Zestaw zadań 1

1.

a) $\sim(\exists x \in \mathbb{R}\ \forall y \in \mathbb{R}\ \ x \geq y)$ zdanie logiczne (prawda)

b) $\phi(p)$: $\forall n \in \mathbb{N}\ (n \neq 1 \wedge n \neq p) \Rightarrow p \bmod n \neq 0$? $p \in \mathbb{N} \wedge p > 1 \wedge \sim[\exists x, y \in \mathbb{N}\ \ y, x > 1 \wedge yx = p]$

c) $a \bmod 8 = 0 \wedge b \bmod 8 = 0 \wedge \sim(\exists n \in \mathbb{R}\ \ n > 8 \wedge a \bmod n = 0 \wedge b \bmod n = 0)$

d) $\sim(\forall n \in \mathbb{N}\ \ 2n + 1 \equiv 0 \bmod 3)$

e) $\forall n \in \mathbb{N}\ \exists r \in \mathbb{R}\ \ n = r^2$

f) $k \in \mathbb{N} \wedge (k \not\equiv 0 \bmod 7 \vee k \equiv 0 \bmod 3)$

g) $\exists x \in \mathbb{R}_-\ \forall y \in \mathbb{R}_- \setminus \{x\}\ \ x > y$

2.

a) tak a') nie istnieje największa liczba naturalna

b) nie, nie istnieje mniejsza liczba naturalna od 1

b') tak, $y = 1$

c) nie, dla $y = x + 1$ nie istnieje takie z

3. $x \in \mathbb{R}$

a) $\varphi(x)$: $x < e \Rightarrow x > \pi$

$x < e \Rightarrow x > \pi$

$\Leftrightarrow \sim(x < e) \vee x > \pi$

$x \geq e \vee x > \pi$

$\{x \in \mathbb{R} : x \geq e \vee x > \pi\}$

$[e, +\infty)$

b) $\varphi(x)$: $x < e \Leftrightarrow x \leq \pi$

$\{x \in \mathbb{R} : (x < e \wedge x \leq \pi) \vee (x \geq e \wedge x > \pi)\}$

$(-\infty, e) \cup (\pi, +\infty)$

c) $\varphi(x)$: $\exists y \in \mathbb{R}\ \ x < \sin(y)$ $-1 < \sin(y) < 1$

$\{x \in \mathbb{R} : \varphi(x)\} = (-\infty, 1)$

d) $\varphi(x)$: $\forall y \in \mathbb{R}\ \ x < y^2 + \pi$

$0 \leq y^2$

$\pi \leq y^2 + \pi$

$\{x \in \mathbb{R} : \varphi(x)\} = (-\infty, \pi)$

e) $\varphi(x)$: $x > e \Rightarrow (\forall y \in \mathbb{R}\ \ x < y^2 + \pi)$

$\{x \in \mathbb{R} : (x > e \wedge \forall y \in \mathbb{R}\ x < y^2 + \pi) \vee (x \leq e)\} = (-\infty, e] \cup (e, \pi) = (-\infty, \pi)$

f) $\varphi(x)$: $(\exists y \in \mathbb{R}\ x < \sin(y)) \Leftrightarrow x > e$

?

~~$\{x \in \mathbb{R} : \varphi(x)\} = [(-\infty, 1) \cap (e, +\infty)] \cup [[1, +\infty) \cap (-\infty, e]] = \emptyset \cup [1, e] = [1, e]$~~

4.

...

5.

a) $\exists x \in R\ [x < \pi \Rightarrow \sin(x) > \pi]$

zdanie prawdziwe, dla $x \in [\pi, +\infty)$ jest $0 \Rightarrow 0$

$\sim(\exists x \in \mathbb{R}\ [x < \pi \Rightarrow \sin(x) > \pi])$

$\forall x \in R\ \sim[x < \pi \Rightarrow \sin(x) > \pi]$

$\forall x \in R\ [x < \pi \wedge \sin(x) \leq \pi]$

b) $(\exists x \in \mathbb{R}\ x < \pi) \Rightarrow (\exists x \in \mathbb{R}\ \sin(x) > \pi)$

zdanie fałszywe $1 \Rightarrow 0$

$\sim[(\exists x \in R\ x < \pi) \Rightarrow (\exists x \in R\ \sin(x) > \pi)]$

$(\exists x \in R\ x < \pi) \wedge \sim(\exists x \in R\ \sin(x) > \pi)$

$(\exists x \in R\ x < \pi) \wedge (\forall x \in R\ \sin(x) \leq \pi)$

c) $\forall x, y \in \mathbb{R}\ [x < y \Rightarrow \exists q \in Q\ (x < q < y)]$

zdanie prawdziwe

$\sim[\forall x, y \in R\ [x < y \Rightarrow \exists q \in Q\ (x < q < y)]]$

$\exists x, y \in R\ \sim[x < y \Rightarrow \exists q \in Q\ (x < q < y)]$

$\exists x, y \in R\ x < y \wedge \sim[\exists q \in Q\ (x < q \wedge q < y)]$

$\exists x, y \in R\ x < y \wedge \forall q \in Q\ (x \geq q \vee q \geq y)$

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c) $\phi(a,b): \quad a,b \in \mathbb{N} \land \exists c \in \mathbb{N} \; a = 8c \land \exists d \in \mathbb{N} \; b = 8d \land \sim\left(\exists e \in \mathbb{N}_2 \; \exists f \in \mathbb{N} \; \exists g \in \mathbb{N} \; ef = a \land eg = b \right)$

d) $\sim\left(\forall x \in \mathbb{N} \; \exists n \in \mathbb{N} \left(x = 2n - 1 \Longrightarrow \exists a \in \mathbb{N} \; x = 3a \right)\right)$

f) $\phi(k): \quad \sim\left(\exists n \in \mathbb{N} \; k = 7n \right) \lor \exists m \in \mathbb{N} \; k = 3m$