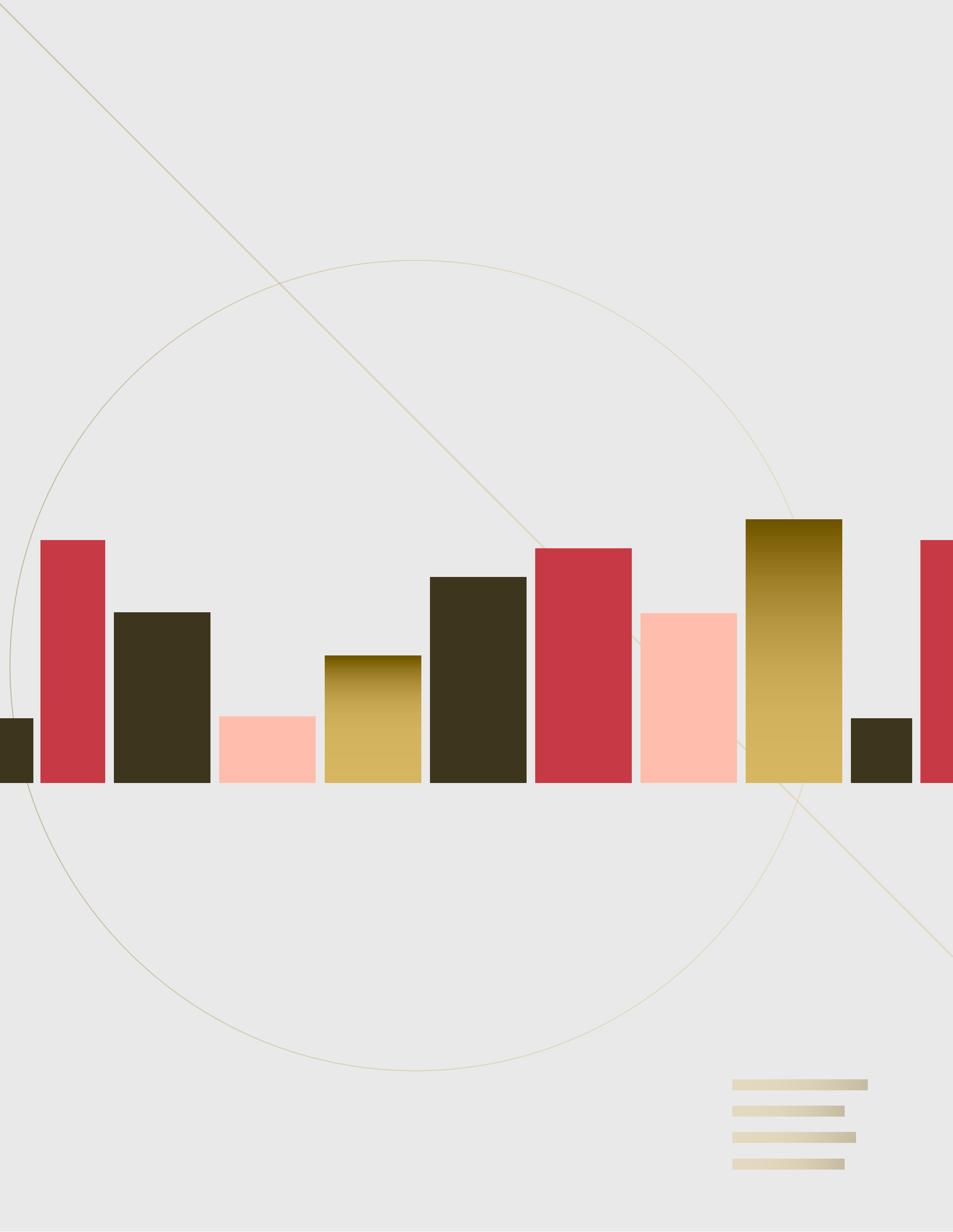
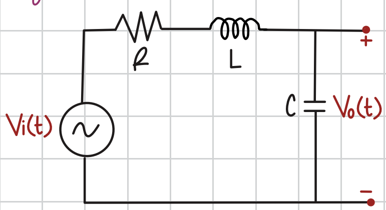


Ejercicios.



$R = 1K\Omega$

$L = 180\,mH$

$C = 120\,\mu F$

① Hallar E.D.O

Si $V_o(t) = V_c(t) = \frac{1}{C}\int_0^t i(\tau)\partial\tau$

$i_c(t) = C\frac{\partial}{\partial t}V_c(t)$

$V_L(t) = L\frac{\partial}{\partial t}i_L(t)$

$V_R(t) = R\,i_R(t)$

Como están en serie: $i(t) = i_R(t) = i_L(t) = i_c(t)$

$V_i(t) = V_R(t) + V_L(t) + V_c(t)$

$V_i(t) = R\cdot C\frac{\partial}{\partial t}V_c(t) + L\frac{\partial}{\partial t}\left(C\frac{\partial}{\partial t}V_c(t)\right) + V_c(t)$

$V_i(t) = RC\frac{\partial}{\partial t}V_c(t) + LC\frac{\partial^2}{\partial t^2}V_c(t) + V_c(t)$

$V_i(t) = LC\frac{\partial^2}{\partial t^2}V_c(t) + RC\frac{\partial}{\partial t}V_c(t) + V_c(t)$ → E.D.O del circuito RLC

② Hallar la función de transferencia y diagrama de bode [dB]

$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{V_o(\omega)}{V_i(\omega)}$

$V_o(t) = V_c(t) = \frac{1}{C}\int_0^t i(\tau)\partial\tau$

$V_i(t) = LC\frac{\partial^2}{\partial t^2}V_c(t) + RC\frac{\partial}{\partial t}V_c(t) + V_c(t)$

$F\left\{LC\frac{\partial^2}{\partial t^2}V_c(t)\right\} = LC\,F\left\{\frac{\partial^2}{\partial t^2}V_c(t)\right\} = LC(j\omega)^2V_c(\omega)$

$F\left\{RC\frac{\partial}{\partial t}V_c(t)\right\} = RC\,F\left\{\frac{\partial}{\partial t}V_c(t)\right\} = RC\,j\omega V_c(\omega)$

$F\{V_c(t)\} = V_c(\omega)$

$V_i(\omega) = LC(j\omega)^2V_c(\omega) + RC(j\omega)V_c(\omega) + V_c(\omega)$

Si $V_o(t) = V_c(t)$

$V_i(\omega) = \left(LC(j\omega)^2 + RC(j\omega) + 1\right)V_o(\omega)$

$$\frac{1}{\left(LC(j\omega)^2 + RC(j\omega) + 1\right)} = \frac{V_o(\omega)}{V_i(\omega)} = H(\omega)$$

$$\frac{1}{\left(-LC\omega^2 + RC(j\omega) + 1\right)} = H(\omega)$$

$$\frac{1}{(1 - LC\omega^2) + jRC\omega} = H(\omega)$$

$$H(\omega) = \frac{1}{(1 - LC\omega^2) + jRC\omega}$$

$$H(\omega) = \frac{1}{(1 - LC\omega^2) + jRC\omega}$$ función de transferencia

Para el diagrama de bode [dB]

$$|H(\omega)| = \frac{1}{\sqrt{(1 - LC\omega^2)^2 + (RC\omega)^2}}$$

Magnitud en dB

$$|H(\omega)|_{dB} = 20\log_{10}\left(\frac{1}{\sqrt{(1 - LC\omega^2)^2 + (RC\omega)^2}}\right)$$

③ Hallar $H(s)=\frac{Y(s)}{X(s)}=\frac{V_o(s)}{V_i(s)}$ y diagrama de polos y ceros

$V_i(t)=LC\frac{\partial^2}{\partial t^2}V_c(t)+RC\frac{\partial}{\partial t}V_c(t)+V_c(t)$

Ahora en el dominio de s

$\mathcal{L}\{V_i(t)\}=V_i(s)$

$\mathcal{L}\{LC\frac{\partial^2}{\partial t^2}V_c(t)\}=LCs^2V_c(s)$

$\mathcal{L}\{RC\frac{\partial}{\partial t}V_c(t)\}=RCsV_c(s)$

$\mathcal{L}\{V_c(t)\}=V_c(s)$

$V_i(s)=LCs^2V_c(s)+RCsV_c(s)+V_c(s)$

Si $V_o(t)=V_c(t)$

$V_i(s)=(LCs^2+RCs+1)V_o(s)$

$$\frac{1}{(LCs^2+RCs+1)}=\frac{V_o(s)}{V_i(s)}$$

$$H(s)=\frac{1}{(LCs^2+RCs+1)}$$ → función de transferencia

Ceros → Raices del numerador (Son las s donde $H(s)=0$)

Polos → Raices del denominador (Son las s donde $H(s)\to\infty$)

- No hay ceros
- Polos: $LCs^2+RCs+1=0$

$2{,}16\times10^{-5}\cdot s^2+0{,}12\cdot s+1=0$

$$s_{1,2}=\frac{-0{,}12\pm\sqrt{(0{,}12)^2-4\cdot 2{,}16\times10^{-5}}}{2\cdot 2{,}16\times10^{-5}}$$

$s_1=-8{,}3459$

$s_2=-5547{,}2097$

Polos en $-8{,}3459$ y $-5547{,}2097$

④ Hallar $h(t)$ desde ③ $X(t)=\delta(t)$ $X(s)=\mathcal{L}\{\delta(t)\}$

$H(s)=Y(s)$ $h(t)=\mathcal{L}^{-1}\{H(s)\}$

Si $X(s)=\mathcal{L}\{\delta(t)\}=1$

$$H(s)=\frac{Y(s)}{X(s)}=\frac{Y(s)}{1}=Y(s)$$

$s_1=-8{,}3459$

$s_2=-5547{,}2097$

$$H(s)=\frac{1}{2{,}16\times10^{-5}\cdot s^2+0{,}12\cdot s+1}=\frac{1}{2{,}16\times10^{-5}\cdot(s+8{,}3459)(s+5547{,}2097)}=\frac{1}{2{,}16\times10^{-5}}\left(\frac{A}{s+8{,}3459}+\frac{B}{s+5547{,}2097}\right)$$

$1=A(s+5547{,}2097)+B(s+8{,}3459)$

Si $s=s_1$ $A=\frac{1}{-8{,}3459+5547{,}2097}=\frac{1}{5538{,}8638}=1{,}80542\times10^{-4}$

$s=s_2$ $B=\frac{1}{-5547{,}2097+8{,}3459}=-A=-1{,}80542\times10^{-4}$

$$H(s)=\frac{1}{2{,}16\times10^{-5}}\left(\frac{A}{s+8{,}3459}+\frac{-A}{s+5547{,}2097}\right)=A\frac{1}{2{,}16\times10^{-5}}\left(\frac{1}{s+8{,}3459}-\frac{1}{s+5547{,}2097}\right)$$

$\mathcal{L}^{-1}\{\frac{1}{s-a}\}=e^{at}u(t)$

$$h(t)=\mathcal{L}^{-1}\{H(s)\}=A\frac{1}{2{,}16\times10^{-5}}\left(\mathcal{L}^{-1}\left[\frac{1}{s+8{,}3459}\right]-\mathcal{L}^{-1}\left[\frac{1}{s+5547{,}2097}\right]\right)=\frac{1{,}80542\times10^{-4}}{2{,}16\times10^{-5}}\left(e^{-8{,}3459}u(t)-e^{-5547{,}2097}u(t)\right)$$

$$=8{,}3584\,u(t)\left(e^{0{,}3459}-e^{-5547{,}2097}\right)$$

⑤ Hallar $h(t)$ desde ③ $X(t)=\delta(t)$ $X(s)=\mathcal{L}\{\delta(t)\}$

$H(s)=Y(s)$ $h(t)=\mathcal{L}^{-1}\{H(s)\}$

$$Y(s)=H(s)X(s)=H(s)\mathcal{L}\{u(t)\}$$

$$Y(t)=\mathcal{L}^{-1}\{H(s)\mathcal{L}\{u(t)\}\} \ ; \text{Si } X(s)=\mathcal{L}\{u(t)\}=\frac{1}{s} \text{ y } H(s)=\frac{1}{2{,}16\times10^{-5}\cdot(s+8{,}3459)(s+5547{,}2097)}$$

$$Y(s)=\frac{1}{s}\cdot\frac{1}{2{,}16\times10^{-5}\cdot(s+8{,}3459)(s+5547{,}2097)}=\frac{1}{2{,}16\times10^{-5}}\left(\frac{A}{s}+\frac{B}{s+8{,}3459}+\frac{C}{s+5547{,}2097}\right)$$

$$A=\frac{1}{1}=1 \quad B=\frac{8{,}3584}{-8{,}3459}\cong-1 \quad C=\frac{-8{,}3584}{-5547{,}2097}\cong0{,}001057$$

$$Y(t)=\mathcal{L}^{-1}\left\{\frac{1}{2{,}16\times10^{-5}}\left(\frac{1}{s}-\frac{1}{s+8{,}3459}+\frac{0{,}001057}{s+5547{,}2097}\right)\right\}$$

$$Y(t)=\frac{1}{2{,}16\times10^{-5}}\cdot u(t)\left(e^{0}-e^{-8{,}3459}+0{,}001057e^{-5547{,}2097}\right)$$