INSTITUTE OF INFORMATION TECHNOLOGY

JAHANGIRNAGAR UNIVERSITY

Number of Assignment : 03

Submission Date : 22/09/2020

Course Tittle : Numerical Analysis

Course Code : ICT - 2105

Submitted To

Dr M Shamim Kaiser

Professor

IIT – JU

Submitted By

MD. Shakil Hossain

Roll – 2023

2nd year 1st Semester

IIT – JU

MD. Shakil Hossain

Roll-2023

3.1 From a table of diffenence for the function $f(x) = x^3 + 6x - 7$ for $x = -1, 0, 1, 2, 3, 4, 5$ continue the table to obtain $f(6)$ and $f(7)$.

Solution:

Given $f(x) = x^3 + 5x - 7$

x	$f(x)$
-1	$(-1)^3 + 5(-1) - 7 = -13$
0	$0^3 + 5(0) - 7 = -7$
1	$1^3 + 5(1) - 7 = -1$
2	$2^3 + 5(2) - 7 = 11$
3	$3^3 + 5(3) - 7 = 35$
4	$4^3 + 5(4) - 7 = 77$
5	$5^3 + 5(5) - 7 = 143$
6	$6^3 + 5(6) - 7 = 239$
7	$7^3 + 5(7) - 7 = 371$

From the table

$f(6) = 239$

$f(7) = 371$ Ans.

2020/9/22 11:06

3.5

Solution:

Let y member of students obtained less than x marks. So that the given table will become.

x	y
40	250
60	$250 + 120 = 370$
80	$370 + 100 = 470$
100	$470 + 70 = 540$
120	$540 + 50 = 590$

Our task is to find out $x(70)$ that is member of students who obtained less than 70 marks.

Using Newton forward difference law:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
60	370			
		100		
80	470		-30	
		70		10
100	570		-20	
		50		
120	490			

here $x = 70$, $x_0 = 60$, $h = 20$

$$p = \frac{x - x_0}{h} = \frac{70 - 60}{20} = \frac{1}{2}$$

2020/9/22 11:06

Using Newton Forward Difference formula.

We obtain

$$x(70) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 370 + \frac{1}{2}\times 100 + \frac{\frac{1}{2}(\frac{1}{2}-1)}{2}(-30) + \frac{\frac{1}{2}(\frac{1}{2}-1)(\frac{1}{2}-2)}{6}\cdot 10$$

$$= 424.375$$

$\therefore x(70) \approx 424$ students.

Therefor we can say there are 424 students who got under 70 marks. and we already know that 370 students got under 60 marks.

$\therefore$ (424-370) or 54 students who got marks between 60 and 70.

2020/9/22 11:07

3.6

Solution:

x	y	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
3	13						
		8					
4	21		2				
		10		0			
5	31		2		0		
		12		0		0	
6	43		2		0		0
		14		0		0	
7	57		2		0		
		16		0			
8	73		2				
		18					
9	91						

Here $h=1$. $x_0=3$

$$p = \frac{x-x_0}{h} = \frac{x-3}{1} = x-3$$

Using Newton's Forward Difference Formula

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y$$

$$= 13 + (x-3)8 + \frac{(x-3)(x-4)}{2}2 + \frac{(x-3)(x-4)(x-5)}{6}\times 0$$

$$= 13 + (x-3)8 + (x-3)(x-4)$$

$$= 13 + (x-3)(8+x-4)$$

$$= 13 + x^2 + x - 12$$

$$\therefore y = x^2 + x + 1$$

2020/9/22 11:07

The Value of the First term $= 1^2+1+1$
$= 3$ Ans.

The Value of the tenth term $= 10^2+10+1$
$= 111$ Ans.

Alternative Solution:

From the above table

$a_0 + a_1 3 + a_2(3)^2 = 13$

$\Rightarrow a_0 + 3a_1 + 9a_2 = 13$

$a_0 + a_1 4 + a_2(4)^2 = 21$

$\Rightarrow a_0 + 4a_1 + 16a_2 = 21$

$a_0 + a_1 5 + a_2(5)^2 = 31$

$\Rightarrow a_0 + 5a_1 + 25a_2 = 31$

Solving using Calculation

$a_0 = 1,\ a_1 = 1,\ a_2 = 1$

$\therefore y(1) = a_0 + a_1(1) + a_2(1)^2 = 1+1+1 = 3$

$y(10) = a_0 + a_1(10) + a_2(10)^2 = 1+10+10^2 = 111$

2020/9/22 11:07

3.7

Solution:

x	$f(x)$	Δ	Δ^2	Δ^3	Δ^4
0.20	1.6596				
		0.0102			
0.22	1.6698		0.0004		
		0.0106		−0.0002	
0.24	1.6804		0.0002		0.0004
		0.0108		0.0002	
0.26	1.6912		0.0004		−0.0003
		0.0112		−0.0001	
0.28	1.7024		0.0003		
		0.0115			
0.30	1.7139				

(i) Here, for stirling formula

$x = 0.23$ $\quad y_0 = 1.6804$ $\quad \Delta y_{-1} = 0.0106$

$x_0 = 0.24$ $\quad \Delta y_0 = 0.0108$ $\quad \Delta^2 y_{-1} = 0.0004$

$h = 0.02$

$$\therefore P = \frac{x - x_0}{h} = \frac{0.23 - 0.24}{0.02} = -0.5$$

Using Stirling formula

$$y(0.23) = y_0 + P\left(\frac{\Delta y_0 + \Delta y_0}{2}\right) + \frac{P^2}{2} \times \Delta^2 y_{+1} + \frac{P(P^2-1)}{3!} \times \left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{+2}}{2}\right) + \cdots$$

2020/9/22 11:07

$$= 1.6804 + (-0.5) \times \left(\frac{0.0108 + 0.0106}{2}\right) + (-0.5)^2 \times 0.0004 + \cdots$$

$\therefore y(0.23) = 1.6751$ Ans.

(ii) for $x = 0.29$ Using Bessel's formula

$x_0 = 0.28$ $\quad y_0 = 1.7024$ $\quad \Delta y_{0-1} = 0.0112$

$x = 0.29$ $\quad \Delta y_0 = 0.0115$ $\quad \Delta^2 y_{-1} = 0.0004$

$h = 0.02$

$$P = \frac{x - x_0}{h} = \frac{0.29 - 0.28}{0.02} = 0.5$$

Using Bessel's formula we get

$$y(0.29) = y_0 + P \times \left(\frac{\Delta y_{0-1} + \Delta y_0}{2}\right) + \frac{P^2}{2} \Delta^2 y_{-1} + \cdots$$

$$= 1.7024 + 0.5 \times \frac{(0.0112 + 0.0115)}{2} + (0.5)^2 \times \frac{1}{2} \times 0.0004 + \cdots$$

$$= 1.708125$$

$\therefore y(0.29) = 1.708125$ Ans.

2020/9/22 11:07

3.9

Solution:

x	$y=x^3$	Δ	Δ^2	Δ^3	Δ^4	Δ^5	Δ^6
6.1	226.981						
		11.347					
6.2	238.328		0.372				
		11.719		0.006			
6.3	250.047		0.378		0		
		12.097		0.006		0	
6.4	262.144		0.384		0		0
		12.481		0.006		0	
6.5	274.625		0.390		0		
		12.871		0.006			
6.6	287.426		0.396				
		13.267					
6.7	300.763						

(i) for $x = 6.36$ using stirling formula

$x_0 = 6.4 \quad y_0 = 262.144 \quad \Delta y_{-1} = 12.097$

$h = 0.1 \quad \Delta y_0 = 12.481 \quad \Delta^2 y_{-1} = 0.378$

$$P = \frac{x - x_0}{h} = \frac{6.36 - 6.4}{0.1} = -0.4$$

$$y(6.36) = y_0 + P\left(\frac{\Delta y_0 + \Delta y_0}{2}\right) + \frac{P^2}{2} \times \Delta^2 y_{-1} + \cdots$$

$$= 262.144 + (-0.4) \times \left(\frac{12.481 + 12.097}{2}\right) + (-0.4)^2 \times \frac{1}{2} \times 0.378$$

$$= 257.71644$$

$\therefore y(6.36) = 257.71644$

2020/9/22 11:07

(ii) for $x = 6.61$ using Bessius formula

$x_0 = 6.6$ $\quad y_0 = 287.496$ $\quad \Delta y_{-1} = 12.871$

$h = 0.1$ $\quad \Delta y_0 = 13.267$ $\quad \Delta^2 y_{-1} = 0.39$

$$P = \frac{x - x_0}{h} = \frac{6.61 - 6.6}{0.1} = 0.1$$

$$y(6.61) = y_0 + P \times \left(\frac{\Delta y_{-1} + \Delta y_0}{2}\right) + \frac{P^2}{2} \times \Delta^2 y_{-1} + \cdots$$

$$= 287.496 + 0.1 \times \left(\frac{12.871 + 13.267}{2}\right) + (0.1)^2 \times \frac{1}{2} \times 0.39 + \cdots$$

$$= 288.80845$$

$\therefore y(6.61) = 288.80845$ Ans.

2020/9/22 11:07

CO

3.12

Solution:

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1921	46				
		20			
1931	66		-5		
		15		2	
1941	81		-3		-3
		12		-1	
1951	93		-4		
		8			
1961	101				

Here $x = 1955$ $x_0 = 1961$ $h = 10$ $y_0 = 101$

$$\delta = \frac{x - x_0}{h} = \frac{1955 - 1961}{10} = -0.6$$

Using Newton Backword difference formula

$$y(1955) = y_0 + \delta y + \delta\frac{(\delta+1)}{2!}\nabla^2 y + \frac{\delta(\delta+1)(\delta+2)}{3!}\times\nabla^3 y + \cdots$$

$$= 101 + (-0.6)\times 8 + (-0.6)(-0.6+1)\times\frac{1}{2}\times(-4)$$

$$+ (-0.6)(-0.6+1)(-0.6+2)\times\frac{1}{6}\times(-1)$$

$$= 96.736$$

$\therefore y(1955) = 96.736$ Thousands Ans.

2020/9/22 11:08

3.13

Solution: As four points are given the given data can be appriximated by a third degree Polynomial in x.

Here $\Delta^4 y_0 = 0$ Substituting $\Delta = E-1$ and simplify.

We get

$$E^4 y_0 - 4E^3 y_0 + 6E^2 y_0 - 4E y_0 + y_0 = 0$$

Since $E^r y_0 = y_r$ the above equation will become

$$y_0 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$$

Substituting for $y_0 \cdot y_1 \cdot y_2$ and y_4 in the above we obtain $y_3 = 31$.

The tabulated function is 3^x and the exact value of $y(3)$ is 27. The error is due to the fact that the exponential function 3^x is approximated by means of a Polynomical in x of degree 3.

2020/9/22 11:08

3.30

Solution:

Given

$x: \quad -2 \quad -1 \quad 2 \quad 3$

$y(x): \quad -12 \quad -8 \quad 3 \quad 5$

From Lagranges formula we know

$$f(x) = \frac{(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)} y_1 + \frac{(x-x_1)(x-x_3)\cdots(x-x_n)}{(x_2-x_1)(x_2-x_3)\cdots(x_2-x_n)} y_2$$

$$+ \cdots + \frac{(x-x_1)(x-x_2)\cdots(x-x_{n-1})}{(x_n-x_1)(x_n-x_2)\cdots(x_n-x_{n-1})} y_n$$

for the given table using Lagranges Formula

$$y(x) = \frac{(x+1)(x-2)(x-3)}{(-2+1)(-2-2)(-2-3)} \times (-12) + \frac{(x+2)(x-2)(x-3)}{(-1+2)(-1-2)(-1-3)} \times (-8)$$

$$+ \frac{(x+2)(x+1)(x-3)}{(2-3)(2+1)(2+2)} \times 3 + \frac{(x-2)(x+1)(x+2)}{(3-2)(3+1)(3+2)} \times 5$$

$$= \frac{3}{5}(x+1)(x-2)(x-3) - \frac{2}{3}(x+2)(x-2)(x-3) - \frac{1}{4}(x+2)$$
$$(x+1)(x-3) + \frac{1}{4}(x-2)(x+1)(x+2)$$

2020/9/22 11:08

$$= (x-2)(x-3)\left(\frac{3}{5}x+\frac{3}{5}-\frac{2}{3}x-\frac{4}{3}\right)+\frac{1}{4}(x+2)(x+1)(x-2-x+3)$$

$$= (x^2-5x+6)\left(-\frac{1}{15}x-\frac{11}{15}\right)+\frac{1}{4}(x^2+3x+2)$$

$$= -\frac{x^3}{15}-\frac{11x^2}{15}+\frac{x^2}{3}+\frac{11x}{3}-\frac{2x}{5}-\frac{22}{5}+\frac{x^2}{4}+\frac{3x}{4}+\frac{1}{2}$$

$$= -\frac{x^3}{15}+x^2\left(\frac{1}{4}+\frac{1}{3}-\frac{11}{15}\right)+x\left(\frac{11}{3}-\frac{2}{5}+\frac{3}{4}\right)-\frac{39}{10}$$

$$\therefore y(x) = -\frac{x^3}{15}-\frac{3}{20}x^2+\frac{241}{60}x-\frac{39}{10} \quad \text{Ans.}$$

2020/9/22 11:08

The End