1.3.3 SEARCHING FOR SOLUTIONS

SEARCH TREE

Having formulated some **problems**,we now need to **solve** them. This is done by a **search** through the **state space**. A **search tree** is generated by the **initial state** and the **successor function** that together define the **state space**. In general,we may have a *search graph* rather than a *search tree*,when the same state can be reached from multiple paths.

Figure 1.23 shows some of the expansions in the search tree for finding a route from Arad to Bucharest.

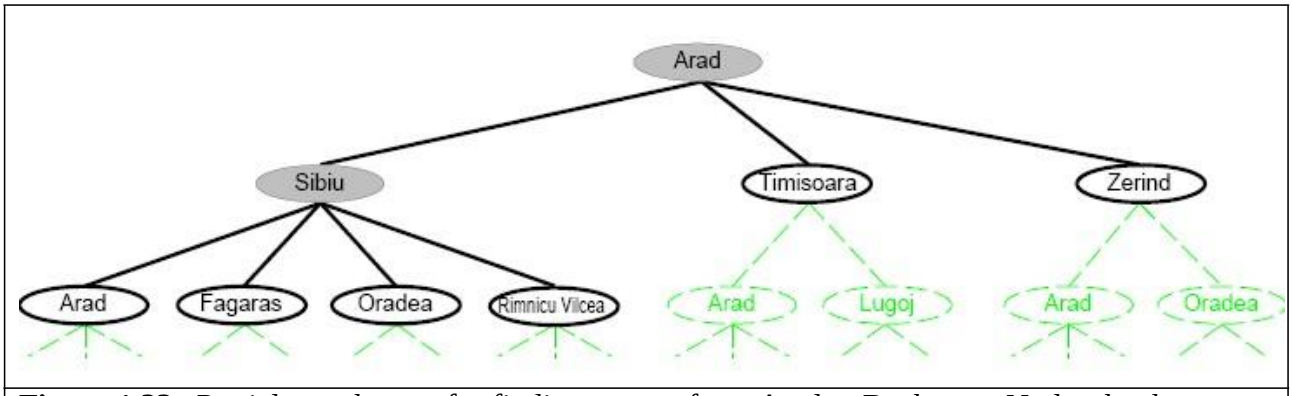

Figure 1.23 Partial search trees for finding a route from Arad to Bucharest. Nodes that have been expanded are shaded.; nodes that have been generated but not yet expanded are outlined in bold;nodes that have not yet been generated are shown in faint dashed line

The root of the search tree is a **search node** corresponding to the initial **state**,In(Arad). The first step is to test whether this is a **goal state**. The current state is expanded by applying the successor function to the current state,thereby generating a new set of states. In this case,we get three new states: In(Sibiu),In(Timisoara),and In(Zerind). Now we must choose which of these three possibilities to consider further. This is the essense of search- following up one option now and putting the others aside for latter,in case the first choice does not lead to a solution.

Search strategy . The general tree-search algorithm is described informally in Figure 1.24
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Tree Search

```
function TREE-SEARCH( problem, strategy) returns a solution, or failure
  initialize the search tree using the initial state of problem
  loop do
      if there are no candidates for expansion then return failure
      choose a leaf node for expansion according to strategy
      if the node contains a goal state then return the corresponding solution
      else expand the node and add the resulting nodes to the search tree
```

Figure 1.24 An informal description of the general tree-search algorithm

The choice of which state to expand is determined by the **search strategy**. There are an infinite number paths in this state space ,so the search tree has an infinite number of **nodes.**
A **node** is a data structure with five components :

- STATE : a state in the state space to which the node corresponds;
- PARENT-NODE : the node in the search tree that generated this node;
- ACTION : the action that was applied to the parent to generate the node;
- PATH-COST :the cost,denoted by g(n),of the path from initial state to the node,as indicated by the parent pointers; and
- DEPTH : the number of steps along the path from the initial state.

It is important to remember the distinction between nodes and states. A node is a book keeping data structure used to represent the search tree. A state corresponds to configuration of the world.

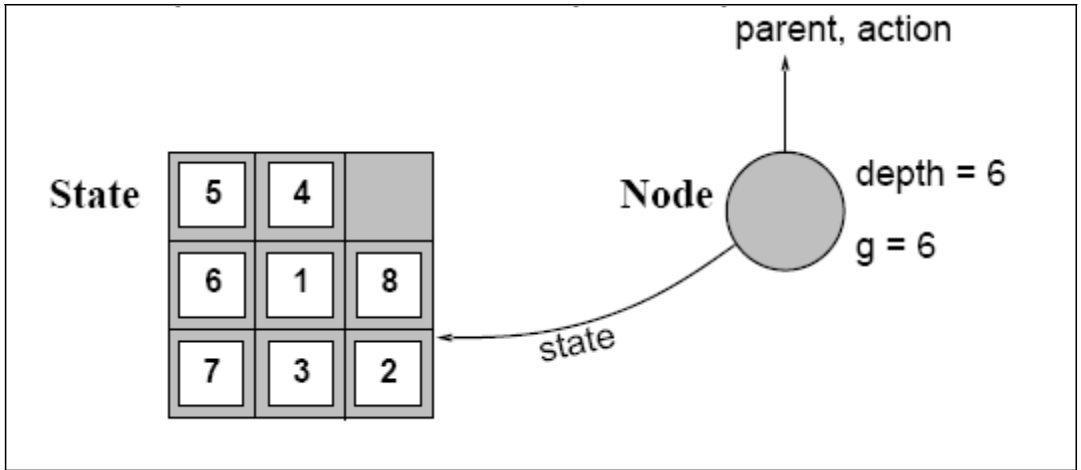

Figure 1.25 Nodes are data structures from which the search tree is constructed. Each has a parent,a state, Arrows point from child to parent.

Fringe

Fringe is a collection of nodes that have been generated but not yet been expanded. Each element of the fringe is a leaf node,that is,a node with no successors in the tree. The fringe of each tree consists of those nodes with bold outlines.
The collection of these nodes is implemented as a **queue.**
The general tree search algorithm is shown in Figure 2.9

```
function TREE-SEARCH(problem, fringe) returns a solution, or failure

  fringe ← INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
      if EMPTY?(fringe) then return failure
      node ← REMOVE-FIRST(fringe)
      if GOAL-TEST[problem] applied to STATE[node] succeeds
          then return SOLUTION(node)
      fringe ← INSERT-ALL(EXPAND(node, problem), fringe)

function EXPAND(node, problem) returns a set of nodes

  successors ← the empty set
  for each ⟨action, result⟩ in SUCCESSOR-FN[problem](STATE[node]) do
      s ← a new NODE
      STATE[s] ← result
      PARENT-NODE[s] ← node
      ACTION[s] ← action
      PATH-COST[s] ← PATH-COST[node] + STEP-COST(node, action, s)
      DEPTH[s] ← DEPTH[node] + 1
      add s to successors
  return successors
```

Figure 1.26 The general Tree search algorithm

The operations specified in Figure 1.26 on a queue are as follows:

- **MAKE-QUEUE(element,…)** creates a queue with the given element(s).
- **EMPTY?(queue)** returns true only if there are no more elements in the queue.
- **FIRST(queue)** returns FIRST(queue) and removes it from the queue.
- **INSERT(element,queue)** inserts an element into the queue and returns the resulting queue.
- **INSERT-ALL(elements,queue)** inserts a set of elements into the queue and returns the resulting queue.

MEASURING PROBLEM-SOLVING PERFORMANCE

The output of problem-solving algorithm is either failure or a solution.(Some algorithms might struck in an infinite loop and never return an output.
The algorithm's performance can be measured in four ways :

- **Completeness** : Is the algorithm guaranteed to find a solution when there is one?
- **Optimality** : Does the strategy find the optimal solution
- **Time complexity** : How long does it take to find a solution?
- **Space complexity** : How much memory is needed to perform the search?

1.3.4 UNINFORMED SEARCH STRATGES

Uninformed Search Strategies have no additional information about states beyond that provided in the **problem definition**.
Strategies that know whether one non goal state is ―more promising‖ than another are called **Informed search or heuristic search** strategies.

There are five uninformed search strategies as given below.

- Breadth-first search
- Uniform-cost search
- Depth-first search
- Depth-limited search
- Iterative deepening search

1.3.4.1 Breadth-first search

Breadth-first search is a simple strategy in which the root node is expanded first,then all successors of the root node are expanded next,then their successors,and so on. In general,all the nodes are expanded at a given depth in the search tree before any nodes at the next level are expanded.
Breath-first-search is implemented by calling TREE-SEARCH with an empty fringe that is a first-in-first-out(FIFO) queue,assuring that the nodes that are visited first will be expanded first. In otherwards,calling TREE-SEARCH(problem,FIFO-QUEUE()) results in breadth-first-search. The FIFO queue puts all newly generated successors at the end of the queue,which means that Shallow nodes are expanded before deeper nodes.

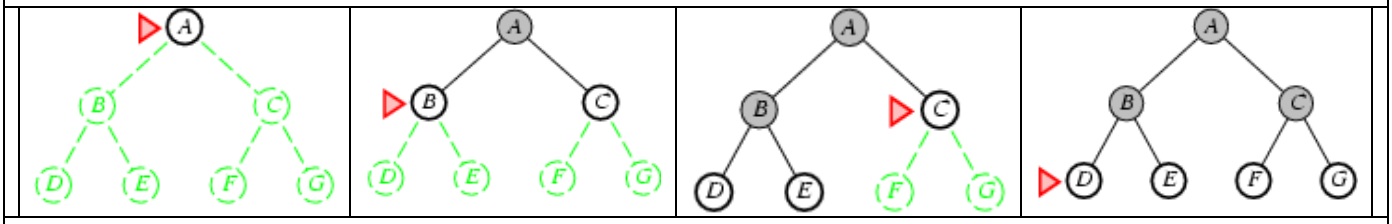

Figure 1.27 Breadth-first search on a simple binary tree. At each stage ,the node to be expanded next is indicated by a marker.

Properties of breadth-first-search

Complete?? Yes (if b is finite)

Time?? $1 + b + b^2 + b^3 + \ldots + b^d + b(b^d - 1) = O(b^{d+1})$, i.e., exp. in d

Space?? $O(b^{d+1})$ (keeps every node in memory)

Optimal?? No, unless step costs are constant

Space is the big problem; can easily generate nodes at 100MB/sec
so 24hrs = 8640GB.

Figure 1.28 Breadth-first-search properties

Time and Memory Requirements for BFS – $O(b^{d+1})$

Example:

- b = 10
- 10000 nodes/second
- each node requires 1000 bytes of storage

Depth	Nodes	Time	Memory
2	1100	.11 sec	1 meg
4	111,100	11 sec	106 meg
6	10^7	19 min	10 gig
8	10^9	31 hrs	1 tera
10	10^{11}	129 days	101 tera
12	10^{13}	35 yrs	10 peta
14	10^{15}	3523 yrs	1 exa

Figure 1.29 Time and memory requirements for breadth-first-search. The numbers shown assume branch factor of b = 10 ; 10,000 nodes/second; 1000 bytes/node

Time complexity for BFS

Assume every state has b successors. The root of the search tree generates b nodes at the first level,each of which generates b more nodes,for a total of b^2 at the second level. Each of these generates b more nodes,yielding b^3 nodes at the third level,and so on. Now suppose,that the solution is at depth d. In the worst case,we would expand all but the last node at level d,generating b^{d+1} - b nodes at level d+1.

Then the total number of nodes generated is

$b + b^2 + b^3 + \ldots + b^d + (b^{d+1} + b) = O(b^{d+1)}$.

Every node that is generated must remain in memory,because it is either part of the fringe or is an ancestor of a fringe node. The space compleity is,therefore ,the same as the time complexity