DFT : Discrete Fourier Transform and IDFT Inverse DFT are mathematical operations used in Signal processing and Image processing.

A seq of discrete data points in t ⟶ DFT ⟶ Sequence of complex no in f-domain ⟶ IDFT ⟶ Sequence t domain

DFT and IDFT are computationally tentensive especially for larger sequence. However, they are widely used for audio / video / Image processing and data analytics. Various fast algorithm like Fast Fourier Transform (FFT) and IFFT are developed to efficiently compute DFT / IDFT and making them practical for real world application

DFT Eq^n

$x(n)$; $0 \leq n \leq N-1$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

Sequence in freq. domain

Discrete time signal

$$= \sum_{n=0}^{N-1} x(n)\, W_N^{kn} \qquad 0 \leq k \leq N-1$$

$W_N = e^{-j\frac{2\pi}{N}}$ is called phase factor or Twiddle factor

IDFT eq^n

$$x(n) = \sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}kn}$$

$$= \sum_{k=0}^{N-1} X(k)\, W_N^{-kn}$$

Compute the N point DFT of the signal (a) $x(n) = \delta(n)$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} \delta(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$\delta(n) = \begin{cases} 1 & n=0 \\ 0 & \text{else} \end{cases}$$

$$= \delta(0)\, e^{-j\frac{2\pi}{N}\cdot k\cdot 0}$$

$$= \delta(0) = 1$$

(b) $x(n) = \delta(n-n_0)$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} \delta(n-n_0)\, e^{-j\frac{2\pi}{N}kn}$$

$n - n_0 = 0$

$\therefore n = n_0$

$$\delta(n-n_0) = \begin{cases} 1 & n=n_0 \\ 0 & \text{else} \end{cases}$$

$$= 1\, e^{-j\frac{2\pi}{N}kn_0}$$

$$= e^{-j\frac{2\pi}{N}kn_0}$$

(c) $x(n) = a^n \qquad 0 \leq n \leq N-1$

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} \cancel{a^n}\, a^n e^{-j\frac{2\pi}{N}kn}$$

$$\sum_{N_1}^{N_2} a^n = \frac{a^{N_1} - a^{N_2+1}}{1-a} \qquad a \neq 1$$

$$= \sum_{n=0}^{N-1} \left(a\, e^{-j\frac{2\pi}{N}k}\right)^n$$

$$X(k) = \frac{1 - \left[a e^{-j\frac{2\pi}{N}k}\right]^N}{1 - a e^{-j\frac{2\pi}{N}k}}$$

$$= \frac{1 - a^N \cdot 1}{1 - a e^{-j\frac{2\pi}{N}k}}$$