1.

α	$-\dfrac{\pi}{2}$	$-\dfrac{\pi}{3}$	$-\dfrac{\pi}{4}$	$-\dfrac{\pi}{6}$	0	$\dfrac{\pi}{6}$	$\dfrac{\pi}{4}$	$\dfrac{\pi}{3}$	$\dfrac{\pi}{2}$
$\cos\alpha$	$\dfrac{\sqrt{0}}{2}$	$\dfrac{\sqrt{1}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{4}}{2}$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{1}}{2}$	$\dfrac{\sqrt{0}}{2}$
$\sin\alpha$	$-\dfrac{\sqrt{4}}{2}$	$-\dfrac{\sqrt{3}}{2}$	$-\dfrac{\sqrt{2}}{2}$	$-\dfrac{\sqrt{1}}{2}$	$\dfrac{\sqrt{0}}{2}$	$\dfrac{\sqrt{1}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{4}}{2}$
	A	B	C	D	E	F	G	H	I

	precizuje z okr. $r=\sqrt{2}$	precizuje z okr. $r=2$
A	$(0,\ -\sqrt{2})$	$(0,\ -2)$
B	$\left(\dfrac{\sqrt{2}}{2},\ -\dfrac{\sqrt{6}}{2}\right)$	$(1,\ -\sqrt{3})$
C	$(1,\ -1)$	$(\sqrt{2},\ -\sqrt{2})$
D	$\left(\dfrac{\sqrt{6}}{2},\ -\dfrac{\sqrt{2}}{2}\right)$	$(\sqrt{3},\ -1)$
E	$(\sqrt{2},\ 0)$	$(2,\ 0)$
F	$\left(\dfrac{\sqrt{6}}{2},\ \dfrac{\sqrt{2}}{2}\right)$	$(\sqrt{3},\ 1)$
G	$(1,\ 1)$	$(\sqrt{2},\ \sqrt{2})$
H	$\left(\dfrac{\sqrt{2}}{2},\ \dfrac{\sqrt{6}}{2}\right)$	$(1,\ \sqrt{3})$
I	$(0,\ \sqrt{2})$	$(0,\ 2)$

	J	K	L	M	N	O	P
α	$\dfrac{2\pi}{3}$	$\dfrac{3\pi}{4}$	$\dfrac{5\pi}{6}$	π	$-\dfrac{5\pi}{6}$	$-\dfrac{3\pi}{4}$	$-\dfrac{2\pi}{3}$
$\cos(\alpha)$	$-\dfrac{1}{2}$	$-\dfrac{\sqrt{2}}{2}$	$-\dfrac{\sqrt{3}}{2}$	-1	$-\dfrac{\sqrt{3}}{2}$	$-\dfrac{\sqrt{2}}{2}$	$-\dfrac{1}{2}$
$\sin(\alpha)$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{2}}{2}$	$\dfrac{1}{2}$	0	$-\dfrac{1}{2}$	$-\dfrac{\sqrt{2}}{2}$	$-\dfrac{\sqrt{3}}{2}$

2. $\quad \alpha \in (-\pi,\ \pi]$

a) $\quad \cos(\alpha) = -\dfrac{\sqrt{3}}{2} \quad \sin(\alpha) = \dfrac{1}{2} \qquad \alpha = \dfrac{5\pi}{6}$

b) $\quad \cos(\alpha) = -\dfrac{\sqrt{2}}{2} \quad \sin(\alpha) = -\dfrac{\sqrt{2}}{2} \qquad \alpha = -\dfrac{3\pi}{4}$

c) $\quad 2\cos(\alpha) = 1 \quad 2\sin(\alpha) = -\sqrt{3} \qquad \alpha = -\dfrac{\pi}{3}$

d) $\quad \sqrt{2}\cos(\alpha) = -1 \quad \sqrt{2}\sin(\alpha) = 1 \qquad \alpha = \dfrac{3\pi}{4}$

3.

a)

b)

c)

d)

e)

4.
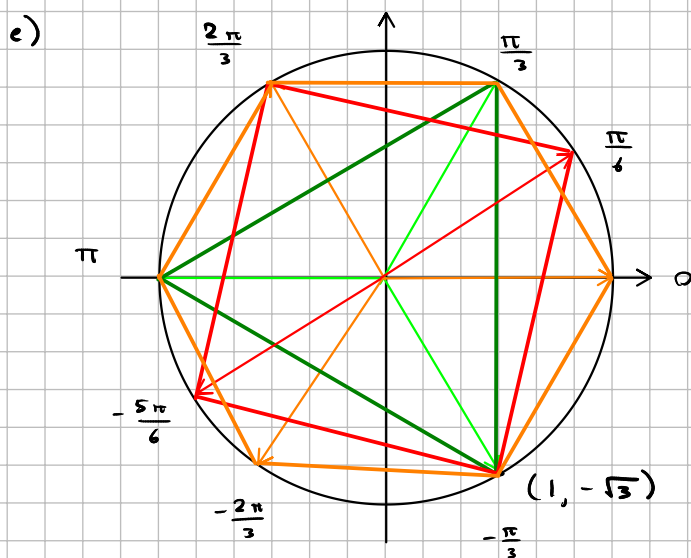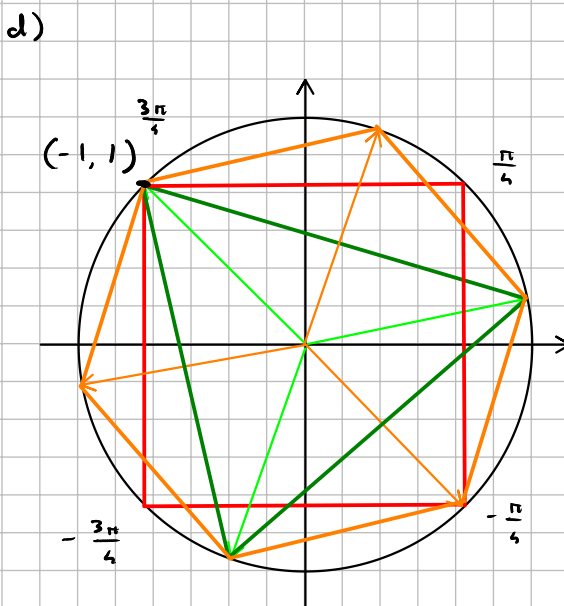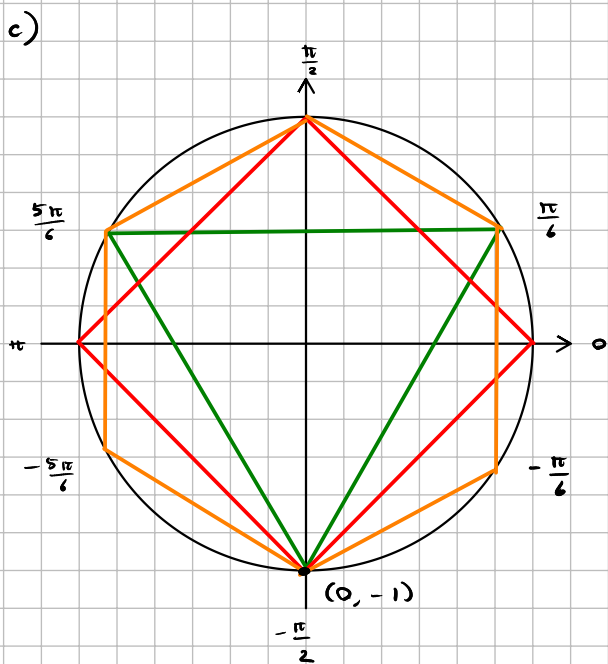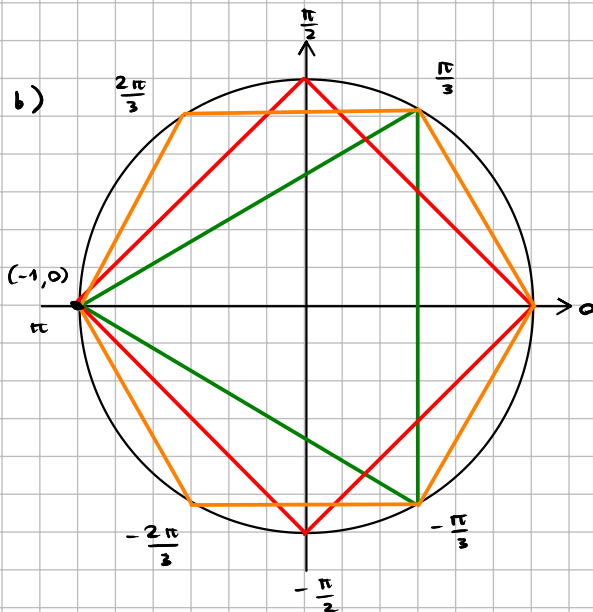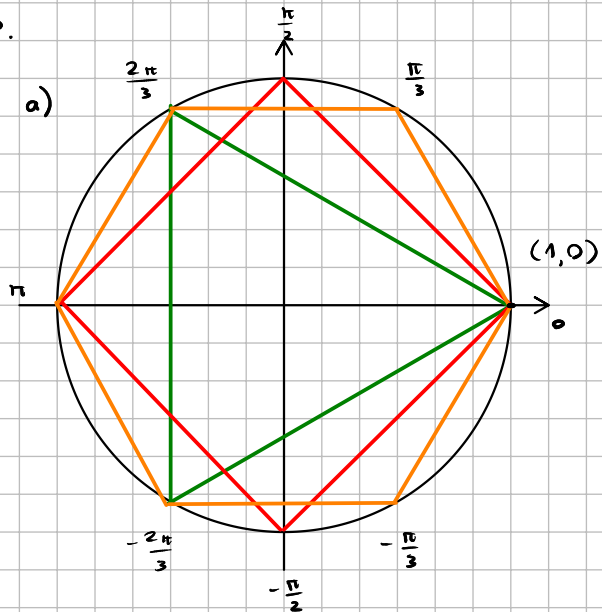
a) $x \circ y := x - y$ $\quad \cup \; \mathbb{Z}$

$a - (b-c) \neq (a-b) - c$ $\qquad$ nie jest łączne

$a - b \neq b - a$ $\qquad$ nie jest przemienne

$\qquad\qquad\qquad\qquad\qquad$ nie istnieje element neutralny

b) $x * y := x + y + xy$ $\quad \cup \; \mathbb{R}$

$a + b + ab = b + a + ba$

$a * (b*c) = a + (b*c) + a(b*c) = a + b + c + bc + a(b+c+bc)$
$\qquad\qquad\qquad\qquad\qquad\qquad = a + b + c + bc + ab + ac + abc$

$(a*b)*c = (a*b) + c + (a*b)c = a + b + ab + c + ac + bc + abc$

$a * 0 = a + 0 + 0 = a$ $\qquad\qquad$ jest łączne
$\qquad\qquad\qquad\qquad\qquad\qquad$ jest przemienne
$\qquad\qquad\qquad\qquad\qquad\qquad$ $e = 0$

c) $x \sqcap y := \max\{x, y\}$ $\quad \cup \; \mathbb{N} = \{1, 2, 3, \dots\}$

$a \sqcap b = \max\{a, b\} = \max\{b, a\} = b \sqcap a$ $\qquad$ jest przemienne

$a \sqcap (b \sqcap c) = (a \sqcap b) \sqcap c = \max\{a, b, c\}$ $\qquad$ jest łączne

$a \sqcap 1 = a = 1 \sqcap a$ $\qquad\qquad\qquad\qquad\qquad$ $e = 1$

d) $(x_1, y_1) \oplus (x_2, y_2) := (x_1 + x_2, y_1 + y_2)$ $\quad \cup \; \mathbb{R} \times \mathbb{R}$

$(a, b) \oplus (c, d) = (a+c, b+d) = (c+a, d+b) = (c, d) \oplus (a, b)$

$(a, b) \oplus [(c, d) \oplus (e, f)] = (a, b) \oplus (c+e, d+f) = (a+c+e, b+d+f)$

$[(a, b) \oplus (c, d)] \oplus (e, f) = (a+c, b+d) \oplus (e, f) = (a+c+e, b+d+f)$

$(a, b) \oplus (0, 0) = (a+0, b+0) = (a, b)$ $\qquad$ jest łączne
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ jest przemienne
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $e = (0, 0)$

e) $(x_1, y_1) \odot (x_2, y_2) := (x_1 x_2 - y_1 y_2, \; x_1 y_2 + x_2 y_1)$

$(a, b) \odot (c, d) = (ac - bd, \; ad + bc)$
$(c, d) \odot (a, b) = (ac - bd, \; bc + ad)$

$(a, b) \odot [(c, d) \odot (e, f)] = (a, b) \odot (ce - df, \; cf + de)$
$\qquad = (ace - adf - bcf - bde, \; acf + ade + bce - bdf)$

$[(a, b) \odot (c, d)] \odot (e, f) = (ac - bd, \; ad + bc) \odot (e, f)$ $\qquad$ jest łączne
$\qquad = (ace - bde - adf - bcf, \; acf - bdf + ade + bce)$ $\qquad$ jest przemienne
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $e = (1, 0)$

$(1, 0) \odot (a, b) = (a, b)$

5.

a) $\quad (c, 0) \oplus (x, 0) = (c + x, 0)$

b) $\quad (c, 0) \odot (x, 0) = (cx, 0)$

c) $\quad (c, 0) \odot (x, y) = (cx, cy)$

d) $\quad (-1, 0) \odot (x, y) = (-x, -y)$

e) $\quad (0, 1) \odot (x, y) = (-y, x)$

f) $\quad (0, 1) \odot (0, 1) = (-1, 0)$