3C1 Signals and Systems **Homework No. 2**

1. A linear time-invariant system has an impulse response $h(t)$ given by

$$h(t) = 2\,e^{-2t}u(t).$$

(i) Determine the frequency response of the system.

(ii) Determine the response of the system to the input signal $x(t)$ given by

$$x(t) = 1 + \cos(\omega_0 t) - 2\,\cos(2\omega_0 t).$$

2. Consider a causal and stable LTI system whose input $x(t)$ and output $y(t)$ are related by the differential equation

$$\frac{dy(t)}{dt} + 3\,y(t) = x(t).$$

(i) Determine the frequency response of the system.

(ii) Find the Fourier series representation of the output $y(t)$ for the input $x(t) = \cos(\omega_0 t)$.

3. Consider an LTI system whose frequency response is

$$H(j\omega) = \begin{cases} 1, & |\omega| \le 50\pi, \\ 0, & \text{otherwise.} \end{cases}$$

When the input to the system is a periodic signal $x(t)$ with fundamental period $T = 0.25$ and Fourier Series coefficients a_k, it is found that the output $y(t)$ is identical to $x(t)$. For what values of k is it guaranteed that $a_k = 0$?

4. Determine the Fourier transform of the following signals:

(i) $x_1(t) = e^{-a\,t}\,u(t), \quad a > 0$

(ii) $x_2(t) = e^{a\,t}\,u(-t), \quad a > 0$

(iii) $x_3(t) = e^{-a\,|t|}, \quad a > 0$

5. Given the following Fourier transforms determine the corresponding signal in each case:

(i) $X(j\omega) = 2\pi\delta(\omega) + \pi[\delta(\omega + \omega_0) + \delta(\omega - \omega_0)]$

(ii) $X(j\omega) = \begin{cases} 1, & |\omega| < \omega_a \\ 0, & |\omega| > \omega_a \end{cases}$