Chapter 2

Measures of Central Tendency

Central Tendency: In a representative sample, the value of a set of data have a tendency to cluster around a certain point usually at the center of the set is usually called central tendency and the numerical measures of the center or middle or location value is called the measures of central tendency or measures of location.

Different Measures of Central Tendency: The following are the important measures of central tendency which are generally used in business:

- Arithmetic mean
- Geometric mean
- Harmonic Mean
- Median
- Mode

Arithmetic mean: Arithmetic mean is defined as the sum of all observations divided by the total number of observations.

Calculation of Arithmetic Mean-Ungrouped Data: For ungrouped data, arithmetic mean may be computed by applying any of the following methods:

- Direct method
- Short-cut method

Direct method: The arithmetic mean, often simply referred to as mean, is the total of the values of a set of observations divided by their total number of observations. Thus, if $X_1, X_2, X_3, ..., X_N$ represent the values of N items or observations, the arithmetic mean denoted by $\overline{X}$ is defined as: $\overline{X} = \frac{X_1 + X_2 + ... + X_n}{n} = \frac{\sum_{i=1}^{n} X_i}{n}$

If the subscripts are dropped, the formula sample mean is: $\overline{X} = \frac{\sum X}{n}$ and the population mean is: $\mu = \frac{1}{N}\sum_{i=1}^{N} X_i$

Short-cut method: According to short-cut method arithmetic mean can be computed by the formula:

$\overline{X} = A + h\overline{U}$, where $U = \frac{X - A}{h}$, here A is called origin and h is called scale.

Example: The monthly expenditure (in taka) of 10 students given as follows:

14870 14930 15020 14460 14750 14920 15720 15160 14680 14890

Find monthly average expenditure.

Solution: Let income be denoted by X

By using calculator,

$\sum X = 149400$

$\overline{X} = \frac{\sum X}{n} = \frac{149400}{10} = 14940$

Hence, the average monthly income Tk.14940

Example: Mr. Peterson is studying the number of minutes used monthly by clients in a particular cell phone rate plan. Random sample of 12 clients showed the following number of minutes used last month.

90	77	94	89	119	112
91	110	92	100	113	83

What is the arithmetic mean number of minutes used?

Solution: Let the minute be denoted by X then $\sum X = 1170$, $\overline{X} = \frac{\sum X}{n} = \frac{1170}{12} = 97.5$

The arithmetic mean number of minutes used last month by the sample of cell phone users is 97.5 minutes.

Calculation of Arithmetic Mean-Grouped Data: For grouped data, arithmetic mean may be computed by applying any of the following methods:

- Direct method
- Short-cut method

Direct Method: When direct method is used

$$\overline{X} = \frac{\sum fX}{n}$$

Where, X = mid-point of the different classes

f = the frequency of each class

n = the total frequency ($\sum fi$)

Note: For computing mean in the case of grouped data the mid points of the various classes are taken as representative of that particular class. The reason is that when the

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data are grouped, the exact frequency with which each of the variable occurs in the distribution is unknown.

Example: The following are the figures of profits earned by 1400 companies during 1999-2000.

Profits (Tk. lakhs)	No. of companies	Profits (Tk. lakhs)	No. of companies
200-400	500	1000-1200	100
400-600	300	1200-1400	80
600-800	280	1400-1600	20
800-1000	120		

Calculate the average profits for all companies.

Solution:

Calculation of average profits

Profits (Tk. lakhs)	Mid-point (X)	No. of companies (f)	fX
200-400	300	500	150000
400-600	500	300	150000
600-800	700	280	196000
800-1000	900	120	108000
1000-1200	1100	100	110000
1200-1400	1300	80	104000
1400-1600	1500	20	30000
		$N=1400$	$\sum fX = 848000$

We know that, $\overline{X} = \dfrac{\sum fX}{n}$ then using data from table, $\overline{X} = \dfrac{848000}{1400} = 605.71$

So the average profit is 605.71 lakhs taka.

Arithmetic mean for two or more related groups: If we have the arithmetic mean and number of observations two or more than two related groups, we can compute combined average of these groups by applying the following formula.

$$\overline{X} = \frac{n_1\overline{X_1} + n_2\overline{X_2}}{n_1 + n_2}$$

Where,

$\overline{X}$ =Combined mean of the two groups, $\overline{X_1}$ =Arithmetic mean of the first group

$\overline{X_2}$ =Arithmetic mean of the second group, n_1 =No. of observations in the first group

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n_2 = No. of observations in the second group

*** If we have to find out the combined mean of three series, the above formula can be extended as follows:

$$\overline{X} = \frac{n_1\overline{X_1} + n_2\overline{X_2} + n_3\overline{X_3}}{n_1 + n_2 + n_3}$$

Example: There are two branches of a company employing 100 and 80 persons respectively. If arithmetic means of the monthly salaries paid by two branches are Tk.1570 and Tk.1750 respectively, find the arithmetic mean of the salaries of the employees of the company as a whole.

Solution: We should compute the combined mean. The formula is $\overline{X} = \frac{n_1\overline{X_1} + n_2\overline{X_2}}{n_1 + n_2}$

Given,

n_1=100, $\overline{X_1}$ =1570, n_2=80 and $\overline{X_2}$ =1750

$$\overline{X} = \frac{(100\times1570 + 80\times1750)}{100+80} = \frac{297000}{180} = 1650$$

Properties of Arithmetic Mean

- Every set of interval-level and ratio-level data has a mean.
- All the values are included in computing the mean.
- A set of data has only one mean. The mean is unique.
- The mean is very useful measure for comparing two or more populations (it is used to compare the performances of employee such as: 1st and 2nd shift).
- It is dependent on scale and origin of measurement.
- The arithmetic mean is the only measure of central tendency where the sum of deviations of each value from mean is zero. Symbolically, $\sum(X-\bar{X}) = 0$.

Limitations of Arithmetic Mean

- Arithmetic mean is highly affected by upper and lower extreme values.
- It is very difficult to compute arithmetic mean in open-end frequency distribution.
- It gives equal weights/importance to each and every observations.

Weighted Arithmetic Mean

One of the limitations of the arithmetic mean is that it gives equal importance to all the observations. But in most of the business problems, there are situations where the relative importance of different items is not same. In this situation, we compute weighted arithmetic mean. The terms 'weight' stands for the relative importance of the different observations.

The weighted arithmetic mean is, $\overline{X}_w = \frac{\sum WX}{\sum W}$

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Where, $\overline{X}_w$ = represents the weighted arithmetic mean, X = the variable, W = weights attached to the variable X. It is especially useful in the problems relating to the construction of index numbers and standardized birth and death rates.

Example: A House Building Construction company in USA pays the employees hourly \$16.50, \$17.50, and \$18.50. There are 26 hourly employees, 14 are paid at the \$16.50 rate, 10 at the \$17.50 rate and 2 at the \$18.50 rate. What is the mean hourly paid the 26 employees?

Solution: We know that, $\overline{X}_w = \dfrac{\sum WX}{\sum W}$, $\overline{X}_w = \dfrac{14(\$16.50) + 10(\$17.50) + 2(\$18.50)}{14 + 10 + 2}$

$= \dfrac{\$443.00}{26} = \$17.038$

The weighted mean hourly wage is rounded to \$17.04

Example: Suppose that, the nearby Wendy's Restaurant sold small, medium and large size soft drinks for \$0.90, \$1.25 and \$1.50 respectively of the last 10 drinks sold, 3 were small, 4 were medium, and 3 were large sized. Find the mean price of the last 10 drinks sold.

Solution: We know that, $\overline{X}_w = \dfrac{\sum WX}{\sum W}$, $\overline{X}_w = \dfrac{3(\$0.90) + 4(\$1.25) + 3(\$1.50)}{3 + 4 + 3}$

$= \dfrac{\$12.20}{10} = \$1.22$

Geometric Mean (For ungrouped data): The geometric mean of a set of n non-zero positive observations is the nth root of their product. Let $X_1, X_2, X_3, ..., X_n$ be non-zero positive observations in a series of data.

Thus, the geometric mean $G.M = (X_1 . X_2 . X_3 .... X_n)^{\frac{1}{n}}$.

The calculation may sometime simplified by taking logarithm, that is

$$Log\ G.M = \frac{1}{n}[\log X_1 + \log X_2 + ... + \log X_n]$$

$$= \frac{\sum_{i-1}^{n} \log X_i}{n}, \quad \therefore G.M = Anti\log\left(\frac{\sum_{i-1}^{n} \log X_i}{n}\right)$$

Example: Calculate the geometric mean for the following data:

6.5	169.0	11.0	112.5	14.2	75.0	35.5	215.0

Solution: We know that,

$$G.M = Anti\log\left(\frac{\sum_{i-1}^{n}\log X_i}{n}\right)$$

Let us consider, the observations is denoted by X .

Now by using calculator we get,

$$\sum \log X = 13.043 \text{ and } n = 8 .$$

$$\therefore\ G.M = Anti\log\ \left(\frac{13.043}{8}\right)$$
$$= Anti\log\ (1.6304)$$
$$= 42.697$$

Example: Calculate the geometric mean of the following price relatives:

Commodity	Price Relatives
Wheat	207
Rice	198
Pulses	156
Sugar	124
Salt	107
Oils	196

Solution: We know that, $G.M = Anti\log\left(\frac{\sum_{i-1}^{n}\log X_i}{n}\right)$

Let us consider, the price relatives is denoted by X , then $\sum \log X = 13.2208$ and $n = 6$.

$$G.M = Anti\log\ \left(\frac{13.2208}{6}\right) \quad = Anti\log\ (2.2035)\ = 159.77$$

For grouped data: In grouped data for calculating geometric mean first we will find the mid-points and then apply the following formula:

$$G.M = Anti\log\ \left(\frac{\sum_{i-1}^{n} f_i \log X_i}{n}\right),$$ **w**here, X = mid-point.

Example: Find out geometric mean from the following data:

X	10	20	30	40	50	60
f	12	15	25	10	6	2

Solution: We know that, $G.M = Anti\log\left(\frac{\sum_{i-1}^{n} f_i \log X_i}{n}\right)$

From the given data we get $\sum f \log X = 98.214$ and $n = 70$.

$$G.M = Anti\log\left(\frac{98.214}{70}\right) = Anti\log\ (1.403)\ = 25.29$$

Example: Calculate geometric mean for the following distribution.

Weight (in lbs)	Frequency
100-104	24
105-109	30
110-114	45
115-119	65
120-124	72

Solution: Calculation of geometric mean

Weight (in lbs)	Midpoint (x)	Frequency
100-104	102	24
105-109	107	30
110-114	112	45
115-119	117	65
120-124	122	72
Total		236

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We know that, $G.M = Anti\log\left(\frac{\sum_{i-1}^{n} f_i \log X_i}{n}\right)$

From the given data we get $\sum f \log X = 485.95$ and $n = 236$.

$G.M = Anti\log\left(\frac{485.95}{236}\right) = Anti\log\ (2.059) = 114.55$

Uses of Geometric Mean:

- It is useful in averaging ratios, percentages and rates of increase between two periods.
- It appropriately useful for computing the average rate of growth of population, average increase in the rate of profits, sales, productions etc.

Limitations of Geometric Mean:

- Compared with AM, it is more difficult to compute and interpret.
- It can not be computed if one or more observation of the series is zero or negative.

Harmonic Mean

It is defined as the reciprocal of the arithmetic mean of the reciprocal of the individuals' observations.

For ungrouped data: The harmonic mean

$$\text{H.M} = \frac{n}{\left(\frac{1}{X_1} + \frac{1}{X_2} + \ldots + \frac{1}{X_n}\right)} = \frac{n}{\sum\left(\frac{1}{X}\right)}$$

i.e. the harmonic mean of a set of n non zero observations $X_1, X_2, \ldots, X_n$ in a series is the reciprocal of the arithmetic mean of the reciprocals.

Example: Calculate the Harmonic mean of the following series of monthly expenditure of a batch of students:

TK.	1250	1300	1750	1000	1450	1500	1550	1400	1500	1150

Solution: We know that, the harmonic mean is given by

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$$\text{H.M} = \frac{n}{\left(\frac{1}{X_1} + \frac{1}{X_2} + \ldots + \frac{1}{X_n}\right)} = \frac{n}{\sum\left(\frac{1}{X}\right)}$$

Let, monthly expenditure denoted by X then we get,

x	$\frac{1}{x}$
1250	0.0008
1300	0.000769
1750	0.000571
1000	0.001
1450	0.00069
1500	0.000667
1550	0.000645
1400	0.000714
1500	0.000667
1150	0.00087
Total	0.007393

Here, $\sum \frac{1}{X} = 0.007393$ and $N = 10$

$$\therefore \text{ H.M} = \frac{10}{0.007393} = 1352.693$$

Example: Calculate the Harmonic mean from the following figures:

9.7 0.0009 178.7 0.874 1238 0.012 89.9 78.4 0.989 0.008

Solution: We know that the harmonic mean

$$\text{H.M} = \frac{n}{\left(\frac{1}{X_1} + \frac{1}{X_2} + \ldots + \frac{1}{X_n}\right)} = \frac{n}{\sum\left(\frac{1}{X}\right)}$$

Let, the observations denoted by X we get, $\sum \frac{1}{X} = 1321.73$ and $N = 10$

$$\therefore \text{H.M} = \frac{10}{1321.73} = 0.008.$$

For grouped data: For grouped data,

$$\text{H.M} = \frac{n}{\sum\left(f \times \frac{1}{X}\right)}$$

Where, $n = \sum f$, X may be considered as the mid values of the class intervals.

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Example: Calculate the harmonic mean from the following data

X	10	12	14	16	18	20
f	5	18	20	10	6	1

Solution: We know that, $\text{H.M} = \dfrac{n}{\sum\left(f \times \dfrac{1}{X}\right)}$

X	f	$f \times \dfrac{1}{X}$
10	5	0.500
12	18	1.500
14	20	1.429
16	10	0.625
18	6	0.333
20	1	0.050
Total		4.437

Here, $\sum\left(f \times \dfrac{1}{X}\right) = 4.437$ and $n = \sum f = 60$ $\quad \therefore \text{H.M} = \dfrac{60}{4.437} = 13.52$

Example: Calculate harmonic mean from the following data:

Marks	**Frequency**
0-10	5
10-20	10
20-30	7
30-40	3
40-50	2

Solution: We know that,

$$\text{H.M} = \frac{n}{\sum\left(f \times \frac{1}{X}\right)}$$

Calculation of harmonic mean

Marks	**Frequency (f)**	**Mid-points (x)**	$f \times \dfrac{1}{X}$
0-10	5	5	1
10-20	10	15	0.667
20-30	7	25	0.280
30-40	3	35	0.086
40-50	2	45	0.044

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Total	27		2.077

We get,

$$\sum\left(f\times\frac{1}{X}\right)=2.077 \text{ and } n=\sum f=27 \quad \therefore \text{H.M}=\frac{27}{2.077}=13.00061=13$$

Uses of Harmonic Mean:

- It is useful in averaging rates and ratios where time factor is variable and the act being performed that is distance is constant.

Theorem: For n non-zero positive observations,

Arithmetic mean (*AM*) $\geq$ Geometric mean (*GM*) $\geq$ Harmonic mean (*HM*)

Example: Find the arithmetic mean, geometric mean and harmonic mean from the following data

5 8 11 10 15 7 11 12 4 6

and hence show that , Arithmetic mean $\geq$ Geometric mean $\geq$ Harmonic mean

Solution: We know that

Arithmetic mean $\overline{X}=\dfrac{X_1+X_2+\text{.........}+X_n}{n}=\dfrac{\sum_{i=1}^{n}X_i}{n}=\dfrac{89}{10}=8.9$

Geometric mean G.M =Antilog $\left(\dfrac{\sum_{i-1}^{n}\log X_i}{n}\right)$=Antilog $\left(\dfrac{9.165}{10}\right)=8.25$

Harmonic mean H.M $=\dfrac{n}{\left(\dfrac{1}{X_1}+\dfrac{1}{X_2}+\ldots+\dfrac{1}{X_n}\right)}=\dfrac{n}{\sum\left(\dfrac{1}{X}\right)}=\dfrac{10}{1.316}=7.598$

Hence we observed that,

Arithmetic mean (*AM*) $\geq$ Geometric mean (*GM*) $\geq$ Harmonic mean (*HM*)

Median

The median is the measure of central tendency which appears in the "middle" of an ordered sequence of values. That is, half of the observations in a set of data are lower than it and half of the observations are greater than it. Median is called positional average.

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For example, if the income of five persons is Tk.1000, 1200, 1500, 1600, 1800 then the median income would be Tk. 1500. Changing any or both of the first two values with any other numbers with value 1500 or less and/or changing any of the last two values to any other values with values of 1500 and more, would not affect the value of the median which would remain 1500.

In contrast, in case of arithmetic mean, the change in value of single observation would cause the value of the mean to be changed.

Calculation of Median –Ungrouped Data: The median is defined as the middle most observations when the observations arranged in order of magnitude.

Arrange the data in ascending or descending order of magnitude. (Both arrangements would give the same answer)

For ungrouped data

- When n (total number of observations) is odd, the middle most observation i.e, $\frac{(n+1)}{2}th$ observation will be the median in the series.
- When n is even, the median will be the arithmetic mean of $\frac{n}{2}th$ and $\left(\frac{n}{2}+1\right)th$ observations in the series.

Example: From the following data of wages of 7 workers, compute the median wage:

Wages (in Tk.) 1600 1650 1580 1690 1660 1606 1640

Solution: Calculation of median

Sl. No.	Wages arranged in ascending order
1	1580
2	1600
3	1606
4	1640
5	1650
6	1660
7	1690

Median = Size of $\frac{n+1}{2}th$ observation

$= \frac{7+1}{2}th = 4^{\text{th}}$ observation

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Value of 4^{th} observation is 1640. Hence median wage = Tk.1640

Example: The flowing table gives the monthly income of 12 families in a village.

House No	Monthly income (Tk.)	House No	Monthly income (Tk.)
1	587	7	805
2	693	8	907
3	595	9	763
4	780	10	865
5	840	11	768
6	760	12	894

Calculate the median income.

Solution: For calculating median the data have to arrange either in ascending or descending order. Here income has been arranged in ascending order.

House No.	Monthly income (Tk.)
1	587
2	595
3	693
4	760
5	763
6	768
7	780
8	805
9	840
10	865
11	894
12	907

We know that, when n is even, the median will be the arithmetic mean of $\frac{n}{2}$th and $\left(\frac{n}{2}+1\right)$th observations in the series.

We have, $n = 12$. Hence, $\frac{n}{2}$th observation $= \frac{12}{2} = 6^{th}$ observation and $\left(\frac{n}{2}+1\right)$observation $= (\frac{12}{2}+1)th = 7^{th}$ observation.

So that, Median $= \frac{(\text{6th observation+7th observation})}{2} = \frac{(768+780)}{2} = 774$

Hence, the median income = Tk.774.

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Calculation of Median –Grouped Data: Apply the following formula for determining the exact value of median:

$$\text{Median} = L + \frac{\frac{n}{2} - p.c.f}{f} \times i$$

Where L = lower limit of median class i.e., the class in which the middle item in the distribution lies.

$p.c.f$ = preceding cumulative frequency to the median class

f = frequency of the median class

i = the class interval of the median class.

Example: Suppose 1500 workers are working in an industrial establishment. Their age is classified as follows:

Age (yrs.)	No. of workers	Age (yrs.)	No. of workers
18-22	120	38-42	184
22-26	125	42-46	162
26-30	280	46-50	86
30-34	260	50-54	75
34-38	155	54-58	53

Calculate the median age.

Solution: Calculation of median age

Age (yrs.)	f	c. f
18-22	120	120
22-26	125	245
26-30	280	525
30-34	260	785
34-38	155	940
38-42	184	1125
42-46	162	1286
46-50	86	1372
50-54	75	1447
54-58	53	1500

Median = Size of $\frac{n}{2}$ th observation $= \frac{1500}{2} = 750th$ observation.

Hence, the median lies in the class 30-34.

$$\text{Median} = L + \frac{\frac{n}{2} - p.c.f}{f} \times i$$

$$= 30 + \frac{750 - 525}{260} \times 4$$

$$= 30 + 3.46$$

$$= 33.46$$

Hence, the median age is 33.46 years.

Example: Calculate the median from the following data:

Value	Frequency
0-10	4
10-20	12
20-30	24
30-40	36
40-50	20
50-60	16
60-70	8
70-80	5

Solution: Calculation of median

Value	f	c. f
0-10	4	4
10-20	12	16
20-30	24	40
30-40	36	76
40-50	20	96
50-60	16	112
60-70	8	120
70-80	5	125

Median = Size of $\frac{n}{2}$ th observation $= \frac{125}{2} = 62.5th$ observation.
Hence, the median lies in the class 30-40.

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$$\text{Median} = L + \frac{\frac{n}{2} - p.c.f}{f} \times i$$

Here, $L = 30$, $\frac{N}{2} = 62.5$, $p.c.f = 40$, $f = 36$, $i = 10$.

$$\text{Median} = 30 + \frac{62.5 - 40}{36} \times 10$$

$= 30 + 6.25$

$= 36.25$ (Ans.)

Properties of Median:

- The median is unique, that is like mean, there is only one median for set of data.
- To determine median arrange the data from low to high, and find the value of the middle observation.
- It is not affected by extreme large or small values and therefore a valuable measure of central tendency when such value do occurs.
- It can be computed for an open ended frequency distribution, if the median does not lie in the open end class.
- It can be computed for ratio-level, interval-level and ordinal-level data.

Limitations of Median:

- Since median is positional average, it's value is not determine by each and every observation.
- It does not capable for algebraic treatment (it cannot be used for computing the combined median of two or more groups).

Mode

Mode is the value of the observation that appears most frequently. For example, if we take the values of six different observations as 5, 8, 10, 8, 5, 8 mode will be 8 as it has occurred maximum number of times, i.e 3 times.

Calculation of mode –Ungrouped data: For determining mode count the numbers of items the various values repeat themselves and the value which occurs the maximum number of times is the modal value.

Example: The following figures relate to the preferences with regard to size of screen in inches of T.V sets of 30 persons selected at random from a locality. Find the modal size of the T.V screen.

12	20	12	24	27	20	12	20	27	24
24	20	12	20	24	27	24	24	20	24
24	20	24	24	12	24	20	27	24	24

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Solution: Calculation of modal size

Size in inches	Tally	Frequency
12	~~IIII~~	5
20	~~IIII~~ III	8
24	IIII IIII III	13
27	IIII	4

Since size 24 occurs the maximum number of items, therefore, the modal size of T.V screen is 24 inches.

Calculation of mode- Grouped data: In case of grouped data the following formula is used for calculating mode:

$$\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i$$

Where,

L = Lower limit of the modal class

Δ_1 = The difference between the frequency of the modal class and the frequency of the pre-modal class ($f_1 - f_0$).

Δ_2 = The difference between the frequency of the modal class and the frequency of the post-modal class ($f_1 - f_2$)..

i = The size of the modal class.

Example: The following data relate to the sales of 100 companies:

Sales(Tk.lakhs)	No. of companies
Below 60	12
60-62	18
62-64	25
64-66	30
66-68	10
68-70	3
70-72	2

Calculate the modal sales.

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Solution: Since the maximum frequency 30 is in the class 64-66, therefore 64-66 is the modal class.

$$\text{Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times i$$

Here, L = 64, $\Delta_1 = (30\text{-}25) = 5$, $\Delta_2 = (30\text{-}10) = 20$, $i = 2$

$$\text{Mode} = 64 + \frac{5}{5+20} \times 2$$

$$= 64 + \frac{10}{25}$$

$$= 64.4$$

Hence, modal sales are Tk.64.4 lakhs.

Properties of Mode:

- The mode is not affected by the extreme values.
- It can be calculated from open ended frequency distribution.
- Mode can be easily used to describe qualitative phenomenon.

Limitations:

- It is not based on all observations.
- It is difficult for algebraic treatments.
- Mode is not clearly defined in case of bi-modal or multimodal distribution.

Uses of Mode:

- It can be used in the problems involving the expression of preference where quantitative measurements are not possible.
- It is used in the comparison of consumer preferences for different kinds of product or advertizing.

Other measure of Location (Positional measure)

- Quartiles
- Deciles
- Percentiles

Quartiles

Quartiles are those values in a series which divide the total frequency into four equal parts when the series is arranged in order of magnitude. The quartiles are denoted by Q. For a grouped frequency distribution the quartiles are given by

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$$Q_i = l_i + \frac{\frac{i \times n}{4} - cf_i}{f_i} \times h$$

where, l_i is the lower limit of the ith quartile class (which contains the $\frac{i \times n}{4}th$ observation)
n is the total number of observation
cf_i is the cumulative frequency of the pre ith quartile class
f_i is the frequency of the ith quartile class and
h is the class interval of ith quartile class

Deciles

Deciles are those values in a series which divide the total frequency into ten equal parts when the series is arranged in order of magnitude. The deciles are denoted by D.

For a grouped frequency distribution the deciles are given by

$$D_i = l_i + \frac{\frac{i \times n}{10} - cf_i}{f_i} \times h$$

where, l_i is the lower limit of the ith deciles class (which contains the $\frac{i \times n}{10}th$ observation)
n is the total number of observation
cf_i is the cumulative frequency of the pre ith decile class
f_i is the frequency of the ith decile class and
h is the class interval of ith decile class

Percentiles

Percentiles are those values in a series which divide the total frequency into four equal parts when the series is arranged in order of magnitude. The quartiles are denoted by p.
For a grouped frequency distribution the percentiles are given by

$$P_i = l_i + \frac{\frac{i \times n}{100} - cf_i}{f_i} \times h$$

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where, l_i is the lower limit of the ith percentile class (which contains the $\frac{i \times n}{100} th$ observation)

n is the total number of observation

cf_i is the cumulative frequency of the pre ith percentile class

f_i is the frequency of the ith percentile class and

h is the class interval of ith percentile class

Q. Which average to use?

We have explained different methods of computing the various types of averages and also their distinctive features. Now among the various measures which of these averages should we use? or which of these is the best average to be used?

The empirical relationship between mean, median and mode

The relationship among mean, median and mode for symmetrical distribution and asymmetrical distribution are as follows:

Symmetrical distribution: **Mean = Mode = Median**

Asymmetrical distribution:

- Positively skewed

  Mean > Median > Mode

- Negatively skewed

  Mode > Median > Mean

Karl Pearson has expressed the relationship as follows.

$$\textbf{Mean-Median} = \frac{1}{3}\textbf{(Mean-Mode)}$$

$$\therefore \textbf{Mode} = \textbf{3} \times \textbf{Median - 2} \times \textbf{Mean}$$

And $$\textbf{Median} = \textbf{Mode} + \frac{2}{3}\textbf{(Mean-Mode)}$$

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