Signals and Operations

IT3105: Signals and systems

Conjugate Symmetric Signals

- Here conjugate means complex conjugate and symmetric signals means when any signal remains same after time reversal.
  - Consider any complex number such as 3+4i, its complex conjugate would be 3-4i. If the imaginary part of any complex number is zero, then the number and its complex conjugate would be same such as 2+0i = 2-0i.
- Let *x*(*t*) be the given CTS. We get *x*(-*t*) after applying time reversal. The complex conjugate of *x*(*t*) can be denoted as $x^*(-t)$. Now *x*(*t*) would be conjugate symmetric signal if $x(t) = x^*(-t)$.
  - Let $x(t) = a(t) + jb(t)$, After performing time reversal we get $x(-t) = a(-t) + jb(-t)$
  - Then the complex conjugate signal is $x^*(-t) = a(-t) - jb(-t)$
  - We can say **R**$[x(t)]$=**R**$[x^*(-t)]$ and **I**$[x(t)]$=**I**$[x^*(-t)]$, which indicate the properties as follows

1. $a(t) = a(-t)$, real part of conjugate symmetric signals is even.
2. $b(t) = -b(-t)$, imaginary part of conjugate symmetric signals is odd.

Conjugate Antisymmetric Signals

- Any CTS can be conjugate antisymmetric if $x(t) = -x^*(-t)$
  - Let $x(t) = a(t) + jb(t)$, After performing time reversal we get $x(-t) = a(-t) + jb(-t)$
  - Then the complex conjugate signal is $x^*(-t) = a(-t) - jb(-t)$
  - After performing amplitude reversal we get the conjugate antisymmetric signal as $-x^*(-t) = -a(-t) + jb(-t)$, then the properties of CAS can be as follows:

  1. $a(t) = -a(-t)$, real part of conjugate antisymmetric signals is odd.
  2. $b(t) = b(-t)$, imaginary part of conjugate antisymmetric signals is even.

- Any signal can be represented as the sum of conjugate symmetric signals and conjugate antisymmetric signals, i.e. $x(t) = x_{CS}(t) + x_{CAS}(t)$; same as even and odd signals. So we can express the components as
  - $x_{CS}(t) = x(t) + x^*(-t)$
  - $x_{CAS}(t) = x(t) - x^*(-t)$

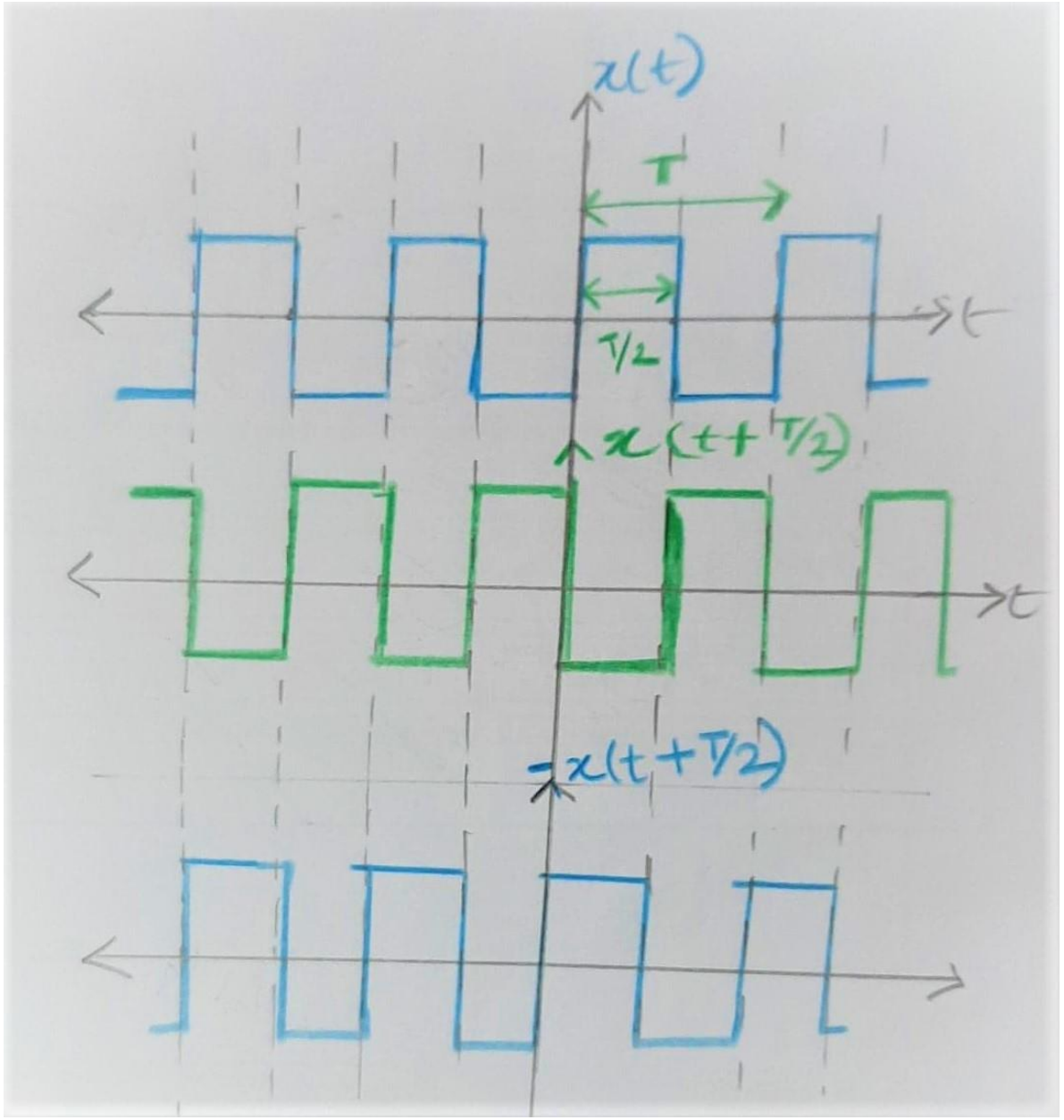
Examples of CS and CAS Signals

- Find if $x(t) = t^2 + jsint$ is CS.
  - t^2or real part is even and $jsint$ or imaginary part is odd of signal x(t). So x(t) is CS signal.
- Find if $x(t) = sint + jt^3$ is CS.
  - Not CS. Why?
- Given $x(t) = 2sint + jt^5$, is CS or CAS?
  - Neither CS nor CAS.
- Given $x(t) = \sin^3 t + jt^2$, is CS or CAS?
  - $\sin^3 t$ is odd and jt^2 is even, so the signal is CAS

Half-Wave Symmetric Signal

- Condition of half-wave symmetric signal is $x(t) = -x(t \pm \frac{T}{2})$; where T is the FTP and T/2 is the half of FTP.
  - We can calculate half wave symmetric signal by time shifting the whole signal by T/2 in both left shifting or time advanced (+) or right shifting or time delay (-).
  - After performing any of the above shifting (not both) we have to do amplitude reversal.
  - After time shifting by T/2 and amplitude reversal, if we get the original signal x(t), then x(t) is said to be half-wave symmetric signal.

Example of square wave



Example of saw tooth wave