

1.

$$P(A) = \frac{2}{5}$$

$$P(B|A) = \frac{1}{4}$$

$$P(C|A \cap B) = \frac{1}{2}$$

$$P(A \cup B) = \frac{3}{5}$$

$$P(C|B) = \frac{1}{3}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A) P(B|A) = \frac{2}{5} \cdot \frac{1}{4} = \frac{1}{10}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(B) = P(A \cup B) - P(A) + P(A \cap B) = \frac{3}{5} - \frac{2}{5} + \frac{1}{10} = \frac{3}{10}$$

$$P(C|B) = \frac{P(B \cap C)}{P(B)}$$

$$P(B \cap C) = P(B) \cdot P(C|B) = \frac{3}{10} \cdot \frac{1}{3} = \frac{1}{10}$$

$$P(C|A \cap B) = \frac{P(A \cap B \cap C)}{P(A \cap B)}$$

$$P(A \cap B \cap C) = P(A \cap B) \cdot P(C|A \cap B) = \frac{1}{10} \cdot \frac{1}{2} = \frac{1}{20}$$

$$P(A|B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)} = \frac{\frac{1}{20}}{\frac{1}{10}} = \frac{1}{2}$$

2.

<u>I</u>	2 monety	$P(O) = \frac{1}{4}$	2 razy losowa moneta
<u>II</u>	2 monety	$P(O) = \frac{1}{5}$	
<u>III</u>	1 moneta	$P(O) = \frac{1}{2}$	

a) A_i - wybrano monety i -tego typu

$$A_1 \cup A_2 \cup A_3 = \Omega$$

 A_i, A_j parami rozłączne B - orzeł wypadł co najmniej raz B' - nie wypadł orzeł

$$P(B') = P(B' | A_1)P(A_1) + P(B' | A_2)P(A_2) + P(B' | A_3)P(A_3)$$

$$P(A_1) = \frac{2}{5} \quad P(A_2) = \frac{2}{5} \quad P(A_3) = \frac{1}{5}$$

$$P(B' | A_1) = \left(1 - \frac{1}{4}\right)^2 = \frac{9}{16}$$

$$P(B' | A_2) = (1 - 1)^2 = 0$$

$$P(B' | A_3) = \left(1 - \frac{1}{2}\right)^2 = \frac{1}{4}$$

$$P(B') = \frac{2}{16} \cdot \frac{2}{5} + 0 \cdot \frac{2}{5} + \frac{1}{5} \cdot \frac{1}{4} = \frac{11}{40}$$

$$P(B) = 1 - P(B') = \frac{29}{40}$$

b)

 C - wypadł 2 razy orzeł

$$P(A_1 \cup A_2 | C) = P(A_1 | C) + P(A_2 | C) = \frac{P(C | A_1)P(A_1)}{P(C)} + \frac{P(C | A_2)P(A_2)}{P(C)}$$

$$= \frac{P(C | A_1)P(A_1) + P(C | A_2)P(A_2)}{P(C | A_1)P(A_1) + P(C | A_2)P(A_2) + P(C | A_3)P(A_3)}$$

$$= \frac{\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{2}{5} + 1 \cdot \frac{2}{5}}{\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{2}{5} + 1 \cdot \frac{2}{5} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{5}} = \frac{17}{19}$$

3.

A - pozytywny

B - chory

$$P(B) = 0.01$$

$$P(A|B) = 0.5$$

$$P(A'|B') = 0.7$$

$$P(A|B') = 0.3$$

$$P(A'|B) = 0.1$$

$$a) \quad P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{0.5 \cdot 0.01}{0.306} = \frac{1}{34}$$

$$P(A) = P(A|B)P(B) + P(A|B')P(B') = 0.5 \cdot 0.01 + 0.3 \cdot 0.99 = 0.306$$

$$b) \quad P(B|A') = \frac{P(A'|B)P(B)}{P(A')} = \frac{0.1 \cdot 0.01}{0.694} = \frac{1}{694}$$

4.

A - wadliwy wyrób

B - odrzucony wyrób

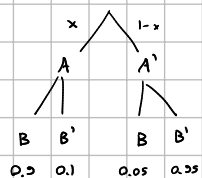
$$P(B|A) = 0.2$$

$$P(B|A') = 0.05$$

$$P(A|B') = 0.01$$

$$P(B'|A) = 0.1$$

$$P(B'|A') = 0.25$$



$$P(A|B') = \frac{P(B'|A)P(A)}{P(B')}$$

$$P(B') = P(B'|A)P(A) + P(B'|A')P(A')$$

$$\frac{1}{100} = \frac{\frac{1}{10}x}{\frac{1}{10}x + \frac{25}{100}(1-x)}$$

$$\frac{1}{100} \cdot \left[\frac{1}{10}x + \frac{25}{100} - \frac{25x}{100} \right] = \frac{1}{10}x \cdot \frac{1}{100}$$

$$\frac{x}{10} + \frac{25}{100} - \frac{25x}{100} = 10x$$

$$10x + 25 - 25x = 1000x$$

$$x = \frac{25}{1025}$$

5.

$$X_1 = 10011$$

 A_k - wystano ciąg X_k

$$P(A_1) = 0.3$$

$$X_2 = 11011$$

$$A_1 \vee A_2 \vee A_3 = \Omega$$

$$P(A_2) = 0.3$$

$$X_3 = 10101$$

 B - otrzymano y

$$P(A_3) = 0.4$$

$$y = 10111$$

$$P(B|A_1) = 0.1 \cdot 0.5^4$$

$$P(B|A_2) = 0.1^2 \cdot 0.5^3$$

$$P(B|A_3) = 0.1 \cdot 0.5^4$$

$$P(A_3|B) = \frac{P(B|A_3)P(A_3)}{P(B)} \approx 0.55$$

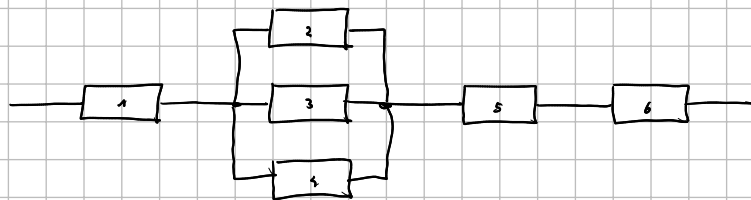
$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3) = 0.048114$$

$$P(A_1|B) \approx 0.41$$

$$P(A_2|B) \approx 0.05$$

Najprawdopodobniej został wystany sygnał X_3

6.



przekształcić do postaci nieskalarnej

$$P(A) = P(A_1) \cdot P(A_2 \cup A_3 \cup A_4) \cdot P(A_5) \cdot P(A_6)$$

$$= P(A_1) \cdot [P(A_2) + P(A_3) + P(A_4) - P(A_2 \cap A_3) - P(A_2 \cap A_4) - P(A_3 \cap A_4) + P(A_2 \cap A_3 \cap A_4)] \cdot P(A_5) \cdot P(A_6)$$

$$= p \cdot (p + p + p - p^2 - p^2 - p^2 + p^3) \cdot p \cdot p$$

$$= p^3 (p^3 - 3p^2 + 3p) = p^6 - 3p^5 + 3p^4$$

$$P(A_2 \cup A_3 \cup A_4) = 1 - P((A_2 \cup A_3 \cup A_4)^c) = 1 - P(A_2^c \cap A_3^c \cap A_4^c) = 1 - (1-p)^3$$