Cramer’s Rule

Cramer’s rule is one of the important methods applied to solve a system of equations. In this method, the values of the variables in the system are to be calculated using the determinants of matrices. Thus, Cramer’s rule is also known as the determinant method.

Consider a system of linear equations with n variables $x_1, x_2, x_3, \ldots, x_n$ written in the matrix form AX = B.

Here,

A = Coefficient matrix (must be a square matrix)

X = Column matrix with variables

B = Column matrix with the constants (which are on the right side of the equations)

Now, we have to find the determinants as:

$D = |A|, Dx_1, Dx_2, Dx_3, \ldots, Dx_n$

Here, Dx_i for i = 1, 2, 3,…, n is the same determinant as D such that the column is replaced with B.

Thus,

$x_1 = Dx_1/D$; $x_2 = Dx_2/D$; $x_3 = Dx_3/D$; ….; $x_n = Dx_n/D$ {where D is not equal to 0}

Let’s have a look at the formulas of Cramer’s rule for 2×2 and 3×3 matrices.

Cramer’s Rule 2×2

Cramer’s rule for the 2×2 matrix is applied to solve the system of equations in two variables.

Let us consider two linear equations in two variables.

$a_1x + b_1y = c_1$

$a_2x + b_2y = c_2$

Let us write these two equations in the form of AX = B.

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

Here,

Coefficient matrix =

$$A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$$

Variable matrix =

$$X = \begin{bmatrix} x \\ y \end{bmatrix}$$

Constant matrix =

$$B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

$$D = |A| = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$$

And

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = c_1b_2 - c_2b_1$$

$$D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = a_1c_2 - a_2c_1$$

Therefore,

$x = D_x/D$ $y = D_y/D$

Cramer's Rule Example – 2×2

Solve the following system of equations using Cramer's rule:

2x – y = 5

x + y = 4

Solution:

Given,

2x – y = 5

x + y = 4

Let us write these equations in the form AX = B.

$$\begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Here,

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \end{bmatrix} \qquad B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Now,

D = |A|

$$= \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} \qquad = 2(1) - (-1)1 \qquad = 2 + 1 \qquad = 3 \neq 0$$

So, the given system of equations has a unique solution.

$$D_x = \begin{vmatrix} 5 & -1 \\ 4 & 1 \end{vmatrix} \qquad = 5(1) - (-1)(4) \qquad = 5 + 4 \qquad = 9$$

$$D_y = \begin{vmatrix} 2 & 5 \\ 1 & 4 \end{vmatrix} \qquad = 2(4) - 5(1) \qquad = 8 - 5 \qquad = 3$$

Therefore,

$x = D_x/D = 9/3 = 3$

$y = D_y/D = 3/3 = 1$

Cramer's Rule 3×3

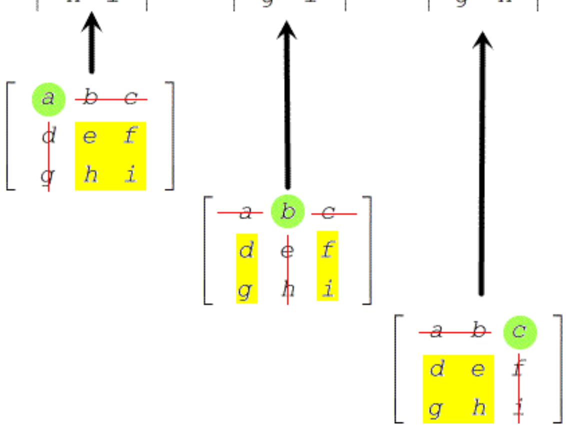
Formula to Find the Determinant of a 3×3 Matrix

- Given a 3×3 matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

- Its determinant can be calculated using the following formula.

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \cdot \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \cdot \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \cdot \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$



Example:

Find the determinant of matrix A

$$A = \begin{bmatrix} 6 & 2 & -4 \\ 5 & 6 & -2 \\ 5 & 2 & -3 \end{bmatrix}$$

$$\begin{aligned} |A| = \begin{vmatrix} 6 & 2 & -4 \\ 5 & 6 & -2 \\ 5 & 2 & -3 \end{vmatrix} &= 6 \cdot \begin{vmatrix} 6 & -2 \\ 2 & -3 \end{vmatrix} - (2) \cdot \begin{vmatrix} 5 & -2 \\ 5 & -3 \end{vmatrix} + (-4) \cdot \begin{vmatrix} 5 & 6 \\ 5 & 2 \end{vmatrix} \\ &= 6\,[-18 - (-4)] - 2\,[-15 - (-10)] - 4\,[10 - 30] \\ &= 6\,(-14) - 2\,(-5) - 4\,(-20) \\ &= -84 + 10 + 80 \\ &= 6 \end{aligned}$$

Now, it's time to go over the procedure on how to use Cramer's Rule in a linear system involving three variables.

Cramer's Rules for Systems of Linear Equations with Three Variables

- Given a linear system

$$\begin{array}{ccccccc} \text{x-column} & & & & \text{z-column} & & \\ \downarrow & & & & \downarrow & & \\ a_1x & + & b_1y & + & c_1z & = & \color{red}{d_1} \\ a_2x & + & b_2y & + & c_2z & = & \color{red}{d_2} \\ a_3x & + & b_3y & + & c_3z & = & \color{red}{d_3} \\ & & \uparrow & & & & \uparrow \\ & & \text{y-column} & & & & \text{constant-column} \end{array}$$

- Labeling each of the four matrices

Coefficient matrix:

$$D = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

X – matrix:

$$D_x = \begin{bmatrix} \color{red}{d_1} & b_1 & c_1 \\ \color{red}{d_2} & b_2 & c_2 \\ \color{red}{d_3} & b_3 & c_3 \end{bmatrix}$$

Y – matrix:

$$D_y = \begin{bmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{bmatrix}$$

Z – matrix:

$$D_z = \begin{bmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{bmatrix}$$

- To solve for x:

$$x = \frac{|D_x|}{|D|} = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

- To solve for y:

$$y = \frac{|D_y|}{|D|} = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

- To solve for z:

$$z = \frac{|D_z|}{|D|} = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}$$

Things to observe from the setup above:

1. The coefficients of variables x, y, and z make use of subscripted a, b, and c, respectively. While the constant terms use subscripted d.
2. The denominators to find the values of x, y, and z are all the same which is the determinant of the coefficient matrix (coefficients coming from the columns of x, y, and z).
3. To solve for x, the coefficients of the x−column is replaced by the constant column (in red).
4. To solve for y, the coefficients of the y−column is replaced by the constant column (in red).
5. In the same manner, to solve for z, the coefficients of the z−column is replaced by the constant column (in red).

Cramer’s Rule Example – 3×3

Question:

Solve the following system of equations using Cramer’s rule:

x + y + z =6

y + 3z = 11

x + z =2y or x – 2y + z = 0

Solution:

Given,

x + y + z =6

y + 3z = 11

x + z =2y or x – 2y + z = 0

Let us write these equations in the form AX = B.

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

Now,

$$D = |A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{vmatrix} = 1(1+6) - 1(0-3) + 1(0-1) = 7 + 3 - 1 = 9$$

D ≠ 0 so the given system of equations has a unique solution.

Also,

$$D_x = \begin{vmatrix} 6 & 1 & 1 \\ 11 & 1 & 3 \\ 0 & -2 & 1 \end{vmatrix} = 6(1+6) - 1(11-0) + 1(-22-0) = 42 - 11 - 22 = 9$$

$$D_y = \begin{vmatrix} 1 & 6 & 1 \\ 0 & 11 & 3 \\ 1 & 0 & 1 \end{vmatrix} = 1(11-0) - 6(0-3) + 1(0-11) = 11 + 18 - 11 = 18$$

$$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 0 & 1 & 11 \\ 1 & -2 & 0 \end{vmatrix} = 1(0+22) - 1(0-11) + 6(0-1) = 22 + 11 - 6 = 27$$

Thus,

$x = D_x/D = 9/9 = 1$

$y = D_y/D = 18/9 = 2$

$z = D_z/D = 27/9 = 3$

Cramer's Rule Questions

1. Solve the following system of equations by Cramer's rule:

   $2x - 3y + 5z = 11$

   $3x + 2y - 4z = -5$

   $x + y - 2z = -3$

2. Solve the following system of linear equations using Cramer's rule:

   $5x + 7y = -2$

   $4x + 6y = -3$

More Examples

1. Solve the following systems of linear equations by Cramer's rule:

(i) $5x - 2y + 16 = 0, x + 3y - 7 = 0$

(ii) $\frac{3}{x} + 2y = 12, \frac{2}{x} + 3y = 13$

(iii) $3x + 3y - z = 11,\ 2x - y + 2z = 9,\ 4x + 3y + 2z = 25$

(iv) $\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0, \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0, \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$

SOLUTION

(i) $5x - 2y + 16 = 0\ ,\ x + 3y - 7 = 0$

The given equations are

$$5x - 2y = -16 \quad \text{-----(1)}$$

$$x + 3y = 7 \quad \text{-----(2)}$$

$$\Delta = \begin{vmatrix} 5 & -2 \\ 1 & 3 \end{vmatrix}$$

$$= 15 + 2 = 17 \neq 0$$

$$\Delta_1 = \begin{vmatrix} -16 & -2 \\ 7 & 3 \end{vmatrix}$$

$$= -48 + 14 = -34$$

$$\Delta_2 = \begin{vmatrix} 5 & -16 \\ 1 & 7 \end{vmatrix}$$

$$= 35 + 16 = 51$$

By Cramer's rule we get

$$x = \frac{\Delta_1}{\Delta} = -\frac{34}{17} = -2$$

$$y = \frac{\Delta_2}{\Delta} = \frac{51}{17} = 3$$

$\therefore$ The solution is $x = -2\ ,\ y = 3$

(ii) $\frac{3}{x} + 2y = 12\ ,\ \frac{2}{x} + 3y = 13$

Put $\frac{1}{x} = a$ in the above equations.

$$3a + 2y = 12 \quad \text{-----(1)}$$

$$2a + 3y = 13 \quad \text{-----(2)}$$

$$\Delta = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 9 - 4 = 5$$

$$\Delta_1 = \begin{vmatrix} 12 & 2 \\ 13 & 3 \end{vmatrix} = 36 - 26 = 10$$

$$\Delta_2 = \begin{vmatrix} 3 & 12 \\ 2 & 13 \end{vmatrix} = 39 - 24 = 15$$

$$a = \frac{\Delta_1}{\Delta} = \frac{10}{5} = 2$$

$$y = \frac{\Delta_2}{\Delta} = \frac{15}{5} = 3$$

$\therefore \quad a = \frac{1}{x} = 2 \quad \Rightarrow \quad x = \frac{1}{2}$

$\therefore$ The solution is $x = \frac{1}{2}$, $y = 3$

(iii) $\mathbf{3x + 3y - z = 11}$, $\mathbf{2x - y + 2z = 9}$,

$\mathbf{4x + 3y + 2z = 25}$

The given equations are

$$3x + 3y - z = 11 \qquad \text{-----}(1)$$

$$2x - y + 2z = 9 \qquad \text{-----}(2)$$

$$4x + 3y + 2z = 25 \qquad \text{-----}(3)$$

$$\Delta = \begin{vmatrix} 3 & 3 & -1 \\ 2 & -1 & 2 \\ 4 & 3 & 2 \end{vmatrix}$$

$$= 3(-2-6) - 3(4-8) - 1(6+4)$$

$$= 3 \times -8 - 3 \times -4 - 1 \times 10$$

$$\Delta = -24 + 12 - 10 = -22$$

$$\Delta_1 = \begin{vmatrix} 11 & 3 & -1 \\ 9 & -1 & 2 \\ 25 & 3 & 2 \end{vmatrix}$$

$$= 11(-2-6) - 3(18-50) - 1(27+25)$$

$$= -88 + 96 - 52 = 96 - 140$$

$$\Delta_1 = -44$$

$$\Delta_2 = \begin{vmatrix} 3 & 11 & -1 \\ 2 & 9 & 2 \\ 4 & 25 & 2 \end{vmatrix}$$

$$= 3(18-50) - 11(4-8) - 1(50-36)$$

$$= 3 \times -32 - 11 \times -4 - 1 \times 14$$

$$= -96 + 44 - 14$$

$$\Delta_2 = -66$$

$$\Delta_3 = \begin{vmatrix} 3 & 3 & 11 \\ 2 & -1 & 9 \\ 4 & 3 & 25 \end{vmatrix}$$

$$= 3[(-25-27) - 3(50-36) + 11(6+4)]$$

$$= \quad 3 \times -52 \; - \; 3 \times 14 \; + \; 11 \times 10$$

$$\Delta_3 = -156 - 42 + 110 = -88$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-44}{-22} = 2$$

$$y = \frac{\Delta_2}{\Delta} = \frac{-66}{-22} = 3$$

$$z = \frac{-88}{-22} = 4$$

$\therefore$ The solution is $x = 2$, $y = 3$, $z = 4$.

(iv) $\dfrac{3}{x} - \dfrac{4}{y} - \dfrac{2}{z} - 1 = 0, \; \dfrac{1}{x} + \dfrac{2}{y} + \dfrac{1}{z} - 2 = 0,$

$\dfrac{2}{x} - \dfrac{5}{y} - \dfrac{4}{z} + 1 = 0$

The given equations are

$$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} = 1$$

$$\frac{1}{x} + \frac{2}{y} + \frac{1}{z} = 2$$

$$\frac{2}{x} - \frac{5}{y} - \frac{4}{z} = -1$$

Put $a = \dfrac{1}{x}$, $b = \dfrac{1}{y}$, $c = \dfrac{1}{z}$

$\therefore$

$$3a - 4b - 2c = 1 \qquad \text{-----(1)}$$

$$a + 2b + c = 2 \qquad \text{-----(2)}$$

$$2a - 5b - 4c = -1 \qquad \text{-----(3)}$$

$$\Delta = \begin{vmatrix} 3 & -4 & -2 \\ 1 & 2 & 1 \\ 2 & -5 & -4 \end{vmatrix}$$

$$= 3(-8+5) + 4(-4-2) - 2(-5-4)$$

$$= 3 \times -3 + 4 \times -6 - 2 - 9$$

$$= -9 - 24 + 18$$

$$= -33 + 18 = -15$$

$$\Delta_1 = \begin{vmatrix} 1 & -4 & -2 \\ 2 & 2 & 1 \\ -1 & -5 & -4 \end{vmatrix}$$

$$= \quad 1(-8+5) + 4(-8+1) - 2(-10+2)$$

$$= \quad 1 \times -3 \; + \; 4 \times -7 \; - \; 2 \times -8$$

$$\Delta_1 \quad = \quad -3 - 28 + 16$$

$$= \quad -31 + 16 \quad = \quad -15$$

$$\Delta_2 \quad = \quad \begin{vmatrix} 3 & 1 & -2 \\ 1 & 2 & 1 \\ 2 & -1 & -4 \end{vmatrix}$$

$$= \quad 3(-8+1) - 1(-4-2) - 2(-1-4)$$

$$= \quad 3 \times -7 \; - \; 1 \times -6 \; - \; 2 \times -5$$

$$= \quad -21 + 6 + 10$$

$$\Delta_2 \quad = \quad -21 + 16 \quad = \quad -5$$

$$\Delta_3 \quad = \quad \begin{vmatrix} 3 & -4 & 1 \\ 1 & 2 & 2 \\ 2 & -5 & -1 \end{vmatrix}$$

$$= \quad 3(-2+10) + 4(-1-4) + 1(-5-4)$$

$$= \quad 3 \times 8 \; + \; 4 \times -5 \; + \; 1 \times -9$$

$$= \quad 24 - 20 - 9$$

$$\Delta_3 \quad = \quad 24 - 29 \quad = \quad -5$$

$$a \quad = \quad \frac{\Delta_1}{\Delta} \quad = \quad \frac{-15}{-15} \quad = \quad 1$$

$$b \quad = \quad \frac{\Delta_2}{\Delta} \quad = \quad \frac{-5}{-15} \quad = \quad \frac{1}{3}$$

$$c \quad = \quad \frac{\Delta_3}{\Delta} \quad = \quad \frac{13}{-15} \quad = \quad \frac{-5}{-15} \quad = \quad \frac{1}{3}$$

$$a \quad = \quad \frac{1}{x} \quad = \quad 1 \quad \Rightarrow \quad x \quad = \quad 1$$

$$b \quad = \quad \frac{1}{y} \quad = \quad \frac{1}{3} \quad \Rightarrow \quad y \quad = \quad 3$$

$$c \quad = \quad \frac{1}{z} \quad = \quad \frac{1}{3} \quad \Rightarrow \quad z \quad = \quad 3$$

$\therefore$ The solutions of the given system equations are

$$x = 1 \quad , \quad y = 3 \quad , \quad z = 3$$

2. In a competitive examination, one mark is awarded for every correct answer while ¼ mark is deducted for every wrong answer. A student answered 100 questions and got 80 marks. How many questions did he answer correctly ? (Use Cramer's rule to solve the problem).

SOLUTION

Total number of questions $= 100$

Let the number of correct questions $= x$

Let the number of wrong questions $= y$

Given $x + y = 100$ ----- (1)

Also given for correct questions marks allotted $= 1$

For wrong questions marks allotted $= -\frac{1}{4}$

$\therefore$ Total marks allotted for x correct questions and y wrong questions $= 80$

$$x \times 1 + y \times -\frac{1}{4} = 80$$

$$x - \frac{y}{4} = 80$$

$$4x - y = 320 \quad \text{----- (2)}$$

Solving equations (1) and (2) using Cramer's rule.

$$\Delta = \begin{vmatrix} 1 & 1 \\ 4 & -1 \end{vmatrix} = -1 - 4 = -5$$

$$\Delta_1 = \begin{vmatrix} 100 & 1 \\ 320 & -1 \end{vmatrix} = -100 - 320 = -420$$

$$\Delta_2 = \begin{vmatrix} 1 & 100 \\ 4 & 320 \end{vmatrix} = 320 - 400 = -80$$

$$x = \frac{\Delta_1}{\Delta} = \frac{-420}{-5} = 84$$

$\therefore$ The number of correct questions $= 84$

3. A chemist has one solution which is 50% acid and another solution which is 25% acid. How much each should be mixed to make 10 litres of a 40% acid solution ? (Use Cramer's rule to solve the problem).

SOLUTION

Let the amount of 50 % acid solution be x litre and the amount of 25 % acid solution be y litre.

By the given data $\quad x + y = 10 \quad$ -----(1)

Also x litres of 50 % acid solution mixed with y litres of 25 % acid solution must give 10 litres of 40 % acid solution.

$$\therefore \quad x \times \frac{50}{100} + y \times \frac{25}{100} = 10 \times \frac{40}{100}$$

$$50x + 25y = 400$$

$$2x + y = 16 \qquad \text{-----(2)}$$

The matrix form of the above equations (1) and (2) is

$$\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 16 \end{bmatrix}$$

$$AX = B$$

where $A = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$, $B = \begin{bmatrix} 10 \\ 16 \end{bmatrix}$

$$X = A^{-1}B$$

$$|A| = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 - 2 = -1$$

$$X = \left(\frac{1}{|A|} \text{adj } A \right) B$$

$$= \frac{1}{-1} \begin{bmatrix} 1 & -2 \\ -1 & 1 \end{bmatrix}^T B$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = - \begin{bmatrix} 1 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 16 \end{bmatrix}$$

$$= - \begin{bmatrix} 10 - 16 \\ -20 + 16 \end{bmatrix} = - \begin{bmatrix} -6 \\ -4 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$$

$\Rightarrow \quad x = 6, \quad y = 4$

$\therefore$ Amount of 50 % acid solution is 6 litres.

Amount of 25 % acid solution is 4 litres.

4. A fish tank can be filled in 10 minutes using both pumps A and B simultaneously. However, pump B can pump water in or out at the same rate. If pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes. How long would it take each pump to fill the tank by itself ? (Use Cramer's rule to solve the problem).

SOLUTION

Let the pump A can fill the tank in x minutes and the pump B can fill the tank in y minutes.

In 1 minute A can fill $\frac{1}{x}$ part of the tank.

In 1 minute B can fill $\frac{1}{y}$ part of the tank.

Given A and B together fill the tank in 10 minutes.

$\therefore$ In minute A and B together fill $\frac{1}{10}$ part of the tank.

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{10} \quad ----- (1)$$

If the pump B is inadvertently run in reverse, then the tank will be filled in 30 minutes.

When A pump water in the tank and B inadvertently run in reverse, then In one minute $\frac{1}{30}$ part of the tank will be filled.

$$\therefore \quad \frac{1}{x} - \frac{1}{y} = \frac{1}{30} \quad ----- (2)$$

Put $\quad a = \frac{1}{x}$ and $b = \frac{1}{y}$

Equations (1) and (2) become

$$a + b = \frac{1}{10} \quad ----- (3)$$

$$a - b = \frac{1}{30} \quad ----- (4)$$

Solving using Cramer's rule,

$$\Delta = \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = -1 - 1 = -2$$

$$\Delta_1 = \begin{vmatrix} \frac{1}{10} & 1 \\ \frac{1}{30} & -1 \end{vmatrix}$$

$$= -\frac{1}{10} - \frac{1}{30} = \frac{-3 - 1}{30}$$

$$= -\frac{4}{30} = -\frac{2}{15}$$

$$\Delta_2 = \begin{vmatrix} 1 & \frac{1}{10} \\ 1 & \frac{1}{30} \end{vmatrix}$$

$$= \frac{1}{30} - \frac{1}{10} = \frac{1 - 3}{30}$$

$$= -\frac{2}{30} = -\frac{1}{15}$$

$$a = \frac{\Delta_1}{\Delta} = \frac{-\frac{2}{15}}{-2} = \frac{2}{15 \times 2} = \frac{1}{15}$$

$$b = \frac{\Delta_2}{\Delta} = \frac{-\frac{1}{15}}{-2} = \frac{1}{2 \times 15} = \frac{1}{30}$$

$$a = \frac{1}{x} = \frac{1}{15} \Rightarrow x = 15$$

$$b = \frac{1}{y} = \frac{1}{30} \Rightarrow y = 30$$

Hence the pump A can fill the tank in 15 minutes and the pump B can fill the tank in 30 minutes.

5. A family of 3 people went out for dinner in a restaurant. The cost of two dosai, three idlies and two vadais is ₹ 150. The cost of the two dosai, two idlies and four vadais is ₹ 200. The cost of five dosai, four idlies and two vadais is ₹ 250. The family has ₹ 350 in hand and they ate 3 dosai and six idlies and six vadais. Will they be able to manage to pay the bill within the amount they had ?

SOLUTION

Let x, y, z denote the dosai, Idly and Vadai. By the given data we have

$$2x + 3y + 2z = 150, \quad 2x + 2y + 4z = 200, \quad 5x + 4y + 2z = 250$$

$$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 2 & 2 & 4 \\ 5 & 4 & 2 \end{vmatrix} = 2[(4-16) - 3(4-20) + 2(8-10)]$$

$$= -24 + 48 - 4 = 20$$

$$\Delta_1 = \begin{vmatrix} 150 & 3 & 2 \\ 200 & 2 & 4 \\ 250 & 4 & 2 \end{vmatrix} = 50\begin{vmatrix} 3 & 3 & 2 \\ 4 & 2 & 4 \\ 5 & 4 & 2 \end{vmatrix}$$

$$= 50[3(4-16) - 3(8-20) + 2(16-10)]$$

$$\Delta_1 = 50[-36 + 36 + 12] = 600$$

$$\Delta_2 = \begin{vmatrix} 2 & 150 & 2 \\ 2 & 200 & 4 \\ 5 & 250 & 2 \end{vmatrix} = 50\begin{vmatrix} 2 & 3 & 2 \\ 2 & 4 & 4 \\ 2 & 5 & 2 \end{vmatrix}$$

$$= 50[2(8-20) - 3(4-20) + 2(10-20)]$$

$$\Delta_2 = 50[-24 + 48 - 20] = 200$$

$$\Delta_3 = \begin{vmatrix} 2 & 3 & 150 \\ 2 & 2 & 200 \\ 5 & 4 & 250 \end{vmatrix} = 50\begin{vmatrix} 2 & 3 & 3 \\ 2 & 2 & 4 \\ 5 & 4 & 5 \end{vmatrix}$$

$$= 50[2(10-16) - 3(10-20) + 3(8-10)]$$

$$\Delta_3 = 50[-12 + 30 - 6] = 600$$

$$x = \frac{\Delta_1}{\Delta} = \frac{600}{20} = 30$$

$$y = \frac{\Delta_2}{\Delta} = \frac{200}{20} = 10$$

$$z = \frac{\Delta_3}{\Delta} = \frac{600}{20} = 30$$

Cost of Dosai Rs. 30, Cost of Idly Rs. 10, Cost of Vadai Rs. 30

Cost of 3 Dosai + 6 Idlies + 6 Vadais $= 3 \times 30 + 6 \times 10 + 6 \times 30$

$$= 90 + 60 + 180 = 330 < 350$$

$\therefore$ The family is able to manage to pay the bill within the amount they had.

Answers for Exercise 1.4:

1. (i) x = -2, y = 3

(ii) x = 1/2 , y = 3

(iii) x = 2, y = 3, z = 4

(iv) x = 1, y = 3, z = 3

2. 84

3. 50% acid is 6 litres, 25% acid is 4 litres

4. Pump A : 15 minutes, Pump B : 30 minutes

5. ₹ 30/-, ₹ 10/-, ₹ 30/-, yes

More Exercise

1. Solve the following equations by using Cramer's rule

(i) 2x + 3y = 7; 3x + 5y = 9

(ii) 5x + 3y = 17; 3x + 7 y = 31

(iii) 2x + y − z = 3, x + y + z = 1, x − 2 y − 3z = 4

(iv) x + y + z = 6, 2x + 3y − z = 5, 6x − 2 y − 3z = − 7

(v) x + 4 y + 3z = 2, 2x − 6 y + 6z = −3, 5x − 2 y + 3z = −5

(i) $2x + 3y = 7, \quad 3x + 5y = 9$

Solution:

$$\Delta = \begin{vmatrix} 2 & 3 \\ 3 & 5 \end{vmatrix} = 10 - 9 = 1 \neq 0.$$

Since $\Delta \neq 0$, we can apply Cramer's rule and the system is consistent with unique solution.

$$\Delta x = \begin{vmatrix} 7 & 3 \\ 9 & 5 \end{vmatrix} = 7(5) - 9\,(3)$$
$$= 35 - 27 = 8$$

$$\Delta y = \begin{vmatrix} 2 & 7 \\ 3 & 9 \end{vmatrix} = 2(9) - 3\,(7)$$
$$= 18 - 21 = -3$$

$$\therefore \quad x = \frac{\Delta x}{\Delta} = \frac{8}{1} = 8$$

$$y = \frac{\Delta y}{\Delta} = \frac{-3}{1} = -3$$

$\therefore$ Solution set is {8, −3)

(ii) $5x + 3y = 17\,; \quad 3x + 7y = 31$

Solution:

$$\Delta = \begin{vmatrix} 5 & 3 \\ 3 & 7 \end{vmatrix} = 5(7) - 3(3)$$
$$= 35 - 9 = 26$$

Since $\Delta \neq 0$, we can apply Cramer's rule and the system is consistent with unique solution.

$$\Delta x = \begin{vmatrix} 17 & 3 \\ 31 & 7 \end{vmatrix} = 17(7) - 31\,(3)$$
$$= 119 - 93 = 26$$

$$\Delta y = \begin{vmatrix} 5 & 17 \\ 3 & 31 \end{vmatrix} = 5\,(31) - 17\,(3)$$
$$= 155 - 51 = 104.$$

$$x = \frac{\Delta x}{\Delta} = \frac{\cancel{26}}{\cancel{26}} = 1$$

$$y = \frac{\Delta y}{\Delta} = \frac{\overset{4}{\cancel{104}}}{\cancel{26}} = 4$$

$\therefore$ Solution set is {1, 4}

(iii) $2x + y - z = 3, x + y + z = 1, x - 2y - 3z = 4$

Solution:

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & -3 \end{vmatrix} = 2 \begin{vmatrix} 1 & 1 \\ -2 & -3 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} - 1 \begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix}$$

$$= 2\,(-3+2) - 1\,(-3-1) - 1\,(-2-1)$$

$$= 2(-1) - 1\,(-4) - 1\,(-3)$$

$$= -2 + 4 + 3 = 5.$$

Since $\Delta \neq 0$, we can apply Cramer's rule and the system is consistent with unique solution.

$$x=\begin{vmatrix}3&1&-1\\1&1&1\\4&-2&-3\end{vmatrix} = 3\begin{vmatrix}1&1\\-2&-3\end{vmatrix} -1\begin{vmatrix}1&1\\4&-3\end{vmatrix} -1\begin{vmatrix}1&1\\4&-2\end{vmatrix}$$

$= 3(-3+2) -1(-3-4) -1(-2-4)$

$= 3(-1) -1(-7) -1(-6)$

$= -3+7+6 = 10.$

$$\Delta y=\begin{vmatrix}2&3&-1\\1&1&1\\1&4&-3\end{vmatrix} = 2\begin{vmatrix}1&1\\4&-3\end{vmatrix} -3\begin{vmatrix}1&1\\1&-3\end{vmatrix} -1\begin{vmatrix}1&1\\1&4\end{vmatrix}$$

$= 2(-3-4) -3(-3-1) -1(4-1)$

$= 2(-7) -3(-4) -1(3)$

$= -14+12-3 = -5$

$$\Delta z=\begin{vmatrix}2&1&3\\1&1&1\\1&-2&4\end{vmatrix}$$

$$= 2\begin{vmatrix}1&1\\-2&4\end{vmatrix} -1\begin{vmatrix}1&1\\1&4\end{vmatrix} +3\begin{vmatrix}1&1\\1&-2\end{vmatrix}$$

$= 2(4+2) -1(4-1) + 3(-2-1)$

$= 2(6) -1(3) + 3(-3)$

$= 12-3-9$

$= 0$

$$\therefore x = \frac{\Delta x}{\Delta} = \frac{\overset{2}{\cancel{10}}}{\cancel{5}} = 2$$

$$y = \frac{\Delta y}{\Delta} = \frac{\cancel{-5}}{\cancel{5}} = -1$$

$$z = \frac{\Delta z}{\Delta} = \frac{0}{5} = 0$$

$\therefore$ Solution set is $\{2, -1, 0\}$

(iv) $x+y+z=6,\ 2x+3y-z=5,\ 6x-2y-3z=-7.$

Solution:

$$\Delta=\begin{vmatrix}1&1&1\\2&3&-1\\6&-2&-3\end{vmatrix}$$

$$= 1\begin{vmatrix}3&-1\\-2&-3\end{vmatrix} -1\begin{vmatrix}2&-1\\6&-3\end{vmatrix} +1\begin{vmatrix}2&3\\6&-2\end{vmatrix}$$

$= 1(-9-2) -1(-6+6) + 1(-4-18)$

$= 1(-11) -1(0) +1(-22)$

$= -11-22 = -33 \neq 0$

Since $\Delta \neq 0$, Cramer's rule can be applied and the system is consistent with unique solution.

$$\Delta x=\begin{vmatrix}6&1&1\\5&3&-1\\-7&-2&-3\end{vmatrix}$$

$$= 6\begin{vmatrix}3&-1\\-2&-3\end{vmatrix} -1\begin{vmatrix}5&-1\\-7&-3\end{vmatrix} +1\begin{vmatrix}5&3\\-7&-2\end{vmatrix}$$

$= 6(-9-2) -1(-15-7) + 1(-10+21)$

$= 6(-11) -1(-22) + 1(11)$

$= -66+22+11 = -33$

$$\Delta y=\begin{vmatrix}1&6&1\\2&5&-1\\6&-7&-3\end{vmatrix}$$

$$= 1\begin{vmatrix}5&-1\\-7&-3\end{vmatrix} -6\begin{vmatrix}2&-1\\6&-3\end{vmatrix} +1\begin{vmatrix}2&5\\6&-7\end{vmatrix}$$

$= 1(-15-7) -6(-6+6) + 1(-14-30)$

$= 1(-22) -6(0) + 1(-44)$

$= -22-44 = -66$

$$\Delta z=\begin{vmatrix}1&1&6\\2&3&5\\6&-2&-7\end{vmatrix}$$

$$= 1\begin{vmatrix}3&5\\-2&-7\end{vmatrix} -1\begin{vmatrix}2&5\\6&-7\end{vmatrix} +6\begin{vmatrix}2&3\\6&-2\end{vmatrix}$$

$= 1(-21+10) -1(-14-30) +6(-4-18)$

$= 1(-11) -1(-44) +6(-22)$

$= -11+44-132 = -99$

$$x = \frac{\Delta x}{\Delta} = \frac{\cancel{-33}}{\cancel{-33}} = 1$$

$$y = \frac{\Delta y}{\Delta} = \frac{\overset{2}{\cancel{-66}}}{\cancel{-33}} = 2$$

$$z = \frac{\Delta z}{\Delta} = \frac{\overset{3}{\cancel{-99}}}{\cancel{-33}} = 3$$

$\therefore$ Solution set is $\{1, 2, 3\}$

(v) $x + 4y + 3z = 2, 2x - 6y + 6z = -3,$

$5x - 2y + 3z = -5$

Solution:

$$\Delta = \begin{vmatrix} 1 & 4 & 3 \\ 2 & -6 & 6 \\ 5 & -2 & 3 \end{vmatrix}$$

$$= 1\begin{vmatrix} -6 & 6 \\ -2 & 3 \end{vmatrix} - 4\begin{vmatrix} 2 & 6 \\ 5 & 3 \end{vmatrix} + 3\begin{vmatrix} 2 & -6 \\ 5 & -2 \end{vmatrix}$$

$= 1(-18 + 12) - 4(6 - 30) + 3(-4 + 30)$

$= 1(-6) - 4(-24) + 3(26)$

$= -6 + 96 + 78 = 168 \neq 0.$

Since $\Delta \neq 0$, the system is consistent with unique solution and Cramer's rule can be applied.

$$\Delta x = \begin{vmatrix} 2 & 4 & 3 \\ -3 & -6 & 6 \\ -5 & -2 & 3 \end{vmatrix}$$

$$= 2\begin{vmatrix} -6 & 6 \\ -2 & 3 \end{vmatrix} - 4\begin{vmatrix} -3 & 6 \\ -5 & 3 \end{vmatrix} + 3\begin{vmatrix} -3 & -6 \\ -5 & -2 \end{vmatrix}$$

$= 2(-18 + 12) - 4(-9 + 30) + 3(6 - 30)$

$= 2(-6) - 4(21) + 3(-24)$

$= -12 - 84 - 72 = -168$

$$\Delta y = \begin{vmatrix} 1 & 2 & 3 \\ 2 & -3 & 6 \\ 5 & -5 & 3 \end{vmatrix} = 1\begin{vmatrix} -3 & 6 \\ -5 & 3 \end{vmatrix} - 2\begin{vmatrix} 2 & 6 \\ 5 & 3 \end{vmatrix} + 3\begin{vmatrix} 2 & -3 \\ 5 & -5 \end{vmatrix}$$

$= 1(-9 + 30) - 2(6 - 30) + 3(-10 + 15)$

$= 1(21) - 2(-24) + 3(5)$

$= 21 + 48 + 15 = 84$

$$\Delta z = \begin{vmatrix} 1 & 4 & 2 \\ 2 & -6 & -3 \\ 5 & -2 & -5 \end{vmatrix}$$

$$= 1\begin{vmatrix} -6 & -3 \\ -2 & -5 \end{vmatrix} - 4\begin{vmatrix} 2 & -3 \\ 5 & -5 \end{vmatrix} + 2\begin{vmatrix} 2 & -6 \\ 5 & -2 \end{vmatrix}$$

$= 1(30 - 6) - 4(-10 + 15) + 2(-4 + 30)$

$= 24 - 4(5) + 2(26)$

$= 24 - 20 + 52 = 56$

$$x = \frac{\Delta x}{\Delta} = \frac{\cancel{-168}}{\cancel{168}} = -1$$

$$y = \frac{\Delta y}{\Delta} = \frac{\cancel{84}}{\underset{2}{\cancel{168}}} = \frac{1}{2}$$

$$z = \frac{\Delta z}{\Delta} = \frac{\cancel{56}}{\underset{3}{\cancel{168}}} = \frac{1}{3}$$

Solution set is $\{-1, \frac{1}{2}, \frac{1}{3}\}$.

2. A commodity was produced by using 3 units of labour and 2 units of capital, the total cost is ₹62. If the commodity had been produced by using 4 units of labour and one unit of capital, the cost is ₹56. What is the cost per unit of labour and capital? (Use determinant method).

Solution:

Let Rs. x represents the cost per unit of labour and Rs. y represents the cost per unit of capital

Given $3x + 2y = 62$

$4x + y = 56$

$$\Delta = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix} = 3(1) - 4(2) = 3 - 8 = -5$$

Since $\Delta \neq 0$, the system is consistent with unique solution and Cramer's rule can be applied.

$$\Delta x = \begin{vmatrix} 62 & 2 \\ 56 & 1 \end{vmatrix} = 62(1) - 56(2) = 62 - 112 = -50$$

$$\Delta y = \begin{vmatrix} 3 & 62 \\ 4 & 56 \end{vmatrix} = 3(56) - 4(62)$$

$= 168 - 248 = -80$

$$\therefore \quad x = \frac{\Delta x}{\Delta} = \frac{\overset{10}{\cancel{-50}}}{\cancel{-5}} = 10$$

$$y = \frac{\Delta y}{\Delta} = \frac{\overset{16}{\cancel{-80}}}{\cancel{-5}} = 16.$$

$\therefore$ Cost per unit of labour is Rs. 10 and the cost per unit of capital is Rs. 16.

3. A total of ₹8,600 was invested in two accounts. One account earned 4 3/4 % annual1 interest and the other earned 6 1/2 % annual interest. If the total interest for one year was ₹431.25, how much was invested in each account? (Use determinant method).

Solution:

Let the amount invested in the two accounts be Rs x and Rs. y respectively

By the given data, $x + y = 8600$ ----- (1)

$$4\frac{3}{4}\times\frac{x}{100}+6\frac{1}{2}\times\frac{y}{100} = 431.25$$

$$\left[\therefore \text{ interest} = \frac{PNR}{100}\right]$$

$$\Rightarrow \frac{19x}{400}+\frac{13y}{200} = 431.25$$

$$\Rightarrow \frac{19x+26y}{400} = 431.25$$

$$19x + 26y = 172500 \quad \text{----- (2)}$$

$$\Delta = \begin{vmatrix} 1 & 1 \\ 19 & 26 \end{vmatrix} = 1(26) - 1(19)$$

$$= 26 - 19 = 7$$

$$\Delta x = \begin{vmatrix} 8600 & 1 \\ 172500 & 26 \end{vmatrix} = 8600(26) - 1(172500)$$

$$= 223600 - 172500 = 51100$$

$$\Delta y = \begin{vmatrix} 1 & 8600 \\ 19 & 172500 \end{vmatrix} = 1(172500) - 19(8600)$$

$$= 172500 - 163400 = 9100$$

$$\therefore \quad x = \frac{\Delta x}{\Delta} = \frac{51100}{7} = 7300$$

$$y = \frac{\Delta y}{\Delta} = \frac{9100}{7} = 1300$$

$\therefore$ Investment in the interest of $4\frac{3}{4}$ % account is Rs. 7300 and investment in the rate of $6\frac{1}{2}$ % account is Rs. 1300.

4. At marina two types of games viz., Horse riding and Quad Bikes riding are available on hourly rent. Keren and Benita spent ₹780 and ₹560 during the month of May.

Name	Number of hours		Total amount spent (in ₹)
	Horse Riding	Quad Bike Riding	
Keren	3	4	780
Benita	2	3	560

Find the hourly charges for the two games (rides). (Use determinant method).

Solution:

Let the hourly charge for horse riding be Rs. x and the hourly charge for quad bike be Rs. y from the given data, $3x + 4y = 780$

$$2x + 3y = 560$$

$$\Delta = \begin{vmatrix} 3 & 4 \\ 2 & 3 \end{vmatrix} = 3(3) - 2(4) = 9 - 8 = 1 \neq 0$$

Since $\Delta \neq 0$, the system is consistent with unique solution and Cramer's rule can be applied.

$$\Delta x = \begin{vmatrix} 780 & 4 \\ 560 & 3 \end{vmatrix} = 780(3) - 4(560)$$

$$= 2340 - 2240$$

$$= 100$$

$$\Delta y = \begin{vmatrix} 3 & 780 \\ 2 & 560 \end{vmatrix} = 3(560) - 2(780)$$

$$= 1680 - 1560$$

$$= 120$$

$$\therefore \quad x = \frac{\Delta x}{\Delta} = \frac{100}{1} = 100$$

$$y = \frac{\Delta y}{\Delta} = \frac{120}{1} = 120$$

$\therefore$ Hourly charges for the two rides are Rs.100 and Rs.120 respectively.

5. In a market survey three commodities A, B and C were considered. In finding out the index number some fixed weights were assigned to the three varieties in each of the commodities. The table below provides the information regarding the consumption of three commodities according to the three varieties and also the total weight received by the commodity

Commodity Variety	Variety I	Variety II	Variety III	Total weight
A	1	2	3	11
B	2	4	5	21
C	3	5	6	27

Find the weights assigned to the three varieties by using Cramer's Rule.

Solution:

Let the weight assigned to the three varieties be Rs. x, Rs. y and Rs. z respectively.

By the given data,

$$x + 2y + 3z = 11$$
$$2x + 4y + 5z = 21$$
$$3x + 5y + 6z = 27$$

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{vmatrix} = 1\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 2\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + 3\begin{vmatrix} 2 & 4 \\ 3 & 5 \end{vmatrix}$$

$= 1(24 - 25) - 2(12 - 15) + 3(10 - 12)$

$= 1(-1) - 2(-3) + 3(-2)$

$= -1 + 6 - 6 = -1 \neq 0.$

Since $\Delta \neq 0$, the system is consistent with unique solution and Cramer's rule can be applied.

$$\Delta x = \begin{vmatrix} 11 & 2 & 3 \\ 21 & 4 & 5 \\ 27 & 5 & 6 \end{vmatrix}$$

$$= 11\begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 2\begin{vmatrix} 21 & 5 \\ 27 & 6 \end{vmatrix} + 3\begin{vmatrix} 21 & 4 \\ 27 & 5 \end{vmatrix}$$

$= 11(24 - 25) - 2(126 - 135) + 3(105 - 108)$

$= 11(-1) - 2(-9) + 3(-3)$

$= -11 + 18 - 9$

$= -2$

$$\Delta y = \begin{vmatrix} 1 & 11 & 3 \\ 2 & 21 & 5 \\ 3 & 27 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 21 & 5 \\ 27 & 6 \end{vmatrix} - 11\begin{vmatrix} 2 & 5 \\ 3 & 6 \end{vmatrix} + 3\begin{vmatrix} 2 & 21 \\ 3 & 27 \end{vmatrix}$$

$= 1(126 - 135) - 11(12 - 15) + 3(54 - 63)$

$= -9 - 11(-3) + 3(-9)$

$= -9 + 33 - 27$

$= -3$

$$\Delta z=\begin{vmatrix}1 & 2 & 11\\ 2 & 4 & 21\\ 3 & 5 & 27\end{vmatrix}=1\begin{vmatrix}4 & 21\\ 5 & 27\end{vmatrix}-2\begin{vmatrix}2 & 21\\ 3 & 27\end{vmatrix}+11\begin{vmatrix}2 & 4\\ 3 & 5\end{vmatrix}$$

$$= 1(108 - 105) - 2(54 - 63) + 11(10 - 12)$$

$$= 1(3) - 2(-9) + 11(-2)$$

$$= 3 + 18 - 22$$

$$= -1$$

$$x = \frac{\Delta x}{\Delta} = \frac{-2}{-1} = 2$$

$$y = \frac{\Delta y}{\Delta} = \frac{-3}{-1} = 3$$

$$\text{and } z = \frac{\Delta z}{\Delta} = \frac{-1}{-1} = 1$$

Hence, the weights assigned to the three varieties are 2, 3 and 1 respectively.

6. A total of ₹8,500 was invested in three interest earning accounts. The interest rates were 2%, 3% and 6% if the total simple interest for one year was ₹380 and the amount invested at 6% was equal to the sum of the amounts in the other two accounts, then how much was invested in each account? (use Cramer's rule).

Solution:

Let the amount invested in the rate of 2%, 3% and 6% be Rs. x, Rs. y and Rs. z respectively.

By the given data,

$$x + y + z = 8500 \quad \text{----- (1)}$$

$$\frac{2x}{100} + \frac{3y}{100} + \frac{6z}{100} = 380$$

$$\Rightarrow \quad \frac{2x + 3y + 6z}{100} = 380$$

$$\boxed{\because \text{Interest} = \frac{PNR}{100} = \frac{x \times 1 \times 2}{100} = \frac{2x}{100}}$$

$$\Rightarrow \quad 2x + 3y + 6z = 38000 \quad \text{----- (2)}$$

Also, $z = x + y$

$$x + y - z = 0 \quad \text{----- (3)}$$

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 6 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 6 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 6 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$$

$$= 1(-3-6) - 1(-2-6) + 1(2-3)$$

$$= 1(-9) - 1(-8) + 1(-1)$$

$$= -9 + 8 - 1 = -2 \neq 0$$

Since $\Delta \neq 0$, Cramer's rule can be applied and the system is consistent with unique solution.

$$\Delta x = \begin{vmatrix} 8500 & 1 & 1 \\ 38000 & 3 & 6 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= 8500\begin{vmatrix} 3 & 6 \\ 1 & -1 \end{vmatrix} - 1\begin{vmatrix} 38000 & 6 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 38000 & 3 \\ 0 & 1 \end{vmatrix}$$

$$= 8500(-3-6) - 1(-38000 \quad -0) + 1(38000 - 0)$$

$$= 8500(-9) - 1(-38000) + 1(38000)$$

$$= -76500 + 38000 + 38000$$

$$= -500$$

$$\Delta y = \begin{vmatrix} 1 & 8500 & 1 \\ 2 & 38000 & 6 \\ 1 & 0 & -1 \end{vmatrix}$$

$$= 1\begin{vmatrix} 38000 & 6 \\ 0 & -1 \end{vmatrix} - 8500\begin{vmatrix} 2 & 6 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 38000 \\ 1 & 0 \end{vmatrix}$$

$$= 1(-38000 - 0) - 8500 \quad (-2-6) + 1(0 - 38000)$$

$$= -38000 - 8500(-8) - 38000$$

$$= -38000 + 68000 - 38000$$

$$= -8000$$

$$\Delta z = \begin{vmatrix} 1 & 1 & 8500 \\ 2 & 3 & 38000 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= 1\begin{vmatrix} 3 & 38000 \\ 1 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 38000 \\ 1 & 0 \end{vmatrix} + 8500\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix}$$

$$= 1(0 - 38000) - 1(0 - 38000) \quad +85000(2-3)$$

$$= -38000 + 38000 + 8500 \quad (-1)$$

$$= -8500$$

$$\therefore x = \frac{\Delta x}{\Delta} = \frac{\overset{250}{\cancel{-500}}}{\cancel{-2}} = +250$$

$$y = \frac{\Delta y}{\Delta} = \frac{\overset{4000}{\cancel{-8000}}}{\cancel{-2}} = 4000$$

$$z = \frac{\Delta z}{\Delta} = \frac{\overset{4250}{\cancel{-8500}}}{\cancel{-2}} = 4250$$

Hence, the amount invested in the three accounts are Rs. 250, Rs. 4000 and Rs. 4250 respectively.

Exercise 1.2

1.(i) $x = 8, y = -3$ (ii) $x = 1, y = 4$ (iii) $(x, y, z) = (2, -1, 0)$

(iv) $(x, y, z) = (1, 2, 3)$ (v) $(x, y, z) = \left(-1, \frac{1}{2}, \frac{1}{3}\right)$

2. Cost per unit of labour is ₹10 Cost per unit of capital is ₹ 16

3. Amount invested at $4\frac{3}{4}\%$ is ₹7,300 Amount invested at $6\frac{1}{2}\%$ is ₹1,300

4. hourly charges for horse riding is ₹100 and AVT riding is ₹120

5. $(x, y, z) = (2, 3, 1)$

6. Amount invested at 2% is ₹250
   Amount invested at 3% is ₹4,000
   Amount invested at 6% is ₹4,250