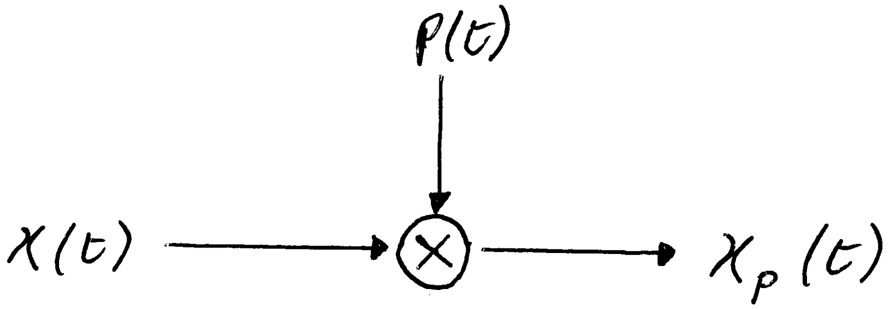
Sampling of Analogue Signals

Impulse - Train Sampling



$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

$$p(t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} e^{jk\omega_s t}$$

where $\omega_s = \frac{2\pi}{T}$

$$x_p(t) = x(t)\, p(t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} x(t)\, e^{jk\omega_s t}$$

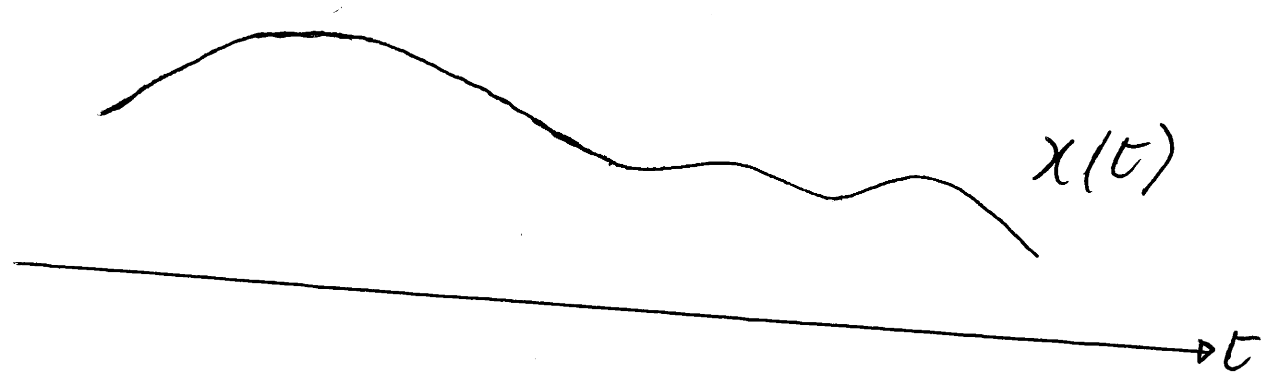
$$X(j\omega) = \mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

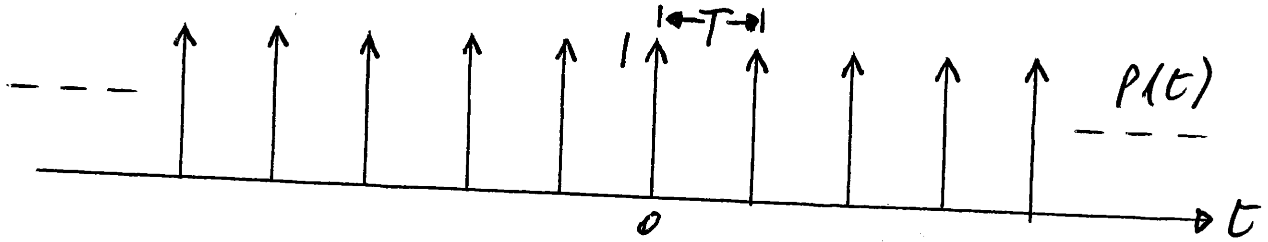
$$X_p(j\omega) = \mathcal{F}\{x_p(t)\} = \int_{-\infty}^{\infty} x(t)\, p(t)\, e^{-j\omega t}\, dt$$

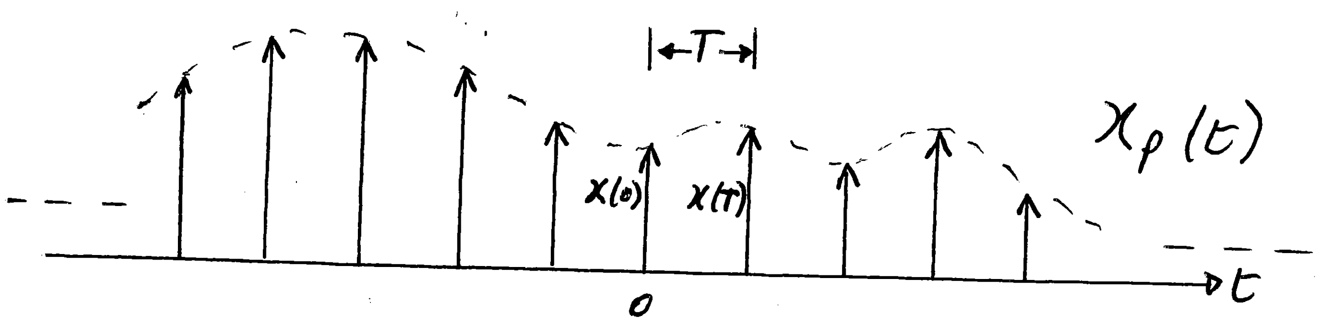
$$X_p(j\omega) = \int_{-\infty}^{\infty} \frac{1}{T} \sum_{k=-\infty}^{\infty} x(t) e^{jk\omega_s t} e^{-j\omega t} dt$$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} \left\{ \int_{-\infty}^{\infty} x(t) e^{-j(\omega - k\omega_s)t} dt \right\}$$

$$= \frac{1}{T} \sum_{k=-\infty}^{\infty} X(j(\omega - k\omega_s))$$

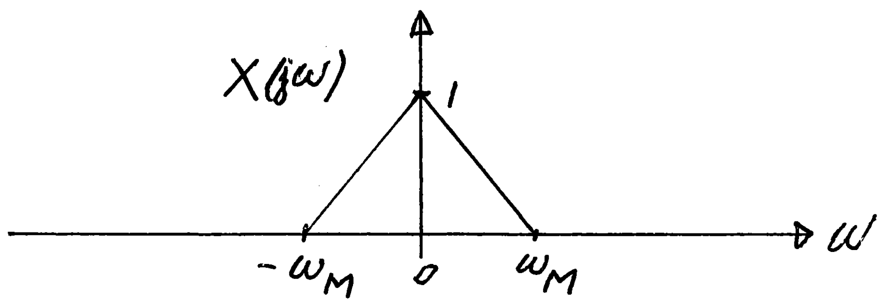




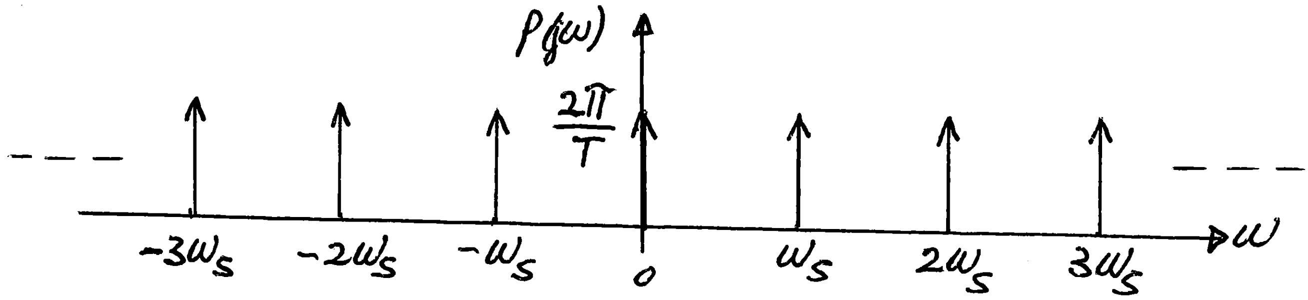


$X_p(j\omega)$ is a periodic function of frequency consisting of a sum of shifted replicas of $X(j\omega)$, scaled by $\frac{1}{T}$. If $\omega_M < (\omega_S - \omega_M)$ or equivalently $\omega_S > 2\omega_M$ there is no overlap between the shifted replicas of $X(j\omega)$ and $x(t)$ can be recovered exactly from $x_p(t)$ by means of an ideal low-pass filter with gain T and a cut-off frequency greater than ω_M and less than $\omega_S - \omega_M$.

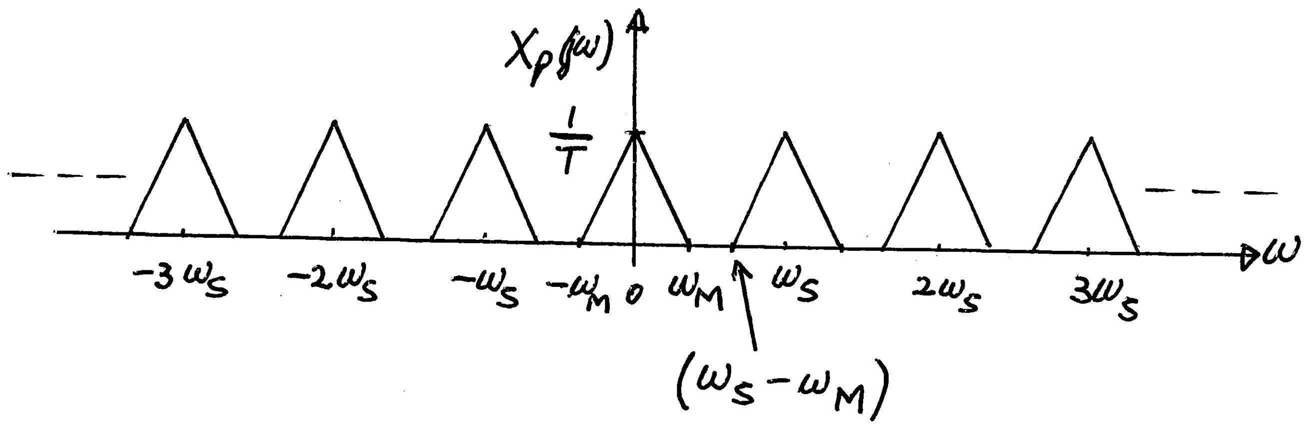
If $\omega_S < 2\omega_M$ the shifted replicas of $X(j\omega)$ overlap and it is therefore not possible to recover the original signal. This overlapping of spectra is referred to as <u>aliasing</u>.



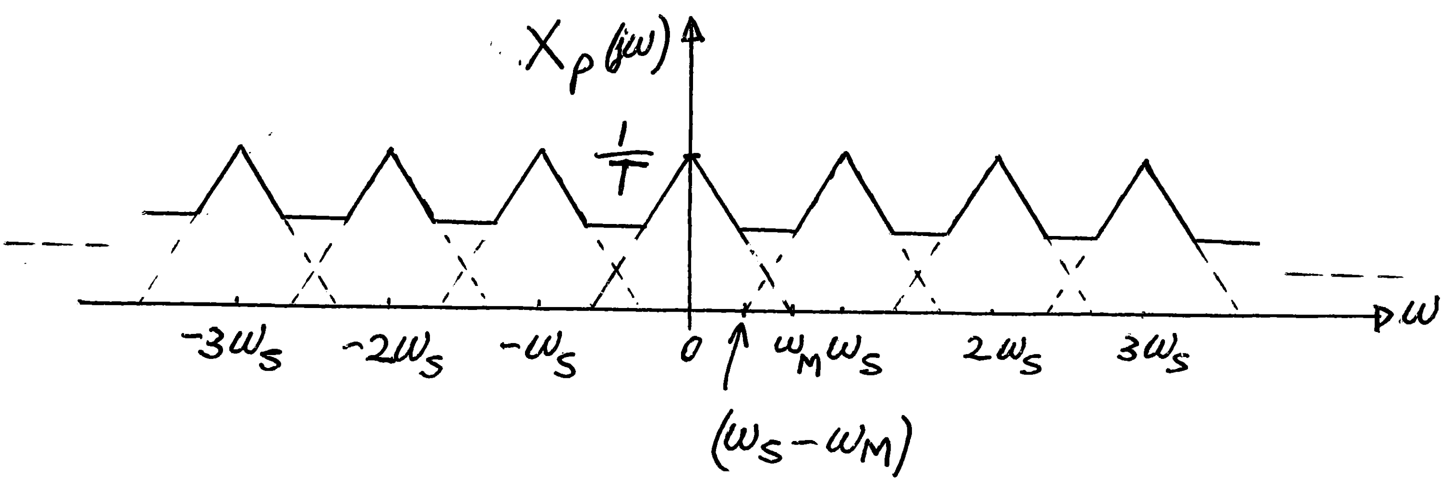
Spectrum of original signal $x(t)$



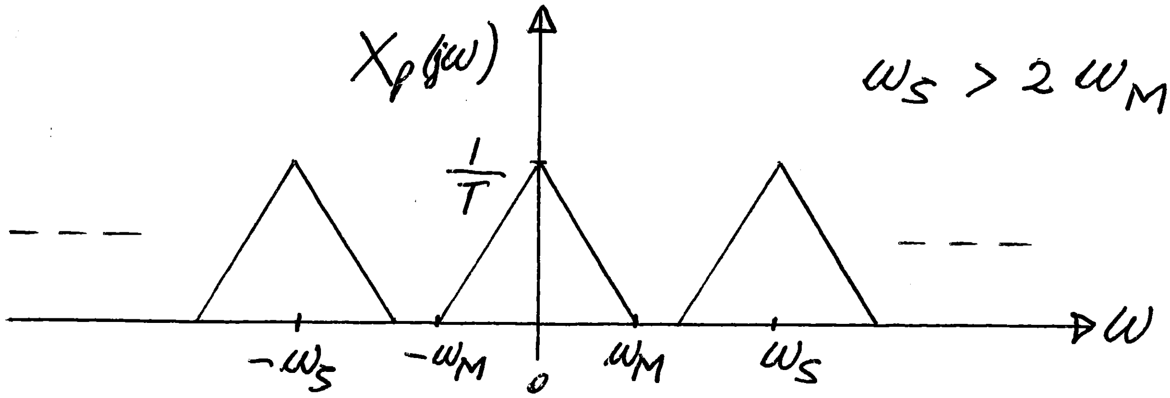
Spectrum of sampling function $p(t)$

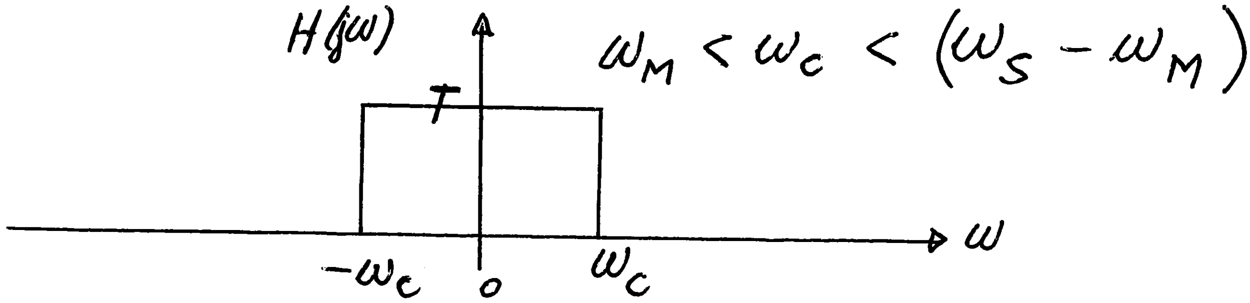


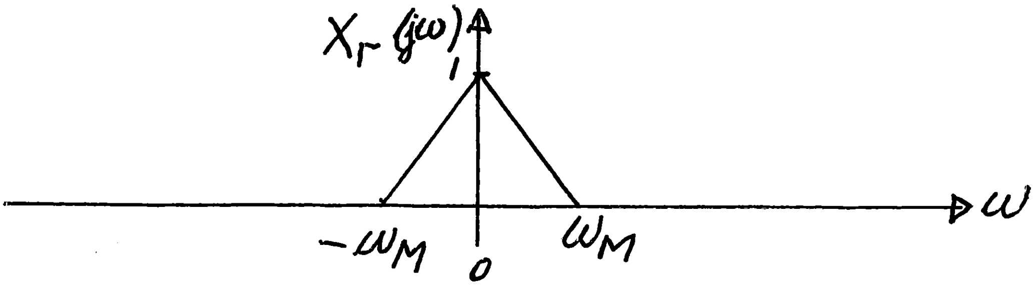
Spectrum of sampled signal with $\omega_s > 2\omega_M$



Spectrum of sampled signal with $\omega_s < 2\omega_M$.







Exact recovery of a continuous-time signal from the sampled signal $x_p(t)$ using an ideal low-pass filter.

Sampling Theorem

If a signal $x(t)$ has a bandlimited Fourier transform $X(j\omega)$, that is $X(j\omega)=0$ for $|\omega|>2\pi f_M$, then $x(t)$ can be uniquely reconstructed without error from equally spaced samples $x(nT)$, $-\infty < n < \infty$, if $f_s > 2f_M$ where $f_s = \frac{1}{T}$ is the sampling frequency.