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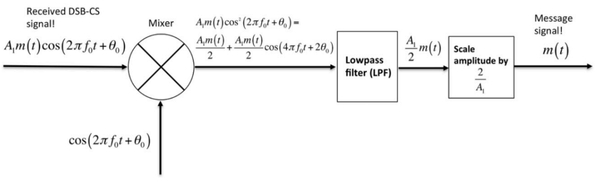
1. Sea el demodulador en amplitud presentado en la siguiente Figura:



Asumiendo $\theta_0 = 0$, determine el espectro de Fourier (teórico) en cada una de las etapas del sistema. Luego, con base en la simulación de modulación en amplitud del Taller 2 y utilizando cinco segundos de una canción de Youtube como mensaje, grafique cada una de las etapas principales del proceso de modulación y demodulación en el tiempo y la frecuencia (reproduzca el segmento de la canción en cada etapa).
Nota: Para la etapa de filtrado pasa bajas, realice su implementación a partir de la transformada rápida de Fourier.

Demulador coherente de AM DSB-SC/DSB-CS

Señal mensaje base: $m(t)\rightarrow M(w)$

Señal recibida: $S_{rec}(t)=A_1m(t)\cos(2\pi f_0t+\theta_0)$

A_1: Ganancia, f_0: frecuencia de la portadora.

Segunda señal que entra al mixer: $\cos(2\pi f_0t+\theta_0)$

$W_0=2\pi f_0=\frac{2\pi}{T}$

Despues del mixer:

$y(t)=A_1m(t)\cos(2\pi f_0t+\theta_0)\cdot\cos(2\pi f_0t+\theta_0)$

$y(t)=A_1m(t)\cos^2(2\pi f_0t+\theta_0)$; $W_0=2\pi f_0$

Si $\cos^2(\theta)=\frac{1+\cos(2\theta)}{2}$

$y(t)=\frac{A_1}{2}m(t)(1+\cos(2(2\pi f_0t+\theta_0)))$

$y(t)=\frac{A_1}{2}m(t)+\frac{A_1}{2}m(t)\cos(4\pi f_0t+2\theta_0)$

Filtro pasa-bajas (LPF):

Entrada: $y(t)$

Salida: $y_f(t)=\frac{A_1}{2}m(t)\rightarrow Y_f(w)=\frac{A_1}{2}M(w)$

Escalador de amplitud:

Entrada: $y_f(t)=\frac{A_1}{2}m(t)$

Ganancia: $G=\frac{2}{A_1}$

Salida: $\hat{m}(t)=\frac{\cancel{A_1}}{\cancel{2}}m(t)\cdot\frac{\cancel{2}}{\cancel{A_1}}=m(t)$

$X(w)=F\{x(t)\}=\int_{-\infty}^{\infty}x(t)e^{-jwt}\partial t$

Espectro de señal recibida al mixer:

$S_{rec}(t)=A_1m(t)\cos(2\pi f_0t+\cancelto{0}{\theta_0})$ Si $\cos(\theta)=\frac{e^{j\theta}+e^{-j\theta}}{2}$ y $W_0=2\pi f_0$

$=A_1m(t)\cos(W_0t)$

$=A_1m(t)\left(\frac{e^{jW_0t}+e^{-jW_0t}}{2}\right)$

$=\frac{1}{2}A_1m(t)(e^{jW_0t}+e^{-jW_0t})$

Si $f(t)e^{jW_0t}\leftrightarrow F(W-W_0)$

$S_{rec}(W)=F\{\frac{1}{2}A_1m(t)(e^{jW_0t}+e^{-jW_0t})\}=\frac{1}{2}A_1F\{m(t)e^{jW_0t}+m(t)e^{-jW_0t}\}=\frac{1}{2}A_1\left(M(W-W_0)+M(W+W_0)\right)$

Espectro de señal de salida del mixer:

$y(t)=\frac{A_1}{2}m(t)+\frac{A_1}{2}m(t)\cos(4\pi f_0t+\cancelto{0}{2\theta_0})$ Si $W_0=2\pi f_0$

$y(t)=\frac{A_1}{2}m(t)+\frac{A_1}{2}m(t)\cos(2W_0t)$

$Y(W)=F\left\{\frac{A_1}{2}m(t)+\frac{A_1}{2}m(t)\cos(2W_0t)\right\}=F\left\{\frac{A_1}{2}m(t)\right\}+F\left\{\frac{A_1}{2}m(t)\cos(2W_0t)\right\}=\frac{A_1}{2}M(W)+F\left\{\frac{A_1}{2}m(t)\cos(2W_0t)\right\}$ Si $\cos(\theta)=\frac{e^{j\theta}+e^{-j\theta}}{2}$

$=\frac{A_1}{2}M(W)+F\left\{\frac{A_1}{2}m(t)\frac{e^{j2W_0t}+e^{-j2W_0t}}{2}\right\}=\frac{A_1}{2}M(W)+\frac{A_1}{4}F\left\{m(t)e^{j2W_0t}+m(t)e^{-j2W_0t}\right\}$ Si $f(t)e^{jW_0t}\leftrightarrow F(W-W_0)$

$=\frac{A_1}{2}M(W)+\frac{A_1}{4}\left(M(W-W_0)+M(W+W_0)\right)$

Espectro de señal de salida del mixer: $H_f(w)=\begin{cases}1, & |w|\leq w_c \\ 0, & |w|>w_c\end{cases}$

$$H_f(w)=\frac{Y_f(w)}{Y(w)}$$

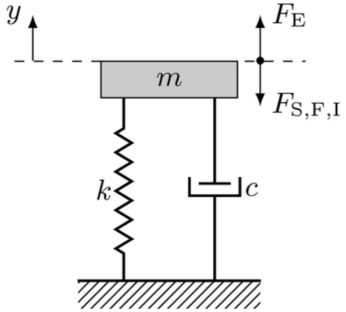
$$y_f(t)=\frac{A_1}{2}m(t)\rightarrow Y_f(w)=\frac{A_1}{2}M(w)$$

Espectro de señal del escalador:

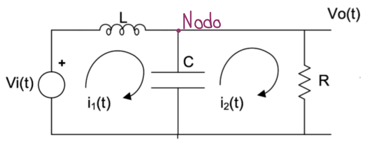
$$\hat{m}(t)=\frac{\cancel{A_1}}{\cancel{2}}m(t)\cdot\frac{\cancel{2}}{\cancel{A_1}}=m(t)$$

$$\hat{m}(w)=F\{m(t)\}=M(w)$$

2. Encuentre la función de transferencia que caracteriza el sistema masa, resorte, amortiguador, presentado en la siguiente Figura (asuma condiciones iniciales cero):



Posteriormente, encuentre el sistema equivalente del modelo masa, resorte, amortiguador, a partir del siguiente circuito eléctrico:



Finalmente, proponga unos valores de m, k y c y sus equivalentes R, L y C, para simular un sistema subamortiguado, sobreamortiguado y de amortiguamiento crítico (determine el factor de amortiguamiento, la frecuencia natural amortiguada, la frecuencia natural no amortiguada, el tiempo pico, tiempo de levantamiento y el tiempo de establecimiento en cada caso). Para cada caso, grafique el diagrama de polos y ceros, el diagrama de Bode, la respuesta impulso, respuesta escalón y respuesta rampa. Repita el proceso para modo lazo cerrado.

masa (bloque): m, resorte: K, amortiguador: C

fuerza externa aplicada sobre la masa: $F_E(t)$ (hacia arriba)

fuerza del resorte: F_S

$F_S = Ky(t)$; (opuesta al desplazamiento)

$F_d = Cy'(t)$; (opuesta a la velocidad) ; $F = m\cdot a$

tomaremos $y(t)$ como el desplazamiento

$$my''(t) = F_E(t) - F_d(t) - F_S(t)$$

$$\boxed{my''(t) + Cy'(t) + Ky(t) = F_E(t)} \quad (1) \rightarrow \text{E.D.O del sistema}$$

$$\mathcal{L}\left\{\frac{\partial^n}{\partial t^n}X(t)\right\} = S^n X(S)$$

$$F_E(S) = \mathcal{L}\{my''(t) + Cy'(t) + Ky(t)\}$$

$$F_E(S) = mS^2Y(S) + CSY(S) + KY(S)$$

$$F_E(S) = Y(S)(mS^2 + CS + K)$$

$$\frac{1}{(mS^2+CS+K)} = \frac{Y(S)}{F_E(S)}$$

$$H(S) = \frac{Y(S)}{F_E(S)} = \frac{1}{(mS^2+CS+K)}$$

$$\boxed{H(S) = \frac{1}{mS^2+CS+K}} \rightarrow \text{función de transferencia}$$

Calculamos la función de transferencia del circuito:

En paralelo: $i_L(t) = i_C(t) + i_R(t) \rightarrow i_L(t) = C\frac{\partial}{\partial t}V_C(t) + \frac{1}{R}V_R(t)$ Si $V_C(t) = V_R(t) = V_o(t)$

$$i_L(t) = C\frac{\partial}{\partial t}V_o(t) + \frac{1}{R}V_o(t)$$

$$V_L(t) = L\frac{\partial}{\partial t}i_L(t) = V_i(t) - V_o(t) \rightarrow \frac{\partial}{\partial t}i_L(t) = \frac{V_i(t)-V_o(t)}{L}$$

Si le aplicamos derivada a: $i_L(t) = C\frac{\partial}{\partial t}V_o(t) + \frac{1}{R}V_o(t)$

$$\frac{\partial}{\partial t}i_L(t) = C\frac{\partial^2}{\partial t^2}V_o(t) + \frac{1}{R}\frac{\partial}{\partial t}V_o(t)$$

$$\frac{V_i(t)-V_o(t)}{L} = C\frac{\partial^2}{\partial t^2}V_o(t) + \frac{1}{R}\frac{\partial}{\partial t}V_o(t) \rightarrow V_i(t) - V_o(t) = CL\frac{\partial^2}{\partial t^2}V_o(t) + \frac{L}{R}\frac{\partial}{\partial t}V_o(t) \rightarrow V_i(t) = CL\frac{\partial^2}{\partial t^2}V_o(t) + \frac{L}{R}\frac{\partial}{\partial t}V_o(t) + V_o(t) \rightarrow \text{E.D.O del sistema}$$

$$V_i(S) = CLS^2V_o(S) + \frac{L}{R}SV_o(S) + V_o(S)$$

$$\boxed{\frac{1}{C}V_i(t) = L\frac{\partial^2}{\partial t^2}V_o(t) + \frac{L}{CR}\frac{\partial}{\partial t}V_o(t) + \frac{1}{C}V_o(t)} \quad (2) \rightarrow \text{E.D.O del sistema}$$

$$V_i(S) = V_o(S)(CLS^2 + \frac{L}{R}S + 1)$$

$$H(S) = \frac{V_o(S)}{V_i(S)} = \frac{1}{(CLS^2 + \frac{L}{R}S + 1)} \cdot \frac{R}{R} = \frac{R}{(RCLS^2 + LS + R)} \longrightarrow \boxed{H(S) = \frac{R}{(RCLS^2+LS+R)}} \rightarrow \text{función de transferencia}$$

Si comparamos los sistemas (1) y (2) $m = L$, $C = \frac{L}{RC}$, $K = \frac{1}{C}$, $F_E(t) = \frac{1}{C}V_i(t)$, $y(t) = V_o(t)$

$$H(s)=\frac{1}{m}\cdot\frac{1}{s^2+\frac{C}{m}s+\frac{K}{m}}$$

Para llevarlo a su forma estandar de segundo orden:

Si La frecuencia natural no amortiguada es: $W_n^2=\frac{K}{m}\longleftrightarrow W_n=\sqrt{\frac{K}{m}}$

$2\zeta W_n=\frac{C}{m}$; ζ es factor de amortiguamiento adimencional

$\zeta=\frac{C}{2mW_n}=\frac{C}{2mW_n}=\frac{C}{2\sqrt{Km}}$; Si no hay amortiguamiento $C=0$

$$H(s)=\frac{1}{m}\cdot\frac{1}{s^2+2\zeta W_n s+W_n^2}$$

Para los polos:

$$s^2+2\zeta W_n s+W_n^2=0$$

$$\text{Con } \frac{-b\pm\sqrt{b^2-4ac}}{2a}\longleftrightarrow P_{1,2}=\frac{-2\zeta W_n\pm\sqrt{(2\zeta W_n)^2-4W_n^2}}{2}=\frac{-2\zeta W_n\pm\sqrt{4\zeta^2W_n^2-4W_n^2}}{2}=\frac{-2\zeta W_n\pm\sqrt{4(\zeta^2W_n^2-W_n^2)}}{2}$$

$$=\frac{-2\zeta W_n\pm2\sqrt{\zeta^2W_n^2-W_n^2}}{2}=-\zeta W_n\pm\sqrt{\zeta^2W_n^2-W_n^2}=-\zeta W_n\pm\sqrt{W_n^2(\zeta^2-1)}=-\zeta W_n\pm W_n\sqrt{\zeta^2-1}$$

Para subamortiguado: $0\leq\zeta<1\rightarrow\zeta^2-1<0$: $\sqrt{\zeta^2-1}=j\sqrt{1-\zeta^2}$

$P_{1,2}=-\zeta W_n\pm jW_n\sqrt{1-\zeta^2}\rightarrow$ La frecuencia amortiguada es la velocidad angular: $W_d=W_n\sqrt{1-\zeta^2}$

Tiempo de establecimiento (2%): $T_s=\frac{4}{\zeta W_n}$

Tiempo de pico: $T_p=\frac{\pi}{W_n\sqrt{1-\zeta^2}}$

Para amortiguamiento critico: $\zeta=1$ $P_{1,2}=-W_n$

Para sobremortiguado: $\zeta>1\rightarrow\zeta^2-1>0$: $P_{1,2}=-\zeta W_n\pm W_n\sqrt{1-\zeta^2}$

Ahora H(s) con lazo cerrado:

$$H(s)=\frac{1}{m}\cdot\frac{1}{s^2+2\zeta W_n s+W_n^2}$$

Para lazo cerrado hay una realimentación:

$$G_{CL}(s)=\frac{\frac{1}{m}\cdot\frac{1}{s^2+2\zeta W_ns+W_n^2}}{1+\frac{1}{m}\cdot\frac{1}{s^2+2\zeta W_ns+W_n^2}}=\frac{\frac{1}{m}\cdot\frac{1}{s^2+2\zeta W_ns+W_n^2}}{\frac{m(s^2+2\zeta W_ns+W_n^2)+1}{m(s^2+2\zeta W_ns+W_n^2)}}=\frac{1}{m(s^2+2\zeta W_ns+W_n^2)}\cdot\frac{m(s^2+2\zeta W_ns+W_n^2)}{m(s^2+2\zeta W_ns+W_n^2)+1}$$

$$=\frac{1}{m(s^2+2\zeta W_ns+W_n^2)+1}=\frac{1}{s^2+2\zeta W_ns+W_n^2+\frac{1}{m}}\cdot\frac{1}{m}$$

Respuesta al impulso (Subamortiguado): Si $F_E(t)=\delta(t)\longrightarrow F_E(s)=1$ $H(s)=\frac{Y(s)}{F_E(s)}=\frac{Y(s)}{1}=Y(s)$

$H(s)=Y(s)$

$$H(s)=\frac{1}{m}\cdot\frac{1}{s^2+2\zeta W_ns+W_n^2}\longrightarrow(s+\zeta W_n)^2+W_n^2(1-\zeta^2)=(s+\zeta W_n)^2+W_d^2$$

$$H(s)=\frac{1}{m}\cdot\frac{1}{(s+\zeta W_n)^2+W_d^2}$$

Si $\frac{1}{(s-a)^2+b^2}\longleftrightarrow e^{-at}\frac{\text{sen}(bt)}{b}$

$H(s)=\mathcal{L}\{h(t)\}$

$\mathcal{L}^{-1}\{H(s)\}=\mathcal{L}^{-1}\{\mathcal{L}\{h(t)\}\}$

$\mathcal{L}^{-1}\{H(s)\}=h(t)$

$$h(t)=\frac{1}{m}e^{-\zeta W_nt}\frac{\text{sen}(W_dt)}{W_d}$$

$$h(t)=\frac{1}{mW_d}e^{-\zeta W_nt}\text{sen}(W_dt)$$

Respuesta al escalón (Subamortiguado): Si $F_E(t)=U(t)\rightarrow F_E(S)=\frac{1}{S}$ $\quad H(S)=\frac{Y(S)}{F_E(S)}\rightarrow Y(S)=F_E(S)H(S)$

$$Y(S)=\frac{1}{m}\cdot\frac{1}{S^2+2\zeta\omega_n S+\omega_n^2}\cdot\frac{1}{S}=\frac{A}{S}+\frac{BS+C}{S^2+2\zeta\omega_n S+\omega_n^2}$$

$\rightarrow BS+C=B((S+\zeta\omega_n)-\zeta\omega_n)+C=B(S+\zeta\omega_n)+(C-B\zeta\omega_n)$

$\rightarrow (S+\zeta\omega_n)^2+\omega_n^2(1-\zeta^2)=(S+\zeta\omega_n)^2+\omega_d^2$

$$\frac{1}{m}=A(S^2+2\zeta\omega_n S+\omega_n^2)+(BS+C)S$$

$$\frac{1}{m}=AS^2+A2\zeta\omega_n S+A\omega_n^2+BS^2+SC$$

$$\frac{1}{m}=(A+B)S^2+(A2\zeta\omega_n+C)+A\omega_n^2$$

Si comparamos coeficientes de ambos lados:
$A+B=0,\ A2\zeta\omega_n+C=0,\ A\omega_n^2=\frac{1}{m}$

$$A=\frac{1}{m\omega_n^2}\ ;\ \text{De } \omega_n^2=\frac{K}{m}\rightarrow\frac{1}{K}=\frac{1}{m\omega_n^2}$$

$$A=\frac{1}{K},\ B=-A=-\frac{1}{m\omega_n^2},\ C=-A2\zeta\omega_n=-2\zeta\omega_n\frac{1}{m\omega_n^2}=-\frac{2\zeta}{m\omega_n}$$

$$\frac{BS+C}{S^2+2\zeta\omega_n S+\omega_n^2}=\frac{BS+C}{(S+\zeta\omega_n)^2+\omega_d^2}=B\frac{(S+\zeta\omega_n)}{(S+\zeta\omega_n)^2+\omega_d^2}+\frac{(C-B\zeta\omega_n)}{(S+\zeta\omega_n)^2+\omega_d^2}$$

$$Y(S)=\frac{1}{m}\cdot\frac{1}{S^2+2\zeta\omega_n S+\omega_n^2}\cdot\frac{1}{S}=A\frac{1}{S}+B\frac{(S+\zeta\omega_n)}{(S+\zeta\omega_n)^2+\omega_d^2}+\frac{(C-B\zeta\omega_n)}{(S+\zeta\omega_n)^2+\omega_d^2}$$

$$Y(S)=A\frac{1}{S}+B\frac{(S+\zeta\omega_n)}{(S+\zeta\omega_n)^2+\omega_d^2}+\frac{(C-B\zeta\omega_n)}{(S+\zeta\omega_n)^2+\omega_d^2}$$

Si usamos: $\mathcal{L}^{-1}\left\{\frac{1}{S}\right\}=1$

$\mathcal{L}^{-1}\left\{\frac{S+a}{(S+a)^2+b^2}\right\}=e^{-at}\cos(bt)$

$\mathcal{L}^{-1}\left\{\frac{b}{(S+a)^2+b^2}\right\}=e^{-at}\operatorname{sen}(bt)$

$\mathcal{L}^{-1}\left\{\frac{1}{(S+a)^2+b^2}\right\}=\frac{1}{b}e^{-at}\operatorname{sen}(bt)$

$$y(t)=\mathcal{L}^{-1}\{Y(S)\}=\mathcal{L}^{-1}\left\{A\frac{1}{S}\right\}+\mathcal{L}^{-1}\left\{B\frac{(S+\zeta\omega_n)}{(S+\zeta\omega_n)^2+\omega_d^2}\right\}+\mathcal{L}^{-1}\left\{\frac{(C-B\zeta\omega_n)}{(S+\zeta\omega_n)^2+\omega_d^2}\right\}$$

$$y(t)=\mathcal{L}^{-1}\{Y(S)\}=A\mathcal{L}^{-1}\left\{\frac{1}{S}\right\}+B\mathcal{L}^{-1}\left\{\frac{(S+\zeta\omega_n)}{(S+\zeta\omega_n)^2+\omega_d^2}\right\}+(C-B\zeta\omega_n)\mathcal{L}^{-1}\left\{\frac{1}{(S+\zeta\omega_n)^2+\omega_d^2}\right\}$$

$$y(t)=A+Be^{-\zeta\omega_n t}\cos(\omega_d t)+\frac{(C-B\zeta\omega_n)}{\omega_d}e^{-\zeta\omega_n}\operatorname{sen}(\omega_d t)$$

$$y(t)=A+e^{-\zeta\omega_n t}\left(B\cos(\omega_d t)+\frac{C-B\zeta\omega_n}{\omega_d}\operatorname{sen}(\omega_d t)\right)$$

$$y(t)=A+e^{-\zeta\omega_n t}\left(B\cos(\omega_d t)+\frac{C-B\zeta\omega_n}{\omega_d}\operatorname{sen}(\omega_d t)\right)$$

$$y(t)=\frac{1}{m\omega_n^2}+e^{-\zeta\omega_n t}\left(-\frac{1}{m\omega_n^2}\cos(\omega_d t)+\left(-\frac{2\zeta}{m\omega_n}-\left(-\frac{1}{m\omega_n^2}\zeta\omega_n\right)\right)\frac{1}{\omega_d}\operatorname{sen}(\omega_d t)\right)$$

$$y(t)=\frac{1}{m\omega_n^2}+e^{-\zeta\omega_n t}\left(-\frac{1}{m\omega_n^2}\cos(\omega_d t)+\left(-\frac{2\zeta}{m\omega_n}-\left(-\frac{1}{m\cancel{\omega_n^2}}\zeta\cancel{\omega_n}\right)\right)\frac{1}{\omega_d}\operatorname{sen}(\omega_d t)\right)$$

$$y(t)=\frac{1}{m\omega_n^2}+e^{-\zeta\omega_n t}\left(-\frac{1}{m\omega_n^2}\cos(\omega_d t)+\left(-\frac{2\zeta}{m\omega_n}+\frac{\zeta}{m\omega_n}\right)\frac{1}{\omega_d}\operatorname{sen}(\omega_d t)\right)$$

$$y(t)=\frac{1}{m\omega_n^2}+e^{-\zeta\omega_n t}\left(-\frac{1}{m\omega_n^2}\cos(\omega_d t)-\frac{\zeta}{m\omega_n\omega_d}\operatorname{sen}(\omega_d t)\right)$$

$$y(t)=\frac{1}{m\omega_n^2}\left(1-e^{-\zeta\omega_n t}\left(\cos(\omega_d t)+\frac{\zeta\omega_n}{\omega_d}\operatorname{sen}(\omega_d t)\right)\right)$$

Si $K=m\omega_n^2$

$$y(t)=\frac{1}{K}\left(1-e^{-\zeta\omega_n t}\left(\cos(\omega_d t)+\frac{\zeta\omega_n}{\omega_d}\operatorname{sen}(\omega_d t)\right)\right)$$

Respuesta al escalón (amortiguamiento crítico): $\zeta=1 \rightarrow \omega_d=0$

$$y(s)=\frac{1}{m}\cdot\frac{1}{s(s+\omega_n)^2}=\frac{A}{s}+\frac{B}{s+\zeta\omega_n}+\frac{C}{(s+\omega_n)^2}$$

$$\frac{1}{m}=A(s+\omega_n)^2+Bs(s+\omega_n)+Cs$$

$$A=\frac{1}{m\omega_n^2},\ B=-\frac{1}{m\omega_n^2},\ C=-\frac{1}{m\omega_n}$$

$$y(t)=\mathcal{L}^{-1}\{y(s)\}=\mathcal{L}^{-1}\left\{\frac{A}{s}\right\}+\mathcal{L}^{-1}\left\{\frac{B}{s+\zeta\omega_n}\right\}+\mathcal{L}^{-1}\left\{\frac{C}{(s+\omega_n)^2}\right\}$$

$$y(t)=A+Be^{-\omega_n t}+Cte^{-\omega_n t}$$

$$y(t)=\frac{1}{m\omega_n^2}-\frac{1}{m\omega_n^2}e^{-\omega_n t}-\frac{1}{m\omega_n}te^{-\omega_n t}$$

$$y(t)=\frac{1}{m\omega_n^2}(1-e^{-\omega_n t}-\omega_n te^{-\omega_n t})$$

Si $K=m\omega_n^2$

$$\boxed{y(t)=\frac{1}{K}(1-e^{-\omega_n t}(1+\omega_n t))}$$

Respuesta al escalón (Sobreamortiguado): $P_{1,2}=-\zeta\omega_n\pm\omega_n\sqrt{1-\zeta^2}$

$$y(s)=\frac{1}{m}\cdot\frac{1}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{1}{s}$$

$$\frac{1}{m}\cdot\frac{1}{s(s-s_1)(s-s_2)}=\frac{A}{s}+\frac{B}{s-s_1}+\frac{C}{s-s_2}$$

$$\frac{1}{m}=A(s-s_1)(s-s_2)+Bs(s-s_2)+Cs(s-s_1)$$

Para $s=0\rightarrow A=\frac{1}{ms_1s_2}=\frac{1}{m\omega_n^2}$

Para $s=s_1\rightarrow B=\frac{1}{ms_1(s_1-s_2)}$

Para $s=s_1\rightarrow C=\frac{1}{ms_2(s_2-s_1)}$

$$y(s)=\frac{A}{s}+\frac{B}{s-s_1}+\frac{C}{s-s_2}$$

$$y(t)=\mathcal{L}^{-1}\{y(s)\}=\mathcal{L}^{-1}\left\{\frac{A}{s}\right\}+\mathcal{L}^{-1}\left\{\frac{B}{s-s_1}\right\}+\mathcal{L}^{-1}\left\{\frac{C}{s-s_2}\right\}$$

$$y(t)=A+Be^{s_1t}+Ce^{s_2t}$$

$$y(t)=\frac{1}{m\omega_n^2}+\frac{1}{ms_1(s_1-s_2)}e^{(-\zeta\omega_n+\omega_n\sqrt{1-\zeta^2})t}+\frac{1}{ms_2(s_2-s_1)}e^{(-\zeta\omega_n-\omega_n\sqrt{1-\zeta^2})t}$$

Respuesta a la rampa:

$$Y(s)=\frac{1}{m}\cdot\frac{1}{s^2+2\zeta\omega_n s+\omega_n^2} \quad \text{Si } F_E(t)=r(t)=t\,u(t)\longrightarrow F_E(s)=R(s)=\frac{1}{s^2}$$

$$H(s)=\frac{Y(s)}{F_E(s)}\rightarrow Y(s)=F_E(s)H(s)=\frac{1}{m}\cdot\frac{1}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{1}{s^2}$$

$$Y(s)=\frac{1}{m}\cdot\frac{1}{s^2(s^2+2\zeta\omega_n s+\omega_n^2)}=\frac{A}{s}+\frac{B}{s^2}+\frac{Cs+D}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$\frac{1}{m}=As(s^2+2\zeta\omega_n s+\omega_n^2)+B(s^2+2\zeta\omega_n s+\omega_n^2)+(Cs+D)s^2$$

$$As(s^2+2\zeta\omega_n s+\omega_n^2)=As^3+A2\zeta\omega_n s^2+A\omega_n^2 s$$

$$B(s^2+2\zeta\omega_n s+\omega_n^2)=Bs^2+B2\zeta\omega_n s+B\omega_n^2$$

$$(Cs+D)s^2=Cs^3+Ds^2$$

$$\frac{1}{m}=(A+C)s^3+(A2\zeta\omega_n+B+D)s^2+(A\omega_n^2+B2\zeta\omega_n)s+B\omega_n^2$$

Si comparamos terminos de cada lado $A+C=0$, $A2\zeta\omega_n+B+D=0$, $A\omega_n^2+B2\zeta\omega_n=0$, $B\omega_n^2=\frac{1}{m}$

$$C=-A \qquad D=-A2\zeta\omega_n-B \qquad A\omega_n^2=-B2\zeta\omega_n \qquad B=\frac{1}{m\omega_n^2}$$

$$C=\frac{2\zeta}{m\omega_n^3} \qquad D=\frac{2\zeta}{m\omega_n^3}2\zeta\omega_n-\frac{1}{m\omega_n^2} \qquad A\omega_n^2=-\frac{1}{m\omega_n^2}2\zeta\omega_n$$

$$D=\frac{4\zeta^2}{m\omega_n^2}-\frac{1}{m\omega_n^2} \qquad A=-\frac{2\zeta}{m\omega_n^3}$$

$$D=\frac{4\zeta^2-1}{m\omega_n^2}$$

Para (Subamortiguado):

$$Y(s)=\frac{A}{s}+\frac{B}{s^2}+\frac{Cs+D}{s^2+2\zeta\omega_n s+\omega_n^2}$$

$$Cs+D=C((s+\zeta\omega_n)-\zeta\omega_n)+D=C(s+\zeta\omega_n)+(D-C\zeta\omega_n)$$

$$s^2+2\zeta\omega_n s+\omega_n^2=(s+\zeta\omega_n)^2+\omega_d^2$$

$$Y(s)=\frac{A}{s}+\frac{B}{s^2}+\frac{C(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2+\omega_d^2}+\frac{(D-C\zeta\omega_n)}{(s+\zeta\omega_n)^2+\omega_d^2}$$

$$y(t)=A\mathcal{L}^{-1}\left\{\frac{1}{s}\right\}+B\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\}+C\mathcal{L}^{-1}\left\{\frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2+\omega_d^2}\right\}+(D-C\zeta\omega_n)\mathcal{L}^{-1}\left\{\frac{1}{(s+\zeta\omega_n)^2+\omega_d^2}\right\}$$

$$y(t)=A+Bt+Ce^{-\zeta\omega_n t}\cos(\omega_d t)+(D-C\zeta\omega_n)\frac{1}{\omega_d}e^{-\zeta\omega_n t}\operatorname{sen}(\omega_d t)$$

$$y(t)=-\frac{2\zeta}{m\omega_n^3}+\frac{t}{m\omega_n^2}+\frac{2\zeta}{m\omega_n^3}e^{-\zeta\omega_n t}\cos(\omega_d t)+\left(\frac{4\zeta^2-1}{m\omega_n^2}-\frac{2\zeta}{m\omega_n^3}\zeta\omega_n\right)\frac{1}{\omega_d}e^{-\zeta\omega_n t}\operatorname{sen}(\omega_d t)$$

$$y(t)=-\frac{2\zeta}{m\omega_n^3}+\frac{t}{m\omega_n^2}+\frac{2\zeta}{m\omega_n^3}e^{-\zeta\omega_n t}\cos(\omega_d t)+\left(\frac{4\zeta^2-1}{m\omega_n^2}-\frac{2\zeta^2}{m\omega_n^2}\right)\frac{1}{\omega_d}e^{-\zeta\omega_n t}\operatorname{sen}(\omega_d t)$$

$$y(t)=-\frac{2\zeta}{m\omega_n^3}+\frac{t}{m\omega_n^2}+\frac{2\zeta}{m\omega_n^3}e^{-\zeta\omega_n t}\cos(\omega_d t)+\frac{2\zeta^2-1}{m\omega_n^2}\cdot\frac{1}{\omega_d}e^{-\zeta\omega_n t}\operatorname{sen}(\omega_d t)$$

$$y(t)=-\frac{2\zeta}{m\omega_n^3}+\frac{t}{m\omega_n^2}+\frac{2\zeta}{m\omega_n^3}e^{-\zeta\omega_n t}\cos(\omega_d t)+\frac{2\zeta^2-1}{m\omega_n^2\omega_d}e^{-\zeta\omega_n t}\operatorname{sen}(\omega_d t)$$

Nota: En los polos del sistema la parte real $\xrightarrow{-\zeta\omega_n}$ determina la tasa de decaimiento exponencial

La parte imaginaria determina la oscilacion angular ω_d

frecuencia $\xrightarrow{\text{natural}}$ amortiguada:

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

ts: tiempo establecimiento (tiempo al rededor del $\pm 5\%$ del valor de equilibrio).

$$t_s = \frac{3}{\zeta\omega_n}$$