



Answer any **Five (05)** from the following questions. Figures at the right indicate the marks.

1. a. Compare the orders of growth of $\frac{1}{2}n(n-1)$ and n^2 3
- b. In an integer array, there is 1 to 100 numbers, out of one is duplicate, how to find? 3
- c. Show that the running time of insertion algorithm in worst case is a *quadratic function*. 4
- d. Describe the worst case running time of the following pseudocode functions in *Big O* notation in terms of the variable n 2

```
void silly(int n) {  
    if (n <= 0) return;  
    System.out.println("n = " + n);  
    silly(n/2);  
}
```

2. a. Show that *mergesort* has time complexity $O(n \log n)$. 3
- b. Compute the worst case running time $T(n)$ for the following algorithm with respect to the input size n . Based on the computed running time give the best *Big-Oh* complexity characterization of $T(n)$. 2

Algorithm Count-Common(A,B,n)

Input: Two integer arrays A and B with both of size n

Output: Number of common elements in A and B

```
1. sort B by merge-sort  
2. e = 0 //number of common elements  
3. for i = 1 to n  
4.     b = binarySearch(A[i], B)  
5.     if b ≠ null  
6.         e = e + 1  
7. return e
```

- c. Discuss the coin-row problem. Solve the instance 5, 1, 2, 10, 6 of the coin-row problem 4
- d. Consider the following list of integers: 3

5, 54, 125, 105, 25, 104, 20, 100, 50, 159

Perform a radix sort on the above list. What is the asymptotic running time for a radix sort on n numbers with d digits each?

3. a. What is the running time of HEAPSORT on an array A of length n that is already sorted in increasing order? What about decreasing order? 2
- b. Describe the backtracking solution to solve 8-Queens problem. 4
- c. Suppose S is the following list of 8 integers: 4

87, 36, 22, 15, 56, 85, 48, 90, 72, 6

Apply the PARTITION subroutine of quick sort algorithm to divide the list into two parts.

- d. Consider the following algorithm. 2

```
for i ← 1 to 100 do  
    for k ← 1 to n do  
        j ← 1 ; m ← n  
        while j < m do  
            m ← (m + j) / 2  
        end while  
    end for  
end for  
for i ← 1 to 30 do  
    for j ← 1 to n do  
        k ← i + j + n  
    end for  
end for  
for i ← 1 to 70 do  
    j ← 2*n + i  
end for
```

What is its *big O* running time with respect to n ?

$= \langle x_1, x_2, \dots, x_m \rangle$ and $Y = \langle y_1, y_2, \dots, y_n \rangle$, and we want to find a maximum-length common subsequence. Write down the algorithm maximum-length common subsequence.

- b. In which situation dynamic programming is preferred over divide-and-conquer approach? Explain with examples. 4
- c. Write down the basic concept of relaxation. 3
- d. Draw the AVL tree that results from inserting the keys: 3, 9, 2, 7, 4, 5, 8 in that order into an initially empty AVL tree. 2

5. a. Consider the following matrix with dimensions 5

Matrix	dimension
A ₁	35x15
A ₂	15x5
A ₃	5x10
A ₄	10x20

Calculate the minimum number of scalar multiplication.

- b. The function f is defined recursively as follows: 3

$$f(n) = \begin{cases} 1 & \text{if } n = 0 \text{ or } n = 1 \\ f(n-1) + 2 * f(n-2) & \text{if } n > 1 \end{cases}$$

Compute $f(4)$ by drawing a recursive tree showing all of the computation required and then use your tree to compute the answer.

- c. Draw the Huffman tree to get an optimal Huffman code for the following set of frequencies, based on the first 8 Fibonacci numbers. 4

a:1 b:1 c:2 d:3 e:5 f:8 g:13 h:21

Generalize your Huffman tree to find the optimal code when the frequencies are the first n Fibonacci numbers.

6. a. Justify the statement — “Fractional knapsack problem has the greedy-choice property”. 2

- b. Show step by step execution of *Prim's algorithm* on the graph provided below in figure 1. 4

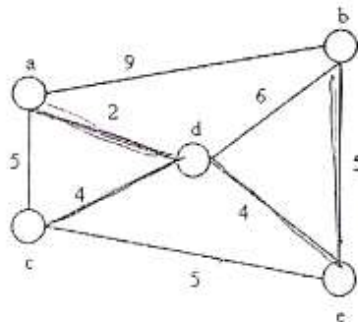


Figure: 1

- c. Write down the *Dijkstra's algorithm* to calculate the single source shortest path. 3
- d. How do you compare *dynamic programming* with *greedy*? 3
7. a. Suppose that the following keys are inserted in the order A B C D E F G into an initially empty linear-probing hash table of size 7, using the following hash function: 4

key	hash(key, 7)
A	3
B	1
C	4
D	1
E	5
F	2
G	5

What is the result of the linear-probing array?

- b. Write down the differences between *NP-hard* and *NP-complete* problem. 3
- c. Write down the *Rollman-Ford algorithm* to calculate single source shortest path of a 2



- 1(a). What is meant by *algorithm*? How do you analyze an algorithm? 1+2
(b). Describe what is meant by big Ω notation in algorithm analysis. 2
(c). What is the time complexity of an algorithm? Show that the running time of insertion algorithm in best case is a *linear function*. 1+3
(d). Show that the expression $10N^2 + 1000N + 1000000$ is $O(N^2)$. 3

2(a). Consider the following algorithm.

```
for i ← 1 to 100 do
  for k ← 1 to n do
    j ← 1 ; m ← n
    while j < m do
      m ← (m + j) / 2
    end while
  end for
end for

for i ← 1 to 30 do
  for j ← 1 to n do
    k ← i + j + n
  end for
end for

for i ← 1 to 70 do
  j ← 2 * n + i
end for
```

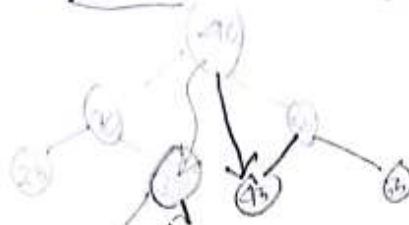
Handwritten notes for algorithm analysis:
 $100 \times n$
 $100 \times (n-1)$
 $100 \times (n-1) \log n$
 $100 \times (n-1) \log(n-1) = 2 \log n$
 $30 \times n$
 $30 \times (n-1)$

- i) What is its exact running time with respect to n ?
ii) What is its *big O* running time? Prove it using its mathematical definition.

- (b). Explain Longest Common Subsequence (LCS) recursive solution. 4
(c). Suppose you are given an array $A[1..n]$ of sorted integers that has been circularly shifted k positions to the right. For example, $[35, 42, 5, 15, 27, 29]$ is a sorted array that has been circularly shifted $k = 2$ positions, while $[27, 29, 35, 42, 5, 15]$ has been shifted $k = 4$ positions. We can obviously find the largest element in A in $O(n)$ time. Describe an $O(\log n)$ algorithm. 4

3(a). Draw binary search trees of height 2, 3, and 4 on the set of keys $\{1, 4, 5, 10, 16, 17, 21\}$ 2x3

- (b). Write down the algorithm to delete an item from the binary search tree. 4
(c). Show that if a node in a binary search tree has two children, then its successor has no left child and its predecessor has no right child. 2



Q(a) How do you compare dynamic programming with greedy? What are their similarities, and what are the differences?

(b) Consider the following matrix with dimensions

6

Matrix	dimension
A_1	30×10
A_2	10×15
A_3	15×10
A_4	10×25

$$m[i, k] + m[k, j] + p_{i+1}$$

(c) Calculate the minimum number of scalar multiplication
Briefly explain the elements of a dynamic programming.

12

5(a). Prove that $(1/2)n(n-1) \in O(n^2)$

3

(b). Proof by Mathematical Induction, for $n \geq 1$,

3

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(n+2)}{6}$$

(c). Proof that $F(n) = \frac{\sqrt{5}}{5} \left\{ \left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right\}$. Also show the golden ratio.

3

(d). Proof from TOWER OF HANOI, $s(n) = 2^n - 1$.

3

Q(a). Does Bucket sort algorithm work for Real Numbers? If yes, then give an example mentioning basic idea.

3

(b). Explain some of the problem types used in the design of algorithm.

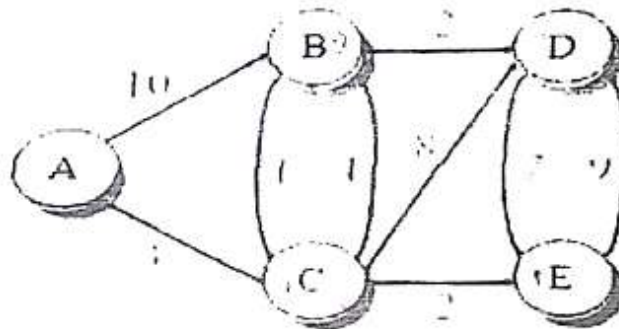
3

(c). Explain about greedy technique

3

(d). Write the Dijkstra's algorithm and implement for the following graph.

3

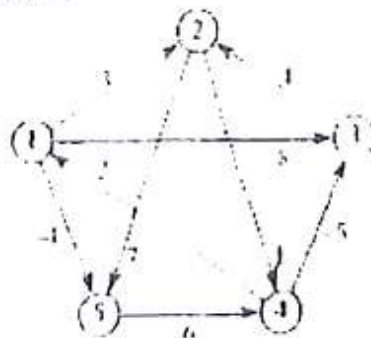


Q(a). Write down the SLOW-ALL-PAIRS-SHORTEST-PATHS algorithm to calculate all pair shortest paths of a graph.

3

(b). Consider the following graph

5



Calculate the all pair shortest path by using SLOW-ALL-PAIRS-SHORTEST-PATHS algorithm.

(c) Given a rod of length n inches and a table of prices p_i for $i = 1, 2, \dots, n$, determine the maximum revenue r_n obtainable by cutting up the rod and selling the pieces. Note that if the price p_n for a rod of length n is large enough, an optimal solution may require no cutting at all.

2

(d). Write down the differences between NP-hard and NP-complete problem.

2



Institute of Information Technology (IIT)
Jahangirnagar University, Savar Dhaka
2nd Year 1st Semester B.Sc. (Hons) Final Examination-2017

Course Code: IT-2109
Course Title: Statistics and Probability Theory

Time: 3 hours
Full Marks: 60

Answer any FIVE Questions

1. a) Briefly explain different types of data collection method. 4
b) What are the different types of measures of location? 4
c) Write the three different situations where geometric mean, harmonic mean and arithmetic mean is appropriate? 4
2. a) Write two differences between Bar chart and Histogram. 2
b) What do you mean by quartile and percentile? Assume that an examination was conducted on 80 students and their scores obtained in a course IT-2109 on 60. The Average is 48.5 with standard deviation 25.8, 3rd quartile is 53.2 and 25th percentile is 30.7, median is 45.2 and mode is 40.5. Explain the above results. 6
c) Define the term standard deviation. Why do most researchers use standard deviation to measure dispersion of data? 4
3. a) What are the conditions of Binomial distribution? 4
b) Write the function of this distribution. When a binomial distribution tends to a Poisson distribution? 4
c) Vehicles pass through a junction on a busy road at an average rate of 300 per hour. 4
 - a. Find the probability that none passes in a given minute.
 - b. What is the expected number passing in two minutes?
 - c. Find the probability that this expected number actually pass through in a given two-minute period.
4. a) In what situations distribution of random variables follow Poisson distribution or exponential distribution? 4
b) Why exponential distribution is called a memoryless distribution? 4
c) The number of calls coming per minute into a hotels reservation center is Poisson random variable with mean. 4
 - i. Find the probability that no calls come in a given 1 minute period.
 - ii. Assume that the number of calls arriving in two different minutes is independent. Find the probability that at least two calls will arrive in a given two minute period.

5. a) Define hypothesis and types of hypotheses. Write down the steps (in flowchart) involved in the hypothesis testing procedure. Define type I and type II errors. 5
- b) When coefficient of variation is an important tool? 2
- c) An insurance broker believes that for a particular contract the probability of making a sale is 0.4. Suppose that the broker has five contracts. 5
- (i) Find the probability that she makes at most one sale.
- (ii) Find the probability that she makes between two and four sales (inclusive).
- Graph the probability distribution function.

6. a) Define stochastic process, birth-death process and markoy chain with suitable example. 4
- b) What do you mean by M/M/1, M/M/C and G/M/1? 3
- c) Compute inter-quartile range and standard deviation from the following data. 5

Class interval	Frequency
10-15	13
15-20	17
20-25	27
25-30	15
30-35	16
35-40	9

7. a) What are the difference between regression and correlation? 2
- b) Explain the situation when a correlation ($r=0$, $r=1$, $r<1$ and $r>1$) 4
- c) A company wants to know how job performance relates to IQ. Data are give in the following table: 6

Serial #	Performance	IQ
1	60	31
2	61	36
3	62	38
4	63	40
5	65	41

Estimate the degree of association between job performance and IQ. Is this association significant?

Estimate a regression equation of Performance on IQ and comment on the results.

Answer any FIVE of the following questions

- 1(a). Define statistics and mention some of the application of statistics in information technology. What are the limitations of statistics? 4
- (b). What do you mean by attributes and variables? Briefly explain discrete and continuous variable. 4
- (c). What is tabulation? Write down the methods of data collection. 4

- 2(a). For the pictorial presentation of frequency distribution, we used to use histogram instead of Bar chart. Why? 2
- (b). Marks obtained by 35 students in a course presents in Table 1. Calculate Mean, Median, Mode, standard deviation and coefficient of variation (CV) and draw comments 10

Marks interval	Mid Value (x)	Frequency (f)	fx	Cf	fx ²
20-30	25	5			
30-40	35	7			
40-50	45	8			
50-60	50	8			
60-70	65	2			

Table 1: Frequency table of age of patients

- 3(a). Define probability, experiment, events, outcomes and sample space with example. 4
- (b). Define conditional probability and independence with example. Write down the theorem of total probability. What is random variable? 4
- (c). An electrical firm manufactures light bulbs what have a life's before burn out that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns between 778 and 834 hours. 4
- 4(a). What are the conditions of Binomial distribution? When a binomial distribution can be approximated by normal distribution? 4
- (b). A manufacturer of meta pistons finds that on the average, 12% of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain?
i. No more than 2 rejects.
ii. At least 2 reject. 4
- (c). Vehicles pass through a junction on a busy road at an average rate of 300 per hour. 4
i. Find the probability that none passes in a given minute.
ii. What is the expected number passing in two minutes?
iii. Find the probability that this expected number actually pass through in a given two-minute period.

- 5(a). What is conceptual difference between Poisson distribution or exponential distribution? Explain with a suitable example. 4
- (b). Why does exponential distribution is called a memoryless distribution? 4
- (c). If the time between accidents follows an exponential distribution with a mean of 900 days, what is the probability that there will be less than 900 days between the next two accidents? 4

- 6(a). Write conceptual differences between correlation and regression. 2
- (b). In an analysis, researchers want to see the relationship between student's anxiety level and test score in the examination. The data of 10 students are below: 8

Anxiety	10	8	2	1	5	6	9	7	6	4
Test Score	2	3	9	7	6	5	3	4	6	7

Calculate correlation coefficient and regress test score on anxiety. Also comment on your results.

- (c). Explain the situation when a) correlation (r) = 0, $r=1$, $r<1$ and $r>1$ 2
- (a). Define statistical hypothesis and null hypothesis with example. 4
- (b). What are the steps to test a hypothesis? 4
- (c). In a survey on hearing levels of schoolchildren with normal hearing it was found that in the frequency 500 cycles per seconds, 62 children tested in the sound-proof room had a mean hearing threshold of 15.5 decibels with a standard deviation of 6.5. 76 comparable children who were tested in the fields had mean threshold of 20.0 decibels with a standard deviation of 6.1. Test, if there is any difference between the hearing levels recorded in the sound proof room and in the field. 4