Lecture: 08

System and it’s Classification

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Representation of a System

- A system is a mathematical model of a physical process that transforms an input (or excitation) signal into an output (or response) signal with properties different from those of the input signal.
- The signals may be of the continuous-time or discrete-time variety, or a mixture of both.
- The interaction between a system and its associated signals is illustrated in the figure below.

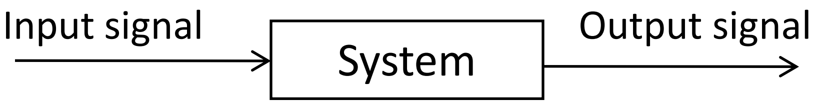


Figure: Block diagram of a system

- Let x and y be the input and output signals, respectively, of a system. Then the system is viewed as a transformation (or mapping) of x into y. This transformation is represented by the mathematical notation **y=Tx**, where T is the operator representing some well-defined rule by which x is transformed into y.

Representation of a System

A system is a defined by the type of input and output it deals with. Since we are dealing with signals, so in our case, our system would be a mathematical model, a piece of code/software, or a physical device, or a black box whose input is a signal and it performs some processing on that signal, and the output is a signal. The input is known as excitation and the output is known as response.

Example: In the figure a system has been shown whose input and output both are signals but the input is an analog signal. And the output is an digital signal. It means our system is actually a conversion system that converts analog signals to digital signals.

Representation of a System

Why do we need to convert an analog signal to digital signal.

- The first and obvious reason is that digital image processing deals with digital images, that are digital signals. So when ever the image is captured, it is converted into digital format and then it is processed.

- The second and important reason is, that in order to perform operations on an analog signal with a digital computer, you have to store that analog signal in the computer. Computers "talk" and "think" in terms of binary digital data. While a microprocessor can analyze analog data, it must be converted into digital form for the computer to make sense of it. And in order to store an analog signal, infinite memory is required to store it. So we convert that signal into digital format and then store it in digital computer and then performs operations on it.

- Digital signals propagate more efficiently than analog signals, largely because digital impulses are well defined and orderly. They're also easier for electronic circuits to distinguish from noise, which is chaotic. That is the chief advantage of digital communication modes.

- A typical telephone modem makes use of ADC to convert the incoming audio from a twisted-pair line into signals the computer can understand. In a digital signal processing system, an analog-to-digital converter is required if the input signal is analog.

Classification of Systems

- Based on different features, several methods of classifying systems are:

    1. Continuous-time and Discrete-time systems
    2. Invertible vs Non-Invertible System
    3. Systems with Memory and without Memory
    4. Casual and Noncasual Systems
    5. Linear and Nonlinear Systems
    6. Time-invariant and Time-varying Systems
    7. Linear Time-invariant Systems
    8. Stable Systems
    9. Feedback Systems

Continuous-Time Vs. Discrete-Time Systems

Continuous-time System:

- If the input signal x and output signal y of a system are continuous-time signals, then the system is called a continuous-time system.
- Figure-1 shows the graphical representation of a continuous-time system.

Discrete-time System:

- If the input signal x and output signal y of a system are discrete-time signals or sequences, then the system is called a discrete-time system.
- Figure-2 shows the graphical representation of a discrete-time system.

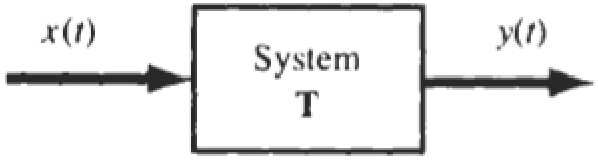


Figure-1: **Continuous-time system**

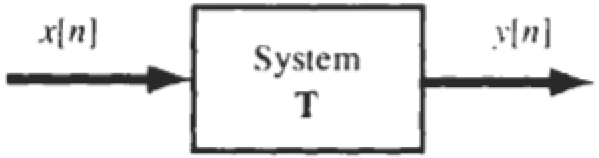


Figure-2: **Discrete-time system**

Continuous-Time Vs. Discrete-Time Systems

Example:

- Consider a discrete-time system whose output signal y[n] is the average of the three most recent values of the input signal x[n]; that is

$$y[n] = 1/3(x[n]+x[n-1]+x[n-2])$$

- Formulate the overall system operator H for this system; hence, develop a block diagram representation for it.

Solution:

- Let the discrete-time shift operator S^k denote a system that shifts the input signal x[n] by k time units to produce an output equal to x[n-k], as depicted in figure below.

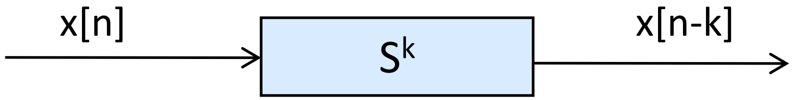


Figure: Discrete-time shift operator

- Accordingly, we may define the overall operator H for the given system as: $H=1/3(1+S+S^2)$.

Continuous-Time Vs. Discrete-Time Systems

- Two different but equivalent implementations of the system are shown in figures below.
    1. Cascade form of implementation: It uses two identical time shifters, namely $S^1 = S$.
    2. Parallel form of implementation: It uses two different time shifters, S and S^2, connected in parallel.
- In both cases, the system is made up of an interconnection of three functional blocks: two time shifters, an adder and a scalar multiplication.

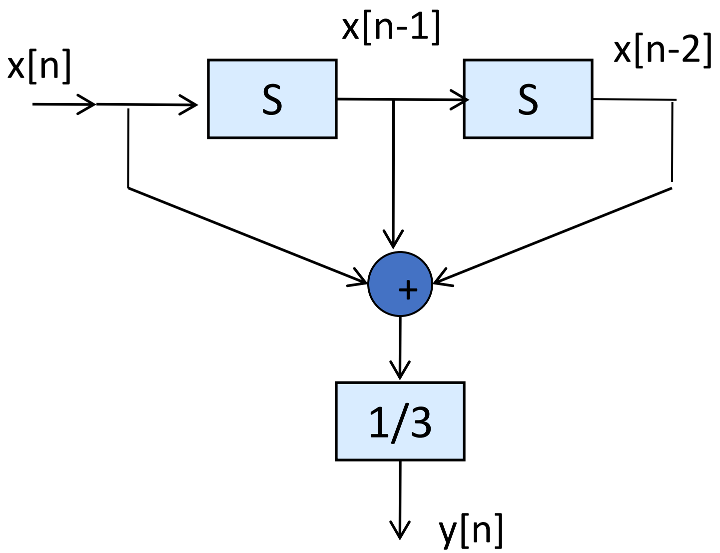


Figure: Cascade form

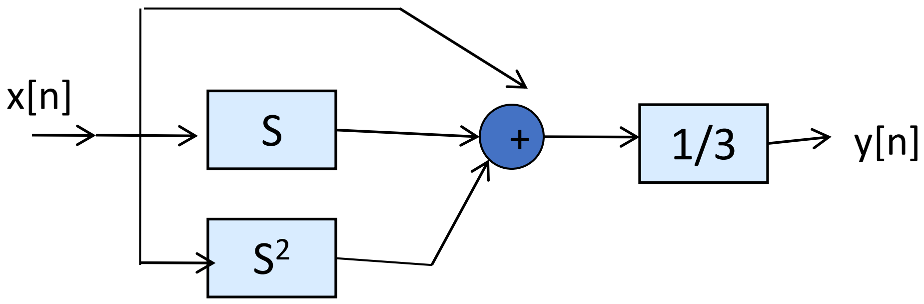


Figure: Parallel form

Invertible vs Non-Invertible System

Invertible System

If a system has a unique relationship between its input and output, the system is called the invertible system. In other words, a system is said to be an invertible system only if an inverse system exists which when cascaded with the original system produces an output equal to the input of the first system. The block diagram representation of an invertible system is shown in Figure-1.

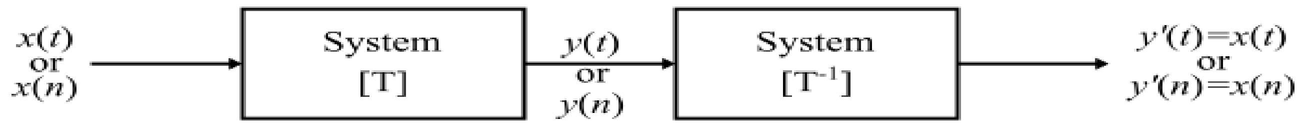


Figure-1

Mathematically, an invertible system is defined as,

$x(t) = T^{-1}[y(t)] = T^{-1}\{T[x(t)]\}$ ... for continuous time system

$x(n) = T^{-1}[y(n)] = T^{-1}\{T[x(n)]\}$ ... for discrete time system

Non-Invertible System

A system is said to be a non-invertible system if the system does not have a unique relationship between its input and output. **In other words**, if there is many to one mapping between input and output at any given instant for system, then the system is known as non-invertible system.

Mathematically, a non-invertible system is represented as,

$x(t) \neq T^{-1}\{T[x(t)]\}$ ... for continuous time system

$x(n) \neq T^{-1}\{T[x(n)]\}$ ... for discrete time system

The block diagram representation of a non-invertible system is shown in Figure-2.

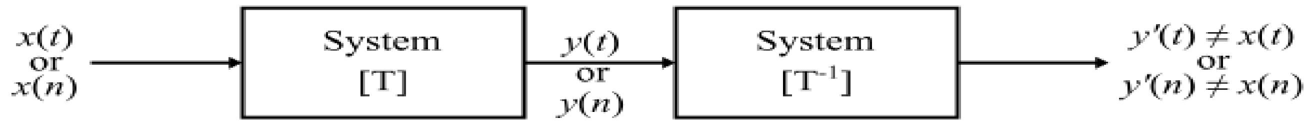


Figure-2

Invertible vs Non-Invertible System

Numerical Example: Find whether the given systems are invertible or non-invertible −

- $y(t) = 5x(t)$
- $y(t) = 3 + x(t)$
- $y(t) = 5x^2(t)$

Solution (1)

The given system is, $y(t) = 5x(t)$

Let, $x(t) = 3$, then the output of the system is, $y(t) = 5 \times 3 = 15$

Let, $x(t) = -3$, then the output of the system is, $y(t) = 5 \times (-3) = -15$

Hence, for different inputs, there is different outputs. Therefore, the system is invertible system.

Solution (2)

The expression describing the system is, $y(t) = 3 + x(t)$

For $x(t) = 10$, the output of the system is, $y(t) = 3 + 10 = 13$

And for $x(t) = -10$, the output of the system is, $y(t) = 3 + (-10) = -7$

Since, for the given system, different inputs lead to a different output. Therefore, the system is an invertible system.

Solution (3)

The given system is, $y(t) = 5x^2(t)$

Let $x(t) = 5$, the output of the system is, $y(t) = 5 \times 5^2 = 125$

Let $x(t) = -5$, then the output of the system is, $y(t) = 5 \times (-5)^2 = 125$

Since, for the given system, different inputs generate same output. Hence, the given system is a non-invertible system.

Systems With Memory Vs. Without Memory

- A system is said to have memory if its output signal depends on past or future values of the input signal.
- A system is said to be memoryless if its output signal at any time depends only on the present value of the input signal. Otherwise, the system is said to have memory.
- An example of a memoryless system is a resistor R, since the current *i(t)* flowing through it in response to the applied voltage *v(t)* is defined by-

$$i(t) = \frac{1}{R} v(t)$$

- The input-output relationship (Ohm's law) of a resistor is
- An example of a system with memory is an inductor L, since the current *i(t)* flowing through it in response to the applied voltage *v(t)* is defined by-

$$i(t) = \frac{1}{L} \int_{-\infty}^{t} v(\tau) d\tau$$

- That is, unlike the current through a resistor, that through an inductor at time t depends on all past values of the voltage *v(t)*; the memory of an inductor extends into the infinite past.

Systems With Memory Vs. Without Memory

Example:

- In a discrete-time system, the output signal y[n] is the average of the three most recent values of the input signal x[n]; that is, $y[n] = 1/3(x[n]+x[n-1]+x[n-2])$. Is the system memoryless?

Solution:

- Since the value of the output signal y[n] at time n depends on the present and two past values of the input signal x[n], the system is not memoryless, it has memory.

Example:

- A system is described by the input-output relation as $y[n]=x^2[n]$. Does the system have memory?

Solution:

- Since the value of the output signal y[n] at time n depends only on the present value of the input signal x[n], the system is memoryless.

Casual Vs. Noncasual Systems

Casual System:

- A system is called causal if its output y(t) at an arbitrary time $t=t_0$ depends only on the input x(t) for $t<=t_0$. That is, in a casual system, the present value of the output signal depends only on the present or past values of the input signal, not on its future values.
- Thus, in a causal system, it is not possible to obtain an output before an input is applied to the system.
- The moving average system described by *y[n]=1/3(x[n]+x[n-1]+x[n-2])* is casual, since the value of the output signal y[n] depends only on the present and two past values of the input signal x[n].

Noncasual System:

- A system is said to be noncasual its output signal depends on one or more future values of the input signal.
- The moving average system described by y[n]=1/3(x[n+1]+x[n]+x[n-1]) is noncasual, since the value of the output signal y[n] depends one future value, one present value and one past value of the input signal x[n].
- Examples of other noncasual systems are:

$$y(t)=x(t+1)$$

$$y[n]=x[-n]$$

$$y(t) = \begin{cases} x(3t) & t < 0 \\ x(t-1) & t \geq 0 \end{cases}$$

Causal

$t < 0$

$y(t) = x(3t)$

$t = -1$

$y(-1) = x(-3)$ ✓ C

past i/p

$t \geq 0$

$t = 0$

$y(0) = x(-1)$ ✓ C

$t = 1$

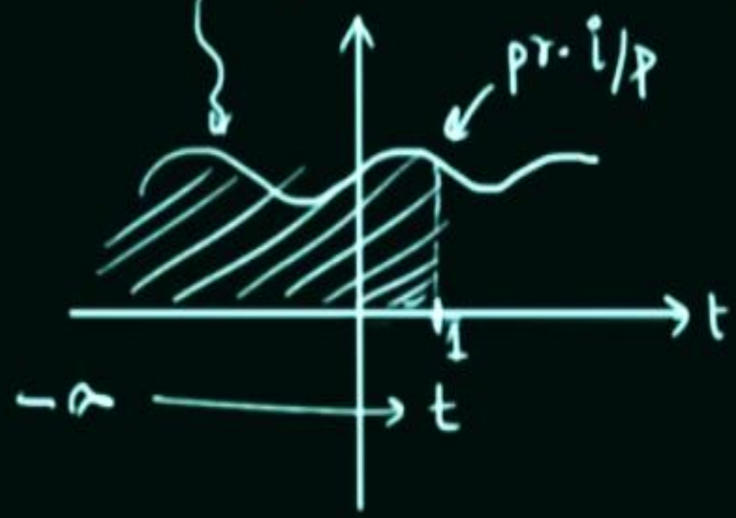
$y(1) = x(0)$ ✓ C

5) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

$= \int_{-\infty}^{t} x(t)\,dt \qquad t \rightarrow \tau$

past i/p

pr. i/p



Causal

6) $y(t) = \int_{-\infty}^{t+1} x(\tau)\,d\tau$

$x(t+1)$

$x(-\infty)$

$t=0 \quad x(1)$

future i/p

past i/p

Non-Causal

7) $y(t) = \int_{-\infty}^{t} x(3\tau)\,d\tau$

$x(3t)$

$x(-\infty)$

$t = 1 \Rightarrow x(3)$

fut. i/p

past i/p

Non-causal

Linear Vs. Nonlinear Systems

- A system is said to be linear in terms of the system input x(t) and the system output y(t) if it satisfies the following two properties:

1. Additivity (or Superposition):
   - Consider a system that is initially at rest. Let the system be subjected to an input $x(t)=x_1(t)$, producing an output $y(t)=y_1(t)$.
   - Suppose next that the same system is subjected to a different input $x(t)=x_2(t)$, producing a corresponding output $y(t)=y_2(t)$.
   - Then for the system to be linear, it is necessary that the composite input $x(t)=x_1(t)+x_2(t)$ produce the corresponding output $y(t)=y_1(t)+y_2(t)$.

$$\left.\begin{array}{l} x(t)=x_1(t) \rightarrow y(t)=y_1(t) \\ x(t)=x_2(t) \rightarrow y(t)=y_2(t) \end{array}\right\} \Rightarrow \begin{array}{l} x(t)=x_1(t)+x_2(t) \\ \rightarrow y(t)=y_1(t)+y_2(t) \end{array}$$

2. Homogeneity (or Scaling):
   - Consider a system that is initially at rest. Suppose that an input x(t) results in an output y(t).
   - Then the system is said to exhibit the property of homogeneity if, whenever the input x(t) is scaled by a constant factor a, the output y(t) is scaled by exactly the same constant factor a.

$$x(t) \rightarrow y(t) \Rightarrow ax(t) \rightarrow ay(t)$$

a = constant factor

$$y(t) = x(\sin t)$$

$$x(t) \longrightarrow \underbrace{\text{sys.}}_{\text{Linear Sys.}} \longrightarrow x(\sin t)$$

1. $y(t) = x(t^2)$

a) Law of add.

$$x_1(t) \longrightarrow \text{sys.} \longrightarrow y_1(t) = x_1(t^2)$$

$$x_2(t) \longrightarrow \text{Sys.} \longrightarrow y_2(t) = x_2(t^2)$$

$$y_1(t) + y_2(t) = x_1(t^2) + x_2(t^2)$$

$$x_1(t) + x_2(t) \longrightarrow \text{Sys.} \longrightarrow y'(t) = x_1(t^2) + x_2(t^2)$$

b) Law of hom.

$$x(t) \longrightarrow \text{sys.} \longrightarrow y(t) \longrightarrow 'k' \longrightarrow ky(t) = kx(t^2)$$

$$x(t) \longrightarrow 'k' \longrightarrow kx(t) \longrightarrow \text{sys.} \longrightarrow y'(t) = kx(t^2)$$

Linear Vs. Nonlinear Systems

EXAMPLE Consider a discrete-time system described by the input–output relation

$$y[n] = nx[n]$$

Show that this system is linear.

Solution: Let the input signal $x[n]$ be expressed as the weighted sum

$$x[n] = \sum_{i=1}^{N} a_i x_i[n]$$

We may then express the resulting output signal of the system as

$$\begin{aligned} y[n] &= n \sum_{i=1}^{N} a_i x_i[n] \\ &= \sum_{i=1}^{N} a_i n x_i[n] \\ &= \sum_{i=1}^{N} a_i y_i[n] \end{aligned}$$

where $y_i[n] = nx_i[n]$ is the output due to each input acting independently.

We thus see that the given system satisfies the principle of superposition and is therefore linear.

Linear Vs. Nonlinear Systems

EXAMPLE Consider next the continuous-time system described by the input–output relation

$$y(t) = x(t)x(t-1)$$

Show that this system is nonlinear.

Solution: Let the input signal $x(t)$ be expressed as the weighted sum

$$x(t) = \sum_{i=1}^{N} a_i x_i(t)$$

Correspondingly, the output signal of the system is given by the double summation

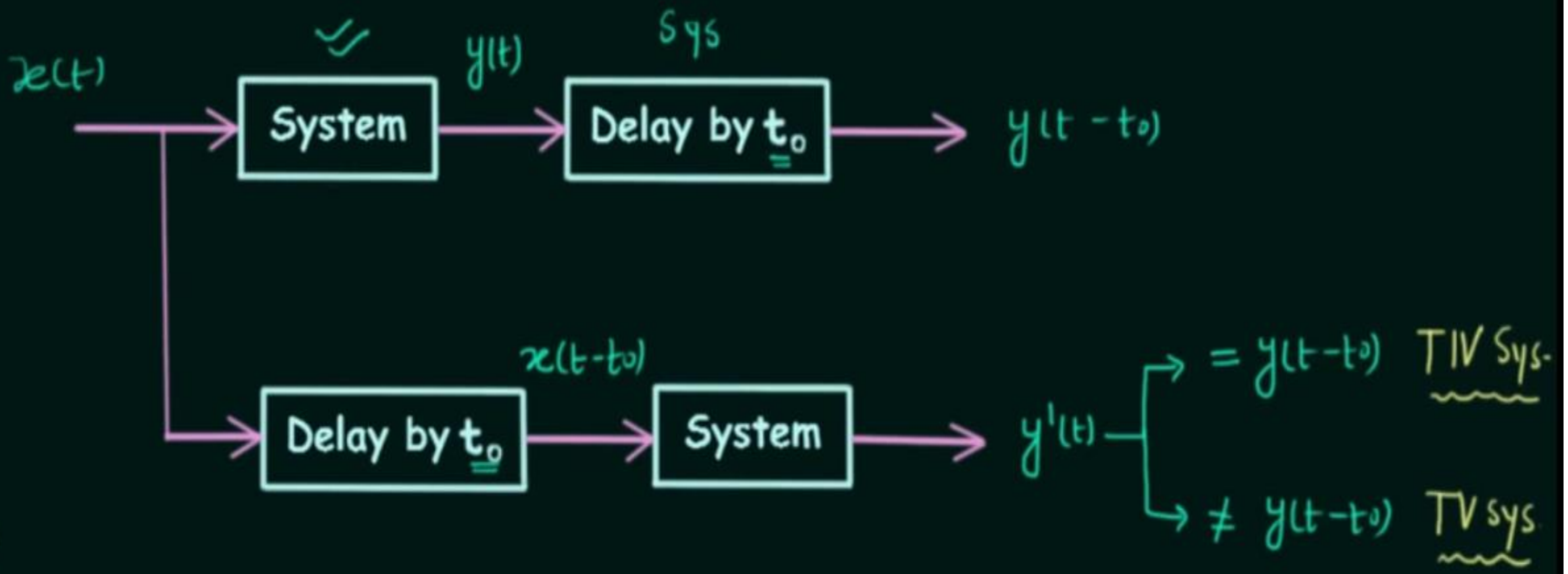
$$\begin{aligned} y(t) &= \sum_{i=1}^{N} a_i x_i(t) \sum_{j=1}^{N} a_j x_j(t-1) \\ &= \sum_{i=1}^{N} \sum_{j=1}^{N} a_i a_j x_i(t) x_j(t-1) \end{aligned}$$

The form of this equation is radically different from that describing the input signal $x(t)$. That is, here we cannot write $y(t) = \Sigma_{i=1}^{N} a_i y_i(t)$. Thus the system violates the principle of superposition and is therefore nonlinear.

Time-Invariant Vs. Time-varying Systems

- A system is called time-invariant if a time shift (delay or advance) in the input signal causes the same time shift in the output signal. That is, the characteristics of a time-invariant system do not change with time.
- Consider a continuous-time system whose input-output relation is described by $y_1(t)=H\{x_1(t)\}$, where H is the system operator, $x_1(t)$ is the input and $y_1(t)$ is the output signal of the system.
- Suppose that the input signal $x_1(t)$ is shifted in time by t_0 seconds, resulting in the new input $x_1(t-t_0)$. This operation may be described by writing $x_2(t)=x_1(t-t_0)=S^t{}_0\{x_1(t)\}$, where the operator $S^t{}_0$ represents a time shift equal to t_0 seconds for the situation at hand.

Time-Invariant Vs. Time-varying Systems



Ex 1: $y(t) = x(2t)$

t-s. ↓

$x(t) \longrightarrow \underbrace{Sys.}_{\text{TIV or TV}} \longrightarrow x(2t) = y(t)$

S-1 $\quad y(t) \xrightarrow{t_0} y(t-t_0) = x[2(t-t_0)] = x(2t-2t_0)$

S-2 $\quad x(t) \xrightarrow{t_0} x(t-t_0) \longrightarrow Sys. \longrightarrow x(2t-t_0)$ $\quad$ TV sys.

Stable Systems

- A system is said to be bounded-input, bounded-output (BIBO) stable if and only if every bounded input results in a bounded output. The output of such a system does not diverge if the input does not diverge.

- To put the condition for BIBO stability on a formal basis, consider a continuous-time system whose input-output relation is y(t)=H{x(t)}. The operator H is BIBO stable if the output signal y(t) satisfies the condition:

$$|y(t)| \le M_y < \infty \quad \text{for all } t$$

Whenever the input signals x(t) satisfy the condition

$$|x(t)| \le M_x < \infty \quad \text{for all } t$$

Both M_x and M_y represent some finite positive numbers.

Note:

- From an engineering perspective, it is important that a system of interest remain stable under all possible operating conditions. Only then is the system guaranteed to produce a bounded output for a bounded input. Unstable systems are usually to be avoided, unless some mechanism can be found to stabilize them.

Feedback Systems

- In a feedback system, the output signal is fed back and added to the input to the system as shown in the figure below.
- This is a special class of systems with great importance.

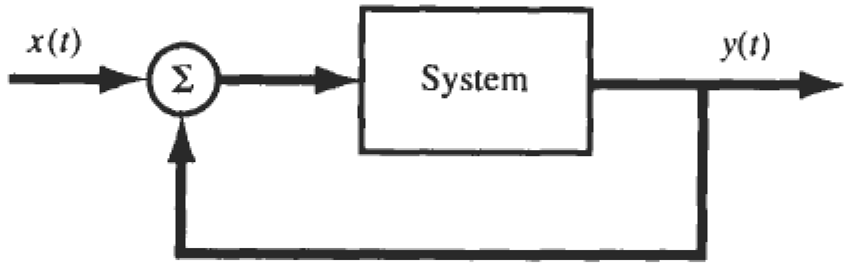


Figure-: Feedback system

Noise

- The term noise is used customarily to designate unwanted signals that tend to disturb the operation of a system and over which we have incomplete control. The sources of noise that may arise in practice depend on the system of interest.
- For example, in a communication system, there are many potential sources of noise affecting the operation of the system.
- In particular, we have the following two broadly defined categories of noise.
    1. External sources of noise
    2. Internal sources of noise

Noise

External sources of noise:

- This type of noise includes atmospheric noise, galactic noise, and human-made noise.

Internal sources of noise:

- This type of noise arises from spontaneous fluctuations of the current or voltage signal in electrical circuits. For this reason, this type of noise is commonly referred to as electrical noise.
- The omnipresence and inevitability of electrical noise in all kinds of electronic systems impose a basic limitation on the transmission or direction of signals.

Noise

➢ Figure below shows a sample waveform of electrical noise generated by a thermionic diode noise generator.

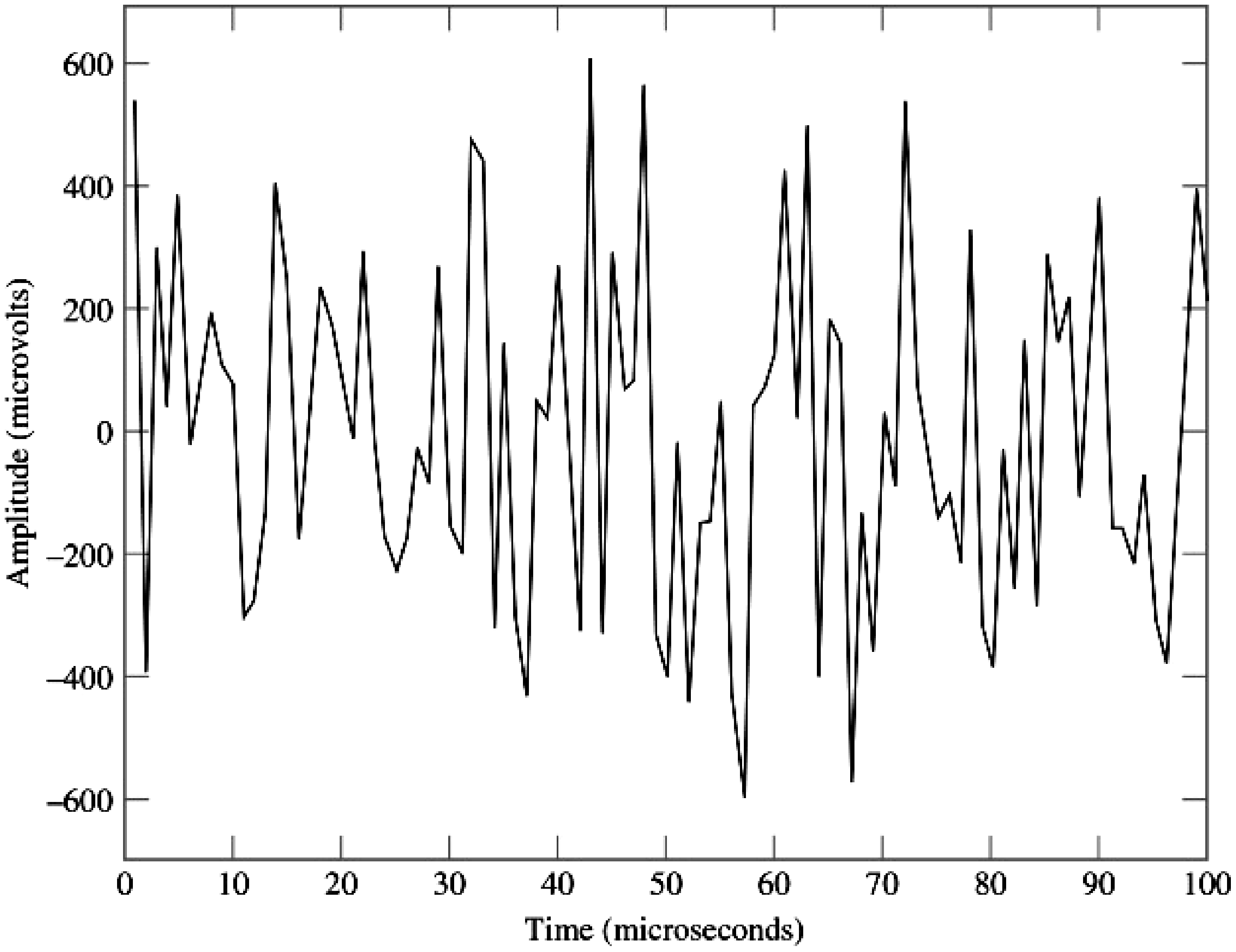


Figure: **Sample waveform of electrical noise**

Thermal Noise

- Thermal noise arises from the random motion of electrons in a conductor.
- It is a ubiquitous form of electrical noise.
- Let v(t) denote the thermal noise voltage appearing across the terminals of a resistor. Then the thermal noise so generated has the following two characteristics:

1. Time-averaged value:

$$\bar{v} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} v(t)dt \qquad (1.94)$$

2T = total observation interval of noise

As T → ∞, $\bar{v} \to 0$ Refer to Fig. 1.60.

2. Time-average-squared value:

$$\overline{v^2} = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} v^2(t)dt \qquad (1.95)$$

Thank You