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Answer to the question no - 1

$$x_1 + x_2 + x_3 = 2$$
$$4x_1 + 3x_2 + x_3 = 6$$
$$3x_1 + 5x_2 + 3x_3 = 4$$

LU decomposition

We have

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 1 \\ 3 & 5 & 3 \end{bmatrix}$$

Let

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 1 \\ 3 & 5 & 3 \end{bmatrix}$$

Now, $u_{11} = 1$, $u_{12} = 1$, $u_{13} = 1$

$l_{21} u_{11} = 4$

$\Rightarrow l_{21} = \frac{4}{1}$

$\therefore L_{21} = 4$

$l_{31} u_{11} = 3$

$\Rightarrow l_{31} = \frac{3}{1}$

$\therefore L_{31} = 3$

$l_{21}u_{12} + u_{22} = 3$

$\Rightarrow u_{22} = 3 - 4\times 1$

$\therefore u_{22} = -1$

$l_{21}u_{13} + u_{23} = 1$

$\Rightarrow u_{23} = 1 - 4\times 1$

$\therefore u_{23} = -3$

Finally

$l_{31}u_{12} + l_{32}u_{22} = 5$

$\Rightarrow l_{32} = \dfrac{5 - 3\times 1}{-1}$

$\therefore l_{32} = -2$

and

$l_{31}u_{13} + l_{32}u_{23} + u_{33} = 3$

$\Rightarrow u_{33} = 3 - 3\times 1 + (-2)\times(-3)$

$\Rightarrow u_{33} = 3 - 3 + 6$

$\therefore u_{33} = 6$

$$\therefore A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -3 \\ 0 & 0 & 6 \end{bmatrix}$$

Now

$LY = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

We written as

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

or

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 3 & -2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 4 \end{bmatrix}$$

where

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & -1 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$\therefore y_1 = 2$, $y_2 = -2$, $y_3 = -6$ [Using Calculator]

$\therefore x_1 = 2$ $y = -1$ $z = 1$

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Answer to the question no - 2

Given that

$$A = \begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$$

$$q_1 = \frac{a_1}{||q_1||} = \frac{1}{2\sqrt{5}} \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$a_2^1 = a_2 - \langle a_2, q_1 \rangle q_1$$

$$= \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \left\langle \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \frac{1}{2\sqrt{5}} \begin{pmatrix} 2 \\ 4 \end{pmatrix} \right\rangle \frac{1}{2\sqrt{5}} \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \left\langle \frac{20}{2\sqrt{5}} \right\rangle \frac{1}{2\sqrt{5}} \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 5 \end{pmatrix} - 2\sqrt{5} \times \frac{1}{2\sqrt{5}} \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 2 \\ 4 \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

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$$q_2 = \frac{a_2^1}{||a_2^1||}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\therefore \; Q = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{2}} \end{pmatrix} \qquad R = \begin{pmatrix} 2\sqrt{5} & 2\sqrt{5} \\ 0 & \sqrt{2} \end{pmatrix}$$

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