

Homework 2

January 22, 2026

These are notes for the directed study on Fourier Analysis from Stein and Shakarchi [1].

Exercises: Complete 2, 4, 5, 6, 8

2.)

4.)

5.)

6.)

8.)

Verify that $\frac{1}{2i} \sum_{n \neq 0} \frac{e^{inx}}{n}$ is the Fourier series of the 2π -periodic sawtooth function, defined by $f(0) = 0$, and

$$f(x) = \begin{cases} -\frac{\pi}{2} - \frac{x}{2} & \text{if } -\pi < x < 0, \\ \frac{\pi}{2} - \frac{x}{2} & \text{if } 0 < x < \pi. \end{cases}$$

Note that this function is not continuous. Show that nevertheless, the series converges for every x (by which we mean, as usual, that the symmetric partial sums of the series converge). In particular, the value of the series at the origin, namely 0, is the average of the values of $f(x)$ as x approaches the origin from the left and the right.

References

- [1] E. M. STEIN AND R. SHAKARCHI, *Fourier Analysis: An Introduction*, vol. 1 of Princeton Lectures in Analysis, Princeton University Press, 2003.