linearity of the D.T.F.T.

$$X_1(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_1[n] e^{-j\Omega n}$$

$$X_2(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_2[n] e^{-j\Omega n}$$

$$x_3[n] = a\,x_1[n] + b\,x_2[n]$$

$$X_3(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_3[n] e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} \left\{ a\,x_1[n] + b\,x_2[n] \right\} e^{-j\Omega n}$$

$$= a \sum_{n=-\infty}^{\infty} x_1[n] e^{-j\Omega n} + b \sum_{n=-\infty}^{\infty} x_2[n] e^{-j\Omega n}$$

$$= a\,X_1(e^{j\Omega}) + b\,X_2(e^{j\Omega})$$

The Shift Property

$$x_2[n] = x_1[n-n_0]$$

$$X_2(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_1[n-n_0]\, e^{-j\Omega n}$$

let $m = n - n_0$, $\quad n = m + n_0$

$$X_2(e^{j\Omega}) = \sum_{m=-\infty}^{\infty} x_1[m]\, e^{-j\Omega(m+n_0)}$$

$$= e^{-j\Omega n_0} \sum_{m=-\infty}^{\infty} x_1[m]\, e^{-j\Omega m}$$

$$= e^{-j\Omega n_0}\, X_1(e^{j\Omega})$$

The Modulation Property

$$x_2[n] = e^{j\alpha n}\, x_1[n]$$

$$X_2(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_1[n]\, e^{j\alpha n}\, e^{-j\Omega n}$$

$$X_2(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_1[n]\, e^{-j(\Omega-\alpha)n}$$

$$= X_1\left(e^{j(\Omega-\alpha)}\right)$$

The Convolution Property

$$x_3[n] = x_1[n] * x_2[n]$$

$$= \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k]$$

$$X_3(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} x_1[k]\, x_2[n-k]\, e^{-j\Omega n}$$

$$= \sum_{k=-\infty}^{\infty} x_1[k] \sum_{n=-\infty}^{\infty} x_2[n-k]\, e^{-j\Omega n}$$

Let $m = n-k$, $n = m+k$

$$X_3(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} x_1[k] \sum_{m=-\infty}^{\infty} x_2[m]\, e^{-j\Omega(m+k)}$$

$$= \sum_{k=-\infty}^{\infty} x_1[k]\, e^{-j\Omega k} \sum_{m=-\infty}^{\infty} x_2[m]\, e^{-j\Omega m}$$

$$= X_1(e^{j\Omega})\, X_2(e^{j\Omega})$$

<u>The multiplication Property</u>

$$x_3[n] = x_1[n]\, x_2[n]$$

$$X_3(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x_1[n]\, x_2[n]\, e^{-j\Omega n}$$

$$x_1[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta})\, e^{j\theta n}\, d\theta$$

$$X_3(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta})\, e^{j\theta n}\, d\theta\; x_2[n]\, e^{-j\Omega n}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta}) \sum_{n=-\infty}^{\infty} x_2[n]\, e^{-j(\Omega-\theta)n}\, d\theta$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta})\, X_2(e^{j(\Omega-\theta)})\, d\theta$$