1.

$$\int e^{4x} \sin(2x)\, dx = \begin{vmatrix} f = e^{4x} & g' = \sin(2x) \\ f' = 4e^{4x} & g = -\frac{1}{2}\cos(2x) \end{vmatrix} = e^{4x} \cdot -\frac{1}{2}\cos(2x) - \int 4 \cdot e^{4x} \cdot -\frac{1}{2}\cos(2x)\, dx =$$

$$= -\frac{1}{2} e^{4x}\cos(2x) + 2\int e^{4x}\cos(2x)\, dx = \begin{vmatrix} f = e^{4x} & g' = \cos(2x) \\ f' = 4e^{4x} & g = \frac{1}{2}\sin(2x) \end{vmatrix} = -\frac{1}{2}e^{4x}\cos(2x) + e^{4x}\sin(2x) - 4\int e^{4x}\sin(2x)\, dx$$

$$\int e^{4x}\sin(2x)\, dx = \frac{1}{2}e^{4x}\left(2\sin(2x) - \cos(2x)\right) - 4\int e^{4x}\sin(2x)\, dx$$

$$5\int e^{4x}\sin(2x)\, dx = \frac{1}{2}e^{4x}\left(2\sin(2x) - \cos(2x)\right) + C$$

$$\int e^{4x}\sin(2x)\, dx = \frac{1}{10}e^{4x}\left(2\sin(2x) - \cos(2x)\right) + C$$

2.

$$\int \frac{-2}{2 + \sqrt{3x}}\, dx = \begin{vmatrix} t = \sqrt{3x} \\ dt = \frac{1}{2\sqrt{3x}}dx \\ dx = 2t\, dt \end{vmatrix} = \int \frac{-4t}{\sqrt{3}\,t + 2}\, dt = \int \frac{-4t - \frac{8}{\sqrt{3}} + \frac{8}{\sqrt{3}}}{\sqrt{3}\,t + 2}\, dt = \int \frac{-\frac{4}{\sqrt{3}}(\sqrt{3}\,t + 2) + \frac{8}{\sqrt{3}}}{\sqrt{3}\,t + 2}\, dt = \int -\frac{4}{\sqrt{3}} + \frac{\frac{8}{\sqrt{3}}}{\sqrt{3}\,t + 2}\, dt$$

$$= -\int \frac{4\sqrt{3}}{3}\, dt + \frac{8}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}\int \frac{dt}{t + \frac{2}{\sqrt{3}}} = -\frac{4\sqrt{3}}{3}t + \frac{8}{3}\ln\left|t + \frac{2}{\sqrt{3}}\right| + C = -\frac{4\sqrt{3}}{3}\sqrt{x} + \frac{8}{3}\ln\left|\sqrt{x} + \frac{2\sqrt{3}}{3}\right| + C$$

$$= -\frac{4\sqrt{3}}{3}\sqrt{x} + \frac{8}{3}\ln\left|\sqrt{x} + \frac{2}{\sqrt{3}}\right| + \frac{8}{3}\ln|\sqrt{3}| + C = -\frac{4\sqrt{3}}{3}\sqrt{x} + \frac{8}{3}\ln\left|\sqrt{3x} + 2\right| + C$$

3.

$$\int \frac{4\,dx}{2 + \sqrt{x}} = \begin{vmatrix} t = \sqrt{x} \\ dt = \frac{1}{2\sqrt{x}}dx \\ dx = 2t\, dt \end{vmatrix} = \int \frac{8t}{t + 2}\, dt = \int \frac{8t + 16 - 16}{t + 2}\, dt = \int 8 - \frac{16}{t + 2}\, dt = \int 8\, dt - 16\int \frac{dt}{t + 2}$$

$$= 8t - 16\ln|t + 2| + C = 8\sqrt{x} - 16\ln|\sqrt{x} + 2| + C$$

4.

$$\int \arcsin^2(x)\, dx = \begin{vmatrix} x = \sin(t) & t \in [-\frac{\pi}{2}, \frac{\pi}{2}] \\ t = \arcsin(x) \\ dx = \cos(t)\, dt \end{vmatrix} = \int t^2 \cos(t)\, dt = \begin{vmatrix} f = t^2 & g' = \cos(t) \\ f' = 2t & g = \sin(t) \end{vmatrix} = t^2\sin(t) - 2\int t\sin(t)\, dt$$

$$= \begin{vmatrix} f = t & g' = \sin(t) \\ f' = 1 & g = -\cos(t) \end{vmatrix} = t^2\sin(t) - 2\left[-t\cos(t) - \int -\cos(t)\, dt\right] = t^2\sin(t) + 2t\cos(t) - 2\int\cos(t)\, dt$$

$$= t^2\sin(t) + 2t\cos(t) - 2\sin(t) + C = x\arcsin^2(x) + 2\sqrt{1-x^2}\,\arcsin(x) - 2x + C$$

$$\cos\left(\arcsin(x)\right) = \sqrt{1 - \sin^2\left(\arcsin(x)\right)} = \sqrt{1 - x^2}$$

5.

$$\int \sin^9(x)\, dx = \int \left(1 - \cos^2(x)\right)^4 \sin(x)\, dx = \begin{vmatrix} t = \cos(x) \\ dt = -\sin(x)dx \end{vmatrix} = -\int (1 - t^2)^4\, dt$$

$$= -\int \left(1 - 2t^2 + t^4\right)^2 dt = -\int 1 - 2t^2 + t^4 - 2t^2 + 4t^4 - 2t^6 + t^4 - 2t^6 + t^8\, dt = -\int 1 - 4t^2 + 6t^4 - 4t^6 + t^8\, dt$$

$$= -t + \frac{4}{3}t^3 - \frac{6}{5}t^5 + \frac{4}{7}t^7 - \frac{1}{9}t^9 + C$$

$$= -\cos(x) + \frac{4}{3}\cos^3(x) - \frac{6}{5}\cos^5(x) + \frac{4}{7}\cos^7(x) - \frac{1}{9}\cos^9(x) + C$$

$$\int \sin^3(x)\, dx = \int \sin^2(x)\sin(x)\, dx = \int \left(1 - \cos^2(x)\right)\sin(x)\, dx = \begin{vmatrix} t = \cos(x) \\ dt = -\sin(x)dx \end{vmatrix} = -\int 1 - t^2\, dt = \int t^2 - 1\, dt = \frac{1}{3}t^3 - t + C = \frac{1}{3}\cos^3(x) - \cos(x) + C$$

6.

$$\int \sin^5(x)\, dx = \int \left(1 - \cos^2(x)\right)^2 \sin(x)\, dx = \begin{vmatrix} t = \cos(x) \\ dt = -\sin(x)dx \end{vmatrix} = -\int (1 - t^2)^2\, dt = -\int 1 - 2t^2 + t^4\, dt = \int -t^4 + 2t^2 - 1\, dt = -\frac{1}{5}\cos^5(x) + \frac{2}{3}\cos^3(t) - \cos(x) + C$$

7.
$$\int \frac{dx}{\sin(6x)\cos^2(6x)} = \int \frac{\sin^2(6x)+\cos^2(6x)}{\sin(6x)\cos^2(6x)}dx = \int \frac{\sin(6x)}{\cos^2(6x)}dx + \int \frac{1}{\sin(6x)}dx$$

$$\int \frac{\sin(6x)}{\cos^2(6x)}dx = \left|\begin{matrix} t=\cos(6x)\\ dt=-6\sin(6x)dx\end{matrix}\right| = -\frac{1}{6}\int \frac{dt}{t^2} = -\frac{1}{6}\int t^{-2}dt = -\frac{1}{6}\cdot(-1)t^{-1}+C = \frac{1}{6\cos(6x)}+C$$

$$\int \frac{1}{\sin(6x)}dx = \int \frac{\sin(6x)}{\sin^2(6x)}dx = \int \frac{\sin(6x)}{1-\cos^2(6x)}dx = \left|\begin{matrix} t=\cos(6x)\\ dt=-6\sin(6x)dx\end{matrix}\right| = -\frac{1}{6}\int \frac{1}{1-t^2}dt =$$

$$\frac{1}{1-t^2} = \frac{-1}{t^2-1} = \frac{A}{t-1}+\frac{B}{t+1} = \frac{A(t+1)+B(t-1)}{t^2-1} = \frac{-\frac{1}{2}}{t-1}+\frac{\frac{1}{2}}{t+1}$$
$$-1 = A(t+1)+B(t-1)$$
$$-1 = -2B \qquad B=\frac{1}{2}$$
$$-1 = 2A \qquad A=-\frac{1}{2}$$

$$\int \frac{1}{\sin(6x)}dx = -\frac{1}{6}\left[\int \frac{-\frac{1}{2}}{t-1}dt + \int \frac{\frac{1}{2}}{t+1}dt\right] = -\frac{1}{6}\left[-\frac{1}{2}\ln|t-1|+\frac{1}{2}\ln|t+1|\right]+C = \frac{1}{12}\ln|\cos(6x)-1| - \frac{1}{12}\ln|\cos(6x)+1|+C$$

$$\int \frac{dx}{\sin(6x)\cos^2(6x)} = \frac{1}{6\cos(6x)} + \frac{1}{12}\ln|\cos(6x)-1| - \frac{1}{12}\ln|\cos(6x)+1|+C = -\frac{1}{12}\ln\left|\frac{\cos(6x)+1}{\cos(6x)-1}\right| + \frac{1}{6\cos(6x)}+C$$

8.
$$\int \frac{7}{(x^2+3)^n}dx = 7\int \frac{dx}{(x^2+3)^n} = \left|\begin{matrix} t=\frac{1}{3}x\\ 3t=x\\ 3dt=dx\end{matrix}\right| = 7\int \frac{3dt}{(3t^2+3)^n} = 21\int \frac{dt}{3^n(t^2+1)^n} = \frac{21}{3^n}\int \frac{dt}{(t^2+1)^n}$$

$$= \frac{21}{3^n}\left[\frac{1}{2(n-1)}\cdot \frac{t}{(1+t^2)^{n-1}} + \frac{2n-3}{2n-2}\int \frac{dt}{(1+t^2)^{n-1}}\right] = \frac{21\cdot\frac{1}{3}x}{3^n\cdot 2(n-1)(1+\frac{1}{3}x^2)^{n-1}} + \frac{21}{3^n}\cdot\frac{2n-3}{2n-2}\int \frac{\frac{1}{3}dx}{(1+\frac{1}{3}x^2)^{n-1}}$$

$$= \frac{7x}{18(n-1)(x^2+3)^{n-1}} + \frac{7}{18}\cdot\frac{2n-3}{n-1}\int \frac{dx}{(x^2+3)^{n-1}}$$

9.
$$\int \frac{dx}{6\cos(x)+10} \qquad \left|\begin{matrix} t=\tan(\frac{x}{2})\\ dt=\frac{1}{\cos^2(\frac{x}{2})}\cdot\frac{1}{2}dx\\ dx=\frac{2dt}{1+t^2}\end{matrix}\right|$$

$$\frac{1}{\cos^2(\frac{x}{2})} = \frac{\cos^2(\frac{x}{2})+\sin^2(\frac{x}{2})}{\cos^2(\frac{x}{2})} = 1+\tan^2(\tfrac{x}{2}) \qquad = \int \frac{\frac{2}{1+t^2}}{6\frac{1-t^2}{1+t^2}+10}dt = \int \frac{\frac{2}{1+t^2}}{\frac{6-6t^2+10+10t^2}{1+t^2}}dt = \int \frac{2}{4t^2+16}dt$$

$$\cos(x) = \frac{1-\tan^2(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}$$

$$= \frac{1}{8}\int \frac{dt}{\frac{1}{4}t^2+1} = \frac{1}{8}\int \frac{dt}{(\frac{1}{2}t)^2+1} = \left|\begin{matrix}u=\frac{1}{2}t\\ du=\frac{1}{2}dt\end{matrix}\right| = \frac{1}{4}\int \frac{du}{u^2+1} = \frac{1}{4}\arctan(u)+C = \frac{1}{4}\arctan(\tfrac{1}{2}t)+C =$$

$$= \frac{1}{4}\arctan\left(\tfrac{1}{2}\tan(\tfrac{x}{2})\right)+C$$

10.
$$\int \frac{3dx}{\sqrt[3]{4x}+\sqrt{4x}} = \left|\begin{matrix} t=(4x)^{\frac{1}{6}}\\ dt=\frac{1}{6}(4x)^{-\frac{5}{6}}\cdot 4dx\\ dx=\frac{6}{4}\cdot(4x)^{\frac{5}{6}}dt=\frac{3}{2}t^5dt\end{matrix}\right| = \int \frac{3\cdot\frac{3}{2}t^5}{t^2+t^3}dt = \frac{27}{2}\int \frac{t^5}{t^2+t^3}dt = \frac{27}{2}\int \frac{t^3}{t+1}dt = \frac{27}{2}\int t^2-t+1-\frac{1}{t+1}dt$$

$$\frac{27}{2}\left[\int t^2-t+1\,dt - \int \frac{dt}{t+1}\right] = \frac{27}{2}\left[\frac{1}{3}t^3-\frac{1}{2}t^2+t-\ln|t+1|\right]+C$$

$$\frac{t^3}{t+1} \,(t+1 \qquad \frac{t^3}{t+1}=t^2-t+1-\frac{1}{t+1}$$
$$\frac{t^2-t+1}{t^3}$$
$$\ominus\ t^3+t^2$$
$$\quad -t^2$$
$$\ominus\ -t^2-t$$
$$\quad\quad t$$
$$\ominus\ t+1$$
$$\quad\quad -1$$

$$= \frac{3}{2}\sqrt{4x} - \frac{27}{4}\sqrt[3]{4x} + \frac{27}{2}\sqrt[6]{4x} - \frac{27}{2}\ln|\sqrt[6]{4x}+1|+C$$

11.

$$\int \sqrt{\frac{x-1}{x-4}} \cdot \frac{dx}{(x-1)^2} = \int \frac{t}{\left(\frac{4t^2-1}{t^2-1}-1\right)^2} \cdot \frac{-6t}{(t^2-1)^2}\, dt = \int \frac{-6t^2\, dt}{\left(\frac{4t^2-1-t^2+1}{t^2-1}\right)^2 \cdot (t^2-1)^2} = \int \frac{-6t^2\, dt}{\frac{3t^4}{(t^2-1)^2}(t^2-1)^2} = \int \frac{-2\, dt}{3\, t^2} = -\frac{2}{3}\int t^{-2}\, dt$$

$$t^2 = \frac{x-1}{x-4} \qquad dx = \frac{8t(t^2-1)-(4t^2-1)\cdot 2t}{(t^2-1)^2}\, dt$$

$$t^2 x - 4t^2 = x-1 \qquad dx = \frac{8t^3-8t-8t^3+2t}{(t^2-1)^2}\, dt$$

$$t^2 x - x = 4t^2-1 \qquad dx = \frac{-6t}{(t^2-1)^2}\, dt$$

$$x = \frac{4t^2-1}{t^2-1}$$

$$= -\frac{2}{3}\cdot(-1)\, t^{-1} + C = \frac{2}{3}\sqrt{\frac{x-4}{x-1}} + C$$