Theoretical Mechanics

HomeWork 7

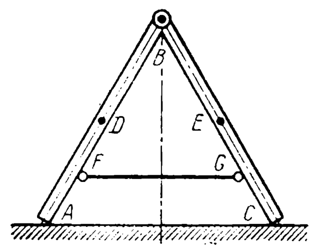
Mikhail Terentev
B22-R-01
m.terentev@innopolis.university
21 March 2024

Task 1

A step ladder ABC, hinged at B, rests on a smooth horizontal floor, as shown on the figure. $AB = BC = 2l$.
The centres of gravity are at the midpoints D and E of the rods. The radius of gyration of each part of the ladder about the axis passing through the center of gravity is p.
The distance between B and the floor is h. At the certain moment the ladder collapses due to the rupture of a ling FG between the two halves of the ladder. Neglecting the effect of friction in the hinge, determine:

1. the velocity v_1 of the point B at the moment, when it hits the floor;

2. the velocity v_2 of point B at the moment, when it is at a distance $\frac{1}{2}h$ from the floor.



Answer: $v_1 = 2l\sqrt{\dfrac{gh}{l^2+p^2}}$, $v_2 = \dfrac{1}{2}\sqrt{gh\dfrac{16l^2-h^2}{2(l^2+p^2)}}$.

Solution:

R.O:

(Due to symmetry of the problem I consider only one half of the ladder)
Body AB - planar motion

Force analysis:

N_y^A, R_x^B, (work = 0)
mg - conservative (work = -$\Pi_1 - \Pi_0$)

Dynamics:

1) $T_1 - T_0 = \sum A_i$

$T_1 = \Pi_0$
$\frac{J^A\omega^2}{2} = mg\frac{h}{2}$

$J^A = J^D + ml^2$
$J^D = m\rho^2$
$J^A = m(\rho^2 + l^2)$

$v^B = 2l\omega^A$

2) $T_1 - T_0 = \sum A_i$

$T_1 = \Pi_0 - \Pi_2$
$\frac{J^c\omega^2}{2} = mg\frac{h}{2} - mg\frac{h}{4}$
(c - instantaneous centre of rotation)

$\frac{J^c\omega^2}{2} = mg\frac{h}{4}$

$J^c = J^D + ml^2$
$J^D = m\rho^2$
$J^c = m(\rho^2 + l^2)$

$v^B = \sqrt{(2l)^2 - (\frac{h}{2})^2}\omega^c$

Solution:

1) $\frac{J^A\omega^2}{2} = mg\frac{h}{2}$

$J^A\omega^2 = mgh$

$(\rho^2 + l^2)\cdot\frac{v^2}{4l^2} = gh$

$v^2 = 4l^2\frac{gh}{\rho^2+l^2}$

$v = 2l\sqrt{\frac{gh}{l^2+\rho^2}}$

2) $\frac{J^c\omega^2}{2} = mg\frac{h}{4}$

$2J^c\omega^2 = mgh$

$2(\rho^2 + l^2)\cdot\frac{v^2}{4l^2 - \frac{h^2}{4}} = gh$

$v^2 = l^2\frac{gh(16l^2-h^2)}{8(\rho^2+l^2)}$

$v = \frac{1}{2}l\sqrt{\frac{gh(16l^2-h^2)}{2(l^2+\rho^2)}}$

Task2

https://colab.research.google.com/drive/1rhJl8J37wvqojFyjYOVA5ANqDxbsWyb-?usp=sharing