Institute Information Technology

Subject : Numerical Techniques Laboratory

Exp. No.-5

Name of the Expt.: Numerical differentiation for equidistant x by Newton's and Stirling's Interpolating Formulae

Theory:

Let there are n+1 number of data points $(x_0, y_0), (x_1, y_1)........(x_n, y_n)$ are given. To find out values of $\frac{dy}{dx} and \frac{d^2y}{dx^2}$ at various given points of x of the table, the methods given below are followed.

- **Points at the beginning of the table:**

To find out $\frac{dy}{dx} and \frac{d^2y}{dx^2}$ for the given points of x at the beginning of the table use Numerical Forward Differentiation formulae using Newton's forward difference table.

Numerical Differentiation formulae using forward Difference Table

$$f'(x_0) = \frac{1}{h} * \left[\Delta y_o - 1/2 * \Delta^2 y_o + 1/3 * \Delta^3 y_o - 1/4 * \Delta^4 y_o + 1/5 * \Delta^5 y_o - \frac{1}{6} * .\Delta^6 y_0 \right] ....(1)$$

$$f''(x_0) = \frac{1}{h^2} * \left[\Delta^2 y_o - \Delta^3 y_0 + \frac{11}{12} \Delta^4 y_o - \frac{5}{6} \Delta^5 y_0 \right] ....(2)$$

Where, h=difference between two successive values of x.

The values of Δy_o, $\Delta^2 y_0$, $\Delta^3 y_0$ .... $\Delta^n y_0$ can be found from the following forward difference Table (Table-1).

Table-1:Forward difference Table(n=degree of polynomial=6)

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
x_0	y_0						
		Δy_0					
x_1	y_1		$\Delta^2 y_0$				
		Δy_1		$\Delta^3 y_0$			
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$		
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$	
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$		$\Delta^6 y_0$
		Δy_3		$\Delta^3 y_2$		$\Delta^5 y_1$	
x_4	y_4		$\Delta^2 y_3$		$\Delta^4 y_2$		
		Δy_4		$\Delta^3 y_3$			
x_5	y_5		$\Delta^2 y_4$				
		Δy_5					
x_6	y_6						

$\Delta y_0 = y_1 - y_0 ; \Delta y_1 = y_2 - y_1 ; \Delta y_2 = y_3 - y_2 ; \Delta^2 y_0 = \Delta y_1 - \Delta y_0 ; \Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$

- **Points at the end of the table:**

For the point x at the end of the table, use Newton's backward difference table.

Numerical Differentiation formula using Backward Difference Table

$$f'(x_n) = \frac{1}{h} * \left[\nabla y_n + 1/2 * \nabla^2 y_n + 1/3 * \nabla^3 y_n + 1/4 * \nabla^4 y_n + 1/5 * \nabla^5 y_n + \frac{1}{6} * .\nabla^6 y_n \right] ....(3)$$

$$f''(x_n) = \frac{1}{h^2} * \left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12} \nabla^4 y_n + \frac{5}{6} \nabla^5 y_n \right] ....(4)$$

Where, h=difference between two successive values of x.

Values of ∇y_n, $\nabla^2 y_n$, $\nabla^3 y_n$ .... $\nabla^n y_n$ can be found from the following backward difference Table (Table-2).

- **Points at the middle of the table(Applicable only when length of given data points are of odd number):**

For the point x at the middle of the table, ***use Stirling formula*** as following (use Table1).

$$f'\left(x_{(n+1)/2}\right)=\frac{1}{h}*\left[\left(\Delta y_2+\Delta y_3\right)/2-\frac{1}{6}(\Delta^3 y_1+\Delta^3 y_2)/2+\frac{1}{30}(\Delta^5 y_o+\Delta^5 y_1)/2\right]\text{..................}(5)$$

$$f''(x_{(n+1)/2})=\frac{1}{h^2}*\left[\Delta^2 y_2-\frac{1}{12}\Delta^4 y_1+\frac{1}{90}\Delta^6 y_o\right]\text{..}(6)$$

Table-2: Backward difference Table (n=5)
and relation between forward and backward elements

x	y	∇	∇^2	∇^3	∇^4	∇^5
x_0	y_0					
		$\nabla y_1=\Delta y_0$				
x_1	y_1		$\nabla^2 y_2=\Delta^2 y_0$			
		$\nabla y_2=\Delta y_1$		$\nabla^3 y_3=\Delta^3 y_0$		
x_2	y_2		$\nabla^2 y_3=\Delta^2 y_1$		$\nabla^4 y_4=\Delta^4 y_0$	
		$\nabla y_3=\Delta y_2$		$\nabla^3 y_4=\Delta^3 y_1$		$\nabla^5 y_5=\Delta^5 y_0$
x_3	y_3		$\nabla^2 y_4=\Delta^2 y_2$		$\nabla^4 y_5=\Delta^4 y_1$	
		$\nabla y_4=\Delta y_3$		$\nabla^3 y_5=\Delta^3 y_2$		
x_4	y_4		$\nabla^2 y_5=\Delta^2 y_3$			
		$\nabla y_5=\Delta y_4$				
x_5	y_5					

$\nabla y_1=y_1-y_0;$ $\nabla y_2=y_2-y_1;$ $\nabla^2 y_2=\nabla y_2-\nabla y_1;$ $\nabla^3 y_3=\nabla^2 y_3-\nabla^2 y_2$ and so on……………

In Matlab the table 1 will appears like following, which can be used for both forward and backward formulae.:

Table-3:Forward difference Table in MATLAB (n=degree of polynomial=6)

X	Y	Dy1	Dy2	Dy3	Dy4	Dy5	Dy6
X(1)	Y(1)						
		Dy1 (1)					
X(2)	Y(2)		Dy2 (1)				
		Dy1 (2)		Dy3 (1)			
X(3)	Y (3)		Dy2 (2)		Dy4 (1)		
		Dy1 (3)		Dy3(2)		Dy5 (1)	
X(4)	Y (4)		Dy2 (3)		Dy4(2)		Dy6 (1)
		Dy1 (4)		Dy3 (3)		Dy5(2)	
X(5)	Y (5)		Dy2 (4)		Dy4 (3)		
		Dy1 (5)		Dy3 (4)			
X(6)	Y (6)		Dy2 (5)				
		Dy1 (6)					
X(7)	Y (7)						

Where , Dy1=diff(Y); Dy2(diff(Dy1)); and Dy3=(diff(Dy2)) and so on.
So, for example, from forward formula (1) becomes,

$$f'\left(x_i\right)=\frac{1}{h}*\left[Dy1(i)-1/2*Dy2(i)+1/3*Dy3(i)-1/4*Dy4(i)+1/5*Dy5(i)-\frac{1}{6}*Dy6(i)\right]\text{..............}(7)$$

From backward formula (3) becomes

$$f'\left(x_i\right)=\frac{1}{h}*\left[Dy1(i-1)-1/2*Dy2(i-2)+1/3*Dy3(i-3)-1/4*Dy4(i-4)+1/5*Dy5(i-5)-\frac{1}{6}*Dy6(i-6)\right]\text{..............}(8)$$

and so on.

Finding out the location of a point (beginning/middle/end);

Now, It is very important to find out in program that whether a point "i", (where i represents the index of the x or y vector), is at the beginning, or end or at the middle point of the table because according to it forward, backward or sterling formula should be applied respectively. Stirling formula is applicable only for the middle point which occurs in the case of odd number of data points, so before applying Stirling Formula it needs to check whether the length of the given number of points is odd or even.

(Hints: now if i<(length(x)+1)/2, use forward differentiation formula, if both rem(length(x),2)=1 and if i= (length(x)+1)/2, use sterling differentiation formula and when i> (length(x)+1)/2, Use backward differentiation formula.)
REM (m,n) finds out the Remainder after division m by n.

Problems and Reports
1. Problem 1.

X	x0=1.0	x1=1.2	x2=1.4	x3=1.6	x4=1.8	x5=2.0	x6=2.2
y	y0=2.7183	y1=3.3201	y2=4.0552	y3=4.9530	y4=6.0496	y5=7.3891	y6=9.0250

Find the value of f′(x) and f′′(x) at x=1.0,1.2, and x= 2.0, 2.2, 1.6 by Numerical Differentiation.
2.Problem 2.
$f(x)=y=x^3-3x^2+2x-1$

Find $f'(2)$, $f''(2)$, $f'(6)$, $f''(6)$,$f'(7)$, $f''(7)$ using Numerical Differentiation formulae, when , $2 \le \zeta \le 7$

3.Discuss on the experiment.