1.

$$F(x) = \begin{cases} 0 & x < -1 \\ 1 - x^2 & x \in [-1, 0) \\ 1 & x \geq 0 \end{cases}$$

$$f(x) = \frac{d}{dx} F(x) = \begin{cases} -2x & x \in [-1, 0) \\ 0 & \text{w p.p.} \end{cases}$$

$$EX = \int_{-1}^{0} x(-2x)\,dx = -2\int_{-1}^{0} x^2\,dx = -\frac{2}{3} \cdot x^3 \Big|_{-1}^{0} = -\frac{2}{3}(0 - (-1)^3) = -\frac{2}{3}$$

$$EX^2 = \int_{-1}^{0} x^2(-2x)\,dx = -2\int_{-1}^{0} x^3\,dx = -\frac{1}{2} \cdot x^4 \Big|_{-1}^{0} = -\frac{1}{2}(0 - (-1)^4) = \frac{1}{2}$$

$$VX = \frac{1}{2} - \frac{4}{9} = \frac{9}{18} - \frac{8}{18} = \frac{1}{18}$$

$$Y = \sum_{k=1}^{1800} X_k \approx N(1800\,EX, 1800\,VX) = N(-1200, 100)$$

$$P(|Y| \leq 1220) = P(-1220 \leq Y \leq 1220) = F(1220) - F(-1220)$$
$$= \phi\left(\frac{1220 + 1200}{10}\right) - \phi\left(\frac{-1220 + 1200}{10}\right)$$
$$= \phi(242) - \phi(-2) \approx 1 - (1 - \phi(2)) = \phi(2)$$
$$\approx 0.9772$$

2.

$X_k \sim P(\lambda = 3.5)$ kolizje w 1 tyg

a) $Y = \sum_{k=1}^{52} X_k \simeq N(52\,EX, 52\,VX) = N(182, 182)$

$$P(Y > 200) = 1 - P(Y \leq 200) = 1 - F(200) = 1 - \phi\left(\frac{200 - 182}{\sqrt{182}}\right)$$
$$\approx 1 - \phi(1.33) \simeq 0.0918$$

b) $Y = \sum_{k=1}^{52} X_k \sim P\left(\sum_{k=1}^{52} 3.5\right) = P(182)$

$$P(Y > 200) = 1 - P(Y \leq 200) = 1 - \sum_{k=0}^{200} P(Y = k)$$
$$= 1 - \sum_{k=0}^{200} e^{-182} \cdot \frac{182^k}{k!} \approx 0.0867$$

3.

$Z = 10\,000 \cdot 30 - X \cdot 2500$ zysk

$X \sim B(n = 10000, p = 0.006)$ liczba uszkodzonych

$$P(Z > 125\,000) = P(300\,000 - 2500X > 125\,000)$$
$$= P(X < 70) = \sum_{k=0}^{69} \binom{10\,000}{k} (0.006)^k (1 - 0.006)^{10\,000 - k}$$
$$\simeq 0.89$$

$$X \simeq N(np, np(1-p)) = N(60, 59.64)$$
$$P(X \leq 69) = F(69) = \phi\left(\frac{69 - 60}{\sqrt{59.64}}\right) = \phi(1.17) \simeq 0.8790$$

4.

$X \sim B(n = 200, p = 0.02)$ – liczba błędów

$\simeq N(4, 3.92)$

$$P(X \leq M) = 0.75 \Rightarrow \phi\left(\frac{M - 4}{\sqrt{3.92}}\right) \approx \phi\left(\frac{2}{3}\right) \Rightarrow M \approx 5.31$$

M powinno być 5 albo 6

5.

$L_1 \sim U([0.07, 0.08])$ ~ stare zužyue $\frac{L}{km}$

$L_2 \sim U([0.06, 0.07])$ ~ nove zužyuic $\frac{L}{km}$

$$X_1 = \sum_{k=1}^{600} L_1 \simeq N(600\,EL_1, 600\,VL_1) = N(45, 600\,VL_1)$$

$$X_2 = \sum_{k=1}^{D} L_2 \simeq N(D\,EL_2, D\,VL_2) = N(0.065D, D\,VL_2)$$

$$P(X_1 \leq 45) = \phi\left(\frac{45-45}{\sqrt{600\,VL_1}}\right) = \phi(0)$$

$$P(X_2 \leq 45) = P(X_1 \leq 45) \Rightarrow \phi\left(\frac{45-0.065D}{\sqrt{D\,VL_2}}\right) = \phi(0) \Rightarrow 45 = 0.065D \Rightarrow D \approx 692$$