

1.

$$P(A) = \frac{1}{3} \quad P(A \cap B) = \frac{1}{4} \quad P(A \cup B) = \frac{2}{3}$$

$$P(B') = \frac{2}{3}$$

$$P(A) = 1 - P(A') = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(A \cap B') = \frac{5}{12}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(B \setminus A) = 0$$

$$P(B) = P(A \cup B) - P(A) + P(A \cap B) = \frac{2}{3} - \frac{2}{3} + \frac{1}{4} = \frac{1}{4}$$

$$P(B') = 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(B \setminus A) = P(B) - P(A \cap B) = \frac{1}{4} - \frac{1}{4} = 0$$

$$A \cap B' = A \cap (\Omega \setminus B) = A \setminus B$$

$$P(A \setminus B) = P(A) - P(A \cap B) = \frac{2}{3} - \frac{1}{4} = \frac{5}{12}$$

2.

3 × C    4 × B    losując 3 kule

A - 3 × C

$$|\Omega| = \binom{7}{3} = \frac{7!}{3!4!} = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} = 35$$

B - 2 × C, 1 × B

C - 3 × C ∨ 3 × B

Ω - 3-elementowe podzbiory zbioru 7-elementowego

$$P(A) = \frac{\binom{3}{3}}{\binom{7}{3}} = \frac{1}{35}$$

$$P(B) = \frac{\binom{3}{2} \binom{4}{1}}{\binom{7}{3}} = \frac{3 \cdot 4}{35} = \frac{12}{35}$$

$$P(C) = \frac{\binom{3}{3} + \binom{4}{3}}{\binom{7}{3}} = \frac{1 + 4}{35} = \frac{1}{7}$$

3.

10 rzutów kostką     $\Omega = \{(x_1, \dots, x_{10}) : x_i \in \{1, 2, 3, 4, 5, 6\}\} = \{1, 2, \dots, 6\}^{10}$

A - nie pojawi się najmniejszy 1 ze ścianek

$A_i$  - nie pojawi się ścianka nr i w żadnym z 10 rzutów

$$A_i = (\{1, 2, 3, 4, 5, 6\} \setminus \{i\})^{10}$$

$$A = A_1 \cup A_2 \cup A_3 \cup \dots \cup A_6$$

$$P(A) = P\left(\bigcup_{i=1}^6 A_i\right) = \sum_{i=1}^6 P(A_i) - \sum_{1 \leq i < j \leq 6} P(A_i \cap A_j) + \sum_{1 \leq i < j < k \leq 6} P(A_i \cap A_j \cap A_k) - \sum_{1 \leq i < j < k < l \leq 6} P(A_i \cap A_j \cap A_k \cap A_l) + \sum_{1 \leq i < j < k < l < m \leq 6} P(A_i \cap A_j \cap A_k \cap A_l \cap A_m) - P\left(\bigcap_{i=1}^6 A_i\right)$$

$$\binom{6}{1} \cdot \frac{5^{10}}{6^{10}} - \binom{6}{2} \cdot \frac{4^{10}}{6^{10}} + \binom{6}{3} \frac{3^{10}}{6^{10}} - \binom{6}{4} \cdot \frac{2^{10}}{6^{10}} + \binom{6}{5} \frac{1^{10}}{6^{10}} - 0$$

$$\frac{1}{6^{10}} (6 \cdot 5^{10} - 15 \cdot 4^{10} + 20 \cdot 3^{10} - 15 \cdot 2^{10} + 6) = \frac{101923}{139212} \approx 0.73$$

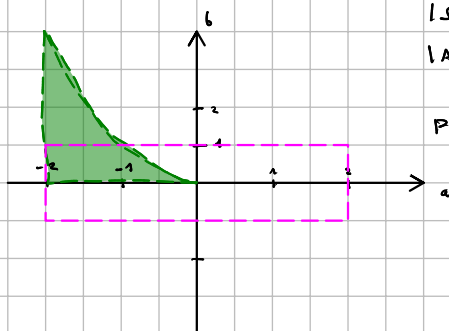
4.

$A: x^2 + 2ax + b = 0$  ma 2 dodatnie pierwiastki rzeczywiste

$$\Omega = \{(a, b): a \in [-2, 2] \wedge b \in [-1, 1]\} = [-2, 2] \times [-1, 1]$$

$$\begin{cases} \Delta > 0 \\ x_1, x_2 > 0 \\ x_1 + x_2 > 0 \end{cases} \Leftrightarrow \begin{cases} 4a^2 - 4b > 0 \\ b > 0 \\ -2a > 0 \end{cases} \Leftrightarrow \begin{cases} a^2 > b \\ b > 0 \\ a < 0 \end{cases}$$

$$A = \{(a, b) \in \Omega: a^2 > b, b > 0, a < 0\}$$



$$|\Omega| = 4 \cdot 2 = 8$$

$$|A| = 1 \cdot 1 \cdot \int_0^1 x^2 dx = 1 + \left[\frac{1}{3}x^3\right]_0^1 = \frac{4}{3}$$

$$P(A) = \frac{\frac{4}{3}}{8} = \frac{1}{6}$$

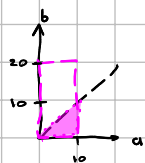
5.

$$\Omega = \{(a, b): a \in [0, 10] \wedge b \in [0, 20]\}$$

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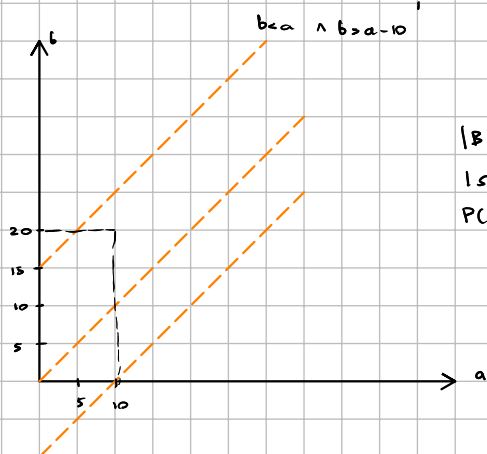
$$A: b < a$$

$$P(A) = \frac{\frac{1}{2} \cdot 10 \cdot 10}{10 \cdot 20} = \frac{1}{4}$$



A	B	A'	B'
A	B	B'	A'
B	A	B'	A'

$$B: a \leq b \leq a+15 \vee b < a < b+10$$



$$|B| = 20 \cdot 10 - \frac{1}{2} \cdot 5 \cdot 5 = \frac{375}{2}$$

$$|\Omega| = 200$$

$$P(B) = 0.9375$$

6.

$$\Omega = \{x, ox, oox, \dots\}$$

x - wypadła 2 lub 3

o - nie wypadła 2 ani 3

$$\Omega = \{1, 2, 3, \dots\}$$

$$P(\{k\}) = \left(\frac{1}{6}\right)^{k-1} \cdot \frac{2}{6}$$

A: rzuty były samo razę

$$P(A) = P(\{1ox\}) + P(\{1oox\}) + P(\{1oooox\}) + \dots$$

$$P(A) = \frac{1}{6} \cdot \frac{2}{6} + \left(\frac{1}{6}\right)^3 \cdot \frac{2}{6} + \left(\frac{1}{6}\right)^5 \cdot \frac{2}{6} + \dots$$

$$= \sum_{i=0}^{\infty} \frac{2}{6} \cdot \left(\frac{1}{6}\right)^{2i+1} = \frac{1}{3} \sum_{i=0}^{\infty} \left(\frac{1}{3}\right)^{2i} \cdot \frac{2}{3} = \frac{2}{9} \sum_{i=0}^{\infty} \left(\frac{1}{9}\right)^i = \frac{2}{9} \cdot \frac{1}{1-\frac{1}{9}} = \frac{2}{9} \cdot \frac{9}{8} = \frac{2}{8} = \frac{1}{4}$$

7.

$$\begin{array}{l} 0-2, 10-12, 20-22, \dots \\ 0-3, 15-18, \dots \end{array}$$

$$\Omega = [0, 60]$$

$$A = [0, 2] \cup [30, 32] \quad |A| = 4$$

$$B = (2, 3] \cup [10, 12] \cup [15, 18] \dots$$

Uzór powtórzenia się po okresie 30 s ( $\text{len}(2+8, 3+12)$ )

Sygnały nakładają się w przedziałach  $[30n, 30n+2]$

$$P(A) = \frac{2(2)}{60} = \frac{2}{30} = \frac{1}{15}$$

$$P(B) = \frac{2(3 \cdot 2 + 2 \cdot 3 - 2 \cdot 2)}{60} = \frac{8}{30} = \frac{4}{15}$$

$$P(C) = \frac{2(3 \cdot 2 + 2 \cdot 3 - 2)}{60} = \frac{10}{30} = \frac{1}{3}$$