Zestaw 1

1.

$x R y \iff \log_2\left(\frac{x}{y}\right) \in \mathbb{Z}$

✓ · zwrotna $\log_2\left(\frac{x}{x}\right) = \log_2(1) = 0 \in \mathbb{Z}$

✓ · symetryczna $\log_2\left(\frac{x}{y}\right) = \log_2\left(\left(\frac{y}{x}\right)^{-1}\right) = -\log_2\left(\frac{y}{x}\right)$

$k \in \mathbb{Z} \iff -k \in \mathbb{Z}$

więc $xRy \implies yRx$

× · antysymetryczna $(1R2 \wedge 2R1) \wedge 1 \neq 2$

× · spójna $1 \not R 3 \wedge 3 \not R 1 \wedge 1 \neq 3$

✓ · przechodnia $xRy \wedge yRz \iff \log_2\left(\frac{x}{y}\right) \in \mathbb{Z} \wedge \log_2\left(\frac{y}{z}\right) \in \mathbb{Z}$

$\implies x = 2^m y \wedge y = 2^n z \quad m,n \in \mathbb{Z}$

$\implies x = 2^{m+n} z$

$\implies \log_2\left(\frac{x}{z}\right) = \log_2\left(\frac{2^{m+n} z}{z}\right) = m+n \in \mathbb{Z}$

$\implies xRz$

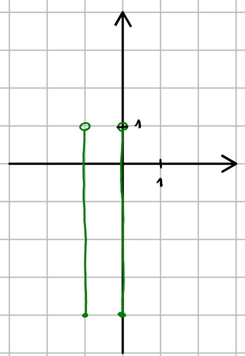
jest relacją równoważności

$[1]_R = \{n \in \mathbb{Z} : 2^n\} = \{\dots \frac{1}{4}, \frac{1}{2}, 1, 2, 4, \dots\}$

2.

$f: \mathbb{R}^2 \to \mathbb{R} \quad f(x,y) = |3x+y|$

a)



$R_f \subset [0,+\infty)$

$f(0,0) = |3\cdot 0 + 0| = 0$

$f(-1,1) = |-3+1| = 2$

$f(x_{min}, y_{min}) = f(-1,-4) = |-3-4| = 7$

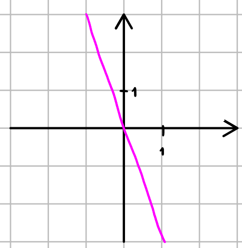
$f(\{0\} \times [-4,1)) = [0,4]$

$f(\{-1\} \times [-4,1)) = (2,7]$

$f(\{-1,0\} \times [-4,1)) = [0,7]$

b) $f^{-1}(\{0\}) = \{(x,y) \in \mathbb{R}^2 : |3x+y| = 0\} = \{(x,-3x) : x \in \mathbb{R}\}$

$|3x+y| = 0 \iff 3x+y = 0 \iff y = -3x$



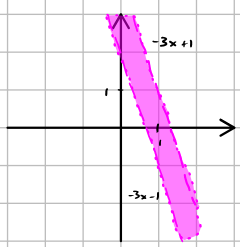
c) $f^{-1}((-\infty,1)) = f^{-1}([0,1))$

$f(x,y) = |3x+y| \geq 0$

$|3x+y| < 1$

$3x+y < 1 \wedge 3x+y > -1$

$y < -3x+1 \wedge y > -3x-1$



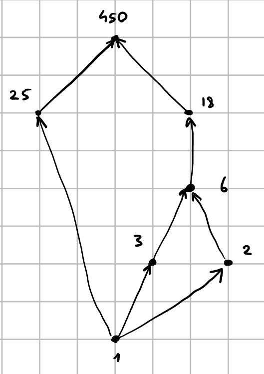
f jest surjekcją $\iff f(\mathbb{R}^2) = \mathbb{R}$

nieprawda, bo $f(\mathbb{R}^2) = [0,+\infty)$

nie jest surjekcją

3.

$(\{1, 2, 3, 6, 18, 25, 450\}, |)$



element największy: 450

element najmniejszy: 1

Łańcuch maksymalnej długości:

$\{1, 3, 6, 18, 450\}$

antyłańcuch maksymalnej długości:

$\{2, 3, 25\}$

Jest kratą bo dla każdej pary istnieje

kres górny - wspólna wielokrotność (450 lub mniejsza)

kres dolny - wspólny dzielnik (1 lub większy)

4.

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} -1 & 2 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow[\frac{1}{2}\cdot w_3]{(-1)\cdot w_1,\ (-1)\cdot w_2} \left[\begin{array}{ccc|ccc} 1 & -2 & 0 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right] \xrightarrow{w_1 + 2w_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & -1 & -2 & 0 \\ 0 & 1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right] \xrightarrow[w_2 + w_3]{w_1 + 2w_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & -2 & 1 \\ 0 & 1 & 0 & 0 & -1 & \frac{1}{2} \\ 0 & 0 & 1 & 0 & 0 & \frac{1}{2} \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} -1 & -2 & 1 \\ 0 & -1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} -1 & -2 & 1 \\ 0 & -1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} -1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

5.

$$\begin{cases} x_1 + x_2 + x_4 + 2x_5 = 1 \\ -2x_1 - 2x_2 - x_3 + x_4 + 2x_5 = 1 \\ 4x_1 + 4x_2 + x_3 - x_4 = 1 \\ -4x_1 - 4x_2 - 2x_3 + x_4 + 3x_5 = 2 \end{cases} \qquad A = \begin{bmatrix} 1 & 1 & 0 & 1 & 2 \\ -2 & -2 & -1 & 1 & 2 \\ 4 & 4 & 1 & -1 & 0 \\ -4 & -4 & -2 & 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 2 & 1 \\ -2 & -2 & -1 & 1 & 2 & 1 \\ 4 & 4 & 1 & -1 & 0 & 1 \\ -4 & -4 & -2 & 1 & 3 & 2 \end{array}\right] \xrightarrow[w_4 + w_3]{w_2 + 2w_1} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & 3 & 6 & 3 \\ 4 & 4 & 1 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & 3 & 3 \end{array}\right] \xrightarrow{w_3 - 4w_1} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & -1 & 3 & 6 & 3 \\ 0 & 0 & 1 & -5 & -8 & -3 \\ 0 & 0 & -1 & 0 & 3 & 3 \end{array}\right] \xrightarrow[w_4 + w_3]{w_2 + w_3} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & -2 & -2 & 0 \\ 0 & 0 & 1 & -5 & -8 & -3 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{-5} & \cancel{-5} & \cancel{0} \end{array}\right]$$

$w_4 = \frac{5}{2} w_2$

$$\xrightarrow{-\frac{1}{2}\cdot w_2} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -5 & -8 & -3 \end{array}\right] \xrightarrow[w_3 + 5w_2]{w_1 - w_2} \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & -3 & -3 \end{array}\right]$$

$\mathrm{rank}(A) = \mathrm{rank}(A|B) = 3$ układ ma nieskończenie wiele rozwiązań zależnych od 2 parametrów

$$\begin{cases} x_1 + x_2 + x_5 = 1 \\ x_4 + x_5 = 0 \\ x_3 - 3x_5 = -3 \end{cases} \qquad \begin{cases} x_2 = 1 - x_1 - x_5 \\ x_4 = -x_5 \\ x_3 = -3 + 3x_5 \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -3 \\ 0 \\ 0 \end{bmatrix} + x_1 \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_5 \begin{bmatrix} 0 \\ -1 \\ 3 \\ -1 \\ 1 \end{bmatrix} \qquad x_1, x_5 \in \mathbb{R}$$

Zestaw 2

1. $R \subset (0,+\infty)^2 \qquad xRy \iff \log_{10}\left(\frac{y}{x}\right) \in \mathbb{Q}$

✓ • zwrotna $\qquad \log_{10}\left(\frac{x}{x}\right) = \log_{10}(1) = 0 \in \mathbb{Q}$

✓ • symetryczna
$$\begin{aligned} xRy &\iff \log_{10}\left(\tfrac{y}{x}\right) \in \mathbb{Q} \\ &\iff -\log_{10}\left(\tfrac{y}{x}\right) \in \mathbb{Q} \\ &\iff \log_{10}\left(\left(\tfrac{y}{x}\right)^{-1}\right) \in \mathbb{Q} \\ &\iff \log_{10}\left(\tfrac{x}{y}\right) \in \mathbb{Q} \\ &\iff yRx \end{aligned}$$

X • antysymetryczna $\qquad 10 R 100 \wedge 100 R 10 \wedge 10 \neq 100$

X • spójna $\qquad 10 R 100 \wedge 100 R 10 \wedge 10 \neq 100$

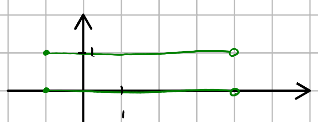
✓ • przechodnia
$$\begin{aligned} xRy \wedge yRz &\iff \log_{10}\left(\tfrac{y}{x}\right) \in \mathbb{Q} \wedge \log_{10}\left(\tfrac{z}{y}\right) \in \mathbb{Q} \\ &\iff y = 10^q x \quad \wedge \quad z = 10^r y \qquad q,r \in \mathbb{Q} \\ &\Rightarrow z = 10^{r+q} x \\ &\Rightarrow \log_{10}\left(\tfrac{z}{x}\right) = \log_{10}\left(\tfrac{10^{r+q}x}{x}\right) = r+q \in \mathbb{Q} \\ &\Rightarrow xRz \end{aligned}$$

jest relacją równoważności

$$[1]_R = \{10^x : x \in \mathbb{Q}\} = \{10, \tfrac{1}{10}, \sqrt[3]{1000}, \dots\}$$

2. $f: \mathbb{R}^2 \to \mathbb{R} \qquad f(x,y) = |x+3y|$

$f([-1,4) \times \{0,1\}) = [0,7)$



$f(-1,1) = |-1+3| = 2$

$f(-1,0) = |-1+0| = 1$

$f(4,1) \to |4+3| = 7$

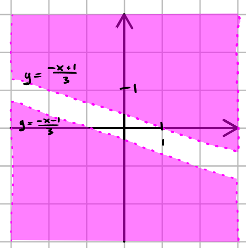
$f(4,0) \to |4+0| = 4$

$f(0,0) = |0+0| = 0$

$f^{-1}((1,+\infty)) = \{(x,y) \in \mathbb{R}^2 : |x+3y| > 1\}$

$x+3y > 1 \quad \vee \quad x+3y < -1$

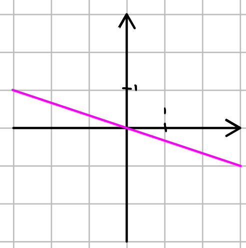
$y > -\frac{1}{3}x + \frac{1}{3} \quad \vee \quad y < -\frac{1}{3}x - \frac{1}{3}$



$f^{-1}(\{0\}) = \{(x,y) \in \mathbb{R}^2 : |x+3y| = 0\}$

$x + 3y = 0$

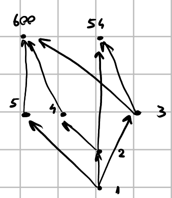
$y = -\frac{1}{3}x$



$f(-1,0) = |-1+0| = 1 = |1+0| = f(1,0)$

$(-1,0) \neq (1,0) \wedge f(-1,0) = f(1,0) \Rightarrow$ nie jest injekcją

3. $(\{1,2,3,4,5,54,600\}, |)$



elementy maksymalne: 54, 600

element najmniejszy: 1

Łańcuch maksymalnej długości

$\{1, 2, 4, 600\}$

antyłańcuch maksymalnej długości

$\{3, 4, 5\}$

zbiór nie jest kratą, bo nie istnieje

$\sup\{54, 600\}$

4.

$$A=\begin{bmatrix}-1&0&0\\2&-1&0\\0&1&-2\end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc}-1&0&0&1&0&0\\2&-1&0&0&1&0\\0&1&-2&0&0&1\end{array}\right]\xrightarrow{w_2+2w_1}\left[\begin{array}{ccc|ccc}-1&0&0&1&0&0\\0&-1&0&2&1&0\\0&1&-2&0&0&1\end{array}\right]\xrightarrow[w_3+w_2]{-1\cdot w_1}\left[\begin{array}{ccc|ccc}1&0&0&-1&0&0\\0&-1&0&2&1&0\\0&0&-2&2&1&1\end{array}\right]\xrightarrow[-\frac{1}{2}\cdot w_3]{-1\cdot w_2}\left[\begin{array}{ccc|ccc}1&0&0&-1&0&0\\0&1&0&-2&-1&0\\0&0&1&-1&-\frac{1}{2}&-\frac{1}{2}\end{array}\right]$$

$$A^{-1}=\begin{bmatrix}-1&0&0\\-2&-1&0\\-1&-\frac{1}{2}&-\frac{1}{2}\end{bmatrix}$$

$$\begin{bmatrix}-1&0&0\\-2&-1&0\\-1&-\frac{1}{2}&-\frac{1}{2}\end{bmatrix}\begin{bmatrix}-1&0&0\\2&-1&0\\0&1&-2\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

5.

$$\left[\begin{array}{ccccc|c}3&3&1&0&0&0\\-2&-2&-1&1&2&1\\-5&-5&-2&1&2&1\\2&2&0&-1&1&2\end{array}\right]\xrightarrow[w_2-w_3]{w_1-w_4}\left[\begin{array}{ccccc|c}1&1&1&1&-1&-2\\3&3&1&0&0&0\\-5&-5&-2&1&2&1\\2&2&0&-1&1&2\end{array}\right]\xrightarrow[\substack{w_3+5w_1\\w_4-2w_1}]{w_2-3w_1}\left[\begin{array}{ccccc|c}1&1&1&1&-1&-2\\0&0&-2&-3&3&6\\0&0&3&6&-3&-9\\\cancel{0}&\cancel{0}&\cancel{-2}&\cancel{-3}&\cancel{3}&\cancel{6}\end{array}\right]\xrightarrow{\frac{1}{3}w_3}\left[\begin{array}{cccccc}1&1&1&1&-1&-2\\0&0&-2&-3&3&6\\0&0&1&2&-1&-3\end{array}\right]\longrightarrow$$

$$\left[\begin{array}{cccccc}1&1&1&1&-1&-2\\0&0&-2&-3&3&6\\0&0&1&2&-1&-3\end{array}\right]\xrightarrow[w_2+2w_3]{w_1-w_3}\left[\begin{array}{ccccc|c}1&1&0&-1&0&1\\0&0&0&1&1&0\\0&0&1&2&-1&-3\end{array}\right]\xrightarrow[w_3-2w_2]{w_1+w_2}\left[\begin{array}{ccccc|c}1&1&0&0&1&1\\0&0&0&1&1&0\\0&0&1&0&-3&-3\end{array}\right]$$

$\text{rank}(A)=\text{rank}(A|B)=3$ jest nieskończenie wiele rozwiązań zależnych od 2 parametrów

$$\begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}=\begin{bmatrix}0\\1\\0\\-3\\0\end{bmatrix}+x_1\begin{bmatrix}1\\-1\\0\\0\\0\end{bmatrix}+x_5\begin{bmatrix}0\\-1\\-1\\3\\1\end{bmatrix}\qquad x_1,x_5\in\mathbb{R}$$

Zestaw 3

1.

$$A = \begin{bmatrix} 0 & 2 & 1 & 0 & 1 \\ 0 & -1 & 3 & 2 & 0 \\ 1 & 2 & 0 & -1 & 3 \\ -1 & 4 & 0 & 3 & -3 \\ 1 & 3 & -1 & 0 & 2 \end{bmatrix} \begin{matrix} w_4+w_3 \\ w_5-w_3 \\ \longrightarrow \\ \\ \\ \end{matrix} \begin{bmatrix} 0 & 2 & 1 & 0 & 1 \\ 0 & -1 & 3 & 2 & 0 \\ 1 & 2 & 0 & -1 & 3 \\ 0 & 6 & 0 & 2 & 0 \\ 0 & 1 & -1 & 1 & -1 \end{bmatrix} \begin{matrix} w_1-2w_5 \\ w_2+w_5 \\ w_3-2w_5 \\ w_4-6w_5 \\ \longrightarrow \end{matrix} \begin{bmatrix} 0 & 0 & 3 & -2 & 3 \\ 0 & 0 & 2 & 3 & -1 \\ 1 & 0 & 2 & -3 & 5 \\ 0 & 0 & 6 & -4 & 6 \\ 0 & 1 & -1 & 1 & -1 \end{bmatrix} \begin{matrix} w_1-w_2 \\ \longrightarrow \\ w_3-w_2 \\ w_4-3w_2 \\ \\ \end{matrix} \begin{bmatrix} 0 & 0 & 1 & -5 & 4 \\ 0 & 0 & 2 & 3 & -1 \\ 1 & 0 & 0 & -6 & 6 \\ 0 & 0 & 0 & -13 & 9 \\ 0 & 1 & -1 & 1 & -1 \end{bmatrix} \begin{matrix} \\ w_2-2w_1 \\ \longrightarrow \\ \\ w_5+w_1 \end{matrix} \begin{bmatrix} 0 & 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 13 & -9 \\ 1 & 0 & 0 & -6 & 6 \\ \cancel{0} & \cancel{0} & \cancel{0} & \cancel{-13} & \cancel{9} \\ 0 & 1 & 0 & -4 & 3 \end{bmatrix}$$

$w_4 = -w_2$

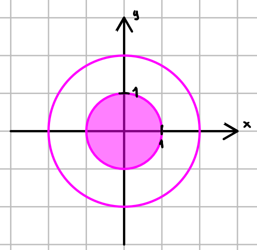
$$\begin{bmatrix} 0 & 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 13 & -9 \\ 1 & 0 & 0 & -6 & 6 \\ 0 & 1 & 0 & -4 & 3 \end{bmatrix} \xrightarrow{\frac{1}{13}w_2} \begin{bmatrix} 0 & 0 & 1 & -5 & 4 \\ 0 & 0 & 0 & 1 & -\frac{9}{13} \\ 1 & 0 & 0 & -6 & 6 \\ 0 & 1 & 0 & -4 & 3 \end{bmatrix} \begin{matrix} w_1+5w_2 \\ \longrightarrow \\ w_3+6w_2 \\ w_4+4w_2 \end{matrix} \begin{bmatrix} 0 & 0 & 1 & 0 & 4-\frac{45}{13} \\ 0 & 0 & 0 & 1 & -\frac{9}{13} \\ 1 & 0 & 0 & 0 & 6-\frac{54}{13} \\ 0 & 1 & 0 & 0 & 3-\frac{36}{13} \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & 0 & 0 & 6-\frac{54}{13} \\ 0 & 1 & 0 & 0 & 3-\frac{36}{13} \\ 0 & 0 & 1 & 0 & 4-\frac{45}{13} \\ 0 & 0 & 0 & 1 & -\frac{9}{13} \end{bmatrix} \quad \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 1 \neq 0$$

rank $A = 4$

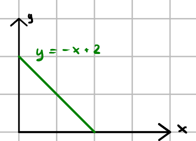
2.

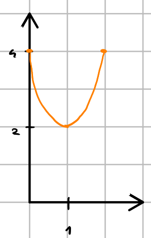
$f: \mathbb{R}^2 \longrightarrow \mathbb{R} \qquad f(x,y) = \max\{1, x^2+y^2\}$

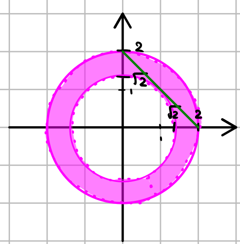
$f^{-1}(\{0, 1, 4\}) = \emptyset \cup \{(x,y)\in\mathbb{R}^2 : x^2+y^2 \leq 1\} \cup \{(x,y)\in\mathbb{R}^2 : x^2+y^2 = 2^2\}$



$f^{-1}(f(\{(x,y)\in\mathbb{R} : x+y=2 \wedge x\in[0,2]\})) = f^{-1}([2,4]) = \{(x,y)\in\mathbb{R}^2 : \sqrt{2}^2 \leq x^2+y^2 \leq 2^2\}$



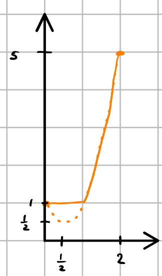




$x^2+y^2 = x^2+(-x+2)^2$
$= x^2+x^2-4x+4 = 2(x-1)^2+2$

$f(\{(x,y)\in\mathbb{R}^2 : x+y=1 \wedge x\in[0,2]\}) = [1,5]$

$x^2+y^2 = x^2+(1-x)^2 = x^2+1-2x+x^2$
$= 2x^2-2x+1 = 2(x-\frac{1}{2})^2+\frac{1}{2}$



3.

$[-2, 2] \qquad x \sim y \iff \lfloor x^2 \rfloor = \lfloor y^2 \rfloor \qquad \lfloor \sqrt{2}^2 \rfloor = \lfloor 2 \rfloor = 2 \qquad \lfloor (\frac{1}{2})^2 \rfloor = \lfloor \frac{1}{4} \rfloor = 0$

$[\sqrt{2}]_\sim = \{x\in[-2,2] : \lfloor x^2 \rfloor = 2\} = \{x\in[-2,2] : x^2\in[2,3)\} = (-\sqrt{3}, -\sqrt{2}] \cup [\sqrt{2}, \sqrt{3})$

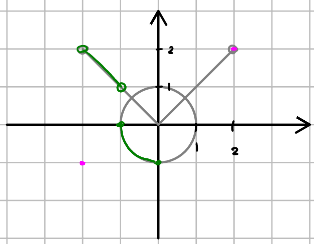
$[\frac{1}{2}]_\sim = \{x\in[-2,2] : \lfloor x^2 \rfloor = 0\} = \{x\in[-2,2] : x^2\in[0,1)\} = (-1, 1)$

Liczba klas abstrakcji $\rightarrow 5 \qquad x\in[-2,2] \Rightarrow x^2\in[0,4] \Rightarrow \lfloor x^2 \rfloor \in \{0,1,2,3,4\}$

4.

$(x_1, y_1) \leq_P (x_2, y_2) \iff x_1 \leq x_2 \wedge y_1 \leq y_2$

$A = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\} \cup \{(x,y) \in \mathbb{R}^2 : y = |x| \wedge x \in (-2,2)\}$



kres górny $(2,2)$

kres dolny $(-2,-1)$

nie ma elementu największego

nie ma elementów maksymalnych

nie ma elementu najmniejszego

elementy minimalne

5.

$$\left[\begin{array}{cccc|c} 1 & 0 & -4 & 8 & 1 \\ 0 & 2 & 1 & -4 & 4 \\ 2 & 1 & -3 & 5 & 4 \\ 3 & -2 & -1 & 4 & -1 \end{array}\right] \xrightarrow[\substack{w_3 - 2w_1 \\ w_4 - 3w_1}]{} \left[\begin{array}{cccc|c} 1 & 0 & -4 & 8 & 1 \\ 0 & 2 & 1 & -4 & 4 \\ 0 & 1 & 5 & -11 & 2 \\ 0 & -2 & 11 & -20 & -4 \end{array}\right] \xrightarrow[w_4 + 2w_3]{w_2 - 2w_3} \left[\begin{array}{ccccc} 1 & 0 & -4 & 8 & 1 \\ 0 & 0 & -9 & 18 & 0 \\ 0 & 1 & 5 & -11 & 2 \\ 0 & 0 & 21 & -42 & 0 \end{array}\right] \xrightarrow[\frac{1}{21} w_4]{-\frac{1}{9} w_2} \left[\begin{array}{ccccc} 1 & 0 & -4 & 8 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 5 & -11 & 2 \\ \cancel{0} & \cancel{0} & \cancel{1} & \cancel{-2} & \cancel{0} \end{array}\right] \xrightarrow[w_3 - 5w_2]{w_1 + 4w_2} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 1 & 0 & -1 & 2 \end{array}\right]$$

$$\begin{array}{cccc} \uparrow & \uparrow & \uparrow & a \end{array}$$

$$\begin{cases} x = 1 \\ y = 2 + a \\ z = 2a \\ t = a \end{cases} \qquad \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix} + a \begin{bmatrix} 0 \\ 1 \\ 2 \\ 1 \end{bmatrix} \qquad a \in \mathbb{R}$$