

1.

rysunek 1 $\rightarrow r = 0.3$

rysunek 2 $\rightarrow r = -0.05$

rysunek 3 $\rightarrow r = 0$

rysunek 4 $\rightarrow r = -1$

rysunek 5 $\rightarrow r = -0.8$

2.

$$\sum_{k=1}^n (x_k - \bar{x})(y_k - \bar{y}) \stackrel{?}{=} \sum_{k=1}^n (x_k - \bar{x}) y_k \stackrel{?}{=} \sum_{k=1}^n (y_k - \bar{y}) x_k$$

$$\bar{x} = \frac{1}{n} \sum_{k=1}^n x_k \quad \bar{y} = \frac{1}{n} \sum_{k=1}^n y_k \quad n\bar{x} = \sum_{k=1}^n x_k$$

$$\sum_{k=1}^n (x_k - \bar{x})(y_k - \bar{y}) = \sum_{k=1}^n x_k y_k - x_k \bar{y} - y_k \bar{x} + \bar{x} \bar{y}$$

$$\sum_{k=1}^n x_k (y_k - \bar{y}) + \bar{x} (\bar{y} - y_k)$$

$$\sum_{k=1}^n \bar{x} (y_k - \bar{y}) = \bar{x} \cdot \sum_{k=1}^n (y_k - \bar{y}) = \bar{x} \cdot \left[\sum_{k=1}^n y_k - \sum_{k=1}^n \bar{y} \right] = \bar{x} (n\bar{y} - n\bar{y}) = 0$$

3.

$$S_x = \{-2, -1, 0, 1, 2\}$$

$$\forall l \in S_x \quad P(X=l) = \frac{1}{5}$$

$$Y = X^2$$

$$\text{cov}(X, Y) = E(X \cdot Y) - E X \cdot E Y = E(X^3) - E X \cdot E(X^2)$$

$$E X = \frac{1}{5}(-2 - 1 + 0 + 1 + 2) = 0$$

$$E X^2 = \frac{1}{5}(4 + 1 + 0 + 1 + 4) = 2$$

$$E X^3 = \frac{1}{5}(-8 - 1 + 0 + 1 + 8) = 0$$

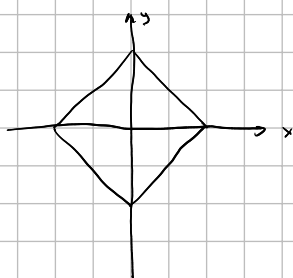
$$\text{cov}(X, Y) = 0 - 0 \cdot 2 = 0 \Rightarrow X, Y \text{ sind unabhängig}$$

$$\left. \begin{array}{l} P(X=0, Y=4) = 0 \\ P(X=0)P(Y=4) = \frac{1}{5} \cdot \frac{2}{5} \end{array} \right\} \text{ nicht unabhängig}$$

4.

$$D = \{(x, y) : |x| + |y| \leq 1\} \quad (X, Y) \sim U(D)$$

$$E(2X+3Y) \quad V(X+Y) \quad |D|=2$$



$$f_{xy}(x, y) = \frac{1}{2} 1_D(x, y)$$

(EX, EY) - střední hodnoty

$$(EX, EY) = (0, 0)$$

$$E(2X+3Y) = 2EX + 3EY = 0$$

$$V(X+Y) = VX + VY + 2\text{cov}(X, Y)$$

$$\begin{aligned} EX^2 &= \iint_{\mathbb{R}^2} x^2 f_{xy}(x, y) dx dy = \iint_D \frac{1}{2} x^2 dx dy = \int_{-1}^1 \int_{|x|-1}^{1-|x|} \frac{1}{2} x^2 dy dx = \int_{-1}^1 \frac{1}{2} x^2 (1-|x| - |x| + 1) dx \\ &= \int_{-1}^1 x^2 (1-|x|) dx = \int_{-1}^1 x^2 - x^2 |x| dx \\ &= 2 \int_0^1 x^2 - x^3 dx = 2 \left(\left(\frac{1}{3} x^3 \right) \Big|_0^1 - \left(\frac{1}{4} x^4 \right) \Big|_0^1 \right) \\ &= 2 \left(\frac{1}{3} - \frac{1}{4} \right) = \frac{1}{6} \end{aligned}$$

$$E(XY) = \iint_{\mathbb{R}^2} xy f_{xy}(x, y) dx dy = \int_{-1}^1 \int_{|x|-1}^{1-|x|} \frac{1}{2} xy dy dx = 0$$

nepárovatelná

$$VX = \frac{1}{6} - 0^2 = \frac{1}{6}$$

$$\text{cov}(X, Y) = EXY - EXEY = 0$$

$$V(X+Y) = \frac{1}{6} + \frac{1}{6} + 0 = \frac{1}{3}$$

5.

$$(X, Y) \sim N\left(\begin{bmatrix} 5 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix}\right)$$

$$EX = 5 \quad VX = 3$$

$$EY = 1 \quad VY = 4$$

$$\text{cov}(X, Y) = -1$$

$$E(X(X+3Y)) = E(X^2 + 3XY) = EX^2 + 3EXY = 28 + 3 \cdot 4 = 40$$

$$\text{cov}(X, Y) = EXY - EX \cdot EY$$

$$EXY = \text{cov}(X, Y) + EX \cdot EY = -1 + 5 \cdot 1 = 4$$

$$VX = EX^2 - (EX)^2$$

$$EX^2 = VX + (EX)^2 = 3 + 25 = 28$$

$$V(6X + Y - 3) = V(6X) + 2 \text{cov}(6X, Y - 3) + V(Y - 3)$$

$$= 36VX + 2 \cdot 6 \text{cov}(X, Y) \cdot VY = 36 \cdot 3 + 12 \cdot (-1) + 4 = 100$$

$$\text{cov}(X + 4Y, 2X - 4Y) =$$

$$= \text{cov}(X, 2X) + \text{cov}(X, -4Y) + \text{cov}(4Y, 2X) + \text{cov}(4Y, -4Y)$$

$$= 2 \text{cov}(X, X) - 4 \text{cov}(X, Y) + 8 \text{cov}(Y, X) - 16 \text{cov}(Y, Y)$$

$$= 2VX - 4 \text{cov}(X, Y) - 16VY$$

$$= 2 \cdot 3 + 4 \cdot (-1) - 16 \cdot 4 = -62$$

Albo uvažujme vzhled na přechodové linece

$$\begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} X + 4Y \\ 2X - 4Y \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 4 \\ 2 & -4 \end{bmatrix}}_A \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$C_{U,V} = A C_{X,Y} A^T$$

$$C_{U,V} = \begin{bmatrix} 1 & 4 \\ 2 & -4 \end{bmatrix} \cdot \begin{bmatrix} 3 & -1 \\ -1 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 4 & -4 \end{bmatrix} = \dots$$

6.

x^y	0	1	2	
1	.1	.1	.2	.4
2	.1	.2	.3	.6
	.2	.3	.5	

$$\text{cov}(X, Y) = EXY - EX \cdot EY = 2.1 - 1.6 \cdot 1.3 = 0.02 > 0 \quad \text{slabavai}$$

skaitinis

$$EXY = .1 + .4 + .4 + 1.2 = 2.1$$

$$EX = .4 + 1.2 = 1.6 \quad EX^2 = .4 + 2.4 = 2.8$$

$$EY = .3 + 1 = 1.3 \quad EY^2 = .3 + 2 = 2.3$$

$$VX = 2.8 - (1.6)^2 = 0.24$$

$$VY = 2.3 - (1.3)^2 = 0.61$$

$$\rho(X, Y) = \frac{\text{cov}(X, Y)}{\sqrt{VX} \sqrt{VY}} \approx 0.05 \quad \text{slabu koreliacija}$$

$$V(X+Y) = VX + VY + 2\text{cov}(X, Y) = 0.87$$

$$E(X+Y) = EX + EY = 2.9$$

7.

 $\{0, 1, 2, 3\}$ a) losujemy z zwracaniem $\Rightarrow X, Y$ niezależne $\Rightarrow \rho_{X,Y} = 0$ b) losujemy bez zwracania $P(X=k, Y=l) = \begin{cases} 0 & k=l \\ \frac{1}{4} \cdot \frac{1}{3} & k \neq l, k, l \in \{0, 1, 2, 3\} \end{cases}$

$x \backslash y$	0	1	2	3	
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$
2	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$
3	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{4}$
	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	

$$EX = EY = \frac{1}{4}(0+1+2+3) = \frac{3}{2}$$

$$EX^2 = \frac{1}{4}(0+1+4+9) = \frac{7}{2}$$

$$VX = VY = \frac{4}{4} - \frac{9}{4} = \frac{5}{4}$$

$$EXY = 2 \cdot \frac{1}{12}(2+3+6) = \frac{11}{6}$$

$$\text{cov}(X, Y) = EXY - EXEY = \frac{11}{6} - \frac{9}{4} = -\frac{5}{12}$$

$$\rho = \frac{-\frac{5}{12}}{\sqrt{\frac{5}{4} \cdot \frac{5}{4}}} = -\frac{1}{3}$$

8.

X - przychód

$$EX = 30\,000$$

$$\sqrt{VX} = 3000$$

Y - koszt

$$EY = 20\,000$$

$$\sqrt{VY} = 4000$$

$$Z = X - Y$$

$$EZ = EX - EY = 10\,000$$

$$\rho_{XY} = 0.75$$

$$\rho_{XY} = \frac{\text{cov}(X, Y)}{\sqrt{VX} \cdot \sqrt{VY}} \Rightarrow \text{cov}(X, Y) = 0.75 \cdot 3000 \cdot 4000 = 9000\,000$$

$$VZ = VX - 2 \text{cov}(X, Y) + VY = 9e6 - 18e6 + 16e6 = 7e6$$

$$\sigma_Z = \sqrt{7} \cdot 1000$$