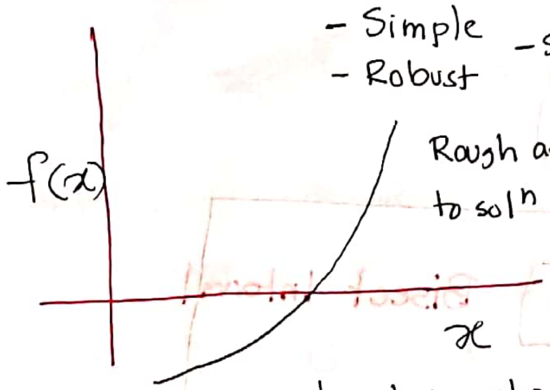
Bisection Method

- Simple
- Robust
- Slow

- Root finding method which applies to continous function for which one knows two values with opposite sign



The method consisting of repeatedly bisecting the interval defined by these values and then selecting the sub interval in which f(x) changes sign and therefore must contain a root

Step 1 Choose x_l and x_r as the initial guesses such that $f(x_l)\, f(x_r) < 0$

Step 2 Find x_m where $x_m = \dfrac{x_l + x_r}{2}$

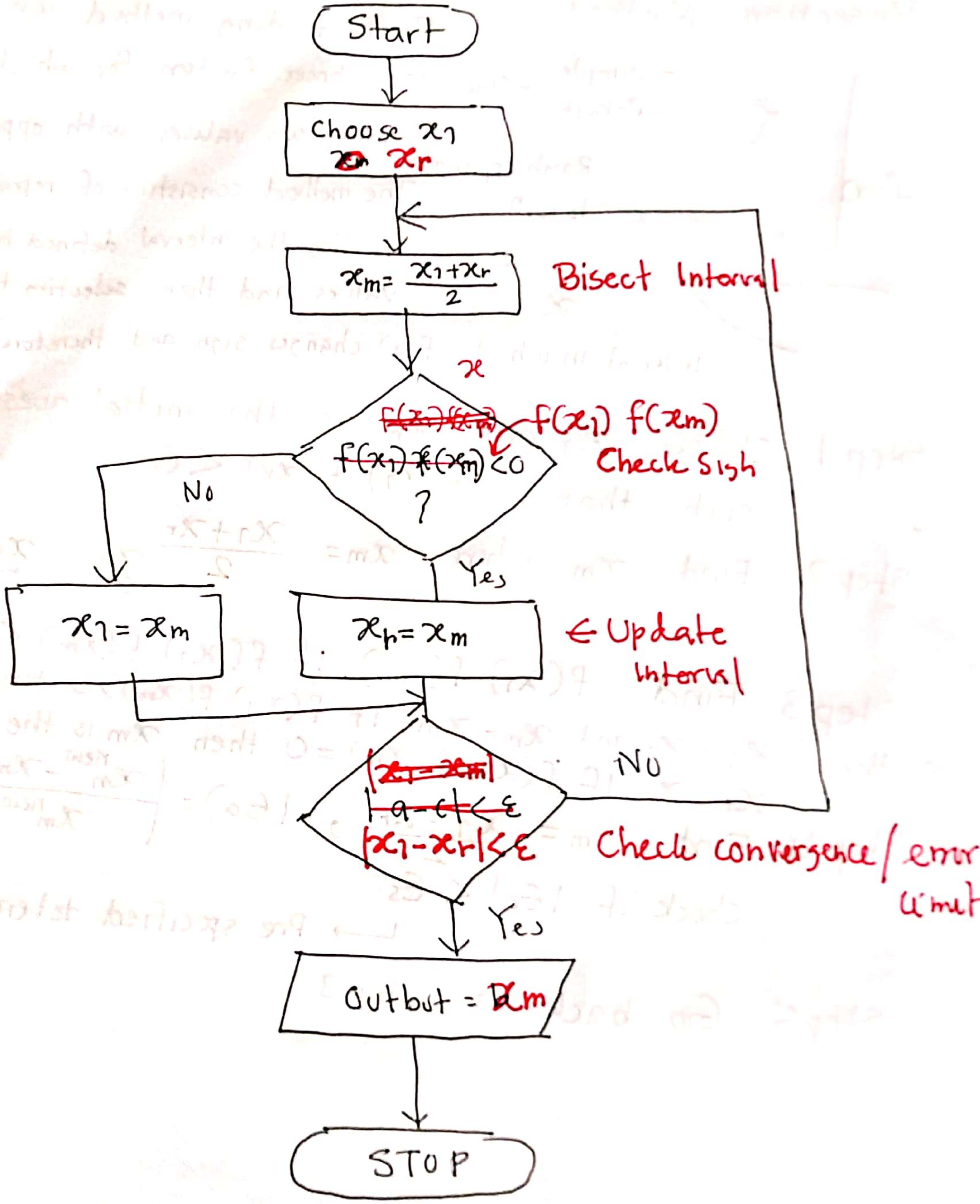
$x_l \quad x_m \quad x_r$

Step 3 Find $f(x_l)\, f(x_m)$. If $f(x_l)\, f(x_m) < 0$ then $x_l = x_l$ and $x_r = x_m$. If $f(x_l)\, f(x_m) > 0$ then $x_l = x_m$, $x_r = x_r$. If $f(x_l) \cdot f(x_m) = 0$ then x_m is the root $\boxed{\text{STOP}}$

Step 4 Find $x_m = \dfrac{x_l + x_r}{2}$, $|\epsilon_a| = \left| \dfrac{x_m^{new} - x_m^{old}}{x_m^{new}} \right| \times 100$

Check if $|\epsilon_a| < \epsilon_s$ ⟶ Pre specified tolerence

Step 5 Go back to step 3


Start
Choose x_l x_r
$x_m = \frac{x_l + x_r}{2}$
Bisect Interval
$f(x_l)\, f(x_m) < 0$?
Check Sign
No
Yes
$x_l = x_m$
$x_r = x_m$
← Update Interval
$|x_l - x_r| < \varepsilon$
No
Check convergence / error limit
Yes
Output = x_m
STOP

Question Find a real root of $x^3 - x = 1$ using Bisection method

Solution: Given $x^3 - x = 1$

$$\Rightarrow x^3 - x - 1 = 0 = f(x)$$

$f(0) = 0$ $f(1) = -1$ $f(2) = 5$

Thus $x_l = 1$ $x_r = 2$

$f(x_m)$ - $-ve$
$x_m = x_l$

$f(x_m)$. $+ve$
$x_m = x_r$

~~a~~ x_l	~~b~~ x_r	~~f(a)~~ $f(x_l)$	$f(x_r)$	$x_m = \frac{x_l + x_r}{2}$	$f(x_m)$
1	2	-1	5	$\frac{1+2}{2} = 1.5$	0.875
1	1.5	-1	0.875	1.25	-0.2968
1.25	1.5	-0.2968	0.875	1.375	0.2246
1.25	1.375	-0.2968	0.2246	1.3125	-0.0515
1.3125	1.375	-0.0515	0.2246	1.3437	0.082
1.3125	1.3437	-0.0515	0.0829	1.3281	-0.014
1.3125	1.3281	-0.0515	0.0144	1.3203	-0.018
1.3203	1.3281	-0.0187	0.0144	1.3242	-0.2075
1.3242	1.3281	-0.2078	0.0144	1.3261	0.0059
1.3242	1.3261	-0.2078	0.0059	1.3251	0.0016
1.3242	1.3251	-0.2078	0.0016	1.3246	0.000

Root 1.324