Sampling rate reduction by an integer factor M.

Consider the continuous-time signal $x_a(t)$ with Fourier transform $X_a(j\omega)$.

$$\text{Let } x[n] = x_a(nT)$$

$$x[n] \longleftrightarrow X(e^{j\Omega})$$

$$X(e^{j\Omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_a\left(j\left(\frac{\Omega}{T} - \frac{2\pi k}{T}\right)\right)$$

Form the new sequence $x_d[n]$:

$$x_d[n] = x[nM]$$
$$= x_a(nMT)$$

$$x[n] \longrightarrow \boxed{\downarrow M} \longrightarrow x_d[n] = x[nM]$$

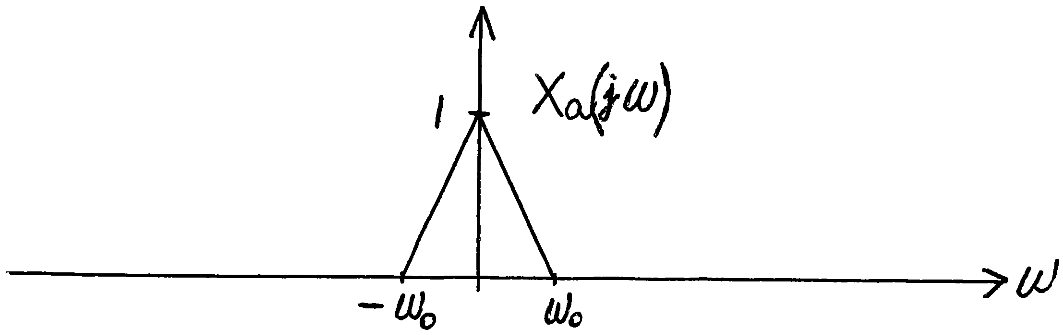
Sampling period T

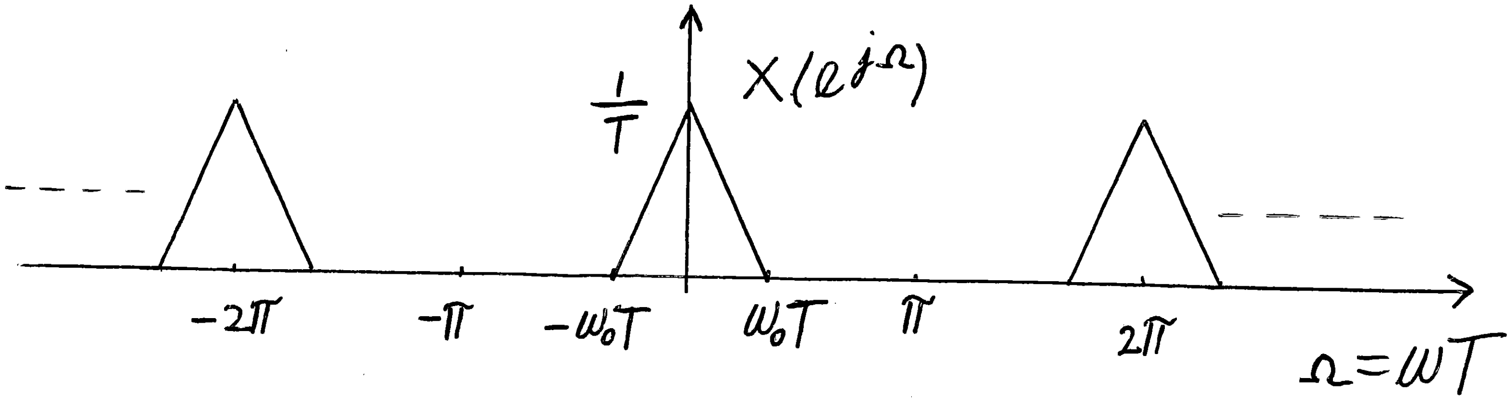
Sampling period $T' = MT$

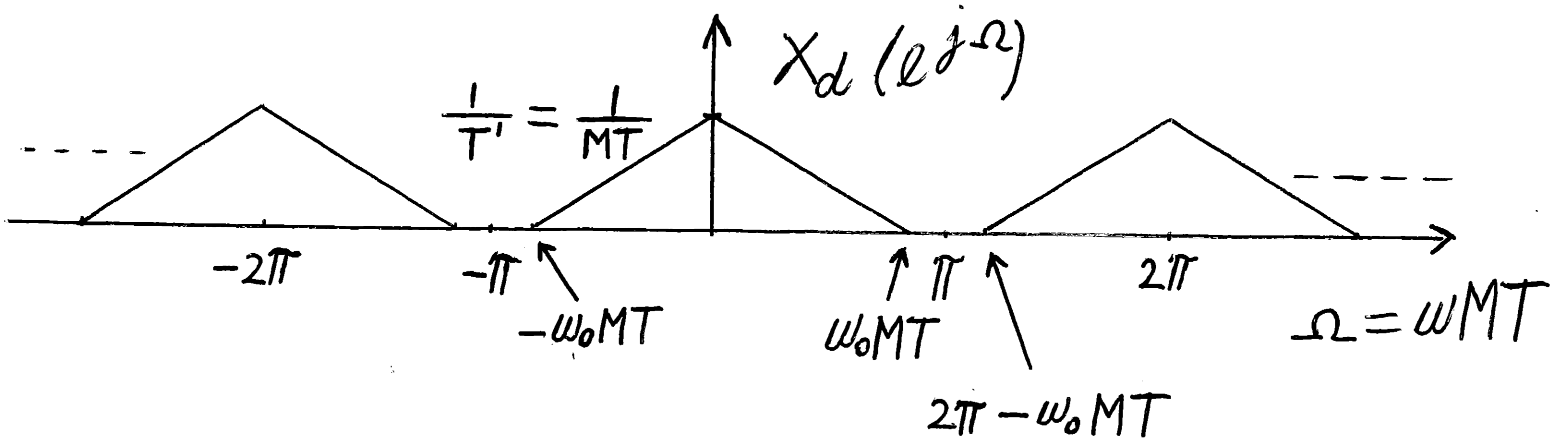
$$x_d[n] = x_a(nT'), \text{ where } T' = MT$$

$$x_d[n] \longleftrightarrow X_d(e^{j\Omega})$$

$$X_d(e^{j\Omega}) = \frac{1}{T'}\sum_{r=-\infty}^{\infty} X_a\left(j\left(\frac{\Omega}{T'} - \frac{2\pi r}{T'}\right)\right)$$

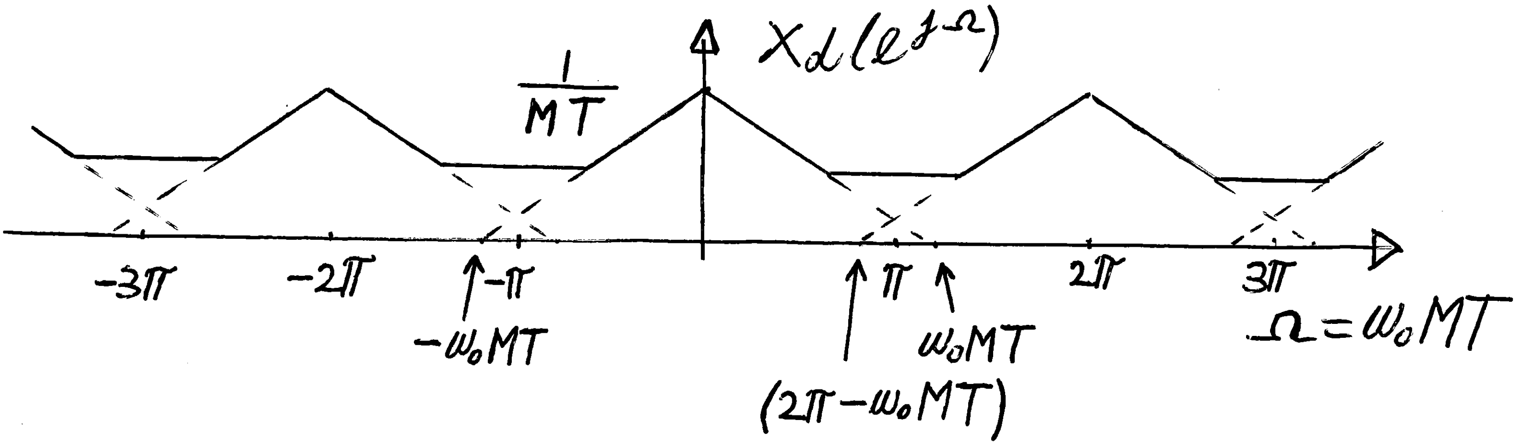






No aliasing occurs if $\omega_0 MT < 2\pi - \omega_0 MT$

i.e. $\omega_0 < \frac{\pi}{MT}$

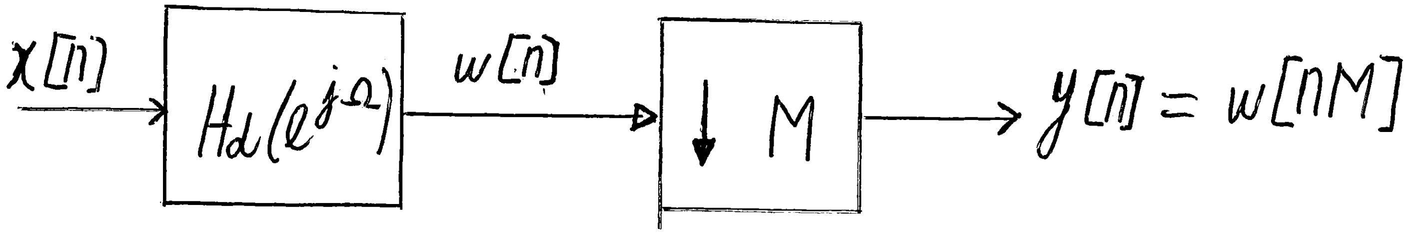


Aliasing occurs when $\omega_0 MT > 2\pi - \omega_0 MT$

ie when $\omega_0 > \frac{\pi}{MT}$

To avoid aliasing use a discrete-time low-pass filter with a gain $= 1$ and a cutoff frequency $\Omega_c = \left(\frac{\pi}{MT}\right)T = \frac{\pi}{M}$

$$H_d(e^{j\Omega}) = \begin{cases} 1, & |\Omega| \leq \frac{\pi}{M} \\ 0, & \frac{\pi}{M} < |\Omega| \leq \pi \end{cases}$$



Sampling period: T $\qquad$ T $\qquad$ $T' = MT$

Block diagram for decimation by an integer factor M.

Increasing the sampling rate by an integer factor

$$x[n] = x_a(nT)$$

$$x_i[n] = x_a(nT')$$

where $T' = \frac{T}{L}$

$$x_i[n] = x[n/L] = x_a(nT/L), \quad n = 0, \pm L, \pm 2L, \ldots$$

$$x[n] \longrightarrow \boxed{\uparrow L} \xrightarrow{\;x_e[n]\;} \boxed{\begin{array}{c}\text{L.P.F.}\\ \text{gain} = L \\ \text{Cutoff} = \frac{\pi}{L}\end{array}} \longrightarrow x_i[n]$$

Sampling period: T $\qquad T' = \frac{T}{L} \qquad\qquad T' = \frac{T}{L}$

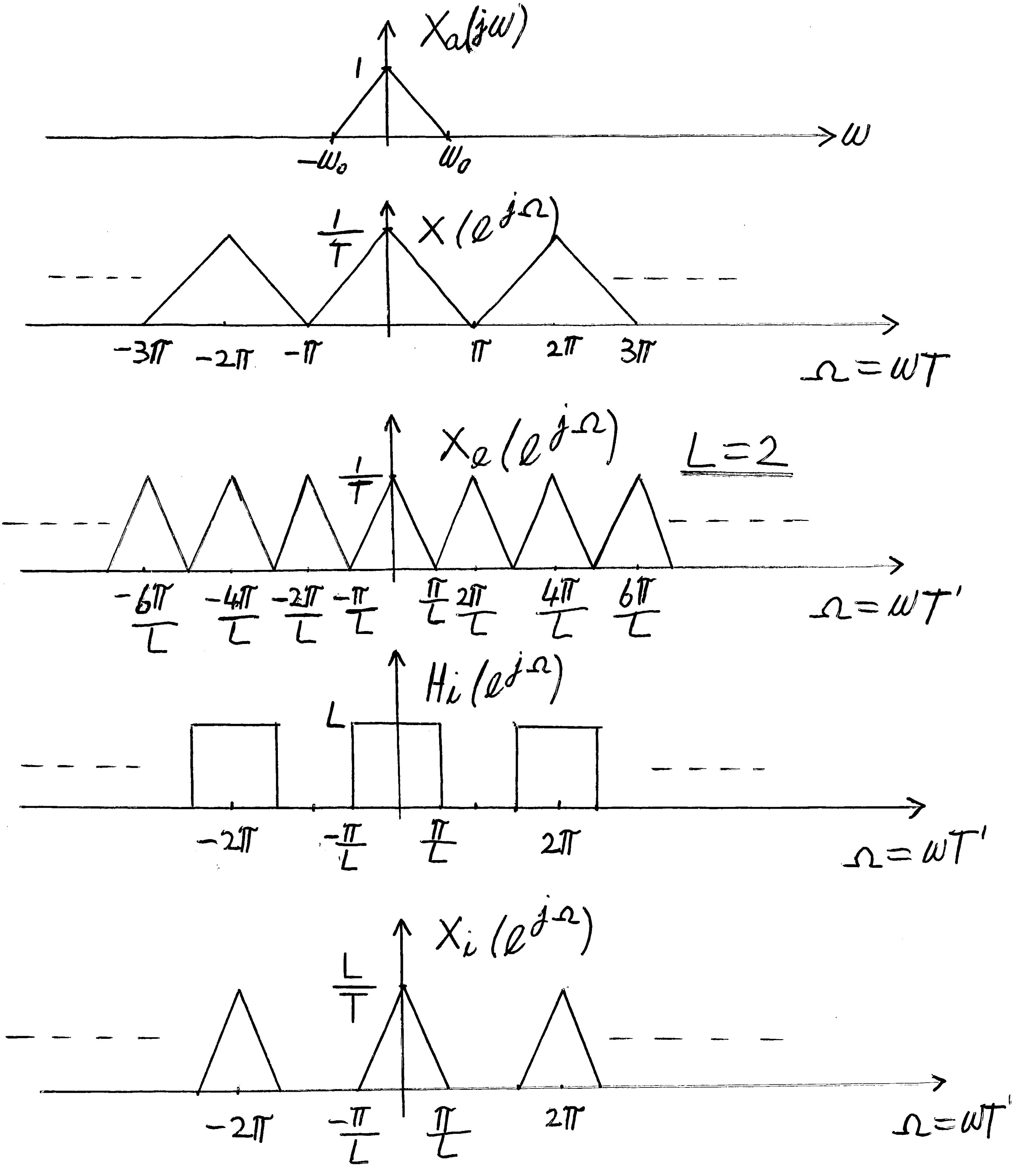
Block diagram for interpolation by an integer factor L.

$$x_e[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \ldots \\ 0, & \text{otherwise} \end{cases}$$

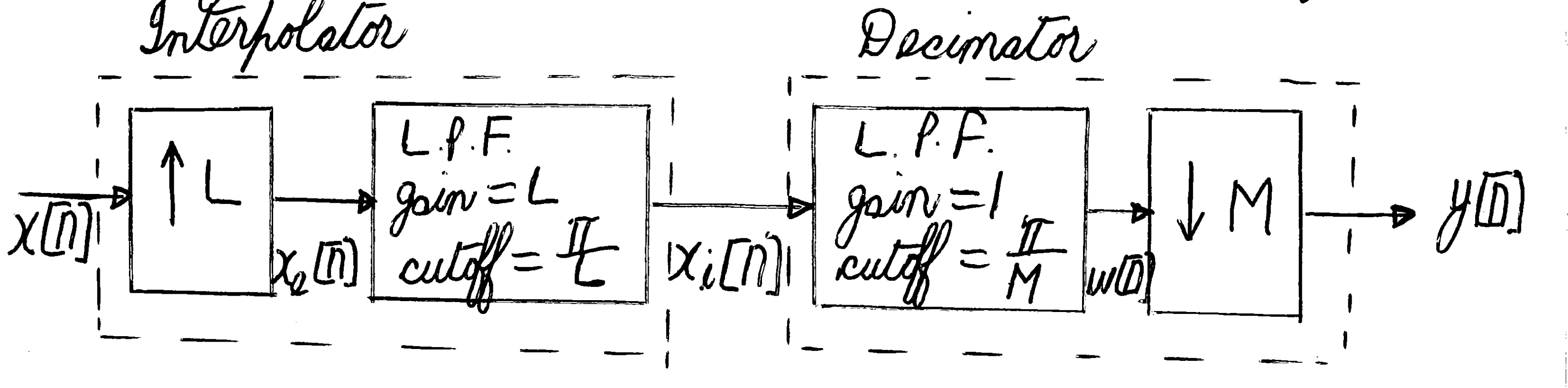
$$X_e(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_e[n]\, e^{-j\Omega n}$$

$$= \sum_{r=-\infty}^{\infty} x[r]\, e^{-j\Omega L r} = X(e^{j\Omega L})$$

where $\Omega = \omega T'$



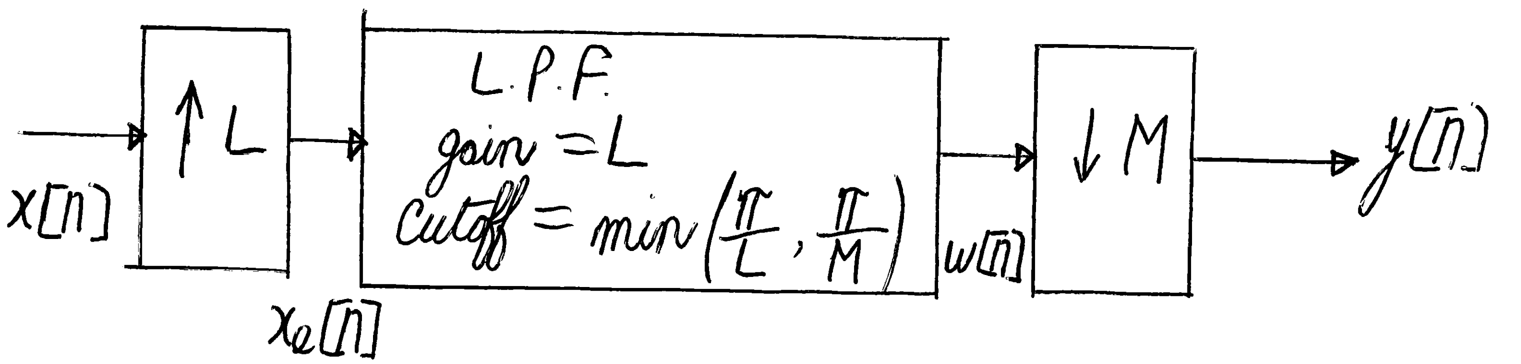
Sampling rate conversion by a non-integer factor L/M



Sampling period:

$T \qquad \frac{T}{L} \qquad \frac{T}{L} \qquad \frac{T}{L} \qquad \frac{TM}{L}$

The decimation and interpolation filters may be combined as shown below.



Sampling Period:

$T \qquad \frac{T}{L} \qquad \frac{T}{L} \qquad \frac{TM}{L}$