1.

a) $f: \mathbb{R} \to [-1,1]$ $\quad f(x) = \sin(x)$

$A = [0, 4\pi]$ $\quad B = [2\pi, 4\pi]$ $\quad A \setminus B = [0, 2\pi)$

$f(A \setminus B) = f([0, 2\pi)) = [-1, 1]$

$f(A) = f([0, 4\pi]) = [-1, 1]$

$f(B) = f([2\pi, 4\pi]) = [-1, 1]$

$f(A) \setminus f(B) = [-1, 1] \setminus [-1, 1] = \emptyset$

$[-1, 1] \not\subseteq \emptyset$

$f(A \setminus B) \not\subseteq f(A) \setminus f(B)$

b) $f: \mathbb{R} \to [-1,1]$ $\quad f(x) = \sin(x)$ $\quad A = [0, 2\pi]$

$f([0, 2\pi]) = [-1, 1]$

$f^{-1}([-1, 1]) = \mathbb{R}$

$\mathbb{R} \not\subseteq [-1, 1]$

$f^{-1}(f([0, 2\pi)) \not\subseteq A$

c) $f: \mathbb{R} \to \mathbb{R}$ $\quad f(x) = \sin(x)$

$C = [-1, 2]$

$f^{-1}([-1, 2]) = \mathbb{R}$

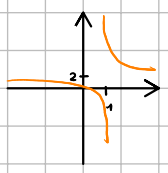
$f(\mathbb{R}) = [-1, 1]$

$[-1, 2] \not\subseteq [-1, 1]$

$C \not\subseteq f(f^{-1}(C))$

2.

a) $f: \mathbb{R}\setminus\{1\} \to \mathbb{R}$ $\quad f(x) = \dfrac{2x+3}{x-1}$



$f(x) = \dfrac{2x-2+5}{x-1} = \dfrac{5}{x-1} + 2$

$R_f = \mathbb{R}\setminus\{2\}$

$y = \dfrac{2x+3}{x-1}$

$yx - y = 2x + 3$

$yx - 2x = y + 3$

$x(y-2) = y + 3$

$x = \dfrac{y+3}{y-2}$

$f^{-1}: \mathbb{R}\setminus\{2\} \to \mathbb{R}\setminus\{1\}$

$f(x) = \dfrac{x+3}{x-2}$

b) $f: [3\pi, 4\pi] \to \mathbb{R}$ $\quad f(x) = e^{1-2\cos(x)}$

$x \in [3\pi, 4\pi] \Rightarrow \cos(x) = [-1, 1] \Rightarrow 1 - 2\cos(x) \in [-1, 3] \Rightarrow f(x) \in [e^{-1}, e^3]$

$R_f = [e^{-1}, e^3]$

$y = e^{1-2\cos(x)}$

$\ln(y) = 1 - 2\cos(x)$

$\cos(x) = \dfrac{1 - \ln(y)}{2}$

$x = \arccos\left(\dfrac{1-\ln(y)}{2}\right) + 3\pi$

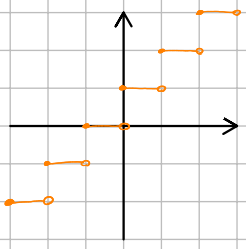
$\arccos: [-1, 1] \to [0, \pi]$

$\arccos(x) + 3\pi \in [3\pi, 4\pi]$

$f^{-1}: [e^{-1}, e^3] \to [3\pi, 4\pi]$ $\quad f^{-1}(x) = \arccos\left(\dfrac{1-\ln(x)}{2}\right) + 3\pi$

3.

a) $f: \mathbb{R} \to \mathbb{Z}$ $\quad f(x) = \lfloor x \rfloor + 1$



niemalejąca

nie jest injekcją $\quad f(1.5) = f(1.51)$

surjekcja $\quad \forall_{n \in \mathbb{Z}} \; f(n-1) = n$

$f^{-1}(\{0\}) = \{ x \in \mathbb{R} : \lfloor x \rfloor = -1 \} = [-1, 0)$

$f^{-1}(\mathbb{Z} \setminus \mathbb{N}) = \{ x \in \mathbb{R} : \lfloor x \rfloor + 1 \in \mathbb{Z} \setminus \mathbb{N} \} = \{ x \in \mathbb{R} : \lfloor x \rfloor < 0 \} = (-\infty, 0)$

$f(0) = 1 \notin \mathbb{Z} \setminus \mathbb{N}$

$f(-0.01) = 0 \in \mathbb{Z} \setminus \mathbb{N}$

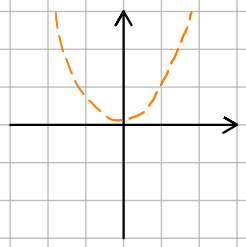
$f((0, +\infty)) = \mathbb{N}$

$f(0) = 1 \quad f(0.001) = 1 \quad f(0.999) = 1$

$f(1) = 2 \quad f(1.001) = 2 \quad f(1.999) = 2$

$f^{-1}(f([-\pi, \pi])) = f^{-1}(\{-3, -2, -1, 0, 1, 2, 3, 4\}) = [-4, 4)$

b) $f: \mathbb{Q} \to \mathbb{Q}$ $\quad f(x) = x^2 + 1$



nie jest injekcją $\quad f(-1) = f(1)$

nie jest surjekcją $\quad \sim \exists_{x \in \mathbb{Q}} \; f(x) = 0 \in \mathbb{Q}$

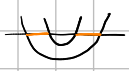
$f^{-1}(\{\frac{13}{9}\}) = \{\frac{2}{3}, -\frac{2}{3}\}$

$f^{-1}([\frac{10}{9}, 2] \cap \mathbb{Q}) = ([-1, -\frac{1}{3}] \cup [\frac{1}{3}, 1]) \cap \mathbb{Q}$

$x^2 + 1 \leq 2 \quad \wedge \quad x^2 + 1 \geq \frac{10}{9}$

$x^2 - 1 \leq 0 \quad \wedge \quad x^2 - \frac{1}{9} \geq 0$

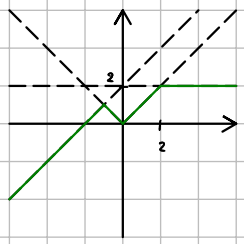
$(x-1)(x+1) \leq 0 \quad \wedge \quad (x - \frac{1}{3})(x + \frac{1}{3}) \geq 0$



$x^2 + 1 = \frac{3}{2} \iff x^2 - \frac{1}{2} = 0 \iff (x - \frac{\sqrt{2}}{2})(x + \frac{\sqrt{2}}{2}) = 0 \iff x \in \{-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\}$

$[1, 2] \cap \mathbb{Q} \not\subseteq f(\mathbb{Q})$ bo $\sim \exists_{x \in \mathbb{Q}} \; f(x) = \frac{3}{2} \; \wedge \; \frac{3}{2} \in [1, 2] \cap \mathbb{Q}$

c) $f: \mathbb{R} \to \mathbb{R}$ $\quad f(x) = \min\{|x|, 2, x+2\}$



$$f(x) = \begin{cases} x+2 & \text{dla } x \in (-\infty, -1] \\ |x| & \text{dla } x \in (-1, 2] \\ 2 & \text{dla } x \in (2, +\infty) \end{cases}$$

nie jest iniekcją $\quad f(2) = f(3)$

nie jest surjekcją $\quad \sim \exists_{x \in \mathbb{R}} \; f(x) = 3$

$f((-1, 3]) = [0, 2]$

$f^{-1}(\{-1, 1, 2\}) = \{-3, -1, 1\} \cup [2, +\infty)$

$f^{-1}(f(\{0, 1, 5\})) = f^{-1}(\{0, 1, 2\}) = \{-2, -1, 0, 1\} \cup [2, +\infty)$

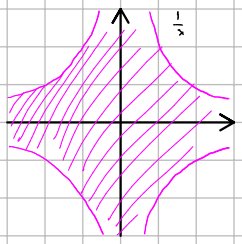
d) $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f((x,y)) = |xy|$

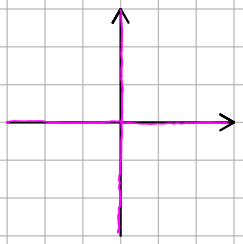
$f^{-1}((-5,2]) = f^{-1}([0,2])$ $\qquad f^{-1}(\{0\})$ $\qquad f((-2,1)\times\{-1\})$

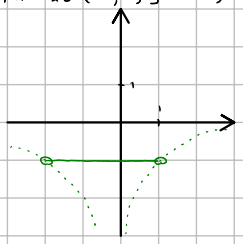
$= \{|x| : x \in (-2,1)\} = [0,2)$

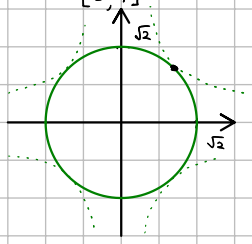
$f(\{(x,y)\in\mathbb{R}^2 : x^2+y^2=2\})$

$= [0,1]$









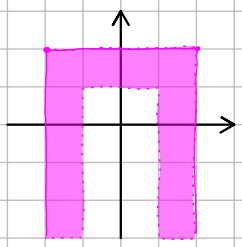
nie jest injekcją $\quad f((1,1)) = f((-1,1))$

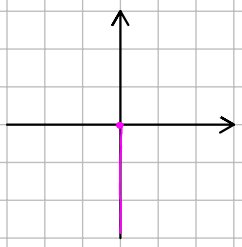
nie jest surjekcją $\quad R_f = [0,+\infty) \neq \mathbb{R}$

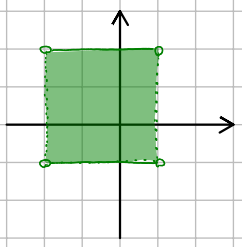
min $f((0,\sqrt{2})) = 0$

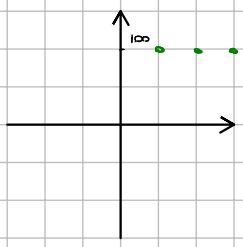
max $f((\sqrt{2}\cos(\frac{\pi}{2}), \sqrt{2}\sin(\frac{\pi}{2}))) = f((1,1)) = 1$

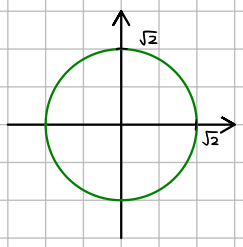
e) $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f((x,y)) = \max\{|x|, y\}$











$f^{-1}((1,2])$ $\qquad f^{-1}(\{0\})$ $\qquad f((-2,1)\times[-1,2])$

$= [0,2]$

$f(\mathbb{N}\times\{100\})$

$= \{\max\{|n|, 100\} : n \in \mathbb{N}\}$

$= \{\max\{n, 100\} : n \in \mathbb{N}\}$

$= \{100, 101, \dots\}$

$f(\{(x,y)\in\mathbb{R}^2 : x^2+y^2=2\})$

$= [0, \sqrt{2}]$

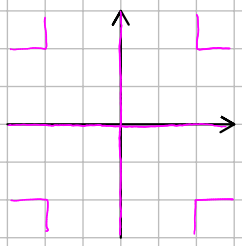
$f((0,-\sqrt{2})) = 0$

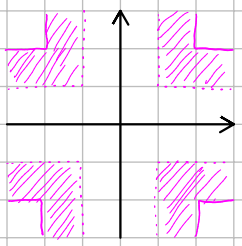
$f((0,\sqrt{2})) = \sqrt{2}$

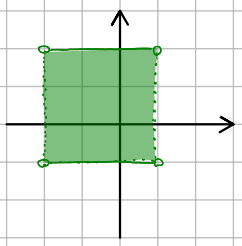
nie jest injekcją bo $\quad f((-2,1)) = f((2,1)) = 2$

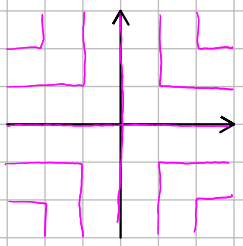
nie jest surjekcją bo $\quad f(\mathbb{R}^2) = [0,+\infty) \neq \mathbb{R}$

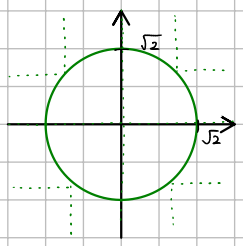
f) $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f((x,y)) = \min\{|x|, |y|\}$











$f^{-1}(\{-2,0,2\})$ $\qquad f^{-1}((1,2])$ $\qquad f((-2,1)\times[-1,2])$

$= [0,2)$

$f(f^{-1}(\{-1,0,1,2\}))$

$= \{0,1,2\}$

$f(\{(x,y)\in\mathbb{R} : x^2+y^2=2\})$

$= [0,\sqrt{2}]$

nie jest injekcją $\quad f((-1,0)) = f((1,0)) = 0$

nie jest surjekcją $\quad R_f = [0,+\infty)$