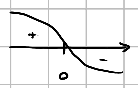
1. $\lim_{x\to 0^-} x^2 e^{-\frac{4}{x}} = \lim_{x\to 0^-} \frac{e^{-\frac{4}{x}}}{x^{-2}} \overset{\frac{\infty}{\infty}}{\underset{H}{=}} \lim_{x\to 0^-} \frac{4x^{-2} e^{-\frac{4}{x}}}{-2x^{-3}} = \lim_{x\to 0^-} \frac{2e^{-\frac{4}{x}}}{-x^{-1}} \overset{\frac{\infty}{\infty}}{\underset{H}{=}} \lim_{x\to 0^-} \frac{2\cdot 4\cancel{x^{-2}} e^{-\frac{4}{x}}}{\cancel{x^{-2}}} = \lim_{x\to 0^-} 8\cdot e^{-\frac{4}{x}} = \left\{ 8\cdot e^{-\frac{4}{0^-}} = 8e^{+\infty} \right\} = +\infty$

$\frac{d}{dx} e^{-\frac{4}{x}} = e^{-\frac{4}{x}} \frac{d}{dx} \frac{-4}{x} = -4e^{-\frac{4}{x}} \frac{d}{dx} x^{-1} = -4e^{-\frac{4}{x}}(-1)x^{-2} = 4e^{-\frac{4}{x}} x^{-2}$

2. $f(x) = e^{-4x^2+3}$

$f'(x) = -8x\, e^{-4x^2+3}$

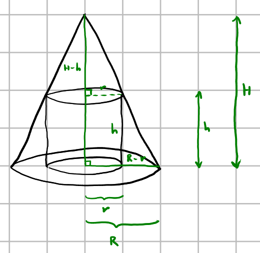


$f \nearrow$ w $(-\infty, 0]$
$f \searrow$ w $[0, +\infty)$

$\forall_{t\in\mathbb{R}}\ e^t > 0$

maximum lokalne $f(0) = e^3$

3.



$V = \pi r^2 h \qquad r \in (0, R)$

$\frac{r}{H-h} = \frac{R-r}{h}$

$rh = RH - Rh - rH + rh$

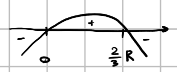
$0 = RH - Rh - rH$

$Rh = RH - rH$

$h = \frac{RH - rH}{R} = H\frac{R-r}{R} = H\left(1 - \frac{r}{R}\right)$

$V = \pi H r^2 \left(1 - \frac{r}{R}\right) = \pi H r^2 - \frac{\pi H}{R} r^3$

$\frac{dV}{dr} = 2\pi H r - 3\frac{\pi H}{R} r^2 = \pi H r \left(2 - \frac{3}{R} r\right) = \frac{3\pi H}{R} r \left(\frac{2R}{3} - r\right)$



$V \nearrow$ w $(0, \frac{2}{3}R]$
$V \searrow$ w $[\frac{2}{3}R, R)$

maximum dla $r = \frac{2}{3}R \qquad h = H\left(1 - \frac{\frac{2}{3}R}{R}\right) = \frac{1}{3}H$

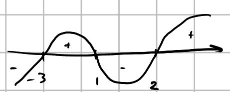
4. $f(x) = \frac{1}{4}x^4 - \frac{7}{2}x^2 + 6x$

$f'(x) = x^3 - 7x + 6 = (x-1)(x^2+x-6) = (x-1)(x-2)(x+3)$

$$\begin{array}{rl}
 & x^2 + x - 6 \\
\hline
 & x^3 \quad - 7x + 6 \;\big(\; x - 1 \\
\ominus & \underline{x^3 - x^2} \\
 & x^2 - 7x \\
\ominus & \underline{x^2 - x} \\
 & -6x + 6 \\
\ominus & \underline{-6x + 6}
\end{array}$$

$\Delta = 1 - 4(-6) = 25 \quad \sqrt{\Delta} = 5$

$x_1 = \frac{-1-5}{2} = -3 \quad x_2 = 2$



$f \nearrow$ w $[-3, 1], [2, +\infty)$
$f \searrow$ w $(-\infty, -3], [1, 2]$

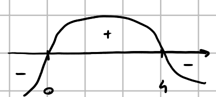
maximum w $x = 1 \quad f(1) = \frac{1}{4} - \frac{7}{2} + 6 = 2\frac{3}{4}$

minimum w $x = -3$ i $x = 2$

5. $f(x) = x^4 e^{-x}$

$f'(x) = 4x^3 e^{-x} - x^4 e^{-x} = x^3 e^{-x}(4 - x) = -x^3(x-4)e^{-x}$

$\forall_{x\in\mathbb{R}}\ e^{-x} > 0$



$f \nearrow$ w $[0, 4]$
$f \searrow$ w $(-\infty, 0], [4, +\infty)$

minimum w $x = 0 \quad f(0) = 0$

maximum w $x = 4 \quad f(4) = 256\, e^{-4}$