1 (a)

If R is reflexive, then

for every x,

since xRx, we have xSx.

$\therefore$ S is reflexive. [Proved]

1 (b)

Given, $xSy \Rightarrow xRy$ and yRx

$$\Rightarrow xRy \wedge yRx$$

$$\Rightarrow yRx \wedge xRy$$

$$\Rightarrow ySx$$

$\therefore$ S is symmetric [Proved]

1(c)

Suppose R is transitive, Then

$$xSy \wedge ySz \Rightarrow (xRy \wedge yRx) \wedge (yRz \wedge zRy)$$

$$\Rightarrow (xRy \wedge yRz) \wedge (zRy \wedge yRz)$$

$$\Rightarrow xRz \wedge zRx$$

$$\Rightarrow xSz$$

$\therefore$ If R is transitive, S is transitive.

[Proved].

1 (d)

If R is antisymmetric,

then $xSy \wedge ySx \Rightarrow xRy \wedge yRx$

$\Rightarrow x = y$

In the same way, S is antisymmetric.

[Proved].

1 (e)

By (a), (b) and (c),

we can say, S is an equivalence relation.

[Proved]

1 (f)

By (a), (c) and (d),

we can see, S is a partial order.

[Proved]

2 (a)

As said in given question,

For any x, xRx xor xRx is false,

so, xSx is also false.

$\therefore$ S is irreflexive [Proved]

2(b)

Given,

$xSy \Rightarrow xRy$ xor yRx

$\Rightarrow yRx$ xor xRy

$\Rightarrow ySx$

$\therefore xSy \Rightarrow ySx$

$\therefore$ S is symmetric.

2(c)

Let R be the subset relation as before.

Then, $\{1\} S \{1,2\}$ and $\{1,2\} S \{2\}$ can happen;

but $\{1\} S \{2\}$ is false.

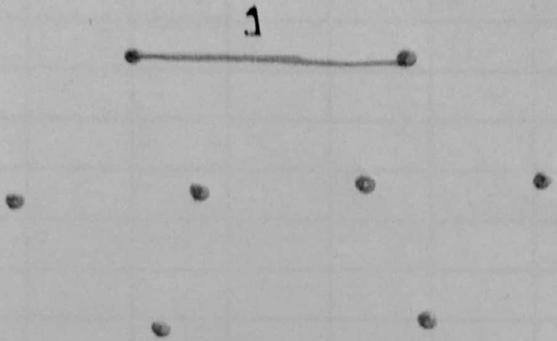
$\therefore$ If R is transitive, S is not necessarily transitive.
[Proved].

(3)

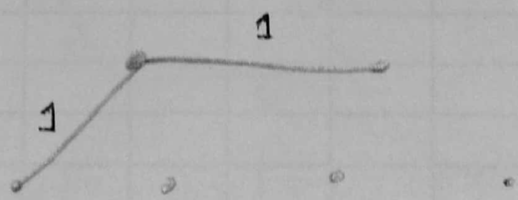
(1)

Using Prim's algorithm,

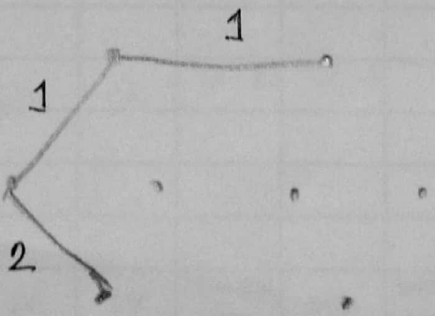
Firstly, we choose edge a, which has the lowest weight 1.



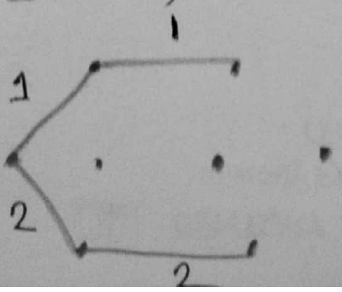
then, we add the adjacent edge with lowest ~~value~~ weight, b, weight 1.



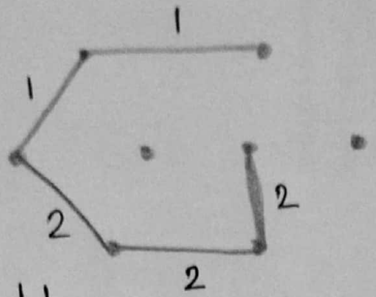
Then we add i.



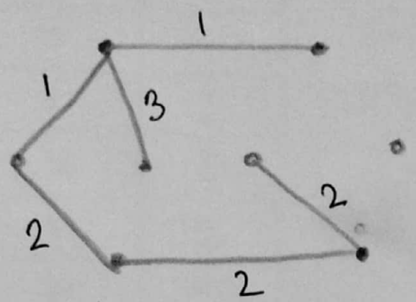
Then, add n,



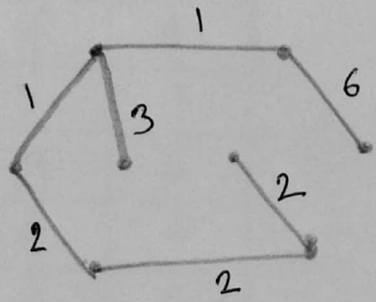
Then, add k.



Then, add c



Then, add e,



The cost $= 1+1+2+2+2+3+6$

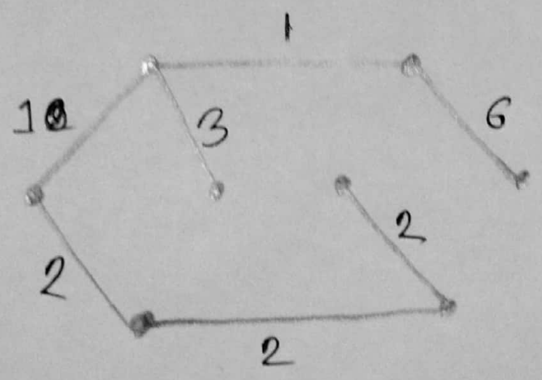
$= 17$

Order $\Rightarrow$ $a \rightarrow b \rightarrow i \rightarrow n \rightarrow k \rightarrow c \rightarrow e$.

(2) Using Kruskal's Algorithm:

The edge with weights are sorted here:

Edge	Weight
a	1
b	1
i	2
k	2
n	2
c	3
d	3
f	4
j	4
g	5
e	6
h	6
m	7



Order $\rightarrow$ a $\rightarrow$ b $\rightarrow$ i $\rightarrow$ n $\rightarrow$ k $\rightarrow$ c $\rightarrow$ e.