1.

$$A = \begin{bmatrix} 4 & -3 & 6 \\ 4 & -1 & 4 \\ -1 & 2 & -3 \end{bmatrix} \qquad \chi(\lambda) = \begin{vmatrix} 4-\lambda & -3 & 6 \\ 4 & -1-\lambda & 4 \\ -1 & 2 & -3-\lambda \end{vmatrix}$$

$$= (4-\lambda)(-1-\lambda)(-3-\lambda) + 12 + 48 + 6(-1-\lambda) - 8(4-\lambda) + 12(-3-\lambda)$$
$$= (4-\lambda)(3+\lambda+3\lambda+\lambda^2) + 60 - 6 - 6\lambda - 32 + 8\lambda - 36 - 12\lambda$$
$$= 12 + 16\lambda + \cancel{4\lambda^2} - 3\lambda - \cancel{4\lambda^2} - \lambda^3 + 60 - 6 - 6\lambda - 32 + 8\lambda - 36 - 12\lambda$$
$$= -\lambda^3 + 3\lambda - 2 = (\lambda-1)(-\lambda^2 - \lambda + 2)$$
$$= -(\lambda-1)^2(\lambda+2)$$

	-1	0	3	-2
1		-1	-1	2
	-1	-1	2	0

$$-\lambda^2 - 2\lambda + \lambda + 2$$
$$= -\lambda(\lambda+2) + \lambda + 2)$$
$$= (-\lambda+1)(\lambda+2)$$

$\lambda_1 = 1 \quad \lambda_2 = -2$

- $\lambda_1 = 1$

$Av = 1 \cdot v$

$Av = Iv$

$(A - I)v = 0$

$$\left[\begin{array}{ccc|c} 3 & -3 & 6 & 0 \\ 4 & -2 & 4 & 0 \\ -1 & 2 & -4 & 0 \end{array}\right] \xrightarrow[v_2 + 4v_3]{v_1 + 3v_3} \begin{bmatrix} 0 & 3 & -6 \\ 0 & 6 & -12 \\ -1 & 2 & -4 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 4 \\ 0 & 1 & -2 \end{bmatrix} \xrightarrow{v_1 + 2v_2} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \end{bmatrix}$$
$$\begin{matrix} \uparrow & \uparrow & t \end{matrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \qquad V_1 = span((0,2,1))$$

- $\lambda_2 = -2$

$(A + 2I)v = 0$

$$\begin{bmatrix} 6 & -3 & 6 \\ 4 & 1 & 4 \\ -1 & 2 & -1 \end{bmatrix} \xrightarrow[+4v_3]{+6v_3} \begin{bmatrix} 0 & 9 & 0 \\ 0 & 9 & 0 \\ -1 & 2 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = t \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$
$$\begin{matrix} \uparrow & \uparrow & t \end{matrix}$$

$$V_2 = span((-1,0,1))$$

1*

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad B = \begin{bmatrix} 2d-3 & 2c \\ 2b & 2a-3 \end{bmatrix}$$

$$\chi_A(t) = (a-t)(d-t) - bc$$
$$= ad - at - dt + t^2 - bc$$
$$= t^2 - (a+d)t + ad - bc$$
$$= t^2 - 4t + 3$$

$$\chi_B(t) = (2d-3-t)(2a-3-t) - 2b \cdot 2c$$
$$= 4ad - 6d - 2dt - 6a + 9 + 3t - 2at + 3t + t^2 - 4bc$$
$$= t^2 + (6 - 2a - 2d)t + 9 + 4(ad-bc) - 6a - 6d$$
$$= t^2 + (6 - 2(a+d))t + 4(ad-bc) - 6(a+d) + 9$$
$$= t^2 + (6-8)t + 4 \cdot 3 - 6 \cdot 4 + 9$$
$$= t^2 - 2t - 3$$

$Av = \lambda v$

$Av - \lambda v = 0$

$(A - I\lambda)v = 0 \iff \chi_A(\lambda) = 0 \iff det(A - \lambda I) = 0$

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = \begin{vmatrix} d & c \\ b & a \end{vmatrix} = ad - bc$$

$\alpha Av = tv$

$\alpha Av - tv = 0$

$(\alpha A - tI)v = 0 \iff \chi_{\alpha A}(t) = 0 \iff det(\alpha A - tI) = 0$

dla $t = \alpha\lambda$

$\chi_{\alpha A}(t) = det(\alpha A - \alpha\lambda I) = \alpha\, det(A - \lambda I) = 0$

$\alpha\lambda$ jest ww αA

$(A + nI)v = tv$

$(A + nI - tI)v = 0$

$\chi_{A+nI}(t) = det(A - I(t-n))$

dla $t = \lambda + n$

$\chi_{A+nI}(t) = det(A - I(\lambda+n-n)) = det(A - \lambda I) = 0$

$\lambda + n$ jest ww $A + nI$

$\chi_A(t) = t^2 - 4t + 3 = t^2 - 3t - t + 3 = (t-3)(t-1)$

$A' = 2A - 3I \qquad 3 \rightarrow 2 \cdot 3 - 3 = 3$

$\qquad\qquad\qquad\quad 1 \rightarrow 2 \cdot 1 - 3 = -1$

$\chi_{A'}(t) = (t-3)(t+1) = t^2 - 3t + t - 3 = t^2 - 2t - 3 = \chi_B(t)$

2.

$A = (-1, 2, 2)$
$B = (-3, -1, 1)$
$C = (1, 0, 2)$

a) $\vec{AB} = \begin{bmatrix} -3+1 \\ -1-2 \\ 1-2 \end{bmatrix} = \begin{bmatrix} -2 \\ -3 \\ -1 \end{bmatrix}$ $\quad \vec{AC} = \begin{bmatrix} 1+1 \\ 0-2 \\ 2-2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix}$

$$n_{ABC} = \vec{AB} \times \vec{AC} = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & -3 & -1 \\ 2 & -2 & 0 \end{bmatrix} = \hat{\imath} \cdot (-2) - \hat{\jmath} \cdot 2 + \hat{k} \cdot 10 = \begin{bmatrix} -2 \\ -2 \\ 10 \end{bmatrix}$$

Płaszczyzna ABC:

$$(x - A) \cdot n_{ABC} = \begin{bmatrix} x+1 \\ y-2 \\ z-2 \end{bmatrix} \circ \begin{bmatrix} -2 \\ -2 \\ 10 \end{bmatrix} = -2x - 2 - 2y + 4 + 10z - 20 = 0$$

$-2x - 2y + 10z = 18$
$x + y - 5z = -9$

Płaszczyzna P: $x = 0$
$n_P = (1, 0, 0)$

$$\sin\sphericalangle(P, ABC) = \sin\sphericalangle(n_{ABC}, n_P) = \frac{|n_{ABC} \times n_P|}{|n_{ABC}| \cdot |n_P|} = \frac{\left\| \begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -2 & -2 & 10 \\ 1 & 0 & 0 \end{matrix} \right\|}{\sqrt{4+4+100} \cdot \sqrt{1}} = \frac{|\hat{\imath} \cdot 0 - \hat{\jmath} \cdot (-10) + \hat{k} \cdot 2|}{\sqrt{108}}$$

$$= \frac{\sqrt{0+100+4}}{\sqrt{108}} = \frac{\sqrt{104}}{\sqrt{108}}$$

b) $|\Delta ABC| = \frac{1}{2} \left| \vec{AB} \times \vec{AC} \right| = \frac{1}{2} \sqrt{4+4+100} = \frac{\sqrt{108}}{2} = \sqrt{27} = 3\sqrt{3}$

c)

$$|OABC| = \frac{1}{6} \left| (OA \times OB) \cdot OC \right| = \frac{1}{6} \left\| \begin{matrix} 1 & 0 & 2 \\ -1 & 2 & 2 \\ -3 & -1 & 1 \end{matrix} \right\| = \frac{1}{6} |1 \cdot 4 - 0 + 2 \cdot 7| = \frac{18}{6} = 3$$

3. $A = (-1, 1, -3)$
$B = (3, -1, 1)$
$C = (-2, 2, -1)$
$D = (2, -1, 1)$

odległość AB do CD

$$\text{wysokość} = \frac{\text{objętość}}{\text{pole podstawy}}$$

$AC = \begin{bmatrix} -2+1 \\ 2-1 \\ -1+3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ $\quad AB = \begin{bmatrix} 3+1 \\ -1-1 \\ 1+3 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 4 \end{bmatrix}$ $\quad CD = \begin{bmatrix} 2+2 \\ -1-2 \\ 1+1 \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \\ 2 \end{bmatrix}$

dla równoległościanu wyznaczonego przez AB, CD i AC

$$\frac{|(AB \times CD) \cdot AC|}{|AB \times CD|} = \frac{\left\| \begin{matrix} -1 & 1 & 2 \\ 4 & -2 & 4 \\ 4 & -3 & 2 \end{matrix} \right\|}{\left\| \begin{matrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 4 & -2 & 4 \\ 4 & -3 & 2 \end{matrix} \right\|} = \frac{|-1 \cdot 8 - 1 \cdot (-8) + 2 \cdot (-4)|}{|\hat{\imath} \cdot 8 - \hat{\jmath} \cdot (-8) + \hat{k} \cdot (-4)|} = \frac{8}{\sqrt{64+64+16}} = \frac{8}{12} = \frac{2}{3}$$

1.

N U D N O O T U

a) $N = \binom{8}{2,\ 2,\ 2,\ 1,\ 1} = \frac{8!}{2!2!2!} = 7! = 5040$

b) 4 samogłoski V
4 spółgłoski C   trudno wymówić kiedy jest zbitka 3 lub 4

$\overline{C}\ \overline{C}\ \overline{C}\ \overline{V}\ \overline{V}\ \overline{C}\ \overline{V}\ \overline{V}$

$$N_2 = \underset{\substack{\downarrow \\ \text{zbitka } 3\times C}}{6} \cdot \underset{\substack{\searrow \\ \text{czwarte } C}}{\binom{5}{1}} \cdot \underset{\text{samogłoski}}{\binom{4}{4}} \cdot \underset{U\ O}{\binom{4}{2,\ 2}} \cdot \underset{N\ D\ T}{\binom{4}{2,\ 1,\ 1}} = 6 \cdot 5 \cdot 1 \cdot \frac{4!}{2!2!} \cdot \frac{4!}{2!} = 30 \cdot 6 \cdot 12 = 2160$$

$N_D = N - N_2 = 5040 - 2160 = 2880$

3.1

N N U D N O T U

a) $= \binom{8}{\underset{N}{3},\ \underset{U}{2},\ \underset{D}{1},\ \underset{O}{1},\ \underset{T}{1}} = \frac{8!}{3!2!} = 3360$

trudno wymówić jest przynajmniej zbitka 3

$\overline{C}\ \overline{V}\ \overline{C}\ \overline{C}\ \overline{C}\ \overline{V}\ \overline{C}\ \overline{V}$

$$N = \underset{\substack{\downarrow \\ \text{zbitka } 3\times C}}{\binom{6}{1}} \cdot \underset{\substack{\text{pozostałe} \\ 2\times C}}{\binom{5}{2,\ 3}} \cdot \underset{N\ D\ T}{\binom{5}{3,\ 1,\ 1}} \cdot \underset{U\ O}{\binom{3}{2,\ 1}} = 6 \cdot \frac{5!}{2!3!} \cdot \frac{5!}{3!} \cdot \frac{3!}{2!}$$

$3\times V$

W miejsca wstawiam grupy spółgłosek

_ V _ V _ V _   grupy C   rozmieszczenie
2, 2, 1 → $\binom{5}{3}$
2, 1, 1, 1 $\binom{5}{4}$
1, 1, 1, 1, 1 $\binom{5}{5}$

$$N_d = \left[\binom{5}{3} + \binom{5}{4} + \binom{5}{5}\right] \cdot \underset{N\ D\ T}{\binom{5}{3,\ 1,\ 1}} \cdot \underset{U\ O}{\binom{3}{2,\ 1}} = 960$$