Proceedings of the American Control Conference
Chicago, Illinois • June 2000

EXPERIMENTAL STUDY OF STOCHASTIC RESONANCE IN ATOMIC FORCE MICROSCOPES

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Abstract

Stochastic resonance (SR) is an interesting phenomenon which can occur in bistable systems subject to periodic and random forcing. This effect produces an improvement in the sensitivity of the bistable system to the periodic signal. In this paper, stochastic resonance for Atomic Force Microscope (AFM) is studied. The experimental results indicate that the AFM can be modeled as a bistable system similar to the Schmitt trigger, for which stochastic resonance has been well studied. The results indicate that stochastic resonance in AFM can be applied in many technological contexts as, for example, in the analysis of the effects of thermal noise in order to optimize the achievable resolution for imaging.

1 Introduction

It has been found that when noise and weak periodic signal are fed together into bistable systems like the Schmitt trigger, (see [1],[2]) the system responds better to the periodic signal when the noise intensity is in a particular range. The phenomenon of cooperation between a stochastic and a periodic signal is termed as stochastic resonance. The minimum amplitude of the periodic signal required to switch the system from one of its bistable states to the other is called the threshold. The periodic signal is termed weak if its amplitude is sub-threshold. The periodic signal has the effect of modulating the system potential curve as shown in Figure 1. The system is depicted as a ball moving in a symmetric bistable potential having a potential barrier ΔV. The potential is shown as being modulated when a sub-threshold periodic signal alone is fed to the system. In this case the system oscillates about its present stable state and there is no transition into the other stable state. When the amplitude of the periodic signal is greater than the threshold, the system oscillates between the two states at a rate equal to the frequency of the periodic signal. When noise alone is fed into the bistable system, the system hops between its stable states at a probabilistic rate known as the Kramer's rate (see [3]). When both signal and noise are fed together, at a particular noise intensity, the system begins to hop between its two states at the frequency of the periodic signal on an average. This is manifested as a peak in the output power spectral density (or equivalently the frequency spectrum) at the frequency of the deterministic signal. The plot of the output at the signal frequency as a function of the noise intensity also displays a peak.

The focus of this paper is to see whether the Atomic Force Microscope (AFM), an instrument widely used to image samples with high resolution, exhibits stochastic resonance. Towards understanding this, the Schmitt trigger, a system already proven to exhibit stochastic resonance is presented. In Section 2 a description of the AFM is given. The possibility of the AFM exhibiting stochastic resonance is shown in Section 3. In Sections 4 and 5 the experiments done on the AFM and the Schmitt trigger are presented. The similarities in the response of both systems are discussed in Section 6 along with further improvements.

2 Introduction to the Atomic Force Microscope

The Atomic Force Microscope has revolutionized high resolution imaging of samples. A schematic of the AFM is shown in Figure 2. The tapping mode operation of the AFM has been widely used to image samples. In this mode, the cantilever is made to tap the sample

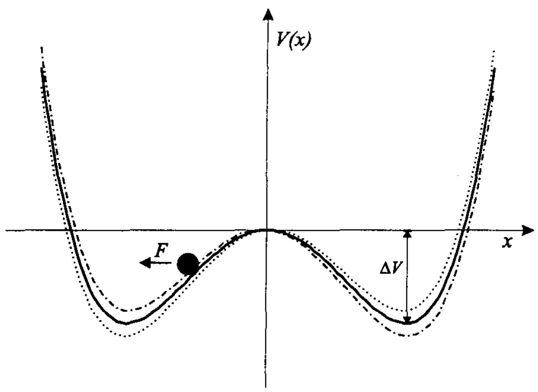

Figure 1: Sketch of a double-well potential function.

surface as it scans the same. The cantilever is mounted on a dither piezoelectric drive. The dither piezo is subjected to a sinusoidal forcing at the resonant frequency of the cantilever. The sample is positioned using a high voltage piezo (Z piezo). Due to the force of interaction between the cantilever and the sample, the amplitude of oscillation changes. The sample can be positioned at any distance from the cantilever by applying voltages to the Z piezo. A LASER beam is focused on the tip of the microcantilever . The reflected beam is detected using a pair of photodiodes, which convert the movement of the cantilever into an equivalent change in voltage. Before performing the experiment, the LASER beam is centered on the photodiode pair. This detection system provides a large optical lever and hence minute variations in the cantilever's displacement are amplified as large voltage signal.

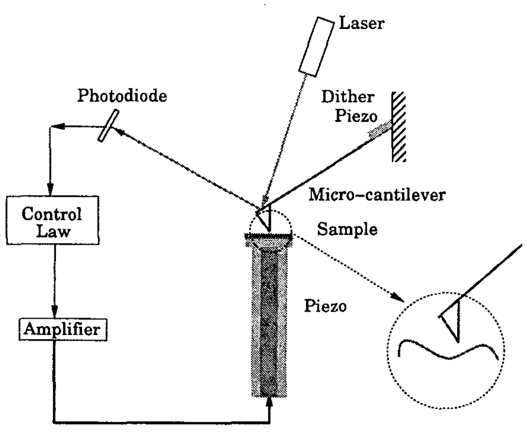

Figure 2: A schematic of the AFM

The cantilever and the sample surface have been well modeled using spring, mass and damper elements as seen in Figure 3 . This model of the AFM has been well studied and proven to be consistent with experimental results(see [7]). Although the cantilever and the sample have been modeled using linear elements, the presence of the sample has been taken into account by the two spring constants only when the cantilever is close to the tip, and hence the sample force on the cantilever is a function of distance, and is modeled as zero for very large distances. This makes the model non-linear.

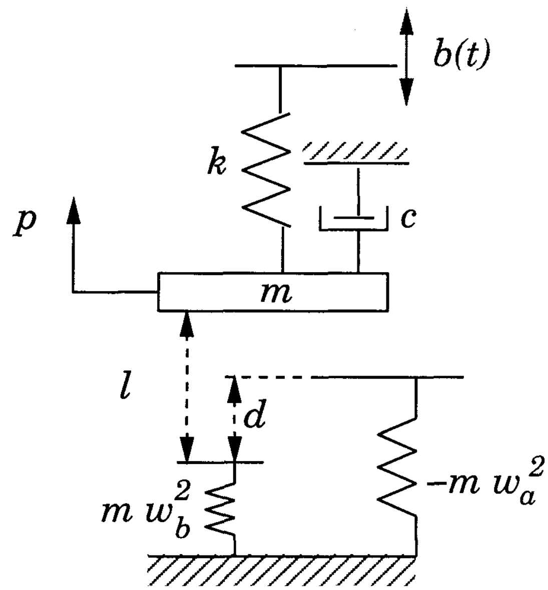

Figure 3: Model of the cantilever and the sample

The dynamical equation for the displacement p of the cantilever when inside the sample potential is given by

$$m\ddot{p} + c\dot{p} + kp = F(t) + kb(t). \tag{1}$$

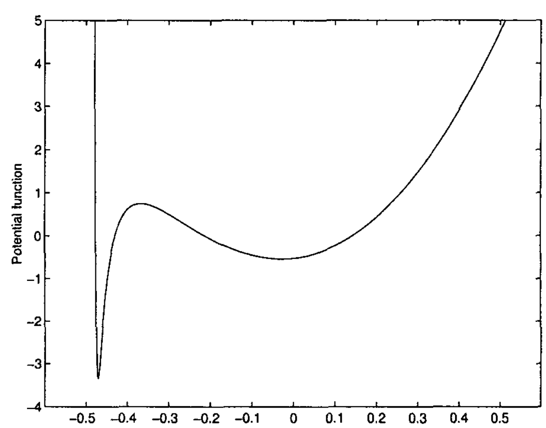

Figure 4: Bistable potential curve of the AFM

3 Stochastic Resonance in the AFM

The AFM when subjected to both noise and a periodic signal can be modeled as the following stochastic process:

$$\begin{cases} \dot{x} = v \\ \dot{v} = -V'(x) - \eta v + \sigma\,\xi(t) + \epsilon \cos \omega_0 t \end{cases} \tag{2}$$

where the random variables x and v denote position and velocity of the cantilever having unitary mass, $V(x)$ is the potential function, η is the damping factor, $\sigma\,\xi(t)$ is a white noise with variance σ^2 and ϵ the amplitude of a sinusoidal modulation of the force. The noise can be thought of as the thermal noise which is present inherently in the AFM. When subjected to noise alone, the transition rate between the two states is equal to the Kramer's rate (see [6]). In [4], systems having asymmetric potential curves are shown to exhibit stochastic resonance, the assumption being that the continuous state system can be approximated as a discrete state system consisting only of the two stable states. This approximation holds under the assumption that the sinusoidal input signal has a period which is large with respect to the *relaxation time*, defined as the time for probability to equilibrate within one well. The discrete state system is modeled by the following rate equation

$$\frac{dn_R}{dt} = -\frac{dn_L}{dt} = W_L(t)n_L - W_R(t)n_R \tag{3}$$

where n_L and n_R are the probabilities of finding the system in the left and right potential wells respectively, and W_L and W_R are the rate of transitions from the left and right wells respectively.

The equation of the potential curve of the AFM when the cantilever is inside the sample potential is given by

$$V(p) = \frac{k}{2}p^2 - \frac{d}{(p+Z)} + \frac{\Sigma^6 d}{210(p+Z)^7}\ . \tag{4}$$

Z characterizes distance between the cantilever and the sample, whereas Σ and d are parameters that depend on the nature of the sample and the cantilever. The potential function is given in Figure 4, which is clearly an asymmetric bistable potential curve. The dependence on Z can be of significant importance for example, in increasing the sensitivity of the AFM by varying Z which, in turn changes the potential curve. The model described in Section 2 also shows bistable behaviour at certain cantilever sample distances, as seen in [7]. In what follows, experiments performed on the AFM and the Schmitt trigger to study stochastic resonance are presented. Two models for the AFM have been discussed so far. Simulations on the model described in the previous section have proven it to be a bistable one (see [7]). The second model described by Equations (2) and (4) (see Figure 4) is again indicative of the fact that the AFM is a bistable system. The explanation for AFM exhibiting stochastic resonance (if proved to be so) can be got from either of these models and is a topic of further research.

4 Experimental setup for the AFM

A cantilever having a resonant frequency of 290.795 KHz was chosen for the experiment. A mica sample was cut into a square size of half an inch side, and the surface was scraped using a gum tape to obtain a new layer. The sample was carefully mounted on to the sample holder (Figure 2). The cantilever was initially subjected to a 10 Volt 1 KHz sinusoidal voltage when outside the sample potential. The sinusoidal voltage was generated using a HP-33120 signal generator. The FFT of the cantilever's response did not show any significant component at 1 KHz (see Figure 5). The response was predominantly noisy as seen in Figure 5. This was as expected because the frequency (1 KHz) was far away from the resonant frequency of the cantilever (290.975 KHz). The cantilever was then introduced into the sample potential in steps of 50 nanometers. This was done by first exciting the cantilever at resonance and then introducing it into the sample potential till the cantilever stopped oscillating. The force curve of the AFM was obtained using the software provided by Digital Instruments Inc. It was confirmed from the force curve that the cantilever was well into the sample. The same 10 Volt 1 KHz signal was fed into the cantilever. It was found that the cantilever, whose response was adversely affected by noise when outside the sample potential, started responding to the same when inside sample potential. The response and the frequency spectrum of the cantilever when inside and outside the sample potential are shown in Figures 5 and 6. The output was seen to be a noisy 1 KHz square wave when the cantilever was kept inside the sample potential. A change in the frequency of the sinusoid resulted in a change in the frequency of the FFT peak of the cantilever response square wave when inside the sample potential.

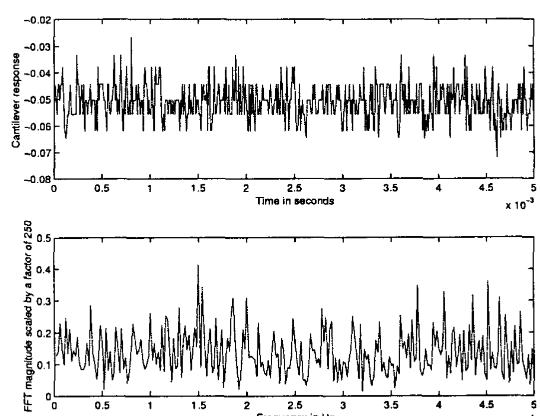

Figure 5: Experimental results on the AFM when cantilever is outside the sample potential

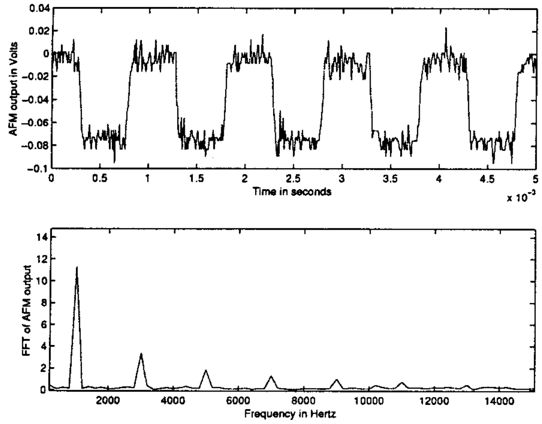

Figure 6: Experimental results on the AFM when cantilever is inside the sample potential

5 Schmitt Trigger

5.1 System description

The Schmitt trigger is an electronic circuit which has two stable states. This is a positive feedback circuit which compares the input voltage to a fixed threshold. A schematic of the Schmitt trigger is shown in Figure 7. The output goes high if the input is higher than the threshold and vice versa. The high and low states of the Schmitt trigger are the two saturation states of the amplifier ($V_{sat} = \pm 12$ Volts). ECG778A high bandwidth (50 MHz) amplifier was used to build the circuit. As shown in Figure 7, the threshold is chosen by fixing the values of the resistors R_1 and R_2 and is given by $V_t = R_1/(R_1+R_2)|V_{sat}|$. When a sinusoidal signal is fed as input, the output is a square wave of same frequency if the amplitude of the input is greater than the threshold. The Schmitt trigger can be in one of its two stable states when no input is fed into it. The two stable states of the Schmitt trigger are the saturation levels of the amplifier. In the following section the results of experiments on the Schmitt trigger circuit which demonstrate stochastic resonance are presented.

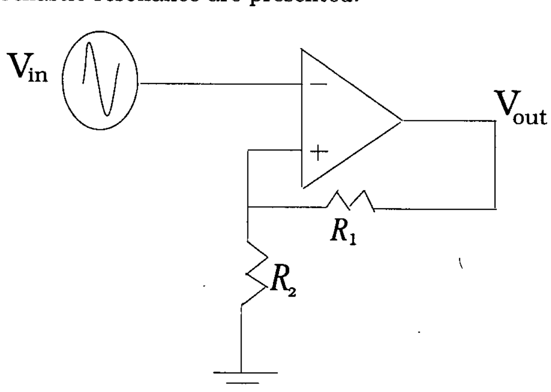

Figure 7: Schematic of a Schmitt Trigger.

5.2 Experimental results

The value of the threshold was fixed at 2 Volts by choosing R_1 to be 2 KΩ and R_2 to be 10 KΩ. A sinusoidal voltage of 1 Volt amplitude and 1 KHz frequency was fed as input. The circuit remained in one of its stable states and no switching was observed as expected. Band-limited white noise was added to the input and the intensity of the noise was varied from 1 Volt to 9 Volts. The signal output at 1 KHz attained a maximum for a noise intensity of approximately 3 Volts as shown in Figure 9. The noise was confirmed to be band limited and white by verifying that its Fast Fourier Transform (FFT) was a flat band till a frequency of 60 MHz. A HP-33120 signal generator was used to generate the two inputs. The FFT's were generated using HP-VEE (Visual Engineering Environment) software. The FFT of the output was averaged 100 times for noise intensities varying from 1 Volt to 9 Volts. The output signal was a square wave at 1 KHz superimposed with noise (see Figure 8). Here again a change in frequency of the sinusoidal input resulted in an equal change in the frequency of the FFT peak.

5.3 Further improvements

Figure 6 shows an output which looks like a deterministic switching between the states. More experiments conducted on the AFM at lower ambient temperatures and reduced levels of sinusoidal voltage can determine the levels of thermal noise inside the AFM at which the increased sensitivity of the AFM to the weak periodic signal can be attained. Position feedback can be used to vary the effective spring constant k, of the system. The modified form of Equation (1) would look like

$$m\ddot{x} + c\dot{x} + (k + k_1)x = F(t) + kb(t) \qquad (5)$$

where k_1 is the change in the original spring constant k of the cantilever due to position feedback. As seen in Equation (4), a change in the spring constant effectively changes the shape of the potential curve, and a change in the potential curve would in turn change the threshold value and could make the ambient thermal noise level optimum enough to produce maximum sensitivity of the AFM to a weak periodic signal. The cantilever can thus be made to respond better to weak periodic signals by varying the effective spring constant using position feedback. The spring constant when varied can make the inherent thermal noise optimum for stochastic resonance to happen. This would mean that for that value of k_1 which is externally set for the controller, the AFM has maximum sensitivity to a weak periodic signal. Hence the sensitivity of the AFM can be optimized by using position feedback. In [4], an expression of the signal to noise ratio has been derived. Further research needs to be done in verifying and understanding the sensitivity of that expression to system parameters.

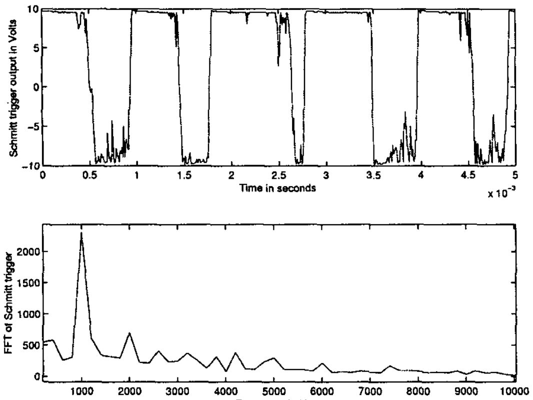

Figure 8: Experimental results on the Schmitt trigger

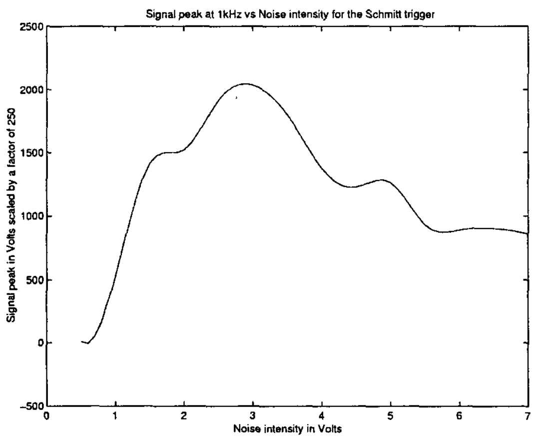

Figure 9: Signal peak vs Noise intensity exhibiting a maximum

6 Discussion

The effect of adding a weak periodic signal to the Schmitt trigger and the AFM look similar as shown in Figures 6 and 8. When noise and a weak sinusoidal signal are fed together, the Schmitt trigger shows switching between its bistable states which, on an average resembles the switching due to the pure sinusoidal signal alone. This is confirmed by the FFT of the Schmitt trigger's response in Figure 8. Similar to this, the cantilever of the AFM when inside the sample potential attains two states as shown in the cantilever response in Figure 6. Also the cantilever whose response is corrupted by noise when outside the sample potential (Figure 5) starts showing a peak in its FFT at the frequency of the sinusoidal input, when inside the sample potential. The two states attained by the cantilever are suspected to be the free state and being stuck to the sample, respectively. A change in the frequency of the sinusoidal input shifted the output FFT peak frequency in both systems. Introducing external noise into the Schmitt trigger improved the deterministic component in the output as shown in Figure 9, which clearly shows a maximum attained by the FFT peak at signal frequency. This leads to the strong belief that the thermal noise present inside the AFM is enough for stochastic resonance to occur.

Equation (4) describes how the sample total potential of the system varies with the cantilever spring constant k and the cantilever sample separation Z. The expression $-V'(x)$ (the negative derivative of the potential function) directly gives us the net force acting on the cantilever, where $V(x)$ is the total potential energy of the cantilever and the sample interaction as a function of x, the displacement of the cantilever. This is another model of the AFM which gives us the dynamics (as opposed to the model shown in Figure 3 which gives Equation (1) as the dynamical equations describing the AFM. Interestingly, the model represented by Figure 3 has also been proved to exhibit bistability (see [7]). The parameters Σ and d in Equation (4), depend on the cantilever and sample properties. This means that different samples will exhibit different potential curves, which in turn will mean that the effect of stochastic resonance on the cantilever output will be different for different samples. This can be used to characterize different samples which is a topic of future research. The fact that the AFM responds in a manner similar to the Schmitt trigger when subjected to sinusoidal and noise inputs (the noise input to the AFM being assumed to be thermal noise) makes it more evident that the AFM can be modeled as a discrete state system in certain range of cantilever-sample distances.

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