IIR Filter

- $y(n)$ depends on $x(n), x(n-1), y(n-1)$

- Difference Eq^n

$$y(n) = b_0 x(n) + b_1 x(n-1) + \dots + b_M (x-M) - a_1 y(n-1) - \dots - a_N y(n-N)$$

- Transfer func

$$H(z) = \frac{b_0 + b_1 z^{-1} + \dots + b_M z^{-M}}{1 + a_1 z^{-1} + \dots + a_N z^{-N}}$$

Poles - inside the unit circle → stable

- Smaller Filter size

- Linear phase is not easy to obtain

- Objectives : Find $b_0 \dots b_M$ $a_1 \dots a_N$ for filter specification

- Adv: Easy design and implementation

- Disadvantage: Non Linear, Not stable, Infinite impulse response.

Bilinear Transformation Design

Digital Filter Specifications

↓ 1. Transformation with freq warping

Analog filter specifications

↓ 2. Transformation by LP prototype filter

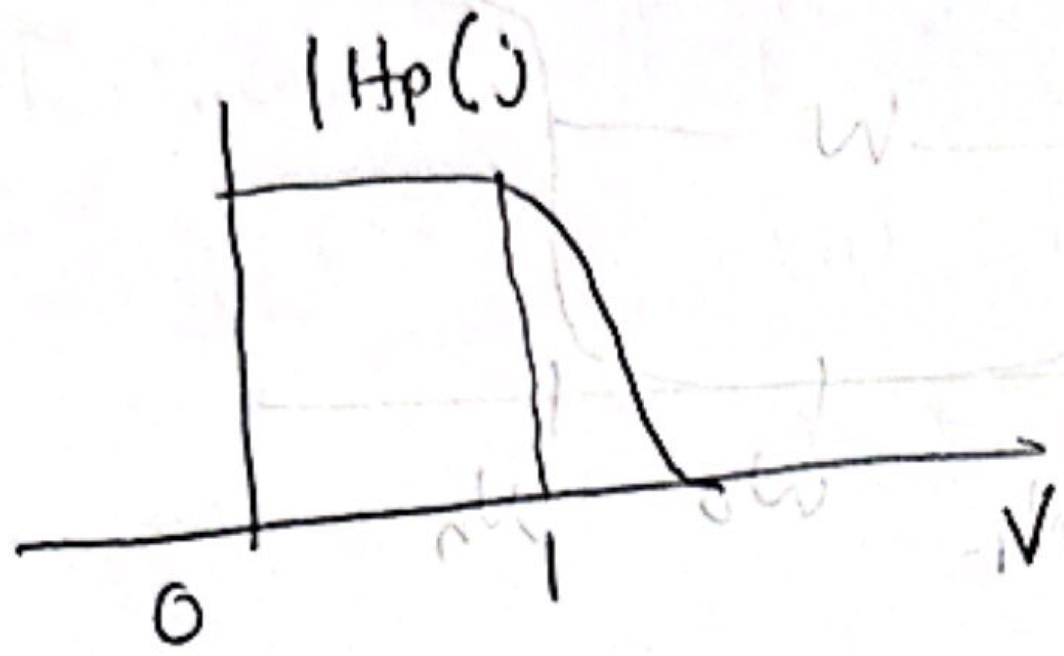
Analog filter transfer func

↓ 3. Bilinear Transformation

Digital Filter transfer func and freq. response verification

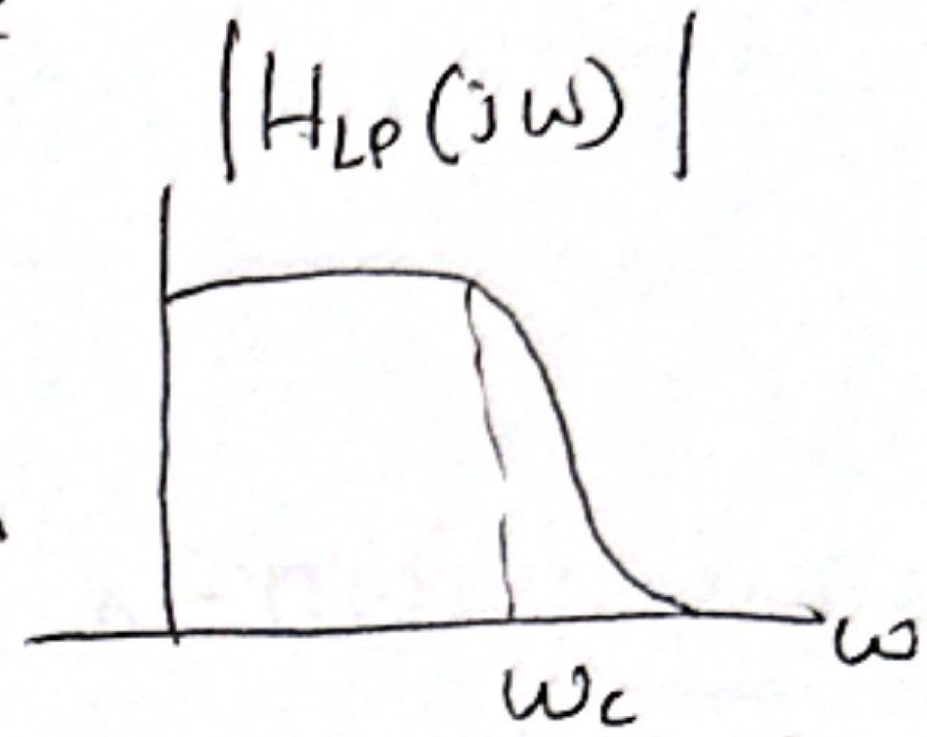
Analog filter with Lowpass prototype

(I) Low pass prototype into LPF



Normalized freq

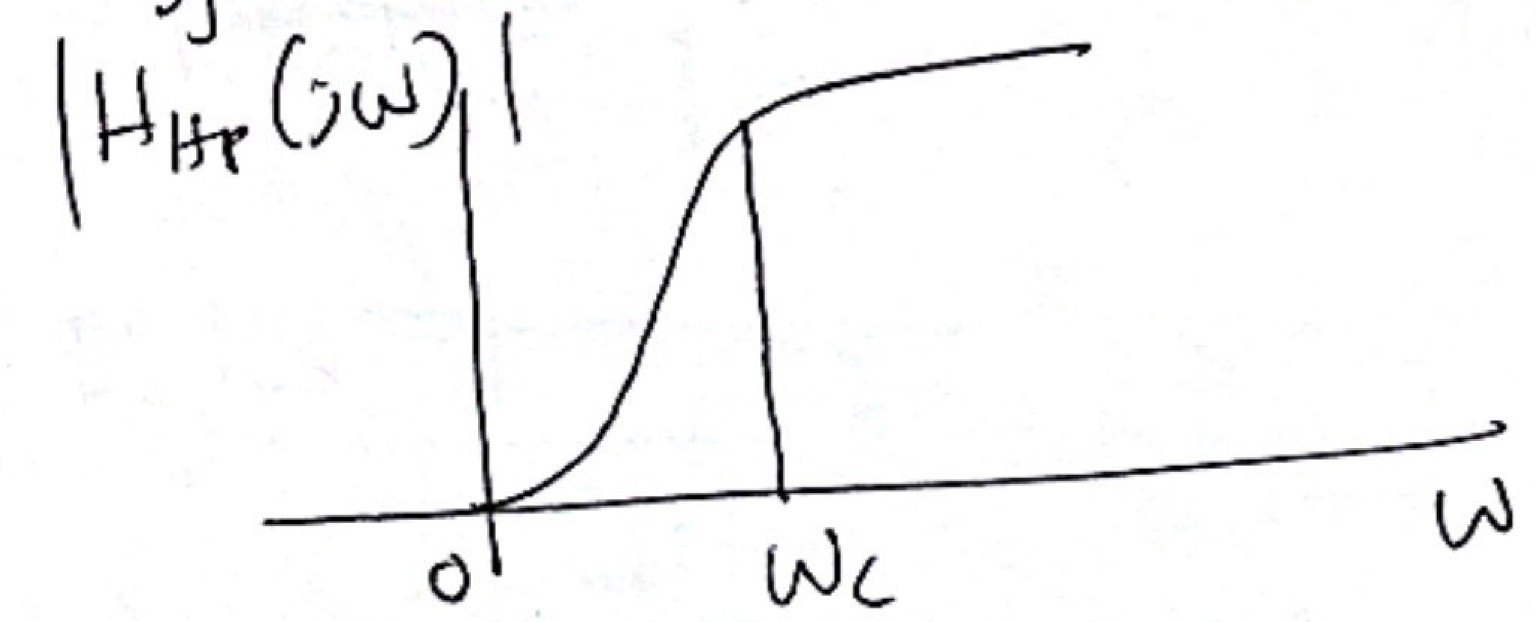
$$s = \frac{s}{w_c}$$



$W_c \rightarrow$ cutoff freq

$$H_{LP}(s) = H_p(s) \Big|_{s = \frac{s}{W_c}}$$

(II) LP prototype into HPF



$$H_{HP}(s) = H_p(s) \Big|_{s = \frac{W_c}{s}}$$

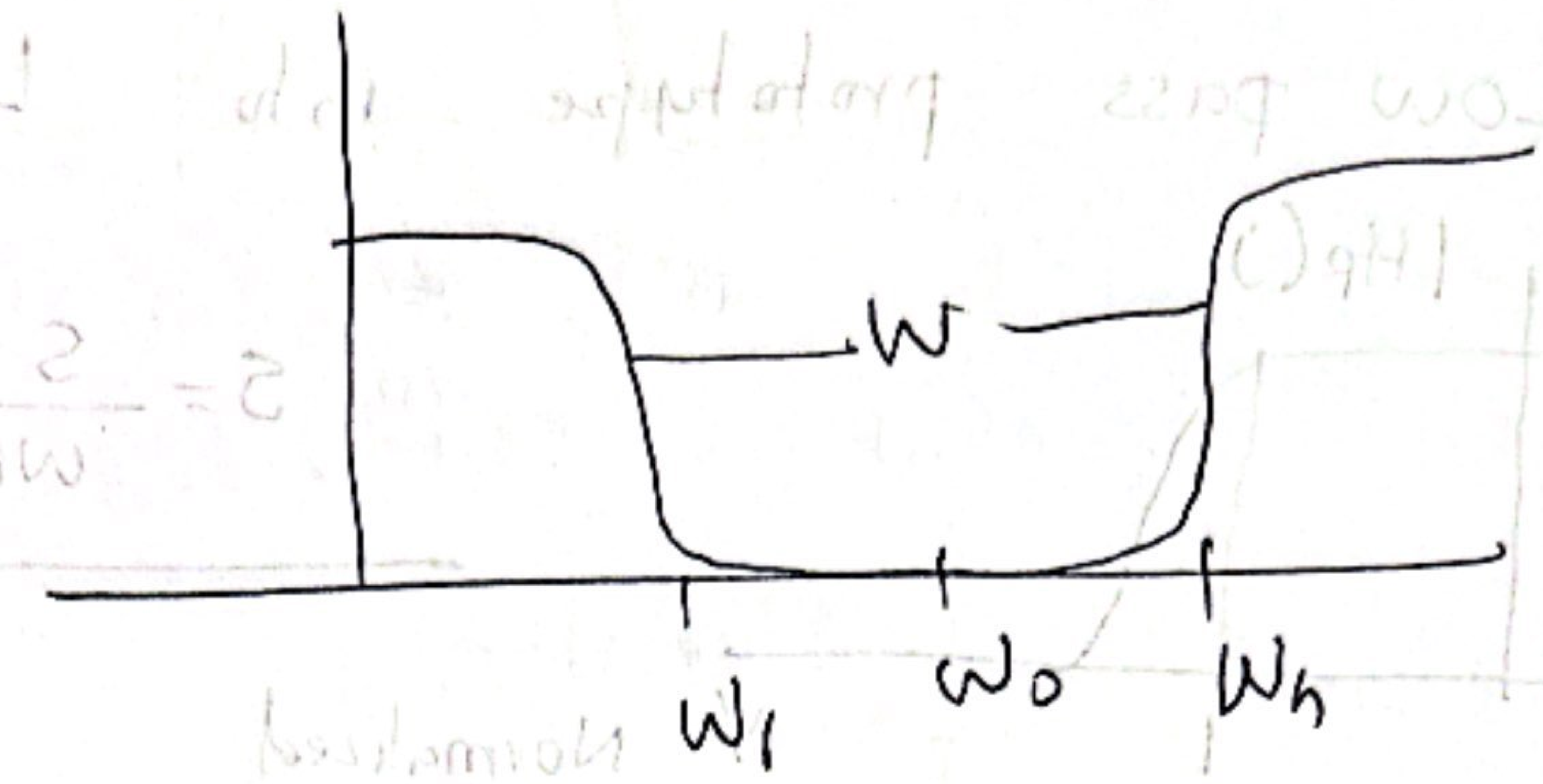
(III) LP prototype into BPF

Geometric centre freq

$$W_0 = \sqrt{W_L \times W_h}$$

$$H_{BP}(s) = H_p(s) \Big|_{s = \frac{s^2 + W_0^2}{sW}}$$

④ Band stop / reject



$$H_{BS}(s) = H_p(s) \Big|_{s = \frac{sW}{(s^2 + W_0)^2}}$$

Given a LP prototype $H_p(s) = \frac{1}{s+1}$ Determine each of the following analog filter and plot their magnitude response from 0 to 200 rad/sec

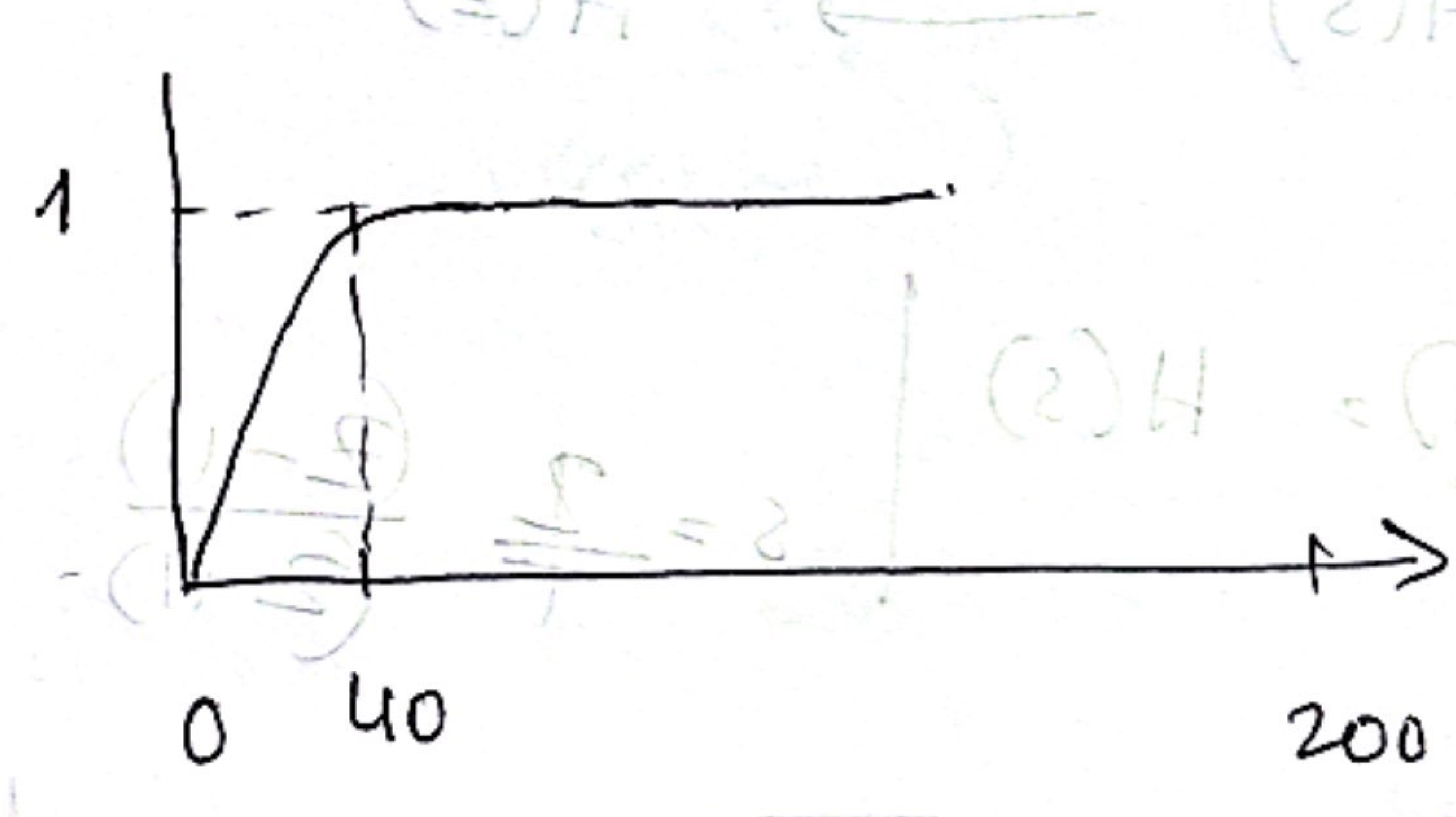
(i) A HPF with $W_c = 40$ rad/sec

(ii) A BPF with $W_c = 100$ rad/sec of BW of 20 rad/sec

(i) High pass filter

$$H_{HP}(s) = \frac{1}{s+1} \Big|_{s = \frac{W_c}{s} = \frac{40}{s}}$$

$$H_{HP}(s) = \frac{1}{\frac{40}{s}+1} = \frac{s}{s+40}$$



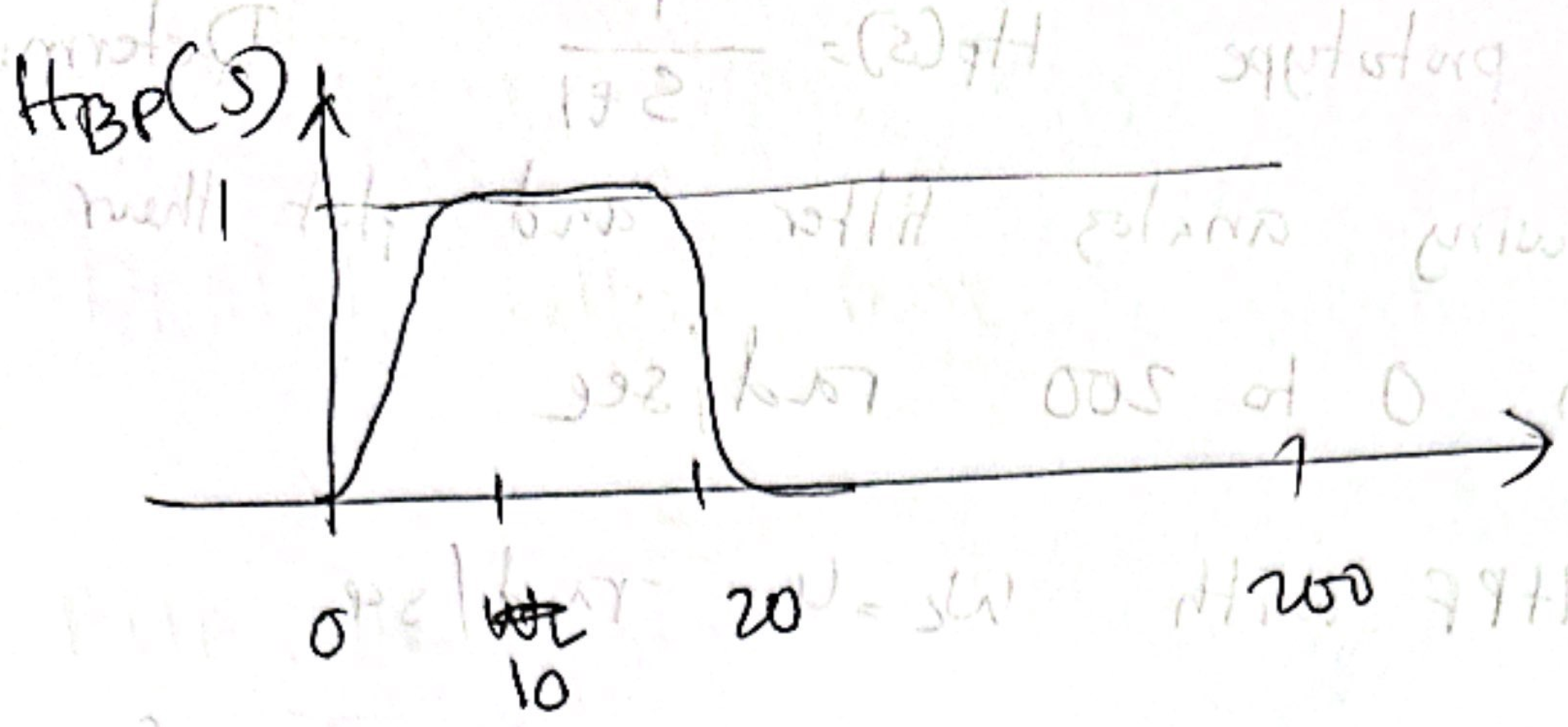
(ii) Band pass filter

$$W_0 = \sqrt{W_L * W_h} = \sqrt{100} \Rightarrow W_0 = 10 \text{ rad/sec}$$

$$W = W_h - W_L = 20 \text{ rad/sec}$$

$$H_p(s) = \frac{1}{s+1} \Big|_{s = \frac{s^2 + W_0^2}{sW}} = \frac{s^2+100}{s20}$$

$$H_{BP}(s) = \frac{1}{\frac{s^2+100}{20s}+1} = \frac{20s}{s^2+20s+100}$$



Bilinear Transformation and Freq wraping

Analog Filter Transfer function

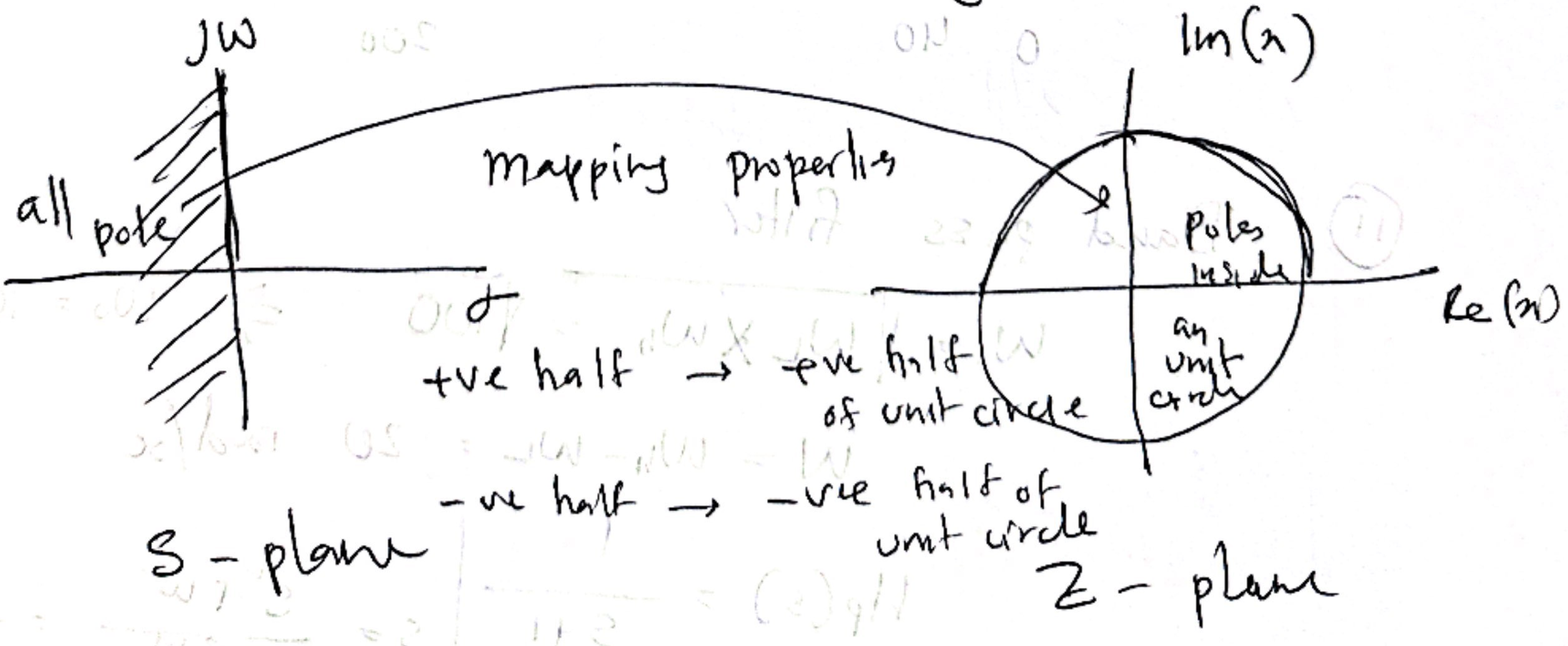
$\downarrow$

Digital Filter Transfer function

$$H(s) \longrightarrow H(z)$$

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\frac{(z-1)}{(z+1)}}$$

$T \rightarrow$ Sampling Period



Stable: Left in s plane and inside unit circle

Problem

$H(s) = \frac{10}{s+10}$ Convert it to digital filter transfer func and difference eq^n when the sampling period is $T = 0.01$ sec

Using Bilinear Transformation

$$H(z) = H(s)\Big|_{s = \frac{2}{T}\frac{z-1}{z+1}}$$

$$= \frac{10}{s+10}\Big|_{s = \frac{2}{0.01}\frac{z-1}{z+1}}$$

$$= \frac{10}{\frac{2(z-1)}{0.01(z+1)} + 10} = \frac{0.05}{\frac{(z-1)}{(z+1)} + 0.05}$$

$$= \frac{0.05z + 0.05}{\underbrace{1.05z}_{\text{it shall be 1}} - 0.95}$$

$$= \frac{(0.05z + 0.05)/1.05z}{(1.05z - 0.95)/1.05z}$$

$$H(z) = \frac{0.0476 + 0.0476z^{-1}}{1 - 0.9048z^{-1}}$$

$$Y(z) = 0.0476\,X(z) + 0.0476\,z^{-1}X(z) + 0.09048\,z^{-1}Y(z)$$

$$y(n) = 0.0476\,x(n) + 0.0476\,x(n-1) + 0.9048\,y(n-1)$$

Bilinear Transformation Design Procedure

(1) Given, Digital filter freq Specification $\longrightarrow$ Pre wrap to Analog freq Specification

Lp and HP Filters

$$W_a = \frac{2}{T} \tan\left(\frac{W_d T}{2}\right) \quad T \text{ is sampling period}$$

Bp and BS filter

$$W_a = \frac{2}{T} \tan\left(\frac{W_l T}{2}\right) \qquad W_{ah} = \frac{2}{T} \tan\left(\frac{W_h T}{2}\right)$$

$$W_0 = \sqrt{W_{al} W_{ah}} \qquad W_{ah} = W_{ah} - W_{al}$$

(2) Perform the prototype transformation using LP prototype

from LP $\rightarrow$ LP $\qquad H(s) = H_p(s) \Big|_{s = \frac{s}{W_a}}$

From LP $\rightarrow$ HP $\qquad H(s)\ H_p(s) \Big|_{s = \frac{W_a}{s}}$

From LP $\rightarrow$ BP $\qquad H(s) = H_p(s) \Big|_{s = \frac{s^2 + W_0^2}{sW}}$

From LP $\rightarrow$ BS $\qquad H_s(s) = H_p(s) = \Big|_{s = \frac{sw}{s^2 + w^2}}$