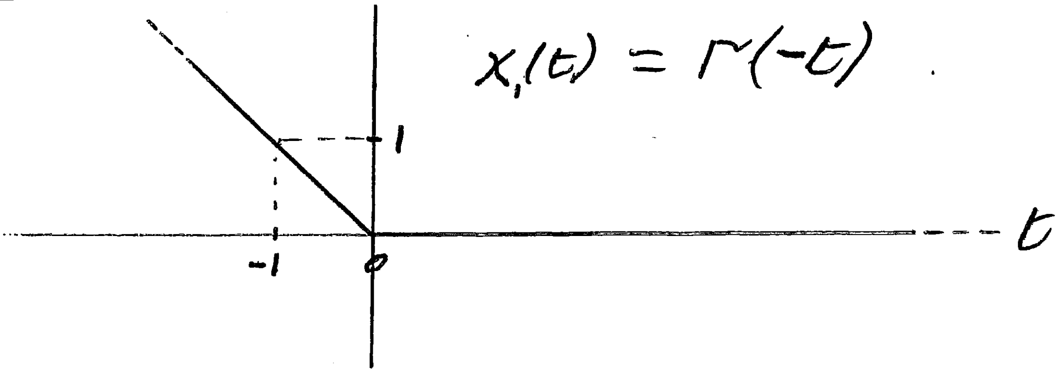
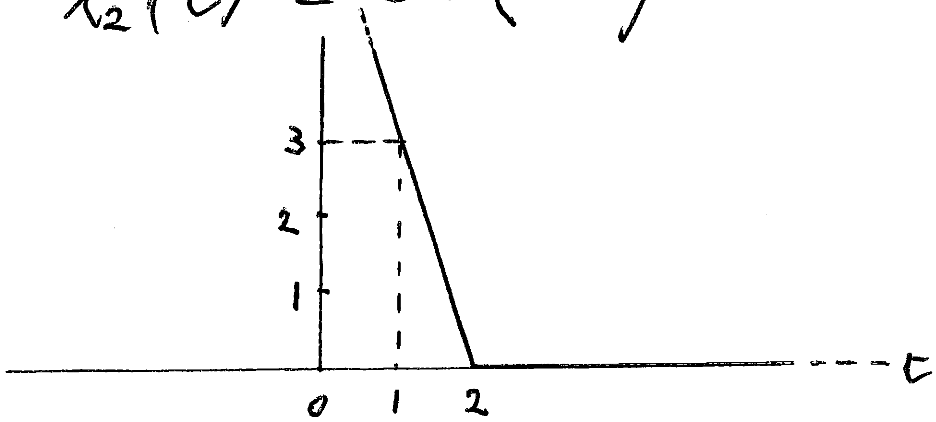
3C1 Homework No. 1 Solutions

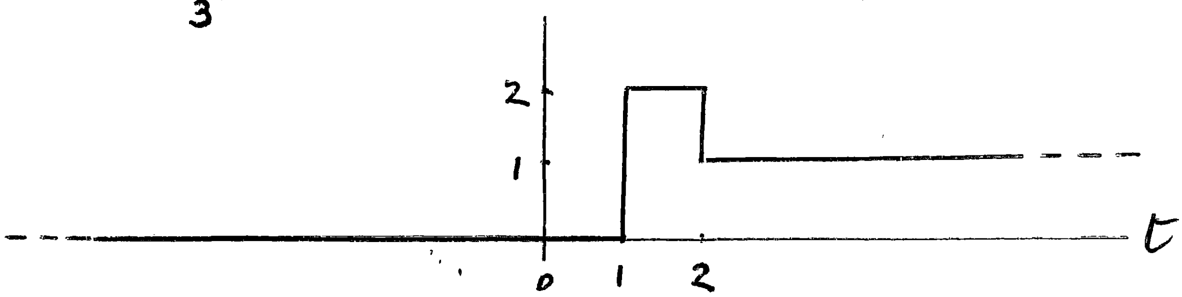
1. (a) $x_1(t) = r(-t)$



(b) $x_2(t) = 3\,r(2-t)$

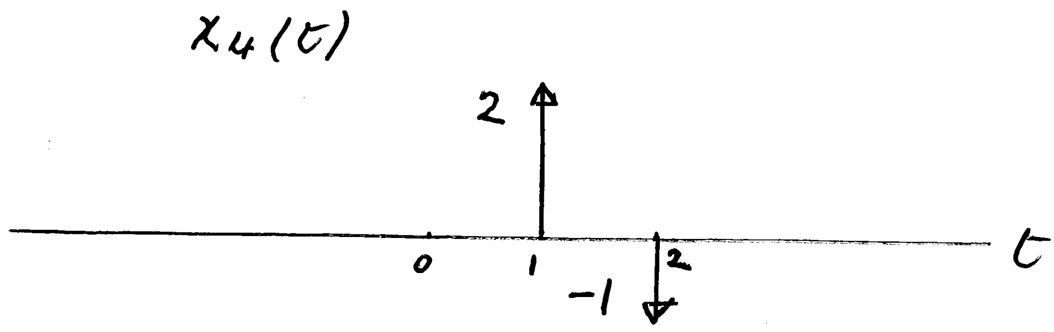


(c) $x_3(t) = 2\,u(t-1) - u(t-2)$

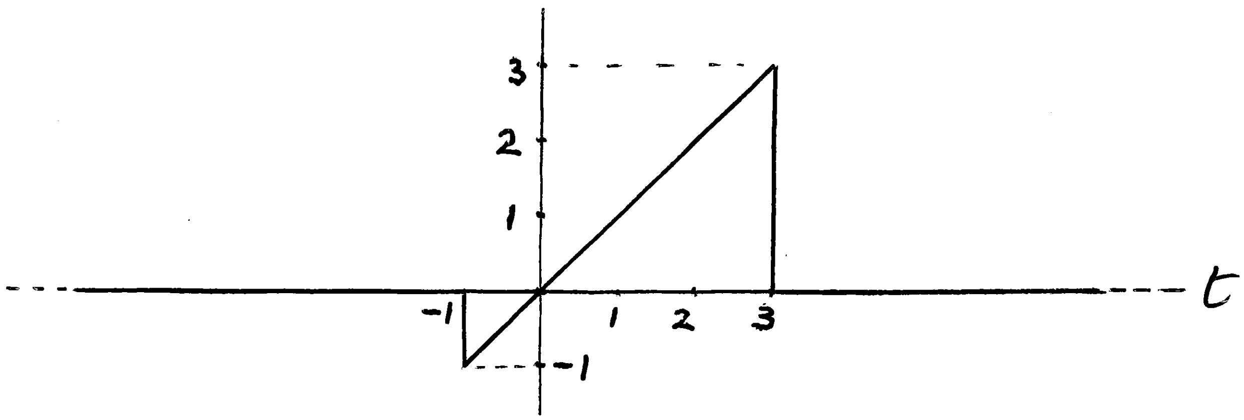


(d) $$x_4(t) = \frac{d x_3(t)}{dt}$$

$$= 2\,\delta(t-1) - \delta(t-2)$$



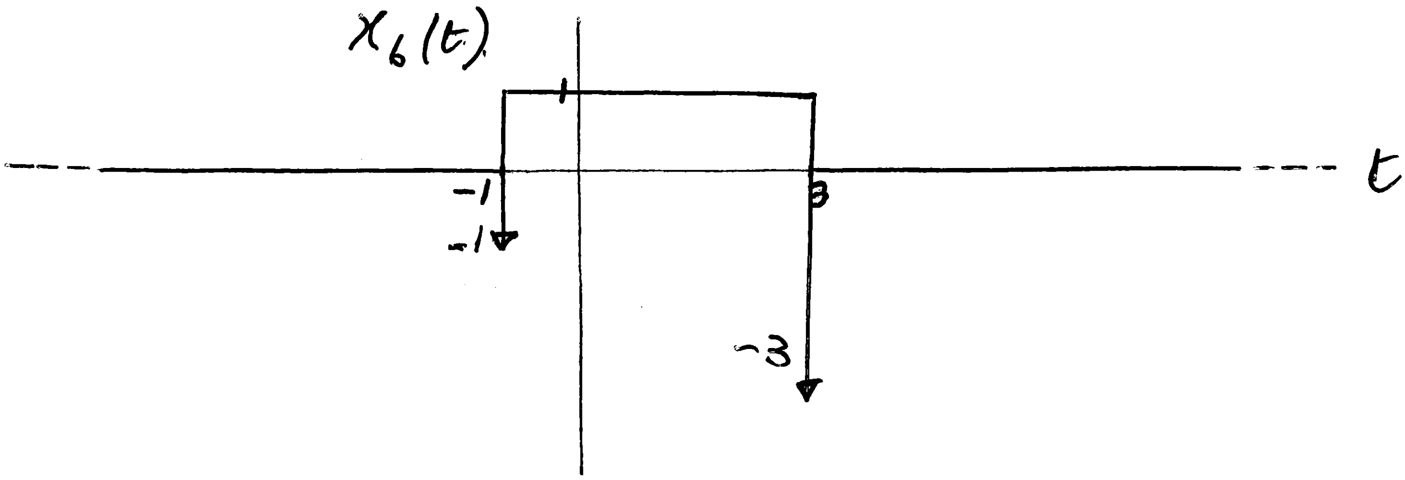
(e) $x_5(t) = t\left[u(t+1) - u(t-3)\right]$



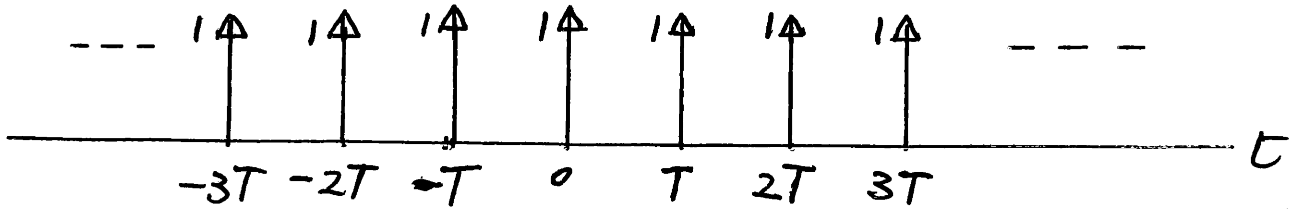
(f) $x_6(t) = \dfrac{dx_5(t)}{dt}$

$= u(t+1) - u(t-3) + t\left[\delta(t+1) - \delta(t-3)\right]$

$= u(t+1) - u(t-3) - \delta(t+1) - 3\,\delta(t-3)$



(g) $x_7(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$



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2. (a) (i) $y_1(t) = t\,x_1(t)$

$$y_2(t) = t\,x_2(t)$$

Let $x_3(t) = a\,x_1(t) + b\,x_2(t)$ where a and b are constants.

$$y_3(t) = t\,x_3(t)$$

$$= a\,t\,x_1(t) + b\,t\,x_2(t)$$

$\therefore$

$$y_3(t) = a\,y_1(t) + b\,y_2(t)$$

The system is linear.

(ii) $y_1(t) = t\,x_1(t)$

Let $x_2(t) = x_1(t-t_0)$ where t_0 is a constant.

$$y_2(t) = t\,x_2(t)$$
$$= t\,x_1(t-t_0)$$
$$\neq y_1(t-t_0).$$

The system is not time-invariant.

(b) (i) $y_1(t) = x_1^2(t)$

$$y_2(t) = x_2^2(t)$$

Let $x_3(t) = a x_1(t) + b x_2(t)$ where a and b are constants.

$$y_3(t) = x_3^2(t)$$
$$= [a x_1(t) + b x_2(t)]^2$$
$$= a^2 x_1^2(t) + 2ab\, x_1(t) x_2(t) + b^2 x_2^2(t)$$
$$\neq a y_1(t) + b y_2(t)$$

The system is not linear.

(ii) $y_1(t) = x_1^2(t)$

Let $x_2(t) = x_1(t-t_0)$ where t_0 is a constant.

$$
\begin{aligned}
y_2(t) &= x_2^2(t) \\
&= x_1^2(t-t_0) \\
&= y_1(t-t_0).
\end{aligned}
$$

The system is time-invariant.

(c) (i) $y_1(t) = x_1(t)+1$

$y_2(t) = x_2(t)+1$

Let $x_3(t) = a x_1(t) + b x_2(t)$ where a and b are constants.

$$
\begin{aligned}
y_3(t) &= x_3(t)+1 \\
&= a x_1(t) + b x_2(t) + 1 \\
&\neq a y_1(t) + b y_2(t)
\end{aligned}
$$

for all constants a and b.

The system is not linear.

(ii) $y_1(t) = x_1(t)+1$

Let $x_2(t) = x_1(t-t_0)$ where t_0 is a constant.

$$
\begin{aligned}
y_2(t) &= x_2(t) + 1 \\
&= x_1(t-t_0) + 1 \\
&= y_1(t-t_0).
\end{aligned}
$$

The system is time-invariant.

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3. (a)

$$
h(t) = \begin{cases} e^{-2t}, & t > 1 \\ 0, & t < 1 \end{cases}
$$

(i) The system is causal since $h(t) = 0,\ t < 0$.

(ii)
$$
\begin{aligned}
\int_{-\infty}^{\infty} |h(t)|\,dt &= \int_{1}^{\infty} e^{-2t}\,dt \\
&= -\frac{1}{2} e^{-2t}\Big|_{1}^{\infty} \\
&= \frac{e^{-2}}{2} < \infty.
\end{aligned}
$$

The system is stable.

(b) (i)

$$
h(t) = \begin{cases} e^{t}, & t < -1 \\ 0, & t > -1 \end{cases}
$$

The system is not causal since $h(t) \neq 0$ for $t < -1$.

(ii)
$$
\int_{-\infty}^{\infty} |h(t)|\,dt = \int_{-\infty}^{-1} e^{t}\,dt
$$

$$= e^{t}\Big|_{-\infty}^{-1}$$

$$= e^{-1} < \infty.$$

The system is stable.

(c) $$h(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

(i) The system is causal since $h(t) = 0,\ t < 0.$

(ii) $\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{0}^{\infty} 1 \cdot dt = \infty$

The system is not stable.

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4. $$y(t) = x(t - t_0)$$

Let $x(t) = \delta(t)$

$$h(t) = \delta(t - t_0)$$

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5. $h(t) = \delta(t-1) - \delta(t-3)$

$$s(t) = u(t) * h(t)$$

$$= h(t) * u(t)$$

$$= \int_{-\infty}^{\infty} h(\tau)\, u(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{t} h(\tau)\, d\tau$$

$$s(t) = \begin{cases} 0, & t<1 \\ 1, & 1<t<3 \\ 0, & t>3 \end{cases}$$