sum\limits_{n=1}^{\infty} \frac{\ln(n)}{n} x^n$

$\lambda = \lim\limits_{n\to\infty} \left| \frac{\frac{\ln(n+1)}{n+1}}{\frac{\ln(n)}{n}} \right| = \lim\limits_{n\to\infty} \frac{\ln(n+1)}{\ln(n)} \cdot \frac{n}{n+1} = 1 \cdot 1 = 1 \rightarrow R = 1$

$\lim\limits_{x\to\infty} \frac{\ln(x+1)}{\ln(x)} \overset{\frac{\infty}{\infty}}{\underset{H}{=}} \lim\limits_{x\to\infty} \frac{\frac{1}{x+1}}{\frac{1}{x}} = \lim\limits_{x\to\infty} \frac{x}{x+1} = 1$

dla $x=1$ $\sum\limits_{n=1}^{\infty} \frac{\ln(n)}{n} \cdot 1^n = \sum\limits_{n=1}^{\infty} \frac{\ln(n)}{n}$

$\int \frac{\ln(x)}{x}\,dx = \left| \begin{matrix} t = \ln(x) \\ dt = \frac{1}{x}dx \end{matrix} \right| = \int t\,dt = \frac{1}{2}\ln^2(x) + C$

$\int\limits_1^{\infty} \frac{\ln(x)}{x}\,dx = \lim\limits_{T\to\infty} \frac{1}{2}\ln^2(T) - \frac{1}{2}\ln(1) = +\infty$ rozbieżna $\rightarrow$ szereg rozbieżny

dla $x=-1$ $\sum\limits_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n}$ $\lim\limits_{n\to\infty} \frac{\ln(n)}{n} = 0$ szereg zbieżny (k. Leibniza)

$\lim\limits_{x\to\infty} \frac{\ln(x)}{x} \overset{\frac{\infty}{\infty}}{\underset{H}{=}} \lim\limits_{x\to\infty} \frac{\frac{1}{x}}{1} = 0$

przedział zbieżności $[-1, 1)$

c) $\sum\limits_{n=1}^{\infty} \frac{n(2n+1)}{6^n} x^{2n}$ wyznaczam granicę w zależności od x

$\lim\limits_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim\limits_{n\to\infty} \left| \frac{\frac{(n+1)(2n+3)}{6^{n+1}} \cdot x^{2n+2}}{\frac{n(2n+1)}{6^n} x^{2n}} \right| = \lim\limits_{n\to\infty} \left| \frac{2n^2+5n+3}{2n^2+n} \cdot \frac{x^{2n+2}}{x^{2n}} \cdot \frac{6^n}{6^{n+1}} \right| = \left| 1 \cdot x^2 \cdot \frac{1}{6} \right| = \frac{|x^2|}{6} = \frac{|x|^2}{6}$

zbieżność dla $0 \le \frac{|x|^2}{6} < 1$ $|x|^2 < 6$ $x \in (-\sqrt{6}, \sqrt{6})$

dla $x = \sqrt{6}$ $\sum\limits_{n=1}^{\infty} \frac{n(n+1)}{6^n} \cdot 6^n = \sum\limits_{n=1}^{\infty} n(n+1)$ rozbieżny

dla $x = -\sqrt{6}$ $\sum\limits_{n=1}^{\infty} \frac{n(n+1)}{6^n} (-6)^n = \sum\limits_{n=1}^{\infty} (-1)^n n(n+1)$ $\lim\limits_{n\to\infty} n(n+1) = +\infty$ rozbieżny

przedział zbieżności $(-\sqrt{6}, \sqrt{6})$

d) $\sum_{n=0}^{\infty} \frac{(6-2x)^n}{3^n+2^n}$

$$\frac{(6-2x)^n}{3^n+2^n} = \frac{[-2(x-3)]^n}{3^n+2^n} = (-1)^n \frac{2^n}{3^n+2^n}(x-3)^n \qquad x_0=3$$

symetryczny względem środka

$$\lim_{n\to\infty} \sqrt[n]{\left|(-1)^n \frac{2^n}{3^n+2^n}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{2^n}{3^n+2^n}} = \frac{2}{3} \rightarrow R = \frac{3}{2} \rightarrow \text{przedział} \quad \left(3-\frac{3}{2}, 3+\frac{3}{2}\right) = \left(\frac{3}{2}, \frac{9}{2}\right)$$

$$\frac{2^n}{3^n+3^n} \leq \frac{2^n}{3^n+2^n} \leq \frac{2^n}{3^n} \qquad \lim_{n\to\infty} \sqrt[n]{\left(\frac{2}{3}\right)^n} = \frac{2}{3} \qquad \lim_{n\to\infty} \sqrt[n]{\frac{1}{2}\cdot\left(\frac{2}{3}\right)^n} = \frac{2}{3}$$

dla $x=\frac{3}{2}$ $\quad \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n+2^n} \cdot \left(-\frac{3}{2}\right)^n = \sum_{n=0}^{\infty} (-1)^n \cdot \frac{2^n}{3^n+2^n} \cdot (-1)^n \cdot \frac{3^n}{2^n} = \sum_{n=0}^{\infty} \frac{3^n}{3^n+2^n}$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{3^n}{3^n+2^n} = \lim_{n\to\infty} \frac{3^n}{3^n(1+(\frac{2}{3})^n)} = \lim_{n\to\infty} \frac{1}{1+(\frac{2}{3})^n} = 1 \neq 0$$

rozbieżny, bo nie spełnia warunku koniecznego

dla $x=\frac{9}{2}$ $\quad \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{3^n+2^n} \left(\frac{3}{2}\right)^n = \sum_{n=0}^{\infty} (-1)^n \frac{3^n}{3^n+2^n}$

$$\lim_{n\to\infty} \frac{3^n}{3^n+2^n} = \lim_{n\to\infty} \frac{1}{1+(\frac{2}{3})^n} = 1 \neq 0$$

rozbieżny z kryterium Leibniza

przedział zbieżności $\left(\frac{3}{2}, \frac{9}{2}\right)$

2. $\frac{x^2}{2} + \frac{x^6}{6} + \dots + \frac{x^{4n+2}}{4n+2} + \dots = \sum_{n=0}^{\infty} \frac{x^{4n+2}}{4n+2}$

$$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{\frac{x^{4n+6}}{4n+6}}{\frac{x^{4n+2}}{4n+2}}\right| = \lim_{n\to\infty} \left|x^4 \cdot \frac{4n+2}{4n+6}\right| = |x|^4$$

$0 \leq |x|^4 < 1 \iff x \in (-1, 1)$

dla $x=1$ $\quad \sum_{n=0}^{\infty} \frac{1}{4n+2}$ rozbieżny (harmoniczny)

dla $x=-1$ $\quad \sum_{n=0}^{\infty} \frac{1}{4n+2}$ rozbieżny

$R>0 \quad x \in (-1, 1)$

$$S(x) = \sum_{n=0}^{\infty} \frac{x^{4n+2}}{4n+2} \qquad S'(x) = \sum_{n=0}^{\infty} \frac{d}{dx} \frac{x^{4n+2}}{4n+2} = \sum_{n=0}^{\infty} \frac{1}{4n+2}(4n+2)x^{4n+1} = \sum_{n=0}^{\infty} x^{4n+1}$$

$$S'(x) = \sum_{n=0}^{\infty} x \cdot x^{4n} = x \sum_{n=0}^{\infty} (x^4)^n = x \cdot \frac{1}{1-x^4} = \frac{x}{1-x^4}$$

szereg geometryczny

$$S(x) = \int S'(x)\,dx = \int \frac{x}{1-x^4}\,dx = -\int \frac{x}{x^4-1}\,dx$$

$$\frac{x}{x^4-1} = \frac{x}{(x^2+1)(x+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x+1} + \frac{D}{x-1} = \frac{Ax(x^2-1)+B(x^2-1)+C(x^2+1)(x-1)+D(x^2+1)(x+1)}{x^4-1}$$

$$x = Ax^3 - Ax + Bx^2 - B + Cx^3 - Cx^2 + Cx - C + Dx^3 + Dx^2 + Dx + D$$

$$x = (A+C+D)x^3 + (B-C+D)x^2 + (-A+C+D)x + (-B-C+D)$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ -1 & 0 & 1 & 1 & 1 \\ 0 & -1 & -1 & 1 & 0 \end{array}\right] \xrightarrow[w_4+w_2]{w_3+w_1} \left[\begin{array}{ccccc} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & -2 & 2 & 0 \end{array}\right] \xrightarrow[w_4+w_3]{w_1-\frac{1}{2}w_3,\ w_2+\frac{1}{2}w_3} \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 2 & \frac{1}{2} \\ 0 & 0 & 2 & 2 & 1 \\ 0 & 0 & 0 & 4 & 1 \end{array}\right] \xrightarrow[\frac{1}{4}w_4]{\frac{1}{2}w_3} \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 2 & \frac{1}{2} \\ 0 & 0 & 1 & 1 & \frac{1}{2} \\ 0 & 0 & 0 & 1 & \frac{1}{4} \end{array}\right] \xrightarrow[w_3-w_4]{w_2-2w_4} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 1 & \frac{1}{4} \end{array}\right]$$

$$\frac{x}{x^4-1} = \frac{-\frac{1}{2}x}{x^2+1} + \frac{\frac{1}{4}}{x+1} + \frac{\frac{1}{4}}{x-1} \qquad \left|\begin{array}{l} t = x^2+1 \\ dt = 2x\,dx \end{array}\right| \quad -\frac{1}{4}\int \frac{dt}{t}$$

$$\int \frac{x}{x^4-1} = -\frac{1}{2}\int \frac{x}{x^2+1}\,dx + \frac{1}{4}\int \frac{dx}{x+1} + \frac{1}{4}\int \frac{dx}{x-1} = -\frac{1}{4}\ln|x^2+1| + \frac{1}{4}\ln|x+1| + \frac{1}{4}\ln|x-1| + C$$

$$S(x) = -\int \frac{x}{x^4-1}\,dx = \frac{1}{4}\left[\ln|x^2+1| - \ln|x+1| - \ln|x-1|\right] = \frac{1}{4}\ln\left|\frac{x^2+1}{x^2-1}\right|$$

$$S(0) = 0 \rightarrow \frac{1}{4}\ln|-1| + C = 0 \rightarrow C = 0$$

3.

a) $f(x) = x^4 \cdot e^{-2x}$ $\qquad e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ $\qquad$ zbieżny w $\mathbb{R}$

$$f(x) = x^4 \cdot \sum_{n=0}^{\infty} \frac{(-2x)^n}{n!} = \sum_{n=0}^{\infty} (-1)^n \frac{2^n}{n!} x^{n+4}$$

b) $f(x) = \frac{1}{1+a^2x^2} = \frac{1}{1-(-a^2x^2)}$ $\qquad \sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ $\qquad$ zbieżny dla $|x|<1$

$a>0$

$$f(x) = \sum_{n=0}^{\infty} (-a^2x^2)^n = \sum_{n=0}^{\infty} (-1)^n (ax)^{2n}$$

zbieżny dla $|(ax)^2|<1 \iff a^2|x|^2<1 \iff |x|^2 < \frac{1}{a^2} \iff x \in \left(-\frac{1}{a}, \frac{1}{a}\right)$

c) $f(x) = 2\sin(x)\sin(3x) = 2 \cdot \frac{1}{2}\left[\cos(-2x) - \cos(4x)\right] = \cos(2x) - \cos(4x)$

$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ $\qquad$ zbieżny w $\mathbb{R}$

$$f(x) = \sum_{n=0}^{\infty} (-1)^n \frac{(2x)^{2n}}{(2n)!} - \sum_{n=0}^{\infty} (-1)^n \frac{(4x)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}(4^n - 16^n)$$

4. $f(x) = \frac{1}{x} = x^{-1}$ $\qquad x_0 = 3$

$f(x) = x^{-1}$ $\qquad f^{(n)}(x) = (-1)^n n!\, x^{-n-1}$

$f' = -x^{-2}$

$f'' = 2x^{-3}$

$f^{(3)} = -6x^{-4}$

$f^{(4)} = 24x^{-5}$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n = \sum_{n=0}^{\infty} \frac{(-1)^n n! \cdot 3^{-n-1}}{n!}(x-3)^n = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}}(x-3)^n$$

$\lambda = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(-1)^n}{3^{n+1}}\right|} = \lim_{n\to\infty} \sqrt[n]{\frac{1}{3^n}\cdot\frac{1}{3}} = \frac{1}{3} \to R = 3 \to$ przedział $(3-3, 3+3) = (0,6)$

dla $x=0$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}}(-3)^n = \sum_{n=0}^{\infty} \frac{1}{3} \cdot \frac{(-1)^{2n} \cdot 3^n}{3^n} = \frac{1}{3}\sum_{n=0}^{\infty} 1$$ rozbieżny

dla $x=6$

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}} \cdot 3^n = \frac{1}{3}\sum_{n=0}^{\infty} (-1)^n$$ rozbieżny

$f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{3^{n+1}}(x-3)^n$ dla $x \in (0,6)$

5. ?

$f(x) = \frac{1-x}{1+x} = \frac{1+x-2x}{1+x} = 1 - 2x\frac{1}{1-(-x)}$

$$f(x) = 1 - 2x\sum_{n=0}^{\infty} (-x)^n = 1 - 2x\sum_{n=0}^{\infty} (-1)^n x^n = 1 - 2\sum_{n=0}^{\infty} (-1)^n x^{n+1}$$

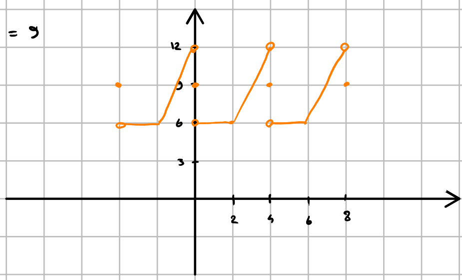
6. $f(x) = \begin{cases} 6 & \text{dla} \quad 0<x<2 \\ 3x & \text{dla} \quad 2<x<4 \end{cases}$

1. uzupełniam punkt nieciągłości

$$f(2) = \frac{1}{2}\left[\lim_{x\to 2^+} f(x) + \lim_{x\to 2^-} f(x)\right] = \frac{1}{2}[6+6] = 6$$

2. uzupełniam krańce przedziału

$$f(0) = f(4) = \frac{1}{2}\left[\lim_{x\to 0^+} f(x) + \lim_{x\to 4^-} f(x)\right] = \frac{1}{2}[6+12] = 9$$



3. rozwinięcie na przedziale $[0,4]$

okres $p = 2l = 4 \rightarrow l = 2$

$$a_0 = \frac{1}{2}\int_0^4 f(x)dx = \frac{1}{2}\int_0^2 6\,dx + \frac{1}{2}\int_2^4 3x\,dx = \frac{1}{2}\cdot 12 + \frac{1}{2}\cdot 18 = 15$$

$$a_n = \frac{1}{2}\int_0^4 f(x)\cos\left(\frac{n\pi x}{2}\right)dx = \frac{1}{2}\int_0^2 6\cos\left(\frac{n\pi}{2}x\right)dx + \frac{1}{2}\int_2^4 3x\cos\left(\frac{n\pi}{2}x\right)dx$$

$$\int \cos\left(\frac{n\pi}{2}x\right)dx = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}x\right) + C$$

$$\int x\cos\left(\frac{n\pi}{2}x\right)dx = \left|\begin{matrix} f=x & g'=\cos\left(\frac{n\pi}{2}x\right) \\ f'=1 & g=\frac{2}{n\pi}\sin\left(\frac{n\pi}{2}x\right)\end{matrix}\right| = \frac{2}{n\pi}x\sin\left(\frac{n\pi}{2}x\right) - \frac{2}{n\pi}\int \sin\left(\frac{n\pi}{2}x\right)dx = \frac{2}{n\pi}x\sin\left(\frac{n\pi}{2}x\right) + \left(\frac{2}{n\pi}\right)^2\cos\left(\frac{n\pi}{2}x\right) + C$$

$$a_n = \frac{6}{n\pi}\cdot \sin\left(\frac{n\pi}{2}x\right)\Big|_0^2 + \frac{3}{n\pi}\cdot x\sin\left(\frac{n\pi}{2}x\right)\Big|_2^4 + \frac{6}{n^2\pi^2}\cdot\cos\left(\frac{n\pi}{2}x\right)\Big|_2^4$$

$$a_n = \frac{6}{n^2\pi^2}\cdot\left[\cos(2\pi n) - \cos(\pi n)\right] = \frac{6}{n^2\pi^2}\left[1-\cos(\pi n)\right] = \begin{cases} \frac{12}{n^2\pi^2} & \text{dla} \quad n=2k+1 \\ 0 & \text{dla} \quad n=2k \end{cases}$$
$$= \frac{6}{n^2\pi^2}\left(1-(-1)^n\right)$$

$$b_n = \frac{1}{2}\int_0^4 f(x)\sin\left(\frac{n\pi}{2}x\right)dx = \frac{1}{2}\int_0^2 6\sin\left(\frac{n\pi}{2}x\right)dx + \frac{1}{2}\int_0^2 3x\sin\left(\frac{n\pi}{2}x\right)dx$$

$$\int \sin\left(\frac{n\pi}{2}x\right)dx = -\frac{2}{n\pi}\cos\left(\frac{n\pi}{2}x\right) + C$$

$$\int x\sin\left(\frac{n\pi}{2}x\right)dx = \left|\begin{matrix} f=x & g'=\sin\left(\frac{n\pi}{2}x\right) \\ f'=1 & g=-\frac{2}{n\pi}\cos\left(\frac{n\pi}{2}x\right)\end{matrix}\right| = -\frac{2x}{n\pi}\cos\left(\frac{n\pi}{2}x\right) + \frac{2}{n\pi}\int\cos\left(\frac{n\pi}{2}x\right)dx = -\frac{2x}{n\pi}\cos\left(\frac{n\pi}{2}x\right) + \frac{4}{n^2\pi^2}\sin\left(\frac{n\pi}{2}x\right) + C$$

$$b_n = -\frac{6}{n\pi}\cdot\cos\left(\frac{n\pi}{2}x\right)\Big|_0^2 - \frac{3}{n\pi}\cdot x\cos\left(\frac{n\pi}{2}x\right)\Big|_2^4 + \frac{6}{n^2\pi^2}\sin\left(\frac{n\pi}{2}x\right)\Big|_2^4$$

$$b_n = -\frac{6}{n\pi}\cdot\left[\cos(n\pi) - 1\right] - \frac{3}{n\pi}\cdot\left[4\cos(2\pi n) - 2\cos(\pi n)\right] + 0$$

$$b_n = \left\{\begin{matrix} -\frac{6}{n\pi} & \text{dla} \quad n=2k \\ \frac{12}{n\pi} - \frac{18}{n\pi} = -\frac{6}{n\pi} & \text{dla} \quad n=2k+1 \end{matrix}\right\} = -\frac{6}{n\pi}$$

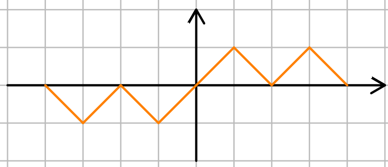
$$f(x) = \frac{15}{2} + \frac{12}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\cos\left(\frac{2n-1}{2}\pi x\right) - \frac{6}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi}{2}x\right)$$

tylko nieparzyste więc zamiast $n \rightarrow 2n-1$

7.

a) szereg sinusowy

$$f(x) = \begin{cases} x & \text{dla } 0 \le x \le 1 \\ 2-x & \text{dla } 1 \le x \le 2 \end{cases}$$



rozwinięcie na $[0, 2]$ $\quad l = 2$

$$b_n = \int_0^2 f(x) \sin\left(\frac{n\pi}{2}x\right) dx = \int_0^1 x \sin\left(\frac{n\pi}{2}x\right) dx + \int_1^2 (2-x) \sin\left(\frac{n\pi}{2}x\right) dx = \int_0^1 x \sin\left(\frac{n\pi}{2}x\right) dx + 2\int_1^2 \sin\left(\frac{n\pi}{2}x\right) dx - \int_1^2 x \sin\left(\frac{n\pi}{2}x\right) dx$$

$$\int \sin\left(\frac{n\pi}{2}\right) dx = -\frac{2}{n\pi} \cos\left(\frac{n\pi}{2}x\right) + C$$

$$\int x \sin\left(\frac{n\pi}{2}x\right) dx = \left| \begin{matrix} f = x & g' = \sin\left(\frac{n\pi}{2}x\right) \\ f' = 1 & g = -\frac{2}{n\pi}\cos\left(\frac{n\pi}{2}x\right) \end{matrix} \right| = -\frac{2x}{n\pi}\cos\left(\frac{n\pi}{2}x\right) + \frac{2}{n\pi}\int \cos\left(\frac{n\pi}{2}x\right) dx = -\frac{2x}{n\pi}\cos\left(\frac{n\pi}{2}x\right) + \frac{4}{n^2\pi^2}\sin\left(\frac{n\pi}{2}x\right) + C$$

$$b_n = -\frac{2}{n\pi} \cdot x\cos\left(\frac{n\pi}{2}x\right)\Big|_0^1 + \frac{4}{n^2\pi^2}\sin\left(\frac{n\pi}{2}x\right)\Big|_0^1 - \frac{4}{n\pi}\cdot\cos\left(\frac{n\pi}{2}x\right)\Big|_1^2 + \frac{2}{n\pi}\cdot x\cos\left(\frac{n\pi}{2}x\right)\Big|_1^2 - \frac{4}{n^2\pi^2}\sin\left(\frac{n\pi}{2}x\right)\Big|_1^2$$

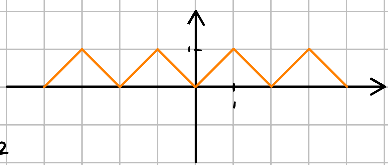
$$= -\frac{2}{n\pi}\cancel{\cos\left(\frac{n\pi}{2}\right)} + \frac{4}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) - \frac{4}{n\pi}\cancel{\cos(\pi n)} + \frac{4}{n\pi}\cancel{\cos\left(\frac{n\pi}{2}\right)} + \frac{4}{n\pi}\cancel{\cos(\pi n)} - \frac{2}{n\pi}\cancel{\cos\left(\frac{n\pi}{2}\right)} + \frac{4}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{8}{n^2\pi^2}\sin\left(\frac{n\pi}{2}\right) = \begin{cases} 0 & \text{dla } n = 2k \\ \frac{8}{n^2\pi^2} & \text{dla } n = 4k-3 \quad (4k+1) \\ \frac{-8}{n^2\pi^2} & \text{dla } n = 4k-1 \quad (4k+3) \end{cases} = \begin{cases} 0 & \text{dla } n = 2k \\ (-1)^{k+1}\frac{8}{n^2\pi^2} & \text{dla } n = 2k-1 \end{cases}$$

$$f(x) = \frac{8}{\pi^2}\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^2}\sin\left(\frac{2n-1}{2}\pi x\right) \qquad (\text{tylko nieparzyste wyrazy})$$

b) szereg cosinusowy

$$f(x) = \begin{cases} x & \text{dla } 0 \le x \le 1 \\ 2-x & \text{dla } 1 \le x \le 2 \end{cases}$$



rozwinięcie na $[0, 2]$ $\quad l = 2$

$$a_0 = \int_0^2 f(x) dx = 1$$

$$a_n = \int_0^2 f(x)\cos\left(\frac{n\pi}{2}x\right) dx = \int_0^1 x\cos\left(\frac{n\pi}{2}x\right) dx + 2\int_1^2 \cos\left(\frac{n\pi}{2}x\right) dx - \int_1^2 x\cos\left(\frac{n\pi}{2}x\right) dx$$

$$\int \cos\left(\frac{n\pi}{2}x\right) dx = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}x\right) + C$$

$$\int x\cos\left(\frac{n\pi}{2}x\right) dx = \left| \begin{matrix} f = x & g' = \cos\left(\frac{n\pi}{2}x\right) \\ f' = 1 & g = \frac{2}{n\pi}\sin\left(\frac{n\pi}{2}x\right) \end{matrix} \right| = \frac{2x}{n\pi}\sin\left(\frac{n\pi}{2}x\right) - \frac{2}{n\pi}\int \sin\left(\frac{n\pi}{2}x\right) dx = \frac{2}{n\pi}x\sin\left(\frac{n\pi}{2}x\right) + \frac{4}{n^2\pi^2}\cos\left(\frac{n\pi}{2}x\right) + C$$

$$a_n = \frac{2}{n\pi}\cancel{\sin\left(\frac{n\pi}{2}\right)} - 0 + \frac{4}{n^2\pi^2}\cancel{\cos\left(\frac{n\pi}{2}\right)} - \frac{4}{n^2\pi^2} + \frac{4}{n\pi}\cancel{\sin(n\pi)} - \frac{4}{n\pi}\cancel{\sin\left(\frac{n\pi}{2}\right)} - \frac{4}{n\pi}\cancel{\sin(n\pi)} + \frac{2}{n\pi}\cancel{\sin\left(\frac{n\pi}{2}\right)} - \frac{4}{n^2\pi^2}\cos(n\pi) + \frac{4}{n^2\pi^2}\cancel{\cos\left(\frac{n\pi}{2}\right)}$$

$$a_n = -\frac{4}{n^2\pi^2}\left(1 + \cos(\pi n)\right) = \begin{cases} -\frac{8}{n^2\pi^2} & \text{dla } n = 2k \\ 0 & \text{dla } n = 2k-1 \end{cases}$$

$$f(x) = \frac{1}{2} - \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{(2n)^2}\cos\left(\frac{2n\pi}{2}x\right) = \frac{1}{2} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(n\pi x)}{n^2}$$