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1. La distancia media entre dos señales periódicas $x_1(t) \in \mathbb{R},\mathbb{C}$ y $x_2(t) \in \mathbb{R},\mathbb{C}$; se puede expresar a partir de la potencia media de la diferencia entre ellas:

$$d^2(x_1,x_2) = \bar{P}_{x_1-x_2} = \lim_{T\to\infty} \frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\, dt.$$

Sea $x_1(t)$ y $x_2(t)$ dos señales definidas como:

$$x_1(t) = Ae^{-jnw_0t}$$

$$x_2(t) = Be^{jmw_0t}$$

con $w_0 = \frac{2\pi}{T}$; $T, A, B \in \mathbb{R}^+$ y $n, m \in \mathbb{Z}$. Determine la distancia entre las dos señales. Compruebe sus resultados con Python.

$x_1(t)=Ae^{-jn\omega_0 t}$, $x_2(t)=Be^{jm\omega_0 t}$; $\omega_0=\frac{2\pi}{T}$

$d^2(x_1,x_2)=\bar{P}_{x_1-x_2}=\lim\limits_{T\to\infty}\frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\,dt$

$$\begin{aligned}
\bar{P}_{x_1-x_2} &= \frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\,dt \\
&= \frac{1}{T}\left(\int_T (x_1(t)-x_2(t))(x_1(t)-x_2(t))^*\right)dt \\
&= \frac{1}{T}\left(\int_T (x_1(t)-x_2(t))(x_1^*(t)-x_2^*(t)\right)dt \\
&= \frac{1}{T}\left(\int_T x_1(t)x_1^*(t)\,dt - \int_T x_1(t)x_2^*(t)\,dt\right. \\
&\qquad \left. -\int_T x_2(t)x_1^*(t)\,dt + \int_T x_2(t)x_2^*(t)\,dt\right) \\
&= \underbrace{\frac{1}{T}\int |x_1(t)|^2 dt}_{\bar{P}_{x_1}} \underbrace{- \frac{2}{T}\int_T x_1(t)x_2^*(t)\,dt}_{C_x} + \underbrace{\frac{1}{T}\int_T |x_2(t)|^2 dt}_{\bar{P}_{x_2}}
\end{aligned}$$

$$\begin{aligned}
\bar{P}_{x_1} &= \frac{1}{T}\int_T x_1(t)x_1^*(t)\,dt = \frac{1}{T}\int_0^T Ae^{-jn\omega_0 t}\left(Ae^{-jn\omega_0 t}\right)^* = \frac{1}{T}\int_0^T Ae^{jn\omega_0 t}Ae^{-jn\omega_0 t}\,dt \\
&= \frac{A^2}{T}\int_0^T e^{\overbrace{(n-n)}^{0}j\omega_0 t}\,dt = \frac{A^2}{T}\int_0^T 1\,dt = \frac{A^2}{T}(T-0) = \frac{A^2}{\cancel{T}}\cdot\cancel{T} = A^2
\end{aligned}$$

$$\begin{aligned}
\bar{P}_{x_2} &= \frac{1}{T}\int_T x_2(t)x_2^*(t)\,dt = \frac{1}{T}\int_0^T Be^{jm\omega_0 t}\left(Be^{jm\omega_0 t}\right)^* = \frac{1}{T}\int_0^T Be^{jm\omega_0 t}Be^{-jm\omega_0 t}\,dt \\
&= \frac{B^2}{T}\int_0^T e^{\overbrace{(m-m)}^{0}j\omega_0 t}\,dt = \frac{B^2}{T}\int_0^T 1\,dt = \frac{B^2}{T}(T-0) = \frac{B^2}{\cancel{T}}\cdot\cancel{T} = B^2
\end{aligned}$$

El cruzado tiene dos casos:

① Si $n \neq -m \rightarrow$

$$\begin{aligned}
C_x &= -\frac{2}{T}\int_T x_1(t)x_2^*(t)\,dt = -\frac{2}{T}\int_0^T Ae^{-jn\omega_0 t}\left(Be^{jm\omega_0 t}\right)^* dt \\
&= -\frac{2}{T}\int_0^T Ae^{-jn\omega_0 t}Be^{-jm\omega_0 t}\,dt = -\frac{2AB}{T}\int_0^T e^{-(n+m)j\omega_0 t}\,dt \\
&= -\frac{2AB}{T}\int_0^T e^{-rj\omega_0 t}\,dt = -\frac{j\cdot 2AB}{j\cdot -jr\omega_0 T}e^{-jr\omega_0 t}\Big|_0^T = -\frac{j2AB}{r\frac{2\pi}{\cancel{T}}\cancel{T}}\left(e^{-jr\frac{2\pi}{\cancel{T}}\cdot\cancel{T}} - e^{\overbrace{-jr\frac{2\pi}{T}\cdot 0}^{0}}\right) \\
&= -\frac{jAB}{r\pi}\left(e^{-jr2\pi}-1\right) = -\frac{jAB}{\pi r}\left(\overbrace{\cos(2\pi r)}^{1} - j\,\overbrace{\mathrm{sen}(2\pi r)}^{0} - 1\right) \\
&= \frac{jAB}{\pi r}(1-0-1) = \frac{jAB}{\pi r}\cdot 0 = 0
\end{aligned}$$

Si $n, m \in \mathbb{Z} \rightarrow n+m = r \in \mathbb{Z}$

② Si $n = -m \rightarrow$

$$\begin{aligned}
C_x &= -\frac{2}{T}\int_T x_1(t)x_2^*(t)\,dt = -\frac{2}{T}\int_0^T Ae^{-jn\omega_0 t}\left(Be^{jm\omega_0 t}\right)^* dt = -\frac{2}{T}\int_0^T Ae^{-j(-m)\omega_0 t}Be^{-jm\omega_0 t}\,dt \\
&= -\frac{2}{T}\int_0^T Ae^{jm\omega_0 t}Be^{-jm\omega_0 t}\,dt = -\frac{2AB}{T}\int_0^T e^{\overbrace{(m-m)}^{0}j\omega_0 t}\,dt = -\frac{2AB}{T}\int_0^T 1\,dt \\
&= -\frac{2AB}{T}(T-0) = -\frac{2AB}{\cancel{T}}\cdot\cancel{T} = -2AB
\end{aligned}$$

$$\bar{P}_{x_1-x_2} = \begin{cases} A^2+B^2 & ; n \neq -m \\ A^2+B^2-2AB & ; n=-m \end{cases}$$

$$d(x_1,x_2) = \begin{cases} \sqrt{A^2+B^2} & ; n \neq -m \\ \sqrt{A^2+B^2-2AB} & ; n=-m \end{cases}$$

2. Encuentre la señal en tiempo discreto al utilizar un conversor análogo digital con frecuencia de muestreo de $5kHz$ y 4 bits de capacidad de representación, aplicado a la señal continua: $x(t) = 3\cos(1000\pi t) + 5\sin(3000\pi t) + 10\cos(11000\pi t)$. Realizar la simulación del proceso de discretización (incluyendo al menos tres periodos de $x(t)$). En caso de que la discretización no sea apropiada, diseñe e implemente un conversor adecuado para la señal estudiada.

$X(t)=3\cos(1000\pi t)+5\,Sen(3000\pi t)+10\cos(11000\pi t)$

$F_s=5\,KHz$, # de estados $=2^{4\,bits}=16$

$t=nT_s \longrightarrow F_s=\frac{1}{T_s}$; $F=\frac{1}{T} \longrightarrow T=\frac{2\pi}{\omega}$

$A\cos(\Omega n)$
$A\cos(2\pi f n)$ $\longrightarrow \Omega=2\pi f \longrightarrow f=\frac{T_s}{T}=\frac{F}{F_s}$

$X(t)=X_1(t)+X_2(t)+X_3(t)\rightarrow$

Para $X_1(t)=3\cos(1000\pi t)$

$\omega_1=1000\pi \longrightarrow T_1=\frac{2\pi}{\omega_1}=\frac{2\pi}{1000\pi}=\frac{1}{500} \longrightarrow F_1=\frac{1}{T}=\frac{1}{\frac{1}{500}}=500\,Hz \longrightarrow X_1[n]=A\cos\left(2\pi n\frac{F}{F_s}\right)=3\cos\left(2\pi\cdot n\cdot\frac{500}{5000}\right)=3\cos\left(\frac{n\pi}{5}\right)$

Para $X_2(t)=5\,Sen(3000\pi t)$

$\omega_2=3000\pi \longrightarrow T_2=\frac{2\pi}{\omega_1}=\frac{2\pi}{3000\pi}=\frac{1}{1500} \longrightarrow F_2=\frac{1}{T}=\frac{1}{\frac{1}{1500}}=1500\,Hz \longrightarrow X_2[n]=A\,sen\left(2\pi n\frac{F}{F_s}\right)=5\,Sen\left(2\pi\cdot n\cdot\frac{1500}{5000}\right)=5\,Sen\left(\frac{3n\pi}{5}\right)$

Para $X_3(t)=10\cos(11000\pi t)$

$\omega_3=11000\pi \longrightarrow T_3=\frac{2\pi}{\omega_1}=\frac{2\pi}{11000\pi}=\frac{1}{5500} \longrightarrow F_3=\frac{1}{T}=\frac{1}{\frac{1}{5500}}=5500\,Hz \longrightarrow X_3[n]=A\cos\left(2\pi n\frac{F}{F_s}\right)=10\cos\left(2\pi\cdot n\cdot\frac{5500}{5000}\right)=10\cos\left(\frac{11n\pi}{5}\right)$

$X[n]=3\cos\left(\frac{n\pi}{5}\right)+5\,Sen\left(\frac{3n\pi}{5}\right)+10\cos\left(\frac{11n\pi}{5}\right)$

$X[n]=3\cos\left(\frac{\pi}{5}n\right)+5\,Sen\left(\frac{\pi}{5}n\right)+10\cos\left(\frac{\pi}{5}n\right)$

$X[n]=13\cos\left(\frac{\pi}{5}n\right)+5\,Sen\left(\frac{\pi}{5}n\right)$

$\frac{\omega_1}{\omega_2}=\frac{1000\pi}{3000\pi}=\frac{1}{3}\in Q$

$\frac{\omega_1}{\omega_3}=\frac{1000\pi}{11000\pi}=\frac{1}{11}\in Q$

$\frac{\omega_2}{\omega_3}=\frac{3000\pi}{11000\pi}=\frac{3}{11}\in Q$

$\hookrightarrow$ Como todas son racionales la señal es cuasiperiodica.

Si $F_s\geq 2F_{max} \longrightarrow 2F_{max}=2\cdot 5500\,Hz=11000\,Hz \longrightarrow F_s=5000\,Hz$.

$\downarrow$ No cumple Nyquist

Vamos a comparar con las originales. $\longrightarrow -\pi\leq\Omega<\pi$

$\Omega_1=\frac{1}{5}\pi$ ✓ , $\Omega_2=\frac{3}{5}\pi$ ✓ , $\Omega_3=\frac{11}{5}\pi$ ✗ $\longrightarrow$ Es una copia / aliasings

$\Omega_{3\,original}=\frac{11}{5}\pi-2\pi=\frac{1}{5}\pi$

Suponemos una frecuencia de muestreo mucho más grande

$F_s=4F_{max}\longrightarrow F_s=4(5500)\,Hz=22\,KHz$ $\qquad T_s=\frac{1}{22000}$

Para $X_1(t)=3\cos(1000\pi t)$

$\omega_1=1000\pi \longrightarrow T_1=\frac{2\pi}{\omega_1}=\frac{2\pi}{1000\pi}=\frac{1}{500} \longrightarrow F_1=\frac{1}{T}=\frac{1}{\frac{1}{500}}=500\,Hz \longrightarrow X_1[n]=A\cos\left(2\pi n\frac{F}{F_s}\right)=3\cos\left(2\pi\cdot n\cdot\frac{500}{22000}\right)=3\cos\left(\frac{\pi}{22}n\right)$

Para $X_2(t)=5\,Sen(3000\pi t)$

$\omega_2=3000\pi \longrightarrow T_2=\frac{2\pi}{\omega_1}=\frac{2\pi}{3000\pi}=\frac{1}{1500} \longrightarrow F_2=\frac{1}{T}=\frac{1}{\frac{1}{1500}}=1500\,Hz \longrightarrow X_2[n]=A\,sen\left(2\pi n\frac{F}{F_s}\right)=5\,Sen\left(2\pi\cdot n\cdot\frac{1500}{22000}\right)=5\,Sen\left(\frac{3\pi}{22}n\right)$

Para $X_3(t)=10\cos(11000\pi t)$

$\omega_3=11000\pi \longrightarrow T_3=\frac{2\pi}{\omega_1}=\frac{2\pi}{11000\pi}=\frac{1}{5500} \longrightarrow F_3=\frac{1}{T}=\frac{1}{\frac{1}{5500}}=5500\,Hz \longrightarrow X_3[n]=A\cos\left(2\pi n\frac{F}{F_s}\right)=10\cos\left(2\pi\cdot n\cdot\frac{5500}{22000}\right)=10\cos\left(\frac{\pi}{2}n\right)$

$X[n]=3\cos\left(\frac{\pi}{22}n\right)+5\,Sen\left(\frac{3\pi}{22}n\right)+10\cos\left(\frac{\pi}{2}n\right)$

Vamos a comparar con las originales. $\longrightarrow -\pi\leq\Omega<\pi$

$\Omega_1=\frac{1}{22}\pi$ ✓ , $\Omega_2=\frac{3}{22}\pi$ ✓ , $\Omega_3=\frac{1}{22}\pi$ ✓ $\longrightarrow$ las tres frecuencias digitales están en los originales.

$X[n]=3\cos\left(\frac{\pi}{22}n\right)+5\,Sen\left(\frac{3\pi}{22}n\right)+10\cos\left(\frac{\pi}{2}n\right)$

Para saber el periodo a gráficar:

$$T = r\frac{2\pi}{w_1} = l\frac{2\pi}{w_2} = m\frac{2\pi}{w_3}$$

$$T = r\frac{2\cancel{\pi}}{1000\cancel{\pi}} = l\frac{2\cancel{\pi}}{3000\cancel{\pi}} = m\frac{2\cancel{\pi}}{11000\cancel{\pi}}$$

$$T = r\frac{1}{500} = l\frac{1}{1500} = m\frac{1}{5500}$$

$$5500T = r\frac{5500}{500} = l\frac{5500}{1500} = m\frac{\cancel{5500}}{\cancel{5500}}$$

$$5500T = r\,11 = l\,\frac{11}{3} = m$$

$$3\cdot 5500T = r\cdot 3\cdot 11 = l\cdot\cancel{3}\cdot\frac{11}{\cancel{3}} = m\cdot 3$$

$$16500T = 33r = 11l = 3m \qquad \text{mcm}\{33, 1, 11\} = 33$$

$$16500\,T = 33$$

$$T = \frac{33}{16500}$$

$$T = \frac{1}{500}$$

3. Sea $x''(t)$ la segunda derivada de la señal $x(t)$, donde $t \in [t_i, t_f]$. Demuestre que los coeficientes de la serie exponencial de Fourier se pueden calcular según:

$$c_n = \frac{1}{(ti - tf)n^2 w_o^2}\int_{t_i}^{t_f} x''(t) e^{-jnw_o t}dt; \quad n \in \mathbb{Z}.$$

¿Cómo se pueden calcular los coeficientes a_n y b_n desde $x''(t)$ en la serie trigonométrica de Fourier?.

Con $X(t)$ para $t \in [t_i, t_f]$

Demostrar: $C_n = \frac{1}{(t_i-t_f)n^2W_0^2}\int_T X''(t)e^{-jnW_0t}\partial t$ con $n \in \mathbb{Z}$

Tenemos: $X(t) = \sum_n C_n e^{jnW_0t}$

$$X'(t) = \frac{\partial}{\partial t}\sum_n C_n e^{jnW_0t} = \sum_n C_n \frac{\partial e^{jnW_0t}}{\partial t}$$

$$X''(t) = \frac{\partial X'(t)}{\partial t} = \frac{\partial}{\partial t}\sum_n C_n \frac{\partial e^{jnW_0t}}{\partial t} = \sum_n C_n \frac{\partial^2 e^{jnW_0t}}{\partial t^2}$$

$$\frac{\partial e^{jnW_0t}}{\partial t} = jnW_0 e^{jnW_0t}; \quad \frac{\partial^2 e^{jnW_0t}}{\partial t^2} = \frac{\partial (jnW_0 e^{jnW_0t})}{\partial t} = jnW_0 \cdot jnW_0 e^{jnW_0t} = -n^2W_0^2 e^{jnW_0t}$$

$$X''(t) = \sum_n C_n \frac{\partial^2 e^{jnW_0t}}{\partial t^2} = \sum_n \underbrace{-C_n n^2 W_0^2}_{C\_n} e^{jnW_0t} = \sum_n C\_n e^{jnW_0t}; \text{ con } C\_n = -C_n n^2 W_0^2$$

Si $C_n = \frac{1}{T}\int_T X(t)e^{-jnW_0t}\partial t$ y $C\_n = \frac{1}{T}\int_T X''(t)e^{-jnW_0t}\partial t \longrightarrow$ Remplazamos el $C\_n \longrightarrow -C_n n^2 W_0^2 = \frac{1}{T}\int_T X''(t)e^{-jnW_0t}\partial t$

$$C_n = \frac{1}{-(t_f-t_i)n^2W_0^2}\int_T X''(t)e^{-jnW_0t}\partial t$$

$$C_n = \frac{1}{(t_i-t_f)n^2W_0^2}\int_T X''(t)e^{-jnW_0t}\partial t$$

$$C_n = \frac{1}{(t_i-t_f)n^2W_0^2}\int_T X''(t)e^{-jnW_0t}\partial t = \frac{1}{(t_i-t_f)n^2W_0^2}\int_T X''(t)(\cos(nW_0t) - j\,\text{sen}(nW_0t))\partial t$$

$$= \frac{1}{(t_i-t_f)n^2W_0^2}\int_T X''(t)\cos(nW_0t)\partial t - j\frac{1}{(t_i-t_f)n^2W_0^2}\int_T X''(t)\,\text{sen}(nW_0t)\partial t$$

Sabiendo que: $a_n = \frac{2}{T}\int_T X(t)\cos(nW_0t)\partial t$; $b_n = \frac{2}{T}\int_T X(t)\,\text{sen}(nW_0t)\partial t$ y $a_n = 2\,\text{Re}\{C_n\}$; $b_n = -2\,\text{Im}\{C_n\}$

$$a_n = 2\,\text{Re}\{C_n\} = 2\frac{1}{(t_i-t_f)n^2W_0^2}\int_T X''(t)\cos(nW_0t)\partial t$$

$$b_n = -2\,\text{Im}\{C_n\} = -2\cdot -\frac{1}{(t_i-t_f)n^2W_0^2}\int_T X''(t)\,\text{sen}(nW_0t)\partial t = 2\frac{1}{(t_i-t_f)n^2W_0^2}\int_T X''(t)\,\text{sen}(nW_0t)\partial t$$

También se pueden calcular con: $X(t) = \sum_n a_n\cos(nW_0t) + b_n\,\text{Sen}(nW_0t)$

$$X(t) = \sum_n a_n\cos(nW_0t) + b_n\,\text{Sen}(nW_0t)$$

$$X'(t) = \sum_n -a_n\,\text{Sen}(nW_0t)nW_0 + b_n\cos(nW_0t)nW_0$$

$$X''(t) = \sum_n -a_n\cos(nW_0t)nW_0\cdot nW_0 - b_n\,\text{Sen}(nW_0t)nW_0\cdot nW_0 = \sum_n \underbrace{-a_n n^2 W_0^2}_{a\_n}\cos(nW_0t) \underbrace{- b_n n^2 W_0^2}_{b\_n}\,\text{Sen}(nW_0t)$$

$$a\_n = -a_n n^2 W_0^2, \quad b\_n = -b_n n^2 W_0^2$$

$$a_n = \frac{2}{T}\int_T X(t)\cos(nW_0t)\partial t \longrightarrow a\_n = \frac{2}{T}\int_T X''(t)\cos(nW_0t)\partial t$$

$$-a_n n^2 W_0^2 = \frac{2}{T}\int_T X''(t)\cos(nW_0t)\partial t$$

$$a_n = \frac{2}{-(t_f-t_i)n^2W_0^2}\int_T X''(t)\cos(nW_0t)\partial t$$

$$a_n = \frac{2}{(t_i-t_f)n^2W_0^2}\int_T X''(t)\cos(nW_0t)\partial t$$

$$b_n = \frac{2}{T}\int_T X(t)\,\text{Sen}(nW_0t)\partial t \longrightarrow b\_n = \frac{2}{T}\int_T X''(t)\,\text{Sen}(nW_0t)\partial t$$

$$-b_n n^2 W_0^2 = \frac{2}{T}\int_T X''(t)\,\text{Sen}(nW_0t)\partial t$$

$$b_n = \frac{2}{-(t_f-t_i)n^2W_0^2}\int_T X''(t)\,\text{Sen}(nW_0t)\partial t$$

$$b_n = \frac{2}{(t_i-t_f)n^2W_0^2}\int_T X''(t)\,\text{Sen}(nW_0t)\partial t$$

4. Encuentre el espectro de Fourier, su parte real, imaginaria, magnitud, fase y el error relativo para $n \in \{0, \pm1, \pm2, \pm3, \pm4, \pm5\}$, a partir de $x''(t)$ para la señal $x(t)$ en la Figura 1. Compruebe el espectro obtenido con la estimación a partir de $x(t)$. Presente las simulaciones de Python respectivas.

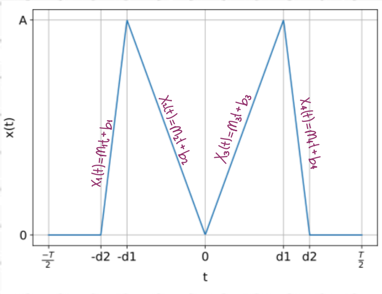


Figura 1: $x(t)$ para el ejercicio 1.4

$$X(t)=\begin{cases} 0 & -\frac{T}{2} \le t < -d_2 \\ \frac{A}{d_2-d_1}(t+d_2) & -d_2 \le t < -d_1 \\ \frac{-A}{d_1}t & -d_1 \le t < 0 \\ \frac{A}{d_1}t & 0 \le t < d_1 \\ -\frac{A}{d_2-d_1}(t+d_2) & d_1 \le t < d_2 \\ 0 & d_2 \le t < \frac{T}{2} \end{cases}$$

Para $X_1(t)=m_1t+b_1 \rightarrow m_1 = \frac{A-0}{-d_1-(-d_2)} = \frac{A}{d_2-d_1}$; En $t=-d_2 \rightarrow X(-d_2) = \frac{A}{d_2-d_1}\cdot -d_2 + b_1$

$$0 = \frac{-d_2 A}{d_2-d_1} + b_1$$

$$b_1 = \frac{d_2}{d_2-d_1}A$$

$$X_1(t) = \frac{A}{d_2-d_1}t + \frac{d_2}{d_2-d_1}A$$

$$X_1(t) = \frac{A}{d_2-d_1}(t+d_2)$$

Para $X_2(t)=m_2t+b_2 \rightarrow m_2 = \frac{0-A}{0-(-d_1)} = \frac{-A}{d_1}$; En $t=-d_1 \rightarrow X(-d_1) = \frac{-A}{d_1}\cdot -d_1 + b_2$

$$X_2(t) = \frac{-A}{d_1}t$$

$$A = A + b_2$$

$$b_2 = A - A = 0$$

Para $X_3(t)=m_3t+b_3 \rightarrow m_3 = \frac{A-0}{d_1-0} = \frac{A}{d_1}$; En $t=0 \rightarrow X(0) = \frac{A}{d_1}\cdot 0 + b_3$

$$X_3(t) = \frac{A}{d_1}t$$

$$0 = 0 + b_3$$

$$b_3 = 0$$

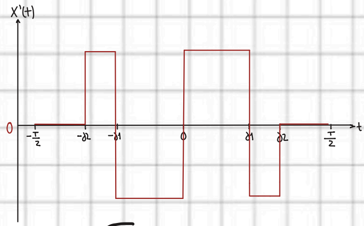
Para $X_4(t)=m_4t+b_4 \rightarrow m_4 = \frac{0-A}{d_2-d_1} = \frac{-A}{d_2-d_1}$; En $t=d_2 \rightarrow X(d_2) = \frac{-A}{d_2-d_1}d_2 + b_4$

$$X_4(t) = \frac{-A}{d_2-d_1}t + \frac{-Ad_2}{d_2-d_1}$$

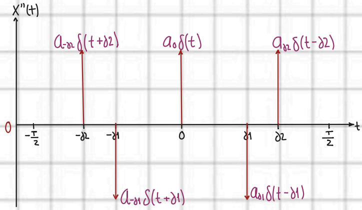
$$0 = \frac{-A}{d_2-d_1}\cdot d_2 + b_4$$

$$b_4 = \frac{-Ad_2}{d_2-d_1}$$

$$X_4(t) = -\frac{A}{d_2-d_1}(t+d_2)$$



$$X'(t)=\begin{cases} 0 & -\frac{T}{2} \le t < -d_2 \\ \frac{A}{d_2-d_1} & -d_2 \le t < -d_1 \\ \frac{-A}{d_1} & -d_1 \le t < 0 \\ \frac{A}{d_1} & 0 \le t < d_1 \\ -\frac{A}{d_2-d_1} & d_1 \le t < d_2 \\ 0 & d_2 \le t < \frac{T}{2} \end{cases}$$



$X''(t)=0$ en cada tramo y en los puntos donde hay cambios tiende a $\pm\infty$ y estos cambios los podemos representar como $a_k\delta(t \pm t_k)$ donde a_k es la diferencia de la derivada por la derecha y la derivada por la izquierda y t_k donde ocurre el salto (cambio instantáneo)

$$a_{-\partial 2} = X'(-\partial 2^{+}) - X'(-\partial 2^{-}) = m1 - 0 = \frac{A}{\partial 2-\partial 1}$$

$$a_{-\partial 1} = X'(-\partial 1^{+}) - X'(-\partial 1^{-}) = m2 - m1 = \frac{-A}{\partial 1} - \frac{A}{\partial 2-\partial 1} = -A\left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)$$

$$a_0 = X'(0^{+}) - X'(0^{-}) = m3 - m2 = \frac{A}{\partial 1} - \frac{-A}{\partial 1} = \frac{A}{\partial 1}(1+1) = \frac{2A}{\partial 1}$$

$$a_{\partial 1} = X'(\partial 1^{+}) - X'(\partial 1^{-}) = m4 - m3 = -\frac{A}{\partial 2-\partial 1} - \frac{A}{\partial 1} = -A\left(\frac{1}{\partial 2-\partial 1} + \frac{1}{\partial 1}\right)$$

$$a_{\partial 2} = X'(\partial 2^{+}) - X'(\partial 2^{-}) = 0 - m4 = 0 - -\frac{A}{\partial 2-\partial 1} = \frac{A}{\partial 2-\partial 1}$$

$$X''(t) = a_{-\partial 2}\,\delta(t+\partial 2) + a_{-\partial 1}\,\delta(t+\partial 1) + a_0\,\delta(t) + a_{\partial 1}\,\delta(t-\partial 1) + a_{\partial 2}\,\delta(t-\partial 2)$$

$$X''(t) = \frac{A}{\partial 2-\partial 1}\delta(t+\partial 2) - A\left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)\delta(t+\partial 1) + \frac{2A}{\partial 1}\delta(t) - A\left(\frac{1}{\partial 2-\partial 1} + \frac{1}{\partial 1}\right)\delta(t-\partial 1) + \frac{A}{\partial 2-\partial 1}\delta(t-\partial 2)$$

$$X''(t) = \frac{A}{\partial 2-\partial 1}\left(\delta(t+\partial 2) + \delta(t-\partial 2)\right) - A\left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)\left(\delta(t+\partial 1) + \delta(t-\partial 1)\right) + \frac{2A}{\partial 1}\delta(t)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\int_T X''(t)\,e^{-jn\omega_0 t}\,\partial t \qquad \text{con } T = t_f - t_i = \frac{T}{2} - \left(-\frac{T}{2}\right) = T \;\text{ y }\; \omega_0 = \frac{2\pi}{T}$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\int_{-T/2}^{T/2}\left(\frac{A}{\partial 2-\partial 1}\left(\delta(t+\partial 2) + \delta(t-\partial 2)\right) - A\left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)\left(\delta(t+\partial 1) + \delta(t-\partial 1)\right) + \frac{2A}{\partial 1}\delta(t)\right)e^{-jn\omega_0 t}\,\partial t$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\int_{-T/2}^{T/2}\frac{A}{2-\partial 1}\left(\delta(t+\partial 2) + \delta(t-\partial 2)\right)e^{-jn\omega_0 t}\,\partial t - \int_{-T/2}^{T/2} A\left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)\left(\delta(t+\partial 1) + \delta(t-\partial 1)\right)e^{-jn\omega_0 t}\,\partial t + \int_{-T/2}^{T/2}\frac{2A}{\partial 1}\delta(t)\,e^{-jn\omega_0 t}\,\partial t\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\int_{-T/2}^{T/2}\frac{A}{\partial 2-\partial 1}\delta(t+\partial 2)e^{-jn\omega_0 t}\,\partial t + \int_{-T/2}^{T/2}\frac{A}{\partial 2-\partial 1}\delta(t-\partial 2)e^{-jn\omega_0 t}\,\partial t - \int_{-T/2}^{T/2} A\left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)\delta(t+\partial 1)e^{-jn\omega_0 t}\,\partial t\right.$$

$$\left. - \int_{-T/2}^{T/2} A\left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)\delta(t-\partial 1)e^{-jn\omega_0 t}\,\partial t + \int_{-T/2}^{T/2}\frac{2A}{\partial 1}\delta(t)\,e^{-jn\omega_0 t}\,\partial t\right) \quad \text{Usando} \rightarrow \int_{-\infty}^{\infty} X(t)\,\delta(t \pm t_0)\,\partial t = X(\mp t_0)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\frac{A}{\partial 2-\partial 1}e^{-jn\omega_0\cdot -\partial 2} + \frac{A}{\partial 2-\partial 1}e^{-jn\omega_0\cdot\partial 2} - A\left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)e^{-jn\omega_0\cdot -\partial 1} - A\left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)e^{-jn\omega_0\cdot\partial 1} + \frac{2A}{\partial 1}\cdot\cancel{e^{0}}\,1\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\frac{A}{\partial 2-\partial 1}\left(\cos(n\omega_0\partial 2) + j\cancel{sen(n\omega_0\partial 2)} + \cos(n\omega_0\partial 2) - j\cancel{sen(n\omega_0\partial 2)}\right)\right.$$
$$\left. - A\left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)\left(\cos(n\omega_0\partial 1) + j\cancel{sen(n\omega_0\partial 1)} + \cos(n\omega_0\partial 1) - j\cancel{sen(n\omega_0\partial 1)}\right) + \frac{2A}{\partial 1}\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\frac{A}{\partial 2-\partial 1}\cdot 2\cos(n\omega_0\partial 2) - A\left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)\cdot 2\cos(n\omega_0\partial 1) + \frac{2A}{\partial 1}\right)$$

$$C_n = \frac{2A}{(t_i-t_f)n^2\omega_0^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1) + \frac{1}{\partial 1}\right)$$

$$C_n = \frac{2A}{\left(-\frac{T}{2} - \frac{T}{2}\right)n^2\frac{2^2\pi^2}{T^2}}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1) + \frac{1}{\partial 1}\right)$$

$$C_n = \frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1} + \frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1) + \frac{1}{\partial 1}\right)$$

Ahora confirmamos con $x(t)$

$$C_n = \frac{1}{T}\int_T x(t)\, e^{-jn\omega_0 t}\, dt \quad ; \quad T = t_f - t_i = \frac{T}{2} - \left(-\frac{T}{2}\right) = \frac{T}{2}(1+1) = T \quad ; \quad \omega_0 = \frac{2\pi}{T}$$

$$C_n = \frac{1}{T}\left(\int_{-\frac{T}{2}}^{-d_2} \cancelto{0}{0\cdot e^{-jn\omega_0 t}}\, dt + \int_{-d_2}^{-d_1}\frac{A}{d_2-d_1}(t+d_2)e^{-jn\omega_0 t}dt + \int_{-d_1}^{0} -\frac{A}{d_1}\, t\, e^{-jn\omega_0 t}dt + \int_{0}^{d_1}\frac{A}{d_1}\, t\, e^{-jn\omega_0 t}dt + \int_{d_1}^{d_2} -\frac{A}{d_2-d_1}(t+d_2)e^{-jn\omega_0 t}dt + \int_{d_2}^{\frac{T}{2}} \cancelto{0}{0\, e^{-jn\omega_0 t}}\, dt\right)$$

$$C_n = \frac{1}{T}\left(\frac{A}{d_2-d_1}\int_{-d_2}^{-d_1}(t+d_2)e^{-jn\omega_0 t}dt - \frac{A}{d_1}\int_{-d_1}^{0} t\, e^{-jn\omega_0 t}dt + \frac{A}{d_1}\int_{0}^{d_1} t\, e^{-jn\omega_0 t}dt - \frac{A}{d_2-d_1}\int_{d_1}^{d_2}(t+d_2)e^{-jn\omega_0 t}dt\right)$$

$$C_n = \frac{1}{T}\left(\frac{A}{d_2-d_1}\left(\underbrace{\int_{-d_2}^{-d_1} t\, e^{-jn\omega_0 t}dt}_{①} + d_2\underbrace{\int_{-d_2}^{-d_1} e^{-jn\omega_0 t}dt}_{②}\right) - \frac{A}{d_1}\underbrace{\int_{-d_1}^{0} t\, e^{-jn\omega_0 t}dt}_{③} + \frac{A}{d_1}\underbrace{\int_{0}^{d_1} t\, e^{-jn\omega_0 t}dt}_{④} - \frac{A}{d_2-d_1}\left(\underbrace{\int_{d_1}^{d_2} t\, e^{-jn\omega_0 t}dt}_{⑤} + d_2\underbrace{\int_{d_1}^{d_2} e^{-jn\omega_0 t}dt}_{⑥}\right)\right)$$

① $\int_{-d_2}^{-d_1} t\, e^{-jn\omega_0 t}dt \longrightarrow u = t \to du = dt$, $dv = e^{-jn\omega_0 t} \to v = \int e^{-jn\omega_0 t}dt = \frac{e^{-jn\omega_0 t}}{-jn\omega_0} \to uv - \int v\, du$

$$= t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{-d_2}^{-d_1} - \int_{-d_2}^{-d_1}\frac{e^{-jn\omega_0 t}}{-jn\omega_0}dt = t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{-d_2}^{-d_1} - \frac{e^{-jn\omega_0 t}}{n^2\omega_0^2}\Big|_{-d_2}^{-d_1} = \left(-d_1\frac{e^{jn\omega_0 d_1}}{-jn\omega_0} - (-d_2)\frac{e^{jn\omega_0 d_2}}{-jn\omega_0}\right) - \left(\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2} - \frac{e^{jn\omega_0 d_2}}{n^2\omega_0^2}\right)$$

$$= d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0} - d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0} - \frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2} + \frac{e^{jn\omega_0 d_2}}{n^2\omega_0^2} = e^{jn\omega_0 d_1}\left(\frac{d_1}{jn\omega_0} - \frac{1}{n^2\omega_0^2}\right) + e^{jn\omega_0 d_2}\left(\frac{1}{n^2\omega_0^2} - \frac{d_2}{jn\omega_0}\right)$$

② $$d_2\int_{-d_2}^{-d_1} e^{-jn\omega_0 t}dt = -d_2\frac{e^{-jn\omega_0 t}}{jn\omega_0}\Big|_{-d_2}^{-d_1} = -d_2\frac{e^{jn\omega_0 d_1}}{jn\omega_0} - (-d_2)\frac{e^{jn\omega_0 d_2}}{jn\omega_0} = d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0} - d_2\frac{e^{jn\omega_0 d_1}}{jn\omega_0} = \frac{d_2}{jn\omega_0}\left(e^{jn\omega_0 d_2} - e^{jn\omega_0 d_1}\right)$$

③ $$\int_{-d_1}^{0} t\, e^{-jn\omega_0 t}dt = t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{-d_1}^{0} - \int_{-d_1}^{0}\frac{e^{-jn\omega_0 t}}{-jn\omega_0}dt = t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{-d_1}^{0} - \frac{e^{-jn\omega_0 t}}{n^2\omega_0^2}\Big|_{-d_1}^{0} = \left(0\cdot\frac{e^{-jn\omega_0 0}}{-jn\omega_0} - (-d_1)\frac{e^{jn\omega_0 d_1}}{-jn\omega_0}\right) - \left(\frac{e^{-jn\omega_0\cdot 0}}{n^2\omega_0^2} - \frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}\right)$$

$$= -d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0} - \left(\frac{1}{n^2\omega_0^2} - \frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}\right)$$

④ $$\int_{0}^{d_1} t\, e^{-jn\omega_0 t}dt = t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{0}^{d_1} - \int_{0}^{d_1}\frac{e^{-jn\omega_0 t}}{-jn\omega_0}dt = t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{0}^{d_1} - \frac{e^{-jn\omega_0 t}}{n^2\omega_0^2}\Big|_{0}^{d_1} = \left(d_1\frac{e^{-jn\omega_0 d_1}}{-jn\omega_0} - 0\,\frac{e^{-jn\omega_0 0}}{-jn\omega_0}\right) - \left(\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2} - \frac{e^{-jn\omega_0\cdot 0}}{n^2\omega_0^2}\right)$$

$$= -d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0} - \left(\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2} - \frac{1}{n^2\omega_0^2}\right)$$

⑤ $$\int_{d_1}^{d_2} t\, e^{-jn\omega_0 t}dt = t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{d_1}^{d_2} - \int_{d_1}^{d_2}\frac{e^{-jn\omega_0 t}}{-jn\omega_0}dt = t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{d_1}^{d_2} - \frac{e^{-jn\omega_0 t}}{n^2\omega_0^2}\Big|_{d_1}^{d_2} = \left(d_2\frac{e^{-jn\omega_0 d_2}}{-jn\omega_0} - d_1\frac{e^{-jn\omega_0 d_1}}{-jn\omega_0}\right) - \left(\frac{e^{-jn\omega_0 d_2}}{n^2\omega_0^2} - \frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}\right)$$

$$= \left(d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0} - d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}\right) - \left(\frac{e^{-jn\omega_0 d_2}}{n^2\omega_0^2} - \frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}\right)$$

⑥ $$d_2\int_{d_1}^{d_2} e^{-jn\omega_0 t}dt = d_2\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{d_1}^{d_2} = d_2\frac{e^{-jn\omega_0 d_2}}{-jn\omega_0} - d_2\frac{e^{-jn\omega_0 d_1}}{-jn\omega_0} = d_2\frac{e^{-jn\omega_0 d_1}}{jn\omega_0} - d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}$$

$$C_n = \frac{1}{T}\left(\frac{A}{d_2-d_1}\left(\left(d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0} - d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0}\right) - \left(\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2} - \frac{e^{jn\omega_0 d_2}}{n^2\omega_0^2}\right) + d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0} - d_2\frac{e^{jn\omega_0 d_1}}{jn\omega_0}\right) - \frac{A}{d_1}\left(-d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0} - \left(\frac{1}{n^2\omega_0^2} - \frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}\right)\right)\right.$$

$$\left. + \frac{A}{d_1}\left(-d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0} - \left(\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2} - \frac{1}{n^2\omega_0^2}\right)\right) - \frac{A}{d_2-d_1}\left(\left(d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0} - d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}\right) - \left(\frac{e^{-jn\omega_0 d_2}}{n^2\omega_0^2} - \frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}\right) + d_2\frac{e^{-jn\omega_0 d_1}}{jn\omega_0} - d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}\right)\right)$$

$$= \frac{1}{T}\left(\frac{A}{d_2-d_1}d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0} - \frac{A}{d_2-d_1}d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0} - \frac{A}{d_2-d_1}\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2} + \frac{A}{d_2-d_1}\frac{e^{jn\omega_0 d_2}}{n^2\omega_0^2} + \frac{A}{d_2-d_1}d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0} - \frac{A}{d_2-d_1}d_2\frac{e^{jn\omega_0 d_1}}{jn\omega_0}\right.$$

$$+ \frac{A}{d_1}d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0} + \frac{A}{d_1}\frac{1}{n^2\omega_0^2} - \frac{A}{d_1}\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2} - \frac{A}{d_1}d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0} - \frac{A}{d_1}\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2} + \frac{A}{d_1}\frac{1}{n^2\omega_0^2}$$

$$\left. - \frac{A\, d_1}{d_2-d_1}\frac{e^{-jn\omega_0 d_1}}{jn\omega_0} + \frac{A\, d_2}{d_2-d_1}\frac{e^{-jn\omega_0 d_2}}{jn\omega_0} + \frac{A}{d_2-d_1}\frac{e^{-jn\omega_0 d_2}}{n^2\omega_0^2} - \frac{A}{d_2-d_1}\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2} - \frac{A}{d_2-d_1}d_2\frac{e^{-jn\omega_0 d_1}}{jn\omega_0} + \frac{A}{d_2-d_1}d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}\right)$$

$$= \frac{1}{T}\left(\frac{\delta_1 A e^{jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} - \frac{\delta_2 A e^{jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} - \frac{A e^{jn\omega_0\delta_1}}{n^2\omega_0^2(\delta_2-\delta_1)} + \frac{A e^{jn\omega_0\delta_2}}{n^2\omega_0^2(\delta_2-\delta_1)} + \frac{\delta_2 A e^{jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} - \frac{\delta_2 A e^{jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} \right.$$

$$+ \frac{A e^{jn\omega_0\delta_1}}{jn\omega_0} + \frac{A}{\delta_1 n^2\omega_0^2} - \frac{A e^{jn\omega_0\delta_1}}{\delta_1 n^2\omega_0^2} - \frac{A e^{-jn\omega_0\delta_1}}{jn\omega_0} - \frac{A e^{-jn\omega_0\delta_1}}{\delta_1 n^2\omega_0^2} + \frac{A}{\delta_1 n^2\omega_0^2}$$

$$\left. - \frac{\delta_1 A e^{-jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} + \frac{\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} + \frac{A e^{-jn\omega_0\delta_2}}{n^2\omega_0^2(\delta_2-\delta_1)} - \frac{A e^{-jn\omega_0\delta_1}}{n^2\omega_0^2(\delta_2-\delta_1)} - \frac{\delta_2 A e^{-jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} + \frac{\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} \right)$$

Acomodamos los terminos por aparte.

$$\frac{\delta_1 A e^{jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} - \frac{\delta_1 A e^{-jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} \longrightarrow \frac{\delta_1 A}{jn\omega_0(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} - e^{-jn\omega_0\delta_1}\right)$$

$$-\frac{\delta_2 A e^{jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} + \frac{\delta_2 A e^{jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} + \frac{\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} + \frac{\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} \longrightarrow \frac{2\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)}$$

$$-\frac{\delta_2 A e^{jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} - \frac{\delta_2 A e^{-jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} \longrightarrow \frac{-\delta_2 A}{jn\omega_0(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right)$$

$$-\frac{A e^{jn\omega_0\delta_1}}{n^2\omega_0^2(\delta_2-\delta_1)} - \frac{A e^{-jn\omega_0\delta_1}}{n^2\omega_0^2(\delta_2-\delta_1)} \longrightarrow \frac{-A}{n^2\omega_0^2(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right)$$

$$+\frac{A}{\delta_1 n^2\omega_0^2} + \frac{A}{\delta_1 n^2\omega_0^2} \longrightarrow \frac{2A}{\delta_1 n^2\omega_0^2}$$

$$+\frac{A e^{jn\omega_0\delta_2}}{n^2\omega_0^2(\delta_2-\delta_1)} + \frac{A e^{-jn\omega_0\delta_2}}{n^2\omega_0^2(\delta_2-\delta_1)} \longrightarrow \frac{A}{n^2\omega_0^2(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_2} + e^{-jn\omega_0\delta_2}\right)$$

$$+\frac{A e^{jn\omega_0\delta_1}}{jn\omega_0} - \frac{A e^{-jn\omega_0\delta_1}}{jn\omega_0} \longrightarrow \frac{A\left(e^{jn\omega_0\delta_1} - e^{-jn\omega_0\delta_1}\right)}{jn\omega_0}$$

$$-\frac{A e^{jn\omega_0\delta_1}}{\delta_1 n^2\omega_0^2} - \frac{A e^{-jn\omega_0\delta_1}}{\delta_1 n^2\omega_0^2} \longrightarrow \frac{-A\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right)}{\delta_1 n^2\omega_0^2}$$

$$\frac{\delta_1 A}{jn\omega_0(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} - e^{-jn\omega_0\delta_1}\right) = \frac{\delta_1 A(\cancel{\cos(n\omega_0\delta_1)} + j\,\mathrm{sen}(n\omega_0\delta_1) - \cancel{\cos(n\omega_0\delta_1)} + j\,\mathrm{sen}(n\omega_0\delta_1))}{jn\omega_0(\delta_2-\delta_1)} = \frac{\delta_1 A\;2j\,\mathrm{sen}(n\omega_0\delta_1)}{jn\omega_0(\delta_2-\delta_1)}$$

$$\frac{2\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} = \frac{2\delta_2 A\cos(n\omega_0\delta_2)}{jn\omega_0(\delta_2-\delta_1)} - \frac{2\delta_2 A j\,\mathrm{sen}(n\omega_0\delta_2)}{jn\omega_0(\delta_2-\delta_1)}$$

$$\frac{-\delta_2 A}{jn\omega_0(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right) = \frac{-\delta_2 A(\cos(n\omega_0\delta_1) + \cancel{j\,\mathrm{sen}(n\omega_0\delta_1)} + \cos(n\omega_0\delta_1) - \cancel{j\,\mathrm{sen}(n\omega_0\delta_1)})}{jn\omega_0(\delta_2-\delta_1)} = \frac{-\delta_2 A\cdot 2\cos(n\omega_0\delta_1)}{jn\omega_0(\delta_2-\delta_1)}$$

$$\frac{-A}{n^2\omega_0^2(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right) = \frac{-A(\cos(n\omega_0\delta_1) + \cancel{j\,\mathrm{sen}(n\omega_0\delta_1)} + \cos(n\omega_0\delta_1) - \cancel{j\,\mathrm{sen}(n\omega_0\delta_1)})}{n^2\omega_0^2(\delta_2-\delta_1)} = \frac{-A\cdot 2\cos(n\omega_0\delta_1)}{n^2\omega_0^2(\delta_2-\delta_1)}$$

$$\frac{2A}{\delta_1 n^2\omega_0^2}$$

$$\frac{A}{n^2\omega_0^2(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_2} + e^{-jn\omega_0\delta_2}\right) = \frac{A(\cos(n\omega_0\delta_2) + \cancel{j\,\mathrm{sen}(n\omega_0\delta_2)} + \cos(n\omega_0\delta_2) - \cancel{j\,\mathrm{sen}(n\omega_0\delta_2)})}{n^2\omega_0^2(\delta_2-\delta_1)} = \frac{A\cdot 2\cos(n\omega_0\delta_2)}{n^2\omega_0^2(\delta_2-\delta_1)}$$

$$\frac{A\left(e^{jn\omega_0\delta_1} - e^{-jn\omega_0\delta_1}\right)}{jn\omega_0} = \frac{A(\cancel{\cos(n\omega_0\delta_1)} + j\,\mathrm{sen}(n\omega_0\delta_1) - \cancel{\cos(n\omega_0\delta_1)} + j\,\mathrm{sen}(n\omega_0\delta_1))}{jn\omega_0} = \frac{A\cdot 2j\,\mathrm{sen}(n\omega_0\delta_1)}{jn\omega_0}$$

$$\frac{-A\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right)}{\delta_1 n^2\omega_0^2} = \frac{-A(\cos(n\omega_0\delta_1) + \cancel{j\,\mathrm{sen}(n\omega_0\delta_1)} + \cos(n\omega_0\delta_1) - \cancel{j\,\mathrm{sen}(n\omega_0\delta_1)})}{\delta_1 n^2\omega_0^2} = \frac{-A\cdot 2\cos(n\omega_0\delta_1)}{\delta_1 n^2\omega_0^2}$$

$$C_n = \frac{1}{T}\left(\frac{\delta_1 A\;2j\,\mathrm{sen}(n\omega_0\delta_1)}{jn\omega_0(\delta_2-\delta_1)} + \frac{2\delta_2 A\cos(n\omega_0\delta_2)}{jn\omega_0(\delta_2-\delta_1)} - \frac{2\delta_2 A j\,\mathrm{sen}(n\omega_0\delta_2)}{jn\omega_0(\delta_2-\delta_1)} - \frac{\delta_2 A\cdot 2\cos(n\omega_0\delta_1)}{jn\omega_0(\delta_2-\delta_1)} + \frac{2A}{\delta_1 n^2\omega_0^2} + \frac{A\cdot 2\cos(n\omega_0\delta_2)}{n^2\omega_0^2(\delta_2-\delta_1)} \right.$$

$$\left. + \frac{A\cdot 2j\,\mathrm{sen}(n\omega_0\delta_1)}{jn\omega_0} - \frac{A\cdot 2\cos(n\omega_0\delta_1)}{\delta_1 n^2\omega_0^2} \right)$$

$$= \frac{1}{T}\left(\frac{2\delta_1 A}{n\omega_0(\delta_2-\delta_1)}\mathrm{sen}(n\omega_0\delta_1) + \frac{2A\delta_2\cdot\cos(n\omega_0\delta_2)}{jn\omega_0(\delta_2-\delta_1)} - \frac{2\delta_2 A}{n\omega_0(\delta_2-\delta_1)}\mathrm{sen}(n\omega_0\delta_2) - \frac{\delta_2 A 2}{jn\omega_0(\delta_2-\delta_1)}\cos(n\omega_0\delta_1) + \frac{2A}{\delta_1 n^2\omega_0^2} \right.$$

$$\left. + \frac{A\cdot 2\cos(n\omega_0\delta_2)}{n^2\omega_0^2(\delta_2-\delta_1)} + \frac{2A}{n\omega_0}\mathrm{sen}(n\omega_0\delta_1) - \frac{2A}{\delta_1 n^2\omega_0^2}\cos(n\omega_0\delta_1) \right)$$

$$= \frac{1}{T}\left(\frac{2A\cdot\mathrm{sen}(n\omega_0\delta_1)}{n\omega_0}\left(\frac{\delta_1}{\delta_2-\delta_1} + 1\right) + \frac{2A\cos(n\omega_0\delta_2)}{n\omega_0(\delta_2-\delta_1)}\left(\frac{\delta_2}{j} + \frac{1}{n\omega_0}\right) \right.$$

$$\left. - \frac{2A\cos(n\omega_0\delta_1)}{n\omega_0}\left(\frac{1}{\delta_1 n\omega_0} + \frac{\delta_2}{\delta_2-\delta_1}\right) - \frac{2\delta_2 A}{n\omega_0(\delta_2-\delta_1)}\mathrm{sen}(n\omega_0\delta_2) + \frac{2A}{\delta_1 n^2\omega_0^2} \right)$$

$$Re\{C_n\} = \frac{-AT}{2 \cdot n^2 \pi^2}\left(\frac{1}{\partial 2 - \partial 1}\cdot \cos(n\omega_0 \partial 2) - \left(\frac{1}{\partial 1} + \frac{1}{\partial 2 - \partial 1}\right)\cdot \cos(n\omega_0 \partial 1) + \frac{1}{\partial 1}\right) \quad y \quad Im\{C_n\} = 0 \longrightarrow \text{Simetría par}$$

$$|C_n| = \sqrt{Re^2\{C_n\} + Im^2\{C_n\}} = \sqrt{\left(\frac{-AT}{2 \cdot n^2 \pi^2}\left(\frac{1}{\partial 2 - \partial 1}\cdot \cos(n\omega_0 \partial 2) - \left(\frac{1}{\partial 1} + \frac{1}{\partial 2 - \partial 1}\right)\cdot \cos(n\omega_0 \partial 1) + \frac{1}{\partial 1}\right)\right)^2}$$

$$\theta_{C_n} = \tan^{-1}\left\{\frac{Im\{C_n\}}{Re\{C_n\}}\right\} = \tan^{-1}\left(\frac{0}{\frac{-AT}{2 \cdot n^2 \pi^2}\left(\frac{1}{\partial 2 - \partial 1}\cdot \cos(n\omega_0 \partial 2) - \left(\frac{1}{\partial 1} + \frac{1}{\partial 2 - \partial 1}\right)\cdot \cos(n\omega_0 \partial 1) + \frac{1}{\partial 1}\right)}\right) = 0$$

$$e_r[\%] = \left(1 - \sum |C_n|^2 \frac{P_n}{P_x}\right)\cdot 100[\%] \quad ; \quad P_n = \frac{1}{T} E_n = \frac{1}{T}\int_T |e^{jn\omega_0 t}|^2 \partial t = \frac{1}{T}\cdot T = 1$$

Como la señal es par solo vamos a integrar de 0 a $\partial 2$ $(x(-t) = x(t))$

$$P_x = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 \partial t = \frac{2}{T}\int_0^{\frac{T}{2}} |x(t)|^2 \partial t \rightarrow \text{Pero de } \partial 2 \text{ a } \tfrac{T}{2} \text{ es } 0 \rightarrow P_x = \frac{2}{T}\int_0^{\partial 2} |x(t)|^2 \partial t = \frac{2}{T}\left(\int_0^{\partial 1}\left(\frac{A}{\partial 1}t\right)^2 \partial t + \int_{\partial 1}^{\partial 2}\left(\frac{A}{\partial 2 - \partial 1}(t + \partial 2)\right)^2 \partial t\right)$$

$$P_x = \frac{2}{T}\left(\int_0^{\partial 1}\left(\frac{A}{\partial 1}t\right)^2 \partial t + \int_{\partial 1}^{\partial 2}\left(\frac{A}{\partial 2 - \partial 1}(t + \partial 2)\right)^2 \partial t\right) = \frac{2}{T}\left(\frac{A^2}{\partial 1^2}\int_0^{\partial 1} t^2 \partial t + \frac{A^2}{(\partial 2 - \partial 1)^2}\int_{\partial 1}^{\partial 2}(t + \partial 2)^2 \partial t\right)$$

$$\int_0^{\partial 1} t^2 \partial t = \frac{t^3}{3}\Big|_0^{\partial 1} = \frac{\partial 1^3}{3} \quad ; \quad \int_{\partial 1}^{\partial 2}(t + \partial 2)^2 \partial t = \int_{\partial 1 + \partial 2}^{2\partial 2} u^2 \partial u = \frac{u^3}{3}\Big|_{\partial 1 + \partial 2}^{2\partial 2} = \frac{(2\partial 2)^3}{3} - \frac{(\partial 1 + \partial 2)^3}{3} = \frac{1}{3}\left(8\partial 2^3 - (\partial 1 + \partial 2)^3\right)$$

$u = t + \partial 2$

$\partial u = \partial t$

Si $t = \partial 1 \rightarrow u = \partial 1 + \partial 2$

Si $t = \partial 2 \rightarrow u = 2\partial 2$

$u \in [\partial 1 + \partial 2, 2\partial 2]$

$$P_x = \frac{2}{T}\left(\frac{A^2}{\partial 1^2}\frac{\partial 1^3}{3} + \frac{A^2}{(\partial 2 - \partial 1)^2}\left(\frac{1}{3}\left(8\partial 2^3 - (\partial 1 + \partial 2)^3\right)\right)\right)$$

$$P_x = \frac{2}{T}\left(\frac{A^2}{\partial 1^2}\frac{\partial 1^3}{3} + \frac{A^2}{3(\partial 2 - \partial 1)^2}\left(8\partial 2^3 - (\partial 1 + \partial 2)^3\right)\right)$$

$$P_x = \frac{2}{T}\left(\frac{A^2}{\cancel{\partial 1^2}}\frac{\cancel{\partial 1^3}}{3} + \frac{A^2}{3(\partial 2 - \partial 1)^2}\left(8\partial 2^3 - (\partial 1 + \partial 2)^3\right)\right)$$

$$P_x = \frac{2A^2 \partial 1}{3T} + \frac{2A^2}{3T(\partial 2 - \partial 1)^2}\left(8\partial 2^3 - (\partial 1 + \partial 2)^3\right)$$

$$|C_n|^2 = \cancel{\sqrt{\left(\frac{-AT}{2 \cdot n^2 \pi^2}\left(\frac{1}{\partial 2 - \partial 1}\cdot \cos(n\omega_0 \partial 2) - \left(\frac{1}{\partial 1} + \frac{1}{\partial 2 - \partial 1}\right)\cdot \cos(n\omega_0 \partial 1) + \frac{1}{\partial 1}\right)\right)^2}}^{\,\cancel{2}}$$

$$= \left(\frac{-AT}{2 \cdot n^2 \pi^2}\left(\frac{1}{\partial 2 - \partial 1}\cdot \cos(n\omega_0 \partial 2) - \left(\frac{1}{\partial 1} + \frac{1}{\partial 2 - \partial 1}\right)\cdot \cos(n\omega_0 \partial 1) + \frac{1}{\partial 1}\right)\right)^2$$

$$e_r(\%) = \left(1 - \sum_n \frac{\left(\frac{-AT}{2 \cdot n^2 \pi^2}\left(\frac{1}{\partial 2 - \partial 1}\cdot \cos(n\omega_0 \partial 2) - \left(\frac{1}{\partial 1} + \frac{1}{\partial 2 - \partial 1}\right)\cdot \cos(n\omega_0 \partial 1) + \frac{1}{\partial 1}\right)\right)^2}{\frac{2A^2 \partial 1}{3T} + \frac{2A^2}{3T(\partial 2 - \partial 1)^2}\left(8\partial 2^3 - (\partial 1 + \partial 2)^3\right)}\right)\cdot 100(\%)$$