1.

$P(A') = \frac{1}{3}$ $\quad P(A \cap B) = \frac{1}{4}$ $\quad P(A \cup B) = \frac{2}{3}$

$P(B') = \frac{3}{4}$

$P(A \cap B') = \frac{5}{12}$

$P(B \setminus A) = 0$

$P(A) = 1 - P(A') = 1 - \frac{1}{3} = \frac{2}{3}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(B) = P(A \cup B) - P(A) + P(A \cap B) = \frac{2}{3} - \frac{2}{3} + \frac{1}{4} = \frac{1}{4}$

$P(B') = 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$

$P(B \setminus A) = P(B) - P(A \cap B) = \frac{1}{4} - \frac{1}{4} = 0$

$A \cap B' = A \cap (\Omega \setminus B) = A \setminus B$

$P(A \setminus B) = P(A) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$

2.

$3 \times C$ $\quad 4 \times B$ $\quad$ losuję 3 kule

A - $3 \times C$

B - $2 \times C$, $1 \times B$

C - $3 \times C \vee 3 \times B$

$|\Omega| = \binom{7}{3} = \frac{7!}{3!4!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} = 35$

Ω - 3-elementowe podzbiory zbioru 7-elementowego

$P(A) = \frac{\binom{3}{3}}{\binom{7}{3}} = \frac{1}{35}$

$P(B) = \frac{\binom{3}{2}\binom{4}{1}}{\binom{7}{3}} = \frac{3 \cdot 4}{35} = \frac{12}{35}$

$P(C) = \frac{\binom{3}{3} + \binom{4}{3}}{\binom{7}{3}} = \frac{1+4}{35} = \frac{1}{7}$

3.

10 rzutów kostką $\qquad \Omega = \{(x_1, \dots, x_{10}) : x_i \in \{1,2,3,4,5,6\}\} = \{1, 2, \dots 6\}^{10}$

A - nie pojawi się conajmniej 1 ze ścianek

A_i - nie pojawi się ścianka nr i w żadnym z 10 rzutów $\qquad A_i = (\{1,2,3,4,5,6\} \setminus \{i\})^{10}$

$A = A_1 \cup A_2 \cup A_3 \cup \dots \cup A_6$

$$P(A) = P\left(\bigcup_{i=1}^{6} A_i\right) = \sum_{i=1}^{6} P(A_i) - \sum_{1 \le i < j \le 6} P(A_i \cap A_j) + \sum_{1 \le i < j < k \le 6} P(A_i \cap A_j \cap A_k) - \sum P(A_i \cap A_j \cap A_k \cap A_l) + \sum P(A_i \cap A_j \cap A_k \cap A_l \cap A_m) - P\left(\bigcap_{i=1}^{6} A_i\right)$$

$$\binom{6}{1} \cdot \frac{5^{10}}{6^{10}} - \binom{6}{2} \cdot \frac{4^{10}}{6^{10}} + \binom{6}{3} \frac{3^{10}}{6^{10}} - \binom{6}{4} \cdot \frac{2^{10}}{6^{10}} + \binom{6}{5} \frac{1^{10}}{6^{10}} - 0$$

$$\frac{1}{6^{10}} (6 \cdot 5^{10} - 15 \cdot 4^{10} + 20 \cdot 3^{10} - 15 \cdot 2^{10} + 6) = \frac{101923}{139968} \simeq 0.73$$

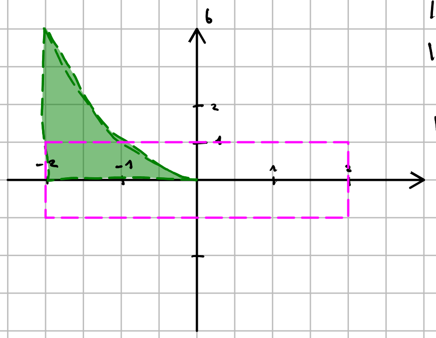
4.

A: $x^2 + 2ax + b = 0$ ma 2 dodatnie pierwiastki rzeczywiste

$$\Omega = \{(a,b) : a \in [-2,2] \wedge b \in [-1,1]\} = [-2,2] \times [-1,1]$$

$$\begin{cases} \Delta > 0 \\ x_1 x_2 > 0 \\ x_1 + x_2 > 0 \end{cases} \iff \begin{cases} 4a^2 - 4b > 0 \\ b > 0 \\ -2a > 0 \end{cases} \iff \begin{cases} a^2 > b \\ b > 0 \\ a < 0 \end{cases}$$

$$A = \{(a,b) \in \Omega : a^2 > b,\ b > 0,\ a < 0\}$$



$$|\Omega| = 4 \cdot 2 = 8$$

$$|A| = 1 \cdot 1 + \int_0^1 x^2 dx = 1 + \left[\tfrac{1}{3}x^3\right]_0^1 = \frac{4}{3}$$

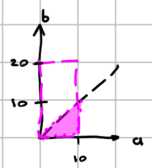
$$P(A) = \frac{\frac{4}{3}}{8} = \frac{1}{6}$$

5.

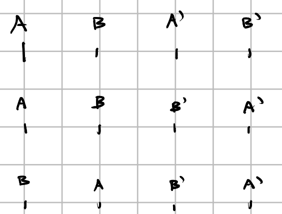
$$\Omega = \{(a,b) : a \in [0,10] \wedge b \in [0,20]\}$$

minuty od 13:00

A: $b < a$

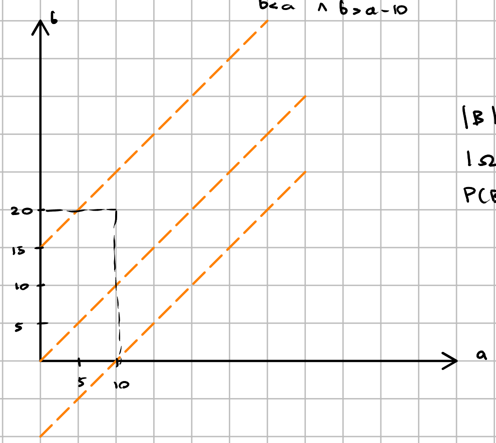


$$P(A) = \frac{\frac{1}{2} \cdot 10 \cdot \cancel{10}}{\cancel{10} \cdot 20} = \frac{1}{4}$$



B: $a \le b \le a + 15 \vee b < a < b + 10$

$b < a \wedge b > a - 10$



$$|B| = 20 \cdot 10 - \frac{1}{2} \cdot 5 \cdot 5 = \frac{375}{2}$$

$$|\Omega| = 200$$

$$P(B) = 0.9375$$

6.

$$\Omega = \{x, 0x, 00x, \dots\}$$

"x" - wypada 2 lub 3

0 - nie wypada 2 ani 3

$$\Omega = \{1, 2, 3, \dots\}$$

$$P(\{k\}) = \left(\frac{4}{6}\right)^{k-1} \cdot \frac{2}{6}$$

A: rzucą tyle samo razy

$$P(A) = P(\{0x\}) + P(\{000x\}) + P(\{00000x\}) + \dots$$

$$P(A) = \frac{4}{6} \cdot \frac{2}{6} + \left(\frac{4}{6}\right)^3 \cdot \frac{2}{6} + \left(\frac{4}{6}\right)^5 \cdot \frac{2}{6} + \dots$$

$$= \sum_{i=0}^{\infty} \frac{2}{6} \cdot \left(\frac{4}{6}\right)^{2n+1} = \frac{1}{3} \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^{2n} \cdot \frac{2}{3} = \frac{2}{9} \sum_{i=0}^{\infty} \left(\frac{4}{9}\right)^n = \frac{2}{9} \cdot \frac{1}{1 - \frac{4}{9}} = \frac{2}{9} \cdot \frac{9}{5} = \frac{2}{5}$$

7.

0-2, 10-12, 20-22　　30-32, 40-42, 50-52　　...

0-3, 15-18　　30-33, 45-48

$\Omega = [0, 60]$

$A = [0,2] \cup [30,32]$　　$|A| = 4$

$B = (2,3] \cup [10,12] \cup [15,18]$ ...

Wzór powtarza się po dwieście 30 s $(\mathrm{lcm}(2+8, 3+12))$

Sygnały nakładają się w przedziałach $[30n, 30n+2]$

$$P(A) = \frac{2(2)}{60} = \frac{2}{30} = \frac{1}{15}$$

bez pokrycia

$$P(B) = \frac{2(3 \cdot 2 + 2 \cdot 3 - 2 \cdot 2)}{60} = \frac{8}{30} = \frac{4}{15}$$

bez liczenia 2 razy

$$P(C) = \frac{2(3 \cdot 2 + 2 \cdot 3 - 2)}{60} = \frac{10}{30} = \frac{1}{3}$$