Institute of Information Technology

Subject: Numerical Techniques Laboratory

Name of the Exp.: Finding Root of an Equation by Newton Raphson Method

Introduction:

In scientific and engineering work, a frequently occurring problem is to find the roots of equations of the form $y = f(x) = 0$, i.e., finding the value of x where the value of $y = f(x)$ is equal to 0. In quadratic, cubic or a biquadratic equation, algebraic formulae are available for expressing the roots in terms of co-efficient. But in the case, where f(x) is a polynomial of higher degree or an expression involving transcendental functions, the algebraic methods are not applicable, and the help of numerical method must be taken to find approximate roots.

Objective of the Experiment:

1. To write a program in order to find out the real roots of a nonlinear equation by Newton Raphson Method.
2. Compare this method with the method of false position for the solution of the same equation.

Theory:

Let x_0 be an approximate root of $f(x)$ =0 and let $x_1 =$ $x_0 + h$ be the correct root so that $f(x_1) = 0$.
Expanding $f(x_1) = f(x_0 + h)$ by Taylor's series, we obtain

$$f(x_0) + hf'(x_0) + \frac{h^2}{2!} f''(x_0) + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots = 0$$

Neglecting the second and higher order derivatives, we get---

$$f(x_0) + hf'(x_0) = 0,$$

which gives

$$h = -\frac{f(x_0)}{f'(x_0)}$$

A better approximation than x_0 is therefore given by x_1, can be written as

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} \qquad \text{[since } x_1 = x_0 + h \text{]}$$

Successive approximations can be obtained by $x_2, x_3, \ldots\ldots\ldots\ldots\ldots, x_{n+1}$, where

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

which is the Newton-Raphson formula.

Now continue this iteration until $|x_{n+1} - x_n| <$ given accuracy.

Problems/Reports:

1. Write programs to find the real root of the following equations by using **Newton Raphson** Method.

   a) $f(x) = x^3 - 3x - 1 = 0$ correct to 5 decimal point, near

   b) $x\sin x + \cos x = 0$; correct to 5 decimal point, near x=3

   c) $x = e^{-x}$ correct to 5decimal point, near x=2

2. How does the program act if the starting value of x is 1? Explain the reason behind it.

3. Solve 1 (a) using **roots, fzero, fsolve** Matlab function
4. Solve 1(b) and 1(c) using **fzero, fsolve** Matlab function
5. What is the number of iteration required to find the root of problem 1(a), 1(b),1(c). Compare its value with that of obtained from the method of False Position to solve the same equation at same condition. Comment on which method is better from the view of number of iteration
6. Write the advantage and disadvantage of Newton Raphson Method.
7. Discuss on the Experiment.

USE OF MATLAB FUNCTION

ROOTS

ROOTS Find l roots of a polynomial.
ROOTS(C) computes the roots of the polynomial whose coefficients are the elements of the vector C.
to find all the roots of a polynomial x^3-3*x-1, use
```
C=[1  0  -3  -1]
X=roots(C)
```

FZERO
Example: to find a root of xsinx near X0=3 use
```
X=fzero(@(x)x*sin(x),3)
```

FSOLVE
Example: to find a root of xsinx near X0=3 use
```
X=FSOLVE(@(x)x*sin(x),3)
```