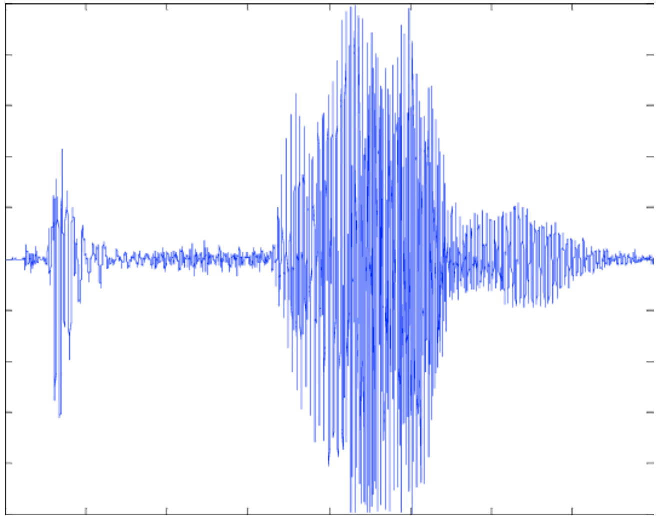
Introduction

IT 3105: Signals and Systems

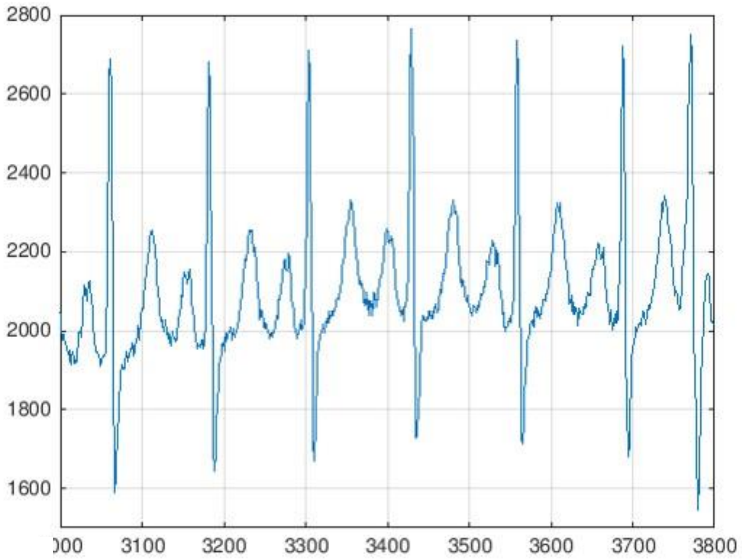
What is signal?

- A signal is formally defined as a function of one or more dependent variables that convoy information on the nature of a physical phenomenon. (Haykin)
- Signal is a time varying physical phenomenon which is intended to convey information.
- Signal is a function of time.
- Signal is a function of one or more independent variables, which contain some information.
- A *signal* is a function representing a physical quantity or variable, and typically it contains information about the behavior or nature of the phenomenon. (Hwei Hsu, Schaum's Series)

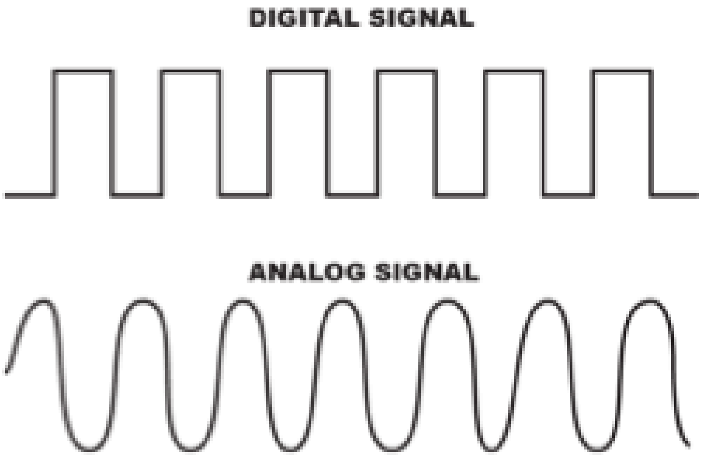
Examples of signals



Voice Signals



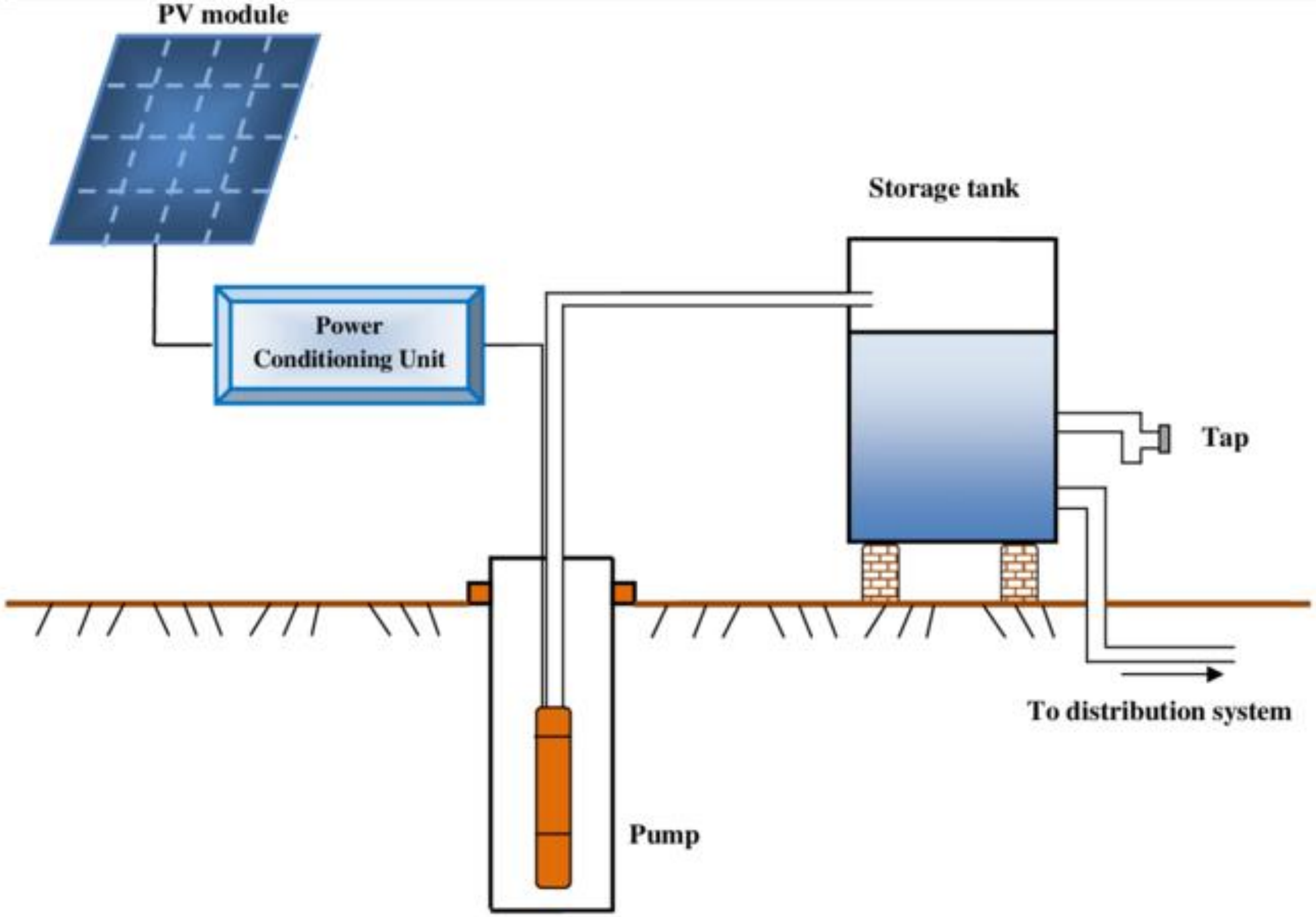
ECG Signals



What is system?

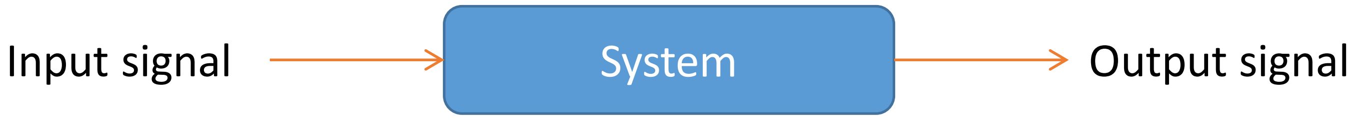
- A system is formally defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals. (Haykin)
- A *system* is a mathematical model of a physical process that relates the *input* (or *excitation)* signal to the *output* (or *response)* signal. (Hwei Hsu, Schaum's Series)
- System is one or more devices, which can operate on signals and produces corresponding response. Input to a system is called as excitation and output from it is called as response.
- For one or more inputs, the system can have one or more outputs.

Example of system



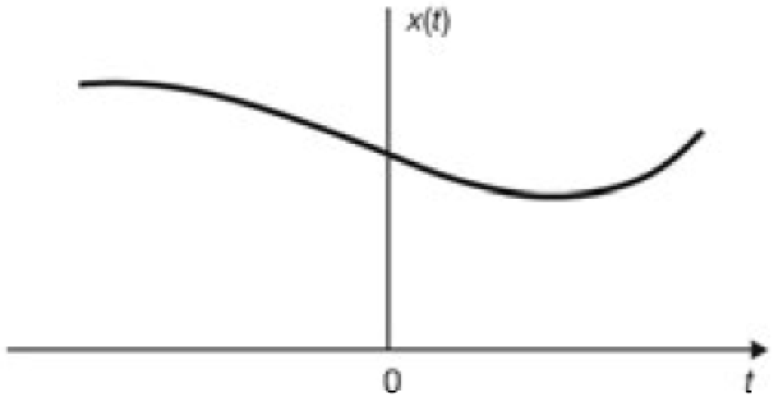
Representation of Signals and Systems

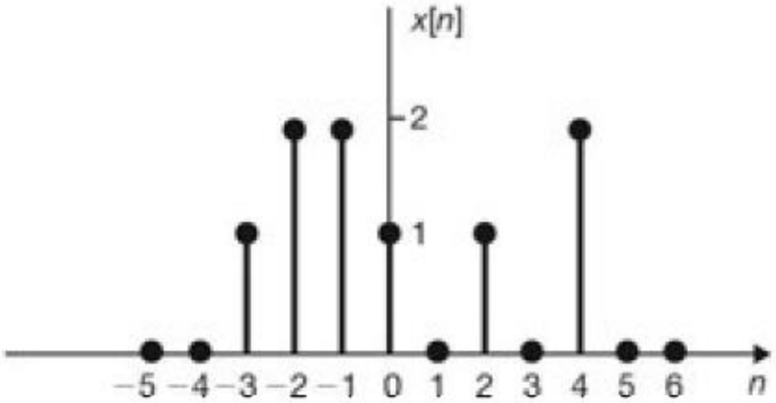
- Signal is dependent variable or function of one or more independent variables.
  - $F(x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n)$; Dependent F, independent $x_1, x_2, x_3, \ldots\ldots\ldots\ldots, x_n$
  - Example: alternating current is a signal as is changes with phase, but direct current is not a signal as it is constant.
  - Single variable/One dimensional signals
  - Multivariable/ Multidimensional signals
- System is the meaningful interconnections of physical devices or components which takes an input signal, process/manipulate it (do some work) then produces an desirable output signal.



Classifications of signals

- Broadly classified into two types
    1. Continuous Time Signal (CTS)
    2. Discrete Time Signal (DTS)
- CTS is specified for every value of independent variable, i.e the variable is continuous. CTS represent as function of continuous variable such as, $x(t)$, where "t" is independent variable and it is continuous.
- DTS is not specified for every value of independent variable rather it is specified at discrete intervals. The function $x[n]$ is a discrete time signal if n is an integer.





CTS & DTS

- DTS can be inherently discrete such as daily closing of stock market, or can be obtained by sampling of a CTS, *x(t)* such as

  $x(t_0), x(t_1), \dots \dots . ., x(t_n)$

  Or, $x[0], x[1], \dots \dots \dots \dots, x[n]$ or $x_0\,, x_1\,, x_2\,, \dots \dots . ., x_n$; where $x_n\ = x[n] = x(t_n)$

- x_n 's are called the samples and time intervals are called sampling intervals. If sample intervals are equal then its called uniform interval, $x_n\ = x[n] = x[nT_s]$, where T_s is the sample interval.

- DTS can be represented as:

1. By calculating the nth value: $x[n] = \begin{cases} \left(\frac{1}{2}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases} = \left\{1, \frac{1}{2}, \frac{1}{4}, \dots \dots \dots, \left(\frac{1}{2}\right)^n, \dots . .\right\}$
2. list the values of the sequence; $x[n] = \left\{\dots \dots . ., 0,0,1,2,2, \underset{\uparrow}{\frac{1}{2}}, 0,1,0,2,0,0, \dots . .\right\} = \left\{1,2,2, \underset{\uparrow}{\frac{1}{2}}, 0,1,0,2\right\}$

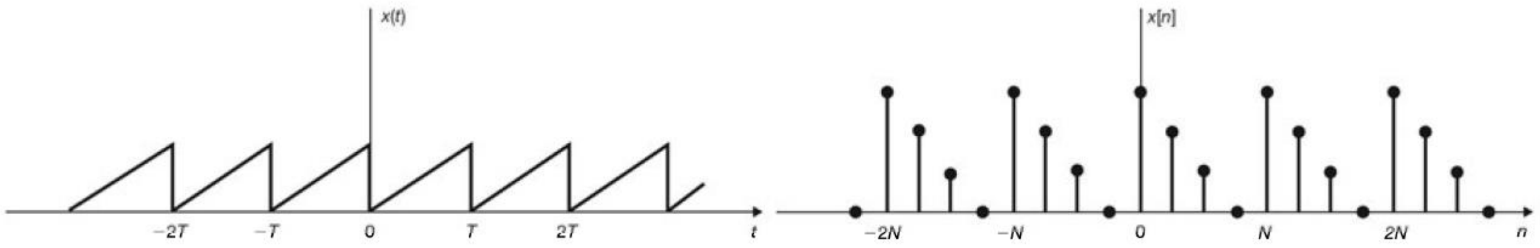
Classifications of signals

- Analog and Digital Signals
- Real and Complex Signals: $x(t) = x_1(t) + jx_2(t)$
- Deterministic and Random Signals
- Even and Odd Signals: $x(t) = x_e(t) + x_o(t); x[n] = x_e[n] + x_o[n];$

  even signal $x(-t) = x(t)$ odd signal $x(-t) = -x(t)$

  $x[-n] = x[n]$ $x[-n] = -x[n]$

- Periodic and Nonperiodic Signals *x(t + T) = x(t)* all *t* and *x[n + N] = x[n]* all *n*



- Energy and Power Signals: The normalized energy and power signals can be defined as
  - For CTS $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$ and $P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{1}{T}}^{\frac{1}{T}} |x(t)|^2 dt$
  - For DTS $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$ and $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$

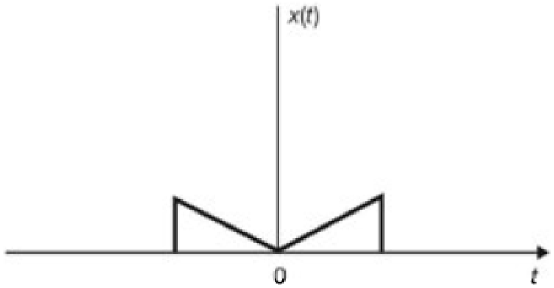
Even & Odd signals

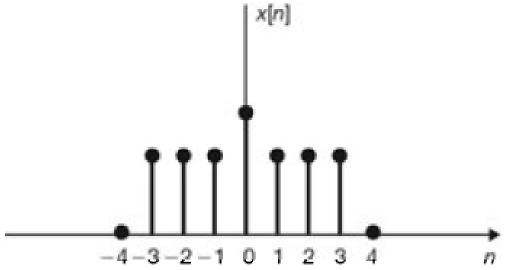
- Even signals remains identical under folding/time reversal/refection operation.

$$x(t) \xrightarrow{T.R} x(-t) = x(t)$$

Example: Prove $x(t) = cos\omega t$ is even.
$x(-t) = \cos(-\omega t) = cos\omega t = x(t)$

we know, $\cos(-\theta) = cos\theta$, so we can say given signal is even as $x(-t) = x(t)$





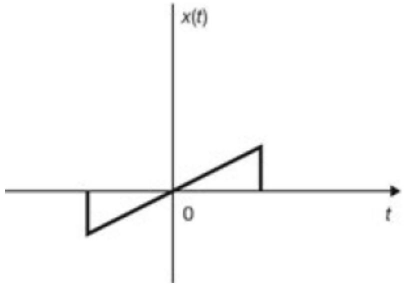
- Odd signals don’t remains identical under folding/time reversal/refection operation.

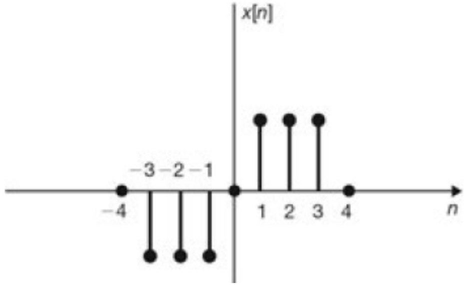
$$x(-t) \neq x(t); x(-t) = -x(t)$$

- Properties of odd signals:
  - At t=0, odd signal should be zero
  - Average value/mean/dc value of odd signals must be zero.

Example: Prove $x(t) = sin\omega t$ is odd.
$x(-t) = \sin(-\omega t) = -\ sin\omega t$

we know, $\sin(-\theta) = -sin\theta$, so we can say given signal is odd as $x(-t) = -x(t)$





Even and Odd Component of a Signal

- Any continuous time signal can be represented as the sum of even and odd components.
- If $x(t)$ is a CTS with even component, $x_e(t)$ and odd component, $x_o(t)$ then we can express as, $x(t) = x_e(t) + x_o(t)$ ------(1)
- Now putting $t = -t$, we get $x(-t) = x_e(-t) + x_o(-t) = x_e(t) - x_o(t)$----(2)
- (1)+(2) we get, $x(t) + x(-t) = 2x_e(t)$, even component $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$
- (1) – (2) we get, $x(t) - x(-t) = 2x_o(t)$, odd component $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$
- Example Problem: Find the even and odd components of the signal $x(t) = e^{-2t}cost$.
- Solution: Replacing t with $-t$ into $x(t)$, we get $x(-t) = e^{-2(-t)}\cos(-t) = e^{2t}\cos t$
- Even component,

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)] = \frac{1}{2}[e^{-2t}cost + e^{2t}\cos t] = \frac{1}{2}[e^{-2t} + e^{2t}]cost = \cosh 2t \cos t$$

- Odd Component,

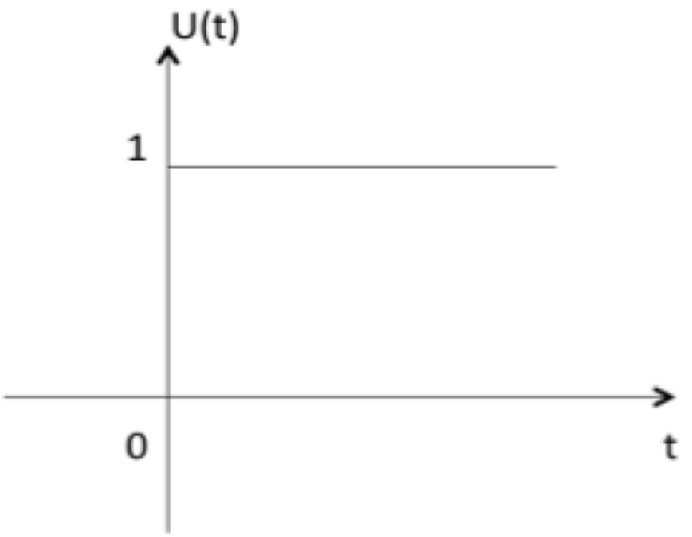
$$x_o(t) = \frac{1}{2}[x(t) - x(-t)] = \frac{1}{2}[e^{-2t}cost - e^{2t}\cos t] = \frac{1}{2}[e^{-2t} - e^{2t}]\cos t = -\sinh(2t)\cos t$$

According to Hyperbolic definitions, $\sinh(x) = (e^x - e^{-x})/2$ and $\cosh(x) = (e^x + e^{-x})/2$

Unit Step Function

- Continuous Unit step function is denoted by u(t). It is defined as,

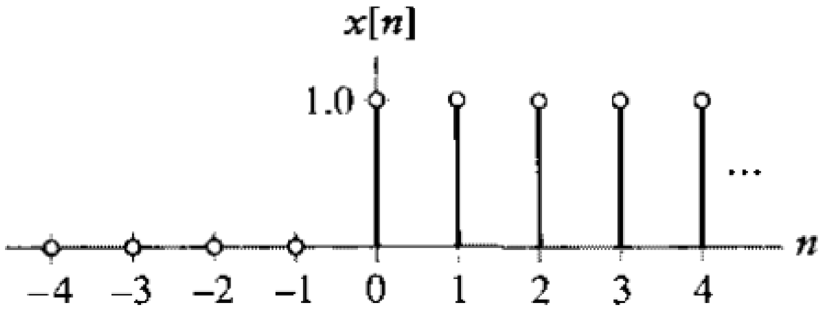
$$u(t)=\begin{cases}1, & t>0\\ 0, & t<0\end{cases}$$



  - It is used as best test signal.
  - Area under unit step function is unity.

- Discrete Unit step function is denoted by u[n],

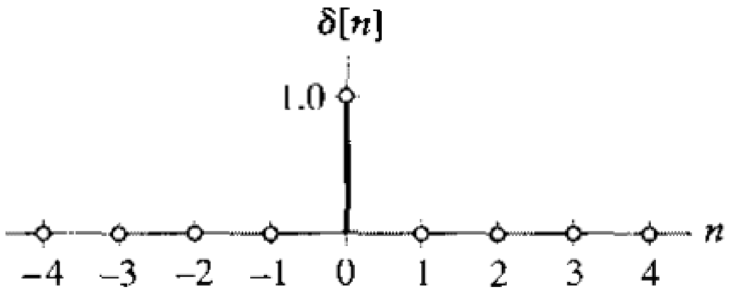
$$u[n]=\begin{cases}1, & n\geq 0\\ 0, & n<0\end{cases}$$



Unit Impulse Function

- The discrete-time version of Impulse function is denoted by $\delta[n]$ as

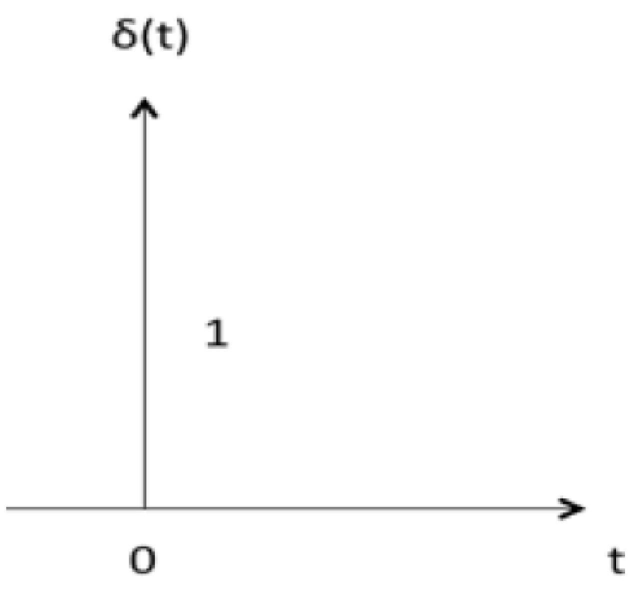
$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



- The continuous –time version of Impulse function is denoted by *δ(t)* as

$$\delta(t) = \{0, \quad for\ t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t) dt = 1$$



- Unit step function and impulse function is related to each other as

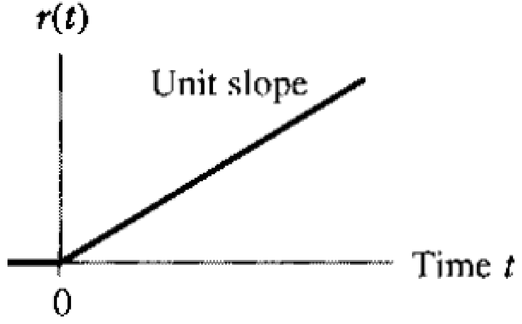
$$\int_{-\infty}^{\infty} \delta(t) dt = u(t)$$

$$\delta(t) = \frac{du(t)}{dt}$$

Ramp Signal

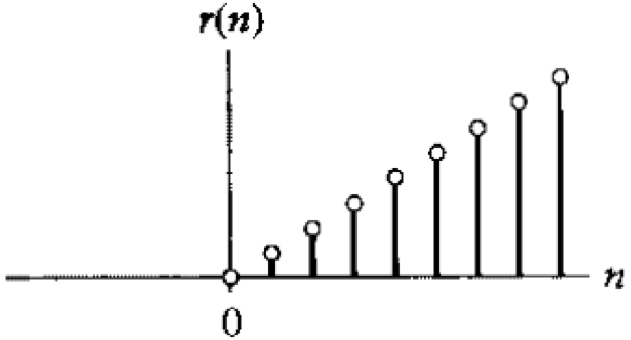
- Ramp signal is denoted by $r(t)$, defined as,
- Equivalently we can write $r(t) = t\,u(t)$.

$$r(t)=\begin{cases} t, & t\geq 0 \\ 0, & t<0 \end{cases}$$



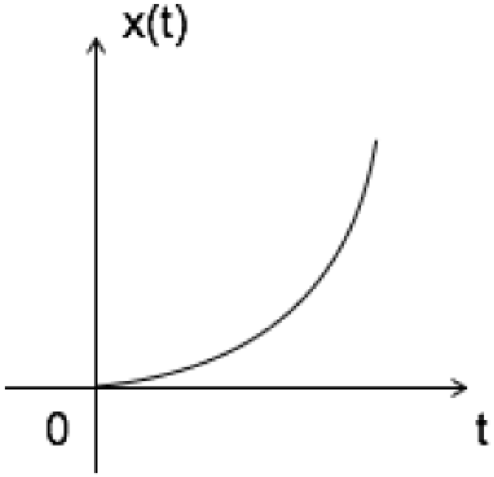
- Discrete-time version of Ramp function is defined as,
- Equivalently we can write $r[n] = n\,u[n]$.

$$r[n]=\begin{cases} n, & n\geq 0 \\ 0, & n<0 \end{cases}$$



Parabolic Signal

- Parabolic signal can be defined as *x*(*t*) as, $x(t) = \begin{cases} \frac{t^2}{2}, & t \geq 0 \\ 0, & t < 0 \end{cases}$



$$\iint u(t)dt = \int r(t)dt = \int tdt = \frac{t^2}{2} = parabolicsignal$$

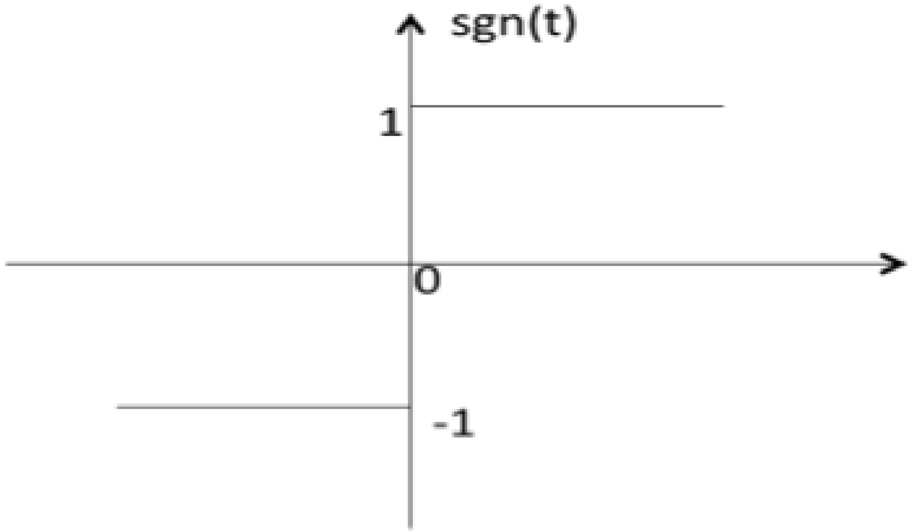
$$\Rightarrow u(t) = \frac{d^2x(t)}{dt^2}$$

$$\Rightarrow r(t) = \frac{dx(t)}{dt}$$

Signum Function

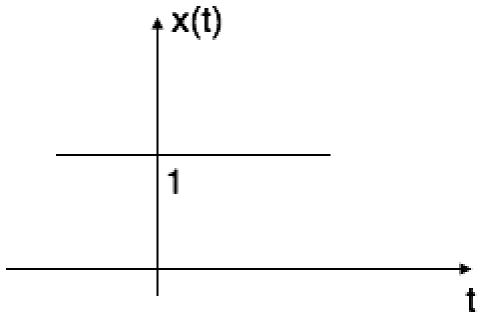
- Signum function is denoted as sgn(t). It is defined as,
- $\mathrm{sgn}(t) = 2u(t) - 1$

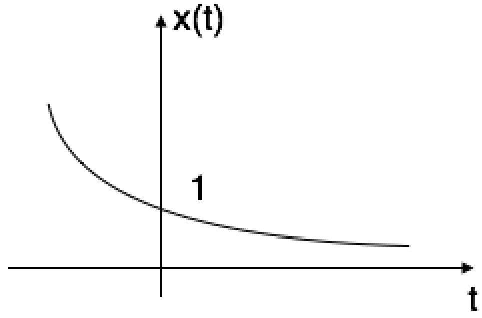
$$\mathrm{sgn}(t) = \begin{cases} 1, & t > 0 \\ 0, & t = 0 \\ -1, & t < 0 \end{cases}$$



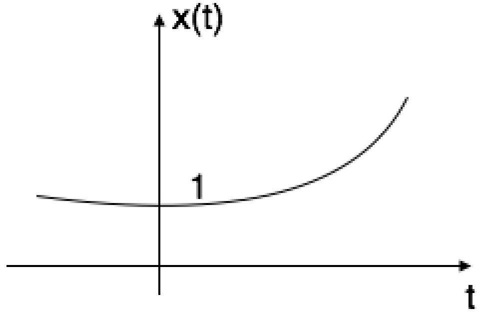
Exponential Signal

- Exponential signal is in the form of $x(t) = e^{\alpha t}$
- The shape of exponential can be defined by α.
- Case I: If $\alpha \to 0$, *i.g.* $x(t) = e^{0} = 1$





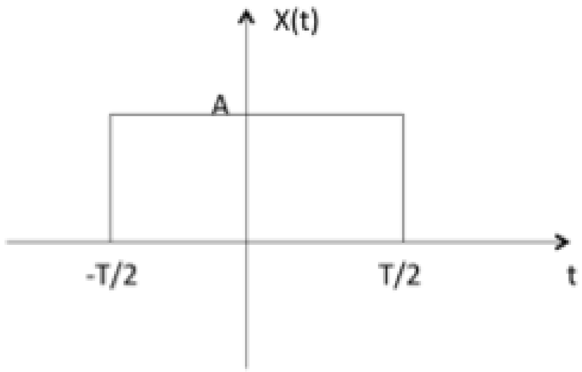
- Case II: If $\alpha < 0$, then $x(t) = e^{-\alpha t}$, The shape is called decaying exponential.
- Case III: If $\alpha > 0$ then, $x(t) = e^{\alpha t}$, The shape is called raising exponential.



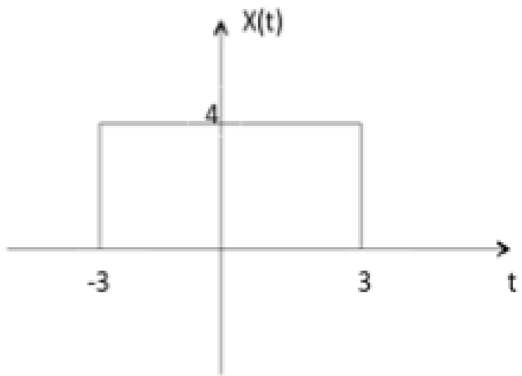
Rectangular Signal

- Let it be denoted as $x(t)$ and it is defined as, $x(t) = A\,rect\left[\frac{r}{t}\right]$ $e.g.$ $x(t) = 4\,rect\left[\frac{r}{6}\right]$

$$x(t) = A\,rect\left[\frac{r}{T}\right]$$



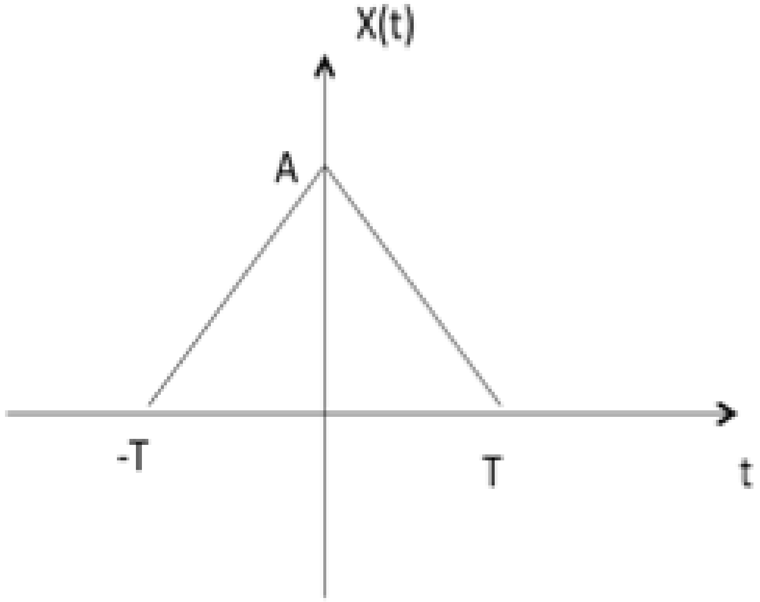
$$\text{ex}: 4\,rect\left[\frac{r}{6}\right]$$



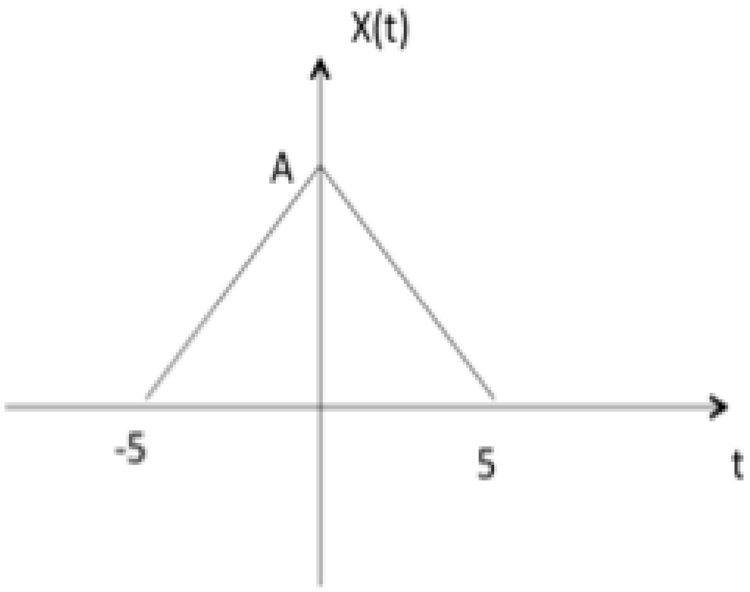
Triangular Signal

- Let it be denoted as $x(t)$.

$$x(t) = A\left[1 - \frac{|t|}{T}\right]$$

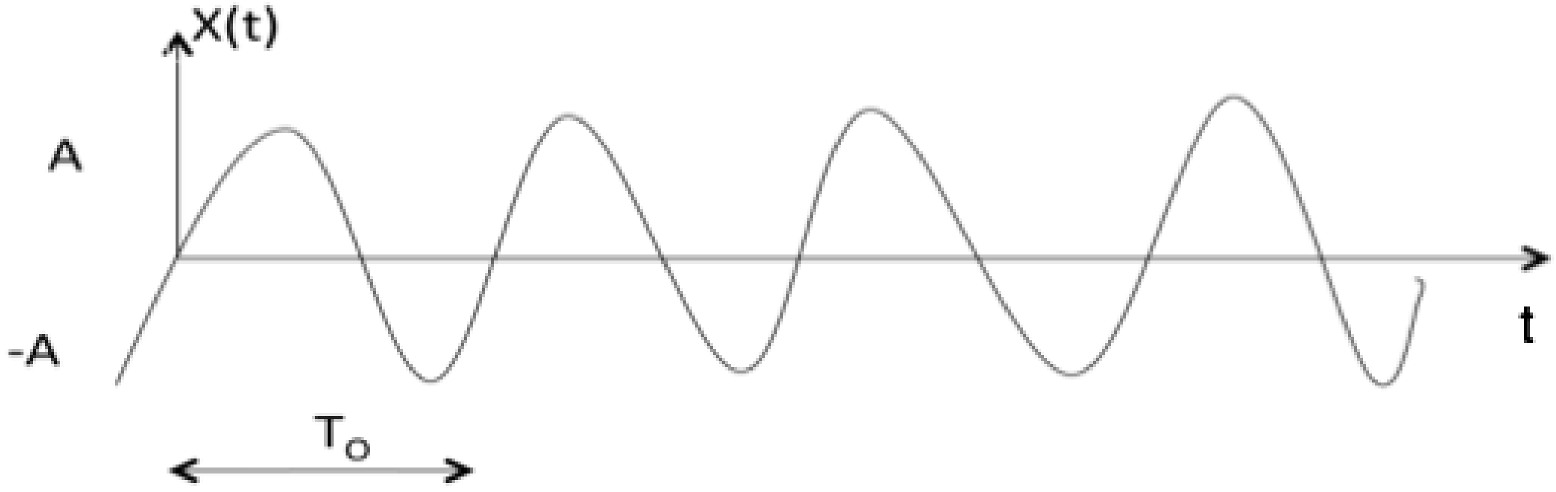


$$\text{ex: } x(t) = A\left[1 - \frac{|t|}{5}\right]$$



Sinusoidal Signal

- Sinusoidal signal is in the form of $x(t) = \mathrm{A}\cos(\omega_0 \pm \phi)$ or $\mathrm{A}\sin(\omega_0 \pm \phi)$

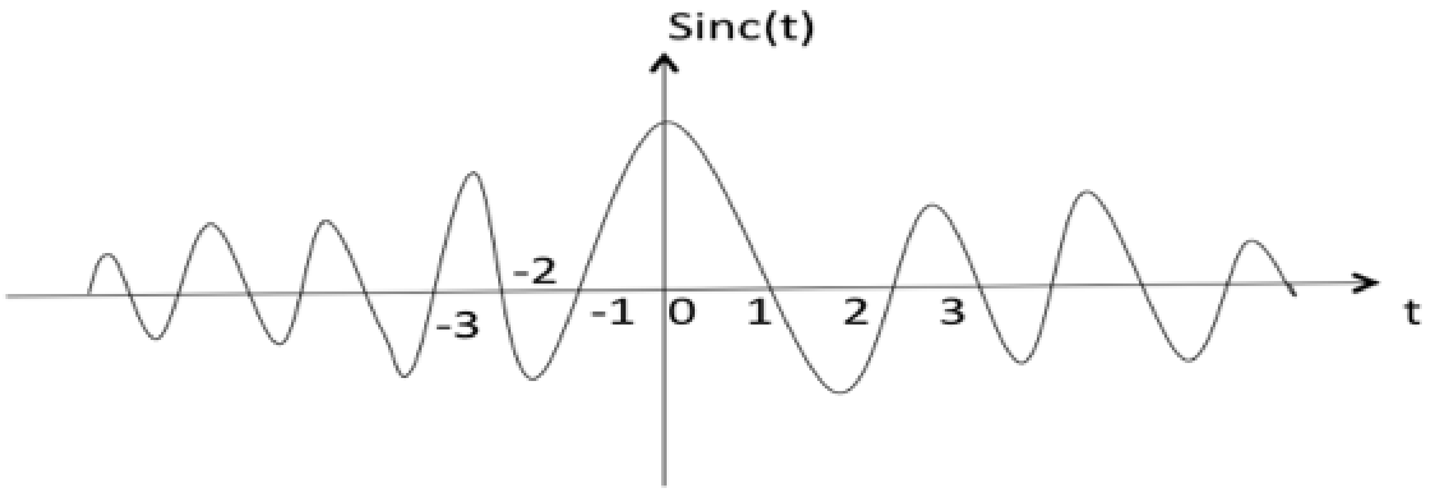


$$T_0 = \frac{2\pi}{\omega_0}$$

Sinc Function

- It is denoted as sinc(t) and it is defined as sinc.

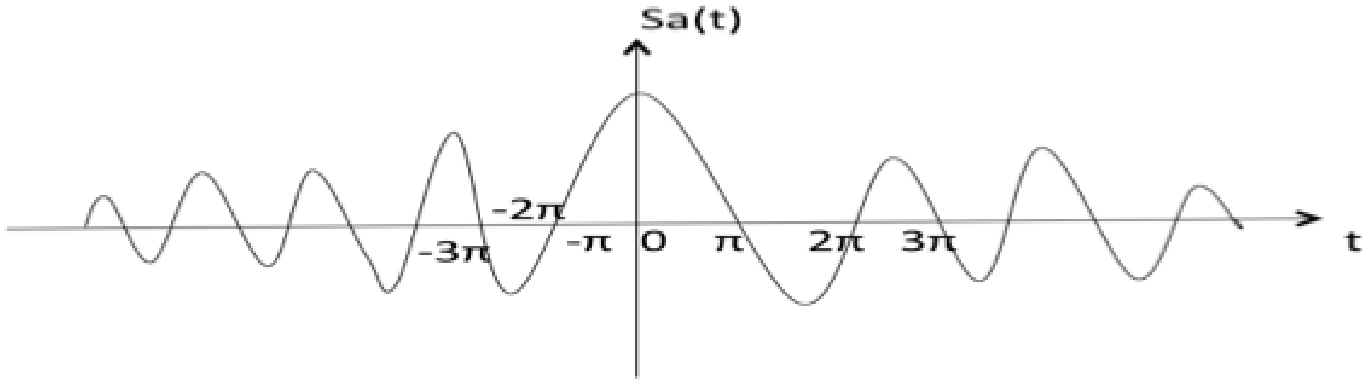
$$\sin c(t) = \frac{\sin \pi t}{\pi t} = 0, \ \ for\ t = \pm 1, \pm 2, \pm 3, \mathrm{K\ K}$$



Sampling Function

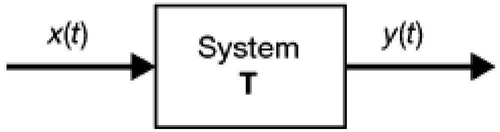
- It is denoted as sa(*t*) and it is defined as

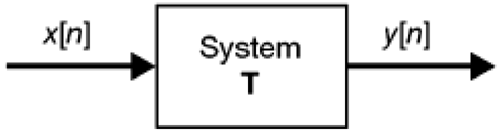
$$sa(t) = \frac{\sin t}{t} = 0,\ \ for\ t = \pm\pi, \pm 2\pi, \pm 3\pi, \Lambda$$



Classification of Systems

- Deterministic and Stochastic Systems
  - If I/O signals are deterministic, system is deterministic
  - If I/O signals are random, system is stoachastic
- Continuous-Time and Discrete-Time Systems





- Systems with Memory and without Memory
  - A system is said to be *memoryless* if the output at any time depends on only the input at that same time. Otherwise, the system is said to have *memory*
  - A memoryless system is a resistor *R* with the input *x(t)* taken as the current and the voltage taken as the output *y(t).* According to Ohm's law *y(t)* = *Rx(t)*
  - A system with memory is a capacitor *C* with the current as the input *x(t)* and the voltage as the output *y(t);* then $y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau$
  - A system memory for any discrete signal, $y[n] = \sum_{k=-\infty}^{n} x[k]$

Classification of Systems

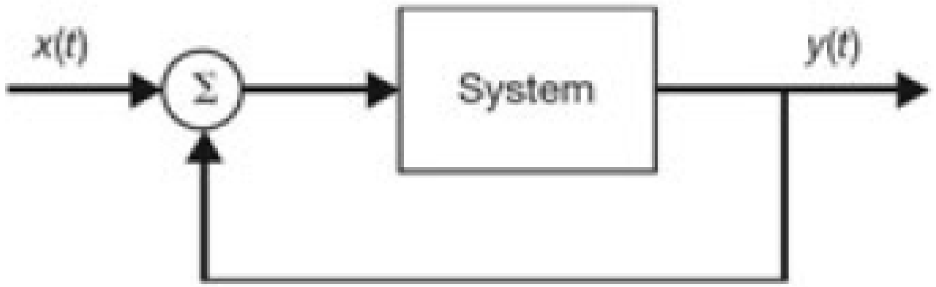
- Causal and Noncausal Systems
  - A system is called *causal* if its output at the present time depends on only the present and/or past values of the input. Thus, in a causal system, it is not possible to obtain an output before an input is applied to the system.
  - A system is called *noncausal* (or *anticipative)* if its output at the present time depends on future values of the input.
  - Example of noncausal systems are $y(t) = x(t + 1)$, $y[n] = x[-n]$
  - All memoryless systems are causal, but not vice versa.
- Linear Systems and Nonlinear Systems
  - If a system can be defined by a linear operator, then the system is linear. If ***T*** satisfied the following two conditions then ***T*** is linear:
    1. Additivity: For any signals x_1 and x_2, if $Tx_1=y_1$ and $Tx_2= y_2$ then $T(x_1+x_2) = y_1+y_2$
    2. Homogeneity (or Scaling): For any signal x and scaler α, $T\{\alpha x\} = \alpha y$
    3. The combination of these two laws is called superposition property, for any two scalers α_1 and α_2 and signals x_1 and x_2; we can write $T\{\alpha_1 x_1 + \alpha_2 x_2\} = \alpha_1 y_1 + \alpha_2 y_2$
  - Example of nonlinear system $y = x^2$ and $y = \cos x$

Classification of Systems

- Time-Invariant and Time-Varying Systems
  - A system is called *time-invariant* if a time shift (delay or advance) in the input signal causes the same time shift in the output signal.
  - For any real value of τ, CT**S: T**$\{x(t - \tau)\} = y(t - \tau)$
  - For any integer n, DTS: **T**$\{x[\mathrm{n} - \tau]\} = y[n - \tau]$
  - Otherwise *time-Varying* Systems
- Linear Time-Invariant Systems
  - If the system is linear and also time-invariant, then it is called a *linear time-invariant* (LTI) system.
- Stable Systems
  - A system is *bounded-input! bounded-output* (BIBO) *stable* if for any bounded input *x* defined by, $|x| \leq k_1$ and the corresponding output *y* is also bounded defined by $|y| \leq k_2$, where k_1 and k_2 are finite real constant.
  - For example, consider the system where output $y[n]$ is given by $y[n] = (n + 1)u[n]$, and input $x[n]$ = $u[n]$ is the unit step sequence. In this case the input $u[n]$ = 1, but the output $y[n]$ increases without bound as *n* increases.

Classification of Systems

- Feedback Systems
  - A special class of systems of great importance consists of systems having *feedback.* In a *feedback system,* the output signal is fed back and added to the input to the system as shown in Fig.



Reference

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- Video tutorials from https://www.youtube.com/user/nesoacademy