1.

$P(A) = \frac{2}{5}$

$P(B|A) = \frac{1}{4}$

$P(C|A\cap B) = \frac{1}{2}$

$P(A\cup B) = \frac{3}{5}$

$P(C|B) = \frac{1}{3}$

$$P(B|A) = \frac{P(A\cap B)}{P(A)}$$

$$P(A\cap B) = P(A)\,P(B|A) = \frac{2}{5}\cdot\frac{1}{4} = \frac{1}{10}$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$P(B) = P(A\cup B) - P(A) + P(A\cap B) = \frac{3}{5} - \frac{2}{5} + \frac{1}{10} = \frac{3}{10}$$

$$P(C|B) = \frac{P(B\cap C)}{P(B)}$$

$$P(B\cap C) = P(B)\cdot P(C|B) = \frac{3}{10}\cdot\frac{1}{3} = \frac{1}{10}$$

$$P(C|A\cap B) = \frac{P(A\cap B\cap C)}{P(A\cap B)}$$

$$P(A\cap B\cap C) = P(A\cap B)\cdot P(C|A\cap B) = \frac{1}{10}\cdot\frac{1}{2} = \frac{1}{20}$$

$$P(A|B\cap C) = \frac{P(A\cap B\cap C)}{P(B\cap C)} = \frac{\frac{1}{20}}{\frac{1}{10}} = \frac{1}{2}$$

2.

$\underline{\mathrm{I}}$ 2 monety $P(O) = \frac{1}{4}$ 2 rzuty Losowa monetę

$\underline{\mathrm{II}}$ 2 monety $P(O) = 1$

$\underline{\mathrm{III}}$ 1 moneta $P(O) = \frac{1}{2}$

a) A_i - wybrano monetę i-tego typu

$A_1 \cup A_2 \cup A_3 = \Omega$

A_i, A_j parami rozłączne

B - orzeł wypadł conajmniej raz

B' - nie wypadł orzeł

$$P(B') = P(B'|A_1)P(A_1) + P(B'|A_2)P(A_2) + P(B'|A_3)P(A_3)$$

$$P(A_1) = \frac{2}{5} \quad P(A_2) = \frac{2}{5} \quad P(A_3) = \frac{1}{5}$$

$$P(B'|A_1) = \left(1 - \frac{1}{4}\right)^2 = \frac{9}{16}$$

$$P(B'|A_2) = (1-1)^2 = 0$$

$$P(B'|A_3) = \left(1 - \frac{1}{2}\right)^2 = \frac{1}{4}$$

$$P(B') = \frac{9}{16} \cdot \frac{2}{5} + 0 \cdot \frac{2}{5} + \frac{1}{5} \cdot \frac{1}{4} = \frac{11}{40}$$

$$P(B) = 1 - P(B') = \frac{29}{40}$$

b)

C - wypadł 2 razy orzeł

$$P(A_1 \cup A_2 | C) = P(A_1|C) + P(A_2|C) = \frac{P(C|A_1)P(A_1)}{P(C)} + \frac{P(C|A_2)P(A_2)}{P(C)}$$

$$= \frac{P(C|A_1)P(A_1) + P(C|A_2)P(A_2)}{P(C|A_1)P(A_1) + P(C|A_2)P(A_2) + P(C|A_3)P(A_3)}$$

$$= \frac{\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{2}{5} + 1 \cdot \frac{2}{5}}{\frac{1}{4} \cdot \frac{1}{4} \cdot \frac{2}{5} + 1 \cdot \frac{2}{5} + \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{5}} = \frac{17}{19}$$

3.

A – pozytywny

B – chory

$P(B) = 0.01$

$P(A|B) = 0.9$

$P(A'|B') = 0.7$

$P(A|B') = 0.3$

$P(A'|B) = 0.1$

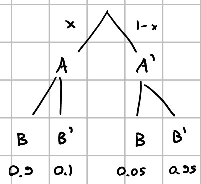
a) $$P(B|A) = \frac{P(A|B)P(B)}{P(A)} = \frac{0.9 \cdot 0.01}{0.306} = \frac{1}{34}$$

$$P(A) = P(A|B)P(B) + P(A|B')P(B') = 0.9 \cdot 0.01 + 0.3 \cdot 0.99 = 0.306$$

b) $$P(B|A') = \frac{P(A'|B)P(B)}{P(A')} = \frac{0.1 \cdot 0.01}{0.694} = \frac{1}{694}$$

4.

A - vadliwy wyrób
B - odrzucony wyrób



$P(B|A) = 0.9$

$P(B|A') = 0.05$

$P(A|B') = 0.01$

$P(B'|A) = 0.1$

$P(B'|A') = 0.95$

$$P(A|B') = \frac{P(B'|A)P(A)}{P(B')}$$

$$P(B') = P(B'|A)P(A) + P(B'|A')P(A')$$

$$\frac{1}{100} = \frac{\frac{1}{10}x}{\frac{1}{10}x + \frac{95}{100}(1-x)}$$

$$\frac{1}{100} \cdot \left[\frac{1}{10}x + \frac{95}{100} - \frac{95x}{100}\right] = \frac{1}{10}x \quad / \cdot 100$$

$$\frac{x}{10} + \frac{95}{100} - \frac{95x}{100} = 10x$$

$$10x + 95 - 95x = 1000x$$

$$x = \frac{95}{1085}$$

5.

$X_1 = 10011$ $\quad$ A_k – wysłano ciąg X_k $\quad$ $P(A_1) = 0.3$

$X_2 = 11011$ $\quad$ $A_1 \vee A_2 \vee A_3 = \Omega$ $\quad$ $P(A_2) = 0.3$

$X_3 = 10101$ $\quad$ B - otrzymano y $\quad$ $P(A_3) = 0.4$

$y = 10111$

$$P(B|A_1) = 0.1 \cdot 0.9^4$$
$$P(B|A_2) = 0.1^2 \cdot 0.9^3$$
$$P(B|A_3) = 0.1 \cdot 0.9^4$$

$$P(A_3|B) = \frac{P(B|A_3)\,P(A_3)}{P(B)} \simeq 0.55$$

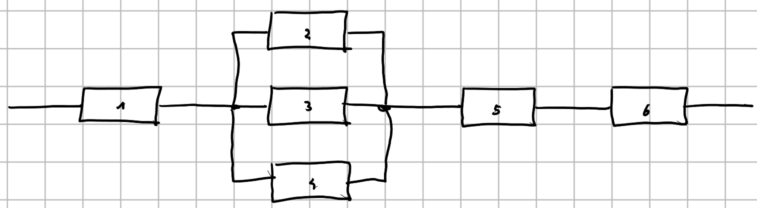
$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3) = 0.048114$$

$$P(A_1|B) \simeq 0.41$$

$$P(A_2|B) \simeq 0.05$$

Najprawdopodobniej został wysłany sygnał X_3

6.



przekaźniki działają niezależnie

$$P(A) = P(A_1) \cdot P(A_2 \cup A_3 \cup A_4) \cdot P(A_5) \cdot P(A_6)$$

$$= P(A_1) \cdot \left[P(A_2) + P(A_3) + P(A_4) - P(A_2 \cap A_3) - P(A_3 \cap A_4) - P(A_2 \cap A_4) + P(A_2 \cap A_3 \cap A_4)\right] \cdot P(A_5) \cdot P(A_6)$$

$$= p \cdot (p + p + p - p^2 - p^2 - p^2 + p^3) \cdot p \cdot p$$

$$= p^3 (p^3 - 3p^2 + 3p) = p^6 - 3p^5 + 3p^4$$

$$P(A_2 \cup A_3 \cup A_4) = 1 - P\left((A_2 \cup A_3 \cup A_4)'\right) = 1 - P(A_2' \cap A_3' \cap A_4') = 1 - (1-p)^3$$