1.

$$A=\begin{bmatrix}4&-3&6\\4&-1&4\\-1&2&-3\end{bmatrix} \qquad \chi(\lambda)=\begin{vmatrix}4-\lambda&-3&6\\4&-1-\lambda&4\\-1&2&-3-\lambda\end{vmatrix}$$

$= (4-\lambda)(-1-\lambda)(-3-\lambda) + 12 + 48 + 6(-1-\lambda) - 8(4-\lambda) + 12(-3-\lambda)$

$= (4-\lambda)(3+\lambda+3\lambda+\lambda^2) + 60 - 6 - 6\lambda - 32 + 8\lambda - 36 - 12\lambda$

$= 12 + 16\lambda + \cancel{4\lambda^2} - 3\lambda - \cancel{4\lambda^2} - \lambda^3 + 60 - 6 - 6\lambda - 32 + 8\lambda - 36 - 12\lambda$

$= -\lambda^3 + 3\lambda - 2 = (\lambda-1)(-\lambda^2-\lambda+2)$

$= -(\lambda-1)^2(\lambda+2)$

	-1	0	3	-2
1		-1	-1	2
	-1	-1	2	0

$-\lambda^2 - 2\lambda + \lambda + 2$
$= -\lambda(\lambda+2) + (\lambda+2)$
$= (-\lambda+1)(\lambda+2)$

$\lambda_1 = 1 \quad \lambda_2 = -2$

- $\lambda_1 = 1$
  $Av = 1\cdot v$
  $Av = Iv$
  $(A-I)v = 0$

$$\left[\begin{array}{ccc|c}3&-3&6&0\\4&-2&4&0\\-1&2&-4&0\end{array}\right] \xrightarrow{\substack{v_1+3v_3\\ v_2+4v_3}} \begin{bmatrix}0&3&-6\\0&6&-12\\-1&2&-4\end{bmatrix} \to \begin{bmatrix}1&-2&4\\0&1&-2\end{bmatrix} \xrightarrow{v_1+2v_2} \begin{bmatrix}1&0&0\\0&1&-2\end{bmatrix}$$

$\uparrow \ \uparrow \ t$

$$\begin{bmatrix}x\\y\\z\end{bmatrix} = t\begin{bmatrix}0\\2\\1\end{bmatrix} \qquad V_1 = \text{span}((0,2,1))$$

- $\lambda_2 = -2$
  $(A+2I)v = 0$

$$\begin{bmatrix}6&-3&6\\4&1&4\\-1&2&-1\end{bmatrix} \xrightarrow{\substack{+6v_3\\ +4v_3}} \begin{bmatrix}0&9&0\\0&9&0\\-1&2&-1\end{bmatrix} \to \begin{bmatrix}1&-2&1\\0&1&0\end{bmatrix} \to \begin{bmatrix}1&0&1\\0&1&0\end{bmatrix} \qquad \begin{bmatrix}x\\y\\z\end{bmatrix} = t\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

$\uparrow \ \uparrow \ t$

$V_2 = \text{span}((-1,0,1))$

1*

$$A=\begin{bmatrix}a&b\\c&d\end{bmatrix} \qquad B=\begin{bmatrix}2d-3&2c\\2b&2a-3\end{bmatrix}$$

$\chi_A(t) = (a-t)(d-t) - bc$
$= ad - at - dt + t^2 - bc$
$= t^2 - (a+d)t + ad - bc$
$= t^2 - 4t + 3$

$\chi_B(t) = (2d-3-t)(2a-3-t) - 2b\cdot 2c$
$= 4ad - 6d - 2dt - 6a + 9 + 3t - 2at + 3t + t^2 - 4bc$
$= t^2 + (6-2a-2d)t + 9 + 4(ad-bc) - 6a - 6d$
$= t^2 + (6-2(a+d))t + 4(ad-bc) - 6(a+d) + 9$
$= t^2 + (6-8)t + 4\cdot 3 - 6\cdot 4 + 9$
$= t^2 - 2t - 3$

$Av = \lambda v$
$Av - \lambda v = 0$
$(A - I\lambda)v = 0 \iff \chi_A(\lambda) = 0 \iff \det(A - \lambda I) = 0$

$$\begin{vmatrix}a&b\\c&d\end{vmatrix} = \begin{vmatrix}d&c\\b&a\end{vmatrix} = ad - bc$$

$\alpha Av = tv$
$\alpha Av - tv = 0$
$(\alpha A - tI)v = 0 \iff \chi_{\alpha A}(t) = 0 \iff \det(\alpha A - tI) = 0$
dla $t = \alpha\lambda$
$\chi_A(t) = \det(\alpha A - \alpha\lambda I) = \alpha \det(A - \lambda I) = 0$
$\alpha\lambda$ jest ww αA

$(A + nI)v = tv$
$(A + nI - tI)v = 0$
$\chi_{A+nI}(t) = \det(A - I(t-n))$
dla $t = \lambda + n$
$\chi_{A+nI}(t) = \det(A - I(\lambda+n-n)) = \det(A - \lambda I) = 0$
$\lambda + n$ jest ww $A + nI$

$\chi_A(t) = t^2 - 4t + 3 = t^2 - 3t - t + 3 = (t-3)(t-1)$

$A' = 2A - 3I \qquad 3 \to 2\cdot 3 - 3 = 3$
$\qquad\qquad\qquad\quad 1 \to 2\cdot 1 - 3 = -1$

$\chi_{A'}(t) = (t-3)(t+1) = t^2 - 3t + t - 3 = t^2 - 2t - 3 = \chi_B(t)$

2.

$A = (-1, 2, 2)$
$B = (-3, -1, 1)$
$C = (1, 0, 2)$

a)

$$\vec{AB} = \begin{bmatrix} -3+1 \\ -1-2 \\ 1-2 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ -1 \end{bmatrix} \qquad \vec{AC} = \begin{bmatrix} 1+1 \\ 0-2 \\ 2-2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$$

$$n_{ABC} = \vec{AB} \times \vec{AC} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & -3 & -1 \\ 2 & -2 & 0 \end{bmatrix} = \hat{\imath} \cdot (-2) - \hat{\jmath} \cdot 2 + \hat{k} \cdot 10 = \begin{bmatrix} -2 \\ -2 \\ 10 \end{bmatrix}$$

Płaszczyzna ABC:

$$(x - A) \cdot n_{ABC} = \begin{bmatrix} x+1 \\ y-2 \\ z-2 \end{bmatrix} \circ \begin{bmatrix} -2 \\ -2 \\ 10 \end{bmatrix} = -2x - 2 - 2y + 4 + 10z - 20 = 0$$

$$-2x - 2y + 10z = 18$$
$$x + y - 5z = -9$$

Płaszczyzna P: $x = 0$

$n_P = (1, 0, 0)$

$$\sin\sphericalangle(P, ABC) = \sin\sphericalangle(n_{ABC}, n_P) = \frac{|n_{ABC} \times n_P|}{|n_{ABC}| \cdot |n_P|} = \frac{\left\|\begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & -2 & 10 \\ 1 & 0 & 0 \end{matrix}\right\|}{\sqrt{4+4+100} \cdot \sqrt{1}} = \frac{|\hat{\imath} \cdot 0 - \hat{\jmath} \cdot (-10) + \hat{k} \cdot 2|}{\sqrt{108}}$$

$$= \frac{\sqrt{0 + 100 + 4}}{\sqrt{108}} = \frac{\sqrt{104}}{\sqrt{108}}$$

b) $$|\triangle ABC| = \frac{1}{2} \left|\vec{AB} \times \vec{AC}\right| = \frac{1}{2}\sqrt{4+4+100} = \frac{\sqrt{108}}{2} = \sqrt{27} = 3\sqrt{3}$$

c)

$$|OABC| = \frac{1}{6}\left|(OA \times OB) \cdot OC\right| = \frac{1}{6}\left\|\begin{matrix} 1 & 0 & 2 \\ -1 & 2 & 2 \\ -3 & -1 & 1 \end{matrix}\right\| = \frac{1}{6}|1 \cdot 4 - 0 + 2 \cdot 7| = \frac{18}{6} = 3$$

3. $A = (-1, 1, -3)$
$B = (3, -1, 1)$
$C = (-2, 2, -1)$
$D = (2, -1, 1)$

odległość AB do CD

$$\text{wysokość} = \frac{\text{objętość}}{\text{pole podstawy}}$$

$$AC = \begin{bmatrix} -2+1 \\ 2-1 \\ -1+3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \qquad AB = \begin{bmatrix} 3+1 \\ -1-1 \\ 1+3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix} \qquad CD = \begin{bmatrix} 2+2 \\ -1-2 \\ 1+1 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$$

dla równoległościanu wyznaczonego przez AB, CD i AC

$$\frac{|(AB \times CD) \circ AC|}{|AB \times CD|} = \frac{\left\|\begin{matrix} -1 & 1 & 2 \\ 4 & -2 & 4 \\ 4 & -3 & 2 \end{matrix}\right\|}{\left\|\begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & 4 \\ 4 & -3 & 2 \end{matrix}\right\|} = \frac{|-1 \cdot 8 - 1 \cdot (-8) + 2 \cdot (-4)|}{|\hat{\imath} \cdot 8 - \hat{\jmath} \cdot (-8) + \hat{k} \cdot (-4)|} = \frac{8}{\sqrt{64+64+16}} = \frac{8}{12} = \frac{2}{3}$$

1.

N U D N O O T U

a) $N = \binom{8}{2,\,2,\,2,\,1,\,1} = \frac{8!}{2!\,2!\,2!} = 7! = 5040$

b) 4 samogłoski V
4 spółgłoski C trudno wymówić kiedy jest zbitka 3 lub 4

C C C V V C V V

$$N_2 = \underset{\text{zbitka } 3\times C}{6} \cdot \underset{\text{czwarte } C}{\binom{5}{1}} \cdot \underset{\text{samogłoski}}{\binom{4}{4}} \cdot \underset{U\ O}{\binom{4}{2,\,2}} \cdot \underset{N\ D\ T}{\binom{4}{2,\,1,\,1}} = 6 \cdot 5 \cdot 1 \cdot \frac{4!}{2!\,2!} \cdot \frac{4!}{2!} = 30 \cdot 6 \cdot 12 = 2160$$

$N_D = N - N_2 = 5040 - 2160 = 2880$

3.1

N N U D N O T U

a) $= \binom{8}{\underset{N}{3},\,\underset{U}{2},\,\underset{D}{1},\,\underset{O}{1},\,\underset{T}{1}} = \frac{8!}{3!\,2!} = 3360$

trudno wymówić jest przynajmniej zbitka 3

C V C C C V C V

$$N = \underset{\text{zbitka } 3\times C}{\binom{6}{1}} \cdot \underset{\text{pozostałe } 2\times C \quad 3\times V}{\binom{5}{2,\,3}} \cdot \underset{N\ D\ T}{\binom{5}{3,\,1,\,1}} \cdot \underset{U\ O}{\binom{3}{2,\,1}} = 6 \cdot \frac{5!}{2!\,3!} \cdot \frac{5!}{3!} \cdot \frac{3!}{2!}$$

W miejsca wstawiam grupy spółgłosek

_ V _ V _ V _

grupy C	rozmieszczenie
2, 2, 1	$\rightarrow \binom{5}{3}$
2, 1, 1, 1	$\binom{5}{4}$
1, 1, 1, 1, 1	$\binom{5}{5}$

$$N_d = \left[\binom{5}{3} + \binom{5}{4} + \binom{5}{5}\right] \cdot \underset{N\ D\ T}{\binom{5}{3,\,1,\,1}} \cdot \underset{U\ O}{\binom{3}{2,\,1}} = 960$$