Continuous-time signals and systems

$$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$$

$$x(t) = \frac{1}{2\pi j}\int_{\sigma-j\infty}^{\sigma+j\infty} X(s)e^{st}\,ds$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,d\omega$$

$$X(j\omega) = X(s)\big|_{s=j\omega}$$

$$x(t) \longrightarrow \boxed{\;h(t)\;} \longrightarrow y(t)$$

L.T.I. system

$$y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

$$Y(s) = X(s)H(s)$$

$$H(s) = \frac{Y(s)}{X(s)}$$

$$Y(j\omega) = X(j\omega)H(j\omega)$$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

$$H(j\omega) = H(s)\big|_{s=j\omega}$$

The system is BIBO stable if

$$\int_{-\infty}^{\infty} |h(t)|\,dt < \infty$$

If the system is stable and causal, all the poles of *H(s)* lie in the left half of the s-plane.

Discrete-time signals and systems

$$X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$x[n] = \frac{1}{2\pi j}\oint_C X(z)z^{n-1}\,dz$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\Omega})e^{j\Omega n}\,d\Omega$$

$$X(e^{j\Omega}) = X(z)\big|_{z=e^{j\Omega}}$$

$$x[n] \longrightarrow \boxed{\;h[n]\;} \longrightarrow y[n]$$

L.T.I. system

$$y[n] = x[n] * h[n]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$Y(z) = X(z)H(z)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(e^{j\Omega}) = X(e^{j\Omega})H(e^{j\Omega})$$

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$$

$$H(e^{j\Omega}) = H(z)\big|_{z=e^{j\Omega}}$$

The system is BIBO stable if

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

If the system is stable and causal:

1)The ROC of H(Z) includes the unit circle and the entire z-plane outside the unit circle.

2)All the poles of H(z) lie within the unit circle.