Three - Dimensional Transformation

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3D Point

- A 3D point **P** with coordinates (x, y, z) is represented as:

$$P = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

3D Transformation

- 3D transformations are represented by 4×4 matrixes:

$$\boxed{\begin{bmatrix} a & b & c & t_x \\ d & e & f & t_y \\ g & h & i & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

Three - Dimensional Transformation

- Manipulation, viewing and construction of three-dimensional graphic images requires the use of three dimensional geometric and coordinate transformations.
- These transformations are formed by composing the basic transformations of translation, scaling and rotation.
- Each of these transformations can be represented as a matrix transformation.
- Transformations are now represented as 4x4 matrices.

Three - Dimensional Transformation

- Very similar to 2D transformation.
- Scaling transformation:

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Geometric Transformations

- Object moves. Coordinate system remains stationary.
- Basic transformations are translation, scaling and rotation.

- Translation

$$V' = V + D$$

- Scaling

$$V' = SV$$

- Rotation

$$V' = RV$$

Translation

- The amount of the translation is added to or subtracted from the x, y, and z coordinates.
- In general, this is done with the equations:

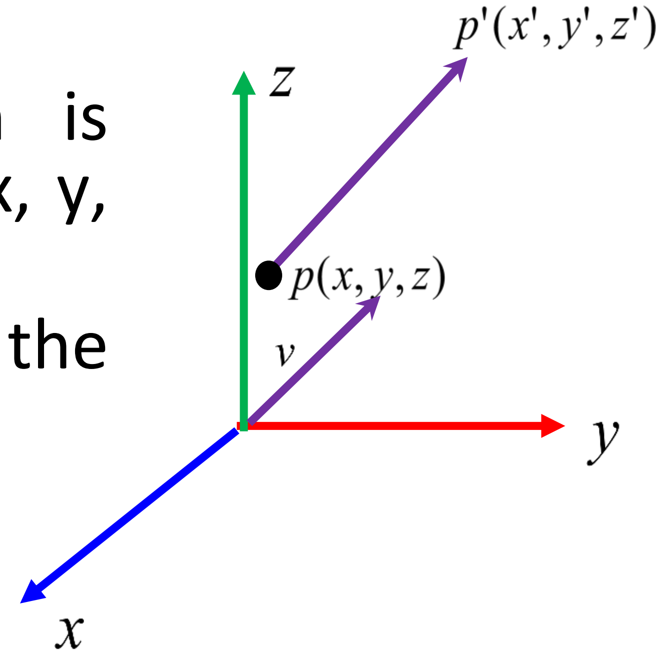
$$x' = x + T_x$$

$$y' = y + T_y$$

$$z' = z + T_z$$

- This can also be done with the matrix multiplication:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$



Scaling

- In general, this is done with the equations:

  $x_n = s_x * x$

  $y_n = s_y * y$

  $z_n = s_z * z$

- This can also be done with the matrix multiplication:

$$\begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation

- 3D rotation is done around a rotation axis.
- Fundamental rotations – rotate about x, y, or z axes.
- Counter-clockwise rotation is referred to as positive rotation.

Rotation

- The matrix form for rotation

– x axis

$$\begin{bmatrix} x \\ y_n \\ z_n \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

– y axis

$$, \begin{bmatrix} x_n \\ y \\ z_n \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

– z axis

$$\begin{bmatrix} x_n \\ y_n \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Rotation

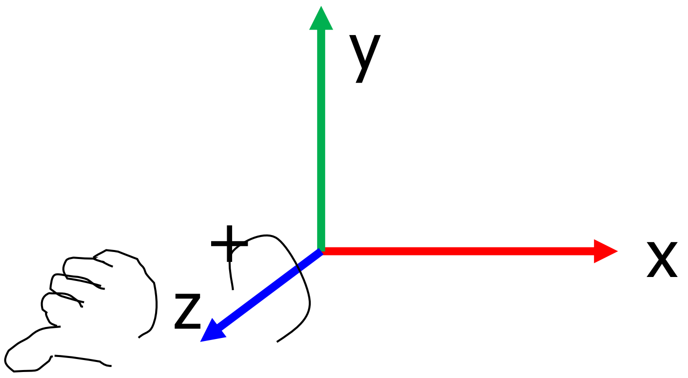
- Rotation about z – similar to 2D rotation.

$$x' = x\cos\alpha - y\sin\alpha$$

$$y' = x\sin\alpha + y\cos\alpha$$

$$z' = z$$

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha & 0 & 0 \\ \sin\alpha & \cos\alpha & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$



3D Reflections

About a plane:

- A reflection through the **xy** plane:

$$\begin{bmatrix} x \\ y \\ -z \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- A reflections through the **xz** and the **yz** planes are defined similarly.

Reflection

- Reflection through the *xy*-plane:

$$[\,T\,] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Reflection through the *yz*-plane:

$$[\,T\,] = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Reflection through the *xz*-plane:

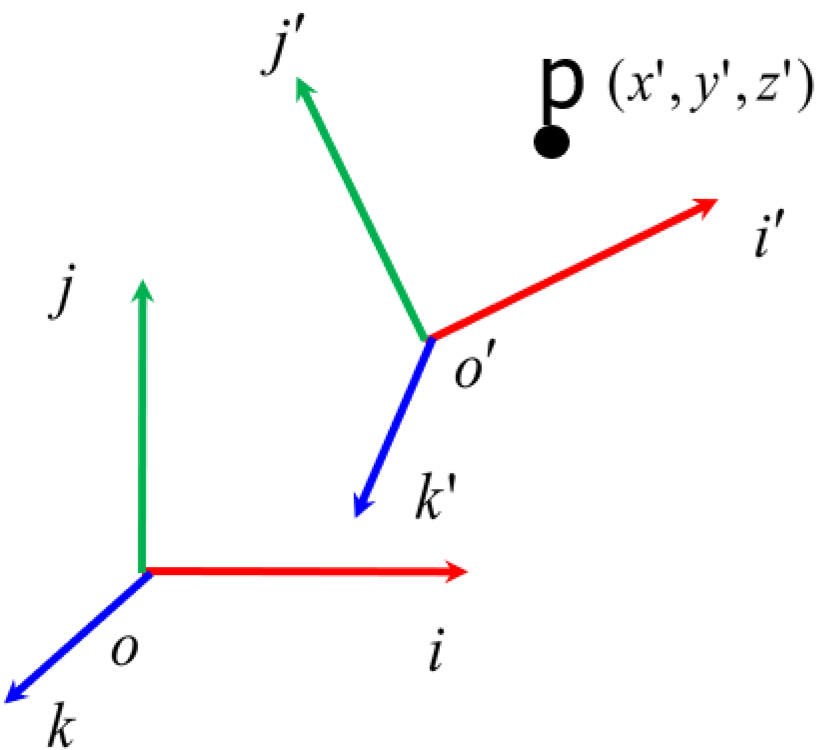
$$[\,T\,] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3D Coordinate Transformation

- Moving the observer who views the object.
- Keeping the object stationary.

3D Coordinate Transformation

- Transform object description.



$$x' = x - T_x$$
$$y' = y - T_y$$
$$z' = z - T_z$$

Composite 3D Transformation

- Equivalent to matrix multiplication or concatenation.
- A series of transformations on an object can be applied as a series of matrix multiplications.

Composite 3D Transformation

- Scaling can be done relative to the object center with a composite transformation.
- Scaling an object centered at (c_x, c_y, c_z) is done with the matrix multiplication:

$$\begin{bmatrix} x_n \\ y_n \\ z_n \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & c_x \\ 0 & 1 & 0 & c_y \\ 0 & 0 & 1 & c_z \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & -c_x \\ 0 & 1 & 0 & -c_y \\ 0 & 0 & 1 & -c_z \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$