Representation of Graphs

There are two principal ways to represent a graph G with the matrix, i.e., adjacency matrix and incidence matrix representation.

(a) Representation of the Undirected Graph:

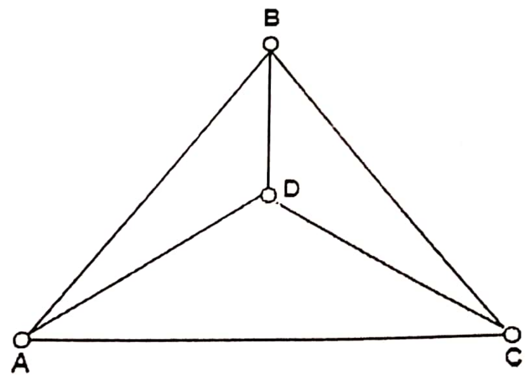
1. Adjacency Matrix Representation: If an Undirected Graph G consists of n vertices then the adjacency matrix of a graph is an n x n matrix A = $[a_{ij}]$ and defined by

$$a_{ij} = \begin{bmatrix} 1, \text{ if } \{V_i, V_j\} \text{ is an edge i.e., } v_i \text{ is adjacent to } v_j \\ 0, \text{ if there is no edge between } v_i \text{ and } v_j \end{bmatrix}$$

If there exists an edge between vertex v_i and v_j, where i is a row and j is a column then the value of $a_{ij}=1$.

If there is no edge between vertex v_i and v_j, then value of $a_{ij}=0$.

Example: Find the adjacency matrix M_A of graph G shown in Fig:



Solution:

Since graph G consist of four vertices. Therefore, the adjacency matrix is a 4 x 4 matrix. The adjacency matrix is as follows in fig:

$$M_A = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 1 & 1 & 1 \\ B & 1 & 0 & 1 & 1 \\ C & 1 & 1 & 0 & 1 \\ D & 1 & 1 & 1 & 0 \end{array}$$

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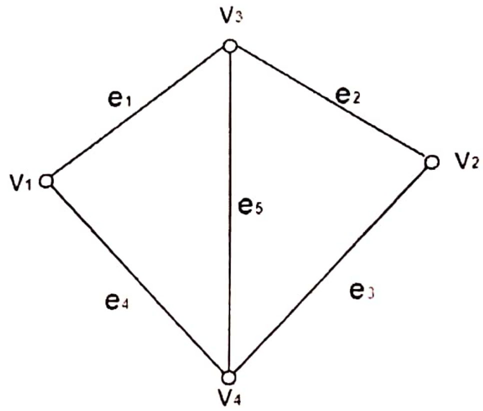
2. Incidence Matrix Representation: If an Undirected Graph G consists of n vertices and m edges, then the incidence matrix is an n x m matrix C = [c_{ij}] and defined by

$$c_{ij} = \begin{bmatrix} 1, \text{if the vertex } V_i \text{ incident by edge } e_j \\ 0, \text{otherwise} \end{bmatrix}$$

There is a row for every vertex and a column for every edge in the incident matrix.

The number of ones in an incidence matrix of the undirected graph (without loops) is equal to the sum of the degrees of all the vertices in a graph.

Example: Consider the undirected graph G as shown in fig. Find its incidence matrix M_1.



Solution:

The undirected graph consists of four vertices and five edges. Therefore, the incidence *matrix is a 4 x 5 matrix,* which is shown in Fig:

$$M_1 = \begin{array}{c|ccccc} & e_1 & e_2 & e_3 & e_4 & e_5 \\ \hline V_1 & 1 & 0 & 0 & 1 & 0 \\ V_2 & 0 & 1 & 1 & 0 & 0 \\ V_3 & 1 & 1 & 0 & 0 & 1 \\ V_4 & 0 & 0 & 1 & 1 & 1 \end{array}$$

(b) Representation of Directed Graph:

1. Adjacency Matrix Representation: If a directed graph G consists of n vertices then *the adjacency matrix* of a graph is an n x n matrix A = [a_{ij}] and defined by

$$a_{ij} = \begin{bmatrix} 1, \text{if } \{V_i, V_j\} \text{is an edge i.e., } V_i \text{is initial vertex and } V_j \text{ is the final vertex} \\ 0, \text{if there is no edge between } V_i \text{ and } V_j \end{bmatrix}$$

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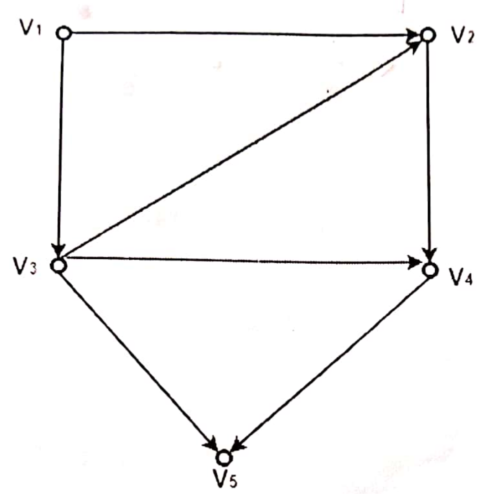
If there exists an edge between vertex V_i and V_j, with V_i as initial vertex and V_j as a final vertex, then the value of $a_{ij}=1$.

If there is no edge between vertex V_i and V_j, then the value of $a_{ij}=0$.

The number of ones in the adjacency matrix of a directed graph is equal to the number of edges.

Example: Consider the directed graph shown in fig. Determine its adjacency matrix M_A.



Solution:

Since the directed graph G consists of five vertices. Therefore, the adjacency matrix will be a 5 x 5 matrix. The adjacency matrix of the directed graphs is as follows:

$$M_A = \begin{array}{c|ccccc} & V_1 & V_2 & V_3 & V_4 & V_5 \\ \hline V_1 & 0 & 1 & 1 & 0 & 0 \\ V_2 & 0 & 0 & 0 & 1 & 0 \\ V_3 & 0 & 1 & 0 & 1 & 1 \\ V_4 & 0 & 0 & 0 & 0 & 1 \\ V_5 & 0 & 0 & 0 & 0 & 0 \end{array}$$

2. Incidence Matrix Representation: If a directed graph G consists of n vertices and m edges, then the incidence matrix is an n x m matrix C = $[c_{ij}]$ and defined by

$$c_{ij} = \begin{bmatrix} 1, \text{ if } V_i \text{ is the initial vertex of edge } e_j \\ -1, \text{ if } V_i \text{ is the final vertex of edge } e_j \\ 0, V_i \text{ is not incident on edge } e_j \end{bmatrix}$$

The number of ones in an incidence matrix is equal to the number of edges in the graph.

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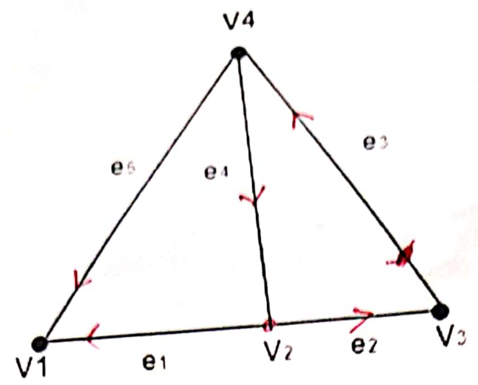
Example: Consider the directed graph G as shown in fig. Find its incidence matrix M_I.



Solution:

The directed graph consists of four vertices and five edges. Therefore, the incidence matrix is a 4 x 5 matrix which is show in fig:

$$M_I = \begin{array}{c|ccccc} & e_1 & e_2 & e_3 & e_4 & e_5 \\ \hline V_1 & -1 & 0 & 0 & 0 & -1 \\ V_2 & 1 & 1 & 0 & -1 & 0 \\ V_3 & 0 & -1 & 1 & 0 & 0 \\ V_4 & 0 & 0 & -1 & 1 & 1 \end{array}$$

(c) Representation of Multigraph:

Represented only by adjacency matrix representation.

(i)Adjacency matrix representation of multigraph: If a multigraph G consists of vertices, then the adjacency matrix of graph is an n x n matrix A = $[a_{ij}]$ and is defined by

$$a_{ij} = \begin{bmatrix} N, \text{ If there are more than one edges between vertex } v_i \text{ and } v_j, \text{ where} \\ N \text{ is the number of edges.} \\ 0, \text{ Otherwise.} \end{bmatrix}$$

If there exist one or more than one edges between vertex v_i and v_j then $a_{ij}=N$, where is the number of edges between v_i and v_j.

If there is no edge between v_i and v_j.

Example: Consider the multigraph shown in Fig, Determine its adjacency matrix.

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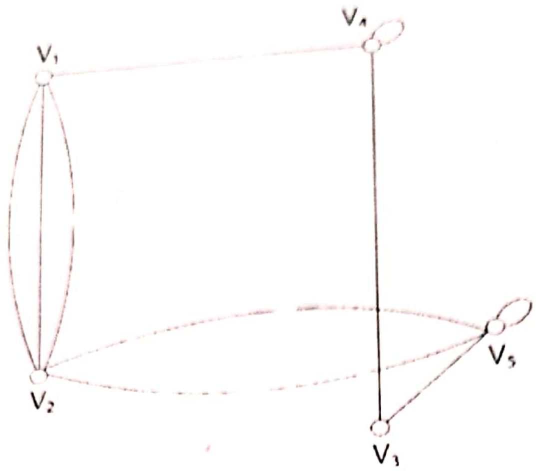

Solution:

Since the multigraph consist of five vertices. Therefore the adjacency matrix will be an 5 x 5 matrix. The adjacency matrix of the multigraph is as follows:

$$M_A = \begin{array}{c|ccccc} & V_1 & V_2 & V_3 & V_4 & V_5 \\ \hline V_1 & 0 & 3 & 0 & 0 & 1 \\ V_2 & 3 & 0 & 0 & 0 & 2 \\ V_3 & 0 & 0 & 0 & 1 & 1 \\ V_4 & 1 & 0 & 1 & 1 & 0 \\ V_5 & 0 & 2 & 1 & 0 & 1 \end{array}$$

Loop = 1

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