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Num PDE

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Homework #1

Question 1. Exercise 8.8.

In the following proof, $||\cdot||_2$ is denoted by $||\cdot||$. According to **Definition 8.4** and **Lemma 8.5**, we have

$$\frac{||\mathbf{e}||}{||\mathbf{u}||} = \frac{||A^{-1}\mathbf{r}||}{||A^{-1}\mathbf{f}||} \tag{1}$$

Since

$$A^{-1}\mathbf{r} = A^{-1} \cdot \mathbf{r}, \quad r = A \cdot A^{-1}\mathbf{r}, \tag{2}$$

we have

$$\frac{||\mathbf{r}||}{||A||} \leq ||A^{-1}\mathbf{r}|| \leq ||A^{-1}\mathbf{r}|| \tag{3}$$

Similarly,

$$\frac{||\mathbf{f}||}{||A||} \leq ||A^{-1}\mathbf{f}|| \leq ||A^{-1}\mathbf{f}|| \tag{4}$$

Substitute 3 and 4 into 1, then we have

$$\frac{1}{\mathrm{cond}(A)} \frac{\|\mathbf{r}\|_2}{\|\mathbf{f}\|_2} \leq \frac{\|\mathbf{e}\|_2}{\|\mathbf{u}\|_2} \leq \mathrm{cond}(A) \frac{\|\mathbf{r}\|_2}{\|\mathbf{f}\|_2} \tag{5}$$

Question 2. Exercise 8.9.

According to **Example 8.2**, we have

$$\begin{aligned} \mathrm{cond}(A) &= \frac{\max \lambda_k(A)}{\min \lambda_k(A)} = \frac{\sin^2 \frac{n-1}{2n}\pi}{\sin^2 \frac{1}{2n}\pi} \\ &= \frac{1}{\tan^2 \frac{1}{2n}\pi} \approx \frac{4n^2}{\pi^2} \end{aligned} \tag{6}$$

For $n = 8$, we have

$$\mathrm{cond}(A) \approx \frac{256}{\pi^2} \tag{7}$$

For $n = 1024$, we have

$$\mathrm{cond}(A) \approx \frac{4194304}{\pi^2} \tag{8}$$

Question 3. Exercise 8.