1.

$$f(x) = x^{e^x} = e^{\ln(x^{e^x})} = e^{e^x \ln(x)}$$

$$\frac{d}{dx} e^{e^x \ln(x)} = e^{e^x \ln(x)} \cdot \frac{d}{dx} e^x \ln(x) = x^{e^x} \cdot \left[e^x \cdot \frac{1}{x} + e^x \ln(x) \right] = e^x x^{e^x} \left(\frac{1}{x} + \ln(x) \right)$$

2.

$$f(t) = \cos\left(\frac{\sin(t)}{5t^2 + 4t + 7} \right)$$

$$\frac{df}{dt} = -\sin\left(\frac{\sin(t)}{5t^2+4t+7} \right) \cdot \frac{d}{dt} \frac{\sin(t)}{5t^2+4t+7}$$

$$= -\sin\left(\frac{\sin(t)}{5t^2+4t+7} \right) \cdot \frac{\cos(t)(5t^2+4t+7) - \sin(t)(10t+4)}{(5t^2+4t+7)^2}$$

3.

$$\lim_{x \to +\infty} \frac{x^n}{e^x} \overset{\frac{\infty}{\infty}}{=} ? \qquad n \in \mathbb{Z}_+$$

dla $n=0$ $\quad \lim_{x \to +\infty} \frac{x^0}{e^x} = \lim_{x \to +\infty} \frac{1}{e^x} = 0$

dla $n>0$ $\quad \lim_{x \to +\infty} \frac{x^n}{e^x} \underset{H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x \to +\infty} \frac{n x^{n-1}}{e^x} = \lim_{x \to +\infty} \frac{n!\, x^{n-n}}{e^x} = \lim_{x \to +\infty} \frac{n!}{e^x} = 0$

po zastosowaniu reguły de l'Hôpitala n razy

4.

$$\lim_{x \to 0^-} \left(\frac{1}{x} - \cot(x) \right) \overset{-\infty - (-\infty)}{=} \lim_{x \to 0^-} \left[\frac{1}{x} - \frac{\cos(x)}{\sin(x)} \right] = \lim_{x \to 0^-} \frac{\sin(x) - x\cos(x)}{x \sin(x)} \underset{H}{\overset{\frac{0}{0}}{=}}$$

$$\lim_{x \to 0^-} \frac{\cos(x) - [-x\sin(x) + \cos(x)]}{x\cos(x) + \sin(x)} = \lim_{x \to 0^-} \frac{x\sin(x)}{x\cos(x) + \sin(x)} \underset{H}{\overset{\frac{0}{0}}{=}} \lim_{x \to 0^-} \frac{x\cos(x) + \sin(x)}{-x\sin(x) + \cos(x) + \cos(x)} = \frac{0}{2} = 0$$

5.

$$\lim_{x \to +\infty} x^{\operatorname{arccot}(x)} = \lim_{x \to +\infty} e^{\ln(x^{\operatorname{arccot}(x)})} = \lim_{x \to +\infty} e^{\operatorname{arccot}(x) \ln(x)}$$

$$\lim_{x \to +\infty} \underset{\searrow 0}{\operatorname{arccot}(x)} \underset{\searrow +\infty}{\ln(x)} = \lim_{x \to +\infty} \frac{\ln(x)}{\frac{1}{\operatorname{arccot}(x)}} \underset{H}{\overset{\frac{\infty}{\infty}}{=}} \lim_{x \to +\infty} \frac{\frac{1}{x}}{\frac{-1}{[\operatorname{arccot}(x)]^2} \cdot \frac{-1}{x^2+1}} = \lim_{x \to +\infty} \frac{\operatorname{arccot}(x)(x^2+1)}{x}$$

$$\lim_{x \to +\infty} \frac{\operatorname{arccot}(x)}{\frac{x}{x^2+1}} \underset{H}{\overset{\frac{0}{0}}{=}} \lim_{x \to +\infty} \frac{\frac{-1}{x^2+1}}{\frac{x^2+1 - x \cdot 2x}{x^4+2x^2+1}} = \lim_{x \to +\infty} \frac{\frac{-1}{x^2+1}}{\frac{-x^2+1}{x^4+2x^2+1}} = \lim_{x \to +\infty} -\frac{x^4+2x^2+1}{(1+x^2)(1-x^2)}$$

$$\lim_{x \to +\infty} \frac{-x^4-2x^2-1}{-x^4+1} = 1$$