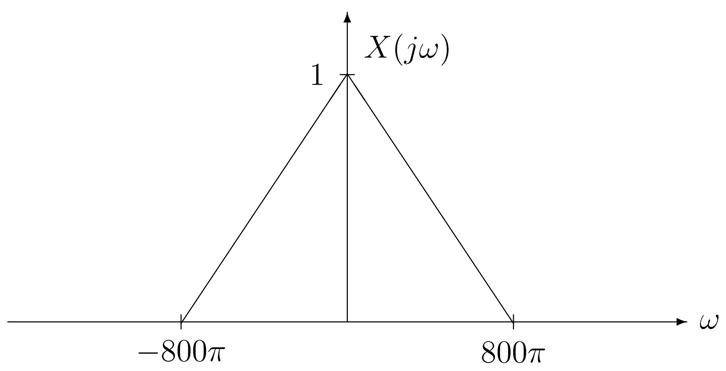
3C1 Signals and Systems **Homework No. 3**

1. Consider a causal LTI system with frequency response
$$H(j\omega) = \frac{1}{1+j\omega}.$$
For a particular input $x(t)$ this system is observed to produce the output
$$y(t) = e^{-t}u(t) - e^{-2t}u(t).$$
Determine $x(t)$.

2. Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$. Let $p(t)$ be a periodic signal with fundamental frequency ω_0 and Fourier series representation
$$p(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}.$$
Obtain an expression for the Fourier transform of $y(t) = x(t)p(t)$.

3.
$$x(t) \leftrightarrow X(j\omega)$$
$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$
$$x_p(t) = x(t)p(t)$$
$$x_p(t) \leftrightarrow X_p(j\omega)$$
(i) Show that
$$X_p(j\omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(j\left(\omega - \frac{2\pi k}{T}\right)\right).$$
(ii) The signal $x(t)$ has the Fourier transform $X(j\omega)$ shown below:



If $T = 10^{-3}$, sketch $X_p(j\omega)$ for $\frac{-3\pi}{T} \le \omega \le \frac{3\pi}{T}$.