c = 7   d = 5

1.

a) $(z+j)^3 + (2+2j)^7 = 0$

1° $z=0$

$j^3 + (2\sqrt{2}\, e^{\frac{\pi}{4}})^7 = -j + 128\cdot 8\cdot \sqrt{2}\, e^{\frac{\pi}{4}} \neq 0$

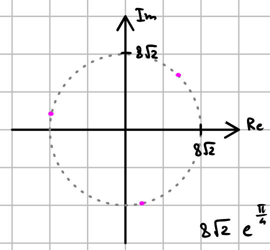
2° $z \neq 0$ $z+j = w = re^{\varphi j}$ $(2\sqrt{2})^7 = 2^{\frac{3}{2}\cdot 7} = 2^{\frac{21}{2}}$

$(z+j)^3 = -(2\sqrt{2}\, e^{\frac{\pi}{4}})^7$

$r^3 e^{3\varphi j} = e^{\pi j}\cdot 2^{\frac{21}{2}} e^{\frac{7\pi}{4}j} = 2^{\frac{21}{2}} e^{\frac{3\pi}{4}j}$

$r^3 = (\sqrt{2})^{21}$ $3\varphi = \frac{3\pi}{4} + 2k\pi$

$r = \sqrt{2}^7 = 8\sqrt{2}$ $\varphi = \frac{\pi}{4} + \frac{2k\pi}{3} = \frac{3\pi + 8k\pi}{12}$ $k \in \mathbb{Z}$



$8\sqrt{2}\, e^{\frac{\pi}{4}j} = 8 + 8j$

b) $k = 5+7+1 = 13$

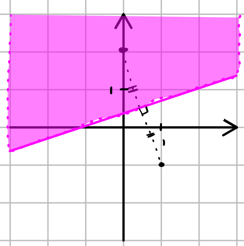
$B = \{ z \in \mathbb{C} : |z-1+j^{13}| \geqslant |z-1+j^{14}| \wedge \arg(z+2j) \in [\frac{\pi}{4}, \frac{\pi}{2}] \}$

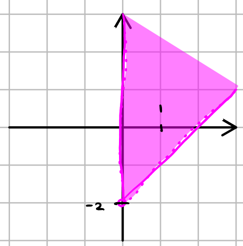
$j^{13} = j$ $j^{14} = j^2 = -1$ $\arg(z-(0-2j)) \in [\frac{\pi}{4}, \frac{\pi}{2}]$

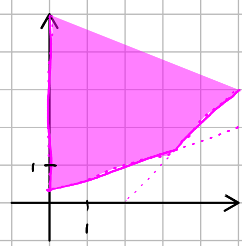
$|z-1+j| \geqslant |z-2|$

$|z-(1-j)| \geqslant |z-(2+0j)|$

$B = B_1 \cap B_2$







2. $m = \sin(\frac{7\pi}{2}) = \sin(\frac{\pi}{2}) = 1$

$$M_{E_3}^{E_3}(\varphi) = \begin{bmatrix}2&1&1\\3&2&-1\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}2&0&0\\0&0&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}2&1&1\\3&2&-1\\0&0&1\end{bmatrix}^{-1} = \begin{bmatrix}4&0&1\\6&0&-1\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}2&-1&-3\\-3&2&5\\0&0&1\end{bmatrix} = \begin{bmatrix}8&-4&-11\\12&-6&-19\\0&0&1\end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc}2&1&1&1&0&0\\3&2&-1&0&1&0\\0&0&1&0&0&1\end{array}\right] \xrightarrow[v_2-v_1]{} \left[\begin{array}{ccc|ccc}2&1&1&1&0&0\\1&1&-2&-1&1&0\\0&0&1&0&0&1\end{array}\right] \xrightarrow{v_1-2v_2} \left[\begin{array}{ccc|ccc}0&-1&5&3&-2&0\\1&1&-2&-1&1&0\\0&0&1&0&0&1\end{array}\right] \xrightarrow[v_2+v_1]{} \left[\begin{array}{ccc|ccc}0&-1&5&3&-2&0\\1&0&3&2&-1&0\\0&0&1&0&0&1\end{array}\right] \xrightarrow[v_2-3v_3]{v_1-5v_3} \left[\begin{array}{ccc|ccc}0&-1&0&3&-2&-5\\1&0&0&2&-1&-3\\0&0&1&0&0&1\end{array}\right]$$

$$\longrightarrow \left[\begin{array}{ccc|ccc}1&0&0&2&-1&-3\\0&1&0&-3&2&5\\0&0&1&0&0&1\end{array}\right]$$

$$M_{E_3}(B) = \begin{bmatrix}2&1&1\\3&2&-1\\0&0&1\end{bmatrix} \quad \left[\begin{array}{ccc|ccc}2&1&1&1&0&0\\3&2&-1&0&1&0\\0&0&1&0&0&1\end{array}\right] \xrightarrow[v_2+v_3]{v_1-v_3} \left[\begin{array}{ccc|ccc}2&1&0&1&0&-1\\3&2&0&0&1&1\\0&0&1&0&0&1\end{array}\right] \xrightarrow[v_2-v_1]{} \left[\begin{array}{ccc|ccc}2&1&0&1&0&-1\\1&1&0&-1&1&2\\0&0&1&0&0&1\end{array}\right] \xrightarrow{v_1-v_2} \left[\begin{array}{ccc|ccc}1&0&0&2&-1&-3\\1&1&0&-1&1&2\\0&0&1&0&0&1\end{array}\right]$$

$$M_B(E_3) = \begin{bmatrix}2&-1&-3\\-3&2&5\\0&0&1\end{bmatrix} \quad \xrightarrow[v_2-v_1]{} \left[\begin{array}{ccc|ccc}1&0&0&2&-1&-3\\0&1&0&-3&2&5\\0&0&1&0&0&1\end{array}\right]$$

$$M_B^B(\varphi) = M_B^{E_3}(id)\, M_{E_3}^{E_3}(\varphi)\, M_{E_3}^{B}(id) = \begin{bmatrix}2&-1&-3\\-3&2&5\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}8&-4&-11\\12&-6&-19\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}2&1&1\\3&2&-1\\0&0&1\end{bmatrix} = \begin{bmatrix}4&-2&-6\\0&0&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}2&1&1\\3&2&-1\\0&0&1\end{bmatrix} = \begin{bmatrix}2&0&0\\0&0&0\\0&0&1\end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 8 & -4 & -11 & 0 \\ 12 & -6 & -19 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{w_2-w_1} \left[\begin{array}{ccc} 8 & -4 & -11 \\ 4 & -2 & -8 \\ 0 & 0 & 1 \end{array}\right] \xrightarrow{w_1-2w_2} \left[\begin{array}{ccc} \cancel{0} & \cancel{0} & \cancel{5} \\ 4 & -2 & -8 \\ 0 & 0 & 1 \end{array}\right] \xrightarrow{w_2+8w_3} \left[\begin{array}{ccc} 0 & 0 & 1 \\ 4 & -2 & 0 \end{array}\right] \to \begin{array}{c} \left[\begin{array}{ccc|c} 0 & 0 & 1 & 0 \\ -2 & 1 & 0 & 0 \end{array}\right] \\ \begin{array}{ccc} \uparrow & & \\ t & y & z \end{array} \end{array} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$\operatorname{Ker}\varphi = \operatorname{span}\{(1,2,0)\}$ $\dim \operatorname{Ker}\varphi = 1$

$\dim \operatorname{Im}\varphi = \operatorname{rank}\varphi = 2$ $\operatorname{Im}\varphi = \operatorname{span}\left\{\begin{bmatrix} 8 \\ 12 \\ 0 \end{bmatrix}, \begin{bmatrix} -11 \\ -19 \\ 1 \end{bmatrix}\right\}$

3. $n = 2 + \sin\left(\frac{5\pi}{2}\right) = 2 + 1 = 3$

$$\left[\begin{array}{cccc|c} 2 & a & a & -2a & 6a \\ 1 & a & 0 & 3a & 4a \\ 3 & 1 & 2 & 5 & 4 \end{array}\right] \qquad \begin{vmatrix} 2 & a & a \\ 1 & a & 0 \\ 3 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & a \\ 1 & a & 0 \\ 3 & 1 & 2 \end{vmatrix} = 2a + 0 + a - 3a^2 - 0 - 0 = -3a^2 + 3a = -3(a^2 - a) = -3a(a-1)$$

1° $a \in \mathbb{R} \setminus \{0, 1\}$

$\operatorname{rank}(A|B) = \operatorname{rank}(A) = 3$

∞ rozwiązań, 1 parametr

2° $a = 0$

$$\left[\begin{array}{cccc|c} 2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 3 & 1 & 2 & 5 & 4 \end{array}\right] \xrightarrow{w_3-3w_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 5 & 4 \end{array}\right]$$

$\operatorname{rank}(A|B) = \operatorname{rank}(A) = 2$

∞ rozwiązań, 2 parametry

3° $a = 1$

$$\left[\begin{array}{cccc|c} 2 & 1 & 1 & -2 & 6 \\ 1 & 1 & 0 & 3 & 4 \\ 3 & 1 & 2 & 5 & 4 \end{array}\right] \xrightarrow[w_3-3w_2]{w_1-2w_2} \left[\begin{array}{ccccc} 0 & -1 & 1 & -8 & -2 \\ 1 & 1 & 0 & 3 & 4 \\ 0 & -2 & 2 & -4 & -8 \end{array}\right] \xrightarrow[w_3-2w_1]{w_2+w_1} \left[\begin{array}{ccccc} 0 & -1 & 1 & -8 & -2 \\ 1 & 0 & 1 & -5 & 2 \\ 0 & 0 & 0 & 12 & -4 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & 1 & -5 & 2 \\ 0 & -1 & 1 & -8 & -2 \\ 0 & 0 & 0 & 12 & -4 \end{array}\right]$$

$\operatorname{rank}(A|B) = \operatorname{rank}(A) = 3$

∞ rozwiązań, 1 parametr

$a = -1$

$$\left[\begin{array}{cccc|c} 2 & -1 & -1 & 2 & -6 \\ 1 & -1 & 0 & -3 & -4 \\ 3 & 1 & 2 & 5 & 4 \end{array}\right] \xrightarrow[w_2+w_3]{w_1+w_3} \left[\begin{array}{cccc|c} 5 & 0 & 1 & 7 & -2 \\ 4 & 0 & 2 & 2 & 0 \\ 3 & 1 & 2 & 5 & 4 \end{array}\right] \xrightarrow[w_3-2w_1]{w_2-2w_1} \left[\begin{array}{ccccc} 5 & 0 & 1 & 7 & -2 \\ -6 & 0 & 0 & -12 & 4 \\ -7 & 1 & 0 & -9 & 8 \end{array}\right] \xrightarrow{-\frac{1}{6}w_2} \left[\begin{array}{ccccc} 5 & 0 & 1 & 7 & -2 \\ 1 & 0 & 0 & 2 & -\frac{2}{3} \\ -7 & 1 & 0 & -9 & 8 \end{array}\right] \xrightarrow[w_3+7w_2]{w_1-5w_2} \begin{array}{c} \left[\begin{array}{cccc|c} 0 & 0 & 1 & -3 & \frac{4}{3} \\ 1 & 0 & 0 & 2 & -\frac{2}{3} \\ 0 & 1 & 0 & 5 & \frac{10}{3} \end{array}\right] \\ \begin{array}{cccc} x_1 & x_2 & x_3 & \uparrow \\ & & & t \end{array} \end{array}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -\frac{2}{3} \\ \frac{10}{3} \\ \frac{4}{3} \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ -5 \\ 3 \\ 1 \end{bmatrix} \quad t \in \mathbb{R}$$