Institute of Information Technology

Subject: Numerical Techniques Laboratory

Exp. No.-4

Name of the Exp.: Interpolating a table of data by Newton's forward and backward difference interpolation formula, Lagrange's Interpolation formula and Inverse Lagrange's Interpolation formula.

<u>Introduction:</u>

Let a set of tabular values $(x_0, y_0), (x_1, y_1)........(x_n, y_n)$ satisfying the relation $y = f(x)$ **,** where the explicit nature of $f(x)$ is not known, it requires to find a simpler function say $\varphi(x)$ such that $f(x)$ and $\varphi(x)$ agree at the set of tabulated points. Such a process is called Interpolation. If $\varphi(x)$ is a polynomial, then the process is called the interpolating polynomial.

Here, if the function $y = f(x)$ is defined by (n+1) numbers of points, the error committed in interpolation is

$$y(x) - \varphi(x) = \frac{x(x-1)(x-2)......(x-n)}{(n+1)!} y^{n+1}(\zeta), x_0 < \zeta < x_n$$

Where, $y^{n+1}(\zeta)$ is the (n+1)th differentiation of y(x) at point, ζ **.**

<u>Objective of the Experiment:</u>

1. To get introduce with different interpolating formulae.
2. To write a program in order to find out the value of y at a point x from a given tabular points by Newton's Forward and backward difference Interpolation formulae for equally spaced points.
3. To write a program in order to find out the value of y at a point x from a given tabular points by Lagrange's interpolation formula for equally or not equally spaced points.
4. To write a program in order to find out the value of x at a point y from a given tabular data by Inverse Lagrange's interpolation formula.

<u>Theory:</u>

<u>Interpolation with evenly spaced data points by Newton's forward and backward difference formulae.</u>

<u>For the points at the beginning of Tabular data</u> Let there are n+1 number of data points, $(x_0, y_0), (x_1, y_1)........(x_n, y_n)$ are given. When values of x are at equal distance and the value of x, for which the value of y is to be determined, is at the beginning of the given data table then use ***Newton's forward difference interpolation Formula*** to find the polynomial y, which is

$$y_n(x) = y_o + p\Delta y_o + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + ... + \frac{p(p-1)(p-2)........(p-n+1)}{(n)!}\Delta^n y_0 \quad .....(1)$$

Where $x = x_0 + ph$ **,** h=difference between two successive values of x.

The values of Δy_o , $\Delta^2 y_0$, $\Delta^3 y_0$ .... $\Delta^n y_0$ can be found from the following forward difference Table (Table-1).

<u>Table-1:Forward difference Table(n=5)</u>

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5
x_0	y_0					
		Δy_0				
x_1	y_1		$\Delta^2 y_0$			
		Δy_1		$\Delta^3 y_0$		
x_2	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$	
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$
x_3	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$	
		Δy_3		$\Delta^3 y_2$		

x_4	y_4		$\Delta^2 y_3$	
		Δy_4		
x_5	y_5			

Where,

$\Delta y_0 = y_1 - y_0 ; \Delta y_1 = y_2 - y_1 ; \Delta y_2 = y_2 - y_3 ; \Delta^2 y_0 = \Delta y_1 - \Delta y_0 ; \Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$

and so on…… . (2)

For the points at the end of the Table: Let there are n+1 number of data points, $(x_0, y_0), (x_1, y_1)........(x_n, y_n)$ are given, when values of x are at equal distance and the value of x, for which the value of y is to be determined, is at the end of the given data table then use ***Newton's backward difference interpolation Formula*** in order to find out the polynomial y which is

$$y_n(x) = y_o + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + ... + \frac{p(p+1)(p+2)........(p+n-1)}{(n)!}\nabla^n y_n$$

….(3)

Where**,** $x = x_n + ph$ **,** h=difference between two successive values of x.

Values of ∇y_n , $\nabla^2 y_n$, $\nabla^3 y_n$ …. $\nabla^n y_n$ can be found from the following backward difference Table (Table-2).

Table-2: Backward difference Table (n=5)
and relation between forward and backward elements

x	y	∇	∇^2	∇^3	∇^4	∇^5
x_0	y_0					
		$\nabla y_1 = \Delta y_0$				
x_1	y_1		$\nabla^2 y_2 = \Delta^2 y_0$			
		$\nabla y_2 = \Delta y_1$		$\nabla^3 y_3 = \Delta^3 y_0$		
x_2	y_2		$\nabla^2 y_3 = \Delta^2 y_1$		$\nabla^4 y_4 = \Delta^4 y_0$	
		$\nabla y_3 = \Delta y_2$		$\nabla^3 y_4 = \Delta^3 y_1$		$\nabla^5 y_5 = \Delta^5 y_0$
x_3	y_3		$\nabla^2 y_4 = \Delta^2 y_2$		$\nabla^4 y_5 = \Delta^4 y_1$	
		$\nabla y_4 = \Delta y_3$		$\nabla^3 y_5 = \Delta^3 y_2$		
x_4	y_4		$\nabla^2 y_5 = \Delta^2 y_3$			
		$\nabla y_5 = \Delta y_4$				
x_5	y_5					

$\nabla y_1 = y_1 - y_0 ;\ \nabla y_2 = y_2 - y_1 ;\ \nabla^2 y_2 = \nabla y_2 - \nabla y_1 ;\ \nabla^3 y_3 = \nabla^2 y_3 - \nabla^2 y_2$ and so on……………
(4)

So from the above table and from (2),(4) it is clear that same number occurs in the same position, whether it is forward or backward difference table.

Interpolation with unevenly spaced points using Lagrange's formula

Newton's interpolation Formulae is not applicable where values of x are unequally spaced. In that case Lagrange's interpolation formula is applicable, which is,

$$y(x) = \frac{(x - x_1)(x - x_2)......(x - x_n)}{(x_0 - x_1)(x_0 - x_2)......(x_0 - x_n)} * y_0 + \frac{(x - x_0)(x - x_2).....(x - x_n)}{(x_1 - x_0)(x_1 - x_2).......(x_1 - x_n)} * y_1 + \frac{(x - x_0)(x - x_1)......(x - x_n)}{(x_2 - x_0)(x_2 - x_1).....(x_2 - x_n)} * y_2$$

$$+ ....+ \frac{(x - x_1)(x - x_2).........(x - x_{n-1})}{(x_n - x_1)(x_n - x_2).......(x_n - x_{n-1})} * y_n \quad .... \quad \textbf{.(5)}$$

Inverse Interpolation by Inverse Lagrange's formula

To find out value of x for a given value of y from a data table we should apply Inverse Lagrange's formula, which is

$$x(y) = \frac{(y-y_1)(y-y_2)......(y-y_n)}{(y_0-y_1)(y_0-y_2)......(y_0-y_n)} * x_0 + \frac{(y-y_0)(y-y_2).....(y-y_n)}{(y_1-y_0)(y_1-y_2).......(y_1-y_n)} * x_1 + \frac{(y-y_0)(y-y_1)......(y-y_n)}{(y_2-y_0)(y_2-y_1).....(y_2-y_n)} * x_2$$

$$+ \;..........+ \frac{(y-y_1)(y-y_2).........(y-y_{n-1})}{(y_n-y_1)(y_n-y_2).......(y_n-y_{n-1})} * x_n \qquad \textbf{..........(6)}$$

Reference Book:
1)Introductory Methods of Numerical Analysis: by S.S. Sastry.
2)Numerical Methods -Gerald/Wheetley

Problems/Reports:

1. Write a program to find out y(10) and y(1) for the following tabular data

x	x_0=3	x_1=4	x_2=5	x_3=6	x_4=7	x_5=8	x_6=9
y	y_0=2.7	y_1=6.4	y_2=12.5	y_3=21.6	y_4=34.3	y_5=51.2	y_6=72.9

2. Write a program to find out y(3) for the following tabular data

x	x_0=0	x_1=1	x_2=2	x_3=4
y	y_0=2	y_1=5	y_2=9	y_3=12

3. If y_1=4, y_3=12, y_4=19 and y_x=7, then write and program to find x.
4. Solve problem 1,2 and 3 by hand calculation.
5. Write the limitation of Newton's Interpolation Formulae.
6. What do you mean by interpolation, extrapolation and inverse interpolation?
7. Discuss on the experiment.

THE NEW MATLAB FUNCTIONS USED IN THIS PROGRAM

1. factorial(N)
Calculates the factorial of N
ie. factorial(N)=1*2*3*4………N

2. length (C)
If C is a vector, n = length(C); returns the size of the longest dimension of C. which is the same as its length.
Examples

```
C = [1  6  4  7   9]
n = length(C)

n =
    5
C = [1  6  4  7   9   4 9  3  2]
n = length(C)

n =
    9
```

3. diff(Y,i)
If Y is a vector, diff(Y) calculates differences between adjacent elements of Y. Then diff(Y) returns a vector, one element shorter than Y, of differences between adjacent elements: [Y(2)-Y(1) Y(3)-Y(2) ... Y(n)-Y(n-1)]

diff(Y,i) applies diff recursively i times, resulting in the ith difference. Thus, diff(Y,2) is the same as diff(diff(Y)).

EXAMPLE:

```
y = [1 3 7 15 13];          % y(1)=1; y(2)=3; y(3)=7;y(4)=15;y(5)=13
dy1 = diff(y)
```

dy1 =
    2   4   8   -2   % dy1(1)=2; dy1(2)=4; dy1(3)=8; dy1(4)= -2

dy2 = diff(y,2)
dy2 =
    2   4   -10   % dy2(1)=2; dy2(2)=4; dy2(3)= -1

NOTE: Difference table can be created by using 'diff(y,i)' matlab function
For this example, considering forward difference table,

Δ $y_{0=}$ y_1- y_0 =dy1(1) =y(2)-y(1)= 3-1=2
Δ $y_{1=}$ y_2–y_1= dy1(2)=y(3)-y(2) =7-3=4
Δ $y_{2=}$ y_3–y_2= dy1(3)=y(4)-y(3)=15-7=8
Δ $y_{3=}$ y_4–y_3= dy1(3)=y(5)-y(4)=13-15= -2

Δ^2 $y_{0=}$ Δ y_1- Δ y_0 =dy2(1) =dy1(2)-dy1(1)= 4-2=2
Δ^2 $y_{1=}$ Δ y_2- Δ y_1 =dy2(2) =dy1(3)-dy1(2)=8- 4=2
Δ^2 $y_{2=}$ Δ y_3- Δ y_2 =dy2(3) =dy2(4)-dy1(3)=-2-8=2