# Box: The approach of Hughes described in the terminology of mathematical sequences

We summarize the key functions of the numerical differentiation of Hughes ([1990](https://www.cs.kent.ac.uk/people/staff/dat/miranda/whyfp90.pdf)) in mathematical terminology. We refer to the original source for a more detailed explanations of these functions. The function easydiff f x h calculates the difference quotient (the aim of numerical differentiation is to approximate the derivative ).

The function differentiate h f x generates the sequence with . Note that the value of is halved from element to element. By definition, this sequence converges to the limit value if the function is continuous.

The function improve aims to speed up this convergence. This function takes a sequence like as argument and generates a new sequence: . Each element of the result sequence is a function of two consecutive elements of the argument: with whereby rounds to the nearest integer.

The function improve can arbitrarily often be composed to itself, i.e. . For example, is the second element of the sequence that result from a threefold composition of improve applied on the difference quotient sequence . Using as argument, the function super generates the sequence:

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