DUBLIN CITY UNIVERSITY

ELECTRONIC AND COMPUTER ENGINEERING

EE517 - Network Analysis And Dimensioning

Assignment 1

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Michael Lenehan

Contents

1 Question 1

In order to draw the feasible region, the following information is required:

$$
\begin{aligned}
4x + y &\leq 5 \\
\therefore \text{Intersects axis at}(0,5)\text{and}(1.25,0) \\
5x - 2y &\leq 3 \\
\therefore \text{Intersects axis at}(0,-1.5)\text{and}(0.6,0) \\
y &\leq 3 \\
\therefore \text{A line parallel to the x-axis at} y &= 3
\end{aligned}
$$

It is also given that the feasible region occurs in the quadrant greater than $x = -1$ and $y = -1$.

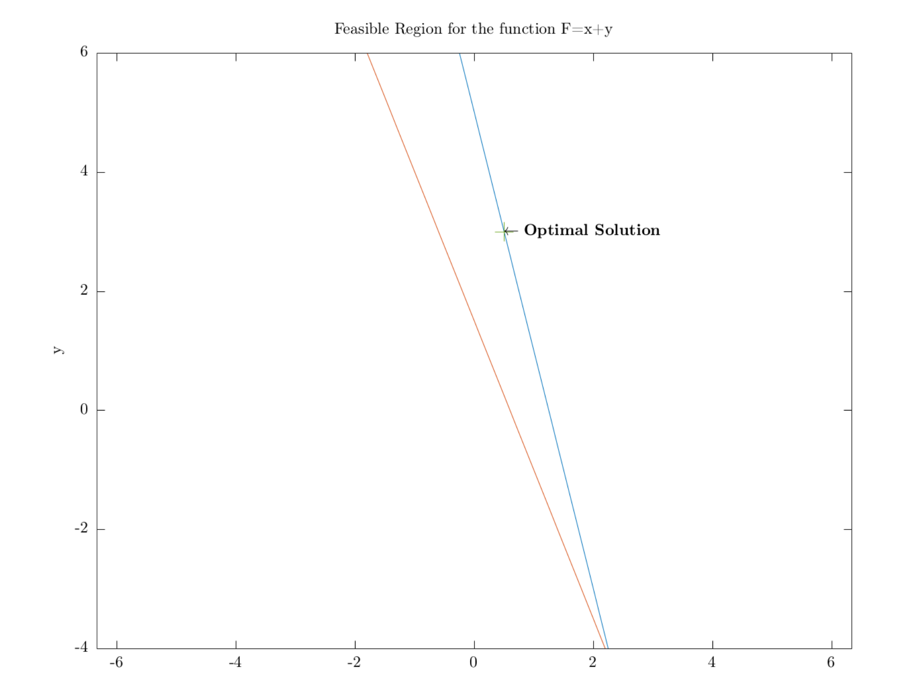

Figure 1: Feasible region for Q1

The maximum value is found to be at the point (0.5, 3.0).

If the objective function is changed to: minimise: $F = x$, the solution would be (-1, 0), as the minimum value of x satisfies the objective function.

2 Question 2

2.1 Part a

Table 1: Demand Volume and Link Capacity

Node-Identifier Notation	Demand-Path-Identifier Notation	Demand Volume	Link Capacity
Demand <1,2>	Demand Label = 1	$h_1 = 14$	-
Demand <1,3>	Demand Label = -	-	-
Demand <2,3>	Demand Label = 2	$h_2 = 13$	-
Link 1-2	Link Label = 1	-	$c_1 = 12$
Link 1-3	Link Label = 3	-	$c_3 = 3$
Link 2-3	Link Label = 2	-	$c_2 = 15$

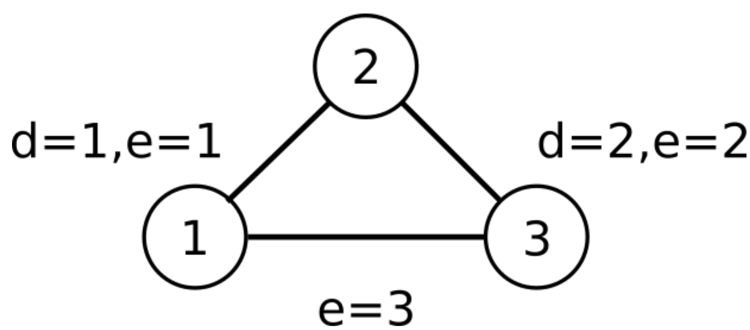

Figure 2: Demands and Links

Table 2: Candidate Paths

Node-Identifier Notation	Demand-Path-Identifier Notation	Candidate Path
Path 1-2	Demand = 1, Path Label = 1	$P_{1,1}$
Path 1-3	Demand = -, Path Label = -	-
Path 2-3	Demand = 2, Path Label = 1	$P_{2,1}$
Path 1-3-2	Demand = 1, Path Label = 2	$P_{1,2}$
Path 1-2-3	Demand = -, Path Label = -	-
Path 2-1-3	Demand = 2, Path Label = 2	$P_{2,2}$

Table 3: Path-Flow Variables

Node-Identifier Notation	Demand-Path-Identifier Notation
$\hat{x}_{1,2}$	$x_{1,1}$
$\hat{x}_{1,3}$	-
$\hat{x}_{2,3}$	$x_{2,1}$
$\hat{x}_{1,3,2}$	$x_{1,2}$
$\hat{x}_{1,2,3}$	-
$\hat{x}_{2,1,3}$	$x_{2,2}$

As the marginal link cost is 1 cost unit, the formulation is as follows:

Minimize (objective Function):

$$F = x_{1,1} + 2x_{1,2} + x_{2,1} + 2x_{2,2}$$

Subject to (Constraints):

$$x_{1,1} + x_{1,2} = h_1$$
$$x_{2,1} + x_{2,2} = h_2$$
$$x_{1,1} + x_{2,2} \leq c_1$$
$$x_{1,2} + x_{2,1} \leq c_2$$
$$x_{1,2} + x_{2,2} \leq c_3$$

2.2 Part b

Four path flow variables are required in total to formulate this problem. This is the case due to the value of the demand between node 1 and 3 ($\hat{h}_{1,3}$) equalling 0. This differs from the node-identifier based formulation as it requires two less variables for the formulation.

2.3 Part c

The demand volumes (in DVUs) are $h_1 = 14$ and $h_2 = 13$. The link capacities are $c_1 = 12$, $c_2 = 15$, and $c_3 = 3$.

In order to minimise the routing costs, try to route all traffic over direct links. There is sufficient capacity to route demand volume h_2 over direct link, as the capacity is 15. This leaves an unused capacity of 2 on the link $e = 2$. Demand volume h_1 can then be routed over link $e = 1$, with the remaining demand volume routed over $e = 2$ and $e = 3$. This results in a capacity of 1 remaining on link $e = 3$.

As such, the optimal values for x_d, p are:

$$
\begin{aligned}
x_{1,1} &= 12 \\
x_{1,2} &= 2 \\
x_{2,1} &= 13 \\
x_{2,2} &= 0
\end{aligned}
$$

2.4 Part d

The link load (y_e) for each path-flow variable ($x_{d,p}$) is shown below:

$$
\begin{aligned}
y_1 &= x_{1,1} + x_{2,2} \\
&= 12 + 0 \\
&= 12 \\
y_2 &= x_{1,2} + x_{2,1} \\
&= 2 + 13 \\
&= 15 \\
y_3 &= x_{1,2} \\
&= 2
\end{aligned}
$$

2.5 Part e

The binary indicator values are as follows:

Table 4: $\delta_{e,d,p}$

$e/P_{d,p}$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$
1	1	0	0	1
2	0	1	1	0
3	0	1	0	1

2.6 Part f

Increasing the value of $\hat{h}_{1,2}$ to 15 would result in an infeasible problem. While the capacity is available on link 3 for this increase in demand, there is no availability for the increase on link 2. The available link capacity remaining on c_3 is 1 DVU, which is the required amount for the increase in demand, however there is 0 availability on the link $e = 2$.

3 Question 3

3.1 Part a

Re-formulating the problem as a modified flow-allocation problem results in the following formulation:

Minimize:

$$z$$

Subject to:

$$\begin{aligned}
x_{1,1} + x_{1,2} &= h_1 \\
x_{2,1} + x_{2,2} &= h_2 \\
x_{1,1} + x_{2,1} &\leq c_1 + z \\
x_{1,2} + x_{2,1} &\leq c_2 + z \\
x_{1,2} + x_{2,2} &\leq c_3 + z \\
x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2} &\geq 0
\end{aligned}$$

3.2 Part b

$$\begin{aligned}
x_{1,1} + x_{1,2} = h_1 &= 15 \\
x_{2,1} + x_{2,2} = h_2 &= 13 \\
x_{1,1} + x_{2,2} &\leq c_1 + z \\
\therefore 12 + 0 &\leq 12 + z \\
x_{1,2} + x_{2,1} &\leq c_2 + z \\
\therefore 3 + 13 &\leq 15 + z \\
x_{1,2} + x_{2,2} &\leq c_3 + z \\
\therefore 3 + 0 &\leq 3 + z
\end{aligned}$$

From these equations it can be seen that for link 1, the value of z is 0, for link 2 the value of z is $\geq$ 1, and for link 3 the value of z is again 0.

Increasing the link capacity of c_2 by 1 DVU - i.e. $c_2 = 16$ - would fulfil the demand volumes given. This would result in flow path variables as follows:

$$x_{1,1} = 12$$
$$x_{1,2} = 3$$
$$x_{2,1} = 13$$
$$x_{2,2} = 0$$

The resulting link loads are as follows:

$$\begin{aligned}
y_e &= \sum_d \sum_p \delta_{e,d,p} x_{d,p} \\
y_1 &= \sum_d \sum_p \delta_{1,d,p} x_{d,p} \\
&= x_{1,1} + x2,2 \\
&= 12 + 0 \\
&= 12 \\
y_2 &= \sum_d \sum_p \delta_{2,d,p} x_{d,p} \\
&= x_{2,1} + x1,2 \\
&= 13 + 3 \\
&= 16 \\
y_3 &= \sum_d \sum_p \delta_{3,d,p} x_{d,p} \\
&= x_{1,2} + x2,2 \\
&= 3 + 0 \\
&= 3
\end{aligned}$$

4 Question 4

4.1 Part a

The given information is as follows:

$$\text{Demand 1: } \mathbb{P}_1 = \{P_{1,1}, P_{1,2}\} = \{\{2,4\},\{1,5\}\}$$
$$\text{Demand 2: } \mathbb{P}_2 = \{P_{2,1}, P_{2,2}\} = \{\{5\},\{4,3\}\}$$
$$\text{Demand 3: } \mathbb{P}_3 = \{P_{3,1}, P_{3,2}\} = \{\{1\},\{3,2\}\}$$
$$\text{Demand Volumes: } \{h_1, h_2, h_3\} = \{17, 17, 12\}$$
$$\text{Marginal Link Costs: } \{\xi_1, \xi_2, \xi_3, \xi_4, \xi_5\} = \{2, 1, 1, 3, 1\}.$$

Demand h_1 has a path cost of 4 on path $P_{1,1}$, and a cost of 3 on path $P_{1,2}$ Therefore, path $P_{1,2}$ should be used to route this demand. This results in the following path flow variables:

$$x_{1,1} = 0$$
$$x_{1,2} = 17.$$

Demand h_2 has a path cost of 1 on path $P_{2,1}$, and a cost of 4 on path $P_{2,2}$. As such, path $P_{2,1}$ should be used to route this demand. This results in the following path flow variables:

$$x_{2,1} = 17$$
$$x_{2,2} = 0.$$

Finally, demand h_3 has a path cost of 2 on path $P_{3,1}$, and a cost of 2 on path $P_{3,2}$. As such, the path flow variables can be expressed as follows:

$$x_{3,1} = a$$
$$x_{3,2} = 12 - a.$$

In order to construct the set of decoupled problems, the $\delta_{e,d,p}$ values are required. The following table shows the δ values:

Table 5: $\delta_{e,d,p}$

$e/Pd,p$	$P_{1,1}$	$P_{1,2}$	$P_{2,1}$	$P_{2,2}$	$P_{3.1}$	$P_{3.2}$
1	0	1	0	0	1	0
2	1	0	0	0	0	1
3	0	0	0	1	0	1
4	1	0	0	1	0	0
5	0	1	1	0	0	0

The following gives the optimisation as a set of decoupled problems:

$$\begin{aligned}
\text{General Form: } F &= \sum_d \sum_p \zeta_{d,p} x_{d,p} \\
\text{Where: } \zeta_{d,p} &= \sum_e \xi_e \delta_{e,d,p} \\
F_1 = \zeta_{1,p} x_{1,p} &= ((\xi_2)\delta_{2,1,1} + (\xi_4)\delta_{4,1,1})x_{1,1} + ((\xi_1)\delta_{1,1,2} + (\xi_5)\delta_{5,1,2})x_{1,2} \\
F_2 = \zeta_{2,p} x_{2,p} &= ((\xi_5)\delta_{5,2,1})x_{2,1} + ((\xi_4)\delta_{4,2,2} + (\xi_3)\delta_{3,2,2})x_{2,2} \\
F_3 = \zeta_{3,p} x_{3,p} &= ((\xi_1)\delta_{1,3,1})x_{3,1} + ((\xi_3)\delta_{3,3,2} + (\xi_2)\delta_{2,3,2})x_{3,2}.
\end{aligned}$$

4.2 Part b

Given the decoupled problems in section a, the following values were calculated:

$$\begin{aligned}
\text{General Form: } F &= \sum_d \sum_p \zeta_{d,p} x_{d,p} \\
\text{Where: } \zeta_{d,p} &= \sum_e \xi_e \delta_{e,d,p} \\
F_1 = \zeta_{1,p} x_{1,p} &= (4(1))0 + (3(1))17 = 51 \\
F_2 = \zeta_{2,p} x_{2,p} &= (1(1))17 + (4(1))0 = 17 \\
F_3 = \zeta_{3,p} x_{3,p} &= (2(1))\frac{12}{2} + (2(1))\frac{12}{2} = 24.
\end{aligned}$$

This gives a total value for the objective function of:

$$\begin{aligned}
F^* &= F_1 + F_2 + F_3 \\
&= 51 + 17 + 24 \\
&= 92.
\end{aligned}$$

The solutions for the flows give the link load values below:

$$\begin{aligned}
x^*_{1,2} + x^*_{3,1} &= \underline{y}_1 = 17 + a \\
x^*_{1,1} + x^*_{3,2} &= \underline{y}_2 = 12 - a \\
x^*_{2,2} + x^*_{3,2} &= \underline{y}_3 = 12 - a \\
x^*_{1,1} + x^*_{2,2} &= \underline{y}_4 = 0 \\
x^*_{1,2} + x^*_{2,1} &= \underline{y}_5 = 34.
\end{aligned}$$

Therefore, the objective function value is confirmed by calculating the objective function using the link loads and link marginal costs:

$$
\begin{aligned}
F^* &= 2 \times (17+a) + 1 \times (12-a) + 1 \times (12-a) + 3 \times (0) + 1 \times (34) \\
&= 34 + 2a + 12 - a + 12 - a + 34 \\
&= 92.
\end{aligned}
$$

Table 6: Shortest Path Allocation

a	y_1	y_1	y_3	y_4	y_5
12	29	0	0	0	34
11	28	1	1	0	34
10	27	2	2	0	34
9	26	3	3	0	34
8	25	4	4	0	34
7	24	5	5	0	34
6	23	6	6	0	34
5	22	7	7	0	34
4	21	8	8	0	34
3	20	9	9	0	34
2	19	10	10	0	34
1	18	11	11	0	34
0	17	12	12	0	34

With a value of 12 for a, there are 3 links which do not require capacity to be provisioned.

5 Question 5

5.1 Part a

It is not guaranteed that there will be a feasible solution to a Mixed Dimensioning/Allocation Problem with Bounded Link Capacities. Because multiple paths can have demands on the same links, the path flow variables are coupled. If none of the link load values are equal to the link capacity values, a feasible solution can be found. The shortest path allocation rule is used for uncapacitated problems, i.e. problems where the demands do not "compete" for a resource, in this case, the link. Therefore the Shortest Path Allocation Rule does not apply to this problem.

5.2 Part b

The mixed problem is still a Linear Programming problem, as the objective function, and the constraint equations are still linear functions of continuous variables in x.

5.3 Part c

The given information is as follows:

$$\text{Demand 1: } \mathbb{P}_1 = \{P_{1,1}\} = \{2,4\}$$
$$\text{Demand 2: } \mathbb{P}_2 = \{P_{2,1}, P_{2,2}\} = \{\{5\},\{4,3\}\}$$
$$\text{Demand 3: } \mathbb{P}_3 = \{P_{3,1}, P_{3,2}\} = \{\{1\},\{3,2\}\}$$
$$\text{Demand Volumes: } \{h_1, h_2, h_3\} = \{17,17,12\}$$
$$\text{Marginal Link Costs: } \{\xi,\xi,\xi,\xi,\xi\} = \{2,1,1,3,1\}$$
$$\text{Link Capacities: } \{c_1, c_2, c_3\} = \{10,20,20\}.$$

Demand h_1 has a path cost of 4 on path $P_{1,1}$. There is a capacity of 20 on link 2. This results in the following path flow variable:

$$x_{1,1} = 17.$$

Demand h_2 has a path cost of 1 on path $P_{2,1}$, and a cost of 4 on path $P_{2,2}$. Path $P_{2,1}$ has no defined capacity, while path $P_{2,2}$ has a capacity of 10 on link 3. This results in the following path flow variables:

$$x_{2,1} = 17$$
$$x_{2,2} = 0.$$

Finally, demand h_3 has a path cost of 2 on path $P_{3,1}$, and a cost of 2 on path $P_{3,2}$. There is a capacity of 20 on link 1, a remaining capacity of 3 on link 2, and a capacity of 10 on link 3. This results in the following path flow variables:

$$x_{3,1} = a + 2$$
$$x_{3,2} = 10 - a.$$

The solutions for these flows give the link load values below:

$$x^*_{1,2} + x^*_{3,1} = \underline{y_1} = a + 2$$
$$x^*_{1,1} + x^*_{3,2} = \underline{y_2} = 27 - a$$
$$x^*_{2,2} + x^*3,2 = \underline{y_3} = 10 - a$$
$$x^*_{2,2} + x^*1,1 = \underline{y_4} = 17$$
$$x^*_{2,1} = \underline{y_5} = 17.$$

This gives the objective function:

$$\begin{aligned} F^* &= 2 \times (a+2) + 1 \times (27 - a) + 1 \times (10 - a) + 3 \times (17) + 1 \times (17) \\ &= 2a + 4 + 27 - a + 10 - a + 51 + 17 \\ &= 109. \end{aligned}$$

Table 7: Shortest Path Allocation

a	$\underline{y_1}$	$\underline{y_2}$	$\underline{y_3}$	$\underline{y_4}$	$\underline{y_5}$
10	12	17	0	17	17
9	11	18	1	17	17
8	10	19	2	17	17
7	9	20	3	17	17
6	8	21	4	17	17
5	7	22	5	17	17
4	6	23	6	17	17
3	5	24	7	17	17
2	4	25	8	17	17
1	3	26	9	17	17
0	2	27	10	17	17

As the link capacities $\{c_1, c_2, c_3\} = \{10, 20, 20\}$ it can be seen that the only solutions for a can be 8 or 7, as all other values give numbers which exceed the capacities for these links. With $a = 8$ the maximum link load is minimised - $a = 8 \therefore \max y_e = 19$ vs $a = 7 \therefore \max y_e = 20$

It is clear from the options presented above that the solution is not unique. The value of 'a' can be chosen based on a secondary constraint, or the demand $d = 2$ could have been routed over path $P_2, 2$, however, this would not provide an optimal solution.

6 Question 6

6.1 Part a

Figure 3 shows all paths for the modified 4-node network problem, containing the following paths:

$$\begin{aligned}
\mathbb{P}_1 &= \{P_{1,1}1, P_{1,2}, P_{1,3}\} = \{\{2,4\},\{1,5\},\{4,3,1\}\} \\
\mathbb{P}_2 &= \{P_{2,1}, P_{2,2}\} = \{\{5\},\{4,3\}\} \\
\mathbb{P}_3 &= \{P_{3,1}, P_{3,2}\} = \{\{1\},\{3,2\}\}.
\end{aligned}$$

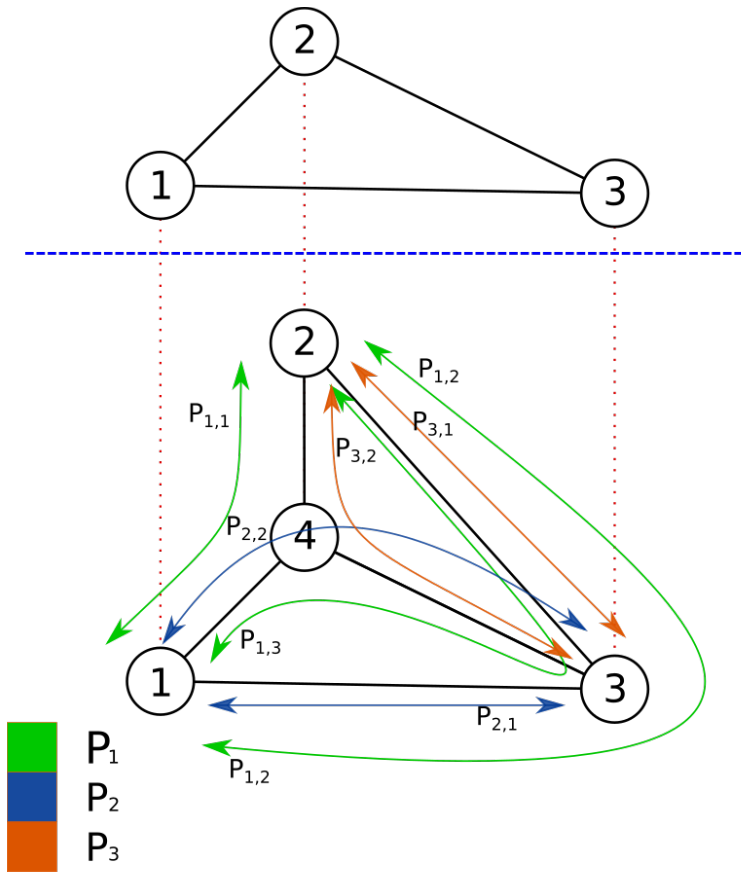

Figure 3: Paths for Question 6 Part a

6.2 Part b

```
/* Objective function */
min: 2y1 + y2 + y3 + 3y4 + y5;

/* Variable bounds */
h1: x1 +x2 + x3 = 17;
h2: x4 + x5 = 17;
h3: x6 + x7 = 12;
```

```
y1 >= x2 + x3 + x6;
y2 >= x1 + x7;
y3 >= x3 + x5 + x7;
y4 >= x1 + x3 + x5;
y5 >= x2 + x4;
c1: y1 <= 20;
c2: y2 <= 20;
c3: y3 <= 20;
c4: y4 <= 20;
c5: y5 <= 15;
```

The minimum value for the objective function obtained via this code is 115. The demand is divided as follows:

$$
\begin{aligned}
x_{1,1} &= 17 \\
x_{2,1} &= 15 \\
x_{2,2} &= 2 \\
x_{3,1} &= 9 \\
x_{3,2} &= 3.
\end{aligned}
$$

6.3 Part c

```
/* Objective function */
min: 2y1 + y2 + y3 + 0*y4 + y5;

/* Variable bounds */
h1: x1 +x2 + x3 = 17;
h2: x4 + x5 = 17;
h3: x6 + x7 = 12;
y1 >= x2 + x3 + x6;
y2 >= x1 + x7;
y3 >= x3 + x5 + x7;
```

```
y4 >= x1 + x3 + x5;
y5 >= x2 + x4;
c1: y1 <= 20;
c2: y2 <= 20;
c3: y3 <= 20;
c4: y4 <= 20;
c5: y5 <= 15;
```

The minimum value for the objective function obtained via this code is 58. The demand is divided as follows:

$$x_{1,1} = 17$$
$$x_{2,1} = 15$$
$$x_{2,2} = 2$$
$$x_{3,1} = 12.$$

6.4 Part d

Due to the capacity constraints placed on the links, there is no way to route the all demands over the available links without bifurcation. At least one route will always be required to be bifurcated.

6.5 Part e

Removing the capacity constraints, as with Question 4, would result in the same outcome, with demand h_3 being bifurcated for an optimal solution. However, there are solutions available in this situation which do not require this demand to be split across links, such as by passing all of demand h_3 over path $P_{3,2}$ or path P_1

7 Question 7

```
/* Objective function */
min: 2y1 + y2 + y3 + 3y4 + y5;

/* Variable bounds */
h1: x11 + x12 + x13 = 17;
h2: x21 + x22 = 17;
h3: x31 + x32 = 12;
x11 <= 17 u11;
x12 <= 17 u12;
x13 <= 17 u13;
x21 <= 17 u21;
x22 <= 17 u22;
x31 <= 12 u31;
x32 <= 12 u32;
6 u11 <= x11;
6 u12 <= x12;
6 u13 <= x13;
6 u21 <= x21;
6 u22 <= x22;
6 u31 <= x31;
6 u32 <= x32;
y1 = x12 + x13 + x31;
y2 = x11 + x32;
y3 = x13 + x22 + x32;
y4 = x11 + x13 + x22;
y5 = x12 + x21;
c1: x12 + x13 + x31 <= 20;
c2: x11 + x32 <= 20;
c3: x13 + x22 + x32 <= 20;
c4: x11 + x13 + x22 <= 20;
```

```
c5: x12 + x21 <= 15;
bin u11,u12,u13,u21,u22,u31,u32;
```

The minimum value for the objective function obtained via this code is 127. The demand is divided as follows:

$$x_{1,1} = 11$$
$$x_{1,2} = 6$$
$$x_{2,1} = 9$$
$$x_{2,2} = 8$$
$$x_{3,1} = 6$$
$$x_{3,2} = 6.$$

As can be seen from these path flow variables, there are no non-zero flows with less than 6 demand volume units.

8 Question 8

8.1 Part a

The given information is as follows:

$$\mathbb{P}_1 = \{P_{1,1}, P_{1,2}, P_{1,3}\} = \{\{2,4\}, \{1,5\}, \{4,3,1\}\}$$
$$\mathbb{P}_2 = \{P_{2,1}, P_{2,2}\} = \{\{5\}, \{4,3\}\}$$
$$\mathbb{P}_3 = \{P_{3,1}, P_{3,2}\} = \{\{1\}, \{3,2\}\}$$
$$(h_1, h_2, h_3) = (17, 17, 12)$$
$$(c_1, c_2, c_3, c_4, c_5) = (20, 20, 20, 20, 20).$$

There is no objective function, and as such there are no link costs, i.e. $\xi_e = 0$.

Each demand must be split across two paths, therefore $k_d = 2$.

The formulation is, therefore, as follows:

$$\sum_p u_{d,p} = k_d$$

$$\sum_d (\sum_p \delta_{e,d,p} u_{d,p}) h_d / k_d \leq c_e.$$

The value of $u_{d,p}$ must be greater than 0 for at least two paths for each demand. Therefore, each path for both $\mathbb{P}_2$ and $\mathbb{P}_3$ must be used.

$$\therefore u_{2,1} + u_{2,2} = 2$$
$$u_{3,1} + u_{3,2} = 2.$$

For path $\mathbb{P}_1$, the paths must be chosen such that the link capacities are not exceeded. Solving for paths $\mathbb{P}_2$ and $\mathbb{P}_3$ will give an insight into the remaining link capacities, allowing for a solution for $\mathbb{P}_1$ to be found.

For path $\mathbb{P}_2$, link 5, 4, and 3 carry some of the demand, with half of the demand over link 5, and the other half over links 4 and 3.

For path $\mathbb{P}_3$, links 1, 3, and 2 carry some of the demand, with half of the demand over link 1, and the other half over links 3 and 2.

This indicates that link 3 is carrying 14.5 units of demand ($\frac{17}{2} + \frac{12}{2}$), which does not leave enough capacity for the remaining demand of demand volume h_1. Therefore, it is assumed that the paths taken from $\mathbb{P}_1$ are $P_{1,1}$ and $P_{1,2}$

8.2 Part b

```
/* Objective function */
min: y1 + y2 + y3 + y4 + y5;

/* Variable bounds */
u11 + u12 + u13 = 2;
u21 + u22 = 2;
u31 + u32 = 2;
y1 = 17 0.5 u12 + 17 0.5 u13 + 12 0.5 u31;
y2 = 17 0.5 u11 + 12 0.5 u32;
```

```
y3 = 17 0.5 u13 + 17 0.5 u22 + 12 0.5 u32;
y4 = 17 0.5 u11 + 17 0.5 u13 + 17 0.5 u22;
y5 = 17 0.5 u12 + 17 0.5 u22;
c1: 17 0.5 u12 + 17 0.5 u13 + 12 0.5 u31 <= 20;
c2: 17 0.5 u11 + 12 0.5 u32 <= 20;
c3: 17 0.5 u13 + 17 0.5 u22 + 12 0.5 u32 <= 20;
c4: 17 0.5 u11 + 17 0.5 u13 + 17 0.5 u22 <= 20;
c5: 17 0.5 u12 + 17 0.5 u22   <= 20;
bin u11, u12, u13, u21, u22, u31, u32;
```

This code unfortunately produced an infeasible solution, and a fix could not be found on time for submission of this assignment.

9 Question 9

9.1 Part a

```
/* Objective function */
min: mW + 8*u11 + 6*u13 + 12*u14 + 4*u21 + 4*u22 + 10*u23
    + 10*u23 + u31 + 4*u32 + 6*u33;

/* Variable bounds */
m = 10;
h1 = 2;
h2 = 2;
h3 = 1;
2u11 + 2u13 + 2u22 + 2u23 + u33 <= W;
2u11 + 2u14 + 2u23 + u23 + u33 <= W;
2u13 + 2u14 + 2u22 + 2u23 <= W;
2u12 + 2u14 + 2u21 + u33 <= W;
2u12 + 2u13 + 2u23 + u31 <= W;
```

```
u11 + u12 + u13 + u14 = 1;
u21 + u22 + u23 = 1;
u31 + u32 + u33 = 1;
```

The value of W achieved using the above code, with $m = 10$, is 3.

```
/* Objective function */
min: mW + 8*u11 + 6*u13 + 12*u14 + 4*u21 + 4*u22 + 10*u23
    + 10*u23 + u31 + 4*u32 + 6*u33;

/* Variable bounds */
m = 0.1;
h1 = 2;
h2 = 2;
h3 = 1;
2u11 + 2u13 + 2u22 + 2u23 + u33 <= W;
2u11 + 2u14 + 2u23 + u23 + u33 <= W;
2u13 + 2u14 + 2u22 + 2u23 <= W;
2u12 + 2u14 + 2u21 + u33 <= W;
2u12 + 2u13 + 2u23 + u31 <= W;
u11 + u12 + u13 + u14 = 1;
u21 + u22 + u23 = 1;
u31 + u32 + u33 = 1;
```

The value of W achieved using the above code, with $m = 0.1$, is 3.

The value of W should increase as the value of m decreases, however, the completed code is incorrect and therefore this did not occur.

9.2 Part b

It is assumed that each demand is routed as a “group of unsplitable channels”. Therefore the demand volume, which is given as the number of light-path connections, enforces that all light paths of a connection follow the same path. Allowing demands to be split across multiple oaths/wavelengths can help to avoid wavelength contention, allowing for more efficient use of existing connections within the network, without requiring new connections to be set up for every new connection request.

10 Question 10

10.1 Part a

This section of code was not completed and therefore no answer can be provided.

10.2 Part b

As no answer is provided for part a, no comparisons can be drawn from for this section.