Lecture 02

Basic Signals Operations

BY
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Objectives of this Lecture:

- To identify two classes of operations on signals:
  - Operations performed on dependent variables
  - Operations performed on independent variables

Basic Operations on Signals

In the study of signals and systems, an important issue is the use of systems to process or manipulate signals. This issue usually involves a combination of some basic operations. Signals can be subjected to several processes to produce new signals. Here, we have provided in-depth details about the following.

1. Signal operations on Amplitude
2. Signal operations on Time

Concerning amplitude, we can perform four basic signal operations, namely-

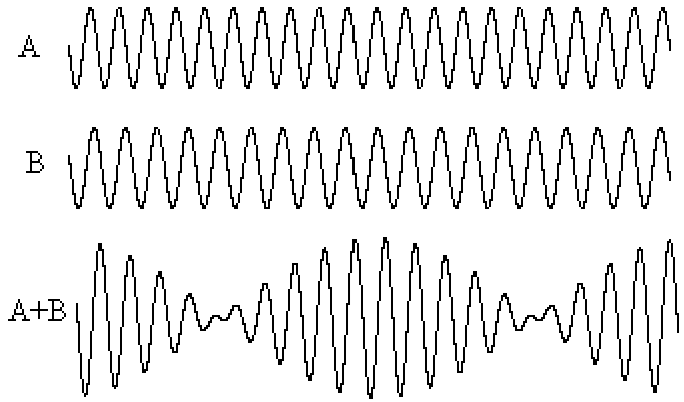
- Addition
- Subtraction
- Amplitude Scaling
- Time shifting
- Multiplication

Basic Operations on Signals: Addition

The particular operation involves the addition of amplitude of two or more signals at each instance of time or any other independent variables which are common between the signals. Addition of signals is illustrated in the diagram below, where $X_1(t)$ and $X_2(t)$ are two time dependent signals, performing the additional operation on them we get,

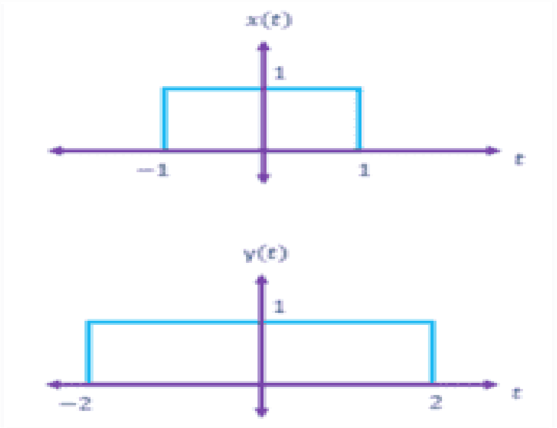
$$Y(t) = X_1(t) + X_2(t)$$

A physical example of a device that adds signals is an audio mixer that combines music and voice signals.

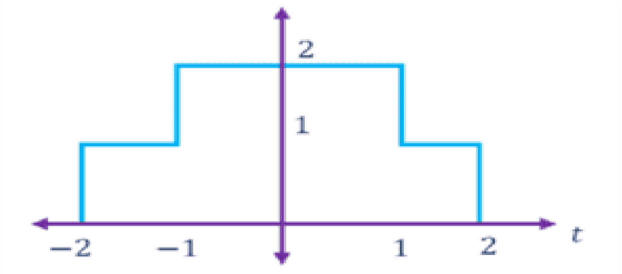


Basic Operations on Signals: Addition

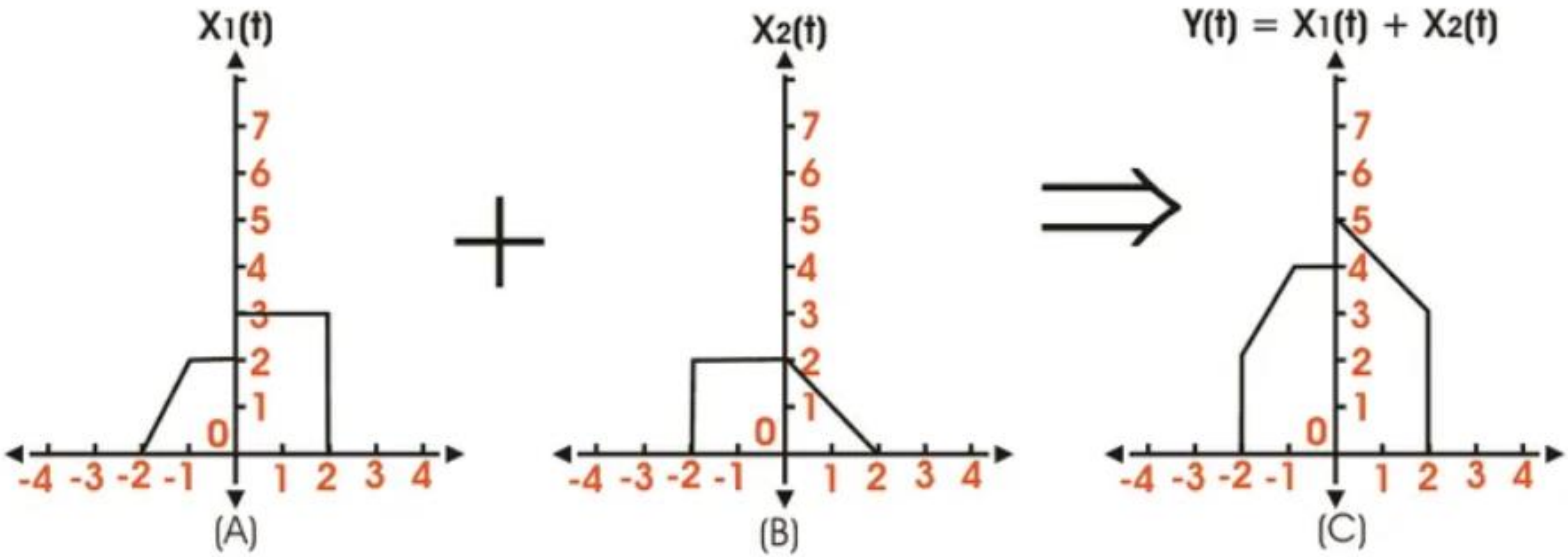
If we add two continuous-time signals, x(t) and y(t), the resultant signal will have an amplitude equal to the sum of their amplitudes. The below example can be used to explain it in a better way.



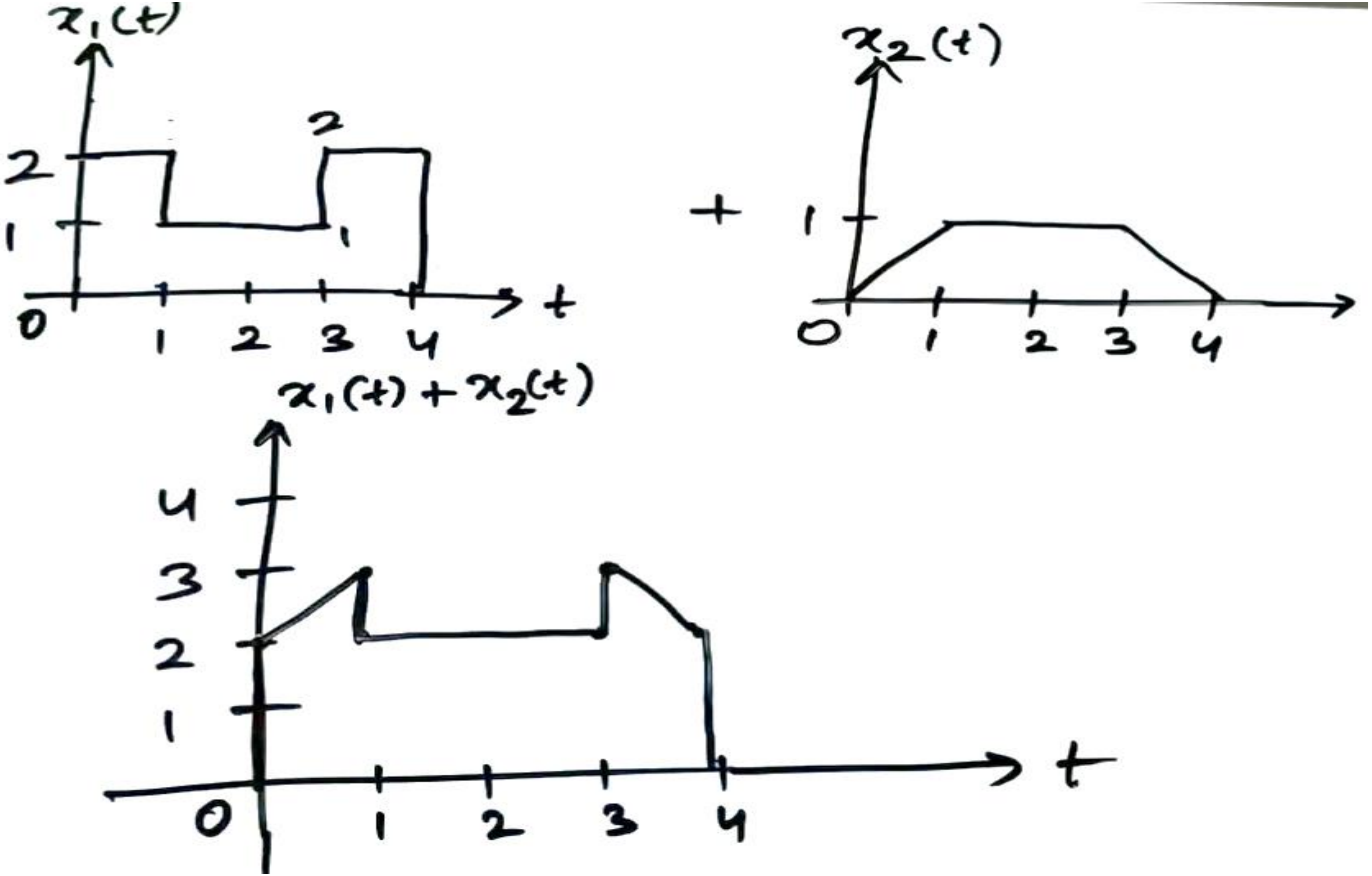
From the above figures, the amplitude of the signal
z(t) = x(t) + y(t) between the interval -2<t<-1 is
z(t) = x(t) + y(t) = 0 + 1 = 1
The amplitude of the signal z(t) between the interval −1<t<1 is
z(t) = x(t) + y(t) = 1 + 1 = 2
The amplitude of the signal z(t) between the interval 1<t<2 is z(t) = x(t) + y(t) = 0 + 1 = 1.



Basic Operations on Signals: Addition



Basic Operations on Signals: Addition



Basic Operations on Signals: Addition

Adding two sinusoids with the same frequency and the same phase (so that the two signals are proportional) **gives a resultant sinusoid with the sum of the two amplitudes**. But when frequency and phases are different we need to apply phasor.

Phasor: Phasor is a complex number representing a sinusoidal function whose amplitude, angular frequency, and initial phase are time-invariant.

From a standard sinusoidal signal, $x(t)=A\cos(\omega t+\varphi)$

Phasor from, $z = r\angle\phi$

$$
\begin{array}{ll}
z = x + jy & \textit{Rectangular form} \\
z = r\angle\phi & \textit{Polar form} \\
z = re^{j\phi} & \textit{Exponential form}
\end{array}
$$

Resultant Amplitude, $r = \sqrt{x^2 + y^2}, \quad \phi = \tan^{-1}\frac{y}{x}$

Basic Operations on Signals: Addition

1. First we have to convert all sinusoidal signals into cosine signals by shifting the phase to 90 degree.

$$R\sin(\omega t + \delta) = R\cos(\omega t + \delta - 90°)$$
$$= R\,\underline{\angle \delta - 90°} \quad \text{(Phasor form)}$$

2.

$$A\angle m = A\cos(m) + j.A\text{sine}(m)$$
$$B\angle n = B\cos(n) + j.B\sin(n)$$

$$\boxed{A\angle m + B\angle n = [A\cos(m) + B\cos(n)] + j.[A\sin(m) + B\sin(n)]}$$

Modulus

Argument

Basic Operations on Signals: Addition

Q:Find Sum of the phasors:

$43\angle 10°$, $7\angle -35°$

ans:

$43\angle 10° = 43.\cos(10) + j.43.\sin(10)$

$= 42.34 + j7.466$

$7\angle -35° = 7.\cos(-35) + 7.\sin(-35)$

$= 5.734 - j4.015$

Now,

$43\angle 10° + 7\angle -35°$

$= 42.34 + j7.466 + 5.734 - j4.015$

$= 48.0808 + j3.4518$

Modulus: $\sqrt{48.0808^2 + 3.4518^2} = \mathbf{48.2}$

Argument: $\tan^{-1}(3.4518/48.0808) = \mathbf{4.1°}$

so, $\mathbf{43\angle 10° + 7\angle -35° = 48.2 \angle 4.1°}$

Basic Operations on Signals: Addition

1. Two voltages v_1 and v_2 appear in series so that their sum is $v = v_1 + v_2$. If $v_1 = 10\cos(50t - \pi/3)$ V and $v_2 = 12\cos(50t + 30°)$ V, find v.

2. Adding two sinusoids of the same frequency but different amplitudes and phases results in another sinusoid (sin or cos) of same frequency. The resulting amplitude and phase are different from the amplitude, and phase of the two original sinusoids, as illustrated with the example below.

Example 6-2: Consider an electrical circuit with two elements R and L connected in series as shown in Fig. 6.15.

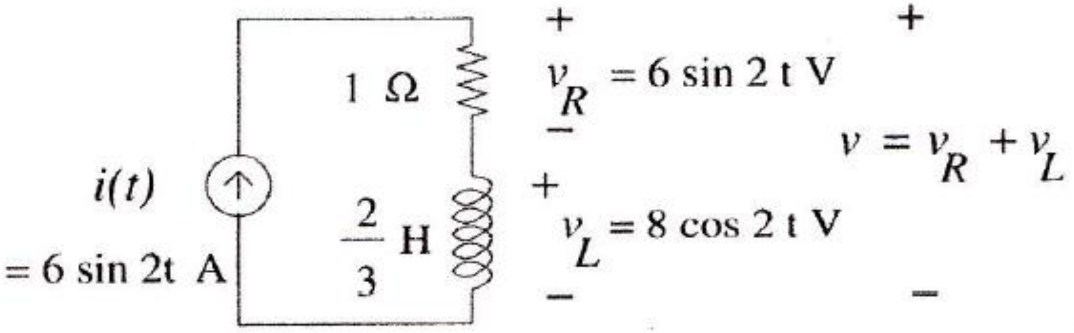
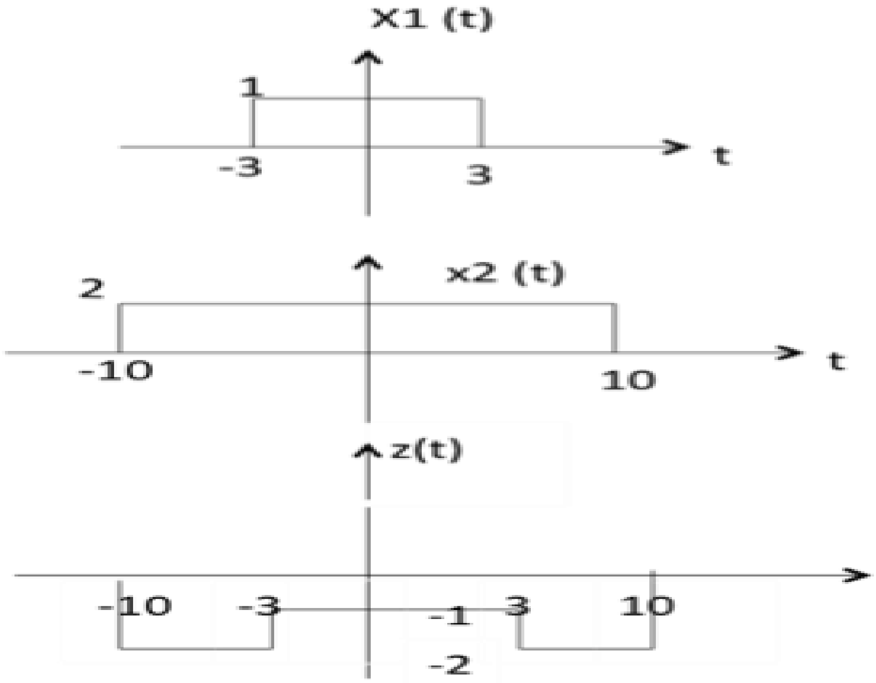


Figure 6.15: Addition of sinusoids in an RL circuit.

Basic Operations on Signals: *Subtraction*

The subtraction operation of two signals is similar to that of addition, as the amplitude of the resultant signal is the value obtained from the subtraction of the amplitudes of the two individual signals in their respective intervals.

Subtraction of two signals is nothing but subtraction of their corresponding amplitudes. This can be best explained by the following example:



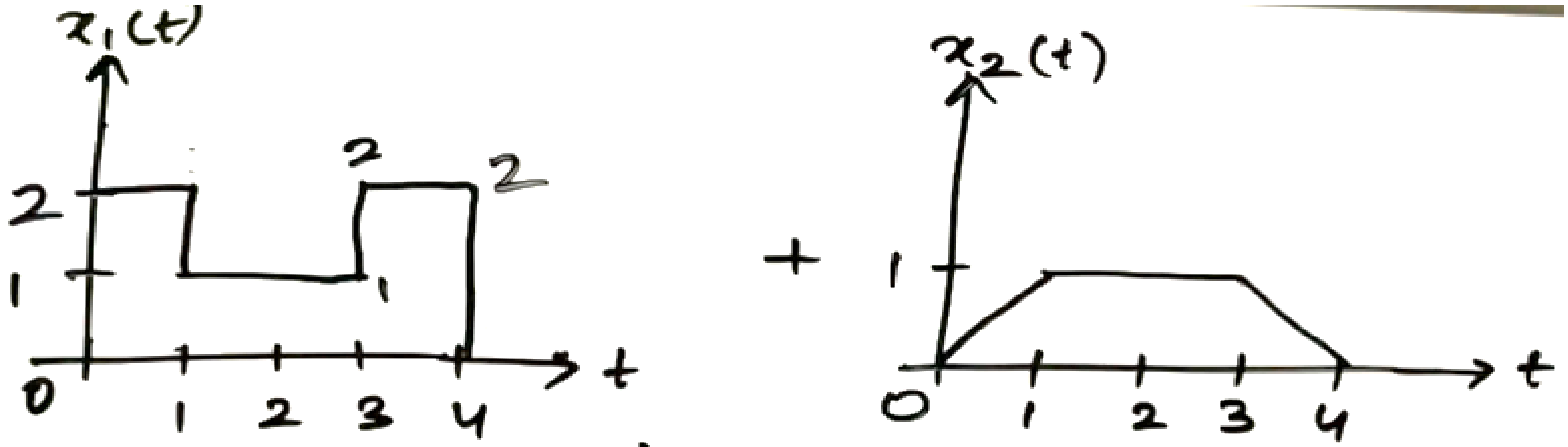
As seen from the diagram above,

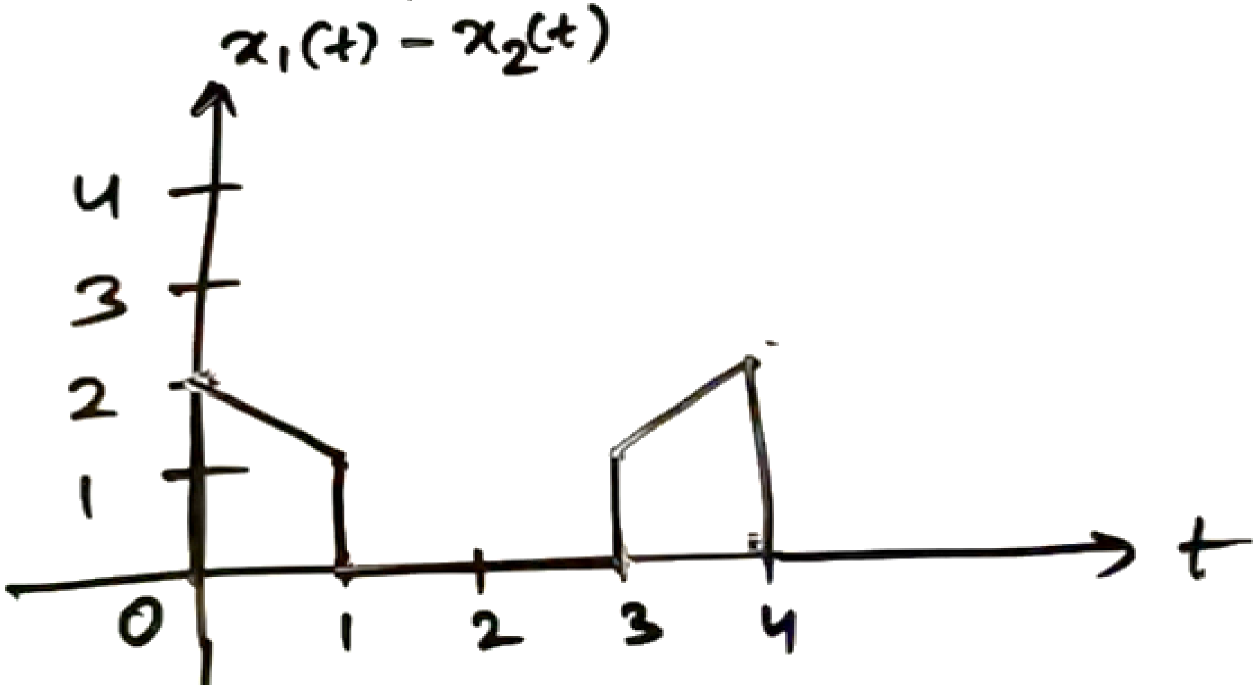
-10 < t < -3 amplitude of z (t) = x1(t) - x2(t) = 0 - 2 = -2

-3 < t < 3 amplitude of z (t) = x1(t) - x2(t) = 1 - 2 = -1

3 < t < 10 amplitude of z (t) = x1(t) + x2(t) = 0 - 2 = -2

Basic Operations on Signals: *Subtraction*





Basic Operations on Signals: *Subtraction*

$$A\angle m = A\cos(m) - j.A\text{sine}(m)$$

$$B\angle n = B\cos(n) - j.B\sin(n)$$

$$A\angle m - B\angle n = [A\cos(m) + B\cos(n)] - j.[A\sin(m) + B\sin(n)]$$

Basic Operations on Signals: Scaling

Scaling of a signal means, a constant is multiplied with the time or amplitude of the signal.

Scaling is a process of readjusting certain internal gain parameters in order to constrain internal signals to a range appropriate to the hardware with the constraint that the transfer function from input to output should not be changed. The filter in Fig. 11.1(a) with unscaled node x has the transfer function.

There are two types of scaling:

1. Time Scaling
2. Amplitude Scaling

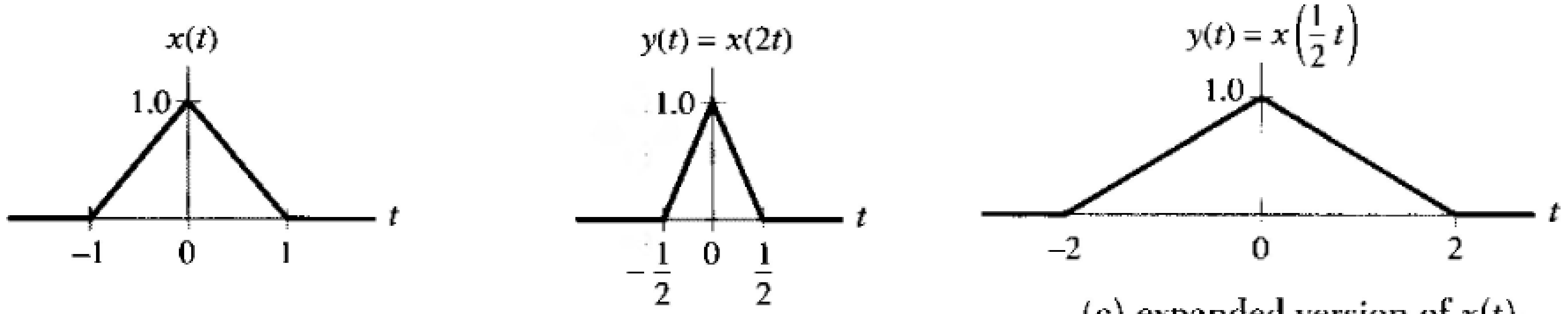
Basic Operations on Signals: Time Scaling

The process of multiplying a constant to the time axis of a signal is known as **time scaling of signal**. The time scaling of signal may be time compression or time expansion depending upon the value of the constant or scaling factor.

Time scaling of continuous-time signal

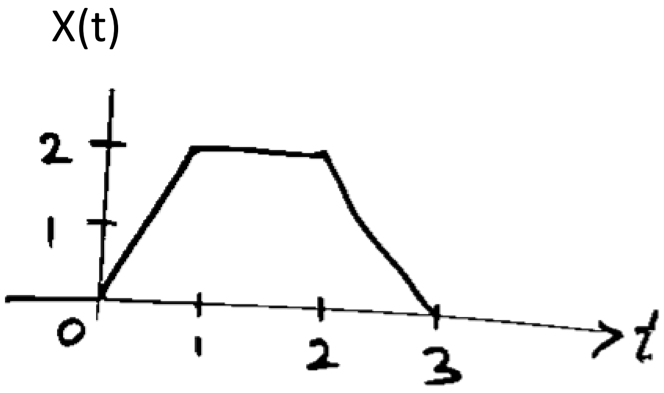
The time scaling of a continuous time signal x(t) can be accomplished by replacing 't' by 'αt' in the function. Mathematically, it is given by, $x(t) \rightarrow y(t) = x(\alpha t)$

Where, α is a constant, called the scaling factor. If α > 1, then the signal is compressed in time by a factor α and the time scaling of the signal is called the time compression. Whereas, if α < 1, then the signal is expanded in time by the factor α and the time scaling is said to be time expansion.



(a) continuous-time signal $x(t)$, (b) compressed version of $x(t)$ (c) expanded version of $x(t)$ by a factor of 2.

Basic Operations on Signals: Time Scaling



$x(0) = 0$ $\quad x(0.5) = 1$

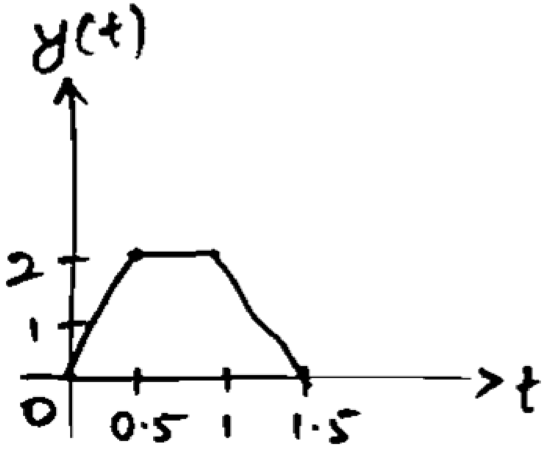
$x(1) = 2$ $\quad x(2.5) = 1$

$x(2) = 2$

$x(3) = 0$

$$y(t) = x(2t)$$

$$\boxed{\begin{aligned} y(0) &= 0 \\ y(0.5) &= 2 \\ y(1) &= 2 \\ y(1.5) &= 0 \end{aligned}}$$



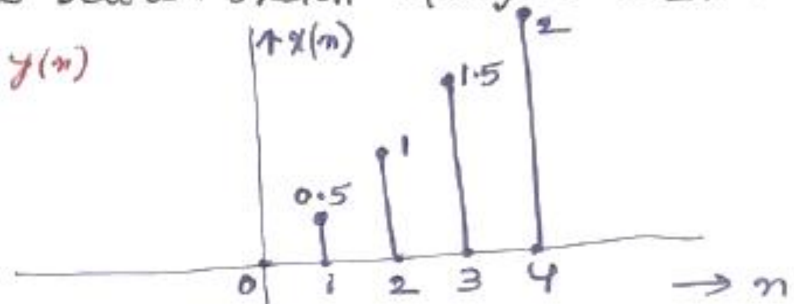
$0.5 = \frac{1}{2}$

$1 = \frac{2}{2}$

$1.5 = 3$

Basic Operations on Signals: Time Scaling

Q. A DT ~~[illegible]~~ triangular pulse is show below. Sketch $x(2n)$ & $x(\frac{n}{2})$.



Sol: Sketching $x(2n)$: → Take this as new signal $y(n)$

$y(n) = x(2n)$

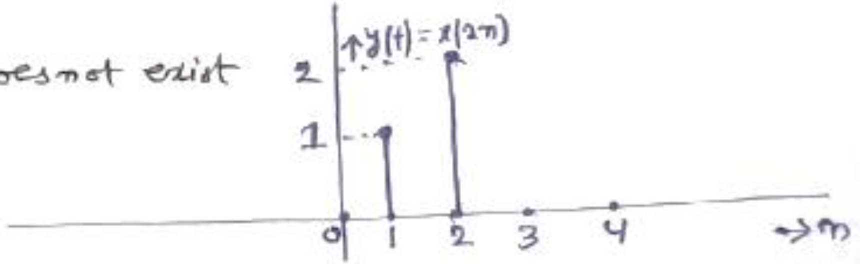
at $n=0 \Rightarrow y(0) = x(2\times 0) = x(0) = 0$

at $n=1 \Rightarrow y(1) = x(2\times 1) = x(2) = 1$

at $n=2 \Rightarrow y(2) = x(2\times 2) = x(4) = 2$

at $n=3 \Rightarrow y(3) = x(2\times 3) = x(6) = 0 \leftarrow$ doesnot exist

at $n=3 \Rightarrow y(4) = x(2\times 4) = x(8) = 0$



Sketching $x(\frac{n}{2})$:

at $n=0 \Rightarrow y(0) = x(\frac{0}{2}) = x(0) = 0$

at $n=1 \Rightarrow y(1) = x(\frac{1}{2}) = x(0.5) =$ Not exist

at $n=2 \Rightarrow y(2) = x(\frac{2}{2}) = x(1) = 0.5$

at $n=3 \Rightarrow y(3) = x(\frac{3}{2}) = x(1.5) =$ Not exist

at $n=4 \Rightarrow y(4) = x(\frac{4}{2}) = x(2) = 1$

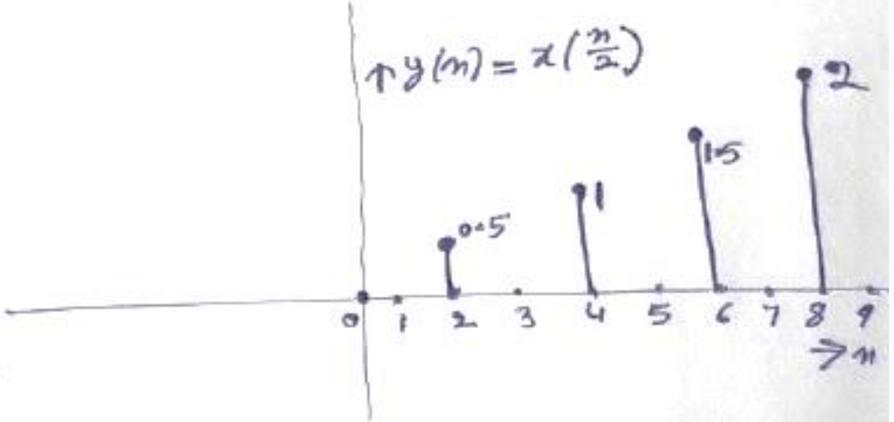
at $n=5 \Rightarrow y(5) = x(\frac{5}{2}) = x(2.5) =$ Not exist

at $n=6 \Rightarrow y(6) = x(\frac{6}{2}) = x(3) = 1.5$

at $n=7 \Rightarrow y(7) = x(\frac{7}{2}) = x(3.5) =$ Not exist

at $n=8 \Rightarrow y(8) = x(\frac{8}{2}) = x(4) = 2$

at $n=9 \Rightarrow y(9) = x(\frac{9}{2}) = x(4.5) =$ Not exist & so on.....



Basic Operations on Signals: Amplitude Scaling

Multiplication of a constant with the amplitude of the signal causes amplitude scaling. Depending upon the value of the constant, it may be either amplitude scaling or attenuation.

Amplitude Scaling of a Continuous-Time Signal

$y(t) = A\,x(t)$

Where, A is a constant. If the value of A is greater than 1 (i.e., A > 1), the signal amplitude scaling is called the amplification of the signal while if A < 1, then the scaling is called the **attenuation of the signal**. Figure-1 shows an arbitrary continuous-time signal x(t) and its amplitude scaled version y(t).

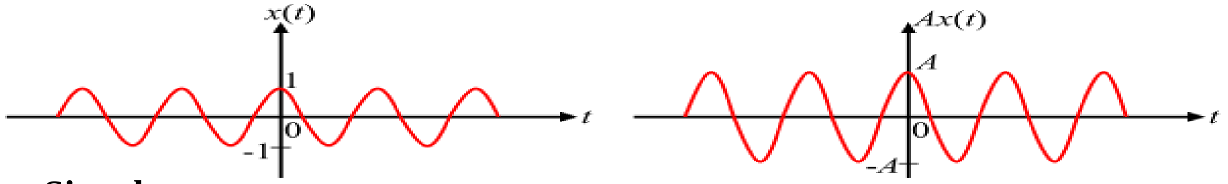


Figure-1

Amplitude Scaling of a Discrete-Time Signal

$y(n) = k\,x(n)$

Where, k is a constant. If k > 1, the scaling is called amplification of the signal, while if k < 1, the scaling is called **attenuation of the signal**. An arbitrary discrete time sequence x(n) and its scaled version y(n) are shown in Figure-2.

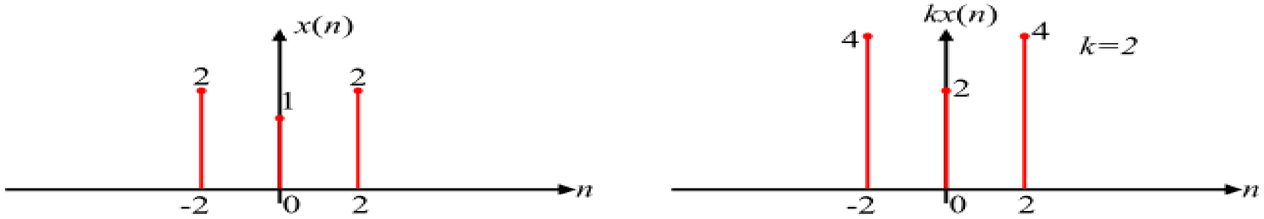
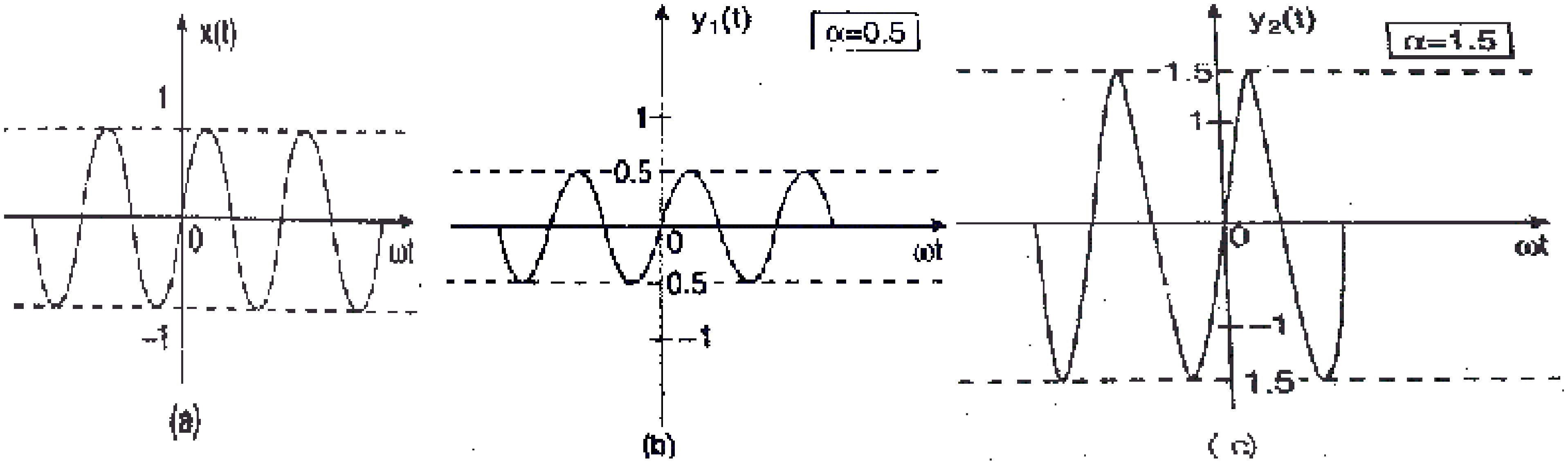
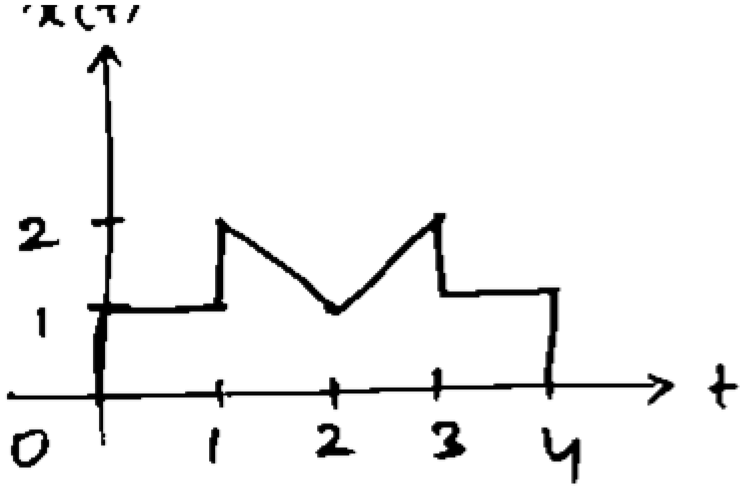


Figure-2

Basic Operations on Signals: Amplitude Scaling



Basic Operations on Signals: Amplitude Scaling + Inversion



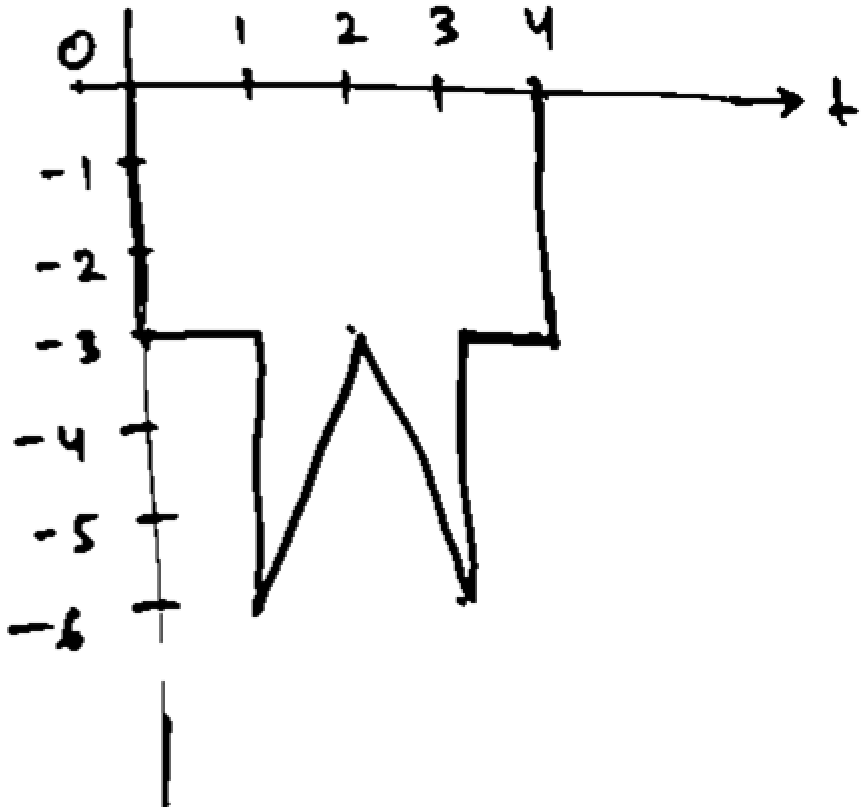
$y(t) = -3x(t)$

$3x(t) \longrightarrow$ Amp scaling

$-3x(t) \longrightarrow$ Amp Invers.

$3x(t)$

$-3x(t)$ $-3x(t)$



Basic Operations on Signals: Time Shifting

Let $x(t)$ denote a continuous-time signal.

The time-shifted version of $x(t)$ is defined by

$$y(t) = x(t - t_0)$$

where t_0 is the time shift.

If $t_0 > 0$, the waveform representing $x(t)$ is shifted intact to the right, relative to the time axis.

If $t_0 < 0$, it is shifted to the left.

In the case of a discrete-time signal $x[n]$, we define its time-shifted version as follows:

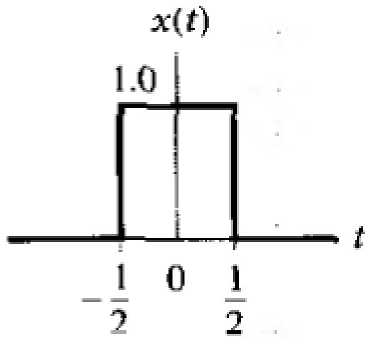
$$y[n] = x[n - m]$$

where the shift m must be an integer; it can be positive or negative.

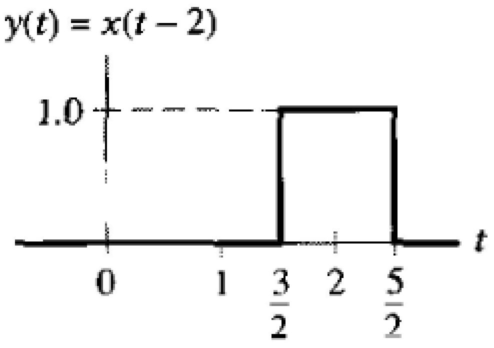
Basic Operations on Signals: Time Shifting

Example 1.3 Figure 1.22(a) shows a rectangular pulse $x(t)$ of unit amplitude and unit duration. Find $y(t) = x(t-2)$.

Solution: In this example, the time shift t_0 equals 2 time units. Hence, by shifting $x(t)$ to the right by 2 time units we get the rectangular pulse $y(t)$ shown in Fig. 1.22(b). The pulse $y(t)$ has exactly the same shape as the original pulse $x(t)$; it is merely shifted along the time axis.



(a) continuous-time signal in the form of a rectangular pulse of amplitude 1.0 and duration 1.0 symmetric about the origin;

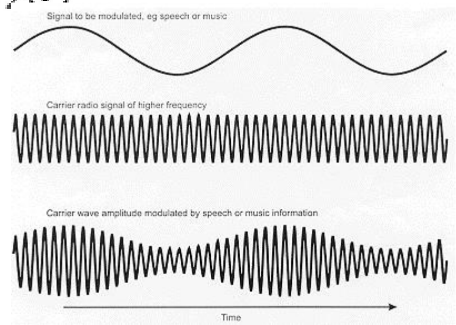


(b) time-shifted version of $x(t)$ by 2 time units.

Basic Operations on Signals: Multiplication

Let $x_1(t)$ and $x_2(t)$ denote a pair of continuous-time signals.

The signal $y(t)$ resulting from the multiplication of $x_1(t)$ by $x_2(t)$ is defined by



$$y(t) = x_1(t)x_2(t) \qquad (1.20)$$

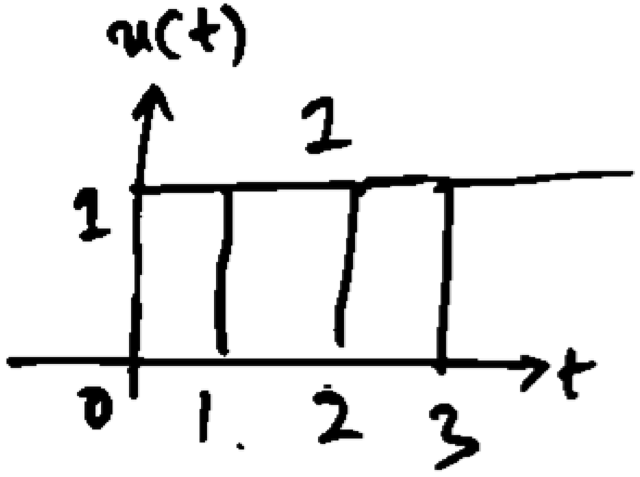
That is, for each prescribed time t the value of $y(t)$ is given by the product of the corresponding values of $x_1(t)$ and $x_2(t)$.

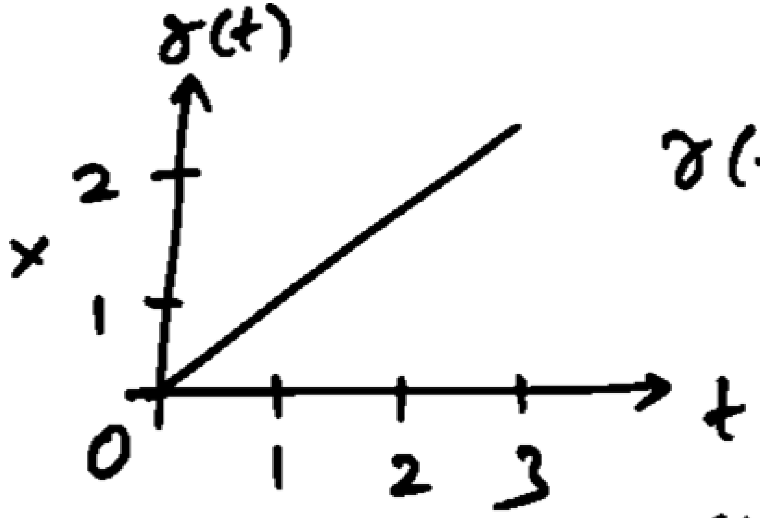
A physical example of $y(t)$ is an *AM radio signal*, in which $x_1(t)$ consists of an audio signal plus a dc component, and $x_2(t)$ consists of a sinusoidal signal called a carrier wave.

In a manner similar to Eq. (1.20), for discrete-time signals we write

$$y[n] = x_1[n]x_2[n]$$

Basic Operations on Signals: Multiplication





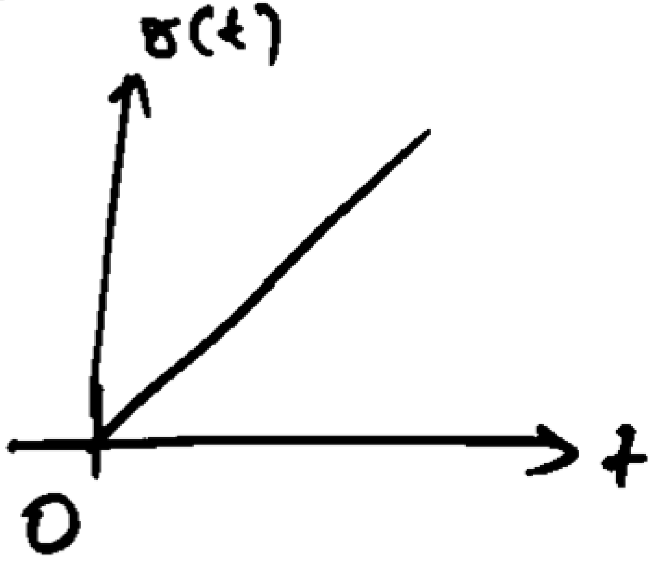
$$r(t) = \begin{cases} t & (t>0) \\ 0 & (t<0) \end{cases}$$

$t=0,\quad 1\times 0 = 0$

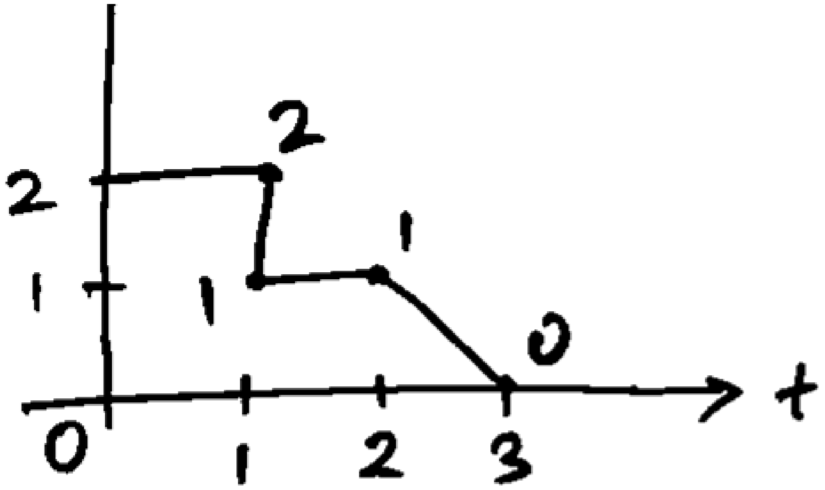
$t=1,\quad 1\times 1 = 1$

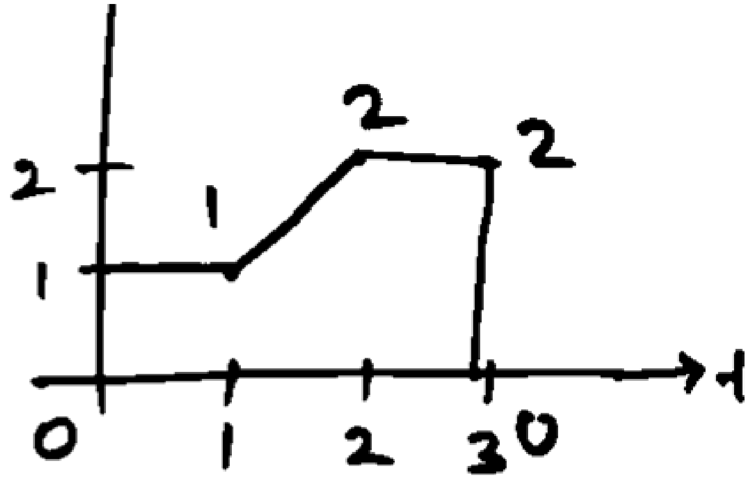
$t=2,\quad 1\times 2 = 2$

$t=3,\quad 1\times 3 = 3$



Basic Operations on Signals: Multiplication





When, t = 0 $2 \times 1 = 2$

$2 \times 1 = 2$ ✓

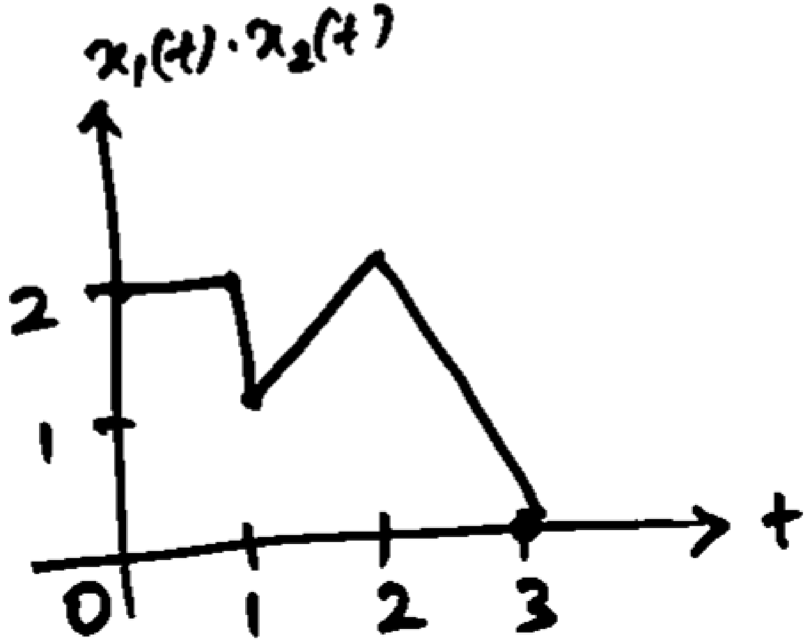
t = 1 $1 \times 1 = 1$ ✓

$1 \times 2 = 2$

t = 2

$0 \times 2 = 0$

t = 3 $0 \times 0 = 0$



Basic Operations on Signals: Multiplication

Phasor Operation

Multiplication of Phasor:

$$A\angle m * B\angle n = AB\angle m+n$$

Division of Phasor:

$$\frac{A\angle m}{B\angle n} = \frac{A}{B}\angle m-n$$

Thank You