Frequency response of an LTI discrete-time system.

$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

$$y[n] = x[n] * h[n]$$

$$Y(e^{j\Omega}) = X(e^{j\Omega})H(e^{j\Omega})$$

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})}$$

Sampling of Continuous-Time Signals

Ideal Impulse Sampling

$$\begin{tikzcd} & p(t) \arrow[d] & \\ x_a(t) \arrow[r] & \otimes \arrow[r] & x_p(t) \end{tikzcd}$$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$x_p(t) = x_a(t)\, p(t)$$

$$X_p(j\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X_a\left(j\left(\omega - \frac{2\pi k}{T}\right)\right) \quad \text{---} \; (1)$$

$$x_p(t) = \sum_{n=-\infty}^{\infty} x_a(nT)\, \delta(t - nT)$$

$$X_p(j\omega) = \mathcal{F}\left[\sum_{n=-\infty}^{\infty} x_a(nT)\, \delta(t-nT)\right]$$

$$= \sum_{n=-\infty}^{\infty} x_a(nT)\, e^{-j\omega nT} \quad \text{---} \; (2)$$

Consider the sequence $x[n] = x_a(nT)$

The discrete-time Fourier transform of $x(n)$ is given by

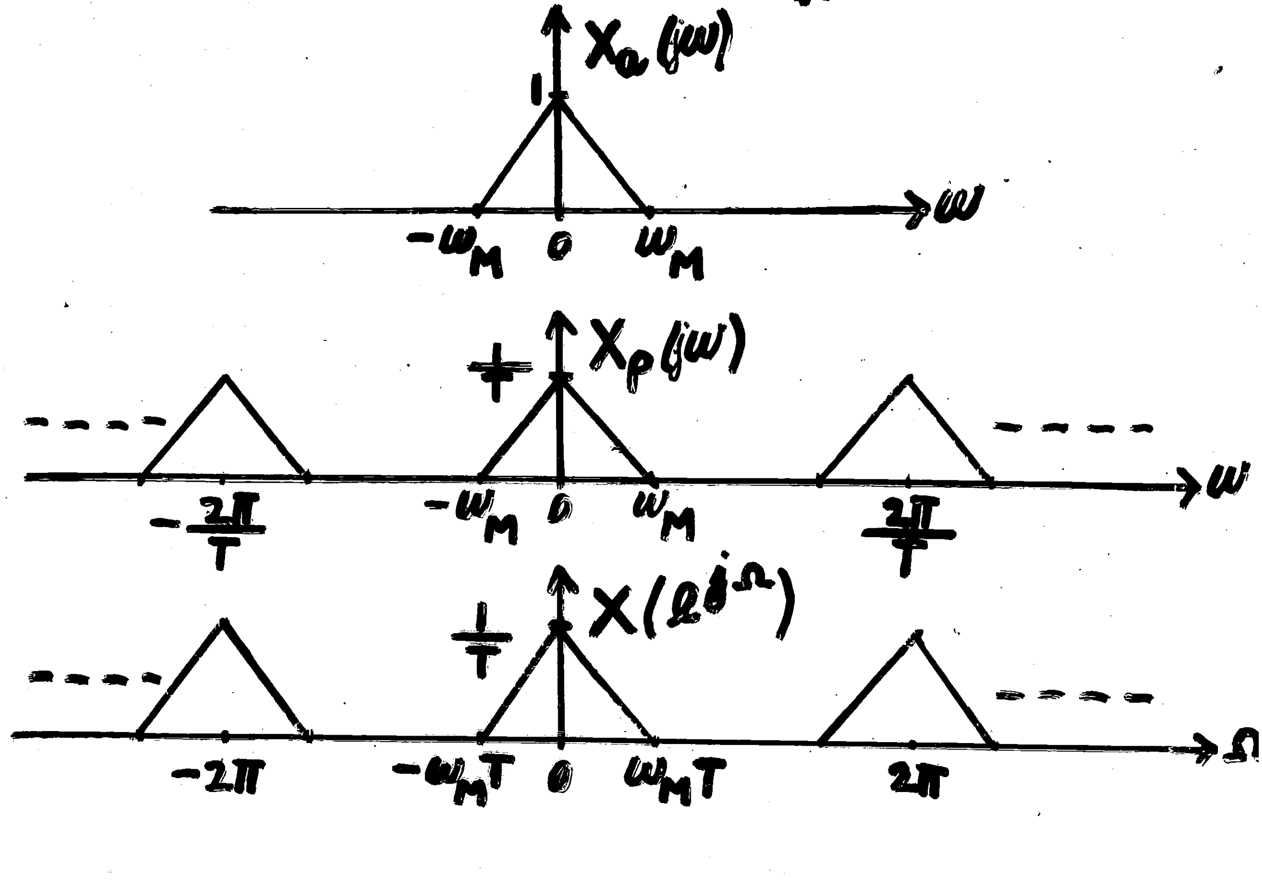
$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n} = \sum_{n=-\infty}^{\infty} x_a(nT)\, e^{-j\Omega n} \quad \text{—} \; (3)$$

Comparing equations ①, ②, and ③ we have

$$X(e^{j\Omega}) = X_p\left(j\left(\frac{\Omega}{T}\right)\right) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_a\left(j\left(\frac{\Omega}{T} - \frac{2\pi k}{T}\right)\right)$$

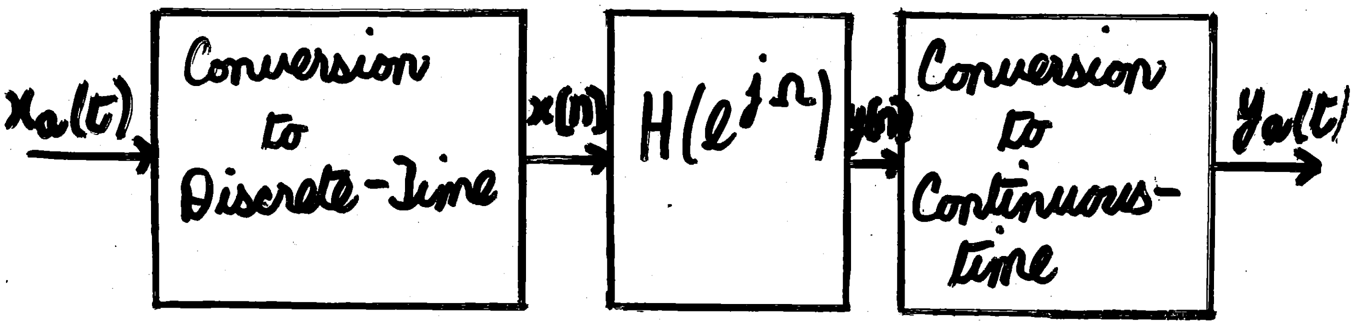
Alternatively this equation can be expressed in terms of the analog frequency variable ω as

$$X(e^{j\omega T}) = X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_a\left(j\left(\omega - \frac{2\pi k}{T}\right)\right)$$



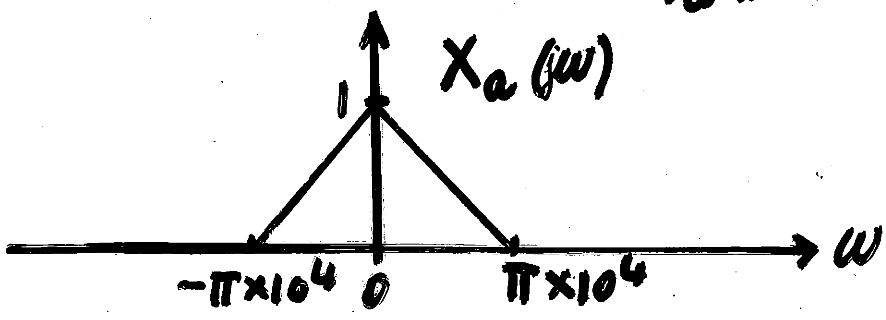
Relationship between $X_a(j\omega)$, $X_p(j\omega)$ and $X(e^{j\Omega})$ for $\omega_s = \frac{2\pi}{T} > 2\omega_M$.

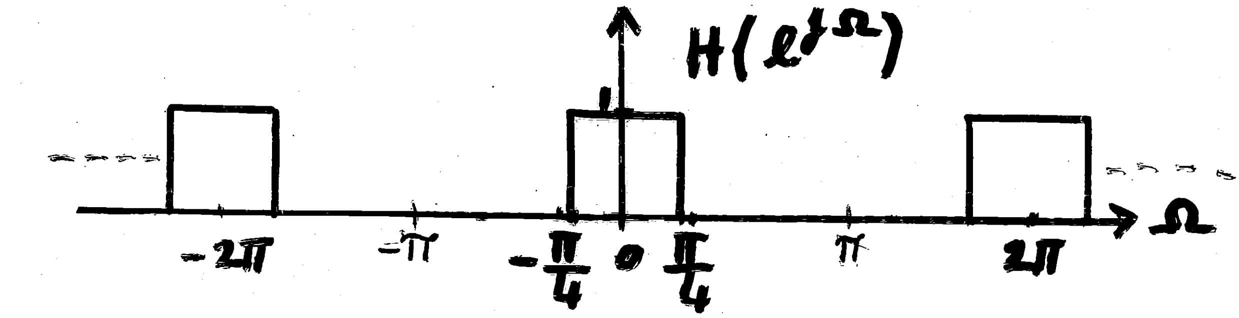
Discrete-time processing of continuous-time signals

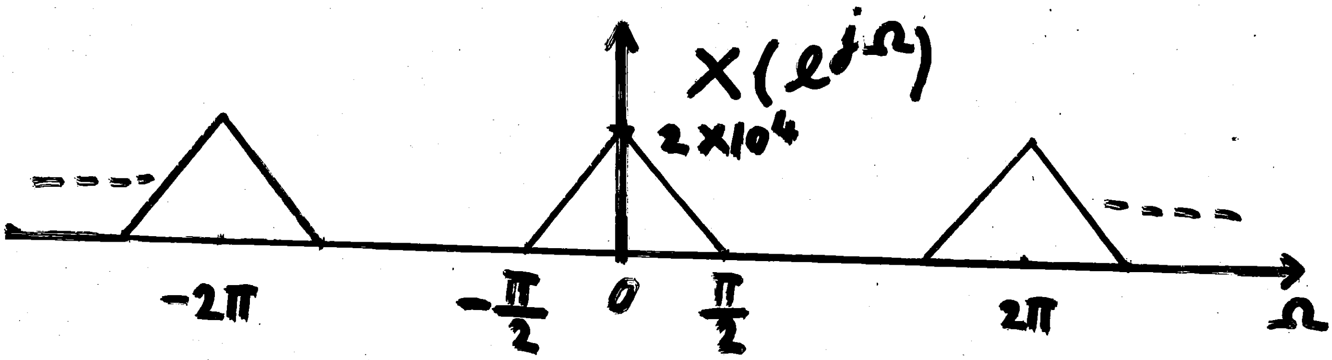


$$x[n] = x_a(nT)$$

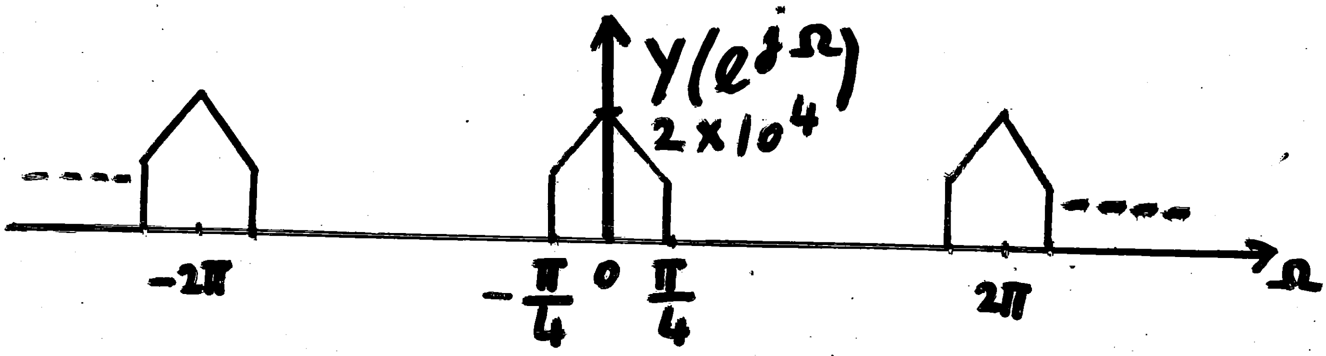
If $X_a(j\omega)$ and $H(e^{j\Omega})$ are as shown below and with $\frac{1}{T} = 20$ kHz, sketch $X(e^{j\Omega})$, $Y(e^{j\Omega})$ and $Y_a(j\omega)$.

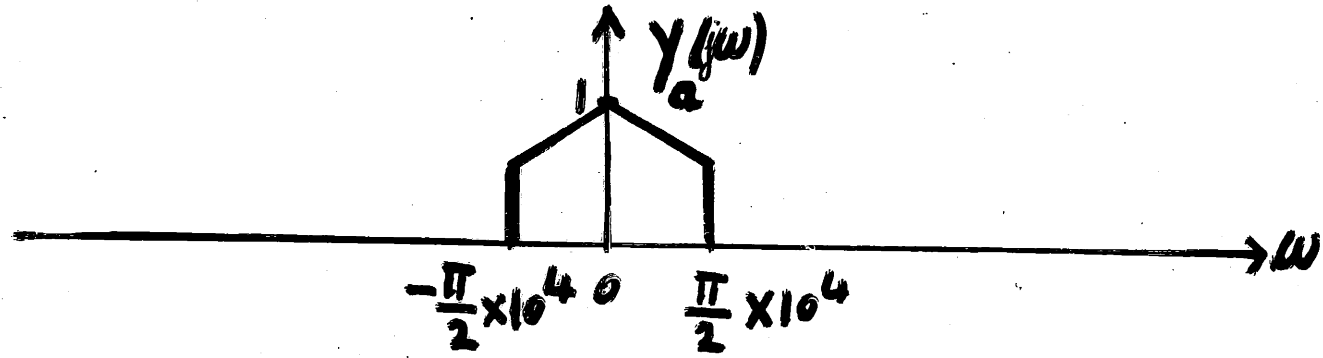






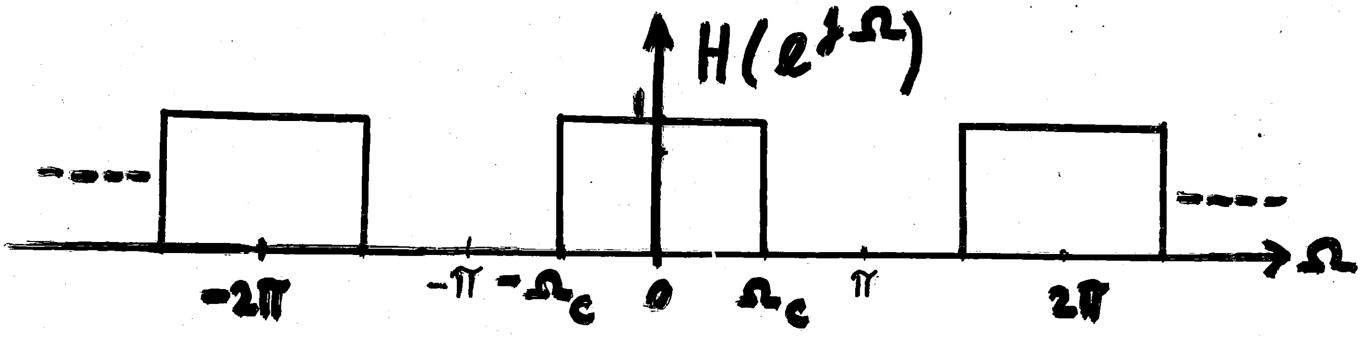
$$Y(e^{j\Omega}) = X(e^{j\Omega})H(e^{j\Omega})$$





Ideal Discrete-time lowpass filter

$$H(e^{j\Omega}) = \begin{cases} 1, & |\Omega| \leq \Omega_c \\ 0, & \Omega_c < |\Omega| \leq \pi \end{cases}$$



$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\Omega})\, e^{j\Omega n}\, d\Omega$$

$$= \frac{1}{2\pi}\int_{-\Omega_c}^{\Omega_c} e^{j\Omega n}\, d\Omega$$

$$h(n) = \frac{\sin(\Omega_c n)}{\pi n}$$

The unit-sample response of an

ideal discrete - time lowpass filter with cutoff frequency $\Omega_c = \frac{\pi}{2}$ is shown below.