ICT 1107: Physics

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In the Subsequent Slides, We Discuss

- ✓ **Waves & Its Types**
- ✓ Progressive Waves
- ✓ Power and Intensity of Wave Motion
- ✓ Stationary Waves
- ✓ Phase Velocity
- ✓ Group Velocity
- ✓ Architectural Acoustics
- ✓ Reverberation
- ✓ Sabine's Reverberation Formula

Waves

Waves are always around us and are present in a variety of forms.

Water waves are in the visible form; however, there are sound waves, radio waves etc which aren't visible but they exist!

Apart from sound waves, there are strong waves, visible light waves, microwaves, stadium waves, earthquake waves, sine waves, cosine waves etc which we encounter in our daily lives.

Waves

A wave is a movement caused in a medium from one point to another when an object comes in contact with it.

Chapter 1: Waves & Oscillations

Waves

There are three types of waves:

1. Mechanical waves (Longitudinal waves and transverse waves)

The different examples of mechanical waves are the vibration of a string, the surface wave generated on the surface of a liquid and solid, tsunami waves, ultrasounds, oscillations in spring, and waves in slink etc.

2. Electromagnetic waves (Examples: Light waves, Radio waves, thermal radiation, etc.)

3. Matter waves (Also known as de Broglie waves)

Chapter 1: Waves & Oscillations

Progressive/Travelling Waves

A **periodic wave** is a periodic disturbance that moves through a medium. The medium itself goes nowhere. The individual atoms and molecules in the medium oscillate about their equilibrium position, but their average position does not change. As they interact with their neighbors, they transfer some of their energy to them. The neighboring atoms in turn transfer this energy to their neighbors down the line. In this way the energy is transported throughout the medium, without the transport of any matter.

Chapter 1: Waves & Oscillations

Progressive/Travelling Waves

Consider a transverse harmonic wave traveling in the positive *x*-direction. Harmonic waves are sinusoidal waves. The displacement *y* of a particle in the medium is given as a function of *x* and *t* by

$$y(x,t) = A\sin(kx - \omega t + \varphi)$$

Here k is the wave number, $k = 2\pi/\lambda$, and $\omega = 2\pi/T = 2\pi f$ is the angular frequency of the wave. φ is called the phase constant.

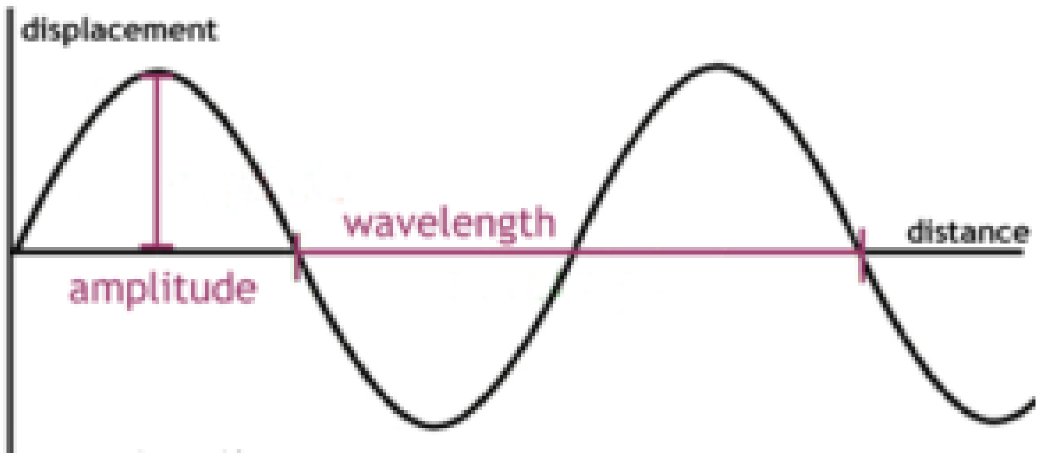
Progressive/Travelling Waves

At a fixed time t the displacement y varies as a function of position x as

$$A\sin(kx) = A\sin[(2\pi/\lambda)x]$$

The phase constant φ is determined by the initial conditions of the motion. If at $t = 0$ and $x = 0$ the displacement y is zero, then $\varphi = 0$ or π. If at $t = 0$ and $x = 0$ the displacement has its maximum value, then $\varphi = \pi/2$. The quantity $kx - \omega t + \varphi$ is called the phase.

Progressive/Travelling Waves

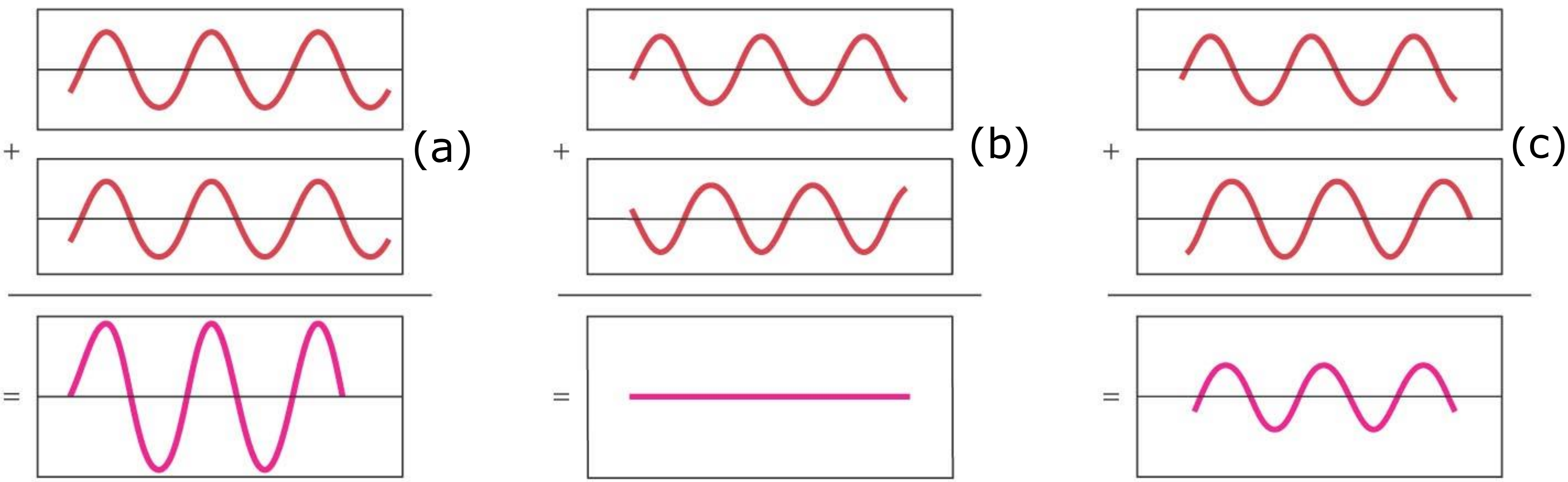


Chapter 1: Waves & Oscillations

Superposition Principle of Waves

The superposition principle states that when two waves pass through the same point, the resultant displacement is the sum of the individual displacements of the two waves.

In the figure below, (a) exhibits constructive interference and (b) exhibits destructive interference, and (c) they add partially destructively.



Chapter 1: Waves & Oscillations

Analytic Treatment of Superposition Principle of Waves

Please see a Text Book

p. 186 (Physics for Engineers Vol. 1)

Power and Intensity of Wave Motion

The equation of a plane progressive wave is given by

$$y = a\sin\frac{2\pi}{\lambda}(vt - x) \;\ldots\ldots\ldots(1)$$

The velocity of the particle is given by

$$u = \frac{dy}{dt} = \frac{2\pi v}{\lambda}a\cos(vt - x) \;\ldots\ldots\ldots\ldots(2)$$

Then the acceleration of the particle is

$$\frac{du}{dt} = \frac{d^2y}{dt^2} = -\frac{4\pi^2v^2}{\lambda^2}\, a\sin\frac{2\pi}{\lambda}(vt - x)$$

$$= -\frac{4\pi^2v^2}{\lambda^2}.y\ldots\ldots(3)$$

The kinetic energy per unit volume of the medium

$$= \frac{1}{2} \times mass \times (velocity)^2 = \frac{1}{2} \times \rho \times (u)^2$$

$$= \frac{1}{2} \times \rho \times \left(\frac{2\pi v}{\lambda} a cos(vt - x) \right)^2$$

$$= \frac{2\rho\pi^2 a^2 v^2}{\lambda^2} cos^2 \left(\frac{2\pi}{\lambda} (vt - x) \right) \ldots\ldots\ldots (4)$$

The work done per unit volume for a small displacement of the layer

$$= force \times displacement$$

$$= mass \times acceleration \times displacement$$

$$= \rho \times \frac{d^2y}{dt^2} \times dy$$

$$= \rho \times \frac{4\pi^2v^2}{\lambda^2}.y \times dy$$

The total work done when the layer is displaced from 0 to *y* is

$$= \int_0^y \rho \frac{4\pi^2v^2y}{\lambda^2}\, dy$$

$$= \frac{4\rho\pi^2v^2}{\lambda^2}\int_0^y y dy = \frac{4\rho\pi^2v^2}{\lambda^2}.\frac{y^2}{2}$$

$$= \frac{2\rho\pi^2 v^2}{\lambda^2} . y^2$$

$$= \frac{2\rho\pi^2 v^2}{\lambda^2} \left(a sin \frac{2\pi}{\lambda} (vt - x) \right)^2$$

$$= \frac{2\rho\pi^2 v^2 a^2}{\lambda^2} \ sin^2 \left(\frac{2\pi}{\lambda} (vt - x) \right)$$

Hence, the potential energy per unit volume of the medium

$$= \frac{2\rho\pi^2 v^2 a^2}{\lambda^2} \ sin^2 \left(\frac{2\pi}{\lambda} (vt - x) \right)$$

Chapter 1: Waves & Oscillations

The energy density of the plane progressive wave is given by

$$E = \frac{2\rho\pi^2 a^2 v^2}{\lambda^2} cos^2\left(\frac{2\pi}{\lambda}(vt - x)\right) + \frac{2\rho\pi^2 v^2 a^2}{\lambda^2}\ sin^2\left(\frac{2\pi}{\lambda}(vt - x)\right)$$

$$= \frac{2\rho\pi^2 a^2 v^2}{\lambda^2} = 2\rho\pi^2 a^2 \left(\frac{v}{\lambda}\right)^2 = 2\pi^2 n^2 a^2 \rho$$

Where *n* is the frequency of the wave.

If v is the velocity of the wave, then a new length v of the medium is set into motion every second; therefore, the energy transferred per second must be the energy contained in length v. This rate of flow of energy per unit area of the wave-front along the direction of propagation of the wave is called the intensity/energy current/ the energy flux of the plane progressive wave and is given by

$$I = E \times v = 2\pi^2 n^2 a^2 \rho v \text{ ergs/sec. cm}^2$$

This indicates that the intensity of the plane progressive wave is directly proportional to the square of the amplitude of the wave.

Stationary Waves

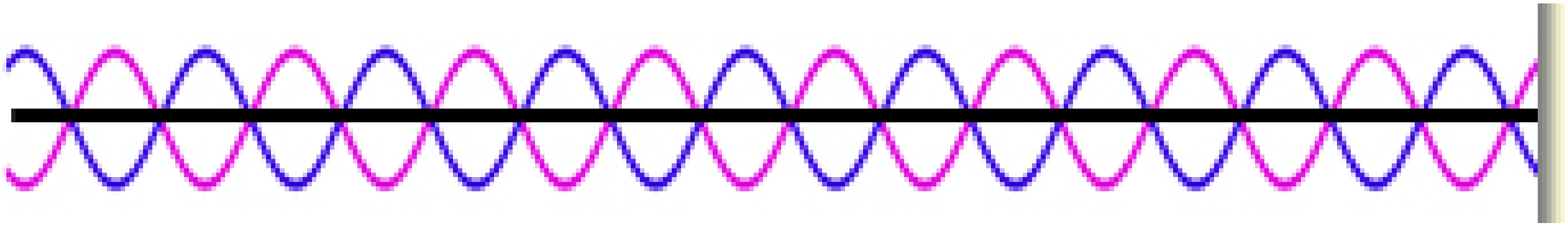
Stationary waves are also known as the standing waves. The conditions for standing waves are:

1. two waves travelling in opposite directions along the same path and in the same plane
2. the waves have the same speed
3. the waves have the same frequency
4. the waves have the same approximate amplitude

Chapter 1: Waves & Oscillations

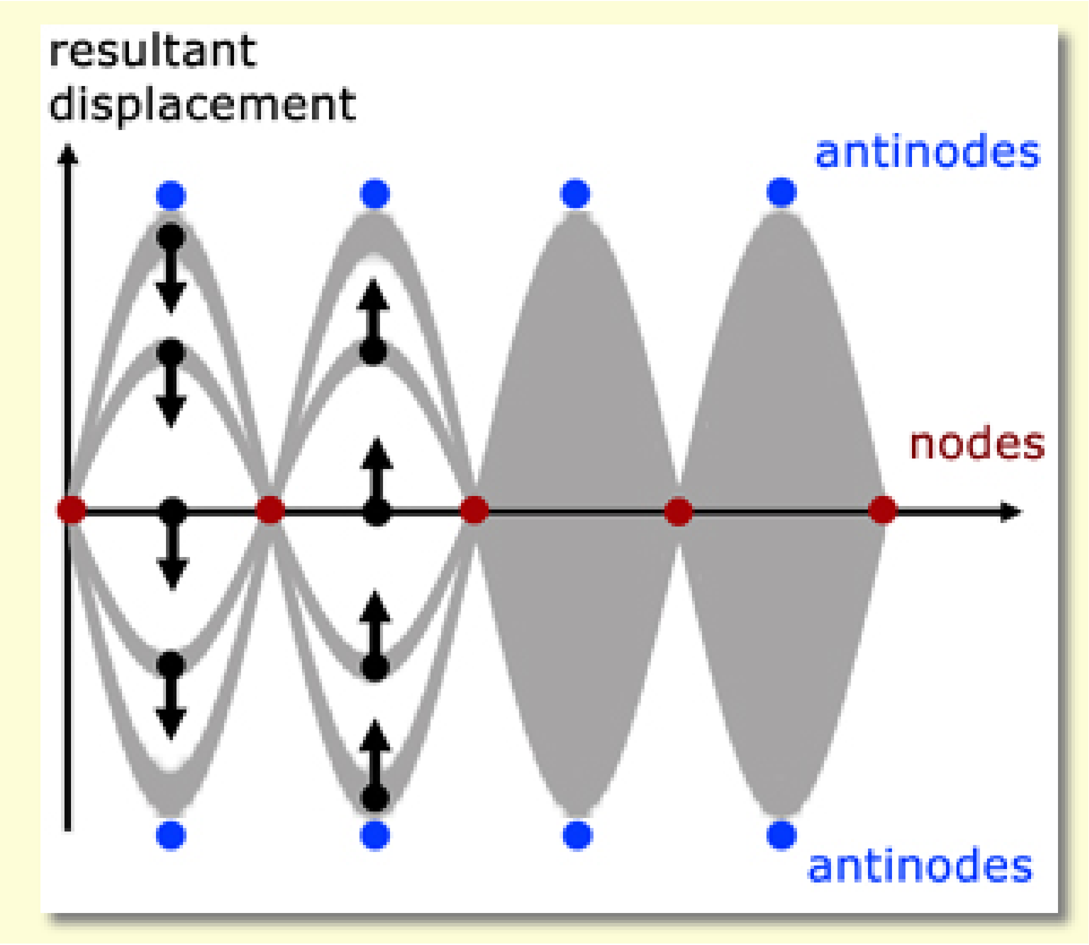
As a result of **superposition** (waves adding/subtracting), a resultant wave is produced. Depending on the phase difference between the waves, this resultant wave appears to move slowly to the right or to the left or disappear completely. It is only when the phase difference is exactly zero, that is when the two waves are exactly out of phase, that 'stationary waves' occur.

Stationary Waves



Chapter 1: Waves & Oscillations

Stationary Waves



Chapter 1: Waves & Oscillations

1. Two waves having the same amplitudes approach each other from opposite directions.

2. The two waves are 180^{o} out of phase with each other and therefore cancel out (black horizontal line).

3. The phase difference between the two waves narrows. The resultant grows but is not in phase with either of the two waves.

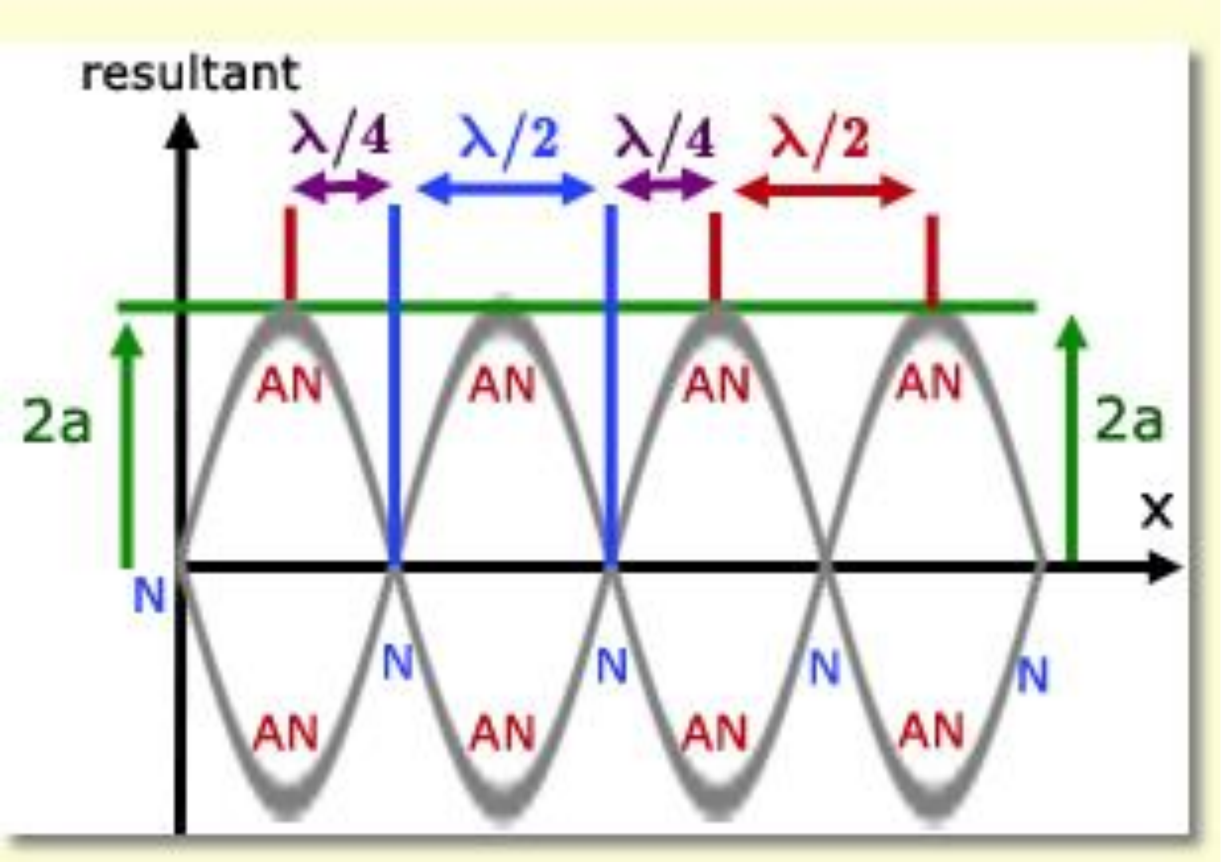
4. The phase difference between the two waves is narrower still. The resultant is larger but is still out of phase with the two waves.

5. The phase difference between the two waves is now zero. The resultant has its maximum value and is in phase with the two waves.

These 'in phase' waves produce an amplitude that is the sum of the individual amplitudes, the region being called an **antinode**.

Between two antinodes is a region where the superposition is zero. This is called a **node**.

Stationary Waves



Properties of Stationary Waves

1. separation of adjacent nodes is half a wavelength $(\lambda/2)$
2. separation of adjacent antinodes is also $\lambda/2$
3. hence separation of adjacent nodes and antinodes is $\lambda/4$
4. the maximum amplitude is 2a (twice that of a single wave)
5. a standing wave does not transfer energy(its two components however, do transfer energy in their respective directions)

Theory of Stationary Waves

Consider two waves:

$$y_R = a\sin(2\pi ft - kx)$$

$$y_L = a\sin(2\pi ft + kx)$$

where, $k = \dfrac{2\pi}{\lambda}$

When the two waves are superposed, the resultant displacement y_T is given by:

$$y_T = y_L + y_R$$

$$y_T = a\sin(2\pi ft - kx) + a\sin(2\pi ft + kx)$$

Therefore,

$$y_T = 2a\sin(2\pi ft)\cos(kx)$$

$$\sin(C-D) + \sin(C+D) = 2\sin(C)\cos(D)$$

C= **2πft** and **D**= ***kx***

Let,

$$A = 2a\cos(kx)$$

Then y_T can be rewritten in a form similar to that of a simple sine wave

$$y_T = A\sin(2\pi ft)$$

The term A takes on the significance of being the vertical displacement of the standing wave.

From the expression for A it can be seen that the magnitude of A depends on the lateral position x.

Consider the magnitude of A at different horizontal displacements (x) along the standing wave.

$A = 0$ at a node, $A = \pm\, 2a$ at an antinode

Phase Velocity

The velocity of a wave can be defined in many different ways, because there are different kinds of waves, and we can focus on different aspects or components of a given wave.

The wave function depends on both time, *t*, and position, *x*, *i.e.*,

$$A = A(x, t) \quad ,$$

where *A* is the amplitude.

Phase Velocity

A pure traveling wave is a function of ω and k as follows:

$$A(t,x) = A_0 \sin(\omega t - kx) \quad ,$$

where A_0 is the maximum amplitude.

A wave packet is formed from the superposition of several such waves, with different A, ω, and k:

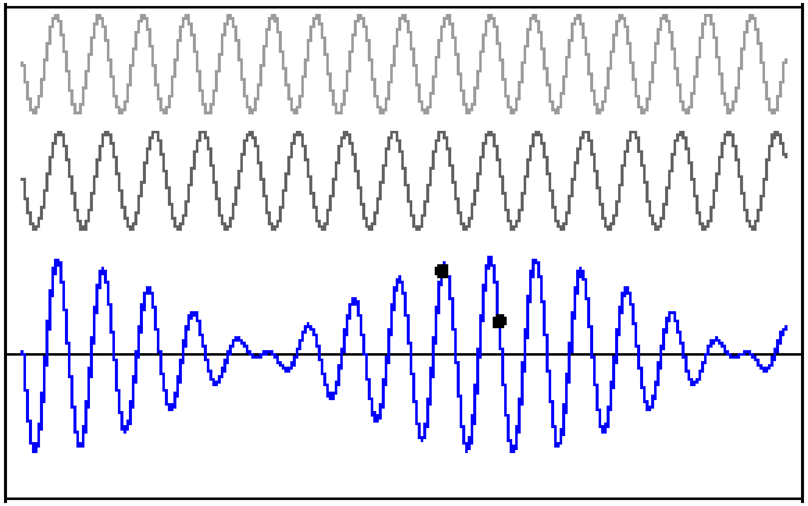
$$A(t,x) = \sum_n A_n \sin(\omega_n t - k_n x) \quad .$$

Phase Velocity

The superposition of two sine waves whose amplitudes, velocities and propagation directions are the same, but their frequencies differ slightly. We can write:

$$A(t) = A\sin(\omega_1 t) + A\sin(\omega_2 t) =$$

$$2A\cos\left(\left(\frac{\omega_1 - \omega_2}{2}\right)t\right)\sin\left(\left(\frac{\omega_1 + \omega_2}{2}\right)t\right).$$



The frequency of the sine term is that of the phase, the frequency of the cosine term is that of the "envelope", *i.e.*, the group velocity.

Phase Velocity

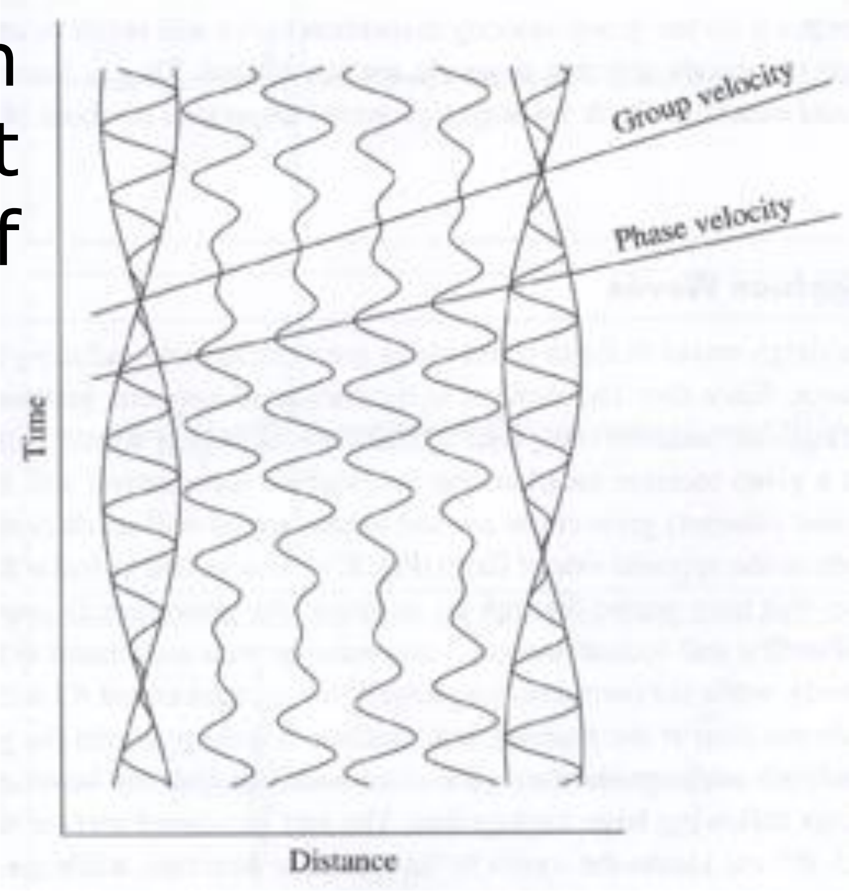
The phase velocity is the velocity at which the phase of any one frequency component of the wave propagates. One could pick one particular phase of the wave (for example the crest) and it would appear to travel at the phase velocity.

The group velocity is the velocity with which the envelope of the wave packet, propagates through space.

Chapter 1: Waves & Oscillations

Phase Velocity & Group Velocity

The speed at which a given phase propagates does not coincide with the speed of the envelope.



Note that the phase velocity is greater than the group velocity.

Phase Velocity & Group Velocity

The velocities of the individual waves which superpose to produce the wave packet representing the particle are different - the wave packet as a whole has a different velocity from the waves that comprise it

- Phase velocity: The rate at which the phase of the wave propagates in space

- Group velocity: The rate at which the envelope of the wave packet propagates

Chapter 1: Waves & Oscillations

Phase Velocity & Group Velocity

The phase velocity is the wavelength / period: $v = \lambda / \tau$

Since $f = 1/\tau$: $\boxed{v = f\,\lambda}$

In terms of k, $k = 2\pi / \lambda$, and
the angular frequency, $\omega = 2\pi / \tau$, this is: $v = \omega / k$

Phase Velocity & Group Velocity

The waveform moves at a rate that depends on the relative position of the component wavefronts as a function of time. This is the group velocity and is

$$v_g = \frac{d\omega}{dk}$$

which can be found if you have

$$\omega = vk = \frac{c}{n(k)}k \quad \text{giving} \quad v_g = v\left(1 - \frac{k}{n}\frac{dn}{dk}\right)$$

“Thank You”