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Question: Explain image sharpening in digital image Processing. If the Image Striping is 5 5 4 3 2 1 0 0 0 6 0 0 0 0 1 3 1 0 0 0 0 7 7 7 7 ..... Find the 1st and 2nd derivatives and the gray level Profile.

Answer: Image sharpening is a technique in digital image Processing that enhances the edges and details of an image. It is done by increasing the Contrast between neighboring Pixels which makes the edges appear sharper and more defined. Sharpening Can be done using a variety of methods.

High-Pass filters: This filters amplify the high-frequency Components of an image.

unsharp masking: This method involves blurring the orginal image and then subtracting the blurred image from the orginal image.

Local adapting sharpening: This method applies sharpening to different parts of the image depending on the local contrast.

Advantage:

1. Improves the visual quality and clearity of image.
2. Makes edges and details more visible and defined.
3. Can help to restore some of the information that was lost or distorted.
4. Can be used to enhance the perception of depth, texture and dimensionality of image.
5. Can be used to make images looks more realistic or appealing.

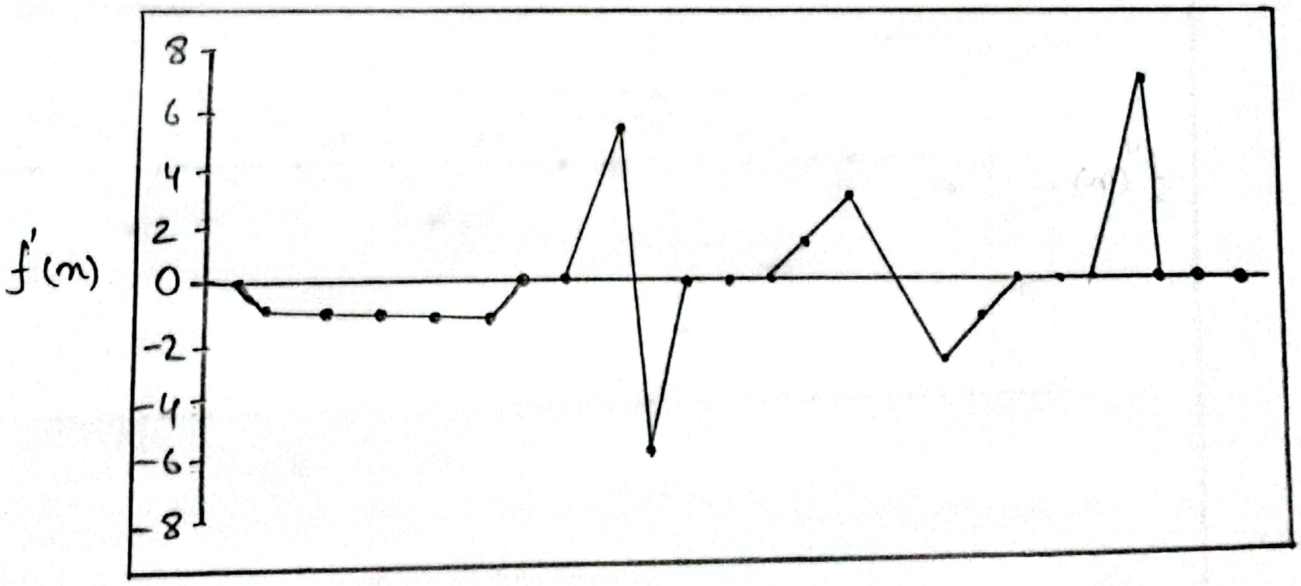
Image strip:

5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7	.	.	.	.	.	.

1^{st} Derivative

The formula for the 1^{st} derivative of a function is as follows

$$\frac{\partial f}{\delta x} = f(x+1) - f(x)$$



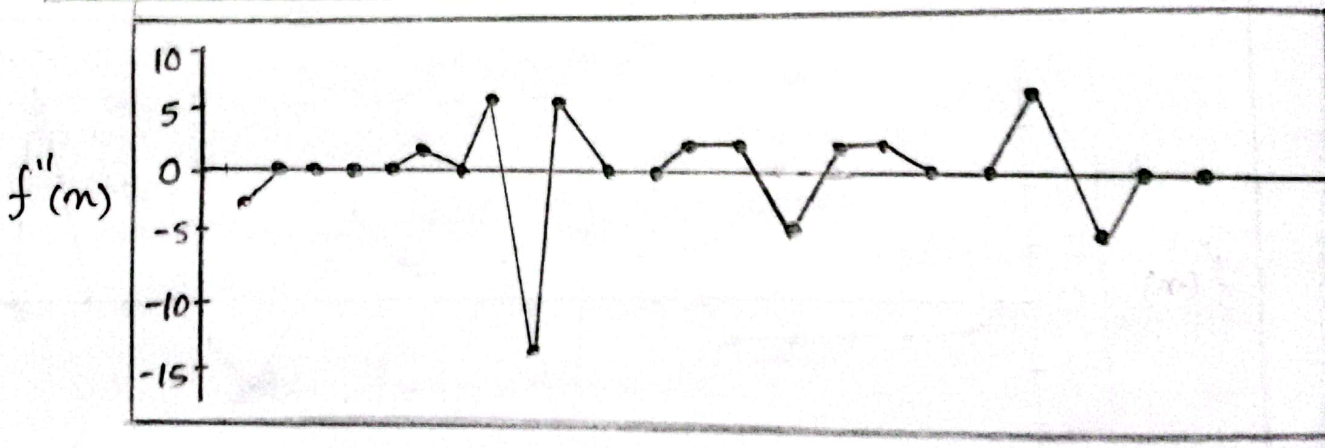
	0	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0	

2nd derivative

The formula for the 2nd derivative of a function is as follows.

$$\frac{\delta^2 f}{\delta x^2} = f(x+1) + f(x-1) - 2f(x)$$

	-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	



Question: Design the symmetric FIR low pass filter whose $H_d(w) = e^{-jwt}$, $|w| \leq w_c$ and $H_d(w) = 0$ otherwise $M=7$ and $w_c = 1$. What will happen if the rectangular window is used.

Answer:

Given that,

$$H_d(w) = \begin{cases} e^{-jwt} & |w| \leq w_c \\ 0 & \text{otherwise} \end{cases} \qquad \begin{matrix} \text{with } M=7 \\ w_c = 1 \text{ rad/sec} \end{matrix}$$

We know,

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(w)\, e^{jwn}\, dw. \quad \text{——— (i)}$$

$$= \frac{1}{2\pi} \int_{-1}^{1} e^{-jwt} \cdot e^{jwn}\, dw$$

$$= \frac{1}{2\pi} \int_{-1}^{1} e^{jw(n-t)}\, dw$$

$$= \frac{1}{2\pi} \left[\frac{e^{jw(n-t)}}{j(n-t)} \right]$$

$$= \frac{1}{2\pi} \frac{e^{j(n-t)} - e^{-j(n-t)}}{j(n-t)}$$

$$= \frac{1}{\pi(n-t)} \left[\frac{e^{j(n-t)} - e^{-j(n-t)}}{2j} \right]$$

$$= \frac{\sin(n-t)}{\pi(n-t)} \quad n \neq t$$

If $n = t$

$$h_d(n) = \frac{1}{2\pi} \int_{-1}^{1} 1 \, d\omega$$

$$= \frac{1}{\pi}$$

$$\therefore h_d(n) = \begin{cases} \frac{\sin(n-t)}{\pi(n-t)} & n \neq t \\ \frac{1}{\pi} & n = t \end{cases}$$

determine the Value of t

$$h(n) = h(M-1-n)$$

$$\Rightarrow h(n) = h_d(n) \cdot W(n)$$

$$\Rightarrow h_d(w)\, w(n) = h_d(M-1-k)\, w(n)$$

$$\Rightarrow h_d(n) = h_d(M-1-n)$$

$$\Rightarrow \frac{-\sin(n-t)}{-\pi(n-t)} = \frac{\sin(M-1-n-t)}{\pi(M-1-n-t)}$$

$$\Rightarrow \frac{\sin(-(n-t))}{\pi(-(n-t))} = \frac{\sin(M-1-n-t)}{\pi(M-1-n-t)}$$

$$\Rightarrow -(n-t) = M-1-n-t$$

$$\Rightarrow -n+t = M-1-n-t$$

$$\therefore t = \frac{M-1}{2}$$

$$h_d(n) = \begin{cases} \dfrac{\sin\left(n-\frac{M-1}{2}\right)}{\pi\left(n-\frac{M-1}{2}\right)} & n \neq \frac{M-1}{2} \\ \dfrac{1}{\pi} & n = \frac{M-1}{2} \end{cases}$$

Since $M=7$

$$h_d(n) = \begin{cases} \dfrac{\sin(n-3)}{\pi(n-3)} & n \neq 3 \\ \dfrac{1}{\pi} & n = 3 \end{cases}$$

Now,

$n = 0$ to 6

$n=0, \quad h_d(0) = 0.01497$

$n=1, \quad h_d(1) = 0.14472$

$n=2, \quad h_d(2) = 0.26786$

$n=3, \quad h_d(3) = \frac{1}{\pi}$

$n=4, \quad h_d(4) = 0.26786$

$n=5, \quad h_d(5) = 0.14472$

$n=6, \quad h_d(6) = 0.01497$

$$h(n) = h_d(n) \cdot W(n)$$

$$W(n) = \begin{cases} 1 & 0 \le n \le 6 \\ 0 & \text{otherwise} \end{cases}$$

$$h(n) = \begin{cases} h_d(n) & ; \ 0 \le n \le 6 \\ 0 & ; \ \text{otherwise} \end{cases}$$

$h(0) = 0.1497$

$h(1) = 0.14472$

$h(2) = 0.26786$

$h(3) = 1/\pi$

$h(4) = 0.26786$

$h(5) = 0.14472$

$h(6) = 0.01497$

Question: Given low Pass Protype $H_p(s) = \frac{1}{s+1}$
Determine each of the following analog filters and Plot their magnitude response from 0 to 200 rad/sec.

~~Answer:~~
(i) A HPF with $W_c = 40$ rad/sec
(ii) A BPF with $W_c = 100$ rad/sec of BW 20 rad/sec

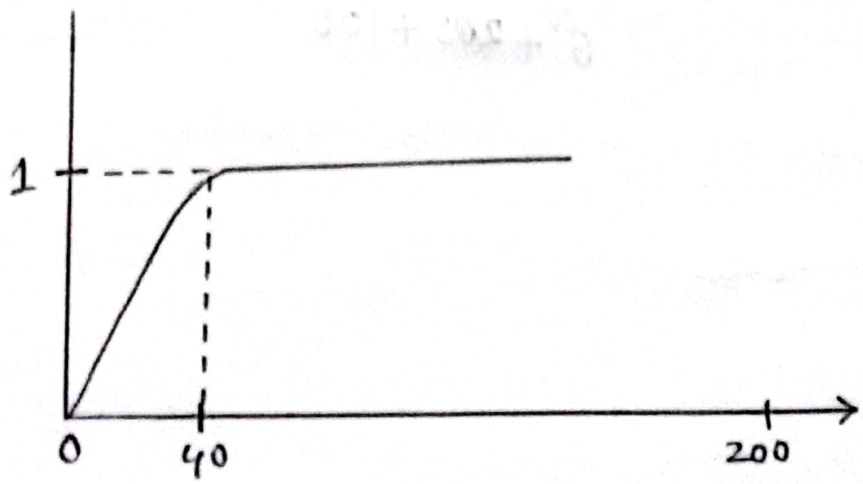
Answer:

(i) High Pass filter

$$H_{HP}(s) = \frac{1}{s+1} \quad \Big| \quad s = \frac{\omega_c}{s} = \frac{40}{s}$$

$$= \frac{1}{\frac{40}{s}+1}$$

$$= \frac{s}{s+40}$$



(ii) Band Pass Filter

$$W_0 = \sqrt{W_L * W_h}$$

$$= \sqrt{100}$$

$$\therefore W_0 = 10 \text{ rad/sec}$$

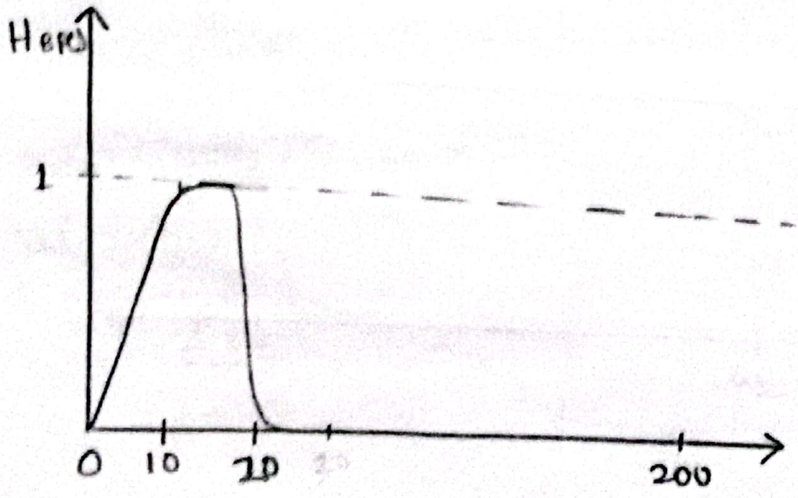
$$W = W_h - W_L = 20 \text{ rad/sec}$$

$$H_p(s) = \frac{1}{s+1}$$

$$s = \frac{s^2+w^2}{sw} = \frac{s^2+100}{s20}$$

$$H_{BP}(s) = \frac{1}{\frac{s^2+100}{s20}+1}$$

$$= \frac{20s}{s^2+20s+100}$$



Question: Write down all Properties of the Linear Time-Invarient System.

Answer: Here are some Properties of linear Time-Invarient system.

1. Commutative:

$$x(t) * h(t) = h(t) * x(t)$$

$$\int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{\infty} x(t-\tau)\, h(t)\, d\tau$$

2. Distributive:

$$x(t) * [h_1(t) + h_2(t)] = x(t) * h_1(t) + x(t) * h_2(t)$$

3. Associative:

$$x(t) * [h_1(t) * h_2(t)] = [x(t) * h_1(t)] * h_2(t)$$

4. Convolution with unit impulse

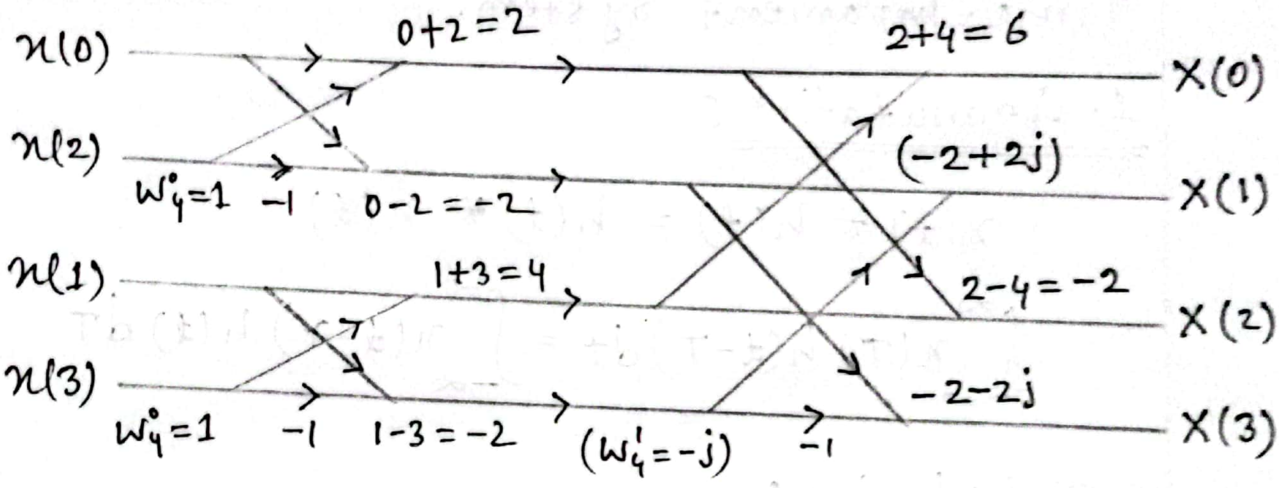
$$x(t) * \delta(t) = x(t)$$

5. Inversion:

$$h_1(t) * h_2(t) = \delta(t)$$
$$h_1[n] * h_2[n] = \delta[n]$$

Question: Find 4-Point DFT of $x(n) = \{0,1,2,3\}$

Answer:



$$\therefore X(k) = \{6, -2+2j, -2, -2-2j\}$$