

restart;

$$EQ := A[i] \cdot x[i-2] + B[i] \cdot x[i-1] + C[i] \cdot x[i] + D[i] \cdot x[i+1] + E[i] \cdot x[i+2] - F[i] = 0$$

$$A_i x_{i-2} + B_i x_{i-1} + C_i x_i + D_i x_{i+1} + E_i x_{i+2} - F_i = 0 \quad (1)$$

$$x(i) := \text{alpha}[i] \cdot x[i+1] + \text{beta}[i] \cdot x[i+2] + \text{gamma}[i];$$

$$i \rightarrow \alpha_i x_{i+1} + \beta_i x_{i+2} + \gamma_i \quad (2)$$

$$EQ1 := \text{subs}(x[i-2]=x(i-2), EQ)$$

$$A_i (\alpha_{i-2} x_{i-1} + \beta_{i-2} x_i + \gamma_{i-2}) + B_i x_{i-1} + C_i x_i + D_i x_{i+1} + E_i x_{i+2} - F_i = 0 \quad (3)$$

$$EQ2 := \text{subs}(x[i-1]=x(i-1), EQ1)$$

$$A_i (\alpha_{i-2} (\alpha_{i-1} x_i + \beta_{i-1} x_{i+1} + \gamma_{i-1}) + \beta_{i-2} x_i + \gamma_{i-2}) + B_i (\alpha_{i-1} x_i + \beta_{i-1} x_{i+1} + \gamma_{i-1}) + C_i x_i + D_i x_{i+1} + E_i x_{i+2} - F_i = 0 \quad (4)$$

$$EQ3 := \text{subs}(x[i]=x(i), EQ2)$$

$$A_i (\alpha_{i-2} (\alpha_{i-1} (\alpha_i x_{i+1} + \beta_i x_{i+2} + \gamma_i) + \beta_{i-1} x_{i+1} + \gamma_{i-1}) + \beta_{i-2} (\alpha_i x_{i+1} + \beta_i x_{i+2} + \gamma_i) + \gamma_{i-2}) + B_i (\alpha_{i-1} (\alpha_i x_{i+1} + \beta_i x_{i+2} + \gamma_i) + \beta_{i-1} x_{i+1} + \gamma_{i-1}) + C_i (\alpha_i x_{i+1} + \beta_i x_{i+2} + \gamma_i) + D_i x_{i+1} + E_i x_{i+2} - F_i = 0 \quad (5)$$

$$EQ4 := \text{collect}(EQ3, [x[i+2], x[i+1]], \text{factor})$$

$$(A_i \alpha_{i-2} \alpha_{i-1} \beta_i + A_i \beta_i \beta_{i-2} + B_i \alpha_{i-1} \beta_i + C_i \beta_i + E_i) x_{i+2} + (A_i \alpha_i \alpha_{i-2} \alpha_{i-1} + A_i \alpha_i \beta_{i-2} + A_i \alpha_{i-2} \beta_{i-1} + B_i \alpha_i \alpha_{i-1} + B_i \beta_{i-1} + C_i \alpha_i + D_i) x_{i+1} + A_i \alpha_{i-2} \alpha_{i-1} \gamma_i + A_i \alpha_{i-2} \gamma_{i-1} + A_i \beta_{i-2} \gamma_i + B_i \alpha_{i-1} \gamma_i + A_i \gamma_{i-2} + B_i \gamma_{i-1} + C_i \gamma_i - F_i = 0 \quad (6)$$

$$\text{first} := \text{coeff}(\text{lhs}(EQ4), x[i+2]) = 0$$

$$A_i \alpha_{i-2} \alpha_{i-1} \beta_i + A_i \beta_i \beta_{i-2} + B_i \alpha_{i-1} \beta_i + C_i \beta_i + E_i = 0 \quad (7)$$

$$\text{second} := \text{coeff}(\text{lhs}(EQ4), x[i+1]) = 0$$

$$A_i \alpha_i \alpha_{i-2} \alpha_{i-1} + A_i \alpha_i \beta_{i-2} + A_i \alpha_{i-2} \beta_{i-1} + B_i \alpha_i \alpha_{i-1} + B_i \beta_{i-1} + C_i \alpha_i + D_i = 0 \quad (8)$$

$$\text{third} := \text{coeff}(\text{coeff}(\text{lhs}(EQ4), x[i+2], 0), x[i+1], 0) = 0;$$

$$A_i \alpha_{i-2} \alpha_{i-1} \gamma_i + A_i \alpha_{i-2} \gamma_{i-1} + A_i \beta_{i-2} \gamma_i + B_i \alpha_{i-1} \gamma_i + A_i \gamma_{i-2} + B_i \gamma_{i-1} + C_i \gamma_i - F_i = 0 \quad (9)$$

$$\text{solve}([\text{first}, \text{second}, \text{third}], [\text{alpha}[i], \text{beta}[i], \text{gamma}[i]])$$

$$\left[ \left[ \alpha_i = - \frac{A_i \alpha_{i-2} \beta_{i-1} + B_i \beta_{i-1} + D_i}{A_i \alpha_{i-2} \alpha_{i-1} + A_i \beta_{i-2} + B_i \alpha_{i-1} + C_i}, \beta_i = - \frac{E_i}{A_i \alpha_{i-2} \alpha_{i-1} + A_i \beta_{i-2} + B_i \alpha_{i-1} + C_i}, \gamma_i = - \frac{A_i \alpha_{i-2} \gamma_{i-1} + A_i \gamma_{i-2} + B_i \gamma_{i-1} - F_i}{A_i \alpha_{i-2} \alpha_{i-1} + A_i \beta_{i-2} + B_i \alpha_{i-1} + C_i} \right] \right] \quad (10)$$