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A Matlab Package for Multiobjective Control Synthesis

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Abstract In this paper, a Matlab package, GMO 1.0, is introduced for synthesizing (sub-)optimal controllers for the general multiobjective (GMO) control problem involving ℓ_1 norm, $\mathcal{H}_2$ norm, $\mathcal{H}_\infty$ norm, time-domain constraint (TDC), and controller structure constraints. Formulated in [1], the GMO framework encompasses a large class of multiobjective problems which can be effectively solved by using GMO 1.0 package (e.g. $\ell_1/\mathcal{H}_\infty$, ℓ_1/TDC, etc). While global convergence is guaranteed for the convex finite dimensional suboptimal problems, this set of routines yields a rational linear time-invariant controller delivering performance within any prescribed tolerance of the optimal cost. In this paper, several multiobjective design problems are solved to demonstrate the effectiveness of the framework and the software.

Keywords $\ell_1/\mathcal{H}_2/\mathcal{H}_\infty$, time-domain constraint, multiobjective control, robust optimal control, linear matrix inequality (LMI), semidefinite programming (SDP)

1 Introduction

In recent years, synthesizing robust multiobjective controllers has attracted significant amount of attention from both industrial and academic communities ([8][9]). This demand is of primary concern in many engineering applications where diverse uncertainties exerted on the system render it necessary to evaluate controllers' performance by employing multiple distinct (often conflicting) quantifiers.

Lately, a new methodology was proposed in [2] and [3] to solve the multi-input multi-output (MIMO) $\mathcal{H}_2/\ell_1$ problem. The novel idea utilized in this methodology is to introduce an ℓ_1 norm bound on the optimizing variable, the Youla parameter Q, to obtain a regularized optimization problem. Then by appealing to the Banach-Alaoglu theorem concerning compactness in the weak-star topology on $\ell_1(= c_0^*)$, a monotonically converging sequence of lower bounds was obtained. In [5], a similar idea was used to obtain a lower bound for the $\mathcal{H}_2/\mathcal{H}_\infty$ problem, where an $\mathcal{H}_\infty$ norm bound was imposed on the Youla parameter Q rather than an ℓ_1 norm bound to act as the regularizing condition.

The framework presented in [2] and [3] has been significantly advanced in [1] to compute solutions to the general multiobjective (GMO) problem where it is now possible to incorporate the ℓ_1 norm, $\mathcal{H}_2$ norm, $\mathcal{H}_\infty$ norm and time-domain constraint (TDC) in the control design process. In this method, the idea of introducing Youla parameter Q as the optimizing variables was used. This has similarity to the $Q-$des method introduced in [6]. For the latter method, however, there is no information provided on how to compute lower bounds for the GMO problem. In contrast, in [1], the existence of an optimal solution to the GMO problem and monotonically converging lower and upper bounds is established.

Although many theoretical results on multiobjective controller synthesis have been obtained, there is a need for a robust design tool (like the $\mathcal{H}_\infty$ and μ design tools incorporated in Matlab toolbox) that is capable of effectively solving multi-objective design problems (see, for example, the comments in [10] and [11]). There is an ℓ_1 software package which is capable of conducting multiobjective controller design concerning linear objectives by using the FMV, FME, and Delay Augmentation (DA) approaches ([12]). In these approaches, zero directions have to be computed to obtain interpolation conditions on the closed-loop maps. Also since the inversion of certain rational matrices is required to recover the optimal controller from the optimal closed-loop map, these approaches commonly suffer from numerical difficulties ([10]). Apart from the computational difficulties, in this framework, convergence to the optimal solution of the upper and lower bounds, in the presence of TDC and $\mathcal{H}_\infty$ constraint, is not guaranteed. In contrast, the framework in [1] guarantees the monotone convergence of upper and lower bounds. Towards fulfilling the need for a computational tool with associated theoretical results, we have developed GMO 1.0, a matlab user-friendly package to implement the algorithm presented in [1] for solving multiobjective problems. Furthermore, it will be illustrated by means of examples that GMO package can effectively solve multiobjective problems involving structure constraints on the Youla parameter Q ([13]).

The outline of the paper is as follows. In Section 2, we give a brief review of the theoretical results on which GMO 1.0 is built. In Section 3, we discuss the software implementation and present an illustrative example. In Section 4, we solve several practical multiobjective problems to illustrate the features and use of GMO 1.0. Finally, in Section 5, we summarize this paper.

2 GMO Problem Formulation and Solution

Consider the general setup in Figure 1, where $G : [w; u] \to [z; v]$ is the generalized discrete-time linear time-invariant plant and K is the controller. r is a given scalar reference input (such as a step) and y is the time response output.

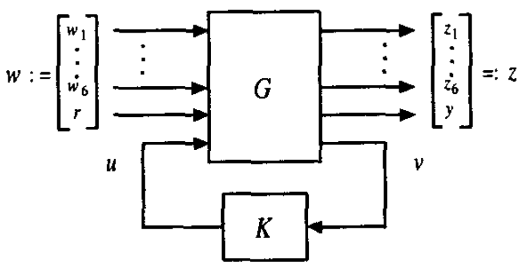

Figure 1: Closed-loop system

By Youla parametrization ([12]), all the achievable closed-loop maps can be characterized as follows:

$$\{R \in \ell_1^{n_z\times n_w} | R = H - U * Q * V \text{ with } Q \in \ell_1^{n_u\times n_v}\} \quad (1)$$

where $H \in \ell_1^{n_z\times n_w}$, $U \in \ell_1^{n_z\times n_u}$, $V \in \ell_1^{n_v\times n_w}$, Q is a free parameter in $\ell_1^{n_u\times n_v}$ and '$*$' denotes the convolution operation. In what follows, w.l.o.g., we shall assume that $\hat{U}$ and $\hat{V}$ have full column and row ranks, respectively. Also it can be assumed that $\hat{H}$, $\hat{U}$, and $\hat{V}$ are polynomial matrices in λ.

To simplify the notations, in the sequel, we shall use $\hat{R}^i$ ($i = 1, \ldots, 6$) to denote the closed-loop transfer matrix from w_i to z_i and $\hat{R}^7$ the transfer function from r to y. For the sake of simplicity, and w.l.o.g., we shall consider the case when the reference input r is a step sequence. Let A_{temp} be defined as:

$$A_{temp} := \begin{pmatrix} 1 & 0 & 0 & \cdots \\ 1 & 1 & 0 & \cdots \\ \vdots & \vdots & \ddots & \ddots \end{pmatrix}.$$

Then the time response of the closed-loop system due to the reference input r is given by $y = R^7 * r = A_{temp}R^7$. The general multiobjective (GMO) problem ([1]) can then be stated as follows: *Given a plant G, constants $c_i > 0$, $i = 1, \ldots, 6$, and two sequences $\{a_{temp}(k)\}_{k=0}^{\infty}$ and $\{b_{temp}(k)\}_{k=0}^{\infty}$, solve the following problem,*

$$\begin{aligned} &\inf_{Q\in\ell_1^{n_u\times n_v}} f(Q) \\ &\text{subject to} \\ &\quad \|R^4(Q)\|_1 \le c_4 \\ &\quad \|R^5(Q)\|_2^2 \le c_5 \\ &\quad \|\hat{R}^6(Q)\|_{\mathcal{H}_\infty} \le c_6 \\ &\quad a_{temp}(k) \le [A_{temp}R^7(Q)](k) \le b_{temp}(k), \forall k. \end{aligned} \quad (2)$$

where $f(Q) := c_1\|R^1(Q)\|_1 + c_2\|R^2(Q)\|_2^2 + c_3\|\hat{R}^3(Q)\|_{\mathcal{H}_\infty}$, $R^i(Q) = H^{ii} - U^i * Q * V^i, i = 1, \ldots, 7$.

GMO problem defined above represents a large class of multiobjective control problems. Many extensively studied multiobjective problems are special cases of the GMO setup, e.g., $\ell_1/\mathcal{H}_\infty$, $\mathcal{H}_2/\mathcal{H}_\infty$, $\mathcal{H}_\infty$/TDC. Furthermore, for the first time, the $\mathcal{H}_\infty/\ell_1$ and $\ell_1/\mathcal{H}_2/\mathcal{H}_\infty$ problems are addressed. The problem formulation in (2) provides a uniform framework for the performance tradeoff study involving the ℓ_1, $\mathcal{H}_2$, $\mathcal{H}_\infty$, and TDC.

In the general case, (2) is difficult to solve. To facilitate the solution of it, an extra one norm bound on the Youla parameter Q is introduced in (2) to obtain its auxiliary problem. Let ν denote the optimal value obtained from this problem. As argued in [1], the extra one norm constraint $\|Q\|_1 \le \beta$ plays an essential role in obtaining solution to (2) and converging lower and upper bounds of ν can be constructed as:

$$\begin{aligned} \nu_n := \; &\inf_{Q\in\ell_1^{n_u\times n_v}} f_n(Q) \\ &\text{subject to} \\ &\|Q\|_1 \le \beta \\ &\|P_n(R^4(Q))\|_1 \le c_4 \\ &\|P_n(R^5(Q))\|_2^2 \le c_5 \\ &\|T_{6,n}(Q)\| \le c_6 \\ &a_{temp}(k) \le [A_{temp}R^7(Q)](k) \le b_{temp}(k) \\ &k = 0, 1, \ldots, n. \end{aligned} \quad (3)$$

$$\begin{aligned} \nu^n := \; &\inf_{Q\in\ell_1^{n_u\times n_v}} f(Q) \\ &\text{subject to} \\ &\|Q\|_1 \le \beta \\ &\|R^4(Q)\|_1 \le c_4 \\ &\|R^5(Q)\|_2^2 \le c_5 \\ &\|\hat{R}^6(Q)\|_{\mathcal{H}_\infty} \le c_6 \\ &a_{temp}(k) \le [A_{temp}R^7(Q)](k) \le b_{temp}(k), \forall k \\ &Q(k) = 0 \text{ if } k > n. \end{aligned} \quad (4)$$

where $f_n(Q) := c_1\|P_n(R^1(Q))\|_1 + c_2\|P_n(R^2(Q))\|_2^2 + c_3\|T_{3,n}(Q)\|$ and

$$T_{i,k}(Q) := \begin{bmatrix} R^i(0) & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & 0 \\ R^i(k) & \cdots & \cdots & R^i(0) \end{bmatrix}, \; i = 3, 6.$$

3 Software Implementation

It is clear that only $Q(0), \ldots, Q(n)$ enter into the optimization of ν_n and ν^n. Thus (3) and (4) are actually two finite dimensional programming problems and they can be effectively solved using well-developed semidefinite programming techniques, e.g., SP ([15]), SDP ([16]), and SDPHA ([17]).

In the case where there is no $\mathcal{H}_\infty$ norm involved in the problem setup (i.e. $c_3 = 0$, $c_6 = \infty$) and no $\mathcal{H}_2$ norm constraint imposed on the closed-loop system($c_5 = 0$), the corresponding GMO problem can be solved in a less computationally expensive manner by using quadratic programming techniques. For the case where neither $\mathcal{H}_2$ norm nor $\mathcal{H}_\infty$ norm are involved in the problem setup, the GMO problem can be efficiently solved by using linear programming techniques.

Partially built on the LMI optimization interface LMITOOL 2.1 ([18]) for SP, SDP, and SDPHA codes, GMO 1.0 solves the problems (3) and (4) and yields (sub-)optimal solutions for the GMO problem. It can run on any IBM-compatible personal computer (PC) system under Windows 98/2000/NT operating system with a Matlab 5.x or higher installed. Besides the basic matlab package, GMO 1.0 requires the *μ − Analysis and Synthesis* toolbox ([19]).

Under the syntax of the GMO package, synthesizing multiobjective controllers can be readily carried out. Take as an example the $\ell_1/\mathcal{H}_\infty$ multi-block problem considered in [14]:

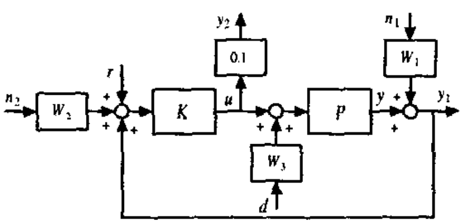

Figure 2: Block diagram of the $\ell_1/\mathcal{H}_\infty$ example

The optimization problem of interest is to solve the problem $\{min \| \Phi \|_1 : \| \Psi \|_\infty \leq 37\}$, where Φ is the transfer matrix from $w_1 := [n_1\ n_2]^T$ to $z_1 := [y_1\ y_2]^T$ and Ψ is the transfer matrix from $w_2 := [r\ d]^T$ to $z_2 := [y\ u]^T$ respectively.

With an FIR supported length of three(lenqind=3), the GMO routines yield a pair of lower and upper bounds [72.5960, 73.0380] with $\| \Phi \|_1 = 72.8220$ achieved by a *9th* order unstable suboptimal controller. The $\mathcal{H}_\infty$ performance of the Ψ subsystem is 36.9583. These results coincide with those obtained in [14], where the optimal ℓ_1 performance is computed to be 72.6418 achieved by a *14th* order optimal controller.

4 Examples

In this section, we present several examples from the literature to illustrate the features of GMO 1.0 to compute multiobjective controllers. All the simulation results shown here were obtained by using GMO 1.0 on a PII-350/312MB/Win2000 PC under Matlab 5.3.

4.1 Active suspension control

The active suspension control design for transport vehicles aims to handle several conflicting goals and can be formulated into a multiobjective design problem. Consider the following two DOF rear suspension system model from [11]:

$$\begin{array}{l} m_2\ddot{q}_2 + b_2(\dot{q}_2 - \dot{q}_1) + k_2(q_2 - q_1) = F \\ m_1\ddot{q}_1 + b_2(\dot{q}_1 - \dot{q}_2) + k_2(q_1 - q_2) \\ +b_1(\dot{q}_1 - \dot{q}_0) + k_1(q_1 - q_0) = -F \end{array} \tag{5}$$

where $m_1 = 1.5e3kg$ denotes the mass of tires, wheels, and real axle, $m_2 = 5.75e3kg$ denotes the sum of mass of the chassis and a half-loaded semitrailer, $b_1 = 1.7e3N/m$ and $b_2 = 5e3N/m$ represent the tire and suspension damping coefficients while $k_1 = 5e6N/m$ and $k_2 = 5e5N/m$ denote the tire and suspension stiffness respectively. q_0, q_1, and q_0 are road level, suspension displacement, and semitrailer displacement respectively.

According to the required performance specifications, the above system can be transformed ([11]) into the generalized plant setup shown in Figure 1, where we choose the exogenous input to be $w := q_0$, control input $u := F$, measured output $y^T := [\ddot{q}_2\ q_2 - q_1]$, and the controlled output $z^T := [\ddot{q}_2\ q_2 - q_1\ q_1 - q_0\ F]$. In this setup, w denotes the road surface level, and z consists of the vertical acceleration $\ddot{q}_2$, suspension deflection $q_2 - q_1$, tire deflection or dynamic tire force $q_1 - q_0$, and actuator force F.

Assume that $w = q_0$ denotes a given deterministic-and-stochastic mixed road profile with a known l_∞ bound and a (spatial) power spectral density. According to the arguments above, the following control design would be of interest to designers:

$$\begin{array}{l} \inf_{Q \in \ell_1^{n_u \times n_v}} \|R_{z_1 w}(Q)\|_2^2 \\ \text{subject to} \\ \|R_{z_2 w}\|_1 \leq c_2 \\ \|R_{z_3 w}\|_1 \leq c_3 \\ \|R_{z_4 w}\|_1 \leq c_4 \end{array} \tag{6}$$

where $c_i (i = 2, 3, 4)$ are parameters to be chosen. Note here we choose to minimize the $\mathcal{H}_2$ norm to address the comfort performance requirement of the driver and cargo. To ascertain the achievable ranges for c_i, we carry out the following study of performance limits:

$$c_i^0 := \inf_{Q \in \ell_1^{n_u \times n_v}} \|R_{z_i w}(Q)\|_p \tag{7}$$

where $\|.\|_p$ denotes $\mathcal{H}_2$ norm for $i = 1$ and ℓ_1 norm for $i = 2,3,4$. This set of problems can be carried out readily by GMO routines. The design yields $c_2^0 = 0.1701$, $c_3^0 = 0.1660$, $c_4^0 = 0.0082$, and the best achievable $\mathcal{H}_2$ norm of $R_{z_1 w}$ is 0.0665 while the other three channels achieve ℓ_1 performance of 0.6733, 0.7296, and 0.0419 respectively. According to the minimum achievable ℓ_1 performance obtained above, $c_i (i = 2,3,4)$ are chosen to be 0.6, 0.6, and 0.1 in (6). And the final resulting $\mathcal{H}_2$ performance of (6) is computed to be 0.0729 with ℓ_1 performance of 0.6026, 0.6037 and 0.0333 in other three channels. It is clear from this example that GMO routine has a flexible structure and various control system design demands can be easily captured in its framework.

4.2 Nested system control design

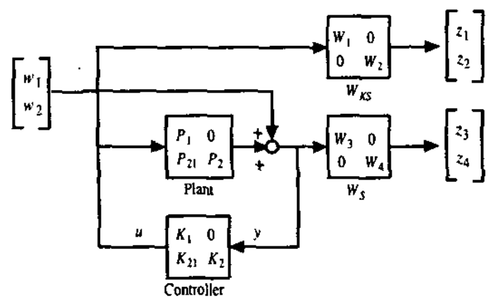

Figure 3: Closed-loop system of the 2-nested design

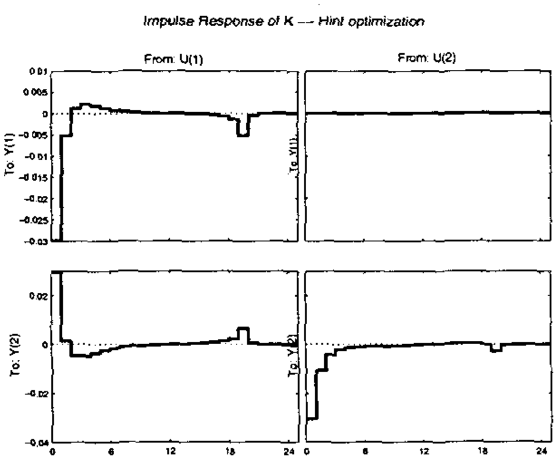

Figure 4: 2-nested control design ($\mathcal{H}_\infty$ case)

In [13], a class of systems with nested structures were studied. In the system connected in the nested fashion, information and control actions of the subsystems go in a particular manner, i.e., from inside to outside, or, the reverse. It was proven ([13]) that the problem of optimal tracking and disturbance rejection for such type of systems is equivalent to a MIMO control problem with structural constraints on the controller, which can be transformed to a model matching problem with convex constraints on the Youla parameter Q. This class of problems can be readily solved by using GMO package. Consider the problem setup shown in Figure 3, where

$$P_1(\lambda) = P_{21}(\lambda) = P_2(\lambda) = \frac{1.5\lambda}{(1+0.15\lambda)}$$
$$W_1(\lambda) = W_2(\lambda) = \frac{0.2452 + 0.2452\lambda}{1 - 0.5095\lambda}$$
$$W_3(\lambda) = W_4(\lambda) = 0.1.$$

Note that for the sake of simplicity, the weighting matrices W_{KS} and W_S in Figure 3 are chosen to be diagonal and equal along the diagonal line. In general cases, they can be any type of weighting matrices, coupling or not. The control objective for this system is to solve the following mixed-sensitivity problem with Toeplitz (block lower triangular) structure constraint on the controller K:

$$\mu := \inf_{K \text{ stabilizing and Toeplitz}} \left\| \begin{array}{c} W_{KS}KS \\ W_S S \end{array} \right\|,$$

where the norm $\|.\|$ can refer to any norm, ℓ_1, $\mathcal{H}_2$, or $\mathcal{H}_\infty$. By Lemma 3.1 of [13], there exists a coprime factorization of the plant P such that the above problem can be equivalently cast as

$$\mu := \inf_{Q \text{ stable and Toeplitz}} \| H - UQV \|.$$

Since P is stable, the desired coprime factorization is obtained by choosing the zero controller. It follows that the above controller design can be readily carried out in the GMO framework and the finally obtained ℓ_1, $\mathcal{H}_2$, and $\mathcal{H}_\infty$ performance are 0.8598, 0.4838, and 0.9803, respectively. The impulse response of the resulting (sub-)optimal controller K are plotted in Figure 4, from which it can be seen clearly that the required Toeplitz structure has been achieved.

4.3 A multiobjective $\mathcal{H}_\infty$ design

This example is taken from [5]. The control objective is to minimize the $\| C_1 \|_\infty + \| C_2 \|_\infty$ performance for an unstable system of the form:

$$\begin{bmatrix} z_1 \\ z_2 \\ y \end{bmatrix} = \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right] \begin{bmatrix} w_1 \\ w_2 \\ u \end{bmatrix}$$

where C_1 and C_2 represent the two performance channels from w_1 to z_1 and w_2 to z_2 respectively. By calling GMO routines (lenqind=12, beta=10000), a lower bound and an upper bound are computed to be 114.4058 and 115.6487 ($\| C_1 \|_\infty = 65.5327$, $\| C_2 \|_\infty = 50.1160$) respectively. The compensator obtained by GMO is a $21th$ order unstable controller.

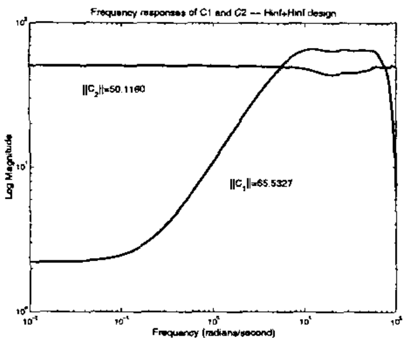

Figure 5: $\mathcal{H}_\infty$+$\mathcal{H}_\infty$ design results

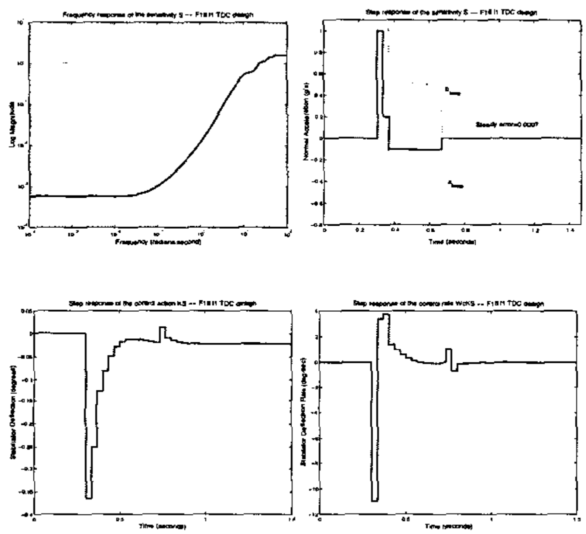

Figure 6: F16 longitudinal design results

4.4 F16 longitudinal control design

The AFTI F-16 control problem([10]) aims to synthesize an ℓ_1 controller for the longitudinal dynamics so as to achieve certain tracking performance while satisfying constraints on control deflection, control rate, and requirements on overshoot and undershoot specifications.

Specifically, the tracking problem is to accurately command a $1-g$ normal acceleration of the aircraft while the stabilator is limited to $\pm 25deg$ deflection angle and $\pm 60deg/s$ deflection rate ([10]). The aircraft model is a concatenation of the actuator servo G_a and the linearized longitudinal equation of motion G_p. The continuous system G_pG_a is discretized at $30Hz$ using a zero-order holder (ZOH). All the simulations are conducted within this hybrid system framework and a step reference input of $1-g$ normal acceleration is applied at 0.3 second(simulation time) to evaluate tracking performance.

To achieve the desired tracking performance, TDC templates a_{temp} and b_{temp} are chosen in such a manner that step response of sensitivity function S is forced to converge to zero as the system proceeds into the steady state. With $lenqind = 25$ and $beta = 100$, GMO routines yield an ℓ_1 performance of 2.2127 achieved by a $15th$ order (sub-)optimal controller. It is interesting to note that there is an integrator (a pole at 0.9966) in this controller, which verifies the result (steady error of -0.0006) shown in Figure 6 from a different viewpoint.

Note that to effectively take out the derivative of the control signal, a discrete-time transfer function (the 'backward Euler transformation') $W_c(z) = (z-1)/Tz$ ($T = 1/30sec$) was applied on the stabilator deflection to generate time-response output in the simulink diagram. To reduce the control action and control rate magnitude, we choose $a_{temp}(1) = a_{temp}(2) = 0.2$ in the ℓ_1/TDC design to prevent the control action becoming too large during the first two sample periods. It is clear from the step response curves of sensitivity S, control action KS, and control rate W_cKS that this objective has been effectively achieved. As a conclusion, the control design has yielded satisfactory tracking performance while satisfying all the prescribed constraints (compared to those obtained in [10]).

4.5 X29 pitch axis control design

To achieve certain desirable aerodynamic characteristics, the wings of the X29 aircraft are designed to be in the forward-swept shape. This renders better maneuverability to the aircraft when compared with classical wing design while leaving the aircraft statically unstable ([12]). The control objective for this plant is to design a stabilizing discrete-time controller to minimize ℓ_1 norm of the transfer function from the disturbance w injected at the plant output to the weighted control signal z_1 and the weighted output z_2:

$$\inf_{K \text{ stabilizing}} \left\| \begin{matrix} W_1KS \\ W_2S \end{matrix} \right\|_1$$

In this example, for the illustrative purpose, we choose $W_1 = 0.01$ and W_2 as a digital Butterworth 2nd order low-pass filter with cut-off frequency $0.1(rad/sec)$. Under this setup, GMO design yields a $32th$ order (sub-)optimal controller with ℓ_1 performance 1.1140 (Dashed curves in Figure 7). Noticing that the step response of the sensitivity function bears an ℓ_∞ norm of 2.3647 and a steady error of 0.3901, we intend to improve the tracking performance by solving the following problem:

$$\inf_{K \text{ stabilizing}} \left\| \begin{matrix} W_1KS \\ W_2S \end{matrix} \right\|_1$$
subject to
$$a_{temp}(k) \leq S * stepIn(k) \leq b_{temp}(k),\ \forall k.$$

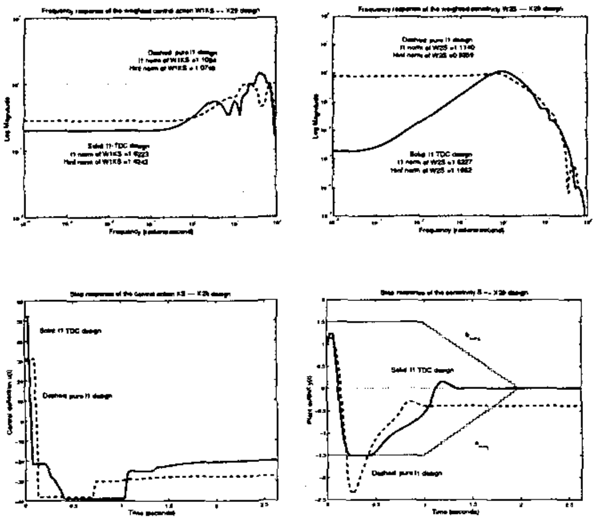

Figure 7: X29 pitch axis design results

where *stepIn* denotes a step and a_{temp} and b_{temp} are two prescribed time-domain template constraint (TDC). In this example, they are chosen such that the maximum absolute magnitude and the steady error of step response of S are constraint within 1.5 and 0.002.

The GMO design yields an ℓ_1 performance of 1.6227 and the step response of the sensitivity function S yields a steady error of -0.0009 with an maximum absolute magnitude of 1.5000 (Solid curves in Figure 7), which implies the desired tracking performance has been achieved. It is interesting to note that there is also an integrator (a pole at 1.0000) in the resulting suboptimal controller.

5 Conclusion

In this paper, we introduce a newly-developed Matlab based package, GMO 1.0, to solve multiobjective problems concerning ℓ_1 norm, $\mathcal{H}_2$ norm, $\mathcal{H}_\infty$ norm, time-domain constraint, and structure constraints on the stabilizing controller. It is shown by means of examples that this package can effectively compute (sub-)optimal solutions to many multiobjective problems.

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