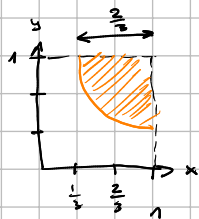


1.

$$\Omega = \{(x, y) \in \mathbb{R}^2 : (x, y) \in [0, 1] \times [0, 1]\}$$

P - prawdopodobieństwo geometryczne

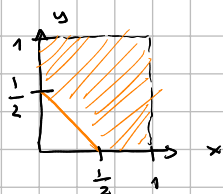
$$a) A = \{(x, y) \in \Omega : (x-1)^2 + (y-1)^2 \leq \frac{2}{3}\}$$



$$P(A) = \frac{|A|}{|\Omega|} = \frac{\frac{1}{4} \cdot \pi \cdot \left(\frac{\sqrt{2}}{2}\right)^2}{1^2} = \frac{\pi}{8}$$

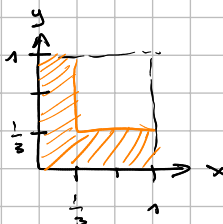
$$b) B = \{(x, y) \in \Omega : x + y > \frac{1}{2}\}$$

$$y > -x + \frac{1}{2}$$



$$P(B) = \frac{|B|}{|\Omega|} = \frac{1^2 - \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}}{1^2} = \frac{7}{8}$$

$$c) C = \{(x, y) \in \Omega : \min(x, y) \leq \frac{1}{3}\}$$



$$P(C) = \frac{|C|}{|\Omega|} = \frac{1^2 - \left(\frac{2}{3}\right)^2}{1^2} = \frac{5}{9}$$

2.

P - prawdopodobieństwo Klasyfikacji

K - kobieta 2000

K' - mężczyzna 1500

R - ogłoda

R' - nie ogłoda

$$P(R|K) = \frac{8}{10}$$

$$P(R'|K) = \frac{2}{10}$$

$$P(R|K') = \frac{45}{100}$$

$$P(R'|K') = \frac{55}{100}$$

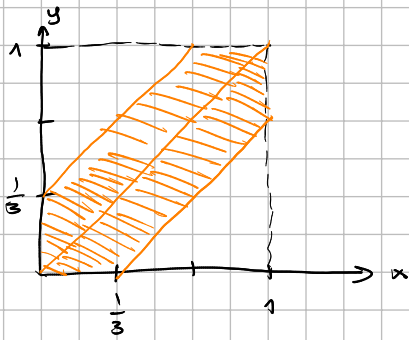
$$a) P(R) = P(R|K)P(K) + P(R|K')P(K') = \frac{8}{10} \cdot \frac{2000}{3500} + \frac{45}{100} \cdot \frac{1500}{3500} = \frac{13}{20}$$

$$b) P(K'|R) = \frac{P(K' \cap R)}{P(R)} = \frac{P(R|K')P(K')}{P(R)} = \frac{\frac{45}{100} \cdot \frac{1500}{3500}}{\frac{13}{20}} = \frac{27}{31}$$

3.

$$\Omega = \{(x, y) \in [0, 1] \times [0, 1]\}$$

$x, y$  - czas przyścia do punktu statowego między 7:30 a 7:45



$$A = \{(x, y) \in \Omega : y \in [x, x + \frac{1}{3}] \vee x \in [y, y + \frac{1}{3}]\}$$

$$y \in [x, x + \frac{1}{3}] \vee x \in [y, y + \frac{1}{3}]$$

$$x \leq y \leq x + \frac{1}{3} \vee y \leq x \leq y + \frac{1}{3}$$

$$(y \geq x \wedge y \leq x + \frac{1}{3}) \vee (y \leq x \wedge y \geq x - \frac{1}{3})$$

$P$  - prawdopodobieństwo geometryczne

$$P(A) = \frac{|A|}{|\Omega|} = \frac{1^2 - 2 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3}}{1^2} = \frac{5}{9}$$

4.

$$\Omega = \{(a, b) \in [-1, 1] \times [-1, 1]\}$$

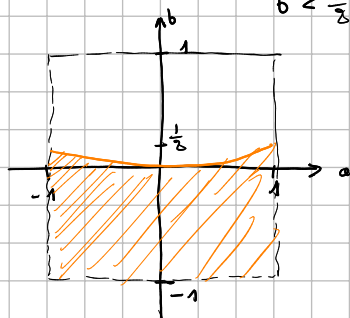
P - geometryczne

a) A -  $x^2 + ax + 2b$  ma 2 różne pierwiastki rzeczywiste

$$\Delta = a^2 - 4 \cdot 1 \cdot 2b = a^2 - 8b$$

$$A = \{(a, b) \in \Omega : a^2 - 8b > 0\}$$

$$b < \frac{1}{8}a^2$$



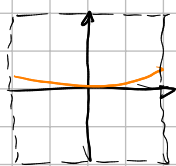
$$|A| = 2 \cdot 1 + \int_{-1}^1 \frac{1}{8} a^2 da = 2 + \frac{1}{8} \cdot \left[ \frac{1}{3} a^3 \right]_{-1}^1 = 2 + \frac{1}{8} \left( \frac{1}{3} - \left(-\frac{1}{3}\right) \right)$$

$$= 2 + \frac{1}{8} \cdot \frac{2}{3} = \frac{25}{12}$$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{\frac{25}{12}}{4} = \frac{25}{48}$$

b) B -  $x^2 + ax + 2b$  ma 1 podwójny pierwiastek rzeczywisty

$$B = \{(a, b) \in \Omega : a^2 - 8b = 0\}$$



$$P(B) = \frac{|B|}{|\Omega|} = \frac{0}{4} = 0$$

5.

D - urządzenie ma defekt

W - metoda informuje o defekcie

$$P(W|D) = 80\% \Rightarrow P(W'|D) = 20\%$$

$$P(W|D') = 1\% \Rightarrow P(W'|D') = 99\%$$

$$P(D) = 1\% \Rightarrow P(D') = 99\%$$

$$\begin{aligned} P(D|W) &= \frac{P(D \cap W)}{P(W)} = \frac{P(W|D)P(D)}{P(W)} = \frac{P(W|D)P(D)}{P(W|D)P(D) + P(W|D')P(D')} \\ &= \frac{0.8 \cdot 0.01}{0.8 \cdot 0.01 + 0.01 \cdot 0.99} \approx 0.45 \end{aligned}$$

6.

$$\Omega = \{0, R\} \times \{1, 2, 3, 4, 5, 6\}$$

Przestrzeń zdarzeń klasyczna  $\forall \omega \in \Omega \quad P(\omega) = \frac{1}{|\Omega|} \quad |\Omega| = 12$

$$X(\omega) = \begin{cases} 3 & \omega = (R, 1) \\ 1 & \omega \in \{0\} \times \{1, 2, 3, 4, 5, 6\} \cup \{R\} \times \{2, 4, 6\} \\ -3 & \omega \text{ p.p.} \end{cases}$$

$$S_X = \{-3, 1, 3\}$$

$i$	3	1	-3
$p_i$	$\frac{1}{12}$	$\frac{3}{12}$	$\frac{2}{12}$

$$EX = 3 \cdot \frac{1}{12} + 1 \cdot \frac{3}{12} - 3 \cdot \frac{2}{12} = \frac{3+3-6}{12} = \frac{1}{2}$$

$$EX^2 = 9 \cdot \frac{1}{12} + 1 \cdot \frac{3}{12} + 9 \cdot \frac{2}{12} = \frac{9+3+18}{12} = \frac{30}{12} = 2.5$$

$$VX = EX^2 - (EX)^2 = 2.5 - \left(\frac{1}{2}\right)^2 = 2.5 - \frac{1}{4} = \frac{10}{4} = 2.5$$

$$V(4X - 10) = 4^2 VX = 16 \cdot \frac{10}{4} = 40$$

7.

$$f_x(x) = \frac{1}{3} \cdot \mathbb{1}_{[-1,0] \cup [1,3]}(x)$$

$$F_x(t) = \int_{-\infty}^t f_x(x) dx = \begin{cases} 0 & t < -1 \\ \frac{1}{3}(t+1) & t \in [-1, 0] \\ \frac{1}{3} & t \in (0, 1) \\ \frac{1}{3} + \frac{1}{3}(t-1) & t \in [1, 3] \\ 1 & t > 3 \end{cases}$$

$$|x-1| > x$$

$\underbrace{x < 1}$	$\underbrace{x \geq 1}$
$-x+1 > x$	$x-1 > x$
$1 > 2x$	$-1 > 0$
$x < \frac{1}{2}$	

$$P(|X-1| > x) = P((X \geq 1 \wedge X-1 > x) \vee (X < 1 \wedge -X+1 > x))$$

$$= P(X \geq 1 \wedge -1 > 0) + P(X < 1 \wedge X < \frac{1}{2})$$

$$= P(X < \frac{1}{2})$$

$$= F_x(\frac{1}{2}) = \frac{1}{3}$$

8.

$$f_x(x) = a(x-1) \cdot 1_{\{0,1\}}(x)$$

$$1. \int_{-\infty}^{+\infty} f_x(x) dx = 1$$

$$2. \forall_{x \in \mathbb{R}} f_x(x) \geq 0$$

$$\int_{-\infty}^{+\infty} f_x(x) dx = \int_0^1 a(x-1) dx = a \left[ \frac{1}{2} x^2 - x \right] \Big|_0^1 = a \left( \frac{1}{2} - 1 - 0 \right) = -\frac{1}{2} a \Rightarrow a = -2$$

$$F_x(t) = \int_{-\infty}^t f_x(x) dx = \begin{cases} 0 & t < 0 \\ \int_0^t -2(x-1) dx = -2 \left[ \frac{1}{2} x^2 - x \right] \Big|_0^t = -2 \cdot \frac{1}{2} t^2 + 2t = 2t - t^2 & t \in [0, 1] \\ 1 & t > 1 \end{cases}$$

$$EX = \int_{-\infty}^{+\infty} x f_x(x) dx = \int_0^1 -2 \cdot x \cdot (x-1) dx = -2 \int_0^1 x^2 - x dx = -2 \left[ \frac{1}{3} x^3 - \frac{1}{2} x^2 \right] \Big|_0^1 = -2 \left( \frac{1}{3} - \frac{1}{2} \right) = \frac{1}{3}$$



9.

$$F_x(x) = \begin{cases} 0 & x < -2 \\ ax+b & -2 \leq x < 0 \\ \frac{1}{3} & 0 \leq x < 1 \\ cx+d & 1 \leq x < 2 \\ 1 & x \geq 2 \end{cases}$$

$F_x$  jest ciągła

$$F_x(-2) = F_x(-2^-) \Rightarrow -2a + b = 0$$

$$a = \frac{1}{6}$$

$$F_x(0) = F_x(0^-) \Rightarrow \frac{1}{3} = b$$

$$b = \frac{1}{3}$$

$$c = \frac{2}{3}$$

$$d = -\frac{1}{3}$$

$$F_x(1) = F_x(1^-) \Rightarrow c + d = \frac{1}{3}$$

$$F_x(2) = F_x(2^-) \Rightarrow 1 = 2c + d$$

$$P(|X| > \frac{1}{2}) = P(X > \frac{1}{2} \vee X < -\frac{1}{2}) = P(X > \frac{1}{2}) + P(X < -\frac{1}{2})$$

$$= 1 - P(X < \frac{1}{2}) - P(X < -\frac{1}{2})$$

$$= 1 - F_x(\frac{1}{2}) - F_x(-\frac{1}{2})$$

$$= 1 - \cancel{\frac{1}{3}} + \frac{1}{6} \cdot (-\frac{1}{2}) + \cancel{\frac{1}{3}} = \frac{11}{12}$$

10.

$$\Omega = \{(a, b) : a, b \in [0, 6]\}$$

P - p. geometryczne

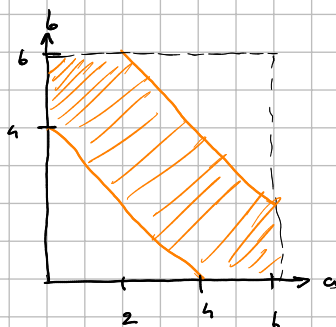
$$a) A = \{(a, b) \in \Omega : \frac{a+b}{2} \in [2, 4]\}$$

$$2 \leq \frac{a+b}{2} \leq 4$$

$$4 \leq a+b \leq 8$$

$$-a+4 \leq b \leq -a+8$$

$$P(A) = \frac{|A|}{|\Omega|} = \frac{6^2 - 2 \cdot \frac{1}{2} \cdot 4 \cdot 4}{6^2} = \frac{20}{36} = \frac{5}{9}$$



$$b) B = \{(a, b) \in \Omega : |a-b| > |6-b|\}$$

$$|a-b| > |6-b|$$

$$|a-b| > 6-b$$

$$a \geq b :$$

$$a < b :$$

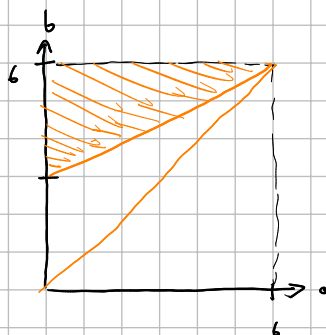
$$a-b > 6-b$$

$$a > 6$$

$$-a+b > 6-b$$

$$2b > a+b$$

$$b > \frac{1}{2}a + 3$$



$$P(B) = \frac{|B|}{|\Omega|} = \frac{\frac{1}{2} \cdot 3 \cdot 6}{6^2} = \frac{1}{4}$$

11.

$$F_X(x) = \begin{cases} 0 & x < -3 \\ a & -3 \leq x < -1 \\ b x + c & -1 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

$$P(X = -3) = P(X = -1) = P(X = 1) = \frac{1}{4}$$

punktly skokove  $-3, -1, 1$

$$P(X = -3) = F_X(-3) - F_X(-3^-) \Rightarrow \frac{1}{4} = a - 0 = a$$

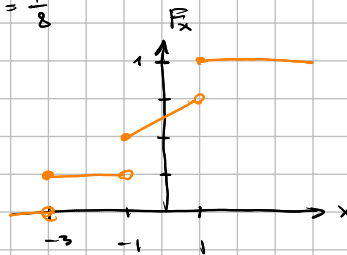
$$P(X = -1) = F_X(-1) - F_X(-1^-) \Rightarrow \frac{1}{4} = -b + c - \frac{1}{4} \Rightarrow c = b + \frac{1}{2}$$

$$P(X = 1) = F_X(1) - F_X(1^-) \Rightarrow \frac{1}{4} = 1 - b - c \Rightarrow c = \frac{3}{4} - b$$

$$b + \frac{1}{2} = \frac{3}{4} - b \Rightarrow 2b = \frac{1}{4} \Rightarrow b = \frac{1}{8}$$

$$c = \frac{1}{8} + \frac{1}{2} = \frac{5}{8}$$

$$a = \frac{1}{4}$$



$$P(-2 \leq X < 1) = F_X(1^-) - F_X(-2^-) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

$$1 \times K_1$$

$$2 \times K_2$$

$$3 \times K_3$$

$$P(O) = \frac{1}{2}$$

Tyle naciśnięć monet, ile cyfr na kuli

$$(O_k | C_n) \sim B(n, \frac{1}{2})$$

$$P(O_k | C_n) = \binom{n}{k} p^k (1-p)^{n-k}$$

$C_i$  - wylosowany kulej oznaczony cyfrą  $i$

$A$  - wypadnięcie co najmniej 2 orły = 2 lub 3

$$P(A) = P(O_2) + P(O_3)$$

$$P(O_2) = P(O_2 | C_1) \cdot P(C_1) + P(O_2 | C_2) \cdot P(C_2) + P(O_2 | C_3) \cdot P(C_3)$$

$$P(O_2) = 0 + \binom{2}{2} \left(\frac{1}{2}\right)^2 \left(\frac{2}{3}\right)^0 \cdot \frac{2}{6} + \binom{3}{2} \left(\frac{1}{2}\right)^2 \left(\frac{2}{3}\right)^1 \cdot \frac{3}{6} = \frac{1}{3} \cdot \frac{2}{6} + 3 \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{2} = \frac{1}{27} + \frac{1}{9} = \frac{4}{27}$$

$$P(O_3) = P(O_3 | C_1) \cdot P(C_1) + P(O_3 | C_2) \cdot P(C_2) + P(O_3 | C_3) \cdot P(C_3)$$

$$P(O_3) = 0 + 0 + \binom{3}{3} \left(\frac{1}{2}\right)^3 \left(\frac{2}{3}\right)^0 \cdot \frac{3}{6} = \frac{1}{27} \cdot \frac{1}{2} = \frac{1}{54}$$

$$P(A) = \frac{4}{27} + \frac{1}{54} = \frac{9}{54} = \frac{1}{6}$$

$B$  - wypadnięcie dokładnie 1 orła =  $O_2$

$$P(C_3 | B) = \frac{P(C_3 \cap B)}{P(B)} = \frac{P(B | C_3) \cdot P(C_3)}{P(B)}$$

$$P(B | C_3) \cdot P(C_3) = \binom{3}{1} \left(\frac{2}{3}\right)^1 \left(\frac{1}{3}\right)^2 \cdot \frac{3}{6} = 3 \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{3}$$

$$P(B) = P(O_2) = \frac{4}{27}$$

$$P(C_3 | B) = \frac{\frac{1}{3}}{\frac{4}{27}} = \frac{3}{4}$$

normal distribution  $a, b \in \mathbb{R}$

$$f_X(x) = \begin{cases} a|x| & -1 \leq x \leq 2 \\ b x & 2 \leq x \leq 4 \\ 0 & \text{v. f.} \end{cases} = \begin{cases} -a x & -1 \leq x < 0 \\ a x & 0 \leq x \leq 2 \\ b x & 2 \leq x \leq 4 \\ 0 & \text{v. f.} \end{cases}$$

$$P(X > 2) = 0.7 \Rightarrow 0.7 = \int_2^{\infty} f_X(x) dx = \int_2^4 b x dx = b \cdot \left[ \frac{1}{2} x^2 \right]_2^4 = b(8 - 4) = 3.5b \Rightarrow b = 0.2$$

$$1 = \int_{-\infty}^{\infty} f_X(x) dx = \int_{-1}^0 -a x dx + \int_0^2 a x dx + \int_2^4 0.2 x dx$$

$$1 = -a \cdot \left[ \frac{1}{2} x^2 \right]_{-1}^0 + a \cdot \left[ \frac{1}{2} x^2 \right]_0^2 + 0.7$$

$$0.3 = -a \cdot (0 - \frac{1}{2}) + a \cdot 2$$

$$0.3 = 2.5a$$

$$a = 0.12$$

$$F_X(t) = \int_{-\infty}^t f_X(x) dx = \begin{cases} 0 & t < -1 \\ \int_{-1}^t -0.12 x dx = -0.12 \cdot \left[ \frac{1}{2} x^2 \right]_{-1}^t = -0.12 \left( \frac{t^2}{2} - \frac{1}{2} \right) & -1 \leq t < 0 \\ \int_{-1}^0 -0.12 x dx + \int_0^t 0.12 x dx = 0.06 + 0.06 t^2 & 0 \leq t \leq 2 \\ \int_{-1}^0 -0.12 x dx + \int_0^2 0.12 x dx = 0.3 & 2 < t < 3 \\ 0.3 + \int_2^t 0.2 x dx = 0.3 + 0.2 \left( \frac{1}{2} t^2 - \frac{3}{2} \right) = 0.3 + 0.1(t^2 - 3) & 3 \leq t \leq 4 \\ 1 & t > 4 \end{cases}$$

$$P(|X-1| > 1) = P(X-1 > 1 \vee X-1 < -1) = P(X > 2 \vee X < 0) = P(X < 0) + P(X > 2)$$

$$= P(X < 0) + 1 - P(X \leq 2)$$

$$= F_X(0) + 1 - F_X(2)$$

$$= 0.06 + 1 - 0.3 = 0.76$$

14.

$$\Omega = \{-1, 0, 1, 2, 3\}$$

$$P(\{1\}) = \frac{1}{3} \quad P(\{2\}) = \frac{1}{6} \quad \text{dla } \omega \in \{-1, 0, 2, 3\}$$

$\omega$	-1	0	1	2	3
X	2	1	0	1	2
Y	0	1	2	1	0

$$X(\omega) = |\omega - 1| \quad Y(\omega) = 2 - |\omega - 1|$$

$$S_X = \{2, 1, 0\}$$

$$S_Y = \{0, 1, 2\}$$

$$P(X=0) = P(\{1\}) = \frac{1}{3}$$

$$P(Y=0) = P(\{-1\}) + P(\{3\}) = \frac{1}{3}$$

$$P(X=1) = P(\{0\}) + P(\{2\}) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$P(X=1) = P(\{0\}) + P(\{2\}) = \frac{1}{3}$$

$$P(X=2) = P(\{-1\}) + P(\{3\}) = \frac{1}{3}$$

$$P(X=2) = P(\{1\}) = \frac{1}{3}$$

Mają taki sam rozkład

$$P(\{\omega \in \Omega : X(\omega) = Y(\omega)\}) = P(\{0, 2\}) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$$

$$V(3X+10) = 9VX$$

$$VX = EX^2 - (EX)^2$$

$$EX^2 = 0^2 \cdot \frac{1}{3} + 1^2 \cdot \frac{1}{3} + 2^2 \cdot \frac{1}{3} = \frac{1}{3} + \frac{4}{3} = \frac{5}{3}$$

$$EX = 0 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3} = \frac{1}{3} + \frac{2}{3} = 1$$

$$VX = \frac{5}{3} - 1^2 = \frac{2}{3}$$

$$V(3X+10) = 9 \cdot \frac{2}{3} = 6$$

15.

$$\Omega = \{2, N, B\}$$

$$P(\{2\}) = \frac{2}{4}$$

$$P(\{N\}) = P(\{B\}) = \frac{1}{4}$$

$X$  - Anzahl nichtestrichen Karten  $\hookrightarrow$  3 rufen

$$X \sim B(n=3, p=\frac{1}{4})$$

$$S_X = \{0, 1, 2, 3\}$$

$$P(X=k) = \binom{3}{k} \left(\frac{1}{4}\right)^k \left(\frac{3}{4}\right)^{3-k}$$

$$P(X=0) = \frac{27}{64}$$

$$P(X=1) = 3 \cdot \frac{1}{4} \cdot \frac{9}{16} = \frac{27}{64}$$

$$P(X=2) = 3 \cdot \frac{1}{16} \cdot \frac{3}{4} = \frac{9}{64}$$

$$P(X=3) = \frac{1}{64}$$

$$V(4X - 500) = 16 V_X = 16 (EX^2 - (EX)^2)$$

$$EX = 0 \cdot \frac{27}{64} + 1 \cdot \frac{27}{64} + 2 \cdot \frac{9}{64} + 3 \cdot \frac{1}{64} = \frac{27+18+3}{64} = \frac{48}{64} = \frac{3}{4}$$

$$EX^2 = 0 \cdot \frac{27}{64} + 1 \cdot \frac{27}{64} + 4 \cdot \frac{9}{64} + 9 \cdot \frac{1}{64} = \frac{27+36+9}{64} = \frac{72}{64} = \frac{9}{8}$$

$$V(4X - 500) = 16 \cdot \left( \frac{9}{8} - \frac{9}{16} \right) = 16 \cdot \frac{9}{16} = 9$$

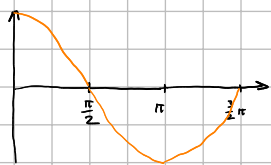
$$\sigma_{4X-500} = \sqrt{9} = 3$$

\*

$[-1, 1]$  - wybieramy 2 punkty  $a, b$

$X$ : liczba rozwiązań  $\cos t = a + b$  dla  $t \in [0, \frac{3}{2}\pi]$

znaleźć rozkład  $X$

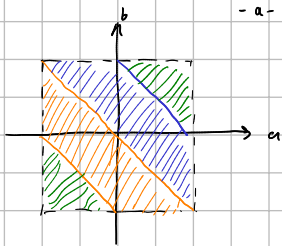


$$X = \begin{cases} 0 & \text{p.p.} \\ 1 & a+b \in (0, 1] \cup \{-1\} \\ 2 & a+b \in (-1, 0) \end{cases}$$

$$\Omega = [-1, 1]^2 \quad |\Omega| = 4$$

$$P(X=2) = P(\{(a,b) \in \Omega : -1 < a+b < 0\})$$

$$-a-1 < b < -a$$

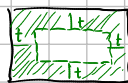


$$P(X=1) = P(\{(a,b) \in \Omega : 0 < a+b < 1\})$$

\*\*

Prostokąt  $2 \times 4$ , losujemy punkt o środku

$T$  - odległość punktu od najbliższego boku



$$P(T \leq t) = \frac{8 - (4-2t)(2-2t)}{8}$$

\*\* \*

Autobusy  
A: 10-11  
B: 10-12

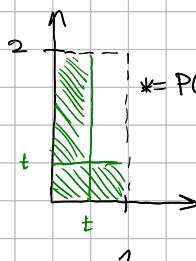
$T$  czas oczekiwania na autobus

$$\Omega = \{(a,b) \in [0,1] \times [0,2]\}$$

$$T(a,b) = \min(a,b) \in [0,1]$$

$$F_T(t) = \begin{cases} 0 & t < 0 \\ * & t \in [0,1] \\ 1 & t > 1 \end{cases}$$

$$T \leq t = \min(a,b) \leq t$$



$$* = P(T \leq t) = \frac{2 - (2-t)(1-t)}{2}$$



1.

 $2 \times B, 3 \times C$  $X$  - liczba czarnych kul spośród 3 wyjętych bez zwracania $P$  - losowanie

$$S_X = \{1, 2, 3\}$$

$$\binom{5}{3} = \frac{5!}{2!3!} = \frac{5 \cdot 4}{2} = 10$$

$$P(X=1) = \frac{\binom{2}{2}\binom{3}{1}}{\binom{5}{3}} = \frac{3}{10}$$

$$P(X=2) = \frac{\binom{2}{1}\binom{3}{2}}{\binom{5}{3}} = \frac{2 \cdot 3}{10} = \frac{6}{10}$$

$$P(X=3) = \frac{\binom{2}{0}\binom{3}{3}}{\binom{5}{3}} = \frac{1}{10}$$

 $Y$  - ile razy wypadły 4 oczka w  $n$  rzutach kostką sześcienną

$$P(X=1 | Y=1) = \frac{P(Y=1 | X=1) P(X=1)}{P(Y=1)}$$

$$P(Y=1) = P(Y=1 | X=1)P(X=1) + P(Y=1 | X=2)P(X=2) + P(Y=1 | X=3)P(X=3)$$

$$= \binom{1}{1} \left(\frac{1}{6}\right)^1 \left(\frac{3}{4}\right)^0 \cdot \frac{3}{10} + \binom{2}{1} \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^1 \cdot \frac{6}{10} + \binom{3}{1} \left(\frac{1}{2}\right)^1 \left(\frac{3}{4}\right)^2 \cdot \frac{1}{10}$$

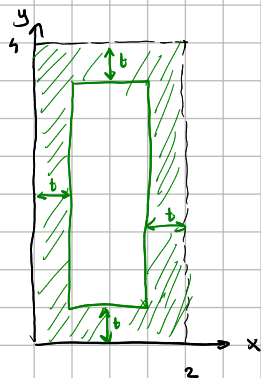
$$= \frac{1}{4} \cdot \frac{3}{10} + 2 \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{6}{10} + 3 \cdot \frac{1}{4} \cdot \frac{9}{16} \cdot \frac{1}{10}$$

$$= \frac{3}{40} + \frac{9}{40} + \frac{27}{640} = \frac{48 + 144 + 27}{640} = \frac{219}{640}$$

$$P(X=1 | Y=1) = \frac{3}{40} \cdot \frac{640}{219} = \frac{48}{219}$$

2.

$$\Omega = \{(x, y) \in [0, 2] \times [0, 1]\}$$

 $T$  - odlegość punktu od najbliższego boku prostokąta

$$F_T(t) = P(T \leq t) = \begin{cases} 0 & t < 0 \\ \frac{8 - (2-2t)(4-2t)}{8} & 0 \leq t \leq 1 \\ 1 & t > 1 \end{cases}$$

 $P$  - geometryczne

$$f_T(t) = \frac{d}{dt} F_T(t) = \begin{cases} \frac{3}{2} - t & t \in [0, 1] \\ 0 & \text{w p.p.} \end{cases}$$

$$\frac{1}{8} (8 - (8 - 4t - 8t + 4t^2)) = \frac{1}{8} (8 - 8 + 12t - 4t^2)$$

$$= \frac{1}{8} (12t - 4t^2) = \frac{1}{2} (3t - t^2)$$

$$\frac{d}{dt} = \frac{1}{2} \cdot (3 - 2t) = \frac{3}{2} - t$$

$$0 \leq t \leq 1$$

$$ET = \int_{-\infty}^{+\infty} t f_T(t) dt = \int_0^1 t \left(\frac{3}{2} - t\right) dt = \int_0^1 -t^2 + \frac{3}{2}t dt = \left[-\frac{1}{3}t^3 + \frac{3}{4}t^2\right]_0^1 = -\frac{1}{3} + \frac{3}{4} = \frac{5}{12}$$

3.

 $T = \text{temperatura}$ 

$$T \sim N(20, \sigma^2)$$

$$P(T > 30) = 15\% \Rightarrow P(T \leq 30) = F_T(30) = 85\%$$

$$P(T < 15) = ?$$

$$F_T(a) = \Phi\left(\frac{a - \mu}{\sigma}\right)$$

$$85\% = \Phi\left(\frac{30 - 20}{\sigma}\right) = \Phi\left(\frac{10}{\sigma}\right)$$

$$\Phi(1.04) \approx 0.85 \Rightarrow \frac{10}{\sigma} \approx 1.04 \Rightarrow -\frac{5}{\sigma} \approx -0.52$$

$$P(T < 15) = F_T(15) = \Phi\left(\frac{15 - 20}{\sigma}\right) = \Phi\left(-\frac{5}{\sigma}\right) \approx \Phi(-0.52) \\ = 1 - \Phi(0.52) \approx 30\%$$

4.

$X \sim \text{verteilt}$  distribution  $\Rightarrow F_x$  ma postiv "schallend"  
 $\Rightarrow a = c = 0$

$$F_x = \begin{cases} 0 & x < 0 \\ b & x \in [0, 1) \\ \frac{3}{4} & x \in [1, 2) \\ d & x \in [2, 3) \\ 1 & x \geq 3 \end{cases}$$

$$EX = 1$$

$$\hookrightarrow P(X=0) = P(X < 3)$$

$$P(X=0) = F_x(0) - F_x(0^-) = b$$

$$P(X < 3) = F_x(3^-) = d$$

$$\hookrightarrow b = d$$

$$S_x = \{0, 1, 2, 3\}$$

i	0	1	2	3
$p_i$	b	$\frac{3}{4} - b$	$4b - \frac{3}{4}$	$1 - 4b$

$$1 = EX = 0 \cdot b + 1 \cdot \left(\frac{3}{4} - b\right) + 2 \cdot \left(4b - \frac{3}{4}\right) + 3 \cdot (1 - 4b)$$

$$1 = \frac{3}{4} - b + 8b - \frac{3}{2} + 3 - 12b$$

$$5b = \frac{5}{4} \quad b = \frac{1}{4}$$

$$V(3 - 4X) = (-4)^2 VX = 16 VX$$

$$EX^2 = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 4 \cdot \frac{1}{4} + 9 \cdot 0 = \frac{1}{2} + 1 = \frac{3}{2}$$

$$V(3 - 4X) = 16 (EX^2 - (EX)^2) = 16 \cdot \left(\frac{3}{2} - 1\right) = 8$$

1.

Uрна 1  
3x B  
1x C

Uрна 2  
2x B  
3x C

$C_1$  - przetworzona czarna kula  
 $C_1'$  - przetworzona biała kula

$X$  - liczba czarnych kul wylosowanych z 2 urny

$$\begin{aligned} P(X=2) &= P(X=2|C_1)P(C_1) + P(X=2|C_1')P(C_1') \\ &= \binom{3}{2} \left(\frac{1}{6}\right)^2 \left(\frac{2}{6}\right)^1 \cdot \frac{1}{4} + \binom{3}{2} \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^1 \cdot \frac{3}{4} \\ &= 3 \cdot \frac{2}{9} \cdot \frac{1}{3} \cdot \frac{1}{4} + 3 \cdot \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{3}{4} \\ &= \frac{1}{9} + \frac{9}{32} \end{aligned}$$

$$P(C_1 | X=2) = \frac{P(X=2|C_1)P(C_1)}{P(X=2)} = \frac{\frac{1}{9}}{\frac{1}{9} + \frac{9}{32}} \approx 0.28$$

2.

$$\Omega = \{(a,b) : a, b \in [-1, 1]\}$$

$X$  - liczba dodatnich pierwiastków równania

$$x^2 - (2a+b)x + a(a+b) = 0$$

$$\Delta = (2a+b)^2 - 4a(a+b) = 4a^2 + 4ab + b^2 - 4a^2 - 4ab = b^2$$

$$X=2 \Leftrightarrow \Delta > 0 \wedge x_1 + x_2 > 0 \wedge x_1 x_2 > 0$$

$$x_1 + x_2 = 2a + b$$

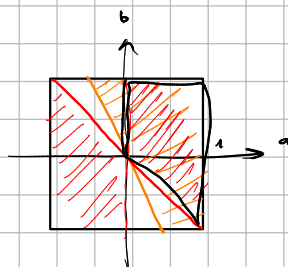
$$X=1 \Leftrightarrow \begin{cases} \Delta > 0 \wedge x_1 x_2 < 0 \\ \vee (\Delta = 0 \wedge x_0 > 0) \end{cases}$$

$$x_1 x_2 = a(a+b)$$

$$x_0 = \frac{2a+b}{2}$$

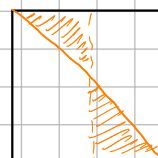
$$S_X = \{0, 1, 2\}$$

$$X=2 \begin{cases} b^2 > 0 \Leftrightarrow b \neq 0 \\ 2a+b > 0 \Leftrightarrow b > -2a \\ a(a+b) > 0 \Leftrightarrow (a > 0 \wedge b > -a) \vee (a < 0 \wedge b < -a) \end{cases}$$



$$P(X=2) = \frac{\frac{3}{2}}{4} = \frac{3}{8}$$

$$X=1 \begin{cases} b^2 > 0 \\ a(a+b) < 0 \end{cases} \Leftrightarrow \begin{cases} b \neq 0 \\ (a < 0 \wedge b > -a) \vee (a > 0 \wedge b < -a) \end{cases}$$



$$\begin{cases} b^2 = 0 \\ a + \frac{1}{2}b < 0 \end{cases} \Leftrightarrow \begin{cases} b = 0 \\ b < -2a \end{cases}$$

$$P(X=1) = \frac{1}{4} + 0 = \frac{1}{4}$$

$$P(X=0) = 1 - \frac{3}{8} - \frac{1}{4} = \frac{3}{8}$$

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{3}{8} & x \in [0, 1) \\ \frac{5}{8} & x \in [1, 2) \\ 1 & x \geq 2 \end{cases}$$

$$EX = 0 \cdot \frac{3}{8} + 1 \cdot \frac{1}{4} + 2 \cdot \frac{3}{8} = 1$$

3.

T - czas przejazdu

$$T \sim N(50, \sigma^2)$$

$$P(T \leq 60) = 0.9 = F_T(60) = \Phi\left(\frac{60-50}{\sigma}\right) = \Phi\left(\frac{10}{\sigma}\right)$$

$$\Phi(1.28) \approx 0.9 \Rightarrow \frac{10}{\sigma} \approx 1.28 \Rightarrow -\frac{\Sigma}{\sigma} \approx -0.64$$

$$P(T > 55) = 1 - P(T \leq 55) = 1 - F_T(55) = 1 - \Phi\left(\frac{55-50}{\sigma}\right) \approx 1 - \Phi(-0.64) \\ = 1 - (1 - \Phi(0.64)) \approx 74\%$$

4.

X ~ rozkład ciągły

$$F(x) = \begin{cases} 0 & x < 0 \\ ax^2 + b & x \in [0, 1) \\ \frac{3}{4} & x \in [1, 2) \\ cx + d & x \in [2, 3) \\ 1 & x \geq 3 \end{cases}$$

F jest ciągła

$$F(0) = F(0^-) \Rightarrow 0 = a \cdot 0 + b \Rightarrow b = 0$$

$$F(1) = F(1^-) \Rightarrow \frac{3}{4} = a \cdot 1 + 0 \Rightarrow a = \frac{3}{4}$$

$$\begin{aligned} F(2) = F(2^-) = 2c + d = \frac{3}{4} \\ F(3) = F(3^-) = 1 = 3c + d \end{aligned} \quad \left. \vphantom{\begin{aligned} F(2) = F(2^-) = 2c + d = \frac{3}{4} \\ F(3) = F(3^-) = 1 = 3c + d \end{aligned}} \right\} \begin{aligned} c &= \frac{1}{4} \\ d &= \frac{1}{4} \end{aligned}$$

$$F(x) = \begin{cases} 0 \\ \frac{3}{4}x^2 \\ \frac{3}{4} \\ \frac{1}{4}x + \frac{1}{4} \\ 1 \end{cases} \quad \text{---} \quad f(x) = \frac{d}{dx} F = \begin{cases} 0 & \text{w p.p.} \\ \frac{3}{2}x & x \in [0, 1) \\ \frac{1}{4} & x \in [2, 3) \end{cases}$$

$$EX = \int_{-\infty}^{+\infty} x f(x) dx = \int_0^1 \frac{3}{2}x^2 dx + \int_2^3 \frac{1}{4}x dx = \frac{1}{2} [x^3]_0^1 + \frac{1}{8} [x^2]_2^3 = \frac{1}{2} + 1 - \frac{1}{2} = 1$$

$$P(|X - 1.5| > 0.5) = P(X - 1.5 > 0.5 \vee X - 1.5 < -0.5) = P(X > 2 \vee X < 1) = P(X > 2) + P(X < 1) = 1 - F(2) + F(1) \\ = 1 - \frac{3}{4} + \frac{3}{4} = 1$$

Prob T 1

2.  $\Omega = \{(p, q) : p, q \in [0, 1]\} \quad |\Omega| = 1$

P - geometrisch

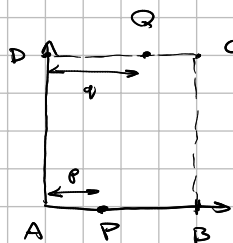
X - pole APQD

$$X(p, q) = \frac{1}{2}(p+q) \in [0, 1]$$

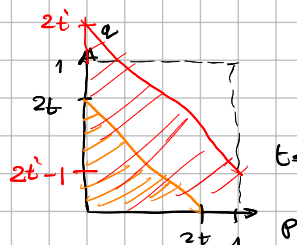
$$F_X(t) = \begin{cases} 0 & t < 0 \\ 2t^2 & t \in [0, \frac{1}{2}] \\ -2t^2 + 4t - 1 & t \in (\frac{1}{2}, 1] \\ 1 & t > 1 \end{cases}$$

$$f_X(t) = \frac{dF_X}{dt}$$

$$f_X(t) = \begin{cases} 4t & t \in [0, \frac{1}{2}] \\ -4t + 4 & t \in (\frac{1}{2}, 1] \\ 0 & \text{sonst} \end{cases}$$



$$P = (p, 0) \\ Q = (q, 1)$$



$$\frac{1}{2}p + \frac{1}{2}q \leq t \\ p + q \leq 2t \\ q \leq -p + 2t$$

$$t \leq \frac{1}{2} : \quad P(X \leq t) = \frac{1}{2} \cdot (2t)^2 = 2t^2$$

$$\begin{aligned} t > \frac{1}{2} : \quad P(X \leq t) &= 1 - \frac{1}{2} (1 - (2t - 1))^2 \\ &= 1 - \frac{1}{2} (2 - 2t)^2 \\ &= 1 - \frac{1}{2} (4 - 8t + 4t^2) \\ &= 1 - 2 + 4t - 2t^2 \\ &= -2t^2 + 4t - 1 \end{aligned}$$

$$-2 \cdot \frac{1}{4} + 4 \cdot \frac{1}{2} - 1 = -\frac{1}{2} + 2 - 1 = \frac{1}{2}$$