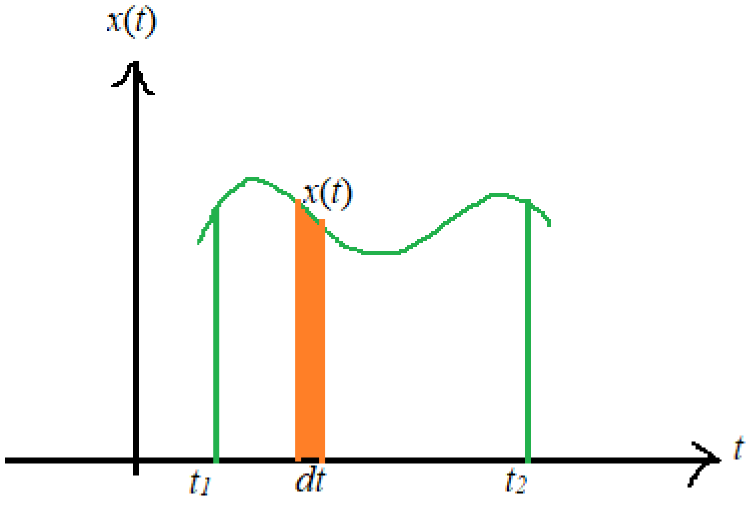
Average Value of CTS Energy, Power, NENP and orthogonal Signals

IT3105: Signals and Systems

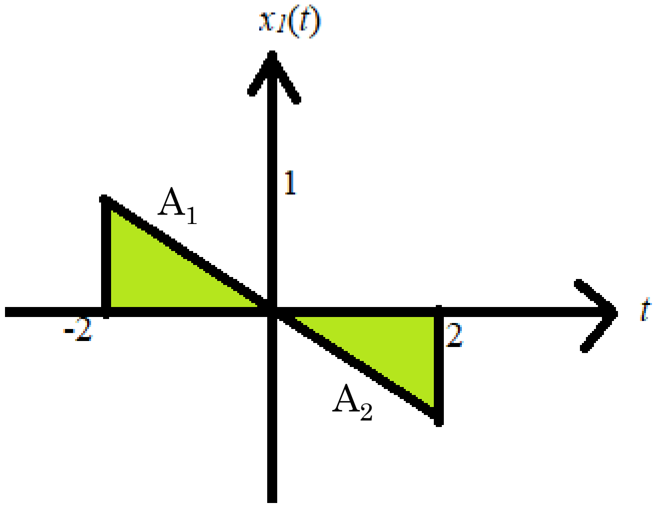
Area of CTS

- Consider any CTS $x(t)$, then the area of that signal is $\int_{-\infty}^{\infty} x(t)dt$
- If $x(t)$ exists between the interval t_1 and t_2 then area, A = $\int_{t_1}^{t_2} x(t)dt$; where $t_1 < t_2$ and $x(t) = 0$ for $t < t_1$ *and* $t > t_2$
- To calculate the area of given signal $x(t)$, let us consider a very rectangle with width dt and height $x(t)$, then Area= $x(t) \times dt$, summation of these small area of rectangles will give us the total area of $x(t)$. Such summations can be represented as the integration $\int_{t_1}^{t_2} x(t)dt$

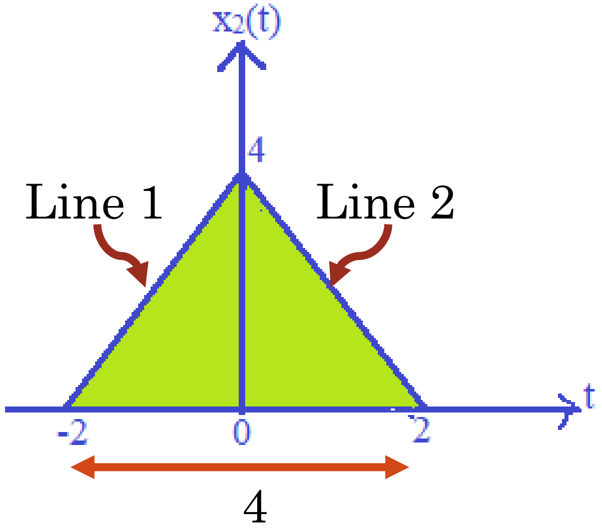


Example of Area measurements

- Area of $x_1(t) = \int_{-2}^{2} x_1(t)dt = A + (-A) = 0$
  - According to fig. area is same but opposite in sign.
  - As we the area of any triangle, $A_1 = \frac{1}{2} \times 2 \times 1 = 1$ and $A_2 = \frac{1}{2} \times 2 \times (-1) = -1$
  - By analyzing the fig, no calculation needed in such cases.



- Area of $x_2(t) = \int_{-2}^{2} x_2(t)dt = \int_{-2}^{0} x_2(t)dt + \int_{0}^{2} x_2(t)dt$
  - According the equation of straight lines we can rewrite the above areas as:
  - $\int_{-2}^{2} x_2(t)dt = \int_{-2}^{0}(2t + 4)dt + \int_{0}^{2}(-2t + 4)dt$
  - After integration and putting limits we get the result as area of $x_2(t)$= 8
  - We can cross check the result by using the law of area of triangles. Area= ½ × 4 × 4 = 8

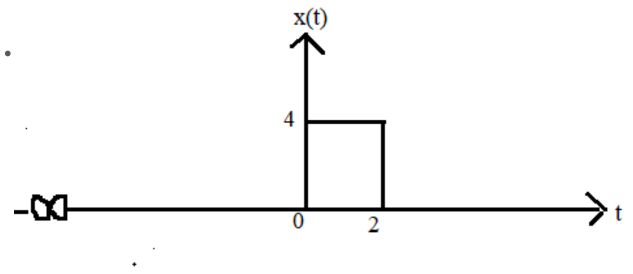


Average Value of CTS

- Average value of CTS = $\frac{total\ area}{total\ time}$
- For periodic signal, (Area) $A = \int_{T_0} x(t)dt$; where T_0 is fundamental period.
- Average value of periodic signals = $\frac{1}{T_0}\int_{T_0} x(t)dt$
- Average value of non-periodic signals= $\lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)dt$
  - Why the limit is (-T/2) to (T/2)?
  - For any non-periodic signal, the average value would be for time T and from - ∞ to ∞, the formula of average will be $\lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)dt$; when we put the limit - ∞ to ∞ over the limit 0 to T we get, 0×(-∞) = 0 and T ×(∞)= ∞. If we do the integration from 0 to ∞, then we only get the half of the signal.

Energy Signal

- Any signal is said to be energy signal of the total power is finite,
- Let us consider the finite duration signal x(t), Energy $E = \int_{-\infty}^{\infty} |x(t)|^2 dt$
- $E = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{0} |x(t)|^2 dt + \int_{0}^{2} |x(t)|^2 dt + \int_{2}^{t} |x(t)|^2 dt = \int_{0}^{2} 16dt$=32J (finite)
- Avg. value of finite duration signal = 0
- Avg. power of finite duration signal =0, Avg. Power $P = \frac{total\ power}{total\ time}$
  - $P = \int_{-\infty}^{\infty} P(t)dt$, will have a finite value for energy signal and total time is infinite. If we divide finite value with an infinite term we get 0.
  - So power can be expressed as $P = \lim_{T \to \infty} \frac{E}{T}$; We know, P(t)=$|x(t)|^2$
  - $P = \int_{-\infty}^{\infty} P(t)dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$=E
  - If any signal is energy signal, then its power will be 0 and vice-versa.



Properties of Energy Signals

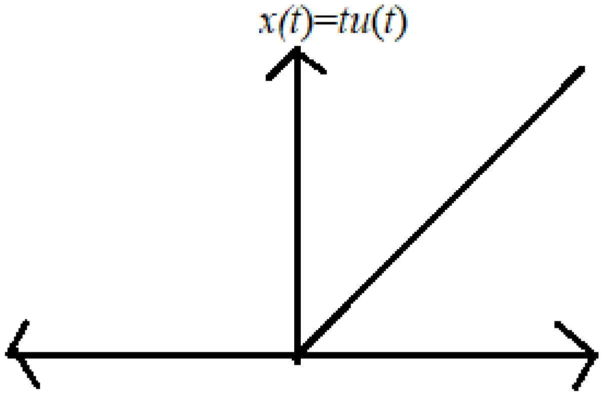
1. Energy signals are absolutely integrable signals, so its Fourier transform exists.
2. Total energy is can be shown under the graph of total area $|x(t)|^2$
3. Power $P = \lim_{T \to \infty} \frac{E}{T}$

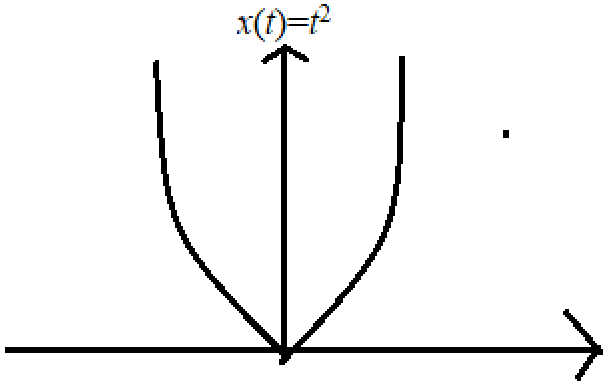
Power Signals

- Any signals is said to be power signal if the energy is infinite.
  - $E = \text{Power} \times \text{Time} = \infty$
- Properties of power signals:
  1. Periodic signals are power signals but vice-versa is not true
  2. Root mean square value, $RMS = \sqrt{P}$ or $P = RMS^2$
  3. If modules of two signals are same, average power of them will be same.

Neither Energy Nor Power Signal (NENP)

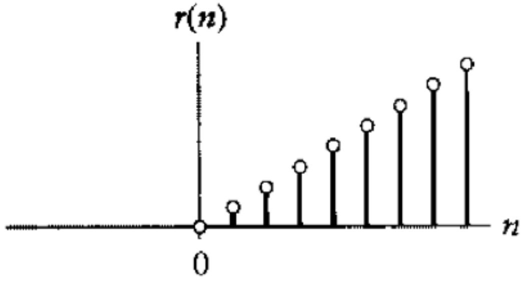
- Any signal is neither energy nor power signal if the magnitude of the signal is infinite at any instant of time.
- To determine the whether the any signal is energy, power or NENP, first of all we have to determine the average power (P) of that signal.
  - If P is zero, then the signal is energy signal, i.e. energy E => finite
  - If P is equal to some finite value then the signal is power signal, i.e. E=> ∞
  - If $P \neq 0$ and P$\neq$ any finite value, P=> ∞ then the signal is NENP





Energy and Power DTS

- For DTS, average power is given as: $P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$
- For DTS, total energy is given as, $E = \sum_{n=-\infty}^{\infty} |x[n]|^2$
  - Example 1: Let x[n] = ramp [n]; $x[n] = \begin{cases} 0; n < 0 \\ n; n \geq 1 \end{cases}$



  - $P = \lim\limits_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$
  - $P = \lim\limits_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{0} 0 + \sum_{n=0}^{N} n^2 = \lim\limits_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} n^2$
  - $P = \lim\limits_{N\to\infty} \frac{1}{2N+1} \frac{N(N+1)(2N+1)}{6} = \frac{\infty}{6} = \infty$
  - $E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{0} 0 + \sum_{n=0}^{\infty} n^2 = 0 + 0 + 1 + 4 + \cdots + \infty = \infty$
  - We can say given signal is a NENP signal

  - Example 2: y[n]=A, identify whether it is energy, power or NENP?

Orthogonal signal

- Orthogonality is where we are allowed to transmit more than one signal over a common channel with successful detection.
- Orthogonal Signal: Two signals are said to be orthogonal if they are mutually independent. So we have to identify if to signals are orthogonal or not.
  - For vector space, if $\vec{a}$ and $\vec{b}$ are two vectors, they will be orthogonal if the dot/scaler product of them are 0, i.e. $\vec{a}.\vec{b}$=0
  - For signal space, if the inner product or definite integral o two signals are zero, then the signals are said to be orthogonal. Let $x_1(t)$ and $x_2(t)$ are two signals, then they are said to be orthogonal if
    - $\int_{-\infty}^{\infty} x_1(t)\, x_2(t)dt = 0$ for non- parodic signal
    - $\int_0^T x_1(t)\, x_2(t)dt = 0$ for periodic signal where T is the FTP.

Properties of Orthogonal Signal

1. Two harmonics of different frequencies are always orthogonal.
2. Sine and Cosine function of same phases and same frequencies are always orthogonal.
3. Dc value and sine functions are always orthogonal.
4. If two signals $x_1(t)$ and $x_2(t)$ are orthogonal and $y(t) = x_1(t)+x_2(t)$, then the average power of y(t) is $P_y = P_{x_1} + P_{x_2}$ and the total energy $E_y = E_{x_1} + E_{x_2}$