1.

$$a_{n+1} = \frac{a_n}{a_n+1} \qquad a_1 = 4$$

$$a_2 = \frac{4}{4+1} = \frac{4}{5}$$

$$a_3 = \frac{4/5}{4/5+1} = \frac{4/5}{9/5} = \frac{4}{9}$$

$$a_4 = \frac{4/9}{4/9+1} = \frac{4/9}{13/9} = \frac{4}{13}$$

$$a_5 = \frac{4/13}{4/13+1} = \frac{4/13}{17/13} = \frac{4}{17}$$

$$a_6 = \frac{4/13}{21/17} = \frac{4}{21}$$

1 5 9 13 17

$$a_n = \frac{4}{4(n-1)+1} = \frac{4}{4n-3}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n} \cdot \frac{4}{4-\frac{3}{n}} = 0$$

2. $$\lim_{n\to\infty} \left(\frac{n-1}{n}\right)^{-4n+3} = \lim_{n\to\infty} \left(1+\frac{1}{(-n)}\right)^{-n\cdot(4-\frac{3}{n})} = e^4$$

$$\lim_{n\to\infty} \left(4-\frac{3}{n}\right) = 4$$

3. $$\lim_{n\to\infty} \sqrt[n]{5^{2n+1}+\sin(n)} = \lim_{n\to\infty} \sqrt[n]{5\cdot 25^n + \sin(n)}$$

$$\sqrt[n]{5\cdot 25^n} \le \sqrt[n]{5\cdot 25^n + \sin(n)} \le \sqrt[n]{5\cdot 25^n \cdot 2}$$

$$\sqrt[n]{5\cdot 25^n} = 25\cdot\sqrt[n]{5} \to 25 \qquad \sqrt[n]{5\cdot 25^n\cdot 2} = 25\cdot\sqrt[n]{10} \to 25$$

$$\lim_{n\to\infty} \sqrt[n]{5\cdot 25^n + \sin(n)} = 25$$

4.

$$a_n = \left(\frac{1}{n}\right)^2 = \frac{1}{n^2}$$

dla jakich n $|a_n| \le \frac{1}{114}$

$n \in \mathbb{N}$ $\quad \left|\frac{1}{n^2}\right| \le \frac{1}{114} \iff \frac{1}{n^2} \le \frac{1}{114}$

$n^2 \ge 114$

$n \ge \sqrt{114} > \sqrt{100} = 10$

5. $$\lim_{n\to\infty} \sqrt[n]{3^n+6^n+n+1} = 6$$

$$\sqrt[n]{6^n+6^n} \le \sqrt[n]{3^n+6^n+n+1} \le \sqrt[n]{n6^n+n6^n+n6^n+n6^n}$$

$$\sqrt[n]{6^n+6^n} = 6\cdot\sqrt[n]{2} \to 6 \qquad \sqrt[n]{n6^n+n6^n+n6^n+n6^n} = 6\cdot\sqrt[n]{4n} \to 6$$