

1.

$$f(x) = x^{e^x} = e^{\ln(x^{e^x})} = e^{e^x \ln(x)}$$

$$\frac{d}{dx} e^{e^x \ln(x)} = e^{e^x \ln(x)} \cdot \frac{d}{dx} e^x \ln(x) = x^{e^x} \cdot \left[e^x \cdot \frac{1}{x} + e^x \ln(x) \right] = e^x x^{e^x} \left(\frac{1}{x} + \ln(x) \right)$$

2.

$$f(t) = \cos\left(\frac{\sin(t)}{5t^2 + 4t + 7}\right)$$

$$\frac{df}{dt} = -\sin\left(\frac{\sin(t)}{5t^2 + 4t + 7}\right) \cdot \frac{d}{dt} \frac{\sin(t)}{5t^2 + 4t + 7}$$

$$= -\sin\left(\frac{\sin(t)}{5t^2 + 4t + 7}\right) \cdot \frac{\cos(t)(5t^2 + 4t + 7) - \sin(t)(10t + 4)}{(5t^2 + 4t + 7)^2}$$

3.

$$\lim_{x \rightarrow +\infty} \frac{x^n}{e^x} \stackrel{0}{=} ? \quad n \in \mathbb{Z}_+$$

$$\text{dla } n=0 \quad \lim_{x \rightarrow +\infty} \frac{x^0}{e^x} = \lim_{x \rightarrow +\infty} \frac{1}{e^x} = 0$$

$$\text{dla } n>0 \quad \lim_{x \rightarrow +\infty} \frac{x^n}{e^x} \stackrel{0}{=} \lim_{x \rightarrow +\infty} \frac{n x^{n-1}}{e^x} = \lim_{x \rightarrow +\infty} \frac{n! x^{n-n}}{e^x} = \lim_{x \rightarrow +\infty} \frac{n!}{e^x} = 0$$

↗
zastosowania reguły de l'Hôpitala n razy

4.

$$-\infty - (-\infty)$$

$$\lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \cot(x) \right) = \lim_{x \rightarrow 0^-} \left[\frac{1}{x} - \frac{\cos(x)}{\sin(x)} \right] \stackrel{0}{=} \lim_{x \rightarrow 0^-} \frac{\sin(x) - x \cos(x)}{x \sin(x)} \stackrel{0}{=} \lim_{x \rightarrow 0^-} \frac{\sin(x) - x \cos(x)}{x \sin(x)}$$

$$\lim_{x \rightarrow 0^-} \frac{\cos(x) - [-x \sin(x) + \cos(x)]}{x \cos(x) + \sin(x)} = \lim_{x \rightarrow 0^-} \frac{x \sin(x)}{x \cos(x) + \sin(x)} \stackrel{0}{=} \lim_{x \rightarrow 0^-} \frac{x \cos(x) + \sin(x)}{-x \sin(x) + \cos(x) + \cos(x)} = \frac{0}{2} = 0$$

5.

$$\lim_{x \rightarrow +\infty} x^{\operatorname{arccot}(x)} = \lim_{x \rightarrow +\infty} e^{\ln(x^{\operatorname{arccot}(x)})} = \lim_{x \rightarrow +\infty} e^{\operatorname{arccot}(x) \ln(x)}$$

$$\lim_{x \rightarrow +\infty} \operatorname{arccot}(x) \ln(x) = \lim_{x \rightarrow +\infty} \frac{\ln(x)}{\frac{1}{\operatorname{arccot}(x)}} \stackrel{0}{=} \lim_{x \rightarrow +\infty} \frac{\frac{1}{x}}{\frac{-1}{[\operatorname{arccot}(x)]^2} \cdot \frac{-1}{x^2 + 1}} = \lim_{x \rightarrow +\infty} \frac{\operatorname{arccot}(x) (x^2 + 1)}{x}$$

$$\lim_{x \rightarrow +\infty} \frac{\operatorname{arccot}(x)}{\frac{x}{x^2 + 1}} \stackrel{0}{=} \lim_{x \rightarrow +\infty} \frac{\frac{-1}{x^2 + 1}}{\frac{x^2 + 1 - x \cdot 2x}{x^4 + 2x^2 + 1}} = \lim_{x \rightarrow +\infty} \frac{\frac{-1}{x^2 + 1}}{\frac{-x^2 + 1}{x^4 + 2x^2 + 1}} = \lim_{x \rightarrow +\infty} \frac{x^4 + 2x^2 + 1}{(1 + x^2)(1 - x^2)}$$

$$\lim_{x \rightarrow +\infty} \frac{-x^4 - 2x^2 + 1}{-x^4 + 1} = 1$$