INSTITUTE OF INFORMATION TECHNOLOGY

JAHANGIRNAGAR UNIVERSITY

Final Assignment

Submission Date : 14/08/2021

Course Tittle : Numerical Analysis

Course Code : ICT - 2105

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Class Roll – 2023

Exam Roll – 192340

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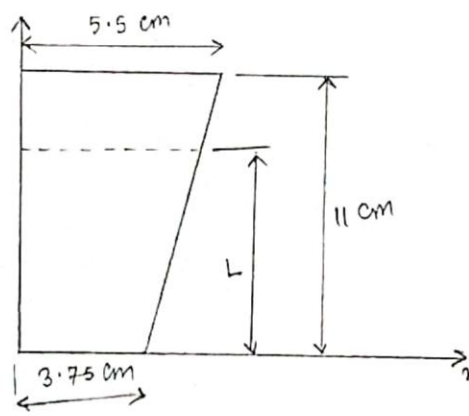
Answer to the question no - 1

We assume the mark on the measuring cup for 500 mililiter (mL) is situated at L as indicated in figure. We thus need to determine the value of L, so that the measuring cup with the content at the height L to be 500 mL.



We have to find the value of L. At first we find the equation of (3.75, 0) and (5.5, 11).

$$\frac{x-3.75}{3.75-5.5} = \frac{y-0}{0-11}$$

$$\Rightarrow x-3.75 = \frac{y}{-11} \times (-1.75)$$

$$\therefore x = 0.16y + 3.75$$

We will use this relation and find out the volume of the cup considering the hieight of the cup is L.

$$V = \pi \int_0^L x^2 \, dy$$

$$= \pi \int_0^L (0.16y + 3.75)^2 \, dy$$

$$= \pi \left[\int_0^L 0.0256 y^2 \, dy + \int_0^L 1.2 \, y \, dy + \int_0^L 14.06 \, dy \right]$$

$$= 0.0268 L^3 + 1.88 L^2 + 44.15 L$$

Since the volume of the measuing cup with the content level L is 500 mL.

$$\therefore \quad 500 = 0.0268 L^3 + 1.88 L^2 + 44.15 L$$

$$\Rightarrow L^3 + \frac{1.88 L^2}{0.0268} + \frac{44.15 L}{0.0268} - \frac{500}{0.0268} = 0$$

$$\Rightarrow L^3 + 70.3 L^2 + 1647.39 L - 18656.72 = 0 \quad \text{(1)}$$

We will solve equation ① using Newton-Raphson's method to determine the value of L. Considering the initial value of L. $L_1 = 4$ cm.

$$f(L) = L^3 + 70.3L^2 + 1647.39L - 18656.72$$

$$f'(L) = 3L^2 + 140.6x + 1647.39$$

$$L_2 = L_1 - \frac{f(L_1)}{f'(L_1)}$$

$$= 4 - \frac{4^3 + 70.3(4^2) + 1647.39 \times 4 - 18656.72}{3 \times (4^2) + 140.3 \times 4 + 1647.39}$$

$$= \cancel{9.22} = 8.818$$

$$L_3 = L_2 - \frac{f(L_2)}{f'(L_2)}$$

$$= 8.818 - \frac{(8.818)^3 + 70.3(8.818)^2 + 1647.39 \times 8.818 - 18659.72}{3 \times (8.818)^2 + 140.3 \times 8.818 + 1647.39}$$

$$= 8.17$$

$$L_4 = L_3 - \frac{f(L_3)}{f'(L_3)}$$

$$= (8.17) - \frac{(8.17)^3 + 70.3 \times (8.17)^2 + 1647.39 \times 8.17 - 18656.7}{3 \times (8.17)^2 + 140.3 \times 8.17 + 1647.39}$$

$$= 8.15$$

We may thus conclude that the mark line of 500 mL for the measuring cup in figure is located at the length $L = 8.15$ cm from the bottom of the cup.

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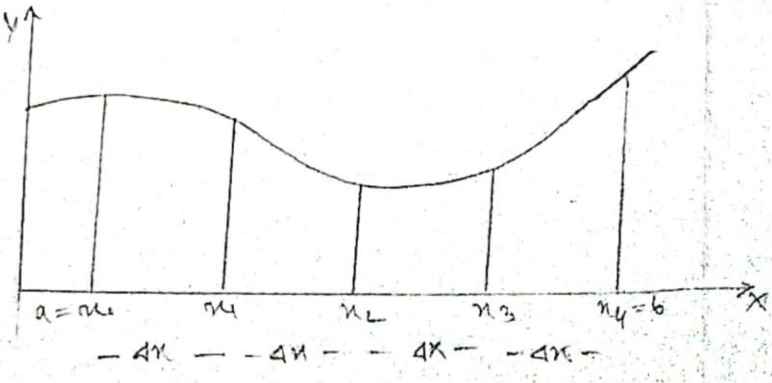
Answer to the question no - 2

The trapezoidal rule for estimating definite integrals uses trapezoids rather than rectangles to approximate the area under a curve.

We know that area of a trapezoid with a height of h and base b_1 and b_2 is given by area $= \frac{1}{2}h(b_1+b_2)$

We see that the first trapezoid has a height Δx and parallel bases of length $f(x_0)$ and $f(x_1)$. Thus the area of the first trapezoid in figure.



$$\frac{1}{2}\Delta x\left(f(x_0)+f(x_1)\right)$$

The areas of the remaining three trapezoids are

$$\frac{1}{2}\Delta x\left(f(x_1)+f(x_2)\right)$$

$$\frac{1}{2}\Delta x\left(f(x_2)+f(x_3)\right)$$

$$\frac{1}{2}\Delta x\left(f(x_3)+f(x_4)\right)$$

area is

$$\Rightarrow \int_a^b f(x)\,dx \approx \frac{1}{2}\Delta x\left(f(x_0)+f(x_1)\right)+\frac{1}{2}\Delta x\left(f(x_1)+f(x_2)\right) + \frac{1}{2}\Delta x\left(f(x_2)+f(x_3)\right)+\frac{1}{2}\Delta x\left(f(x_3)+f(x_4)\right)$$

$$=\frac{1}{2}\Delta x\left(f(x_0)+2f(x_1)+2f(x_2)+2f(x_3)+f(x_4)\right).$$

calculate the error when using the trapezoidal rule the following formula is used.

$$\text{Error} \leq \frac{K_T(b-a)^3}{12n^2}$$

So the error for calculating area under the given curve

$$e_{(t)} \le \frac{K_T(x_4 - x_0)^3}{12 \times 4^2}$$

$$e_{(t)} \le \frac{K_T(x_4 - x_0)^3}{192}$$

for finding out the area under the curve using simpson's rule. Over the first pair of subintervals we approximate $\int_{x_0}^{x_2} f(x)\,dx$ with $\int_{x_0}^{x_2} P(x)\,dx$ where $P(x) = Ax^2 + Bx + C$ is the quadratic function passing through $(x_0 . f(x_0)) . (x_1 . f(x_1))$ and $(x_2 , f(x_2))$.

$$x_2 - x_0 = 2\Delta x$$

$$x_1 = \frac{x_2 + x_0}{2}$$

$$\int_{x_0}^{x_2} f(x)\,dx = \int_{x_0}^{x_2} P(x)\,dx$$

$$= \int_{x_0}^{x_2} (Ax^2+Bx+C)\,dx$$

$$= \left[\frac{A}{3}x^3 + \frac{B}{2}x^2 + Cx\right]_{x_0}^{x_2}$$

$$= \frac{A}{3}(x_2^3 - x_0^3) + \frac{B}{2}(x_2^2 - x_0^2) + C(x_2 - x_0)$$

$$= \frac{A}{3}(x_2 - x_0)(x_2^2 + x_2x_0 + x_0^2) + \frac{B}{2}(x_2 - x_0)(x_2 + x_0)$$
$$+ C(x_2 - x_0)$$

$$= \frac{x_2 - x_0}{6}\left(2A(x_2^2 + x_2x_0 + x_0^2)\,3B(x_2+x_0) + 6C\right)$$

$$= \frac{\Delta x}{3}\left((Ax_2^2 + Bx_2 + C) + (Ax_0^2 + Bx_0 + C) +\right.$$
$$A(x_2^2 + 2x_2x_0 + x_0^2) + 2B(x_2 + x_0) + 4C \quad \text{--- (1)}$$

Now. $f(x) = P(x) = Ax^2 + Bx + C$

$f(x_0) = P(x_0) = Ax_0^2 + Bx_0 + C$

$f(x_1) = P(x_1) = Ax_1^2 + Bx_1 + C$

$f(x_2) = P(x_2) = Ax_2^2 + Bx_2 + C$

From equation (i)

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{4x}{3}\left(f(x)+f(x_0)+A(x_2+x_0)^2+2B(x_2+x_0)+4C\right)$$

$$= \frac{4x}{3}\left(f(x_2)+f(x_0)+A(2x_1)^2+2B(2x_1)+4C\right)$$

$[\because x_2 - x_0 = 2x_1]$

$$= \frac{4x}{3}\left(f(x_2)+f(x_0)+4(Ax_1^2+Bx_1+C)\right)$$

$$= \frac{4x}{3}\left(f(x_2)+f(x_0)+4f(x_1)\right) \text{ --- (ii)}$$

Similarly it can be determined that

$$\int_{x_2}^{x_4} f(x)\,dx = \frac{4x}{3}\left(f(x_4)+4f(x_3)+f(x_2)\right) \text{ --- (iii)}$$

Combining (ii) and (iii) we get

$$\int_{x_0}^{x_4} f(x)\,dx = \frac{4x}{3}\left(f(x_2)+4f(x_1)+f(x_0)+f(x_4)+4f(x_3)+f(x_2)\right)$$

or.

$$\int_{x_0}^{x_4} f(x)\,dx = \frac{4x}{3}\left(f(x_0)+f(x_4)+2f(x_2)+4(f(x_1)+f(x_3))\right)$$

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Calculate the error when using the simpson's rule the following formula is used.

$$\text{Error} \leq \frac{K_5 (b-a)^5}{180 n^4}$$

So the error for calculating area under the given curve

$$e_{(s)} \leq \frac{k_5 (x_4 - x_0)^5}{180 \times 4^2}$$

$$e_{(s)} \leq \frac{k_5 (x_4 - x_0)^5}{2880}$$