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Beating Landauer's bound by Memory Erasure using Time Multiplexed Potentials

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Abstract:
The Landauer's Principle, proposed by Rolf Landauer in 1961, states that the logically irreversible operation of erasing a single bit of information requires *atleast* k_bT ln 2 amount of energy, k_b and T being Boltzmann's constant and temperature respectively. However, Landauer's bound holds only for erasure mechanisms that are perfect. In this article we investigate the effect of imperfections in erasure mechanisms. If the proportion of successful erasures is quantified by p, we show that the minimum energy needed to erase a bit of information is given by $k_bT[\ln 2+p\ln p+(1-p)\ln(1-p)]$, also known as the Generalized Landauer bound. Furthermore, we provide a mechanism for realizing a memory bit by multiplexing an optical trap rapidly and propose a mechanism of erasure, for various success proportions p. Using our framework, we show using Monte Carlo simulations that heat dissipation lower than the Landauer's bound is achievable by reducing p. Thus, we establish an independent method of beating Landauer's bound by resorting to partially successful erasures.

Keywords: Landauer's Principle, Landauer's bound, Generalized Landauer's bound, Memory erasure, Langevin Equation

1. INTRODUCTION

Advancement of the semiconductor industry and modern Information and Computer Technology (ICT) devices was enabled by its capability to reduce the size of CMOS-FET (Taur and Ning (2013)) switches, while consistently increasing their computational abilities. A critical aspect of an ICT device is its energy consumption during computations. An area of intense research focuses on the limits to the energy efficiency of computations, an idea that resonanates with the Carnot limit for heat engine efficiency proposed by Carnot et al. (1890). Groundbreaking work by Neumann and Burks (1966), Bennett (1982) and Landauer (1961) established a connection between the seemingly disparate fields of Thermodynamics and Information Theory. Landauer's Principle states that the logically irreversible step of erasing a single bit of information is accompanied, on an average, by dissipation of at least k_bT ln2 amount of energy. Landauer argued that erasure of information lowers the entropy of the overall system and thus, is accompanied by an average heat dissipation of at least $k_bT\ln 2$ amount of energy per erased bit. Bennet further utilized Landauer's argument (Bennett (1982); Lef and Rex (2003)) to explain Maxwell's Demon and avoid a potential threat to the second law of thermodynamics.

Following the original work of Landauer (1961), numerous studies have considered an over damped Brownian particle in a symmetric double well potential as a model for single bit memories. For example, Gammaitoni (2011) proposed that the two-well, one barrier model (Landauer (1961)) can be used as a valid abstraction for switching devices based on electron transport. Field Effect Transistors (FET's) can be modeled as two wells (signifying source and drain) separated by distance L and a potential energy barrier of E, as shown in Fig. 1. Several theoretical studies such as Shizume (1995); Dillenschneider and Lutz (2009); Lambson et al. (2011) employed similar approaches to model one bit memory and demonstrate the existence of Landauer's bound (LB); yet the experimental study by Bérut et al. (2012) provided strong evidence toward the principle. Further corroboration by Jun et al. (2014); Hong et al. (2016); Neri and López-Suárez (2016) have demonstrated an intimate link between thermodynamics and information theory. A crucial assumption in obtaining the minimum heat dissipation limit of $k_bT\ln 2$ is that the erasure process is perfect.

In this article, we present an extension of the Landauer's bound to incorporate partially successful erasures. We begin by introducing the basic notions of thermodynamics, following which we justify the existence of Landauer's bound for fully as well as partially successful erasures. The minimum heat dissipation bound for partially successful erasures with probability p is referred to as the Generalised Landauer bound(GLB). We model a one bit memory similar to Fig. 1 based on optical tweezers, to create the required bistable potential landscape. Using this model, we propose a novel method to implement memory erasure and quantify the associated heat dissipation,

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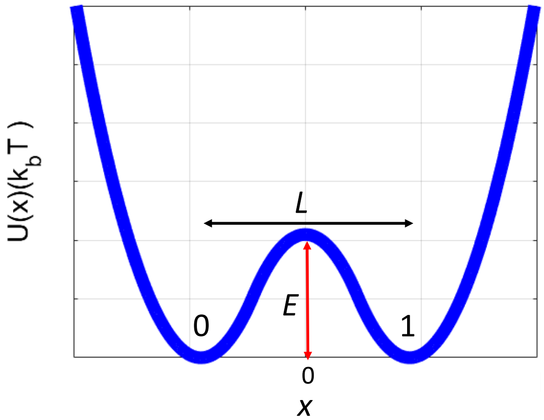

Fig. 1. Schematic for a bistable switch, as described by Gammaitoni (2011). The x-axis denotes the position and the y-axis denotes the energy of the particle. The two wells are separated by distance L and a barrier height of E. Particle in right well ($x > 0$) represents logic state 1 and particle in left well ($x < 0$) represents logic state 0.

while maintaining a handle on the success proportion p. We thereby demonstrate that by resorting to imperfect erasures, it is possible to significantly reduce the energy expenditure in erasing one bit of information. We further show that the energy expenditures validate the existence of the Generalised Landauer's bound.

2. TERMINOLOGY

In order to understand work and heat associated with erasure of a single bit memory we briefly summarise below some terminology related to thermodynamics, which are needed for subsequent developments. See Sekimoto (2010) for more details.

The thermodynamic entropy of a system in equilibrium with a heat bath a temperature T is given by $S = -k_b \int p(x)\ln(p(x))dx$, where $p(x)$ is the equilibrium probability distribution(also known as Canonical distribution) of the system and x denotes the state of the system.

The 1^{st} Law of Thermodynamics states that, $dU = W - Q_d$, that is, change in energy of the system is equal to the difference between the work done on the system, W and heat dissipated by the system to the surroundings, Q_d. It is a statement about conservation of energy.

The 2^{nd} Law of Thermodynamics states that change in entropy of the system undergoing a process which changes the state of the system from i to f is given by, $\Delta S = S_f - S_i \geq -\frac{\langle Q_d \rangle}{T}(S_i - S_f \leq \frac{\langle Q_d \rangle}{T})$, where $\langle Q_d \rangle$ is the average heat dissipated by the system to the surroundings, S_i and S_f are the initial and final thermodynamic entropy of the system respectively. The equality holds in the 2^{nd} Law of Thermodynamics if the system is undergoing a quasi-static process.

3. ERASURE AND GENERALIZED LANDAUER BOUND

In this section we begin by briefly describing the process of erasure and Landauer bound, following which we arrive at the Generalized Landauer bound.

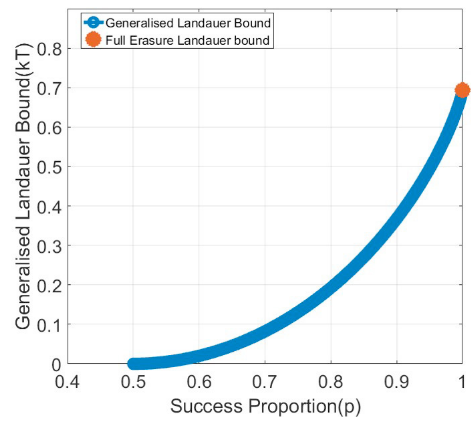

Fig. 2. Generalised Landauer bound as a function of success proportion p.

Bennett (1982) described *erasure* as a logically irreversible operation, where the final state of the memory is zero irrespective of its initial state (also referred to as 'reset-to-zero' operation). A logically irreversible operation is one where it is not possible to know the initial state based on the knowledge of the final state. In case of a memory bit, the initial state can be either zero or one with equal probability. However at the end of the 'reset-to-zero' erasure process, the state of the memory *must* be *zero*.

Landauer proposed that the erasure of one bit of information is accompanied by an average heat dissipation of at least $k_b T \ln 2$, also known as the Landauer bound. The Landauer bound holds in the case of completely successful erasures. Let p denote the probability of the erasure being successful. It is proposed by Maroney (2009); Gammaitoni (2011); Bérut et al. (2012, 2015) that the average minimum heat dissipation for erasures with success proportion p reduces sharply and logarithmically with p through the relation $k_b T[\ln 2 + p\ln p + (1-p)\ln(1-p)]$, which is referred to as the *Generalized Landauer's bound*. Note that as p reduces from 1 to 0.5, the Generalized Landauer bound reduces rapidly from $k_b T \ln 2$ to 0 as shown in Figure 2.

The state of the memory before erasure can be modeled as a Bernoulli random variable X_i with parameter $q = \frac{1}{2}$ and state space $\Omega = \{0, 1\}$, while the state of the memory after undergoing an erasure with success proportion p can be thought of as a Bernoulli random variable X_f with parameter $q = p$ and state space $\Omega = \{0, 1\}$. The decrease in thermodynamic entropy due to erasure with success proportion p is $k_b(\ln 2 + p\ln p + (1-p)\ln(1-p))$, which is the Generalised Landauer bound Gammaitoni (2011); Maroney (2009). However, this approach assumes discrete probability distributions which is unrealistic from the perspective of a 1 bit memory, which has a continuous state space. The discussion in the remainder of this section is focused on derivation of the Generalised Landauer bound without resorting to the discrete probability distribution viewpoint and hence is generic.

Landauer (1961) modeled a single bit memory (binary device) as a Brownian particle in a bistable potential well as shown in Fig. 1. The particle is assumed to be in thermodynamic equilibrium with its surrounding (heat bath), which is assumed to be at a constant temperature T. The state of a single bit memory M takes two values, one and zero, which is modeled by the particle occupying the right and left potential well respectively. Shizume (1995) proved the existence of the Landauer bound by resorting to a Brownian particle in a double well potential model of a single bit memory. We extend Shizume's approach to justify the existence of the Generalized Landauer bound.

Let the equilibrium probability distribution of the particle in state one be $f_1(x)$ and in state zero be $f_0(x)$, with $-k_b\int_{-\infty}^{\infty} f_0(x)\ln(f_0(x))dx = -k_b\int_{-\infty}^{\infty} f_1(x)\ln(f_1(x))dx$, that is, the two states of the 1 it memory are identical and have similar equilibrium probability distributions. Next we introduce the notion of 'insignificant overlap' which is needed for demonstrate the existence of the Generalized Landauer bound.

Definition 1. *(Insignificant Overlap) Given two distributions $f_0(x)$ and $f_1(x)$, there is insignificant overlap between $f_0(x)$ and $f_1(x)$ if $X_0 \cap X_1 = \phi$, where, $X_0 := \{x \in \mathbb{R} : f_0(x) \neq 0\}, X_1 := \{x \in \mathbb{R} : f_1(x) \neq 0\}$.*

Remark: If $f_0(x)$ and $f_1(x)$ are Gaussian distributions, then this condition will never be met exactly.

Initially it is equally likely for the state of the memory, M to be zero or one, that is, $P(M = 0) = P(M = 1) = \frac{1}{2}$. The probability of finding the Brownian particle between x and $x + dx$ is given by,

$$\begin{aligned} P(X \in (x, x+dx)) &= P(M=0)P(X \in (x, x+dx)|M=0) \\ &\quad + P(M=1)P(X \in (x, x+dx)|M=1), \\ &= \frac{1}{2}f_0(x)dx + \frac{1}{2}f_1(x)dx \end{aligned} \tag{1}$$

Thus, the probability distribution function of the particle before undergoing erasure is given by, $p_i(x) = \frac{1}{2}f_0(x) + \frac{1}{2}f_1(x)$. It is shown by Shizume (1995), that assuming insignificant overlap, the initial entropy S_i can be obtained by,

$$\begin{aligned} S_i &= -k_b\int_{-\infty}^{\infty}(\frac{1}{2}f_0(x) + \frac{1}{2}f_1(x))\ln(\frac{1}{2}f_0(x) + \frac{1}{2}f_1(x))dx, \\ &\approx -k_b(\int_{-\infty}^{\infty}\frac{1}{2}f_0(x)\ln(\frac{1}{2}f_0(x))dx \\ &\quad + \int_{-\infty}^{\infty}\frac{1}{2}f_1(x)\ln(\frac{1}{2}f_1(x))dx), \\ &= k_b(-\int_{-\infty}^{\infty}f_0(x)\ln(f_0(x))dx + \ln 2). \end{aligned}$$

After erasing a bit with success proportion p (that is, $P(M = 0) = p$) the probability of finding the particle between x and $x + dx$ is given by,

$$\begin{aligned} P(X \in (x, x+dx)) &= P(M=0)P(X \in (x, x+dx)|M=0) \\ &\quad + P(M=1)P(X \in (x, x+dx)|M=1), \\ &= pf_0(x)dx + (1-p)f_1(x)dx \end{aligned} \tag{2}$$

Thus, the probability distribution function of the particle after undergoing a an erasure with success proportion p is given by, $p_f(x) = pf_0(x)dx + (1-p)f_1(x)$. The final entropy of the system after undergoing erasure with success proportion p, based on the 'insignificant overlap 'assumption, is given by,

$$\begin{aligned} S_f &= -k_b\int_{-\infty}^{\infty}(pf_0(x) + (1-p)f_1(x))\ln(pf_0(x) \\ &\quad + (1-p)f_1(x))dx, \\ &\approx -k_b(\int_{-\infty}^{\infty}pf_0(x)\ln(pf_0(x))dx \\ &\quad + \int_{-\infty}^{\infty}(1-p)f_1(x)\ln((1-p)f_1(x))dx), \\ &= -k_b(\int_{-\infty}^{\infty}f_0(x)dx + p\ln p + (1-p)\ln(1-p)). \end{aligned}$$

Using, the 2^{nd} Law of Thermodynamics, the average heat dissipated, $\langle Q_d\rangle \geq T(S_i - S_f) = k_bT(\ln 2 + p\ln p + (1-p)\ln(1-p))$, which is the Generalized Landauer bound.

Note that the decrease in thermodynamic entropy associated with imperfect erasure is a scalar multiple (adjusted to appropriate units) of the decrease in information entropy due to partial information erasure.

In the next section we describe a methodology to create a bistable potential well by multiplexing an optical trap between two locations using an 'Optical Tweezer', similar to the approach used by Bérut et al. (2012). It allows for high precision nanoscale measurements of the position of the Brownian particle in the optical trap, enabling the quantification of heat dissipation in the *zeptojoule* (10^{-21}J) range. We then utilize the method to model a single bit memory and propose a novel protocol to erase it, by suitably manipulating the optical fields. The design of the protocol allows the user to control the proportion of successful erasures p. We then utilize the 'stochastic thermodynamics' framework of Sekimoto (1997, 1998) to compute the energy expenditure during the erasure process. Our results indicate that erasing one bit of information can be accomplished by expending energy less than the Landauer bound of $k_bT\ln 2$. Thus, it is important to note that the Landauer's bound, a very small quantity (approximately equal to 10^{-21} joules), can now be overcome by introducing imperfections in the erasure process. Additionally, our approach is based on the working of a physical system (namely, Optical Tweezers), thus making it applicable to any standard optical tweezer setup.

4. BEATING LANDAUER'S BOUND

4.1 Model for single bit memory

Bhaban et al. (2016) utilized an optical trapping system, where a 1064 nm 50 mW infrared laser is used to trap a polystyrene bead ($1\mu m$ in diameter) near the focus of the beam, while remaining suspended in a solution of deionized water. The bead in an optical trap experiences a harmonic potential $\frac{1}{2}kx^2$ up to finite width w on either sides of locally stable equilibrium point, where k is the trap stiffness and x is the bead position measured experimentally using a photo-diode. Stiffness of the optical trap k is deduced by the Equipartition Theorem, $\frac{1}{2}k\langle x^2\rangle = \frac{1}{2}k_bT$ (Neuman and Block (2004)), where T is the temperature of the surrounding and k_b is the Boltzmann constant. To characterize the nature of the well outside the width w, Bhaban et al. (2016) demonstrated experimentally that the bead effectively sees a flat potential, indicating that the

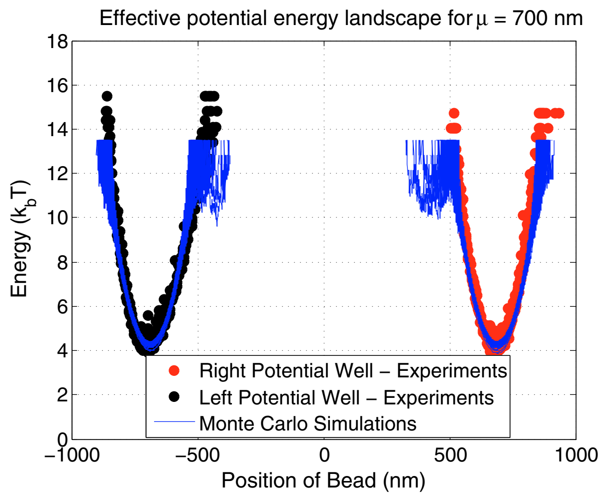

Fig. 3. Double well potential for $\mu = 700\ nm$ from Bhaban et al. (2016), obtained using Monte Carlo simulations and experiments. Close match between Monte Carlo simulations and experiments is seen.

bead dynamics are dictated primarily by thermal noise.It thus enables the potential $U(x)$ to be modeled as,

$$U(x) = \begin{cases} \frac{1}{2}kx^2 + U_r, & \text{if } |x| \leq w \\ \frac{1}{2}kw^2 + U_r, & \text{if } |x| > w. \end{cases} \tag{3}$$

We model the bead dynamics in deionised water under the influence of a laser trap by the overdamped Langevin equation Gardiner et al. (1985) described below.

$$-\gamma\frac{dx}{dt} + \xi(t) - \frac{\partial U(x)}{\partial x} = 0. \tag{4}$$

Here, the coefficient of viscosity γ is obtained experimentally by step response method of Visscher and Block (1998), $U(x)$ is the potential formed due to optical trap and $\xi(t)$ is a zero mean uncorrelated Gaussian noise force, with $\langle\xi(t)\rangle = 0$, $\langle\xi(t), \xi(t^{'})\rangle = 2D\delta(t - t^{'})$. $D = \gamma k_b T$ denotes the diffusion coefficient, with T being the temperature of the heat bath. The measured position of the bead is used to determine the potential $U(x)$ experienced by the bead using the canonical distribution $U(x) = -\ln\left(\frac{P(x)}{C}\right)$, where $P(x)$ is the probability distribution of the bead and C is a normalizing constant. The probability distribution function of position of the particle can be approximated by a Gaussian distribution $P(x)$ due to the harmonic nature of the potential experienced by the particle in the optical trap. The mean μ of the distribution is the location of the trap focus and variance σ^2 $(= \frac{k_b T}{k})$ is obtained from the measured position of the bead. Bhaban et al. (2016) showed a good agreement between experimental observations and simulation results obtained from (3) and (4), demonstrating the efficacy the above mentioned model.

The existing model for single parabolic potential is utilized to create a model for double well potential by switching the laser between two locations $-\mu$ and μ. The time scales of switching are maintained in the order of 10 μs, which is much lower than the time constants of the bead dynamics that are in the order of 1 ms; effectively creating two optical traps. Of the total cycle time T_{cycle}, the laser is kept ON at location $-\mu$ for a time T_{on} and OFF for a time $T_{off} = T_{cycle} - T_{on}$. During the time T_{off}, the laser remains ON at location μ and OFF at $-\mu$. Let $r(t)$ denote the ON or OFF state of the laser at the location μ. $r(t) = 1$ if laser at μ is ON at the time instant t, while $r(t) = 0$ if laser at μ is OFF at the time instant t.The potential realized by switching of this nature by utilizing the parameters k, U_r and w (obtained from (3) using experimental data for single trap) is modeled as follows:

$$U(x, r(t)) = \begin{cases} \frac{1}{2}k(x-\mu)^2 + U_r, & \text{if } |x-\mu| \leq w \ , \ r(t) = 1 \\ \frac{1}{2}k(x+\mu)^2 + U_r, & \text{if } |x+\mu| \leq w \ , \ r(t) = 0, \\ \frac{1}{2}kw^2 + U_r, & \text{otherwise.} \end{cases} \tag{5}$$

By defining the duty ratio $d = \frac{T_{on}}{T_{cycle}}$ and maintaining $d = 0.5$, a symmetric double well potential is created as shown in Figure 3. Note that, the stiffness k used to simulate the double well potential is the same value of stiffness obtained from the experimental data of the single well potential. The potential obtained using position information by simulation of (4) based on the potential described in (5) and experimentally (by laser multiplexing at duty ratio of 0.5) from measured bead position results in a symmetric double well potential as seen in Figure 3 and shows an excellent match between simulations and experiments. It enables us to use this framework to model a single bit memory, where the bead in the right well (i.e. the well with its minima at μ) denotes the memory state, $M = 1$ and the bead in the left well (i.e. the well with its minima at $-\mu$) denotes the memory state, $M = 0$.

In the next section we provide a 'protocol' to erase the information content of a single bit memory modeled as described above and quantify the energetics for the erasure process. We then resort to Monte Carlo simulations to employ the protocol and implement imperfect erasures by maintaining a handle on the success proportion p. We demonstrate that, while one can spend energy lesser than the Landauer's bound of $k_b T \ln 2$ by resorting to imperfect erasures, the Generalized Landauer's bound is still obeyed.

4.2 Erasure Protocol

Erasure of a single bit memory modeled using the approach described in the preceding section can be achieved by following a protocol involving the manipulation of the duty ratio d. Consider a single bit memory element modeled by a particle in a double well potential as described in the previous section. For a particle in the right or left well, an operation $Op(d_i, T_i)$ involves maintaining a duty ratio $d = d_i$ for time duration $T = T_i$. Then, an *erasure protocol* defines a sequence of three operations $[Op(d_1, T_1) \rightarrow Op(d_2, T_2) \rightarrow Op(d_1, T_1)]$, where $d_1 = 0.5$, $d_2 > 0.5$. The choice of T_1 and T_2 have been described in detail in Talukdar et al. (2016).

After the initiation of the protocol, the operation $Op(d_1, T_1)$ creates a symmetric double well potential similar to Fig. 3 and ensures that the particle (in our case, a spherical polystyrene bead of diameter $1\mu m$) is in the left or the right well with equal probability, that is, the initial state of the memory is either zero or one. The operation $Op(d_2, T_2)$ raises the well at location μ and lowers the well at $-\mu$, thereby creating an asymmetric double well potential (see Fig. 4). Such a potential favors the transfer of the bead to the left well. An appropriate choice of (d_2, T_2) ensures

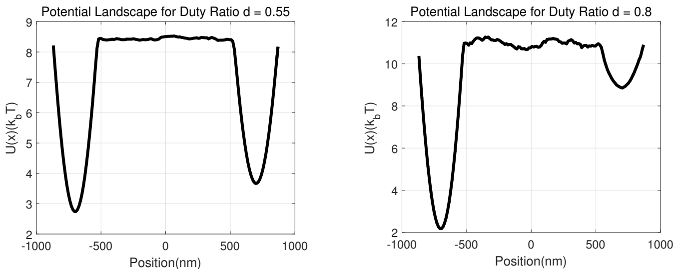

Fig. 4. Asymmetric double well potentials, formed by maintaining the duty ratio $d > 0.5$. It is evident that a higher duty ratio in (b) than in (a) leads to a higher asymmetry.

that such a transfer happens with a high probability. The operation $(Op(d_1, T_1))$ is then performed once again to create a symmetric double well potential, thus bringing the potential back to the original condition.

Next, we present an approach for computation of heat dissipation in erasing a single bit memory realized using the model described in section 4.1 and erased using the erasure protocol described in section 4.2.

4.3 Erasure Thermodynamics

We utilize the thermodynamics framework of Sekimoto (1997, 1998) for systems governed by Langevin dynamics, to compute heat dissipated Q_d in each realization of erasure process as described in Definition (4.2). This approach has been utilized in many recent articles related to experimental verification Landauer's bound (Bérut et al. (2012); Jun et al. (2014)). For the particular case considered in this paper, the work done on the bead is given as,

$$W = \sum_j [U(x(t_j), d(t_j^+)) - U(x(t_j), d(t_j^-)], \qquad (6)$$

where d denotes the duty ratio, t_j denotes the time instant when the duty ratio was changed, t_j^- and t_j^+ denote the instants just before and after changing the duty ratio respectively. The above expression can be interpreted as work done on the particle due to changing of duty ratio is equal to the change in energy of the particle on manipulation of duty ratio. Note that the work done on the Brownian particle can be computed using position information of the particle and time instants of modulation of duty ratio, both of which can be measured in an optical tweezer setup. Utilizing the fact that erasure is a cyclic thermodynamic process and the 1^{st} Law of Thermodynamics, the average heat dissipated over several realizations of erasure, $\langle Q_d \rangle$ is equal to the average work done on the Brownian particle over several realization of erasure, $\langle W \rangle$.

5. RESULTS

In this section we implement the thermodynamics framework described above on the double well potential memory model (Section 4.1) and erasure protocol (Section 4.2) to demonstrate erasures with heat dissipation lower than the Landauer's Bound. The average heat dissipated $\langle Q_d \rangle$ for various success proportions obtained using Monte Carlo simulations of the stochastic dynamics of a bead in a

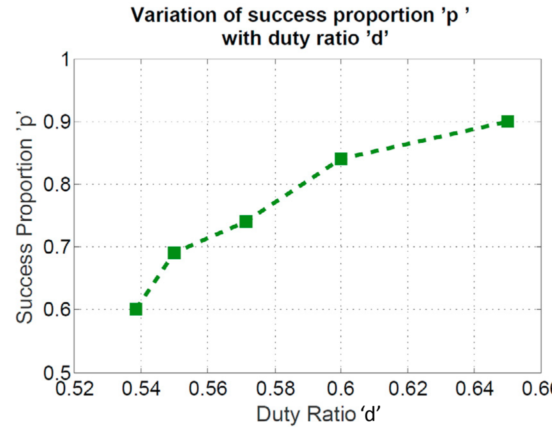

Fig. 5. The variation of success proportion p with duty ratios $0.53, 0.55, 0.57, 0.6$ and 0.65 is shown here. Note that as $d \uparrow 1$, $p \uparrow 1$. The variation of p for duty ratios $d > 0.65$ can be found in Talukdar et al. (2016).

double well potential are shown in Figure 6. It is seen that average heat dissipation less than the Landauer bound of $k_b T \ln 2$ can be achieved by compromising on the accuracy of erasure. The decreasing trend of the average heat dissipation with decreasing success proportion reflects the trend shown by the Generalised Landauer bound curve and is explained by the fact that lower success proportion is caused by a lower duty ratio and hence, lesser asymmetry in the double well potential(lesser transport of mass from the initial to final probability distribution). Note that, the average heat dissipation for various success proportions lies above the Generalised Landauer bound curve, indicating that the Generalised Landauer bound is respected. In the Monte Carlo simulations, the duration of the erasure process is 30 seconds, which is not a quasi-static process, while the Generalised Landauer bound can be achieved only by a quasi-static process. For the duration of 30 seconds, lower success proportion erasure processes are closer to a quasi static process than the high success proportion erasures. Hence, the average heat dissipation for lower success proportions is closer to the Generalised Landauer bound curve as compared to the higher success proportions. The departure of the average heat dissipated obtained using simulations from the Generalised Landauer bound curve is usually corrected by applying a finite time correction, to obtain the heat dissipation in a quasi-static process from a finite time process as described in (Sekimoto and Sasa (1997); Bérut et al. (2012); Jun et al. (2014)) but is not pursued here.

6. CONCLUSION

In this article, we establish the existence of the Generalised Landauer bound for partially successful erasures and demonstrate it to be lower than the Landauer's bound of $k_b T \ln 2$. We employ an Optical Tweezer to present a model for a single bit memory and propose a protocol to erase it, while simultaneously regulating the proportion of success of the erasure process. We utilize the protocol to demonstrate that an average heat dissipation less than the Landauer bound of $k_b T \ln 2$ can be achieved by compromising on the accuracy of the erasure operation. A crucial point to note that the Generalised Landauer bound reduces rapidly and exponentially with the proportion of success. Thus,

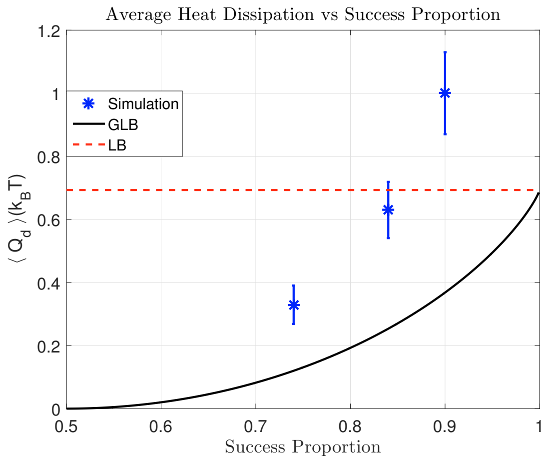

Fig. 6. Average heat dissipation as a function of success proportion obtained using Monte Carlo simulations of (4). The simulations are performed for duty ratio of $0.57, 0.60, 0.65$ and success proportions are computed for each duty ratio after analyzing the trajectory of the particle for all simulated realizations. For each duty ratio 150 realizations of the erasure process is simulated with 75 being initialized in the state $M = 1$ (right well) and the other 75 being initialized in the state $M = 0$ (left well). In this article, we perform simulations with the parameters $\mu = 700\ nm$ and $\sigma = 36\ nm$.

significant reductions in the energy expenditure per bit can be possibly achieved by a relatively lesser compromise on the proportion of success. This could possibly be applied to computations involving erasure of least significant bits, where a compromise on accuracy is acceptable.

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