If we have to design the PDA accepted by the empty stack, the λ function is

$$\delta(\mathrm{q_1}, \lambda, \mathrm{z_0}) \rightarrow (\mathrm{q_1}, \lambda)$$

If we have to design the PDA accepted by the final state, the λ function is

$$\delta(\mathrm{q_1}, \lambda, \mathrm{z_0}) \rightarrow (\mathrm{q_f}, \lambda)$$

11. Construct an NPDA that accepts the language generated by the productions S $\rightarrow$ aSa/bSb/c. Show an instantaneous description of this string abcba for this problem. [WBUT 2007]

Solution: The production rules are not in GNF. So, we need to first convert it into GNF. The production rules are

$$\mathrm{S} \rightarrow \mathrm{aSa} \mid \mathrm{bSb} \mid \mathrm{c}$$

Let us introduce two new productions $\mathrm{C_a} \rightarrow \mathrm{a}$, $\mathrm{C_b} \rightarrow \mathrm{b}$

The new production rules become

$$\mathrm{S} \rightarrow \mathrm{aSC_a} \ \mathrm{S} \rightarrow \mathrm{bSC_b} \ \mathrm{S} \rightarrow \mathrm{c} \ \mathrm{C_a} \rightarrow \mathrm{a} \ \mathrm{C_b} \rightarrow \mathrm{b}$$

Now, all the productions are in GNF. Now, from these productions, a PDA can be easily constructed. First, the start symbol S is pushed into the stack by the following production

$$\delta(\mathrm{q_0}, \epsilon, \mathrm{z_0}) \rightarrow (\mathrm{q_1}, \mathrm{Sz_0})$$

For the production $\mathrm{S} \rightarrow \mathrm{aSC_a}$, the transitional function is

$$\delta(\mathrm{q_1}, \mathrm{a}, \mathrm{S}) \rightarrow (\mathrm{q_1}, \mathrm{SC_a})$$

For the production $\mathrm{S} \rightarrow \mathrm{bSC_b}$, the transitional function is

$$\delta(\mathrm{q_1}, \mathrm{b}, \mathrm{S}) \rightarrow (\mathrm{q_1}, \mathrm{SC_b})$$

For the production S $\rightarrow$ c, the transitional function is

$$\delta(\mathrm{q_1}, \mathrm{c}, \mathrm{S}) \rightarrow (\mathrm{q_1}, \lambda)$$

For the production $\mathrm{C_a} \rightarrow \mathrm{a}$, the transitional function is

$$\delta(\mathrm{q_1}, \mathrm{a}, \mathrm{C_a}) \rightarrow (\mathrm{q_1}, \mathrm{Sz_0})$$

For the production $\mathrm{C_b} \rightarrow \mathrm{b}$, the transitional function is

$$\delta(\mathrm{q_1}, \mathrm{b}, \mathrm{C_b}) \rightarrow (\mathrm{q_1}, \lambda)$$

For acceptance, the transitional function is

$$\delta(\mathrm{q_1}, \lambda, \mathrm{z_0}) \rightarrow (\mathrm{q_f}, \mathrm{z_0}) \text{ // accepted by the final state}$$

$$\delta(\mathrm{q_1}, \lambda, \mathrm{z_0}) \rightarrow (\mathrm{q_1}, \lambda) \text{ // accepted by the empty stack}$$

ID for the String ‘abcba’

$\delta(\mathrm{q_0}, \underline{\epsilon}\mathrm{abcba}, \mathrm{z_0}) \rightarrow \delta(\mathrm{q_1}, \mathit{a}\mathrm{bcba}, \mathrm{Sz_0}) \rightarrow \delta(\mathrm{q_1}, \mathrm{a\underline{b}cba}, \mathrm{SC_a z_0}) \rightarrow \delta(\mathrm{q_1}, \mathrm{ab\underline{c}ba}, \mathrm{SC_b C_a z_0}) \rightarrow \delta(\mathrm{q_1}, \mathrm{abc\underline{b}a}, \mathrm{C_b C_a z_0}) \rightarrow \delta(\mathrm{q_1}, \mathrm{abcb\underline{a}}, \mathrm{C_a z_0}) \rightarrow \delta(\mathrm{q_1}, \mathrm{abcbaB}, \mathrm{z_0}) \rightarrow (\mathrm{q_f}, \mathrm{z_0})$ (Acceptance by FS).

12. Construct a PDA, A, equivalent to the following context-free grammar

$$\mathrm{S} \rightarrow \mathrm{0BB}, \ \mathrm{B} \rightarrow \mathrm{0S} \mid \mathrm{1S} \mid 0$$

Test whether 0104 is in N(A).

Solution: The CFG is S $\rightarrow$ 0BB, B $\rightarrow$ 0S | 1S | 0

All the production rules of the grammar are in GNF. First, the start symbol S is pushed into the stack by the following production

$$\delta(\mathrm{q_0}, \epsilon, \mathrm{z_0}) \rightarrow (\mathrm{q_1}, \mathrm{Sz_0})$$

For the production S $\rightarrow$ 0BB, the transitional function is

$$\delta(\mathrm{q_1}, 0, \mathrm{S}) \rightarrow (\mathrm{q_1}, \mathrm{BB})$$

For the production B $\rightarrow$ 0S, the transitional function is

$$\delta(q_1, 0, B) \rightarrow (q_1, S)$$

For the production $B \rightarrow 1S$, the transitional function is

$$\delta(q_1, 1, B) \rightarrow (q_1, S)$$

For the production $B \rightarrow 0$, the transitional function is

$$\delta(q_1, 0, B) \rightarrow (q_1, \lambda)$$

For acceptance, the transitional functions are

$$\delta(q_1, \lambda, z_0) \rightarrow (q_f, z_0) \text{ // accepted by the final state}$$

$$\delta(q_1, \lambda, z_0) \rightarrow (q_1, \lambda) \text{ // accepted by the empty stack}$$

The ID for the String 010000

$(q_0, \underline{\epsilon}10000, z_0) \rightarrow (q_1, \underline{0}10000, Sz_0) \rightarrow (q_1, 0\underline{1}0000, BBz_0) \rightarrow (q_1, 01\underline{0}000, SBz_0) \rightarrow (q_1, 010\underline{0}00, BBBz_0) \rightarrow (q_1, 0100\underline{0}0, BBz_0) \rightarrow (q_1, 01000\underline{0}, Bz_0) \rightarrow (q_1, 010000\epsilon, z_0) \rightarrow (q_f, 010000\epsilon, z_0)$ (Accepted by the final state).

13. Show that the language $L = \{0^n1^n \mid n \geq 1\} \bigcup \{0^n1^{2n} \mid n \geq 1\}$ is a context-free language that is not accepted by any DPDA. [UPTU 2005]

Solution: The context-free grammar for the language is

$$S \rightarrow S_1 \mid S_2$$

$$S_1 \rightarrow 0S_11 \mid 01$$

$$S_2 \rightarrow 0S_211 \mid 011$$

The GNF equivalent to the grammar is

$$S \rightarrow 0S_1A \mid 0A \mid 0S_2A \mid 0AA$$

$$A \rightarrow 1$$

The transitional functions of the PDA equivalent to the grammar are

$$\delta(q_0, \epsilon, z_0) \rightarrow (q_1, Sz_0)$$

$$\delta(q_1, 0, S) \rightarrow (q_1, S_1A)$$

$$\delta(q_1, 0, S) \rightarrow (q_1, A)$$

$$\delta(q_1, 0, S) \rightarrow (q_1, S_2A)$$

$$\delta(q_1, 0, S) \rightarrow (q_1, AA)$$

$$\delta(q_1, 1, A) \rightarrow (q_1, \lambda)$$

$$\delta(q_1, \lambda, z_0) \rightarrow (q_f, z_0) \text{ // accepted by the final state}$$

$$\delta(q_1, \lambda, z_0) \rightarrow (q_1, \lambda) \text{ // accepted by the empty stack}$$

The PDA is an NPDA, as for the combination $(q_1, 0, S)$, there are four transitional functions.

14. Convert the CFG into an equivalent PDA. [Cochin University 2006]

$$S \rightarrow aAA$$

$$A \rightarrow aS \mid bS \mid a$$

Solution: The grammar is in GNF.

First, the start symbol S is pushed into the stack by the following production

$$\delta(q_0, \epsilon, z_0) \rightarrow (q_1, Sz_0)$$

For the production $S \rightarrow aAA$, the transitional function is

$$\delta(q_1, a, S) \rightarrow (q_1, AA)$$

For the production $A \rightarrow aS$, the transitional function is

$$\delta(q_1, a, A) \to (q_1, S)$$

For the production A $\to$ bS, the transitional function is

$$\delta(q_1, b, S) \to (q_1, S)$$

For the production A $\to$ a, the transitional function is

$$\delta(q_1, a, A) \to (q_1, \lambda)$$

For acceptance, the transitional functions are

$$\delta(q_1, \lambda, z_0) \to (q_f, z_0) \text{ // accepted by the final state}$$

$$\delta(q_1, \lambda, z_0) \to (q_1, \lambda) \text{ // accepted by the empty stack}$$

15. Construct a PDA equivalent to the grammar

$$S \to aAA$$

$$A \to aS \mid b$$

[Andhra University 2007]
Solution: The grammar is in GNF.

First, the start symbol S is pushed into the stack by the following production

$$\delta(q_0, \epsilon, z_0) \to (q_1, Sz_0)$$

For the production S $\to$ aAA, the transitional function is

$$\delta(q_1, a, S) \to (q_1, AA)$$

For the production A $\to$ aS, the transitional function is

$$\delta(q_1, a, A) \to (q_1, S)$$

For the production A $\to$ b, the transitional function is

$$\delta(q_1, b, A) \to (q_1, \lambda)$$

For acceptance, the transitional functions are

$$\delta(q_1, \lambda, z_0) \to (q_f, z_0) \text{ // accepted by the final state}$$

$$\delta(q_1, \lambda, z_0) \to (q_1, \lambda) \text{ // accepted by the empty stack}$$

16. Construct a PDA equivalent to the following grammar. [JNTU 2008]

$$S \to aBc$$

$$A \to abc$$

$$B \to aAb$$

$$C \to AB$$

$$C \to c$$

Solution: The grammar is not in GNF. The grammar is converted into GNF by replacing c by C , adding the production D $\to$ b, and replacing b by D and replacing A of C $\to$ ABby aDC. The final grammar is

$$S \to aBC$$

$$A \to aDC$$

$$B \to aAD$$

$$C \to aDCB$$

$$C \to c$$

$$D \to b$$

(Now convert it to an equivalent PDA.)

17. Convert the PDA

$$P = (\{p, q\}, \{0, 1\}, (x, z_0), \delta, q, z_0)$$

to a CFG, if λ is given as

$$\delta(q, 1, z_0) \rightarrow (q, xz_0) \text{ [UPTU 2005]}$$

Solution: The PDA contains two states, p and q. Thus, the following two production rules are added to the grammar.

$$S \rightarrow (q\ z_0\ q) \mid (q\ z_0\ p)$$

For the transitional function $\delta(q, 1, z_0) \rightarrow (q, xz_0)$, the production rules are

$$\delta(q, 1, z_0) \rightarrow (q, xz_0)$$

$$(q\ z_0\ q) \rightarrow 1(q\ x\ q)\ (\ q\ z_0\ q)$$

$$(q\ z_0\ q) \rightarrow 1(q\ x\ p)\ (\ p\ z_0\ q)$$

$$(q\ z_0\ q) \rightarrow 1(q\ x\ q)\ (\ q\ z_0\ p)$$

$$(q\ z_0\ q) \rightarrow 1(q\ x\ p)\ (\ p\ z_0\ p)$$