1.

$S_x = \{-2, -1, 0, 1, 3\}$

$$p_X(x) = \begin{cases} \frac{x}{a} & x \in \{-2, -1, 0\} \\ \frac{x}{a^2} & x \in \{1, 3\} \\ 0 \end{cases}$$

$a \in \mathbb{R}$

$$\sum_{x_i \in S_x} p_X(x_i) = 1$$

$$\frac{-2}{a} + \frac{-1}{a} + \frac{0}{a} + \frac{1}{a^2} + \frac{3}{a^2} = 1$$

$-3a + 4 = a^2$

$a^2 + 3a - 4 = 0$

$a^2 + 4a - a - 4 = 0$

$(a+4)(a-1) = 0$

$a = 1 \lor a = -4$

$p_X : S_x \to [0, 1] \implies a = -4$

x	-2	-1	0	1	3
p_X	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{16}$	$\frac{3}{16}$

$$F_X(t) = \begin{cases} 0 & t < -2 \\ \frac{1}{2} & -2 \le t < -1 \\ \frac{3}{4} & -1 \le t < 1 \\ \frac{13}{16} & 1 \le t < 3 \\ 1 & t \ge 3 \end{cases}$$

2.

$$F_X(x) = \begin{cases} 0 & x < 1 \\ \frac{1}{8} & 1 \le x < 2 \\ \frac{1}{2} & 2 \le x < 3 \\ \frac{3}{4} & 3 \le x < 4 \\ 1 & x \ge 4 \end{cases}$$

$$P(X = a) = F_X(a) - \lim_{t \to a^-} F_X(t)$$

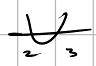
$$\text{ozn} \quad F_X(a) - F_X(a^-)$$

$$p_X(x) = P(X = x) = F_X(x) - F_X(x^-)$$

$$S_X = \{1, 2, 3, 4\}$$

$$p_X(x) = \begin{cases} \frac{1}{8} & x = 1 \\ \frac{3}{8} & x \in \{2, 3\} \\ \frac{1}{4} & x = 4 \\ 0 & \text{w p.p.} \end{cases}$$

$$x^2 - 5x + 6 = x^2 - 3x - 2x + 6$$
$$x(x-3) - 2(x-3)$$
$$(x-3)(x-2)$$



$$P(X^2 - X = 0) = P(X = 0 \vee X = 1) = P(X = 0) + P(X = 1) = p_X(0) + p_X(1) = 0 + \frac{1}{8} = \frac{1}{8}$$

$$P(X^2 - 5X + 6 \le 0) = P(2 \le X \le 3) = F_X(3) - \lim_{t \to 2^-} F_X(t) = \frac{3}{4} - \frac{1}{8} = \frac{5}{8}$$

3.

$\Omega = \{0, 1, 2, 3\}$

$P(\{\omega\}) = \frac{1}{4}$

$X(\omega) = \sin(\frac{\pi}{2}\omega)$ $\qquad S_X = \{-1, 0, 1\}$

$Y(\omega) = \cos(\frac{\pi}{2}\omega)$ $\qquad S_Y = \{-1, 0, 1\}$

a)

$p_X(x) = \begin{cases} \frac{1}{4} & \text{dla } x \in \{-1, 1\} \\ \frac{1}{2} & \text{dla } x = 0 \end{cases}$

ω	0	1	2	3
X	0	1	0	-1
Y	1	0	-1	0

$p_Y(y) = \begin{cases} \frac{1}{4} & \text{dla } y \in \{-1, 1\} \\ \frac{1}{2} & \text{dla } y = 0 \end{cases}$

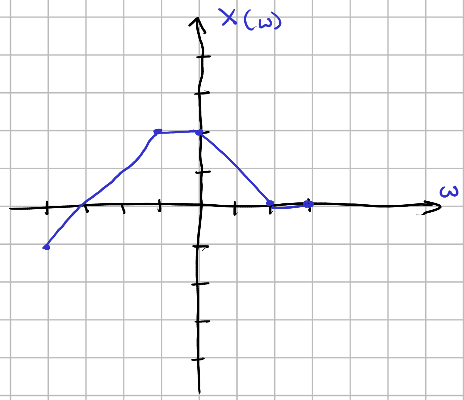
Mają takie same rozkłady

b) $P(\{\omega \in \Omega : X(\omega) = Y(\omega)\}) = P(\emptyset) = 0$

4.

$$\Omega = [-4, 3]$$

$$X(\omega) = \begin{cases} \omega + 3 & -4 \le \omega \le -1 \\ 2 & -1 < \omega \le 0 \\ 2 - \omega & 0 < \omega \le 2 \\ 0 & 2 < \omega \le 3 \end{cases}$$



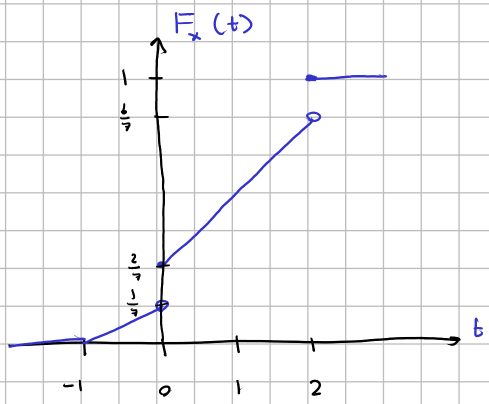
P – prawdopodobieństwo geometryczne

$$X^{-1}((-\infty, t]) = \begin{cases} \emptyset & t < -1 \\ [-4, t-3] & -1 \le t < 0 \\ [-4, t-3] \cup [2-t, 3] & 0 \le t < 2 \\ [-4, 3] & t \ge 2 \end{cases}$$

$$F_X(t) = P(X^{-1}((-\infty, t])) = \begin{cases} 0 & t < -1 \\ \frac{t+1}{7} & -1 \le t < 0 \\ \frac{2t+2}{7} & 0 \le t < 2 \\ 1 & t \ge 2 \end{cases}$$

$$\frac{|[-4, t-3]|}{|[-4, 3]|} = \frac{t-3-(-4)}{7} = \frac{t+1}{7}$$

$$\frac{|[-4, t-3] \cup [2-t, 3]|}{7} = \frac{t+1+(3-(2-t))}{7} = \frac{2t+2}{7}$$



Rozkład nie jest ani ciągły ani dyskretny

$$S_X = [-1, 2]$$

5.

10 monet
- 2 z dwoma orłami N'
- 8 normalnych N

inne zadanie

X – liczba rzutów

a) rzucamy do pierwszego orła

$$p_X(1) = P(O|N')P(N') + P(O|N)P(N) = 1\cdot\frac{2}{10} + \frac{1}{2}\cdot\frac{8}{10} = \frac{6}{10}$$

$$p_X(2) = \left(0\cdot\frac{2}{10} + \frac{1}{2}\cdot\frac{8}{10}\right)\cdot\left(1\cdot\frac{2}{10} + \frac{1}{2}\cdot\frac{8}{10}\right) = \frac{4}{10}\cdot\frac{6}{10}$$

$$p_X(3) = \left(0\cdot\frac{2}{10} + \frac{1}{2}\cdot\frac{8}{10}\right)^2\cdot\left(1\cdot\frac{2}{10} + \frac{1}{2}\cdot\frac{8}{10}\right)$$

$$p_X(x) = \left[P(O'|N')P(N') + P(O'|N)P(N)\right]^{x-1}\cdot\left[P(O|N')P(N') + P(O|N)P(N)\right] = \left(\frac{2}{5}\right)^{x-1}\cdot\frac{3}{5}$$

b) rzucamy do pierwszego orła ale max 3 razy

$$p_X(3) = \left(\frac{2}{5}\right)^{3-1}\cdot\frac{3}{5} + \left(\frac{2}{5}\right)^3 = \left(\frac{2}{5}\right)^2\left(\frac{3}{5}+\frac{2}{5}\right)$$ 2 razy reszka i raz (orzeł v reszka)

$$p_X(x) = \begin{cases} \left(\frac{2}{5}\right)^{x-1}\cdot\frac{3}{5} & \text{dla } x\in\{1,2\} \\ \left(\frac{2}{5}\right)^2 & \text{dla } x=3 \\ 0 & \text{w p.p.} \end{cases}$$

c) rzucamy do wypadnięcia drugiego orła

$$p_X(x) = \underbrace{\binom{x-1}{1}}_{\text{za którym z pierwszych } x-1 \text{ razy będzie 1szy orzeł}}\underbrace{\left(\frac{2}{5}\right)^{x-2}}_{x-2 \text{ reszki}}\underbrace{\left(\frac{3}{5}\right)^2}_{2 \text{ orły}} \qquad x\geq 2$$

6.

Dystrybuanta spełnia warunki

- $F : \mathbb{R} \to [0, 1]$

- $\lim\limits_{t \to -\infty} F(t) = 0 \ \wedge \ \lim\limits_{t \to +\infty} F(t) = 1$

- F jest niemalejąca

- F jest co najmniej prawostronnie ciągła

a) $F(t) = \arctan(t)$ ✗

$\lim\limits_{t \to \infty} \arctan(t) = \frac{\pi}{2}$

b) $F(t) = \begin{cases} 0 & t < 0 \\ 1 - e^{-2t} & t > 0 \end{cases}$ ✗

nieokreślona dla $t = 0$

c) $F(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2} & t = 0 \\ 1 & t > 0 \end{cases}$ ✗

$\lim\limits_{t \to 0^+} F(t) = 1 \neq F(0) = \frac{1}{2}$

nie jest prawostronnie ciągła

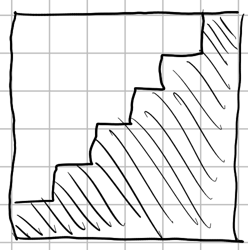
d) $F(t) = \begin{cases} 0 & t < 0 \\ \frac{1}{2}(t + \frac{1}{4}) & 0 \leq t < 1 \\ 1 & t > 1 \end{cases}$ ✗

nieokreślona dla $t = 1$

7.

Rzucamy kostką, aż suma oczek przekroczy 6

X – liczba rzutów kostką



$F_X(1) = p_X(1) = 0$

$F_X(2) = p_X(2) + p_X(1) = \frac{21}{36} + 0 = \frac{21}{36}$

$F_X(7) = 1$ bo $X = 7$ jest zdarzeniem pewnym (?)