"The reverberation time is the time that sound takes to fall to one millionth of its original intensity"

It is mathematically given as.....

$$I_t = I_0 \times 10^{-6} \quad \underline{OR} \quad \frac{I_t}{I_0} = 10^{-6}$$

Where, I_0 = Original Intensity

I_t = decreased Intensity.

Average reverberation is about one second.

The exponential decrease in this intensity is due to the absorption of sound. The sound waves are absorbed by the surrounding so the intensity falls exponentially.

According to the difinition of absorption co-efficient,

$$A = \frac{I_A}{I_0} \quad --- (1)$$

where A = Absorption co-efficient
I_A = absorbed intensity
I_0 = Original intensity.

$\therefore I_A = A \cdot I_0$ (I_R = Reflected Intensity)

$\therefore I_R = I_0 - AI_0$

$= I_0(1-A) \quad --- (2)$

The sound waves is partially absorbed and partially reflected for a second absorption. so the intensity of the first reflected wave is I_{R_1}.
For second reflected wave, reflected intensity becomes I_{R_2}.

$\therefore I_{R_2} = I_0(1-A)^2$ $\left[\because A = \frac{I_{A_2}}{I_0(1-A)} \Rightarrow I_A = AI_0(1-A)\right.$

so, $I_{R_2} = I_0(1-A) - AI_0(1-A)$
$\left.= I_0(1-A)^2\right]$

Similarly,

$I_{R_3} = I_0(1-A)^3$

$I_{R_4} = I_0(1-A)^4$ And so on...

Suppose there are n reflections before the sound becomes 10^{-6} times its original intensity

$\therefore I_{Rn} = I_0(1-A)^n$ ---(3)

$\therefore I_0 \times 10^{-6} = I_0(1-A)^n$

$\therefore (1-A)^n = 10^{-6}$ ---(4)

taking nature log on both sides.

$\therefore n \log_e(1-A) = \log_e 10^{-6}$

Here, $A << 1$, so we can expand $\log_e(1-A)$ as follows:-

$\log_e(1-A) = -A - \frac{A^2}{2} - \frac{A^3}{3}$ .... (by series expansion)

But as $A << 1$, we can ~~expand~~ neglect A^2, A^3 ... as $A^2 \approx A^3 \approx 0$

$\therefore n(-A) = \log_e 10^{-6}$

$\therefore -nA = 2.303 \times (-6) \log_{10} 10$

$\therefore -nA = -2.303 \times 6$

$\therefore nA = 2.303 \times 6 = 13.82$

$\therefore A = \frac{1}{n} \times 13.82$ ---(5)

Thus, We have an equation for calculating the absorption co-efficient A.

$A = \frac{1}{n} \times 13.82$

If the number of reflections before the fall in intensity to 10^{-6} times is known, the absorption coefficient can be calculat

Now, average distance travelled by sound or in other words, the difference between two reflections is given by,

$$d = \frac{4V}{S} \quad \cdots\cdots (6)$$

where, V = Volume of the room
S = Total surface area of the room

now, velocity $= \frac{dist}{time} = \frac{d}{t} = \frac{4V}{St}$

$\therefore t = \frac{4V}{S \times velocity}$ (let velocity = u)

$$\therefore t = \frac{4V}{Su} \quad \cdots\cdots (7)$$

So, for n reflections, time will be nxt
i.e. $\frac{4nV}{Su}$.

$$\therefore t_n = \frac{4nV}{Su}$$

$$\therefore n = \frac{t_n Su}{4V} \quad \cdots\cdots (8)$$

Substituting the value of n from above eqn. to eqn (4), we get

$$(1-A)^{\frac{t_n Su}{4V}} = 10^{-6}$$

taking loge on both sides, we get,

$$\frac{t_n S \cdot u}{4V} \log_e (1-A) = -2.303 \times 6 \log_{10} 10 \quad \text{---(9)}$$

$$\therefore \log_e (1-A)\, t_n = -\frac{2.303 \times 6 \times 4V}{S \cdot u}$$

Expanding the l.h.s. of above eqn. we have the following result.

$$\left\{-A - \frac{A^2}{2} - \frac{A^3}{3} \cdots \right\} t_n = -\frac{2.303 \times 6 \times 4V}{S \cdot U}$$

but $A << I$, so $-A^2, -A^3 \approx 0$ so, we can neglect them.

$$\therefore -A \cdot t_n = -\frac{2.303 \times 6 \times 4V}{S \cdot U}$$

$$\therefore t_n = \frac{2.303 \times 6 \times 4}{U} \cdot \frac{V}{S \cdot A}$$

$$\approx \frac{0.05V}{S \cdot A} \qquad \boxed{V = 1100}$$

Here, S is the total surface area and A is the absorption coefficient.

$$\therefore t_n = T = \frac{0.05 \times V}{\Sigma A_i S_i}$$

$$\boxed{\therefore T = \frac{0.05 \times V}{\Sigma a_i S_i}} \quad \text{----(10)}$$