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Answer to the question no — 1

Given that

$$3\frac{dy}{dx} + xy^2 = \sin x$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sin x - xy^2}{3}$$

and $y(0) = 5$

$$\Rightarrow x_0 = 0, \; y_0 = 5$$

Let, $h = 0.1$

Now,

$$x_1 = x_0 + h$$
$$= 0 + 0.1$$
$$= 0.1$$

$$y_1 = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

Then,

$$k_1 = hf(x_0, y_0)$$

$$= 0.1 \times \left(\frac{\sin x_0 - x_0 y_0^2}{3}\right)$$

$$= 0.1 \times \frac{\sin 0 - 0 \times 5^2}{3}$$

$$= 0$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$= 0.1 \times \left(\frac{\sin\left(x_0 + \frac{h}{2}\right) - \left(x_0 + \frac{h}{2}\right)\left(y_0 + \frac{k_1}{2}\right)^2}{3}\right)$$

$$= 0.1 \times \left(\frac{\sin \frac{0.1}{2} - \left(0 + \frac{0.1}{2}\right)\left(5 + \frac{0}{2}\right)^2}{3}\right)$$

$$= -0.042$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$= 0.1 \times \left(\frac{\sin\left(x_0 + \frac{h}{2}\right) - \left(x_0 + \frac{h}{2}\right)\left(y_0 + \frac{k_2}{2}\right)^2}{3}\right)$$

$$= 0.1 \times \left(\frac{\sin\left(0 + \frac{0.1}{2}\right) - \left(0 + \frac{0.1}{2}\right)\left(5 + \frac{-0.042}{2}\right)^2}{3}\right)$$

$$= -0.412$$

$$k_4 = hf(x_0+h, y_0+k_3)$$

$$= 0.1 \times \left(\frac{\sin(x_0+h) - (x_0+h)(y_0+k_3)^2}{3} \right)$$

$$= 0.1 \times \left(\frac{\sin(0+0.1) - (0+0.1)(5-0.0412)^2}{3} \right)$$

$$=$$

Answer to the question no-2

Given that

$$V(t) = 55.8 \tanh(0.17t), \quad t \geq 0$$

The Runge-Kutta 4th order method is

$$S_{i+1} = S_i + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)h$$

$\therefore K_1 = f(t_i, S_i)$

$$K_2 = f\left(t_i + \frac{1}{2}h, S_i + \frac{1}{2}K_1 h\right)$$

$$K_3 = f\left(t_i + \frac{1}{2}h, S_i + \frac{1}{2}K_2 h\right)$$

$$K_4 = f(t_i + h, S_i + K_3 h)$$

for $i = 0$, $t_0 = 2$, $S_0 = 0$

$$K_1 = f(t_0, S_0)$$

$$= f(2, 0)$$

$$= 55.8 \tanh(0.17 \times 2)$$

$$= 18.2732$$

$$K_2 = f\left(t_0 + \frac{1}{2}h,\ S_0 + \frac{1}{2} K_1 h\right)$$

$$= f\left(2 + \frac{1}{2} \times 5,\ 0 + \frac{1}{2} \times 18.2732 \times 5\right)$$

$$= f(5.5, 45.683)$$

$$= 55.8 \tanh(0.17 \times 4.5)$$

$$= 35.9359$$

$$K_3 = f\left(t_0 + \frac{1}{2}h,\ S_0 + \frac{1}{2} K_2 h\right)$$

$$= f\left(2 + \frac{1}{2} \times 5,\ 0 + \frac{1}{2} \times 35.9359 \times 5\right)$$

$$= f(4.5, 89.8398)$$

$$= 55.8 \tanh(0.17 \times 4.5)$$

$$= 35.9359$$

$$k_4 = f(t_0 + h, s_0 + k_3 h)$$

$$= f(2+5,\ 0 + 35.9359 \times 5)$$

$$= f(7, 179.68)$$

$$= 55.8 \tanh(0.19 \times 7)$$

$$= 46.3463$$

$$s_1 = s_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$= 0 + \frac{1}{6}(18.2732 + 2 \times 35.9359 + 2 \times 35.9359 + 46.3463) \times 5$$

$$= 173.636$$

$$t_1 = t_0 + h$$

$$= 2 + 5$$

$$= 7$$

for $i=1$, $t_1=7$, $S_1=173.636$

$$k_1 = f(t_1, S_1)$$
$$= f(7, 173.636)$$
$$= 55.8 \tanh(0.17 \times 7)$$
$$= 46.3463$$

$$k_2 = f\left(t_1 + \tfrac{1}{2}h,\ S_1 + \tfrac{1}{2}k_1 h\right)$$
$$= f\left(7 + \tfrac{1}{2} \times 5,\ 173.636 + \tfrac{1}{2} \times 46.3463 \times 5\right)$$
$$= f(9.5,\ 289.502)$$
$$= 55.8 \tanh(0.17 \times 9.5)$$
$$= 51.5534$$

$$k_3 = f\left(t_1 + \tfrac{1}{2}h,\ S_1 + \tfrac{1}{2}k_2 h\right)$$
$$= f\left(7 + \tfrac{1}{2} \times 5,\ 173.636 + \tfrac{1}{2} \times 51.5534 \times 5\right)$$
$$= f(9.5,\ 302.52)$$
$$= 55.8 \tanh(0.17 \times 9.5)$$
$$= 51.5534$$

$$k_4 = f(t_1+h, S_1+k_3 h)$$

$$= f(7+5, 173.636 + 51.5534 \times 5)$$

$$= f(12, 431.403)$$

$$= 55.8 \tanh(0.17 \times 12)$$

$$= 53.9445$$

$$S_2 = S_1 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)h$$

$$= 173.636 + \frac{1}{6}(46.3463 + 2\times 51.5534 + 2\times 51.5534 + 53.9445)\times 5$$

$$= 429.05 \text{ m}$$

$$t_2 = t_1 + h$$

$$= 7+5$$

$$= 12$$

$\therefore$ Distance $= S_2 - S_0$

$$= S(t_2) - S(t_0)$$

$$= S(10) - S(2)$$

$$= 429.05 - 0$$

$$= 429.05 \text{ m}$$ Ans