UNIVERSITY OF DUBLIN

TRINITY COLLEGE

FACULTY OF ENGINEERING, MATHEMATICS & SCIENCE

SCHOOL OF ENGINEERING

Electronic & Electrical Engineering

Senior Sophister **Trinity Term, 2014**
Engineering
Annual Examinations

Digital Signal Processing (4C5)

Date: Friday 9th May **Venue: Luce Upper** **Time: 14.00 – 16.00**

Mr. W. Dowling

Answer THREE questions
All questions carry equal marks

Permitted Materials:

Calculator
Drawing Instruments
Mathematical Tables
Graph Paper

Q.1 (a) A continuous-time signal, $x_a(t)$, has the Fourier transform $X_a(j\omega)$.

The discrete-time signal $x[n]$ is derived from $x_a(t)$ by periodic sampling:

$$x[n] = x_a(nT),$$

where T is a positive constant.

Let $X(e^{j\Omega})$ denote the discrete-time Fourier transform of $x[n]$. Show that

$$X(e^{j\Omega}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_a\left(j\left(\frac{\Omega}{T} - \frac{2\pi k}{T}\right)\right).$$

[11 marks]

(b) The continuous-time signal $y(t)$ is given by

$$y(t) = \sum_{n=-\infty}^{\infty} x[n]h(t-nT),$$

where
$$h(t) = \begin{cases} 1, & 0 < t < T \\ 0.5, & t = 0, T \\ 0, & \text{elsewhere.} \end{cases}$$

Let $Y(j\omega)$ and $H(j\omega)$ denote the Fourier transforms of $y(t)$ and $h(t)$ respectively.

(i) Obtain an expression for $H(j\omega)$ and sketch $|H(j\omega)|$ for $|\omega| \le \frac{2\pi}{T}$.

[5 marks]

(ii) Show that $Y(j\omega) = X(e^{j\Omega})\Big|_{\Omega = \omega T} H(j\omega)$.

[4 marks]

Q.2 **(a)** Show that a linear, time-invariant, discrete-time system is stable in the bounded-input bounded-output sense if, and only if, the unit-sample response of the system, $h[n]$, is absolutely summable, that is

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty .$$

[9 marks]

(b) A causal discrete-time system has a unit-sample response, $h[n]$, which is absolutely summable. Let $H(z)$ denote the z-transform of $h[n]$. Show that the region of convergence of $H(z)$ includes the unit circle and the entire z-plane outside the unit circle. **[7 marks]**

(c) The transfer function, $H(z)$, of a causal linear time-invariant discrete-time system is

$$H(z) = \frac{1 + z^{-1}}{1 - \alpha z^{-1}},$$

where α is a real constant.

(i) Determine the unit-sample response of the system, $h[n]$. **[1 mark]**

(ii) For what range of values of α is the system stable in the bounded-input bounded-output sense? **[1 mark]**

(iii) The input to the system is

$$x[n] = \begin{cases} (0.5)^n, & n \ge 0, \\ 0, & n < 0. \end{cases}$$

If $\alpha = 0.8$, determine the output $y[n]$. **[2 marks]**

Q.3 **(a)** A discrete-time filter has a unit-sample response $h[n]$ which is zero for $n < 0$ and $n > N-1$. Let $H(e^{j\Omega})$ denote the frequency response of the filter.

If $h[n] = h[N-1-n]$ and N is odd, show that

$$H(e^{j\Omega}) = e^{-j\Omega[(N-1)/2]}\left\{ h\left[\frac{N-1}{2}\right] + \sum_{n=0}^{\left(\frac{N-1}{2}\right)-1} 2h[n]\cos\left[\Omega\left(n - \frac{N-1}{2}\right)\right]\right\}$$

[8 marks]

(b) An ideal discrete-time band-pass filter has a frequency response, $H_{id}(e^{j\Omega})$, given by

$$H_{id}(e^{j\Omega}) = \begin{cases} 0, & |\Omega| < \dfrac{\pi}{4} \\ 1, & \dfrac{\pi}{4} < |\Omega| < \dfrac{3\pi}{4} \\ 0, & \dfrac{3\pi}{4} < |\Omega| \le \pi \end{cases}$$

Obtain an expression for the unit-sample response of this filter.

[7 marks]

(c) A nine point Hamming window, $w_H[n]$, is given by

$$w_H[n] = \begin{cases} 0.54 + 0.46\cos\left(\dfrac{\pi}{4}n\right), & -4 \le n \le 4, \\ 0, & \text{otherwise.} \end{cases}$$

Using the Hamming window, design a causal, nine point finite impulse response filter that approximates the magnitude response of the ideal band-pass filter in part (b).

[5 marks]

Q.4 (a) Let $H_c(s)$ denote the transfer function of a continuous-time filter. The transfer function of a discrete-time filter, $H(z)$, is obtained from $H_c(s)$ by the following transformation:

$$H(z) = H_c(s)\Big|_{s=(1-z^{-1})/(1+z^{-1})}.$$

Show that the frequency responses of the discrete-time and continuous-time filters are related by

$$H(e^{j\Omega}) = H_c(j\omega)\Big|_{\omega = \tan(\Omega/2)}.$$

[8 marks]

(b) A discrete-time high-pass filter with frequency response $H(e^{j\Omega})$ is to be designed to meet the following specifications:

$$0.89 \le \left|H(e^{j\Omega})\right| \le 1, \quad 0.6\pi \le |\Omega| \le \pi,$$

and

$$\left|H(e^{j\Omega})\right| \le 0.18, \quad |\Omega| \le 0.2\pi.$$

The filter is to be designed by applying the bilinear transformation to the transfer function of an appropriate Butterworth continuous-time filter. Verify that a second order filter is sufficient to meet the specifications. Determine the transfer function, $H(z)$, of the discrete-time filter. Note that the transfer function of a second order Butterworth low-pass prototype filter is

$$H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}.$$

[12 marks]

Q.5 (a) The sequence $x[n]$ is zero outside the interval $0 \le n \le N-1$. Assume that $N = 2^{\nu}$, where ν is a positive integer. Let $x_1[n] = x[2n]$, and $x_2[n] = x[2n+1]$. Show that the N-point discrete Fourier transform (DFT) of the sequence $x[n]$ can be obtained by appropriately combining the $N/2$-point DFTs of the sequences $x_1[n]$ and $x_2[n]$.

[8 marks]

(b) Consider two finite duration signals $x[n]$ and $h[n]$ where both are zero for $n < 0$ and where

$$x[n] = 0, \quad n \ge 32$$

$$h[n] = 0, \quad n \ge 8.$$

The 32-point DFTs of each of the signals are multiplied and the inverse DFT computed. Let $r[n]$ denote this inverse DFT.

The sequence $y[n]$ is obtained by linearly convolving $x[n]$ and $h[n]$.

Specify the values of n for which $r[n]$ is guaranteed to be equal to $y[n]$.

[7 marks]

(c) A 15,000 point sequence is to be linearly convolved with a sequence that is 80 points long. The convolution is to be implemented using DFTs and inverse DFTs of length 512. If the overlap-add method is used, what is the minimum number of 512-point DFTs and the minimum number of 512-point inverse DFTs needed to implement the convolution for the entire 15,000 point sequence?

[5 marks]