DUBLIN CITY UNIVERSITY

ELECTRONIC AND COMPUTER ENGINEERING

EE517 - Network Analysis And Dimensioning

Assignment 2

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Signed: ____________________ Date: xx/xx/20xx

Michael Lenehan

Contents

1 Question 1

Given:

$$
\begin{aligned}
\text{Poisson Distribution: } p(k) &= \frac{\lambda^k e^{-\lambda}}{k!} \\
\text{Probability Generating Function: } G(z) &\triangleq E[z^k] = \sum_{\forall k \in \Omega} z^k px(k) \\
\text{nth Central Moment: } \mu_n &= E[(X - E[X])^2] \\
G(z) = \sum_{k=0}^{\infty} z^k \frac{\lambda^k e^{-\lambda}}{k!} &= e^{-\lambda} \sum_{k=0}^{\infty} \frac{(z\lambda)^k}{k!} \\
e^{-\lambda} e^{z\lambda} &= e^{(z-1)\lambda} \\
\text{ith Moment: } \overline{X^i} &= \left. \frac{d}{dz}(z\frac{d}{dz})^{i-1} G(z) \right|_{z=1} \\
\text{First Moment: } \overline{X} = \left. \frac{d}{dz} G(z) \right|_{z=1} &= \left. \frac{d}{dz} e^{(z-1)\lambda} \right|_{z=1} \\
&= \lambda \\
\text{Second Moment: } \overline{X^2} &= \left. \frac{d}{dz} z \frac{d}{dz} G(z) \right|_{z=1} \\
&= \left. \frac{d}{dz} z (\frac{d}{dz} e^{(z-1)\lambda}) \right|_{z=1} \\
&= \lambda^2 + \lambda \\
\therefore \text{Third Moment: } \overline{X^3} &= \left. \frac{d}{dz}(z\frac{d}{dz})^2 G(z) \right|_{z=1} \\
&= \frac{d}{dz} z (\frac{d}{dz} \lambda e^{(z-1)\lambda}) \\
&= \lambda^3 + 3\lambda^2 + \lambda \\
\text{Third Central Moment: } \mu_3 &= E[(X - E[X])^3] \\
&= E[X^3] - 3E[X^2]E[X] + 3E[x]E^2[X] - E^3[X] \\
&= \lambda^3 + 3(\lambda^2 + \lambda) - 3(\lambda^2 + \lambda)(\lambda) + 3(\lambda)(\lambda)^2 - \lambda^3 \\
&= \lambda^3 + 3\lambda^3 - 3\lambda^3 - \lambda^3 + 3\lambda^2 - 3\lambda^2 + \lambda \\
&= \lambda.
\end{aligned}
$$

2 Question 2

This process is a Markov chain, as it meets the definition of a Markoc chain, having both a discrete state space and is in discrete time.

Table 1: caption

0	1	2	3	4	...
0	0.5	0.5	0	0	...
0	0	0.5	0.5	0	...
0	0	0	0.5	0.5	...
0	0	0	0	0.5	0.5

$$\begin{aligned}
\pi^{(n)} &= \pi^{(n-1)}P \\
\text{Given: } n &= 3 \\
\pi^{0} &= 1 \\
\pi^{(1)} &= \pi^{(0)}P = 1(0.5) \\
&= 0.5 \\
\pi^{(2)} &= \pi^{(1)}P = 0.5(0.5) \\
&= 0.25 \\
\pi^{(3)} &= \pi^{(2)}P = 0.25(0.5) \\
&= 0.125 \\
\therefore P(X = 4|n = 3) &= 1 - 0.125 \\
&= \frac{7}{8}.
\end{aligned}$$

3 Question 3

$\tau.$

4 Question 4

5 Question 5

Table 2: Blocking Probabilities (As given by Erlang-B Chart)

Number of Channels (W)	Blocking Probability (B)	Initialisation Cost (1.2 x W)	Blocking Cost (3.1 x 10 x B)	Total Cost (IC + BC)
4	0.6467	4.8	20.047	24.847
8	0.3383	9.6	10.488	20.088
12	0.1197	14.4	3.712	18.119
16	0.0223	19.2	0.6914	19.891
20	0.0019	24	0.0589	24.058

The following plot shows the total overall costs for between zero and 25 channels.

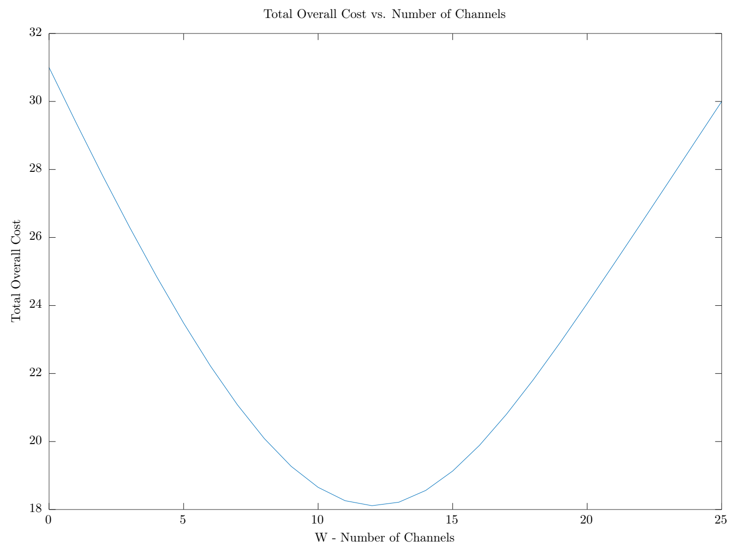

Figure 1: Plot of the Total Overall Cost within the given system

Therefore, the number of channels which minimizes the total overall cost is 12.

The above answers were obtained using the Erlang-B Iterative Formula, programmed using GNU-Octave. This code is shown below.

```
x = 0;
Ex = 1;
A = 10;
B = zeros(0,25);
IC = zeros(0,25);
BC = zeros(0,25);
Total = zeros(0,25);
f1 = fopen('erlang.txt', 'w');
f2 = fopen('costs.txt', 'w');
while(x<26)
        Ex = (A*Ex)/(x+A*Ex);
        fprintf(f1, "W: %d \t B: %d\n", x, Ex);
```

```
        x++;
        B(x) = Ex;
        IC(x) = 1.2*(x-1);
        BC(x) = 3.1*A*Ex;
        Total(x) = IC(x)+BC(x);
        fprintf(f2, "Installation: %d \t Blocking: %d \t
            Total: %d\n", IC(x),
        BC(x), Total(x));
endwhile
h=figure();
plot(0:1:25, Total);
xlabel("W - Number of Channels");
ylabel("Total Overall Cost");
title("Total Overall Cost vs. Number of Channels");
print(h, "Q5.pdf", "-dpdflatexstandalone")
system("pdflatex Q5.pdf")
system("pdftoppm Q5.pdf Q5 -png");
pause();
```

6 Question 6

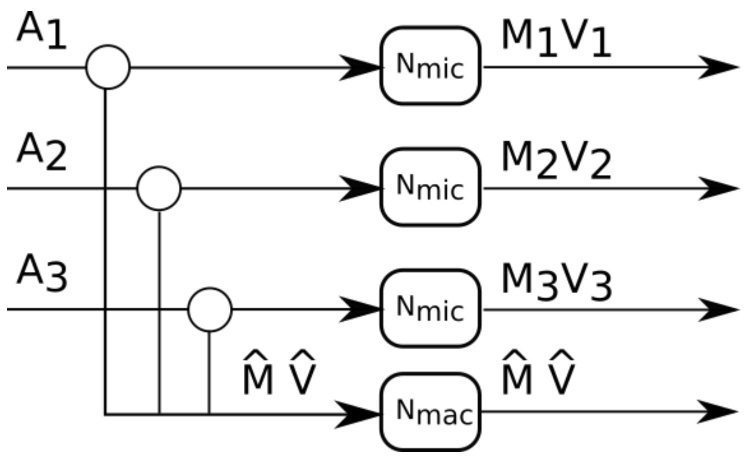

Figure 2: images/Q6