Problems Project

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Construct an NPDA that accepts the language generated by the productions $S \rightarrow aSa/bSb/c$. Show an instantaneous description of this string abcba for this problem. [WBUT 2007]
[Introduction to Automata Theory, Formal Language and Computation, Shyamalendu Kandar - Page 421]

Problem I: Solution

The production rules are not in GNF.So, we need to first convert it into GNF. The production rules are

$$S \rightarrow aSa \mid bSb \mid c$$

Let us introduce two new productions $C_a \rightarrow a$, $C_b \rightarrow b$ The new production rules become

$$S \rightarrow aSC_a$$

$$S \rightarrow bSC_b$$

$$S \rightarrow c$$

$$C_a \rightarrow a$$

$$C_b \rightarrow b$$

Now, all the productions are in GNF. Now, from these productions, a PDA can be easily constructed. First, the start symbol S is pushed into the stack by the following production

$$\delta(q_0,\ \epsilon,\ z_0) \rightarrow (q_1, Sz_0)$$

Problem I: Solution, Cont.

For the production $S \rightarrow aSC_a$, the transitional function is

$$\delta(q_1, a, S) \rightarrow (q_1, SC_a)$$

For the production $S \rightarrow bSC_b$, the transitional function is

$$\delta(q_1,b,S) \rightarrow (q_1, SC_b)$$

For the production $S \rightarrow c$, the transitional function is

$$\delta(q_1, c, S) \rightarrow (q_1, \lambda)$$

For the production $C_a \rightarrow a$, the transitional function is

$$\delta(q_1, a, C_a) \rightarrow (q_1,Sz_0)$$

For the production $C_b \rightarrow b$, the transitional function is

$$\delta(q_1, b, C_b) \rightarrow (q_1, \lambda)$$

For acceptance, the transitional function is

$$\delta(q_1, \lambda, z_0) \rightarrow (q_f,z_0) \text{ // accepted by the final state}$$

$$\delta(q_1, \lambda, z_0) \rightarrow (q_1, \lambda) \text{ // accepted by the empty stack}$$

ID for the String 'abcba'

$\delta(q_0, \underline{\epsilon}abcba, z_0) \rightarrow \delta(q_1, abcba, Sz_0) \rightarrow \delta(q_1, a\underline{b}cba, SC_a z_0) \rightarrow \delta(q_1, ab\underline{c}ba, SC_b C_a z_0) \rightarrow \delta(q_1, abc\underline{b}a, C_b C_a z_0) \rightarrow \delta(q_1, abcb\underline{a}, C_a z_0) \rightarrow \delta(q_1, abcbaB, z_0) \rightarrow (q_f, z_0)$ (Acceptance by FS).

Problem II: Statement

Construct a PDA, A, equivalent to the following context-free grammar

$$S \to 0BB,\ B \to 0S \mid 1S \mid 0$$

Test whether 0104 is in N(A).
[Introduction to Automata Theory, Formal Language and Computation, Shyamalendu Kandar - Page 422]

Problem II: Solution

Solution: The CFG is $S \to 0BB$, $B \to 0S \mid 1S \mid 0$
All the production rules of the grammar are in GNF. First, the start symbol S is pushed into the stack by the following production

$$\delta(q_0, \epsilon, z_0) \to (q_1, Sz_0)$$

Problem II: Solution, Cont.

For the production $S \rightarrow 0BB$, the transitional function is

$$\delta(q_1, 0, S) \rightarrow (q_1, BB)$$

For the production $B \rightarrow 0S$, the transitional function is

$$\delta(q_1, 0, B) \rightarrow (q_1, S)$$

For the production $B \rightarrow 1S$, the transitional function is

$$\delta(q_1, 1, B) \rightarrow (q_1, S)$$

For the production $B \rightarrow 0$, the transitional function is

$$\delta(q_1, 0, B) \rightarrow (q_1, \lambda)$$

For acceptance, the transitional functions are

$$\delta(q_1, \lambda, z_0) \rightarrow (q_f, z_0) \text{ // accepted by the final state}$$

$$\delta(q_1, \lambda, z_0) \rightarrow (q_1, \lambda) \text{ // accepted by the empty stack}$$

The ID for the String 010000

$(q_0, \underline{\epsilon}10000, z_0) \rightarrow (q_1, \underline{0}10000, Sz_0) \rightarrow (q_1, 0\underline{1}0000, BBz_0) \rightarrow (q_1, 01\underline{0}000, SBz_0) \rightarrow (q_1, 010\underline{0}00, BBBz_0) \rightarrow (q_1, 0100\underline{0}0, BBz_0) \rightarrow (q_1, 01000\underline{0}, Bz_0) \rightarrow (q_1, 010000\epsilon, z_0) \rightarrow (q_f, 010000\epsilon, z_0)$ (Accepted by the final state).

References

An Introduction to Automata Theory, Formal Language and Computation, Shyamalendu Kandar - Pages 421-422

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