常用 ACM 模板

目录

一、算法

1、手写二分搜索

(1) lower_bound

```
//[low, high]
int lower_bound0(int *a, int low, int high, int key){
    int mid;
    while(low < high){
        mid = (high - low) / 2 + low;
        if(a[mid] >= key)high = mid;
        else low = mid + 1;
    }
    return low;
}
```

(2) upper_bound

```
/*[low, high]
ȧa[high] <= key, return high;
*/
int upper_bound0(int *a, int low, int high, int key){
    while(low < high){
        int mid = (high - low) / 2 + low;
        if(a[mid] <= key)low = mid + 1;
        else high = mid;
    }
    return low;
}
```

2、LIS 系列

(1) NlogN 时间最长递增子序列：lower_bound

(2) NlogN 时间最长非递减子序列：把 lower_bound 改为 upper_bound

(3) NlogN 时间输出所有 LIS 的方案

```
#include<bits/stdc++.h>
using namespace std;
```

```cpp
const int MAXN = 1e5 + 10;

int n, a[MAXN], dp[MAXN], par[MAXN];
map<int, int> mp;
vector<int> v[MAXN];

void print(int x, int y) {
    if(x > 0) {
        for(int i = 0; i < v[x - 1].size(); ++i) {
            if(a[v[x - 1][i]] < a[v[x][y]]){
                print(x - 1, i);
                break;
            }
        }
    }
    printf("%d ", a[v[x][y]]);
}

int main() {
    freopen("input.txt", "r", stdin);
    freopen("output2.txt", "w", stdout);
    while(~scanf("%d", &n)) {
        for(int i = 0; i < n; ++i)scanf("%d", a + i);
        for(int i = 0; i < n; ++i)v[i].clear();
        memset(dp, 0x3f, sizeof(dp));
        memset(par, -1, sizeof(par));
        mp.clear();
        for(int i = 0; i < n; ++i) {
            int p = lower_bound(dp, dp + n, a[i]) - dp;
            dp[p] = a[i];
            mp[a[i]] = i;
            if(p > 0)par[i] = mp[dp[p - 1]];
            v[p].push_back(i);
        }
        int len = lower_bound(dp, dp + n, 0x3f3f3f3f) - dp;
        printf("%d\n", len);
        print(len - 1, 0);
        printf("\n\n");
    }
    return 0;
}
```

（4）NlogN 时间求下标以 i 结尾的最大长度：线段树维护区间最大值优化朴素 dp

3、康托展开

```
#include<bits/stdc++.h>
using namespace std;
const int facts[] = {1, 1, 2, 6, 24, 120, 720, 5040, 40320, 362880};

/*
返回一个排列的字典序排序。[0, n-1]
*/
int encode(vector<int> seq) {
    int res = 0, i, j, cnt, len = seq.size();
    for(i = 0; i < len; ++i) {
        cnt = seq[i] - 1;
        for(j = 0; j < i; ++j)if(seq[j] < seq[i])--cnt;
        res += cnt * facts[len - i - 1];
    }
    return res;
}
/*
    在由 1 到 n 组成的前 n 个数的全排列中，找字典序排第 m 的序列。
    字典序最小的序列 123...n，排名为 0；
    字典序最大的序列 n...321，排名为(n!-1)。
    如，在由 1、2、3 组成的序列的全排列中，
        序列 123 的排名是 0，序列 321 的排名是 5。
    另外：代码中，facts[i]为 i 的阶乘；
*/
vector<int> decode(int n, int m) {
    vector<int> res;
    long long board = 0;
    int i, t, r;
    /*
    注意，如果需要的排名是[1, n]，
    即在由 1、2、3 组成的序列的全排列中，要求序列 123 的排名是 1，
    那么，这里加一句：m--。
    */
    for(t = n; t > 0; --t) {
        r = m / facts[t - 1];
        m %= facts[t - 1];
        for(i = 1; i <= n; ++i) {
            if(!((board >> i) & 1)) {
                if(r == 0)break;
                else --r;
            }
        }
        res.push_back(i);
        board |= 1 << i;
    }
    return res;
}

int main() {
    for(int i = 0; i < 6; ++i) {
        vector<int> res = decode(3, i);
        for(int j = 0; j < res.size(); ++j)printf("%d ", res[j]);
        printf("\n");
    }
    vector<int> v{3, 2, 1};
    cout << encode(v) << endl;
    return 0;
}
```

4、0-1 背包记录路径(以 Installing Apps 题为例)

```
#include<bits/stdc++.h>
using namespace std;
struct app {
    int d, s, id;
} p[510];
bool cmp(app a1, app a2) {
    return max(a1.d, a1.s) - a1.s > max(a2.d, a2.s) - a2.s;
}
int N, C, dp[510][10010], ans[510], acnt;
bool path[510][10010];
```

```
int main() {
    scanf("%d%d", &N, &C);
    for(int i = 1; i <= N; ++i)scanf("%d%d", &p[i].d, &p[i].s), p[i].id = i;
    for(int i = 0; i <= N; ++i) {
        for(int j = 0; j <= C; ++j)dp[i][j] = 0;
    }
    sort(p + 1, p + N + 1, cmp);
    for(int i = 1; i <= N; ++i) {
        for(int j = 0; j <= C; ++j) {
            dp[i][j] = dp[i - 1][j];
            if(j >= p[i].s) {
                int last = j - p[i].s;
                if(C - last >= p[i].d) {
                    if(dp[i][j] < dp[i - 1][last] + 1) {
                        dp[i][j] = dp[i - 1][last] + 1;
                        path[i][j] = 1;
                    }
                }
            }
        }
    }
    int V, aa = 0;
    for(int j = C; j >= 0; --j) {
        if(aa < dp[N][j]) {
            aa = dp[N][j], V = j;
        }
    }
    for(int i = N, j = V; i > 0; --i) {
        if(path[i][j]) {
            ans[++acnt] = p[i].id;
            j -= p[i].s;
        }
    }
    reverse(ans + 1, ans + acnt + 1);
    printf("%d\n", acnt);
    for(int i = 1; i <= acnt; ++i)printf("%d ", ans[i]);
    return 0;
}
```

5、多重背包二进制优化转 0-1 背包(以焦作网络赛 Transport Ship 题为例)

```
#include<bits/stdc++.h>
using namespace std;
typedef long long LL;
const LL mod = 1000000007;
int v[510], n, q, ncnt;
LL dp[510][10010];
int main() {
    int T, x, y;
    for(scanf("%d", &T); T--;) {
        ncnt = 0;
        scanf("%d%d", &n, &q);
        for(int i = 1; i <= n; ++i) {
            scanf("%d%d", &x, &y);
            y = (1 << y) - 1;
            for(int i = 1; i <= y; i <<= 1) {
                v[++ncnt] = i * x;
            }
        }
        for(int i = 0; i <= ncnt; ++i) {
            for(int j = 0; j <= 10000; ++j)dp[i][j] = 0;
        }
        dp[0][0] = 1;
        for(int i = 1; i <= ncnt; ++i) {
            for(int j = 0; j <= 10000; ++j) {
                dp[i][j] = dp[i - 1][j];
                if(j >= v[i])dp[i][j] = (dp[i][j] + dp[i - 1][j - v[i]]) % mod;
            }
        }
        while(q--) {
            scanf("%d", &x);
            printf("%lld\n", dp[ncnt][x]);
        }
    }
    return 0;
}
```

6、斜率优化 dp

(1) 一维斜率优化 dp(以打印单词 hdu3507 那个经典入门题为例)

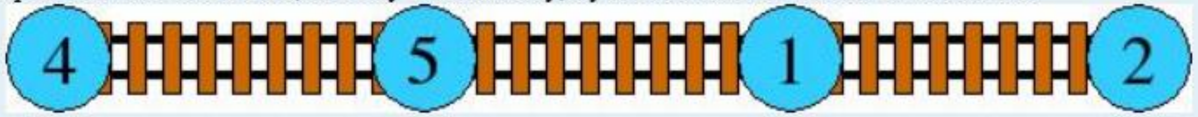
```
#include<bits/stdc++.h>
using namespace std;
#define MAXN 500010
typedef long long LL;
LL a[MAXN], s[MAXN], dp[MAXN];
int n, m;
```

```
LL f(int x) {
    return dp[x] + s[x] * s[x];
}
LL sq(LL x) {
    return x * x;
}
int dq[MAXN * 2];
int main() {
    int sz;
    while(~scanf("%d%d", &n, &m)) {
        for(int i = 1; i <= n; ++i)scanf("%lld", a + i), s[i] = s[i - 1] + a[i];
        memset(dp, 0x3f, sizeof(dp[0]) * (n + 2));
        dp[0] = 0;
        int head = 1, tail = 0;
        dq[++tail] = 0;
        for(int i = 1; i <= n; ++i) {
            while(tail - head >= 1 && (f(dq[head + 1]) - f(dq[head])) < (s[dq[head + 1]] -
s[dq[head]])*s[i] * 2)head++;
            dp[i] = dp[dq[head]] + sq(s[i] - s[dq[head]]) + m;
            while(tail - head >= 1 && (f(i) - f(dq[tail])) * (s[dq[tail]] - s[dq[tail -
1]]) <= (f(dq[tail]) - f(dq[tail - 1])) * (s[i] - s[dq[tail]]))tail--;
            dq[++tail] = i;
        }
        printf("%lld\n", dp[n]);
    }
    return 0;
}
```

（2）二维斜率优化 dp(以炸铁路 hdu2829 那个经典入门题为例)

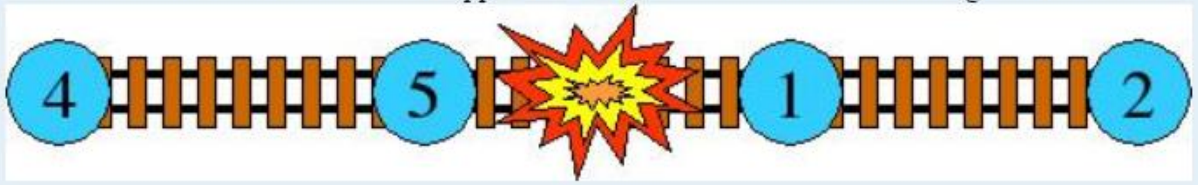
T. E. Lawrence was a controversial figure during World War I. He was a British officer who served in the Arabian theater and led a group of Arab nationals in guerilla strikes against the Ottoman Empire. His primary targets were the railroads. A highly fictionalized version of his exploits was presented in the blockbuster movie, "Lawrence of Arabia".

You are to write a program to help Lawrence figure out how to best use his limited resources. You have some information from British Intelligence. First, the rail line is completely linear---there are no branches, no spurs. Next, British Intelligence has assigned a Strategic Importance to each depot---an integer from 1 to 100. A depot is of no use on its own, it only has value if it is connected to other depots. The Strategic Value of the entire railroad is calculated by adding up the products of the Strategic Values for every pair of depots that are connected, directly or indirectly, by the rail line. Consider this railroad:

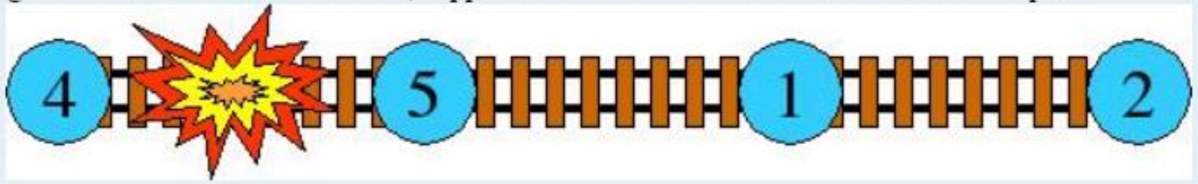


Its Strategic Value is 4*5 + 4*1 + 4*2 + 5*1 + 5*2 + 1*2 = 49.

Now, suppose that Lawrence only has enough resources for one attack. He cannot attack the depots themselves---they are too well defended. He must attack the rail line between depots, in the middle of the desert. Consider what would happen if Lawrence attacked this rail line right in the middle:



The Strategic Value of the remaining railroad is 4*5 + 1*2 = 22. But, suppose Lawrence attacks between the 4 and 5 depots:



The Strategic Value of the remaining railroad is 5*1 + 5*2 + 1*2 = 17. This is Lawrence's best option.

Given a description of a railroad and the number of attacks that Lawrence can perform, figure out the smallest Strategic Value that he can achieve for that railroad.

```
/*
dp[i][j]表示在前 i 个火车站中炸毁 j 处铁路所需的最小代价。
朴素的 dp 式子是：
for(int i=1;i<=n;++i){
    for(int j=1;j<=m;++j){
        for(int k=1;k<i;++k){
            dp[i][j]=min(dp[i][j],dp[k][j-1]+sum[k+1][i]);
        }
    }
}
斜率优化 dp 注意点：
1、在朴素的 dp 中，i 的循环应该写在 j 的循环外面。但是在斜率优化 dp 里面，如果有多层 for 循环(俗称多维斜率优化 dp))，
单调队列里面的元素的循环应该置于内层，而原来处于内层的应放置在外层。
2、单调队列里面的初始元素，一定是当前“阶段”第一次进入内层循环修改 dp 值的前一个“状态”。以下面的程序为例，单调队列里面的初始元素是 j，也就是说，一定要保证要可以炸毁 j-1 处铁路，至少需要 j 个火车站。
3、多维斜率优化 dp，两个 while 循环传递的“阶段”都是上一个。也就是说，以下面的程序为例，如果当前阶段是 j，那么上一个阶段是 j-1,在 f 函数传参的时候传递的是 j-1，不是 j。
4、求的是 dp 的最小值<=，所以是维护下凸包。相反，如果求的是 dp 的最大值>=，维护的是上凸包。
5、单调队列维护完成以后，一定要记得把当前“阶段”下的当前“状态”放到单调队列中。
6、由于 dp 数组的元素不需要取 min，所以，不需要初始化为 inf。
**/
#include<bits/stdc++.h>
using namespace std;
typedef long long LL;
#define MAXN 1010
int n, m, a[MAXN];
LL pre[MAXN], s[MAXN], dp[MAXN][MAXN];
int dq[2020];
```

```
template<class T> inline T sq(T x) {
    return x * x;
}
inline LL f(int x, int j) {
    return 2 * dp[x][j] + sq(pre[x]) + s[x];
}
int main() {
//    freopen("input.txt", "r", stdin);
    while(scanf("%d%d", &n, &m), n || m) {
        for(int i = 1; i <= n; ++i)scanf("%d", a + i), pre[i] = pre[i - 1] + a[i], s[i] =
s[i - 1] + a[i] * a[i];
        for(int i = 1; i <= n; ++i)dp[i][0] = (sq(pre[i]) - s[i]) / 2;
        for(int j = 1; j <= m; ++j) {
            int head = 1, tail = 0;
            dq[++tail] = j;
            for(int i = j + 1; i <= n; ++i) {
                while(tail - head >= 1 && f(dq[head + 1], j - 1) - f(dq[head], j - 1) <=
(pre[dq[head + 1]] - pre[dq[head]]) * 2 * pre[i])head++;
                dp[i][j] = dp[dq[head]][j - 1] + (sq(pre[i] - pre[dq[head]]) - (s[i] -
s[dq[head]])) / 2;
                while(tail - head >= 1 &&
                        (f(i, j - 1) - f(dq[tail], j - 1)) * (pre[dq[tail]] - pre[dq[tail -
1]]) <=
                        (f(dq[tail], j - 1) - f(dq[tail - 1], j - 1)) * (pre[i] -
pre[dq[tail]]))tail--;
                dq[++tail] = i;
            }
        }
        printf("%lld\n", dp[n][m]);
    }
    return 0;
}
```

7、三分

（1）整数

```
#include<bits/stdc++.h>
using namespace std;
/*
上突。
**/
int fd3(int a[], int l, int r) {
    int lmid, rmid, low = l, high = r, res = -1;
    while(low < high) {
        lmid = (low + high) / 2;
        rmid = (lmid + high) / 2;
        if(a[lmid] > a[rmid])high = rmid;
        else if(a[lmid] == a[rmid])low = lmid, high = rmid;
        else low = lmid;
    }
    res = low;
    if(a[res] < a[high])res = high;
    if(a[res] < a[l])res = l;
    if(a[res] < a[r])res = r;
    return res;
}
```

```
int main() {
    int a[] = {0, 8, 7, 6, 6, 5, 5, 4, 2, 2, 1}, n = sizeof(a) / sizeof(a[0]) - 1;
    int idx = fd3(a, 1, n);
    for(int i = 1; i <= n; ++i)printf("%d ", a[i]); printf("\n");
    printf("##%d\n", a[idx]);
    return 0;
}
```

(2) 分数

```
#include<bits/stdc++.h>
using namespace std;
/*
上突。
**/
int main(){
    double low=下界,high=上界,lmid,rmid;
    while(low<high-1e-8){
        lmid=(low+high)/2;
        rmid=(lmid+high)/2;
        if(calc(lmid)>calc(rmid))high=rmid;
        else low=lmid;
    }
    double ans=max(calc(low),calc(high));
    return 0;
}
```

二、数据结构

1、LCA

(1) 二分搜索版

```cpp
#include<bits/stdc++.h>
using namespace std;
const int MAXN = 100010;
const int LOGN = 20;
struct edge {
    int to, next;
    edge(int to = 0, int next = 0): to(to), next(next) {}
} es[MAXN * 2];
int head[MAXN], ecnt, n, m;
int parent[MAXN][LOGN], deep[MAXN];

void add(int from, int to) {
    es[++ecnt] = edge(to, head[from]), head[from] = ecnt;
}

void init() {//确保在 n 输入之后调用
    memset(head, 0, sizeof(head[0]) * (n + 5));
    ecnt = 0;
}

void dfs4lca(int root, int par, int d) {
    parent[root][0] = par, deep[root] = d;
    for(int i = head[root]; i != 0; i = es[i].next) {
        int to = es[i].to;
        if(to != par) {
            dfs4lca(to, root, d + 1);
        }
    }
}
void init4lca() {
    dfs4lca(1, 0, 1);
    for(int j = 0; j + 1 < LOGN; ++j) {
        for(int i = 1; i <= n; ++i) {
            if(parent[i][j] == 0)parent[i][j + 1] = 0;
            else parent[i][j + 1] = parent[parent[i][j]][j];
        }
    }
}

int lca(int a, int b) {
    if(deep[a] < deep[b])swap(a, b);
    for(int i = 0; i < LOGN; ++i) {
        if(((deep[a] - deep[b]) >> i) & 1)a = parent[a][i];
    }
    if(a == b)return a;
    for(int i = LOGN - 1; i >= 0; --i) {
        if(parent[a][i] != parent[b][i]) {
            a = parent[a][i], b = parent[b][i];
        }
    }
    return parent[a][0];
}
```

(2) tarjan 版

```
#include<iostream>
#include<cstdio>
#include<cstring>
#include<vector>
using namespace std;
const int MAXN = 1010;
typedef struct {
    int to, next;
} Edge;
Edge tree[MAXN * 2];
int head[MAXN], cnt;
typedef struct S1 {
    int bro;
    int anc;//和 to 的祖先
    S1(): bro(0), anc(0) {}
    S1(int arg1, int arg2) {
        bro = arg1;
        anc = arg2;
    }
} Node;
vector<Node> query[MAXN];
bool vis[MAXN];
int n, m;
/*并查集部分*/
int f[MAXN];
int lookup(int x) {
    if(f[x] == x)return x;
    else return f[x] = lookup(f[x]);
}
```

```
void unite(int root, int to) {
    if(lookup(root) == lookup(to))return;
    else f[to] = root;
}
/*并查集部分*/

void add(int from, int to) {
    tree[cnt].to = to;
    tree[cnt].next = head[from];
    head[from] = cnt++;
}

void init() {
    memset(head, -1, sizeof(head));
    memset(vis, 0, sizeof(vis));
    for(int i = 0; i < MAXN; ++i)query[i].clear();
    cnt = 0;
    /*并查集部分*/
    for(int i = 1; i <= MAXN; ++i)f[i] = i;
    /*并查集部分*/
}

void tarjan_lca(int root) {
    for(int i = head[root], to = -1; i != -1; i = tree[i].next) {
        to = tree[i].to;
        tarjan_lca(to);
        unite(root, to);
        vis[to] = true;
    }
    for(int i = 0, sz = query[root].size(), bro = -1; i < sz; ++i) {
        bro = query[root][i].bro;
        if(vis[bro])query[root][i].anc = lookup(bro);
    }
}

int main() {
    freopen("input.txt", "r", stdin);
    int a, b;
    scanf("%d%d", &n, &m);
    init();
    for(int i = 1; i < n; ++i) { //construct tree.
        scanf("%d%d", &a, &b);
        add(a, b);
    }
    for(int i = 1; i <= m; ++i) {//m times queries.
        scanf("%d%d", &a, &b);
        query[a].push_back(Node(b, -1));
        query[b].push_back(Node(a, -1));
    }

    tarjan_lca(1);

    for(int i = 1;i <= n;++i){
        for(int j = 0, sz = query[i].size();j < sz;++j){
            if(query[i][j].anc == -1)continue;
            printf("LCA(%d, %d) = %d\n", i, query[i][j].bro, query[i][j].anc);
        }
    }
    return 0;
}
```

2、并查集

```
#define MAXN 100010
typedef struct {
    int par[MAXN], rk[MAXN];
    void init() {
        for(int i = 0; i < MAXN; ++i)par[i] = i, rk[i] = 0;
    }
    int root(int v) {
        return v == par[v] ? v : par[v] = root(par[v]);
    }
    bool same(int u, int v) {
        return root(u) == root(v);
    }
    void unite(int u, int v) {
        int x = root(u), y = root(v);
        if(x == y)return;
        if(rk[x] < rk[y])par[x] = y;
        else {
            par[y] = x;
            if(rk[x] == rk[y])rk[x]++;
        }
    }
} UniFinder;
```

3、树状数组

（1）一维区间更新单点查询

```
#include<cstring>
#include<cstdio>
#define MAXN 50010
using namespace std;
typedef long long LL;
int n, m;
LL bit[MAXN];

LL query(int x) {
    LL res = 0;
    for(int i = x; i > 0; i -= i & -i)res += bit[i];
    return res;
}

void update(int x, int val) {
    for(int i = x; i <= n; i += i & -i)bit[i] += val;
}

int main() {
    freopen("input.txt", "r", stdin);
    freopen("output2.txt", "w", stdout);
    int a, b, v;
    memset(bit, 0, sizeof(bit));
    scanf("%d%d", &n, &m);
    for(int i = 1;i <= n;++i){
        scanf("%d", &v);
        update(i, v);
        update(i + 1, -v);
    }
    for(int i = 1;i <= m;++i){
        scanf("%d%d%d", &a, &b, &v);
        update(a, v);
        update(b + 1, -v);
    }
    for(int i = 1;i <= n;++i)printf("%I64d\n", query(i));
    return 0;
}
```

(2) 一维区间更新区间查询

```
#include<iostream>
#include<cstdio>
#include<cstring>
#define MAXN 50010
using namespace std;
```

```
typedef long long LL;
int n, m, q;
LL d[MAXN], f[MAXN];

LL query(LL* arr, int x) {
    LL res = 0;
    for(int i = x; i > 0; i -= (i & -i))res += arr[i];
    return res;
}

void update(LL* arr, int x, LL val) {
    for(int i = x; i <= n; i += (i & -i))arr[i] += val;
}

inline LL getsum(int x) {
    return (x + 1) * query(d, x) - query(f, x);
}

int main() {
    freopen("input.txt", "r", stdin);
    freopen("output2.txt", "w", stdout);
    memset(d, 0, sizeof(d));
    memset(f, 0, sizeof(f));
    scanf("%d%d%d", &n, &m, &q);
    int x, y;
    LL v, v0 = 0;
    for(int i = 1; i <= n; ++i) {
        scanf("%I64d", &v);
        int t = v - v0;
        update(d, i, t);
        update(f, i, t * i);
        //这里要注意：不能多写。
        v0 = v;
    }
    for(int i = 1; i <= m; ++i) {
        scanf("%d%d%I64d", &x, &y, &v);
        update(d, x, v);
        update(d, y + 1, -v);
        update(f, x, v * x);
        update(f, y + 1, -v * (y + 1));
    }
    for(int i = 1; i <= q; ++i) {
        scanf("%d%d", &x, &y);
        printf("%I64d\n", getsum(y) - getsum(x - 1));
    }
    return 0;
}
```

(3) 二维树状数组区间更新区间查询

```
#include<bits/stdc++.h>
using namespace std;
struct bit2d {
    typedef long long LL;
#define MAXN 2050
    LL b[MAXN][MAXN], bi[MAXN][MAXN], bj[MAXN][MAXN], bij[MAXN][MAXN];
    int N, M;
    void init(int n, int m){
        N = n, M = m;
    }
    void clear(){
        memset(b, 0, sizeof(b));
        memset(bi, 0, sizeof(bi));
        memset(bj, 0, sizeof(bj));
        memset(bij, 0, sizeof(bij));
    }
private:
    void add0(LL a[][MAXN], int x, int y, LL val) {
        for(int i = x; i <= N; i += i & -i) {
            for(int j = y; j <= M; j += j & -j)a[i][j] += val;
        }
    }
private:
    void add(int x, int y, LL val) {
        add0(b, x, y, val);
        add0(bi, x, y, 1LL * x * val);
        add0(bj, x, y, 1LL * y * val);
        add0(bij, x, y, 1LL * x * y * val);
    }
public:
    void update(int x1, int y1, int x2, int y2, LL val) {
        add(x1, y1, val);
        add(x1, y2 + 1, -val);
        add(x2 + 1, y1, -val);
        add(x2 + 1, y2 + 1, val);
    }
private:
    LL sum0(LL a[][MAXN], LL x, LL y) {
        LL res = 0;
        for(int i = x; i > 0; i -= i & -i) {
            for(int j = y; j > 0; j -= j & -j)res += a[i][j];
        }
        return res;
    }
private:
    LL sum(int x, int y) {
        return sum0(b, x, y) * (x * y + x + y + 1)
               - sum0(bi, x, y) * (y + 1)
               - sum0(bj, x, y) * (x + 1)
               + sum0(bij, x, y);
    }
public:
    LL query(int x1, int y1, int x2, int y2) {
        return sum(x2, y2) - sum(x2, y1 - 1) - sum(x1 - 1, y2) + sum(x1 - 1, y1 - 1);
    }
};

bit2d bd;
int n, m;
int main() {
//    freopen("input.txt", "r", stdin);
```

```
    int x1, y1, x2, y2, val;
    char op[5];
    scanf("%s%d%d", op, &n, &m);
    bd.init(n, m);
    while(~scanf("%s", op)) {
        scanf("%d%d%d%d", &x1, &y1, &x2, &y2);
        if(op[0] == 'L') {
            scanf("%d", &val);
            bd.update(x1, y1, x2, y2, val);
        }
        else printf("%lld\n", bd.query(x1, y1, x2, y2));
    }
    return 0;
}
```

4、可持久化线段树(主席树)

（1）只可查询区间第 k 小、不可修改（时间复杂度：$NlogN + QlogN$，空间复杂度：$NlogN + QlogN$）

```
/**
只支持一棵线段树一个值。
如果要实现一棵线段树插入多个值，需要修改 update0 函数的 if。
**/
struct PerSegTree {
#define MAXN 100010
    int N, M, root[MAXN], ncnt;
    struct seg {
        int lch, rch, cnt;
    } segs[MAXN * 20];
```

```
public: //an:线段树的数量; am:线段树叶子节点的数量;
    void init(int an, int am) {
        N = an, M = am;
        ncnt = 0;
        memset(root, 0, sizeof(root[0]) * (an + 3));
    }
private:
    void update0(int& croot, int proot, int val, int l, int r) {
        if(croot == 0) {
            croot = ++ncnt;
            segs[croot].cnt = segs[proot].cnt + 1;
            segs[croot].lch = segs[croot].rch = 0;
        }
        if(l == r)return;
        int mid = (l + r) >> 1;
        if(val <= mid) {
            segs[croot].rch = segs[proot].rch;
            update0(segs[croot].lch, segs[proot].lch, val, l, mid);
        }
        else {
            segs[croot].lch = segs[proot].lch;
            update0(segs[croot].rch, segs[proot].rch, val, mid + 1, r);
        }
    }
public:
    void insert(int x, int val) {
        update0(root[x], root[x - 1], val, 1, M);
    }
private:
    int query0(int qlr, int qrr, int k, int l, int r) {
        if(l == r)return l;
        int mid = (l + r) >> 1, tmp = segs[segs[qrr].lch].cnt - segs[segs[qlr].lch].cnt;
        if(k <= tmp) {
            return query0(segs[qlr].lch, segs[qrr].lch, k, l, mid);
        }
        else {
            return query0(segs[qlr].rch, segs[qrr].rch, k - tmp, mid + 1, r);
        }
    }
public:
    int query(int ql, int qr, int k) {
        return query0(root[ql - 1], root[qr], k, 1, M);
    }
} pst;
```

(2) 例题(POJ2104)

```
#include<bits/stdc++.h>
using namespace std;
/**
只支持一棵线段树一个值。
如果要实现一棵线段树插入多个值，需要修改 update0 函数的 if。
**/
struct PerSegTree {
#define MAXN 100010
    int N, M, root[MAXN], ncnt;
    struct seg {
        int lch, rch, cnt;
    } segs[MAXN * 20];
public: //an:线段树的数量; am:线段树叶子节点的数量;
    void init(int an, int am) {
        N = an, M = am;
        ncnt = 0;
        memset(root, 0, sizeof(root[0]) * (an + 3));
    }
private:
    void update0(int& croot, int proot, int val, int l, int r) {
        if(croot == 0) {
            croot = ++ncnt;
            segs[croot].cnt = segs[proot].cnt + 1;
            segs[croot].lch = segs[croot].rch = 0;
        }
        if(l == r)return;
        int mid = (l + r) >> 1;
        if(val <= mid) {
            segs[croot].rch = segs[proot].rch;
            update0(segs[croot].lch, segs[proot].lch, val, l, mid);
        }
        else {
            segs[croot].lch = segs[proot].lch;
            update0(segs[croot].rch, segs[proot].rch, val, mid + 1, r);
        }
    }
public:
    void insert(int x, int val) {
        update0(root[x], root[x - 1], val, 1, M);
    }
```

```
private:
    int query0(int qlr, int qrr, int k, int l, int r) {
        if(l == r)return l;
        int mid = (l + r) >> 1, tmp = segs[segs[qrr].lch].cnt - segs[segs[qlr].lch].cnt;
        if(k <= tmp) {
            return query0(segs[qlr].lch, segs[qrr].lch, k, l, mid);
        }
        else {
            return query0(segs[qlr].rch, segs[qrr].rch, k - tmp, mid + 1, r);
        }
    }
public:
    int query(int ql, int qr, int k) {
        return query0(root[ql - 1], root[qr], k, 1, M);
    }
} pst;
int a[MAXN], b[MAXN], n, q;
int main() {
    freopen("a.in", "r", stdin);
    freopen("a2.out", "w", stdout);
    int T;
    for(scanf("%d", &T); T--;) {
        scanf("%d%d", &n, &q);
        for(int i = 1; i <= n; ++i)scanf("%d", a + i), b[i] = a[i];
        sort(b + 1, b + n + 1);
        int m = unique(b + 1, b + n + 1) - b - 1;
        pst.init(n, m);
        for(int i = 1; i <= n; ++i)pst.insert(i, lower_bound(b + 1, b + m + 1, a[i]) -
b);
```

```
        while(q--) {
            int l, r, k;
            scanf("%d%d%d", &l, &r, &k);
            printf("%d\n", b[pst.query(l, r, k)]);
        }
    }
    return 0;
}
```

（3）区间查询第 k 小，可单点修改（时间复杂度：$NlogN + Qlog^2N$，空间复杂度：$NlogN + Qlog^2N$）

```
#define MAXN 50010
#define MAXQ 10010
/*
n:数字序列长度;
q:操作的个数;
m:对 b 去重后的数字个数;
a:输入序列;
b:输入序列+更新数字的集合。要排序，用于离散化。

*/
int n, q, m, a[MAXN], b[MAXN + MAXQ];
/*操作需要先全部记录下来*/
struct oper {
    int type;
    int l, r, k;
    int x, val;
```

```
} p[MAXQ];

/*先 insert，再 add*/
typedef struct {
    int root0[MAXN], root1[MAXN], qrt[MAXN], ncnt;
    struct seg {
        int lch, rch, cnt;
    } segs[MAXN * 16 + MAXQ * 256];
    /*MAXN * log2(MAXN) + MAXQ * log2(MAXN) * log2(MAXN)*/
public:
    /*通常情况下，传递参数为 n。*/
    void init(int size) {
        ncnt = 0;
        memset(root0, 0, sizeof(root0[0]) * (size + 1));
        memset(root1, 0, sizeof(root0[1]) * (size + 1));
        segs[0].lch = segs[0].rch = segs[0].cnt = 0;
    }
private:
    int update0(int proot, int val, int d, int l = 1, int r = m) {
        int nroot = ++ncnt;
        segs[nroot].cnt = segs[proot].cnt + d;
        if(l == r)return nroot;
        int mid = (l + r) >> 1;
        if(val <= mid) {
            segs[nroot].rch = segs[proot].rch;
            segs[nroot].lch = update0(segs[proot].lch, val, d, l, mid);
        }
        else {
            segs[nroot].lch = segs[proot].lch;
            segs[nroot].rch = update0(segs[proot].rch, val, d, mid + 1, r);
        }
        return nroot;
    }
public:
    void add(int x, int val, int d) {
        for(int i = x; i <= n; i += (i & -i))root1[i] = update0(root1[i], val, d);
    }
public:
    /*用法：insert(i, a[i])*/
    void insert(int x, int val){
        root0[x] = update0(root0[x - 1], val, 1);
    }
private:
    int getsum(int x){
        int res = 0;
        for(int i = x;i > 0;i -= (i & -i))res += segs[segs[qrt[i]].lch].cnt;
        return res;
    }
private:
    int query0(int ql, int qr, int rl, int rr, int k, int l = 1, int r = m) {
        if(l == r)return l;
        int sum = getsum(qr) - getsum(ql - 1) + segs[segs[rr].lch].cnt -
segs[segs[rl].lch].cnt, mid = (l + r) >> 1;
        if(k <= sum) {
            for(int i = ql - 1; i > 0; i -= (i & -i))qrt[i] = segs[qrt[i]].lch;
            for(int i = qr; i > 0; i -= (i & -i))qrt[i] = segs[qrt[i]].lch;
            return query0(ql, qr, segs[rl].lch, segs[rr].lch, k, l, mid);
        }
        else {
            for(int i = ql - 1; i > 0; i -= (i & -i))qrt[i] = segs[qrt[i]].rch;
            for(int i = qr; i > 0; i -= (i & -i))qrt[i] = segs[qrt[i]].rch;
            return query0(ql, qr, segs[rl].rch, segs[rr].rch, k - sum, mid + 1, r);
        }
    }
public:
    int query(int ql, int qr, int k) {
        for(int i = ql - 1; i > 0; i -= (i & -i))qrt[i] = root1[i];
        for(int i = qr; i > 0; i -= (i & -i))qrt[i] = root1[i];
        return query0(ql, qr, root0[ql - 1], root0[qr], k);
    }
} ModPerSegTree;
```

(4) 例题(ZOJ2112)

```
#pragma comment(linker, "/STACK:102400000,102400000")
#include<bits/stdc++.h>
using namespace std;
#define MAXN 50010
```

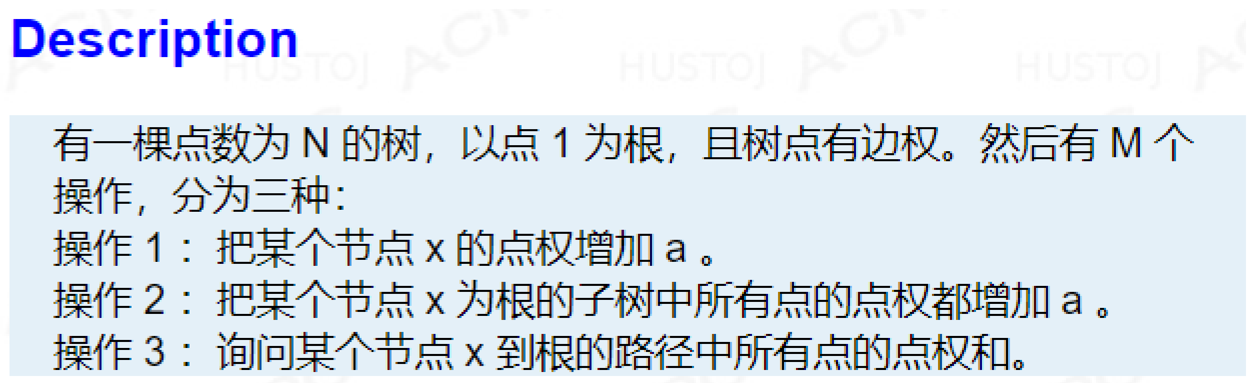
```
#define MAXQ 10010
int n, q, m, a[MAXN], b[MAXN + MAXQ];

struct oper {
    int type;
    int l, r, k;
    int x, val;
} p[MAXQ];

typedef struct {
    int root0[MAXN], root1[MAXN], qrt[MAXN], ncnt;
    struct seg {
        int lch, rch, cnt;
    } segs[MAXN * 16 + MAXQ * 160];
public:
    void init(int size) {
        ncnt = 0;
        memset(root0, 0, sizeof(root0[0]) * (size + 1));
        memset(root1, 0, sizeof(root0[1]) * (size + 1));
        segs[0].lch = segs[0].rch = segs[0].cnt = 0;
    }
private:
    int update0(int proot, int val, int d, int l = 1, int r = m) {
        int nroot = ++ncnt;
        segs[nroot].cnt = segs[proot].cnt + d;
        if(l == r)return nroot;
        int mid = (l + r) >> 1;
        if(val <= mid) {
            segs[nroot].rch = segs[proot].rch;
            segs[nroot].lch = update0(segs[proot].lch, val, d, l, mid);
        }
        else {
            segs[nroot].lch = segs[proot].lch;
            segs[nroot].rch = update0(segs[proot].rch, val, d, mid + 1, r);
        }
        return nroot;
    }
public:
    void add(int x, int val, int d) {
        for(int i = x; i <= n; i += (i & -i))root1[i] = update0(root1[i], val, d);
    }
public:
    void insert(int x, int val){
        root0[x] = update0(root0[x - 1], val, 1);
    }
private:
    int getsum(int x){
        int res = 0;
        for(int i = x;i > 0;i -= (i & -i))res += segs[segs[qrt[i]].lch].cnt;
        return res;
    }
private:
    int query0(int ql, int qr, int rl, int rr, int k, int l = 1, int r = m) {
        if(l == r)return l;
        int sum = getsum(qr) - getsum(ql - 1) + segs[segs[rr].lch].cnt -
segs[segs[rl].lch].cnt, mid = (l + r) >> 1;
        if(k <= sum) {
            for(int i = ql - 1; i > 0; i -= (i & -i))qrt[i] = segs[qrt[i]].lch;
            for(int i = qr; i > 0; i -= (i & -i))qrt[i] = segs[qrt[i]].lch;
            return query0(ql, qr, segs[rl].lch, segs[rr].lch, k, l, mid);
        }
        else {
            for(int i = ql - 1; i > 0; i -= (i & -i))qrt[i] = segs[qrt[i]].rch;
            for(int i = qr; i > 0; i -= (i & -i))qrt[i] = segs[qrt[i]].rch;
            return query0(ql, qr, segs[rl].rch, segs[rr].rch, k - sum, mid + 1, r);
        }
    }
public:
    int query(int ql, int qr, int k) {
        for(int i = ql - 1; i > 0; i -= (i & -i))qrt[i] = root1[i];
        for(int i = qr; i > 0; i -= (i & -i))qrt[i] = root1[i];
        return query0(ql, qr, root0[ql - 1], root0[qr], k);
    }
} ModPerSegTree;

ModPerSegTree mpst;
int main() {
    freopen("input.txt", "r", stdin);
    freopen("output5.txt", "w", stdout);
    int T;
    char cmd[5];
    for(scanf("%d", &T); T--;) {
        m = 0;
        scanf("%d%d", &n, &q);
        mpst.init(n);
        for(int i = 1; i <= n; ++i)scanf("%d", a + i), b[++m] = a[i];
        for(int i = 1; i <= q; ++i) {
            scanf("%s", cmd);
            if(cmd[0] == 'Q') {
                p[i].type = 0;
                scanf("%d%d%d", &p[i].l, &p[i].r, &p[i].k);
            }
            else {
                p[i].type = 1;
                scanf("%d%d", &p[i].x, &p[i].val);
                b[++m] = p[i].val;
            }
        }
        sort(b + 1, b + m + 1);
        m = unique(b + 1, b + m + 1) - (b + 1);
        for(int i = 1; i <= n; ++i)mpst.insert(i, lower_bound(b + 1, b + m + 1, a[i]) -
b);
        for(int i = 1; i <= q; ++i) {
            if(p[i].type == 0)printf("%d\n", b[mpst.query(p[i].l, p[i].r, p[i].k)]);
            else {
                mpst.add(p[i].x, lower_bound(b + 1, b + m + 1, a[p[i].x]) - b, -1);
                mpst.add(p[i].x, lower_bound(b + 1, b + m + 1, p[i].val) - b, 1);
                a[p[i].x] = p[i].val;
            }
        }
    }
    return 0;
}
```

5、树链剖分(以 BZOJ4034 为例)


Description

有一棵点数为 N 的树，以点 1 为根，且树点有边权。然后有 M 个操作，分为三种：
操作 1 ：把某个节点 x 的点权增加 a 。
操作 2 ：把某个节点 x 为根的子树中所有点的点权都增加 a 。
操作 3 ：询问某个节点 x 到根的路径中所有点的点权和。

```
#include<bits/stdc++.h>
using namespace std;
#define MAXN 100010
typedef long long LL;
struct edge {
    int to, next;
    edge(int t = 0, int nx = 0): to(t), next(nx) {}
} es[MAXN * 2];
```

```
int head[MAXN], ecnt;
int f[MAXN], deep[MAXN], size[MAXN], zson[MAXN], top[MAXN], o2n[MAXN], dfscnt;
int n, q;
LL a[MAXN];
struct node {
    LL sum, lazy;
} ns[MAXN * 4];

void add(int from, int to) {
    es[++ecnt] = edge(to, head[from]), head[from] = ecnt;
}

void init() {
    memset(head, 0, sizeof(head[0]) * (n + 3));
    ecnt = 0;
    memset(zson, 0, sizeof(zson[0]) * (n + 3));
    dfscnt = 0;
    memset(ns, 0, sizeof(ns));
}

void pushdown(int rt, int l, int r) {
    if(ns[rt].lazy) {
        int lch = rt << 1, rch = rt << 1 | 1, mid = (l + r) >> 1;
        ns[lch].sum += (mid - l + 1) * ns[rt].lazy;
        ns[lch].lazy += ns[rt].lazy;
        ns[rch].sum += (r - mid) * ns[rt].lazy;
        ns[rch].lazy += ns[rt].lazy;
        ns[rt].lazy = 0;
    }
```

```
}

void update(int ul, int ur, LL v, int rt = 1, int l = 1, int r = n) {
    if(l > ur || r < ul)return;
    if(l >= ul && r <= ur) {
        ns[rt].sum += LL(r - l + 1) * v, ns[rt].lazy += v;
        return;
    }
    pushdown(rt, l, r);
    int mid = (l + r) >> 1, lch = rt << 1, rch = rt << 1 | 1;
    if(ul <= mid)update(ul, ur, v, lch, l, mid);
    if(ur > mid)update(ul, ur, v, rch, mid + 1, r);
    ns[rt].sum = ns[lch].sum + ns[rch].sum;
}

LL query0(int ql, int qr, int rt = 1, int l = 1, int r = n) {
    if(l > qr || r < ql)return 0LL;
    if(l >= ql && r <= qr)return ns[rt].sum;
    pushdown(rt, l, r);
    int mid = (l + r) >> 1, lch = rt << 1, rch = rt << 1 | 1;
    LL res = 0;
    if(ql <= mid)res += query0(ql, qr, lch, l, mid);
    if(qr > mid)res += query0(ql, qr, rch, mid + 1, r);
    return res;
}

LL query(int u, int v) {
    int tu = top[u], tv = top[v];
    LL res = 0;
    while(tu != tv) {
        if(deep[tu] >= deep[tv]) {
            res += query0(o2n[tu], o2n[u]);
            u = f[tu], tu = top[u];
        }
        else {
            res += query0(o2n[tv], o2n[v]);
            v = f[tv], tv = top[v];
        }
    }
    if(o2n[u] <= o2n[v])res += query0(o2n[u], o2n[v]);
    else res += query0(o2n[v], o2n[u]);
    return res;
}

void dfs1(int root, int par) {
    deep[root] = deep[par] + 1, f[root] = par, size[root] = 1;
    int ms = 0;
    for(int i = head[root]; i; i = es[i].next) {
        int to = es[i].to;
        if(to != par) {
            dfs1(to, root);
            size[root] += size[to];
            if(ms < size[to]) {
                ms = size[to], zson[root] = to;
            }
        }
    }
}
```

```
void dfs2(int root, int par, int tp) {
    top[root] = tp;
    o2n[root] = ++dfscnt;
    update(dfscnt, dfscnt, a[root]);
    if(!zson[root])return;
    dfs2(zson[root], root, tp);
    for(int i = head[root]; i; i = es[i].next) {
        int to = es[i].to;
        if(to != par && to != zson[root]) {
            dfs2(to, root, to);
        }
    }
}

int main() {
//    freopen("input.txt", "r", stdin);
    int op, x, y, z;
    while(~scanf("%d%d", &n, &q)) {
        init();
        for(int i = 1; i <= n; ++i)scanf("%lld", a + i);
        for(int i = 1; i < n; ++i) {
            scanf("%d%d", &x, &y);
            add(x, y), add(y, x);
        }
        dfs1(1, 0);
        dfs2(1, 0, 1);
        while(q--) {
            scanf("%d", &op);
            if(op == 1) {
                scanf("%d%d", &x, &z);
                update(o2n[x], o2n[x], z);
            }
            else if(op == 2) {
                scanf("%d%d", &x, &z);
                update(o2n[x], o2n[x] + size[x] - 1, z);
            }
            else {
                scanf("%d", &x);
                printf("%lld\n", query(1, x));
            }
        }
    }
    return 0;
}
```

6、线段树

（1）同时支持区间加、区间乘、区间求和

```
namespace st {
    struct node {
        ULL sum, slazy, plazy;
        node(ULL a1 = 0, ULL a2 = 0, ULL a3 = 0): sum(a1), slazy(a2), plazy(a3) {}
    } ns[MAXN * 4];
    void init() {
        for(int i = 0, t = 4 * n + 5; i < t; ++i)ns[i] = node(0, 0, 1);
    }
```

```
void pushdown(int rt, int l, int r) {
    int lch = rt << 1, rch = rt << 1 | 1, mid = (l + r) >> 1;
    if(ns[rt].plazy != 1 || ns[rt].plazy != 0) {
        ns[lch].sum = ns[lch].sum * ns[rt].plazy + (mid - l + 1) * ns[rt].slazy;
        ns[rch].sum = ns[rch].sum * ns[rt].plazy + (r - mid) * ns[rt].slazy;
        ns[lch].plazy *= ns[rt].plazy;
        ns[rch].plazy *= ns[rt].plazy;
        ns[lch].slazy = ns[lch].slazy * ns[rt].plazy + ns[rt].slazy;
        ns[rch].slazy = ns[rch].slazy * ns[rt].plazy + ns[rt].slazy;
        ns[rt].plazy = 1, ns[rt].slazy = 0;
    }
}
void update(int ul, int ur, ULL x, int type, int rt = 1, int l = 1, int r = n) {
    if(l > ur || r < ul)return;
    if(l >= ul && r <= ur) {
        if(type == 2)ns[rt].sum += x * ULL(r - l + 1), ns[rt].slazy += x;
        else ns[rt].sum *= x, ns[rt].plazy *= x, ns[rt].slazy *= x;
        return;
    }
    pushdown(rt, l, r);
    int mid = (l + r) >> 1;
    if(ul <= mid)update(ul, ur, x, type, rt << 1, l, mid);
    if(ur > mid)update(ul, ur, x, type, rt << 1 | 1, mid + 1, r);
    ns[rt].sum = ns[rt << 1].sum + ns[rt << 1 | 1].sum;
}
ULL query(int ql, int qr, int rt = 1, int l = 1, int r = n) {
    if(l > qr || r < ql)return 0LL;
    if(l >= ql && r <= qr)return ns[rt].sum;
    pushdown(rt, l, r);
```

```
        int mid = (l + r) >> 1;
        ULL res = 0;
        if(ql <= mid)res += query(ql, qr, rt << 1, l, mid);
        if(qr > mid)res += query(ql, qr, rt << 1 | 1, mid + 1, r);
        return res;
    }
};
```

（2）区间加等差数列，单点查询

```
#include<bits/stdc++.h>
using namespace std;
#define MAXN 100010
typedef long long LL;
struct node {
    LL f, d;
} ns[MAXN * 4];
int n, m;

void pushdown(int rt, int l, int r) {
    if(ns[rt].f || ns[rt].d) {
        int mid = (l + r) >> 1, lch = rt << 1, rch = rt << 1 | 1;
        LL f = ns[rt].f, d = ns[rt].d;
        ns[lch].f += ns[rt].f, ns[lch].d += ns[rt].d;
        ns[rch].f += ns[rt].f + (mid - l + 1) * d, ns[rch].d += ns[rt].d;
        ns[rt].f = ns[rt].d = 0;
    }
}
```

```cpp
void update(int ul, int ur, LL f, LL d, int rt = 1, int l = 1, int r = n) {
    if(r < ul || l > ur)return;
    if(l >= ul && r <= ur) {
        ns[rt].f += f + 1LL * (l - ul) * d, ns[rt].d += d;
        return;
    }
    pushdown(rt, l, r);
    int mid = (l + r) >> 1;
    if(ul <= mid)update(ul, ur, f, d, rt << 1, l, mid);
    if(ur > mid)update(ul, ur, f, d, rt << 1 | 1, mid + 1, r);
}

LL query(int x, int rt = 1, int l = 1, int r = n) {
    if(l == r && l == x)return ns[rt].f;
    pushdown(rt, l, r);
    int mid = (l + r) >> 1;
    if(x <= mid)return query(x, rt << 1, l, mid);
    else return query(x, rt << 1 | 1, mid + 1, r);
}

int main() {
    int x, op, l, r, f, d;
    scanf("%d%d", &n, &m);
    for(int i = 1; i <= n; ++i) {
        scanf("%d", &x);
        update(i, i, x, 0);
    }
    while(m--) {
        scanf("%d", &op);
        if(op == 1) {
            scanf("%d%d%d%d", &l, &r, &f, &d);
            if(n == 0)printf("0");
            else update(l, r, f, d);
        }
        else {
            scanf("%d", &x);
            if(n == 0)printf("0");
            else printf("%lld\n", query(x));
        }
    }
    return 0;
}
```

（3）区间加等差数列，区间查询

```
#include<bits/stdc++.h>
using namespace std;
#define MAXN 100010
typedef long long LL;
struct node {
    LL f, d, sum;
} ns[MAXN * 4];
int n, m;

inline LL calc(LL a1, LL d, int len) {
    return len * a1 + d * len * (len - 1) / 2;
}
```

```cpp
void pushdown(int rt, int l, int r) {
    if(ns[rt].f || ns[rt].d) {
        int mid = (l + r) >> 1, lch = rt << 1, rch = rt << 1 | 1;
        LL f = ns[rt].f, d = ns[rt].d;
        ns[lch].f += f, ns[lch].d += d;
        ns[lch].sum += calc(f, d, mid - l + 1);
        ns[rch].f += f + (mid - l + 1) * d, ns[rch].d += d;
        ns[rch].sum += calc(f + (mid - l + 1) * d, d, r - mid);
        ns[rt].f = ns[rt].d = 0;
    }
}

void update(int ul, int ur, LL f, LL d, int rt = 1, int l = 1, int r = n) {
    if(r < ul || l > ur)return;
    if(l >= ul && r <= ur) {
        LL tf = f + 1LL * (l - ul) * d, td = d;
        ns[rt].f += tf, ns[rt].d += td;
        ns[rt].sum += calc(tf, td, r - l + 1);
        return;
    }
    pushdown(rt, l, r);
    int mid = (l + r) >> 1;
    if(ul <= mid)update(ul, ur, f, d, rt << 1, l, mid);
    if(ur > mid)update(ul, ur, f, d, rt << 1 | 1, mid + 1, r);
    ns[rt].sum = ns[rt << 1].sum + ns[rt << 1 | 1].sum;
}

LL query(int ql, int qr, int rt = 1, int l = 1, int r = n) {
    if(l > qr || r < ql)return 0;
    if(l >= ql && r <= qr) {
        return ns[rt].sum;
    }
    pushdown(rt, l, r);
    int mid = (l + r) >> 1;
    LL sum = 0;
    if(ql <= mid)sum += query(ql, qr, rt << 1, l, mid);
    if(qr > mid)sum += query(ql, qr, rt << 1 | 1, mid + 1, r);
    return sum;
}

int main() {
//    freopen("2.in", "r", stdin);
    int x, op, l, r, f, d;
    scanf("%d%d", &n, &m);
    for(int i = 1; i <= n; ++i) {
        scanf("%d", &x);
        update(i, i, x, 0);
    }
    while(m--) {
        scanf("%d", &op);
        if(op == 1) {
            scanf("%d%d%d%d", &l,&r,&f, &d);
            update(l, r, f, d);
        }
        else {
            scanf("%d%d", &l,&r);
            printf("%lld\n", query(l, r));
        }
    }
    return 0;
}
```

7、树分治(以 poj1741 统计树上距离不超过 k 的点对为例)

```
/*统计树上距离不超过 k 的点对，(x,y)和(y,x)只记 1 次。**/
#include<cstdio>
#include<cstring>
#include<cstdlib>
#include<cassert>
#include<iostream>
#include<vector>
#include<algorithm>
using namespace std;
#define MAXN 100010
typedef long long LL;
struct edge {
    int to, next, wt;
    edge(int t = 0, int n = 0, int w = 0): to(t), next(n), wt(w) {}
} es[MAXN * 2];
int head[MAXN], ecnt;
int n, k;
int siz[MAXN], maxsonsiz[MAXN], G, subn;
vector<int> dis;
bool vis[MAXN];
LL ans;

void add(int from, int to, int wt) {
    es[++ecnt] = edge(to, head[from], wt), head[from] = ecnt;
}

void init() {
    memset(head, 0, sizeof(head[0]) * (n + 3));
    memset(vis, 0, sizeof(vis[0]) * (n + 3));
    ecnt = 0;
    G = 0, subn = n, maxsonsiz[G] = subn;
}

void dfs4siz(int root, int par) {
    siz[root] = 1, maxsonsiz[root] = 0;
    for(int i = head[root]; i; i = es[i].next) {
        int to = es[i].to;
        if(to != par && !vis[to]) {
            dfs4siz(to, root);
            siz[root] += siz[to];
            maxsonsiz[root] = max(maxsonsiz[root], siz[to]);
        }
    }
    maxsonsiz[root] = max(maxsonsiz[root], subn - siz[root]);
    if(maxsonsiz[root] < maxsonsiz[G])G = root;
}

void dfs4dis(int root, int par, int dd) {
    dis.push_back(dd);
    for(int i = head[root]; i; i = es[i].next) {
        int to = es[i].to;
        if(to != par && !vis[to]) {
            dfs4dis(to, root, dd + es[i].wt);
        }
    }
}

LL calc(int root, int dd) {
    dis.clear();
    dfs4dis(root, 0, dd);
    sort(dis.begin(), dis.end());
    LL res = 0;
    for(int i = 0, j = (int)dis.size() - 1; i < j;) {
        if(dis[i] + dis[j] > k) {
            j--;
        }
        else {
            res += j - i;
            i++;
        }
    }
    return res;
}

void dfs(int root, int par) {
    dfs4siz(root, par);
    assert(G != 0);
    vis[G] = 1;
    ans += calc(G, 0);
    for(int i = head[G]; i; i = es[i].next) {
        int to = es[i].to;
        if(!vis[to]) {
            ans -= calc(to, es[i].wt);
            G = 0, subn = siz[to], maxsonsiz[G] = subn;
            dfs(to, G);
        }
    }
}

int main() {
    int x, y, z;
    while(scanf("%d%d", &n, &k), n || k) {
        init();
        for(int i = 1; i < n; ++i) {
            scanf("%d%d%d", &x, &y, &z);
            add(x, y, z), add(y, x, z);
        }
        ans = 0;
        dfs(1, 0);
        printf("%lld\n", ans);
    }
    return 0;
}
```

三、图论

1、最短路

(1) Dijkstra(不能处理负边图)

```
#define MAXN 10010
typedef struct Edge0 {
    int to, wt;
    Edge0(int t, int w) {
        to = t;
        wt = w;
    }
} Edge;
vector<Edge> g[MAXN];
int V, E, S;

typedef struct Node0{
    int p, d;
    Node0(int _p, int _d){
        p = _p;
        d = _d;
    }
    bool operator > (const Node0& n) const{
        return d > n.d;
    }
}Node;
int d[MAXN];
priority_queue<Node, vector<Node>, greater<Node> > que;

void dijkstra(int s) {
    memset(d, 0x3f, sizeof(d));
    while(!que.empty())que.pop();
    d[s] = 0, que.push(Node(s, 0));
    while(!que.empty()) {
        Node nd = que.top();
        que.pop();
        for(int i = 0; i < g[nd.p].size(); ++i) {
            Edge& e = g[nd.p][i];
            if(d[e.to] > d[nd.p] + e.wt){
                d[e.to] = d[nd.p] + e.wt;
                que.push(Node(e.to, d[e.to]));
            }
        }
    }
}
```

(2) SPFA-LLF 优化(能处理负边图)

```
#define MAXN 200010
typedef struct Edge0 {
    int to, wt;
    Edge0(int t, int w) {
        to = t, wt = w;
    }
} Edge;
vector<Edge> g[MAXN];
```

```cpp
int V, E, S;
long long d[MAXN];
bool inque[MAXN];
deque<int> que;

void spfa(int s) {
    memset(d, 0x3f, sizeof(d));
    memset(inque, 0, sizeof(inque));
    while(!que.empty())que.pop_front();
    d[s] = 0, inque[s] = true, que.push_back(s);
    while(!que.empty()) {
        int v = que.front();
        que.pop_front();
        inque[v] = false;
        for(int i = 0; i < g[v].size(); ++i) {
            Edge& e = g[v][i];
            if(d[e.to] > d[v] + e.wt) {
                d[e.to] = d[v] + e.wt;
                if(!inque[e.to]) {
                    if(que.empty() || d[e.to] > d[que.front()])que.push_back(e.to);
                    else que.push_front(e.to);
                    inque[e.to] = true;
                }
            }
        }
    }
}
```

(3) SPFA-LLL 优化(能处理负边图)

```
#define MAXN 200010
typedef struct Edge0 {
    int to, wt;
    Edge0(int t, int w) {
        to = t, wt = w;
    }
} Edge;
vector<Edge> g[MAXN];
int V, E, S;
long long d[MAXN], sum;
bool inque[MAXN];
deque<int> que;

void spfa(int s) {
    memset(d, 0x3f, sizeof(d));
    memset(inque, 0, sizeof(inque));
    while(!que.empty())que.pop_front();
    d[s] = 0, inque[s] = true, que.push_back(s), sum = 0;
    while(!que.empty()) {
        int v = que.front();
        while(d[v] * que.size() > sum) {
            que.pop_front();
            que.push_back(v);
            v = que.front();
        }
        que.pop_front();
        inque[v] = false;
        sum -= d[v];
        for(int i = 0; i < g[v].size(); ++i) {
            Edge& e = g[v][i];
            if(d[e.to] > d[v] + e.wt) {
                d[e.to] = d[v] + e.wt;
                if(!inque[e.to]) {
                    if(que.empty() || d[e.to] > d[que.front()])que.push_back(e.to);
                    else que.push_front(e.to);
                    inque[e.to] = true;
                    sum += d[e.to];
                }
            }
        }
    }
}
```

2、最小生成树

```
typedef struct Edge0{
    int u, v, w;
    Edge0(int _u, int _v, int _w){
        u = _u, v = _v, w = _w;
    }
    bool operator < (const Edge0& e) const{
        return w < e.w;
    }
}Edge;

vector<Edge> edges;
UniFinder uf; //用之前记得要初始化 uf.init();
int kruskal(){
    int res = 0;
    sort(edges.begin(), edges.end());
    for(int i = 0;i < edges.size();++i){
        Edge& e = edges[i];
        if(!uf.same(e.u, e.v)){
            res += e.w;
            uf.unite(e.u, e.v);
        }
    }
    return res;
}
```

四、数学

1、组合数(lucas 定理+费马小定理，要求模数 p 是素数)

```
#define MAXN 100100
using namespace std;
typedef long long LL;
LL facts[MAXN], p;

LL qpow(LL a, LL x) {
    LL res = 1;
    while(x > 0) {
        if(x & 1)res = (res * a) % p;
        a = (a * a) % p;
        x >>= 1;
    }
    return res;
}

/*调用 lucas(n, m)前一定初始化。同时保证 p 是素数。*/
void init() {
    facts[0] = 1;
    for(int i = 1; i < MAXN; ++i) {
        facts[i] = (facts[i - 1] * i) % p;
    }
}

LL C(LL n, LL m) {
    if(n < m)return 0;
    if(n == m || m == 0)return 1;
    if(m == 1)return n % p;
    return facts[n] * qpow(facts[m], p - 2) % p * qpow(facts[n - m], p - 2) % p;
}

LL lucas(LL n, LL m) {
    if(n < p && m < p)return C(n, m);
    else return lucas(n / p, m / p) * lucas(n % p, m % p) % p;
}
```

2、快速拉格朗日插值法求和

```
#include<bits/stdc++.h>
using namespace std;
#define MAXN 1010
typedef long long LL;
const LL mod = 1e9 + 7;

LL powmod(LL aa, LL x) {
    LL res = 1;
    for(; x > 0; x >>= 1) {
        if(x & 1)res = (res * aa) % mod;
        aa = (aa * aa) % mod;
    }
    return res;
}

struct lagrange {
```

```
#define ll long long
#define rep(i,a,n) for (int i=a;i<n;i++)
#define per(i,a,n) for (int i=n-1;i>=a;i--)
#define D 2010 //D 比 MAXN 大 100 就行
    ll a[D], f[D], g[D], p[D], p1[D], p2[D], b[D], h[D][2], C[D];
    void init(int M) {//初始化：参数填 MAXN + 20
        f[0] = f[1] = g[0] = g[1] = 1;
        rep(i, 2, M + 5) f[i] = f[i - 1] * i % mod;
        g[M + 4] = powmod(f[M + 4], mod - 2);
        per(i, 1, M + 4) g[i] = g[i + 1] * (i + 1) % mod;
    }
    /*给定一组样本数据 a[]，规模为 0-d，计算出第 n 项*/
    ll calcn(int d, ll *a, ll n) {
        if (n <= d) return a[n];
        p1[0] = p2[0] = 1;
        rep(i, 0, d + 1) {
            ll t = (n - i + mod) % mod;
            p1[i + 1] = p1[i] * t % mod;
        }
        rep(i, 0, d + 1) {
            ll t = (n - d + i + mod) % mod;
            p2[i + 1] = p2[i] * t % mod;
        }
        ll ans = 0;
        rep(i, 0, d + 1) {
            ll t = g[i] * g[d - i] % mod * p1[i] % mod * p2[d - i] % mod * a[i] % mod;
            if ((d - i) & 1) ans = (ans - t + mod) % mod;
            else ans = (ans + t) % mod;
        }
        return ans;
    }
    /*
    给定一组观测点(0, a[0]), (1, a[1]), ..., (m, a[m])，、
    样本点的个数为 f(x)的最高次+1。
    求在该函数模型下，a[0]+a[1]+...+a[n]的和。
    */
    ll ta[D];
    ll polysum(ll m, ll *a, ll n) { // a[0].. a[m] \sum_{i=0}^{n} a[i]
        memcpy(ta, a, sizeof(a[0]) * (m + 1));
        ta[m + 1] = calcn(m, ta, m + 1);
        rep(i, 1, m + 2)ta[i] = (ta[i - 1] + ta[i]) % mod;
        return calcn(m + 1, ta, n);
    }
};

lagrange lag;
int main(){
    lag.init(MAXN + 20);
    return 0;
}
```

(1)例题(计算通式x^3+x^2+x+1的前3×10^8的和)

```
#include<bits/stdc++.h>
using namespace std;
#define MAXN 1010
typedef long long LL;
const LL mod = 1e9 + 7;

LL powmod(LL aa, LL x) {
    LL res = 1;
    for(; x > 0; x >>= 1) {
        if(x & 1)res = (res * aa) % mod;
        aa = (aa * aa) % mod;
    }
    return res;
}

struct lagrange {
#define ll long long
#define rep(i,a,n) for (int i=a;i<n;i++)
#define per(i,a,n) for (int i=n-1;i>=a;i--)
#define D 2010 //D 比 MAXN 大 100 就行
    ll a[D], f[D], g[D], p[D], p1[D], p2[D], b[D], h[D][2], C[D];
    void init(int M) {//初始化：参数填 MAXN + 20
        f[0] = f[1] = g[0] = g[1] = 1;
        rep(i, 2, M + 5) f[i] = f[i - 1] * i % mod;
        g[M + 4] = powmod(f[M + 4], mod - 2);
        per(i, 1, M + 4) g[i] = g[i + 1] * (i + 1) % mod;
    }
    /*给定一组样本数据 a[]，规模为 0-d，计算出第 n 项*/
```

```
ll calcn(int d, ll *a, ll n) {
    if (n <= d) return a[n];
    p1[0] = p2[0] = 1;
    rep(i, 0, d + 1) {
        ll t = (n - i + mod) % mod;
        p1[i + 1] = p1[i] * t % mod;
    }
    rep(i, 0, d + 1) {
        ll t = (n - d + i + mod) % mod;
        p2[i + 1] = p2[i] * t % mod;
    }
    ll ans = 0;
    rep(i, 0, d + 1) {
        ll t = g[i] * g[d - i] % mod * p1[i] % mod * p2[d - i] % mod * a[i] % mod;
        if ((d - i) & 1) ans = (ans - t + mod) % mod;
        else ans = (ans + t) % mod;
    }
    return ans;
}
/*
给定一组观测点(0, a[0]), (1, a[1]), ..., (m, a[m])，、
样本点的个数为 f(x)的最高次+1。
求在该函数模型下，a[0]+a[1]+...+a[n]的和。
*/
ll ta[D];
ll polysum(ll m, ll *a, ll n) { // a[0].. a[m] \sum_{i=0}^{n} a[i]
```

```
        memcpy(ta, a, sizeof(a[0]) * (m + 1));
        ta[m + 1] = calcn(m, ta, m + 1);
        rep(i, 1, m + 2)ta[i] = (ta[i - 1] + ta[i]) % mod;
        return calcn(m + 1, ta, n);
    }
};

lagrange lag;
int main(){
    lag.init(MAXN + 20);
    return 0;
}

int main(){
    lag.init(MAXN + 20);
    typedef long long LL;
    LL a[] = {1, 4, 15, 40};//x^3+x^2+x+1
    LL ans = lag.polysum(3, a, 300000000);
    cout << ans << endl;
    LL ans2 = 0;
    for(LL i = 0;i <= 300000000LL;++i){
        ans2 += (i * i % mod * i + i * i % mod + i + 1);
        ans2 %= mod;
    }cout << ans2 << endl;
    return 0;
}
```

3、预处理1~1M内数字*i*和阶乘的逆元，计算组合数

```
#include<bits/stdc++.h>
using namespace std;
typedef long long LL;
const LL mod = 998244353;
#define MAXN 1000100
LL fact[MAXN], inv[MAXN], finv[MAXN];

LL qpow(LL a, LL x) {
    LL res = 1;
    for(; x > 0; x >>= 1) {
        if(x & 1)res = (res * a) % mod;
        a = (a * a) % mod;
    }
    return res;
}

void init() {
    fact[0] = fact[1] = 1;
    inv[0] = inv[1] = 1;
    for(int i = 2; i < MAXN; i++){
        fact[i] = fact[i - 1] * i % mod;
        inv[i] = (mod - mod / i) * inv[mod % i] % mod;;
    }
    finv[MAXN - 1] = qpow(fact[MAXN - 1], mod - 2);
    for(int i = MAXN - 2; i >= 0; --i) finv[i] = finv[i + 1] * (i + 1) % mod;
}
```

```
LL C(int n, int m){
    if(n == m || m == 0)return 1LL;
    return (fact[n] * finv[n - m] % mod) * finv[m] % mod;
}

int main(){
    return 0;
}
```

4、线性基

（1）查第 k 小（不支持动态插入）

```
#include <cstdio>
#include <vector>
const int MAXN = 100010;
const int MAXL = 62;
struct LinearBasis {
    std::vector<long long> v;
    int n;
    void build(long long *x, int n) {
        this->n = n;
        std::vector<long long> a(MAXL + 1);
        for (int i = 1; i <= n; i++) {
            long long t = x[i];
            for (int j = MAXL; j >= 0; j--) {
                if ((t & (1ll << j)) == 0) continue;
                if (a[j]) t ^= a[j];
                else {
                    for (int k = 0; k < j; k++) if (t & (1ll << k)) t ^= a[k];
                    for (int k = j + 1; k <= MAXL; k++) if (a[k] & (1ll << j)) a[k] ^= t;
                    a[j] = t;
                    break;
                }
            }
        }
        v.clear();
        for (int i = 0; i <= MAXL; i++) if (a[i]) v.push_back(a[i]);
    }

    long long kmin(long long k) {//从 1 开始的第 k 小
        if (int(v.size()) != n) {
            // 可能是 0
            k--;
        }
        // 如果 k 超过的所有不同异或和的数量
        if (k > (1ll << v.size()) - 1) return -1;
        long long ans = 0;
        for (size_t i = 0; i < v.size(); i++) {
            if (k & (1ll << i)) {
                ans ^= v[i];
            }
        }
        return ans;
    }
} lb;

int main() {
    int n, T;
    scanf("%d", &T);
    for(int ca = 1; ca <= T; ++ca) {
        scanf("%d", &n);
        static long long a[MAXN + 1];
        for (int i = 1; i <= n; i++) scanf("%lld", &a[i]);
        lb.build(a, n);
        int m;
        scanf("%d", &m);
        printf("Case #%d:\n", ca);
        while (m--) {
            long long k;
            scanf("%lld", &k);
            printf("%lld\n", lb.kmin(k));
        }
    }
    return 0;
}
```

(2) 查最大、最小、合并（支持动态插入）

```
#include<bits/stdc++.h>
using namespace std;
struct LinearBasis {
#define MAXL 62
    long long a[MAXL + 1];
    LinearBasis() {
        std::fill(a, a + MAXL + 1, 0);
    }
    LinearBasis(long long *x, int n) {
        build(x, n);
    }
    void insert(long long t) {
        for (int j = MAXL; j >= 0; j--) {
            if (!t) return;
            if (!(t & (1ll << j))) continue;
            if (a[j]) t ^= a[j];
            else {
                for (int k = 0; k < j; k++) if (t & (1ll << k)) t ^= a[k];
                for (int k = j + 1; k <= MAXL; k++) if (a[k] & (1ll << j)) a[k] ^= t;
                a[j] = t;
                break;
            }
        }
    }

    // 数组 x 表示集合 S，下标范围 [1...n]
    void build(long long *x, int n) {
        memset(a, 0, sizeof(a));
        for (int i = 1; i <= n; i++) {
            insert(x[i]);
        }
    }

    long long queryMax() {
        long long res = 0;
        for (int i = 0; i <= MAXL; i++) res ^= a[i];
        return res;
    }
    void mergeFrom(const LinearBasis &other) {
        for (int i = 0; i <= MAXL; i++) insert(other.a[i]);
    }

    static LinearBasis merge(const LinearBasis &a, const LinearBasis &b) {
        LinearBasis res = a;
        for (int i = 0; i <= MAXL; i++) res.insert(b.a[i]);
        return res;
    }
} lb;

long long a[55];
int main() {
    int n;
    scanf("%d", &n);
    for(int i = 1; i <= n; ++i)scanf("%lld", a + i);
    lb = LinearBasis(a, n);
    printf("%lld\n", lb.queryMax());
    return 0;
}
```

5、FWT

```
#include<bits/stdc++.h>
using namespace std;
typedef long long LL;
const LL mod = 1e9 + 7;
/*
```

```
如果不需要取模，直接去掉取模，乘逆元变成除法，不影响最终结果。
调用时，只需要改变 solve()函数中的 type 即可。
**/
struct FFF {
#define AND 0
#define OR 1
#define XOR 2
    //下标从 0 开始,N 为 2 的幂次
    int N, type;
public:
    void init(int n) {
        N = n;
        type = XOR;
    }
private:
    template<class T> void FWT(T a[]) {
        for(int i = 1; i < N; i <<= 1) {
            for(int p = i << 1, q = 0; q < N; q += p) {
                for(int j = 0; j < i; j++) {
                    T x = a[q + j], y = a[q + j + i];
                    //and
                    if(type == AND)a[q + j] = (x + y) % mod;
                    //or
                    else if(type == OR)a[q + j + i] = (x + y) % mod;
                    //xor
                    else if(type == XOR)a[q + j] = (x + y) % mod, a[q + j + i] = (x - y +
mod) % mod;
                }
            }
        }
    }
private:
    template<class T> void UFWT(T a[]) {
        T inv2 = (mod + 1) >> 1;
        for(int i = 1; i < N; i <<= 1) {
            for(int p = i << 1, q = 0; q < N; q += p) {
                for(int j = 0 ; j < i; j++) {
                    T x = a[q + j], y = a[q + j + i];
                    //and
                    if(type == AND)a[q + j] = (x - y + mod) % mod;
                    //or
                    else if(type == OR)a[q + j + i] = (y - x + mod) % mod;
                    //xor
                    else if(type == XOR)a[q + j] = 1LL * (x + y) * inv2 % mod, a[q + j + i]
= (1LL * (x - y) * inv2 % mod + mod) % mod;
                }
            }
        }
    }
public:
    template<class T> void solve(T a[], T b[], T c[]) {
        FWT(a), FWT(b);
        for(int i = 0; i < N; i++) c[i] = 1LL * a[i] * b[i] % mod;
        UFWT(c);
    }
} fwt;
```

五、字符串

六、其他

1、输入挂

(1)slowIO

```
template <class T> inline void read(T &x) {
    char t;
    bool sign = false;
    while((t = getchar()) != '-' && (t < '0' || t > '9'));
    if(t == '-') sign = true, t = getchar();
    x = t - '0';
    while((t = getchar()) >= '0' && t <= '9') x = x * 10 + t - '0';
    if(sign) x = -x;
}
```

(2)fastIO

```
namespace fastIO {
    #define BUF_SIZE 100000
    //fread -> read
    bool IOerror = 0;
    inline char nc() {
        static char buf[BUF_SIZE], *p1 = buf + BUF_SIZE, *pend = buf + BUF_SIZE;
        if(p1 == pend) {
            p1 = buf;
            pend = buf + fread(buf, 1, BUF_SIZE, stdin);
            if(pend == p1) {
                IOerror = 1;
                return -1;
            }
        }
        return *p1++;
    }
    inline bool blank(char ch) {
        return ch == ' ' || ch == '\n' || ch == '\r' || ch == '\t';
    }
    template<class T>
    inline void read(T &x) {
        bool sign = 0; char ch = nc(); x = 0;
        for (; blank(ch); ch = nc());
        if (IOerror)return;
        if (ch == '-')sign = 1, ch = nc();
        for (; ch >= '0' && ch <= '9'; ch = nc())x = x * 10 + ch - '0';
        if (sign)x = -x;
    }
    struct Ostream_fwrite {
        char *buf, *p1, *pend;
        Ostream_fwrite() {buf = new char[BUF_SIZE]; p1 = buf; pend = buf + BUF_SIZE;}
        void out(char ch) {
            if (p1 == pend) {
                fwrite(buf, 1, BUF_SIZE, stdout); p1 = buf;
            }
            *p1++ = ch;
        }
```

```
        template<class T> void print(T x) {
            static char s[25], *s1; s1 = s;
            if (!x)*s1++ = '0';
            if (x < 0)out('-'), x = -x;
            while(x)*s1++ = x % 10 + '0', x /= 10;
            while(s1-- != s)out(*s1);
        }
        void print(char *s) {while (*s)out(*s++);}
        void flush() {if (p1 != buf) {fwrite(buf, 1, p1 - buf, stdout); p1 = buf;}}
        ~Ostream_fwrite() {flush();}
    } Ostream;
    template<class T>
    inline void print(T x) {Ostream.print(x);}
    template<class T>
    inline void println(T x) {Ostream.print(x);Ostream.out('\n');}
    inline void print(char *s) {Ostream.print(s);}
    inline void println(char *s) {Ostream.print(s);Ostream.out('\n');}
};
using namespace fastIO;
```

2、常用函数

(1) 字符数组分割 strtok

```
#include<cstdio>
#include<cstring>
#include<cstdlib>
char str[] = "12#34#56#78";
int main(){
    char *tmp = strtok(str, "#");
    do{
        printf("%d\n", atoi(tmp));
        tmp = strtok(NULL, "#");
    }while(tmp != NULL);
    return 0;
}
```

（2）数字转字符数组 sprintf

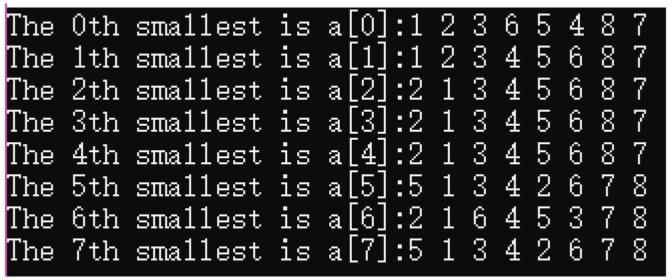
```
char buf[70];//确保足够
sprintf(buf, "%d", 123456);
```

（3）string 转字符数组

```
string s("123456");
char buf[70];//确保足够
strcpy(buf, s.data());
```

（4）nth_element(把第 n 小的元素放在第 n 位置)

```
#include<bits/stdc++.h>
using namespace std;
int main() {
    int a[] = {1, 7, 5, 3, 2, 8, 4, 6};
    for(int k = 0; k < 8; ++k) {
        nth_element(a, a + k, a + 8);
        printf("The %dth smallest is a[%d]:", k, k);
        for(int i = 0; i < 8; ++i)printf("%d ", a[i]); cout << endl;
    }
    return 0;
}
```



```
The 0th smallest is a[0]:1 2 3 6 5 4 8 7
The 1th smallest is a[1]:1 2 3 4 5 6 8 7
The 2th smallest is a[2]:2 1 3 4 5 6 8 7
The 3th smallest is a[3]:2 1 3 4 5 6 8 7
The 4th smallest is a[4]:2 1 3 4 5 6 8 7
The 5th smallest is a[5]:5 1 3 4 2 6 7 8
The 6th smallest is a[6]:2 1 6 4 5 3 7 8
The 7th smallest is a[7]:5 1 3 4 2 6 7 8
```

（5）编译器内置的二进制处理函数

```
– Built-in Function: int __builtin_ffs (unsigned int x)
Returns one plus the index of the least significant 1-bit of x, or if x is zero, returns
zero.
返回右起第一个‘1’的位置。

– Built-in Function: int __builtin_clz (unsigned int x)
Returns the number of leading 0-bits in x, starting at the most significant bit position.
If x is 0, the result is undefined.
返回左起第一个‘1’之前 0 的个数。

– Built-in Function: int __builtin_ctz (unsigned int x)
Returns the number of trailing 0-bits in x, starting at the least significant bit
position. If x is 0, the result is undefined.
返回右起第一个‘1’之后的 0 的个数。

– Built-in Function: int __builtin_popcount (unsigned int x)
Returns the number of 1-bits in x.
```

返回‘1’的个数。

– Built-in Function: int __builtin_parity (unsigned int x)
Returns the parity of x, i.e. the number of 1-bits in x modulo 2.
返回‘1’的个数的奇偶性。

– Built-in Function: int __builtin_ffsl (unsigned long)
Similar to __builtin_ffs, except the argument type is unsigned long.

– Built-in Function: int __builtin_clzl (unsigned long)
Similar to __builtin_clz, except the argument type is unsigned long.

– Built-in Function: int __builtin_ctzl (unsigned long)
Similar to __builtin_ctz, except the argument type is unsigned long.

– Built-in Function: int __builtin_popcountl (unsigned long)
Similar to __builtin_popcount, except the argument type is unsigned long.

– Built-in Function: int __builtin_parityl (unsigned long)
Similar to __builtin_parity, except the argument type is unsigned long.

– Built-in Function: int __builtin_ffsll (unsigned long long)
Similar to __builtin_ffs, except the argument type is unsigned long long.

– Built-in Function: int __builtin_clzll (unsigned long long)
Similar to __builtin_clz, except the argument type is unsigned long long.

– Built-in Function: int __builtin_ctzll (unsigned long long)
Similar to __builtin_ctz, except the argument type is unsigned long long.

```
– Built-in Function: int __builtin_popcountll (unsigned long long)
Similar to __builtin_popcount, except the argument type is unsigned long long.

– Built-in Function: int __builtin_parityll (unsigned long long)
Similar to __builtin_parity, except the argument type is unsigned long long.
```

3、BM 算法(用于求线性递推的第n项)

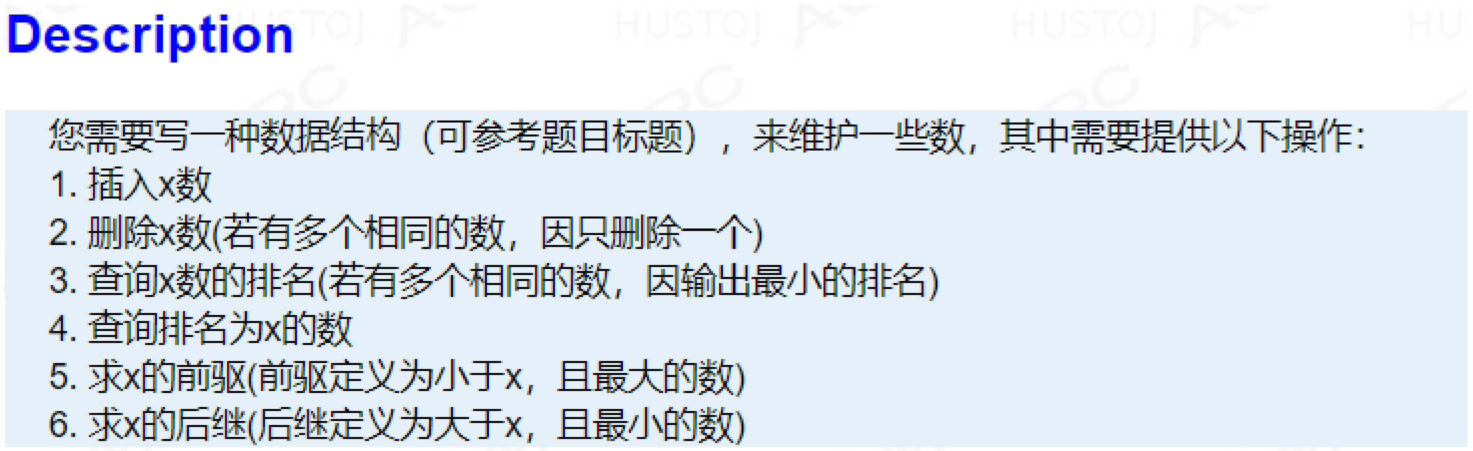
```
#include<bits/stdc++.h>
using namespace std;
#define rep(i,a,n) for (int i=a;i<n;i++)
#define per(i,a,n) for (int i=n-1;i>=a;i--)
#define pb push_back
#define mp make_pair
#define all(x) (x).begin(),(x).end()
#define fi first
#define se second
#define SZ(x) ((int)(x).size())
typedef long long ll;
typedef vector<ll> VI;
typedef pair<int, int> PII;
const ll mod = 1000000007;
ll powmod(ll a, ll b) {
    ll res = 1;
    a %= mod;
    assert(b >= 0);
    for(; b; b >>= 1) {
        if(b & 1)res = res * a % mod;
        a = a * a % mod;
    }
    return res;
}
// head

namespace linear_seq {
    const int N = 10010;
    ll res[N], base[N], _c[N], _md[N];

    vector<int> Md;
    void mul(ll *a, ll *b, int k) {
        rep(i, 0, k + k) _c[i] = 0;
        rep(i, 0, k) if (a[i]) rep(j, 0, k) _c[i + j] = (_c[i + j] + a[i] * b[j]) % mod;
        for (int i = k + k - 1; i >= k; i--) if (_c[i])
                rep(j, 0, SZ(Md)) _c[i - k + Md[j]] = (_c[i - k + Md[j]] - _c[i] *
_md[Md[j]]) % mod;
        rep(i, 0, k) a[i] = _c[i];
    }
    int solve(ll n, VI a, VI b) { // a 系数 b 初值 b[n+1]=a[0]*b[n]+...
//        printf("SIZE %d\n",SZ(b));
        ll ans = 0, pnt = 0;
        int k = SZ(a);
        assert(SZ(a) == SZ(b));
        rep(i, 0, k) _md[k - 1 - i] = -a[i]; _md[k] = 1;
        Md.clear();
        rep(i, 0, k) if (_md[i] != 0) Md.push_back(i);
        rep(i, 0, k) res[i] = base[i] = 0;
        res[0] = 1;
        while ((1ll << pnt) <= n) pnt++;
        for (int p = pnt; p >= 0; p--) {
            mul(res, res, k);
            if ((n >> p) & 1) {
                for (int i = k - 1; i >= 0; i--) res[i + 1] = res[i]; res[0] = 0;
                rep(j, 0, SZ(Md)) res[Md[j]] = (res[Md[j]] - res[k] * _md[Md[j]]) % mod;
            }
        }
        rep(i, 0, k) ans = (ans + res[i] * b[i]) % mod;
        if (ans < 0) ans += mod;
        return ans;
    }
    VI BM(VI s) {
        VI C(1, 1), B(1, 1);
        int L = 0, m = 1, b = 1;
        rep(n, 0, SZ(s)) {
            ll d = 0;
            rep(i, 0, L + 1) d = (d + (ll)C[i] * s[n - i]) % mod;
            if (d == 0) ++m;
            else if (2 * L <= n) {
                VI T = C;
                ll c = mod - d * powmod(b, mod - 2) % mod;
                while (SZ(C) < SZ(B) + m) C.pb(0);
                rep(i, 0, SZ(B)) C[i + m] = (C[i + m] + c * B[i]) % mod;
                L = n + 1 - L; B = T; b = d; m = 1;
            }
            else {
                ll c = mod - d * powmod(b, mod - 2) % mod;
                while (SZ(C) < SZ(B) + m) C.pb(0);
                rep(i, 0, SZ(B)) C[i + m] = (C[i + m] + c * B[i]) % mod;
                ++m;
            }
        }
        return C;
    }
    int gao(VI a, ll n) {
        VI c = BM(a);
        c.erase(c.begin());
        rep(i, 0, SZ(c)) c[i] = (mod - c[i]) % mod;
        return solve(n, c, VI(a.begin(), a.begin() + SZ(c)));
    }
};
```

4、pb_ds 库

(1)红黑树(以 BZOJ3224 为例)



Description

您需要写一种数据结构（可参考题目标题），来维护一些数，其中需要提供以下操作:
1. 插入x数
2. 删除x数(若有多个相同的数，因只删除一个)
3. 查询x数的排名(若有多个相同的数，因输出最小的排名)
4. 查询排名为x的数
5. 求x的前驱(前驱定义为小于x，且最大的数)
6. 求x的后继(后继定义为大于x，且最小的数)

```
#include<bits/stdc++.h>
#include<ext/pb_ds/assoc_container.hpp>
```

```
#include<ext/pb_ds/tree_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
typedef long long LL;
/*g++ 4.4 以下的编译器版本第二个参数为:null_mapped_type*/
typedef tree<LL, null_type, less<LL>, rb_tree_tag, tree_order_statistics_node_update>
rb_tree;
rb_tree st;
typedef unordered_map<LL,int> ump;
ump cnt;
int main() {
//    freopen("input.txt", "r", stdin);
    int q, op;
    LL x;
    scanf("%d", &q);
    while(q--) {
        scanf("%d%lld", &op, &x);
        if(op == 1) {
            st.insert(x << 32 | (++cnt[x]));
        }
        else if(op == 2) {
            LL v = x << 32 | (cnt[x]--);
            assert(st.find(v) != st.end());
            st.erase(v);
        }
        else if(op == 3) {
            printf("%d\n", (int)st.order_of_key(x << 32 | 1) + 1);
        }
        else if(op == 4) {
            LL v = *st.find_by_order(x - 1);
            v >>= 32;
            printf("%lld\n", v);
        }
        else if(op == 5) {
            rb_tree::iterator it = st.lower_bound(x << 32);
            assert(it != st.begin());
            --it;
            printf("%lld\n", (*it) >> 32);
        }
        else if(op == 6) {
            rb_tree::iterator it = st.upper_bound(x << 32 | (cnt.find(x) != cnt.end() ?
cnt[x] : 0));
            assert(it != st.end());
            printf("%lld\n", (*it) >> 32);
        }
    }
    return 0;
}
```

(2)gp_hash_table(像数组一样使用)

```
#include<bits/stdc++.h>
#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/hash_policy.hpp>
using namespace std;
using namespace __gnu_pbds;
gp_hash_table<int,bool> tab;

int main(){
    return 0;
}
```

5、快速随机数

```cpp
#include<iostream>
#include<chrono>
#include<random>
using namespace std;
/*mt19937:随机产生 unsigned int 范围内的整数;
mt19937_64:随机产生 unsigned long long 范围内的整数;
*/
unsigned test1() {
    unsigned seed = chrono::system_clock::now().time_since_epoch().count();
    mt19937 gen(seed);  // mt19937 is a standard mersenne_twister_engine
    return gen();
}
```

6、单样例输入对拍程序/批处理脚本

(1)脚本: windows

```
@echo off
:again
dataCreator.exe>a.in
sol.exe<a.in>a1.out
bf.exe<a.in>a2.out
fc a1.out a2.out
if %errorlevel%==0 goto again
```

```
pause
```

(2)程序：windows

```
#include<bits/stdc++.h>
using namespace std;
int main(){
    while(1){
        system("dataCreator.exe>a.in");
        system("sol.exe<a.in>a1.out");
        system("bf.exe<a.in>a2.out");
        if(system("fc a1.out a2.out"))break;
    }
    return 0;
}
```

(3)程序：Ubuntu

```
#include<bits/stdc++.h>
using namespace std;
int main(){
    while(1){
        system("./dataCreator>a.in");
        system("./sol<a.in>a1.out");
        system("./bf<a.in>a2.out");
        if(system("diff a1.out a2.out"))break;
        else puts("AC");
    }
    return 0;
}
```

7、集合的整数操作汇总

```
#include<bits/stdc++.h>
using namespace std;
int main() {
    /*1、枚举子集*/
    for(int n = 5, s = 0; s < 1 << n; ++s) {
        for(int j = s; j; j = (j - 1)&s) {
            /*j 为 s 的所有子集。
            例如 s=11, 二进制表示为 1011, 那么, j 会从大到小遍历 11,10,9,8,3,2,1
            **/
        }
    }

    /*2、枚举大小为 k 的子集*/
    for(int n = 5, k = 3, sub = (1 << k) - 1; sub < 1 << n;) {
        cout << bitset<8>(sub) << endl;
        int x = sub & -sub, y = sub + x;
        sub = ((sub & ~y) / x >> 1) | y;
    }
    return 0;
}
```