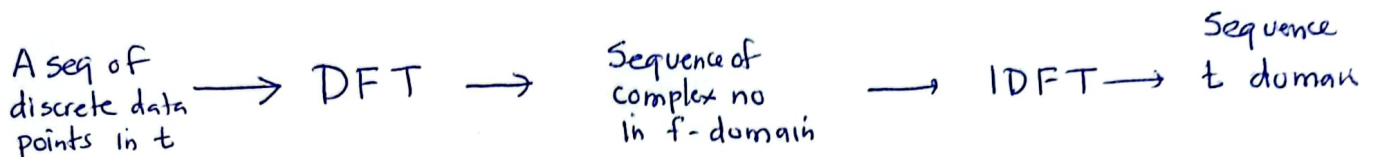


DFT : Discrete Fourier Transform and IDFT Inverse DFT are mathematical operations used in signal processing and image processing.



DFT and IDFT are computationally intensive especially for larger sequence. However, they are widely used for audio/video/image processing and data analytics. Various fast algorithm like Fast Fourier Transform (FFT) and IFFT are developed to efficiently compute DFT/IDFT and making them practical for real world application.

DFT Eqⁿ

$$x(n); 0 \leq n \leq N-1$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

Sequence in freq. domain \rightarrow Discrete time signal

$$= \sum_{n=0}^{N-1} x(n) W_N^{kn} \quad 0 \leq k \leq N-1$$

$$W_N = e^{-j \frac{2\pi}{N}}$$

is called phase factor or Twiddle factor

IDFT eqⁿ

$$x(n) = \sum_{k=0}^{N-1} X(k) e^{j \frac{2\pi}{N} kn}$$

$$= \sum_{k=0}^{N-1} X(k) W_N^{-kn}$$

Compute the N point DFT of the signal (a) $x(n) = \delta(n)$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

$$= \sum_{n=0}^{N-1} \delta(n) e^{-j \frac{2\pi}{N} kn}$$

$$= \delta(0) e^{-j \frac{2\pi}{N} \cdot k \cdot 0}$$

$$= \delta(0) = 1$$

$$\delta(n) = \begin{cases} 1 & n=0 \\ 0 & \text{else} \end{cases}$$

(b) $x(n) = \delta(n - n_0)$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

$$= \sum_{n=0}^{N-1} \delta(n - n_0) e^{-j \frac{2\pi}{N} kn}$$

$$= 1 e^{-j \frac{2\pi}{N} k n_0}$$

$$= e^{-j \frac{2\pi}{N} k n_0}$$

$$n - n_0 = 0$$

$$\therefore n = n_0$$

$$\delta(n - n_0) = \begin{cases} 1 & n = n_0 \\ 0 & \text{else} \end{cases}$$

(c) $x(n) = a^n \quad 0 \leq n \leq N-1$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

$$= \sum_{n=0}^{N-1} a^n a^n e^{-j \frac{2\pi}{N} kn}$$

$$= \sum_{n=0}^{N-1} \left(a e^{-j \frac{2\pi}{N} k} \right)^n$$

$$\sum_{n=0}^{N-1} a^n = \frac{a^N - a^{N+1}}{1 - a} \quad a \neq 1$$

$$X(k) = \frac{1 - [a e^{-j\frac{2\pi}{N}k}]^N}{1 - a e^{-j\frac{2\pi}{N}k}}$$

$$= \frac{1 - a^N \cdot 1}{1 - a e^{-j\frac{2\pi}{N}k}}$$