Structured optimal control with applications to network flow coordination *

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Abstract In this paper the design of controllers that incorporate structural and multiobjective performance requirements is considered. The control structures under study cover nested, chained, hierarchical, delayed interaction and communications, and symmetric systems. Such structures are strongly related to several modern-day and future applications including integrated flight propulsion systems, platoons of vehicles, micro-electro-mechanical systems, networked control, control of networks, production lines and chemical processes. It is shown that the system classes presented have the common feature that all stabilizing controllers can be characterized by convex constraints on the Youla-Kucera parameter. Using this feature, a solution to a general optimal performance problem that incorporates time domain and frequency domain constraints is obtained. A synthesis procedure is provided which at every step yields a feasible controller together with a measure of its performance with respect to the optimal. Convergence to the optimal performance is established. An example of a multi-node network congestion control problem is provided that illustrates the effectiveness of the developed methodology.
Keywords Structure, input-output, ℓ_1

1 Introduction

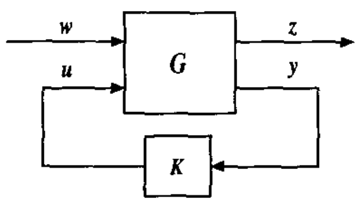

Figure 1: Standard Framework

In large, complex and distributed systems there is often the need of considering a specific structure on the overall control scheme (e.g., [1]). In this paper we consider the general framework of Figure 1 where G may represent a complex system consisting of subsystems interacting with each other. The overall controller for G is K. Both G and K are assumed to be linear, discrete-time systems. The controller K has to respect a specific structure that may be imposed by interaction and communication constraints. A typical example of structure that has been studied extensively in the literature is when K is decentralized. However, the optimal performance problem when structural constraints are present still remains a challenge, notably the lack of a convex characterization of the problem (see for example [2, 3, 4] and references therein). Taking an input-output point of view and parameterizing all stabilizing K via the Youla-Kucera [5] parameter Q one can see as a major source of difficulty is that structural constraints on K may lead to non-convex constraints on Q. A main theme in the paper is the identification of specific classes of problems for which the constraints on Q are convex with the appropriate choice of the coprime factors of G. The various classes of systems identified include what are herein referred to as nested, chained, hierarchical, delayed interaction and communication, and, symmetric systems. They are associated with several practical applications such as integrated flight propulsion systems, platoons of vehicles, networked control, production lines, and chemical processes. Common to all these problems is that G_{22}, the part of the system that relates controls u to measurements y, has a specific structure. It is the structure of G_{22} that matters for convexity; the remaining part of G can be unstructured.

In the classes that we consider, G_{22} has the same structure as the one imposed on the controller K. This by itself is not in general a necessary or sufficient condition for the problem to be convex. As indicated in [6] where these structures were initially reported, there is an algebraic property between the K's and G_{22}'s under consideration. That is, with the exception of the symmetric case, they form a ring as the structure is preserved in products, additions and in $(I - G_{22}K)^{-1}$ whenever the inverse exists, as it should, for well-posedness. A very interesting relaxation of this ring property is possible as reported in [7] where it is shown that structures that are so-called quadratically invariant, lead to convex closed loop maps in the term $K(I - G_{22}K)^{-1}$, which is the Youla-Kucera parameter in the case of stable G. However, by itself convexity in $K(I - G_{22}K)^{-1}$ does not provide a method to effectively synthesize controllers that incorporate structure and specifications on the closed-loop performance in terms of various measures that may reflect various frequency and time domain concerns. This is precisely what this paper achieves. In particular, in this article we explicitly identify the coprime factors to be employed in the Youla-Kucera parameterization that result in the parameter Q inheriting the structure of the controller K. The results obtained hold for stable as well as unstable plants G_{22}. The constructive nature of these results allows one to pose the problem of optimal performance as an optimization problem in the parameter Q. For various different specifications and performance measures on the desired closed-loop behavior the resulting optimization problem takes a convex form, which we solve within any prespecified tolerance from the optimal via tractable finite dimensional convex optimization problems.

The need to design control systems that perform well with

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respect to a variety of performance criteria is important in its own right and has led to the development of a number of multiobjective techniques over the last decade (see for example [8], [9] and the references therein). The majority of these methods concentrate on optimizing a combination of two-criteria, selected among standard measures such as $\mathcal{H}_\infty$, $\mathcal{H}_2$, ℓ_1 closed loop norms. Moreover, these results do not address the concerns of controllers that need to satisfy structure constraints. Here, we consider the general multi-objective (GMO) control synthesis problem that can simultaneously incorporate multiple measures on the closed-loop in terms of the $\ell_1, \mathcal{H}_2, \mathcal{H}_\infty$ norms and can incorporate time-domain template constraints on the response of the closed-loop due to specific inputs while respecting the structure constraints on the controller. As stated before the results on translating the structure constraints on the controller K to structure constraints on the Youla-Kucera parameter Q makes it possible to use Q as the optimizing variable. An important advantage of incorporating Q as an optimization variable is that the controller can be easily retrieved from the optimal solution. This is in contrast to methods where interpolation constraints are imposed on the closed-loop map to characterize their achievability. This method has similarity to the basic $Q-$des method introduced in [10]. However, in this article emphasis is placed on obtaining a sequence of tractable finite dimensional convex optimization problems which provide converging upper and lower bounds to the optimal cost. Thus in this methodology, every step furnishes a feasible controller that meets the specifications together with the distance from the optimal performance. The development presented here is inspired in part by some of the ideas presented in [11] for the special case of the unstructured $\mathcal{H}_2/\ell_1$ problem. Our approach, apart from providing solutions to hitherto unaddressed multiobjective problems, relies only on the primal formulation of the problem. Thus there is no need for constructing the dual and to perform the ensuing analysis of the dual and its relationship to the primal. This approach makes it straightforward to add new performance objectives into the problem formulation and its solution. Finally, it needs to be noted that problems with structured constraints have only recently been considered in [12, 13, 14, 15, 16] where the Youla-Kucera parameterization is used. These works all fall within the general classes of problems considered here.

The paper is organized as follows. In Section 2, we present the various control structures of interest. In Section 3 to 5, we introduce the controller parameterization and describe the solution procedure to the structural multiobjective optimal control synthesis problem. In Section 6, we illustrate the effectiveness of the proposed framework with a network congestion control example. In Section 7, we conclude the paper.

2 Classes of System Structures

2.1 Triangular Structures

Nested systems: simple triangular structures

This class represents the case where a subsystem is inside another and there is only one-way interaction, from inside

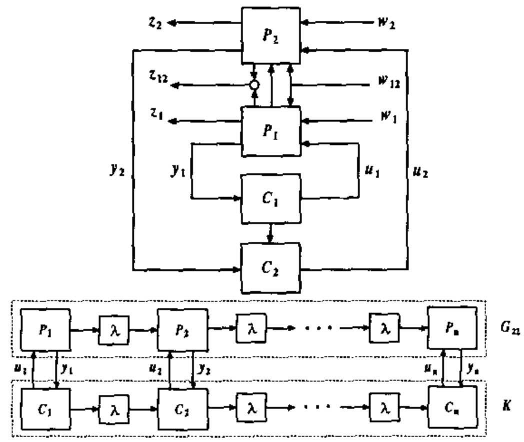

Figure 2: Nested structure and chain structure

to outside, or, the reverse. The two-nests case is shown in Figure 2(up) where it can be seen that the plant G_{22} and the controller K have the following lower (block) triangular (l.b.t.) structure:

$$G_{22} := \begin{bmatrix} g_1 & 0 \\ g_{21} & g_2 \end{bmatrix}, \quad K = \begin{bmatrix} k_1 & 0 \\ k_{12} & k_2 \end{bmatrix}.$$

Chains

In the chain (or string) system of Figure 2(down), bringing in the general $G-K$ framework we have that the structure of G_{22} is

$$G_{22} = \begin{bmatrix} g_1 & & & \\ \lambda g_{21} & g_2 & & \\ \vdots & \ddots & \ddots & \\ \lambda^n g_{n1} & \cdots & \lambda g_{nn-1} & g_n \end{bmatrix}$$

with K as in G_{22} by replacing g's with k's. This implies that this problem can be addressed using an equivalent simple triangular structure.

Hierarchical structures

This type of structure is depicted in Figure 3(up), for which the controller K and the G_{22} admit the following upper block triangular (u.b.t.) structure:

$$K = \begin{bmatrix} * & * & \dots & * \\ 0 & * & \dots & * \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & * \end{bmatrix}, \quad G_{22} = \begin{bmatrix} * & * & \dots & * \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

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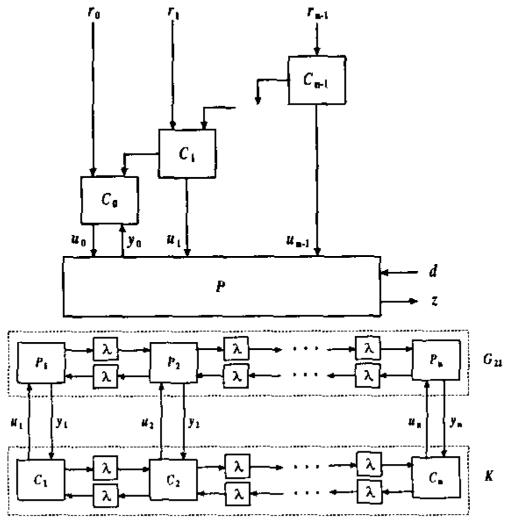

Figure 3: Hierarchical structure and delayed interaction and communication structure

Toeplitz triangular structures

An additional triangular structure that is interest that requires fewer building blocks is that of Toeplitz, i.e.,

$$G_{22} = \begin{bmatrix} g_1 & & & \\ \lambda g_2 & g_1 & & \\ \vdots & \ddots & \ddots & \\ \lambda^{n-1} g_n & \dots & \lambda g_2 & g_1 \end{bmatrix}$$

with K as in G_{22} by replacing g's with k's.

2.2 Delayed Interaction and Communication Networks: Band Structure

The network in this case is as in Figure 3(down). In the $G - K$ framework the structure is reflected as

$$G_{22} = \begin{bmatrix} g_{11} & \lambda g_{12} & \cdots & \lambda^{n-1} g_{1n} \\ \lambda g_{21} & g_{22} & \cdots & \lambda^{n-2} g_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda^{n-1} g_{n1} & \cdots & \cdots & g_{nn} \end{bmatrix}$$

with K as in G_{22} by replacing g's with k's. There are some other structure patterns of interest (like the delayed observation sharing structure and symmetric structures) that can be addressed in a similar manner and are omitted due to space limits.

3 Controller Parameterization

Via Youla-Kucera parameterization, all stabilizing K are given by[5]:

$$K = (Y_r - M_r Q)(X_r - N_r Q)^{-1} = (X_\ell - Q N_\ell)^{-1}(Y_\ell - Q M_\ell)$$

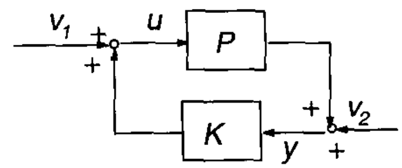

Figure 4: Feedback interconnection of $P = G_{22}$ and K.

where Q is a stable free parameter and Y_r, D_r, X_r, N_r, X_l, N_l, Y_l, D_l can be obtained from a coprime factorization (e.g., [17, 18]) of G_{22}. These coprime factors are highly non-unique. However, there is a particular choice of these factors such that the structural constraints on K transform to the exactly same constraints in Youla parameter Q.

3.1 Triangular Structures

Lemma 3.1 *Let the plant P that maps the control inputs $u = (u_1, \dots, u_n)^T$ to the measured outputs $y = (y_1, \dots, y_n)^T$ be lower triangular. Assume that the i^{th} row of P denoted by $P_i = \{P_{i1} \dots P_{ii}\ 0 \dots, 0\}$ admits a realization $\left[\begin{array}{c|c} A_i & B_i \\ \hline C_i & D_i \end{array}\right]$ with $B_i = \{B_{i1} \dots B_{ii}\ 0 \dots 0\}$, $D_i = (D_{i1}\ D_{i2} \dots D_{ii}\ 0 \dots 0)$ and $\left[\begin{array}{c|c} A_i & B_{ii} \\ \hline C_i & D_{ii} \end{array}\right]$ the inherited state space realization of P_{ii} is stabilizable and detectable. Then there exist stable parameters Y_r, M_r, X_r, N_r, X_ℓ, N_ℓ, Y_ℓ, and M_ℓ such that the following statements are equivalent:*

- *K is lower triangular and it internally stabilizes the interconnection depicted in Figure (4).*
- *there exists a stable, lower triangular Q such that*

$$K = (Y_r - M_r Q)(X_r - N_r Q)^{-1} = (X_\ell - Q N_\ell)^{-1}(Y_\ell - Q M_\ell)$$

For all other classes of structures discussed in the previous subsection, similar conclusions as Lemma 3.1 can be drawn and the presentations of results are omitted due to space constraints.

4 Multiple Objective Optimal Performance in the Presence of Structure Constraints

Consider the system shown in Figure 5. As indicated, for the classes of structural problems described in the previous sections, a suitable choice of the Youla parameterization leads to subspace type of restrictions on Q. We denote by $\mathcal{S}$ the closed subspace of stable systems $Q \in \ell_1^{n_u \times n_y}$ that have the required structure. Then, all the achievable closed-loop maps can be given as follows (Youla parametrization):

$$\{R \in \ell_1^{n_z \times n_w} | R = H - U * Q * V \text{ with } Q \in \mathcal{S}\}$$

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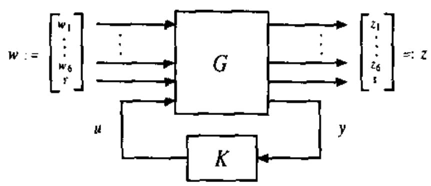

Figure 5: Closed-loop system

where $H \in \ell_1^{n_z \times n_w}, U \in \ell_1^{n_z \times n_u}, V \in \ell_1^{n_y \times n_w}$, Q is a free parameter in S and '$*$' denotes the convolution operation.

We use $\hat{R}^i$ $(i = 1, \ldots, 6)$ to denote the closed-loop transfer matrix from w_i to z_i and $\hat{R}^7$ the transfer function from r to s. For simplicity, and w.l.o.g., we shall consider the case when the reference input r is a step sequence. Define

$$A_{temp} := \begin{pmatrix} 1 & 0 & 0 & \cdots \\ 1 & 1 & 0 & \cdots \\ \vdots & \vdots & \ddots & \ddots \end{pmatrix}.$$

The GMO problem studied in this paper can be stated as: *Given a plant P, constants $c_i > 0$, $i = 1, \ldots, 6$, and two sequences $\{a_{temp}(k)\}_{k=0}^{\infty}$ and $\{b_{temp}(k)\}_{k=0}^{\infty}$, solve the following problem,*

$$\begin{array}{ll} \mu = & \inf_{Q \in S} f(Q) \\ \text{subject to} & \|R^4(Q)\|_1 \leq c_4 \\ & \|R^5(Q)\|_2^2 \leq c_5 \\ & \|\hat{R}^6(Q)\|_{\mathcal{H}_\infty} \leq c_6 \\ & a_{temp}(k) \leq [A_{temp} R^7(Q)](k) \leq b_{temp}(k), \forall k. \end{array}$$

where $f(Q) := c_1\|R^1(Q)\|_1 + c_2\|R^2(Q)\|_2^2 + c_3\|\hat{R}^3(Q)\|_{\mathcal{H}_\infty}$, $R^i(Q) = H^{ii} - U^i * Q * V^i, i = 1, \ldots, 7.$

4.1 An auxiliary problem

To facilitate the solution of μ, we define an auxiliary problem closely related to it: *Given constants $\gamma > 0$, $c_i > 0$, $i = 1, \ldots, 6$, and two sequences $\{a_{temp}(k)\}_{k=0}^{\infty}$ and $\{b_{temp}(k)\}_{k=0}^{\infty}$, solve the following problem,*

$$\begin{array}{ll} \nu = & \inf_{Q \in S} f(Q) \\ \text{subject to} & \|Q\|_1 \leq \gamma \\ & \|R^4(Q)\|_1 \leq c_4 \\ & \|R^5(Q)\|_2^2 \leq c_5 \\ & \|\hat{R}^6(Q)\|_{\mathcal{H}_\infty} \leq c_6 \\ & a_{temp}(k) \leq [A_{temp} R^7(Q)](k) \leq b_{temp}(k), \forall k. \end{array}$$

It can be shown that for most cases of interest we have $\nu = \mu$ and for other cases the solution of μ would admits more advantages than that of ν from a computational point of view. So we shall solely focus on the solution of problem μ from now.

5 Problem Solution

5.1 Existence of an optimal solution and converging lower bounds

$$\nu_n := \inf_{Q \in S} f_n(Q)$$

subject to

$$\begin{array}{l} \|Q\|_1 \leq \gamma \\ \|P_n(R^4(Q))\|_1 \leq c_4 \\ \|P_n(R^5(Q))\|_2^2 \leq c_5 \\ \|T_{6,n}(Q)\| \leq c_6 \\ a_{temp}(k) \leq [A_{temp} R^7(Q)](k) \leq b_{temp}(k), \forall k. \end{array}$$

where $f_n(Q) := c_1\|P_n(R^1(Q))\|_1 + c_2\|P_n(R^2(Q))\|_2^2 + c_3\|T_{3,n}(Q)\|$, P_n denotes the truncation operator on the space of sequences defined as $P_n(x(0)\ x(1)\ \ldots) = (x(0)\ x(1)\ \ldots\ x(n)\ 0\ 0\ \ldots)$, and

$$T_{i,k}(Q) := \begin{bmatrix} R^i(0) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 \\ R^i(k) & \cdots & \cdots & R^i(0) \end{bmatrix}, \ i = 3, 6.$$

Theorem 5.1 *There is an optimal solution Q^0 in $\ell_1^{n_u \times n_v}$ to problem ν. Moreover, $\nu_n \nearrow \nu$, as $n \to \infty$.*

5.2 Converging upper bounds

$$\nu^n := \inf_{Q \in S} f(Q)$$

subject to

$$\begin{array}{l} \|Q\|_1 \leq \gamma \\ \|R^4(Q)\|_1 \leq c_4 \\ \|R^5(Q)\|_2^2 \leq c_5 \\ \|\hat{R}^6(Q)\|_{\mathcal{H}_\infty} \leq c_6 \\ a_{temp}(k) \leq [A_{temp} R^7(Q)](k) \leq b_{temp}(k), \ \forall k \\ Q(k) = 0 \text{ if } k > n. \end{array}$$

Theorem 5.2 *$\{\nu^n\}$ forms a monotonically decreasing sequence of upper bounds of ν. Furthermore,*

$$\nu^n \searrow \nu, \ \text{as } n \to \infty.$$

6 Example of Optimal Control Design for ABR Network

Figure 6 depicts a setup for the congestion control of a 3-node ABR network ([19]). In Figure 6, r_1, r_2 and r_3 denote the flow rates from data sources into network nodes 1,2,3 respectively. r_{12} denotes the rate of flow from node 1 to node 2 and r_{23} denotes the rate of flow from node 2 to node 3. w represents the total capacity available for the three data sources. q_i denotes the buffer length at the i^{th} node. The network exerts control over the network traffic by assigning the rate for each data source. In particular, there are three (nodal) subcontrollers C_1, C_2, C_3 that dictate respectively r_1, (r_{12}, r_2), and (r_{21}, r_3). Moreover, there is a one-step delay in passing nodal information (q_i) from one nodal subcontroller C_i to its preceding one C_{i-1}, while each C_i does not receive information from any of the preceding nodes. The

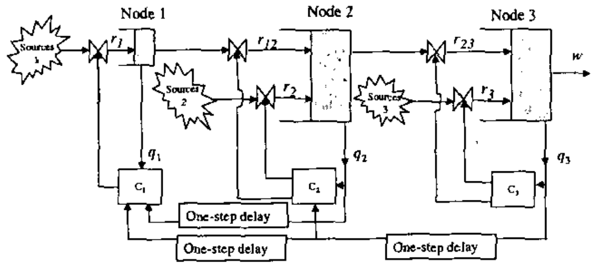

Figure 6: 3-nodal ABR Network

goals are to prevent the node buffers from overflowing so as to avoid possible data loss ('stabilization goal'), and to optimally utilize the available transfer capacity w such that the sum of the data rates $r_i (i = 1, 2, 3)$ matches w as closely as possible ('optimality goal'). The exogenous input signal is identified as the available capacity w and the control input and measured output signals are identified as:

- $u = [\ r_1\ \ r_{12}\ \ r_2\ \ r_{23}\ \ r_3\ ]^T$
- $y = [\ q_1\ \ q_2\ \ q_3\ ]^T$.

The goal of the congestion control for the network can be captured by identifying the regulated output as:

- $z = [\ q_1\ \ q_2\ \ q_3\ \ r_1 - wa_1\ \ r_2 - wa_2\ \ r_3 - wa_3\ ]^T$.

where a_i is a prescribed constant representing the ratio of available resource assigned to i_{th} source. Suppose also that steps are the typical exogenous input signals w we would like to optimally track. Then, we can impose TDCs on $z_i (i = 4, 5, 6)$ such that the step response of $z_i (i = 4, 5, 6)$ is forced to stay within a prescribed envelope. Clearly, the plant G_{22} and the controller K are upper triangular operators of the following form:

$$G_{22} = \left[\begin{array}{c|cc|cc} * & \lambda * & \lambda * & \lambda^2 * & \lambda^2 * \\ \hline 0 & * & * & \lambda * & \lambda * \\ \hline 0 & 0 & 0 & * & * \end{array}\right] \quad K = \left[\begin{array}{c|c|c} * & \lambda * & \lambda^2 * \\ \hline 0 & * & \lambda * \\ 0 & * & \lambda * \\ \hline 0 & 0 & * \\ 0 & 0 & * \end{array}\right]$$

In this example we provide a tradeoff study between ℓ_1 and $\mathcal{H}_2$ performance of the closed-loop system by solving the following multiobjective problem:

$$\begin{array}{rll} \nu := & \inf & c_1 \|R(K)\|_1 + c_2 \|R(K)\|_2^2 \\ & \text{s.t.} & K \text{ is stabilizing} \\ & & K \text{ satisfies structure constraints above} \\ & & z_i (i = 4, 5, 6) \text{ satisfies prescribed TDCs.} \end{array}$$

where c_1 and c_2 are prescribed weighting constants. Following the framework established in Section 3, we have

$$\begin{array}{rll} \nu := & \inf & c_1 \|R(Q)\|_1 + c_2 \|R(Q)\|_2^2 \\ & \text{subject to} & \\ & & Q \text{ is stable} \\ & & Q \in \mathcal{S} \\ & & z_i (i = 4, 5, 6) \text{ satisfies prescribed TDCs.} \end{array}$$

where $\mathcal{S}$ characterizes the structure constraints and TDC are the time domain constraints that the three error signals z_4, z_5, and z_6 due to step inputs lie within the upper and lower templates b_{temp} and a_{temp}:

$$\begin{array}{l} b_{temp}[0] = \ 0.5, \ldots b_{temp}[5] = \ 0.5, b_{temp}[6] = 0.01 \ldots \\ a_{temp}[0] = \ -0.5, \ldots a_{temp}[5] = \ -0.5, a_{temp}[6] = -0.01 \ldots \end{array}$$

The fairness index a_i is taken to be $a_1 = a_2 = a_3 = 1/3$ and the upper bounds of $\|Q\|_1$ are chosen to be $\gamma = 100$.

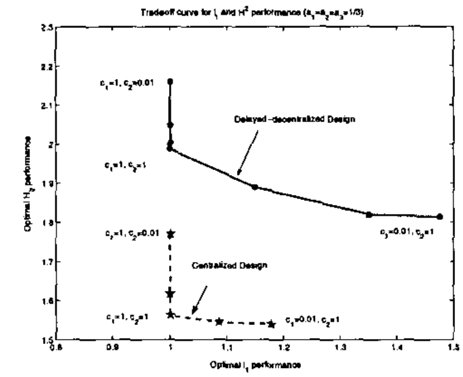

Figure 7: Tradeoff Curve between ℓ_1 and $\mathcal{H}_2$ norms

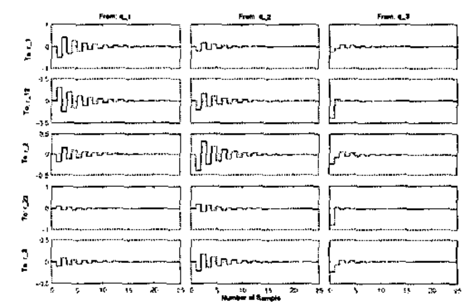

Figure 8: Impulse Response of Centralized Controller

The plots of $\|R(Q)\|_1$ versus $\|R(Q)\|_2^2$ are shown in Figure 7, where the dashed curve denotes the cases of centralized design with no information transfer delay while the solid curve denotes the cases where there exists transfer delay. From these two curves, we conclude that the structure constraints imposed on the stabilizing controllers induce a significant loss of the closed-loop system performance.

The impulse responses of the centralized sub-optimal controller and the decentralized, delayed sub-optimal controller (case $c_1 = c_2 = 1$) are plotted in Figure 8 and Figure 9, respectively. From the last figure, it can be clearly observed that controller admits the required upper block triangular structure while the centralized controller does not admit such a structure. In Figure 10, the step response of the closed-loop system with structured controller is plotted, where the dash-dotted lines denote the TDC envelops imposed on the step responses. It is clear from the response plots that the time

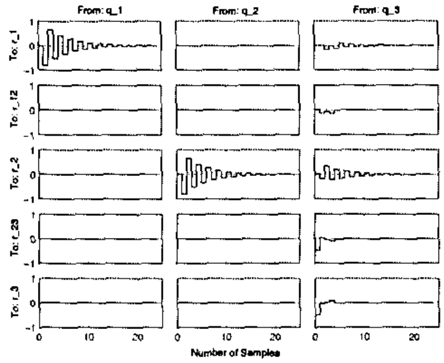

Figure 9: Impulse Response of Decentralized, Delayed Controller

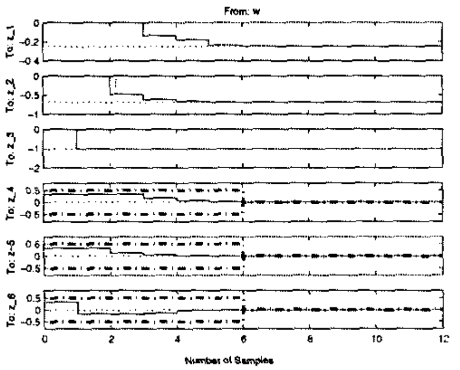

Figure 10: Step Response of Closed-loop system with Decentralized, Delayed Controller

response of $z_i (i = 4, 5, 6)$ satisfies the requirement of zero steady value, which implies that the optimality goal of the congestion control mechanism is achieved.

7 Conclusions and Discussion

In this paper we presented a convex optimization approach for optimal synthesis in systems in which the overall control scheme is required to have certain structure. These structures correspond to various classes of controlled systems such as nested, chained, hierarchical, delayed interaction and communication, and, symmetric systems. The common thread in all of these classes is that by taking an input-output point of view we can characterize all stabilizing controllers in terms of convex constraints in the Youla-Kucera parameter. We present an attractive methodology to solve the general multiobjective (GMO) control problem concerning the ℓ_1 norm, $\mathcal{H}_2$ norm, $\mathcal{H}_\infty$ norm and time-domain constraints in the presence of structural constraints. We showed that the regularized GMO problem admits a minimizing solution in ℓ_1 and, more importantly, from an engineering viewpoint, we showed how to obtain arbitrarily close to optimal controllers within any prespecified accuracy. Furthermore, the GMO problem presented here furnishes us with a uniform framework under which many multiobjective and structured problems can be addressed and solved by using the techniques proposed in this paper. The use of this method for simultaneous parameter and control design is under investigation.

References

[1] D. D. Siljak, *Large-Scale Dynamic Systems*. North-Holland, New York, 1978.

[2] D. D. Sourlas and V. Manousiouthakis, "Best achievable decentralized performance," *IEEE TAC, Vol. 40, pp. 1858-1871*, 1995.

[3] K. A. Unyelioglu and U. Ozguner, "$\mathcal{H}_\infty$ sensitivity minimization using decentralized feedback: 2-input 2-output systems," *Sys. & Contr. Letters*, 1994.

[4] A. N. Gundes and C. A. Desoer, *Algebraic Theory of Linear Feedback Systems with Full and Decentralized Compensators*. Springer Verlag,, 1990.

[5] D. C. Youla, H. A. Jabr, and J. J. Bongiorno, "Modern Wiener-Hopf design of optimal controllers–part 2: The multivariable case," *IEEE TAC, Vol. 21*, June, 1976.

[6] P. G. Voulgaris, "A Convex Characterization of Classes of Problems in Control with Sepcific Interaction and Communication Structures," *ACC01*, June, 2001.

[7] M. Rotkowitz and S. Lall, "Decentralized control information structures preserved under feedback," *CDC 02*, 2002.

[8] M. V. Salapaka and M. Dahleh, *Multiple Objective Control Synthesis*. Springer-Verlag, New York (Monograph), 1999.

[9] C. W. Scherer, "Lower Bounds in Multi-Objective $\mathcal{H}_2/\mathcal{H}_\infty$ Problems," *CDC99, pp. 3605-3610*, 1999.

[10] S. P. Boyd and C. H. Barratt, *Linear Controller Design: Limits of Performance*. Prentice Hall, Englewood Cliffs, New Jersey 07632, 1991.

[11] M. V. Salapaka, M. Khammash, and M. Dahleh, "Solution of MIMO $\mathcal{H}_2/\ell_1$ Problem without Zero Interpolation," *SIAM J. on Contr. and Opt., Vol. 37, No. 6, pp. 1865-1873*, November, 1999.

[12] P. G. Voulgaris, "Control under a hierarchical decision making structure," *ACC99*, June, 1999.

[13] G. C. Goodwin, M. M. Seron, and M. E. Salgado, "$\mathcal{H}_2$ design of decentralized controllers," *ACC99*, 1999.

[14] P. G. Voulgaris, "Control of nested systems," *ACC00*, June, 2000.

[15] C. W. Scherer, "Design of structured controllers with applications," *CDC00*, December, 2000.

[16] G.-H. Yang and L. Qiu, "Optimal summetric $\mathcal{H}_2$ controllers for systems with collocated sensors and actuators," *CDC01*, December, 2001.

[17] B. A. Francis, *A Course in $\mathcal{H}_\infty$ Control Theory*. Springer-Verlag, 1987.

[18] M. Vidyasagar, *Control Systems Synthesis: A Factorization Approach*. MIT press, 1985.

[19] E. Altman, T. Basar, and R. Srikant, "Congestion control as a stochastic control problem with action delays," *Automatica 35, pp. 1937-1950*, 1999.

[20] M. V. Salapaka, M. Dahleh, and P. G. Voulgaris, "Mixed Objective Control Synthesis: Optimal $\ell_1/\mathcal{H}_2$ Control," *SIAM J. on Contr. and Opt., Vol. 35 No. 5, pp. 1672-1689*, September, 1997.