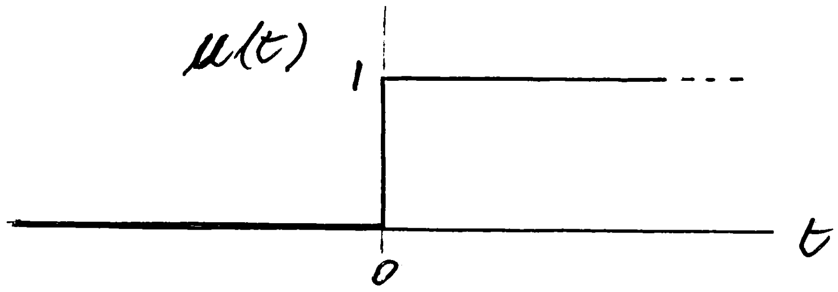
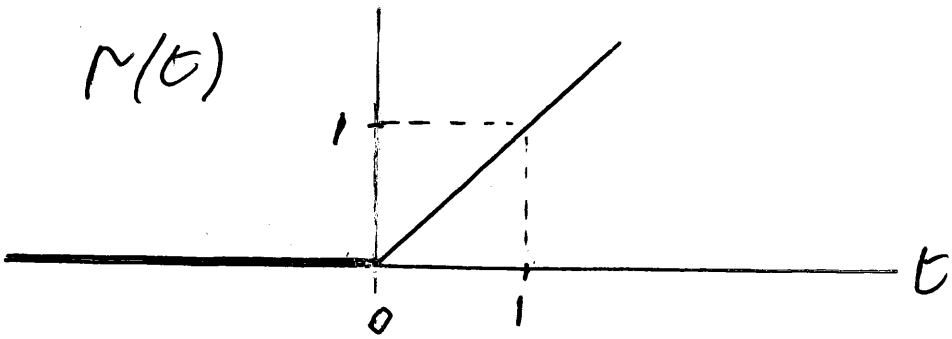
Signals

Unit step function, $u(t)$



$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

Unit ramp function, $r(t)$



$$r(t) = \begin{cases} t, & t > 0 \\ 0, & t < 0 \end{cases}$$

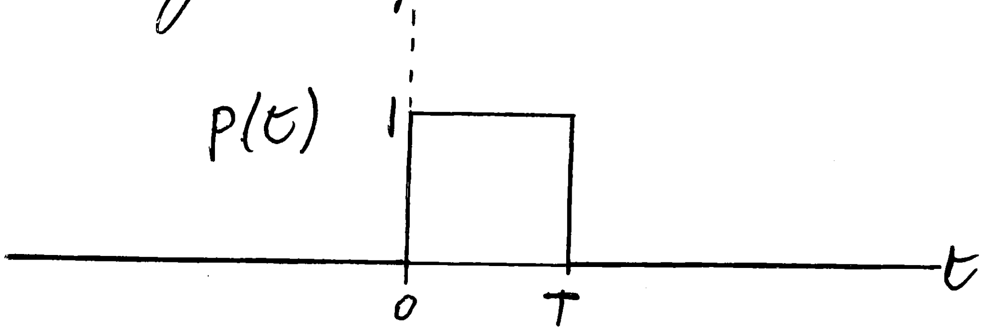
$$r(t) = t\,u(t)$$

Note that $r(t) = \int_{-\infty}^{t} u(\tau)\, d\tau$

and $u(t) = \dfrac{d\, r(t)}{dt}, \quad t \neq 0.$

Rectangular Pulse, $p(t)$

The rectangular pulse, $p(t)$, shown below



can be expressed as:

$$p(t) = u(t) - u(t-T)$$

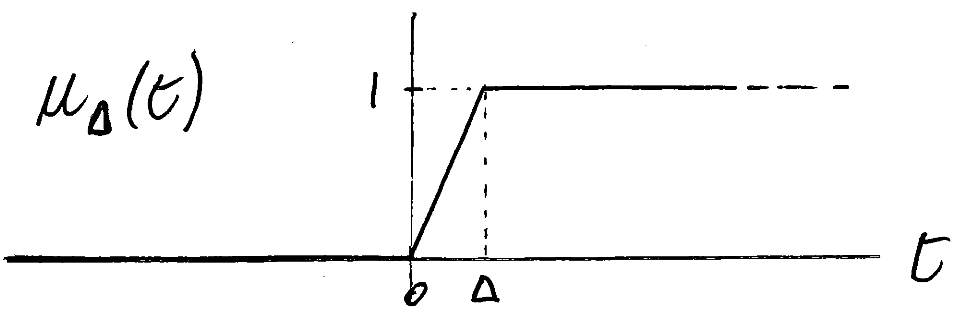
Many elementary signals such as the unit step function and unit ramp function are either discontinuous or have discontinuous derivatives.

These signals are referred to as <u>singularity functions</u>.

<u>The impulse (delta) function, $\delta(t)$</u>

The intuitive interpretation of the unit impulse function is that it is an idealization of a very narrow pulse having unit area.

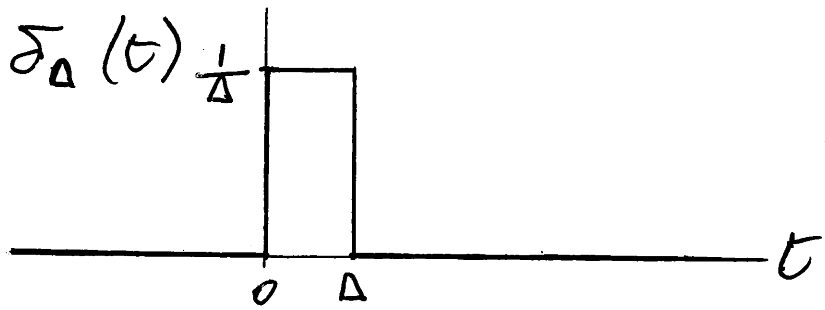
Let $u_\Delta(t)$ denote an approximation to the unit step function.



$$u(t) = \lim_{\Delta \to 0} u_\Delta(t)$$

Consider the derivative

$$\delta_\Delta(t) = \frac{d\, u_\Delta(t)}{dt}$$



[<u>Note</u> In this section we use the term "derivative" to mean the actual derivative except at those points at which a unique derivative does not exist.]

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$

The unit impulse function can be thought of as the first derivative of the unit step function : $\delta(t) = \frac{d\, u(t)}{dt}$

[Note that since $u(t)$ is discontinuous at $t = 0$, it is not differentiable in the strict mathematical sense.]

Properties of the unit-impulse function

1. $\delta(t - t_0) = 0, \quad t \neq t_0$

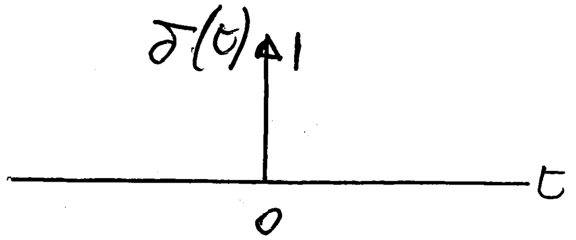
2. $\int_{t_1}^{t_2} \delta(t - t_0)\, dt = 1, \quad t_1 < t_0 < t_2$

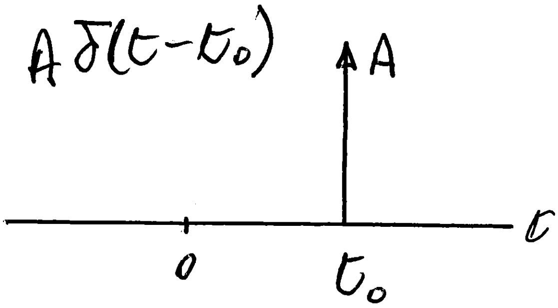
3. $\int_{-\infty}^{\infty} f(t)\delta(t - t_0)\, dt = f(t_0),$

   $f(t)$ continuous at t_0.

Property 3 is called the sifting property.

Graphical representation of impulses





Complex sinusoid

$$x(t) = A e^{j\phi} e^{j\omega_0 t}$$

A and ω_0 are real-valued.

$A e^{j\phi}$ is referred to as the complex amplitude of the complex exponential $e^{j\omega_0 t}$.

$$x(t) = A e^{j(\omega_0 t + \phi)}$$

Using Euler's formula we may write

$$x(t) = A \cos(\omega_0 t + \phi) + jA \sin(\omega_0 t + \phi)$$

The real sinusoid $x_1(t) = A \cos(\omega_0 t + \phi)$ with A real can be expressed in terms of the complex sinusoid:

$$x_1(t) = \mathrm{Re}\left\{ A e^{j(\omega_0 t + \phi)} \right\}$$

or equivalently

$$x_1(t) = \frac{A}{2}\left\{ e^{j(\omega_0 t + \phi)} + e^{-j(\omega_0 t + \phi)} \right\}$$

Continuous-time convolution

The convolution of $x(t)$ and $h(t)$ to produce $y(t)$ is denoted by

$$y(t) = x(t) * h(t)$$

and is defined by

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \,.$$

The Commutative Property

Convolution is a commutative operation:

$$x(t) * h(t) = h(t) * x(t)$$

The Distributive Property

Convolution distributes over addition:

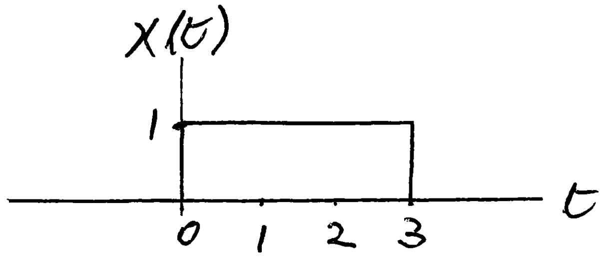
$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$

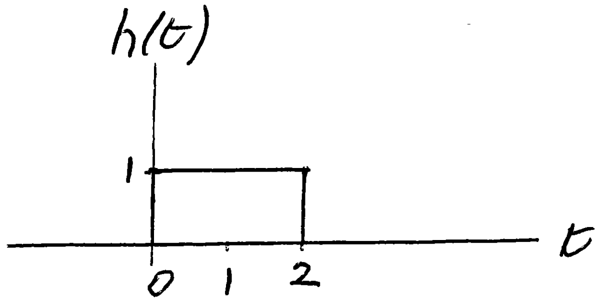
The Associative Property

$$x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t)$$

The Convolution of two functions

Consider the two functions $x(t)$ and $h(t)$ shown below.



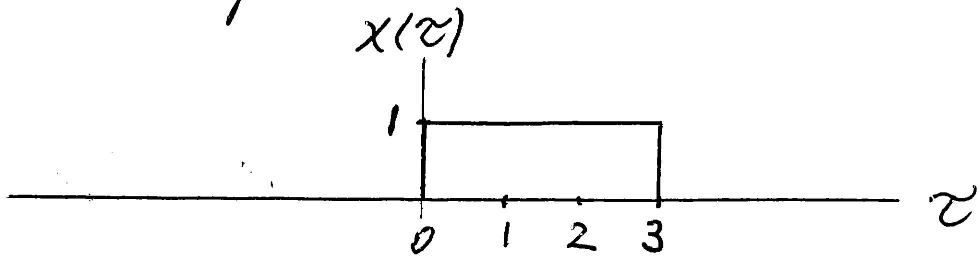


Let $y(t)$ denote the convolution of $x(t)$ and $h(t)$

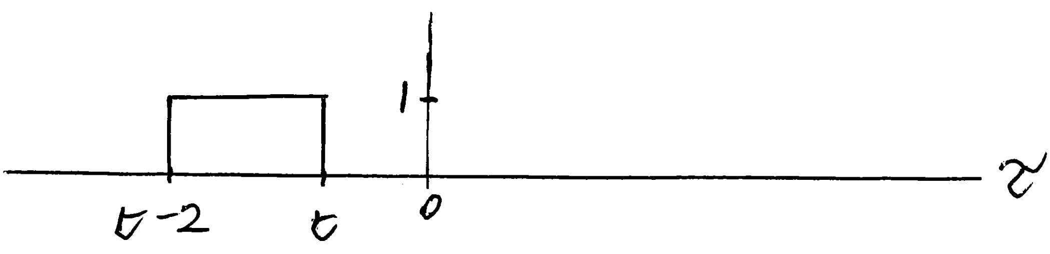
$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

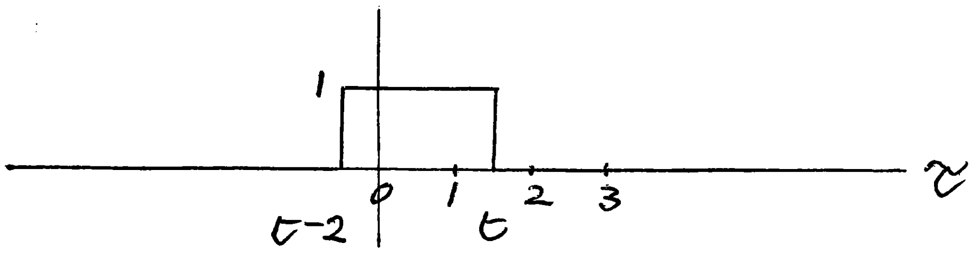
It is convenient to consider the evaluation of $y(t)$ in separate intervals.



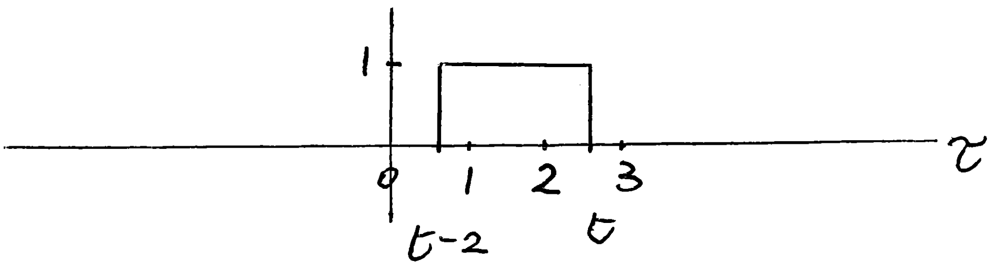
$h(t-\tau)$ $\qquad\qquad t<0$



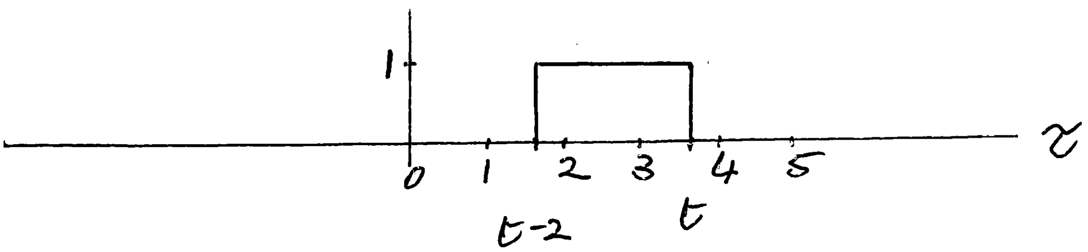
$h(t-\tau)$ $\qquad$ $0 < t < 2$



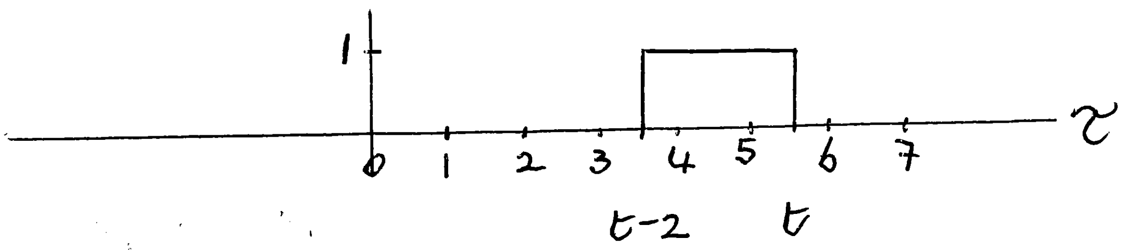
$h(t-\tau)$ $\qquad$ $2 < t < 3$



$h(t-\tau)$ $\qquad$ $3 < t < 5$

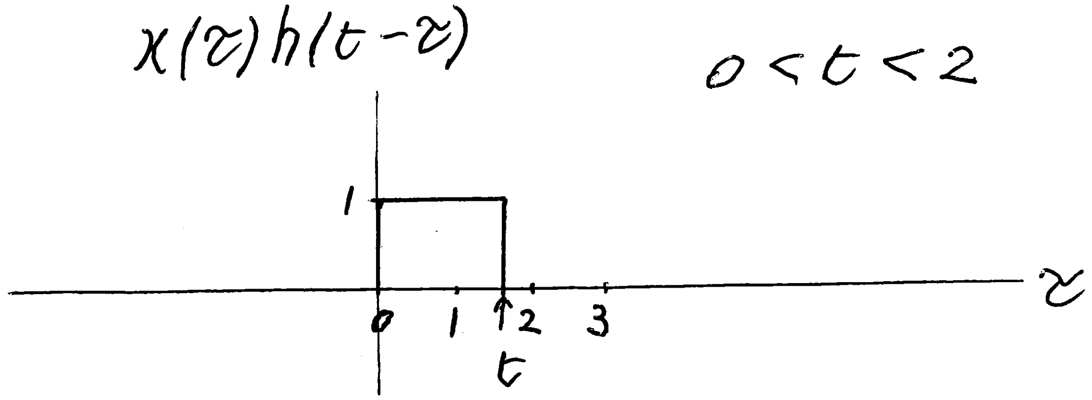


$h(t-\tau)$ $\qquad$ $t > 5$

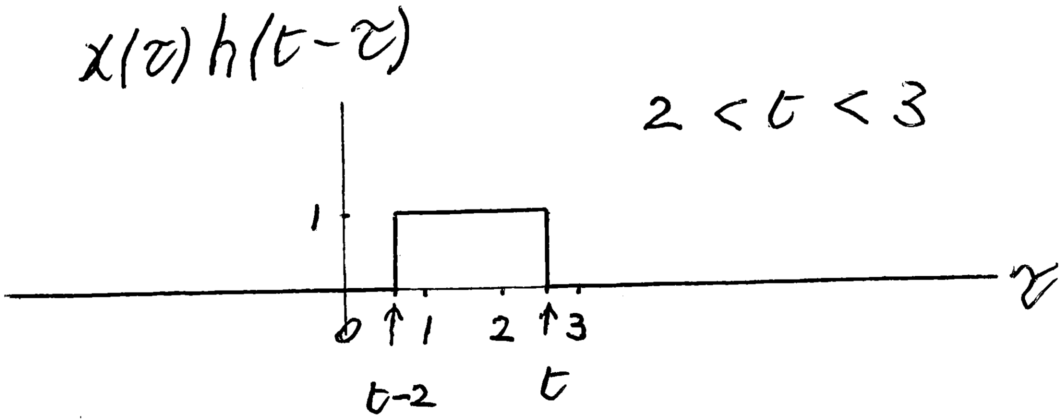


For $t < 0$ and for $t > 5$, $x(\tau)h(t-\tau) = 0$ for all values of τ, and consequently $y(t) = 0$.

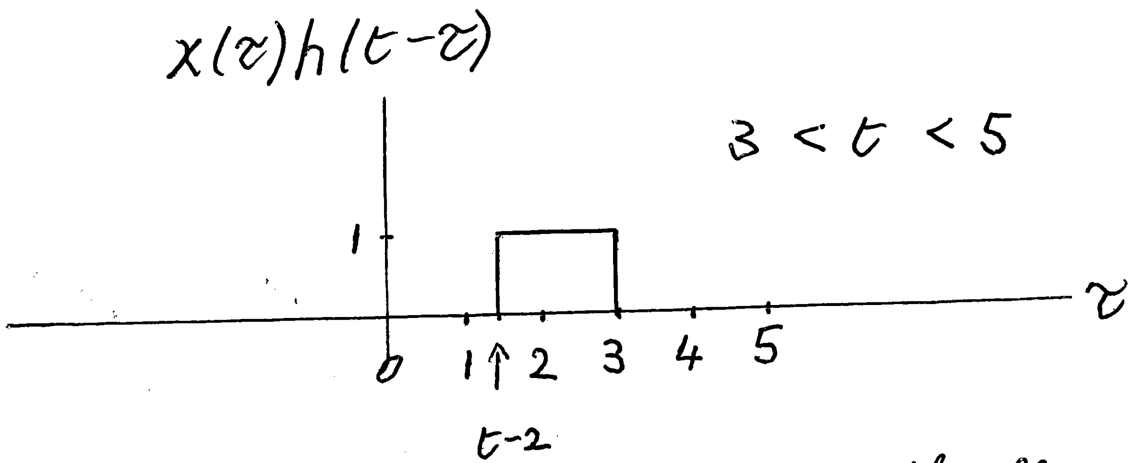
For the other intervals, the product $x(\tau)h(t-\tau)$ is shown below.



$0 < t < 2$



$2 < t < 3$



$3 < t < 5$

The integration can be carried out graphically.

$$y(t) = \begin{cases} 0, & t < 0 \\ t, & 0 < t < 2 \\ 2, & 2 < t < 3 \\ 5-t, & 3 < t < 5 \\ 0, & t > 5 \end{cases}$$