1.

a) $3a_{n+1} - 5a_n = 0$

$3r - 5 = 0 \qquad r_1 = \frac{5}{3}$

$3 \cdot C \cdot \frac{5^{n+1}}{3^{n+1}} - 5 \cdot C \cdot \frac{5^n}{3^n} = 5 \cdot \frac{5^n}{3^n} C - 5 \cdot \frac{5^n}{3^n} C = 0$

RS: $a_n = \left(\frac{5}{3}\right)^n$

RO: $C \cdot \left(\frac{5}{3}\right)^n$

b) $a_{n+2} = 6a_{n+1} - 8a_n$

$a_{n+2} - 6a_{n+1} + 8a_n = 0$

$L = C_1 \cdot 2^{n+2} + C_2 \cdot 4^{n+2} = 4C_1 \cdot 2^n + 16C_2 \cdot 4^n$

$r^2 - 6r + 8 = 0$

$r^2 - 4r - 2r + 8 = r(r-4) - 2(r-4) = (r-2)(r-4)$

$R = 6\left[C_1 2^{n+1} + C_2 4^{n+1}\right] - 8\left[C_1 2^n + C_2 4^n\right]$

$= 12C_1 2^n + 24C_2 4^n - 8C_1 2^n - 8C_2 4^n$

$= 4C_1 2^n + 16C_2 4^n$

$a_n = 2^n \quad \vee \quad a_n = 4^n$

$L = R$

RO: $a_n = C_1 2^n + C_2 4^n$

c) $a_{n+2} = 6a_{n+1} - 5a_n$

$a_{n+2} - 6a_{n+1} + 5a_n = 0$

$L = C_1 + 25C_2 \cdot 5^n$

$r^2 - 6r + 5 = r^2 - 5r - r + 5 = (r-1)(r-5) = 0$

$R = 6\left[C_1 + 5C_2 \cdot 5^n\right] - 5\left[C_1 + C_2 \cdot 5^n\right]$

$= C_1 + 25C_2 \cdot 5^n$

RS: $a_n = 1^n \quad \vee \quad a_n = 5^n$

$L = R$

RO: $a_n = C_1 + C_2 5^n$

d) $a_{n+2} = 6a_{n+1} - 9a_n$

$a_{n+2} - 6a_{n+1} + 9a_n = 0$

$r^2 - 6r + 9 = (r-3)^2 = 0$

RS: $a_n = 3^n \quad \vee \quad a_n = n3^n$

RO: $a_n = C_1 \cdot 3^n + C_2 \cdot n3^n$

$C_1 \cdot 3^{n+2} + C_2 \cdot (n+2) \cdot 3^{n+2} - 6\left[C_1 \cdot 3^{n+1} + C_2(n+1) \cdot 3^{n+1}\right] + 9\left[C_1 \cdot 3^n + C_2 \cdot n \cdot 3^n\right]$

$9C_1 \cdot 3^n + 9nC_2 \cdot 3^n + 18C_2 3^n - 18C_1 \cdot 3^n - 18nC_2 3^n - 18C_2 \cdot 3^n + 9C_1 \cdot 3^n + 9nC_2 \cdot 3^n = 0$

e) $a_{n+2} = 6a_{n+1} - 10a_n$

$a_{n+2} - 6a_{n+1} + 10a_n = 0$

$r^2 - 6r + 10 = 0 \qquad \Delta = 36 - 40 = -4$

$r_1 = \frac{6 + 2i}{2} = 3 + i \qquad r_2 = 3 - i$

RS: $a_n = (3+i)^n \quad \vee \quad a_n = (3-i)^n$

RO: $a_n = C_1 (3+i)^n + C_2 (3-i)^n$

f) $a_{n+2} = 2a_{n+1} - a_n$

$a_{n+2} - 2a_{n+1} + a_n = 0$

$r^2 - 2r + 1 = (r-1)^2 = 0$

RS: $a_n = 1^n \quad \vee \quad a_n = n \cdot 1^n$

RO: $C_1 + C_2 n$

$C_1 + C_2(n+2) - 2\left[C_1 + C_2(n+1)\right] + C_1 + C_2 n$

$= C_1 + nC_2 + 2C_2 - 2C_1 - 2nC_2 - 2C_2 + C_1 + C_2 n = 0$

2.

a) $3a_{n+1} - 5a_n = 5^n - 3^n + 2^{n+1} - 4$

$3r - 5 = 0$

RSRJ: $a_n = \left(\frac{5}{3}\right)^n$

RORJ: $a_n = C_1 \left(\frac{5}{3}\right)^n$

RSRN:

$a_n = A\cdot 5^n - B\cdot 3^n + C\cdot 2^n + D$

$3a_{n+1} - 5a_n = 15A\cdot 5^n - 9B\cdot 3^n + 6C\cdot 2^n + 3D$

$5^n \quad -3^n \quad +2\cdot 2^n \quad -4$

$a_n = \frac{1}{15}\cdot 5^n - \frac{1}{9}\cdot 3^n + \frac{1}{3}\cdot 2^n - \frac{4}{3}$

RORN: $a_n = \frac{1}{15}\cdot 5^n - \frac{1}{9}\cdot 3^n + \frac{1}{3}\cdot 2^n - \frac{4}{3} + C_1\left(\frac{5}{3}\right)^n$

b) $a_{n+1} - a_n = n+1$

RSRJ:

$r - 1 = 0$

$a_n = 1^n$

RORJ: $a_n = C$

RSRN:

$a_n = An^2 + Bn + C$

$a_{n+1} - a_n = A(n+1)^2 + B(n+1) + C - An^2 - Bn - C$

$= \cancel{An^2} + 2An + A \cancel{+Bn} + B \cancel{+C} \cancel{-An^2} \cancel{-Bn} \cancel{-C}$

$n+1 = 2An + (A+B)$

$$\begin{cases} 1 = 2A \\ 1 = A+B \end{cases} \qquad \begin{matrix} A = \frac{1}{2} \\ B = \frac{1}{2} \end{matrix}$$

$a_n = \frac{1}{2}n^2 + \frac{1}{2}n = \frac{1}{2}n(n+1)$

RORJ: $\frac{1}{2}n(n+1) + C$

c) $a_{n+1} - a_n = (n+1)^2$

$r - 1 = 0$

RORJ: $a_n = C$

RSRN:

$(n+1)^2 = \underbrace{1^n\cdot(n^2+2n+1)}$ 1-krotny pierwiastek wielomianu charakterystycznego

$a_n = n^1\cdot(An^2 + Bn + C)$

$a_{n+1} - a_n = A(n+1)^3 + B(n+1)^2 + C(n+1) - An^3 - Bn^2 - Cn$

$= \cancel{An^3} + 3An^2 + 3An + A + \cancel{Bn^2} + 2Bn + B + \cancel{Cn} + C \cancel{-An^3} \cancel{-Bn^2} \cancel{-Cn}$

$n^2 + 2n + 1 = 3An^2 + (3A + 2B)n + (A+B+C)$

$$\begin{cases} 1 = 3A \\ 2 = 3A + 2B \\ 1 = A + B + C \end{cases} \qquad \begin{cases} A = \frac{1}{3} \\ B = \frac{1}{2} \\ C = \frac{1}{6} \end{cases}$$

RORN: $a_n = \frac{1}{3}n^3 + \frac{1}{2}n^2 + \frac{1}{6}n + C$

d) $a_{n+2} - 6a_{n+1} + 8a_n = 3^n + 1$

RORJ: $a_n = C_1 2^n + C_2 4^n$

RSRN:

$a_n = A\cdot 3^n + B$

$a_{n+2} - 6a_{n+1} + 8a_n = 9A\cdot 3^n + B - 18A\cdot 3^n - 6B + 8A\cdot 3^n + 8B$

$3^n + 1 = -A\cdot 3^n + 3B$

$A = -1 \quad B = \frac{1}{3}$

RORN: $-3^n + \frac{1}{3} + C_1\cdot 2^n + C_2\cdot 4^n$

e) $a_{n+2} - 2a_{n+1} + a_n = 1$

$r^2 - 2r + 1 = (r-1)^2 = 0$

RORJ: $a_n = C_1 + C_2 n$

RSRN:

1 jest 2-krotnym pierwiastkiem

$a_n = A\cdot n^2$

$a_{n+2} - 2a_{n+1} + a_n = A(n+2)^2 - 2A(n+1)^2 + An^2$

$= \cancel{An^2} + \cancel{4An} + 4A - \cancel{2An^2} - \cancel{4An} - 2A + \cancel{An^2}$

$1 = 2A \quad A = \frac{1}{2}$

RORN: $\frac{1}{2}n^2 + C_1 + C_2 n$

f) $a_{n+2} = 6a_{n+1} - 5a_n + 1$

$a_{n+2} - 6a_{n+1} + 5a_n = 1$

RORJ: $a_n = C_1 + C_2 5^n$

RSRN:

$a_n = A\cdot n^1$

$a_{n+2} - 6a_{n+1} + 5a_n = A(n+2) - 6A(n+1) + 5An$

$= \cancel{An} + 2A - \cancel{6An} - 6A + \cancel{5An}$

$1 = -4A \quad A = -\frac{1}{4}$

RORN: $a_n = -\frac{1}{4}n + C_1 + C_2\cdot 5^n$

g) $a_{n+2} = a_{n+1} + a_n + 2^n$

$a_{n+2} - a_{n+1} - a_n = 2^n$

$r^2 - r - 1 = 0 \quad \Delta = 5$

$r_1 = \frac{1-\sqrt{5}}{2} \quad r_2 = \frac{1+\sqrt{5}}{2}$

RORJ: $a_n = C_1\left(\frac{1-\sqrt{5}}{2}\right)^n + C_2\left(\frac{1+\sqrt{5}}{2}\right)^n$

RSRN:

$a_n = A\cdot 2^n$

$a_{n+2} - a_{n+1} - a_n = 4A\cdot 2^n - 2A\cdot 2^n - A\cdot 2^n = A\cdot 2^n$

$2^n = A\cdot 2^n$

$A = 1$

RORJ: $a_n = 2^n + C_1\left(\frac{1-\sqrt{5}}{2}\right)^n + C_2\left(\frac{1+\sqrt{5}}{2}\right)^n$

h) $a_{n+2} - 2a_{n+1} + a_n = n$

$(r-1)^2 = 0$

RORJ: $a_n = C_1 + C_2 n$

RSRN

$a_n = (An+B)\cdot n^2 = An^3 + Bn^2$

$a_{n+2} - 2a_{n+1} + a_n = A(n+2)^3 + B(n+2)^2 - 2A(n+1)^3 - 2B(n+1)^2 + An^3 + Bn^2$

$= \cancel{An^3} + \cancel{6An^2} + 12An + 8A + \cancel{Bn^2} + \cancel{4Bn} + 4B$

$- \cancel{2An^3} - \cancel{6An^2} - 6An - 2A - \cancel{2Bn^2} - \cancel{4Bn} - 2B + \cancel{An^3} + \cancel{Bn^2}$

$n = 6An + (6A + 2B)$

$\begin{cases} 1 = 6A \\ 0 = 6A + 2B \end{cases} \quad A = \frac{1}{6} \quad B = -\frac{1}{2}$

RORN: $\frac{1}{6}n^3 - \frac{1}{2}n^2 + C_1 + C_2 n$

3.

a) $\begin{cases} a_1 = 0 \\ a_2 = 0 \\ a_{n+2} - 2a_{n+1} + a_n = n \quad \text{dla } n \geq 1 \end{cases}$

$a_n = \frac{1}{6}n^3 - \frac{1}{2}n^2 + C_1 + C_2 n$ (zad. 2h)

$a_1 = \frac{1}{6} - \frac{1}{2} + C_1 + C_2 = 0$

$a_2 = \frac{8}{6} - 2 + C_1 + 2C_2 = 0$

$$\left[\begin{array}{cc|c} 1 & 1 & \frac{1}{3} \\ 1 & 2 & \frac{2}{3} \end{array}\right] \xrightarrow[w_2 - w_1]{} \left[\begin{array}{cc|c} 1 & 1 & \frac{1}{3} \\ 0 & 1 & \frac{1}{3} \end{array}\right] \xrightarrow{-w_2} \left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & \frac{1}{3} \end{array}\right] \quad \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{3} \end{bmatrix}$$

$a_n = \frac{1}{6}n^3 - \frac{1}{2}n^2 + \frac{1}{3}n$

b) $\begin{cases} a_0 = 0 \\ a_1 = 1 \\ a_{n+2} = 2(a_{n+1} - a_n) \quad \text{dla } n \geq 0 \end{cases}$

$a_{n+2} - 2a_{n+1} + 2a_n = 0$

$r^2 - 2r + 2 = 0 \qquad \Delta = -4$

$r_1 = \frac{2-2i}{2} = 1 - i \qquad r_2 = 1 + i$

$a_n = C_1(1-i)^n + C_2(1+i)^n$

$a_0 = C_1 + C_2 = 0$

$a_1 = C_1(1-i) + C_2(1+i) = 1$

$C_2 = -C_1$

$C_1(1-i) - C_1(1+i) = 1$

$C_1(1-i-1-i) = 1$

$-2iC_1 = 1$

$C_1 = -\frac{1}{2} \cdot \frac{1}{i} = -\frac{1}{2} \cdot \frac{i}{i^2} = \frac{i}{2}$

$a_n = \frac{1}{2}i(1-i)^n - \frac{1}{2}i(1+i)^n$