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1. La distancia media entre dos señales periódicas $x_1(t) \in \mathbb{R},\mathbb{C}$ y $x_2(t) \in \mathbb{R},\mathbb{C}$; se puede expresar a partir de la potencia media de la diferencia entre ellas:

$$d^2(x_1,x_2) = \bar{P}_{x_1-x_2} = \lim_{T\to\infty} \frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\,dt.$$

Sea $x_1(t)$ y $x_2(t)$ dos señales definidas como:

$$x_1(t) = Ae^{-jnw_0 t}$$

$$x_2(t) = Be^{jmw_0 t}$$

con $w_0 = \frac{2\pi}{T}$; $T, A, B \in \mathbb{R}^+$ y $n, m \in \mathbb{Z}$. Determine la distancia entre las dos señales. Compruebe sus resultados con Python.

$x_1(t)=Ae^{-jn\omega_0 t}$, $x_2(t)=Be^{jm\omega_0 t}$; $\omega_0=\frac{2\pi}{T}$

$d^2(x_1,x_2)=\bar P_{x_1-x_2}=\lim_{T\to\infty}\frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\partial t$

$$\bar P_{x_1-x_2} = \frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\partial t$$

$$=\frac{1}{T}\left(\int_T (x_1(t)-x_2(t))(x_1(t)-x_2(t))^*\right)\partial t$$

$$=\frac{1}{T}\left(\int_T (x_1(t)-x_2(t))(x_1^*(t)-x_2^*(t))\partial t\right)$$

$$=\frac{1}{T}\left(\int_T x_1(t)x_1^*(t)\partial t-\int_T x_1(t)x_2^*(t)\partial t - \int_T x_2(t)x_1^*(t)\partial t+\int_T x_2(t)x_2^*(t)\partial t\right)$$

$$=\underbrace{\frac{1}{T}\int_T |x_1(t)|^2\partial t}_{\bar P_{x_1}}-\underbrace{\frac{2}{T}\int_T x_1(t)x_2^*(t)\partial t}_{C_x}+\underbrace{\frac{1}{T}\int_T |x_2(t)|^2\partial t}_{\bar P_{x_2}}$$

$$\bar P_{x_1}=\frac{1}{T}\int_T x_1(t)x_1^*(t)\partial t=\frac{1}{T}\int_0^T Ae^{-jn\omega_0 t}(Ae^{-jn\omega_0 t})^*=\frac{1}{T}\int_0^T Ae^{jn\omega_0 t}Ae^{-jn\omega_0 t}\partial t$$

$$=\frac{A^2}{T}\int_0^T e^{(n-n)j\omega_0 t}\partial t=\frac{A^2}{T}\int_0^T 1\,\partial t=\frac{A^2}{T}(T-0)=\frac{A^2}{\cancel{T}}\cdot\cancel{T}=A^2$$

$$\bar P_{x_2}=\frac{1}{T}\int_T x_2(t)x_2^*(t)\partial t=\frac{1}{T}\int_0^T Be^{jm\omega_0 t}(Be^{jm\omega_0 t})^*=\frac{1}{T}\int_0^T Be^{jm\omega_0 t}Be^{-jm\omega_0 t}\partial t$$

$$=\frac{B^2}{T}\int_0^T e^{(m-m)j\omega_0 t}\partial t=\frac{B^2}{T}\int_0^T 1\,\partial t=\frac{B^2}{T}(T-0)=\frac{B^2}{\cancel{T}}\cdot\cancel{T}=B^2$$

El cruzado tiene dos casos:

① Si $n\neq -m \rightarrow C_x=-\frac{2}{T}\int_T x_1(t)x_2^*(t)\partial t=-\frac{2}{T}\int_0^T Ae^{-jn\omega_0 t}(Be^{jm\omega_0 t})^*\partial t$

$$=-\frac{2}{T}\int_0^T Ae^{-jn\omega_0 t}Be^{-jm\omega_0 t}\partial t=-\frac{2AB}{T}\int_0^T e^{-(n+m)j\omega_0 t}\partial t$$

→ Si $n,m\in\mathbb{Z}\rightarrow n+m=r\in\mathbb{Z}$

$$=-\frac{2AB}{T}\int_0^T e^{-rj\omega_0 t}\partial t=-\frac{j\cdot 2AB}{j\cdot -jr\omega_0 T}e^{-jr\omega_0 t}\Big|_0^T=-\frac{j2AB}{r\frac{2\pi}{\cancel{T}}\cancel{T}}\left(e^{-jr\frac{2\pi}{\cancel{T}}\cdot\cancel{T}}-e^{-jr\frac{2\pi}{T}\cdot 0}\right)$$

$$=-\frac{jAB}{r\pi}\left(e^{-jr2\pi}-1\right)=-\frac{jAB}{\pi r}\left(\cos(2\pi r)-j\,\mathrm{sen}(2\pi r)-1\right)$$

$$=\frac{jAB}{\pi r}(1-0-1)=\frac{jAB}{\pi r}\cdot 0=0$$

② Si $n=-m\rightarrow C_x=-\frac{2}{T}\int_T x_1(t)x_2^*(t)\partial t=-\frac{2}{T}\int_0^T Ae^{-jn\omega_0 t}(Be^{jm\omega_0 t})^*\partial t=-\frac{2}{T}\int_0^T Ae^{-j(-m)\omega_0 t}Be^{-jm\omega_0 t}\partial t$

$$=-\frac{2}{T}\int_0^T Ae^{jm\omega_0 t}Be^{-jm\omega_0 t}\partial t=-\frac{2AB}{T}\int_0^T e^{(m-m)j\omega_0 t}\partial t=-\frac{2AB}{T}\int_0^T 1\,\partial t$$

$$=-\frac{2AB}{T}(T-0)=-\frac{2AB}{\cancel{T}}\cdot\cancel{T}=-2AB$$

$$\bar P_{x_1-x_2}=\begin{cases}A^2+B^2 & ;\ n\neq -m\\ A^2+B^2-2AB & ;\ n=-m\end{cases}$$

$$d(x_1,x_2)=\begin{cases}\sqrt{A^2+B^2} & ;\ n\neq -m\\ \sqrt{A^2+B^2-2AB} & ;\ n=-m\end{cases}$$

2. Encuentre la señal en tiempo discreto al utilizar un conversor análogo digital con frecuencia de muestreo de $5kHz$ y 4 bits de capacidad de representación, aplicado a la señal continua: $x(t) = 3\cos(1000\pi t) + 5\sin(3000\pi t) + 10\cos(11000\pi t)$. Realizar la simulación del proceso de discretización (incluyendo al menos tres periodos de $x(t)$). En caso de que la discretización no sea apropiada, diseñe e implemente un conversor adecuado para la señal estudiada.

$X(t) = 3\cos(1000\pi t) + 5\,\text{Sen}(3000\pi t) + 10\cos(11000\pi t)$

$F_s = 5\,KHz$, $\#$ de estados $= 2^{4\,Bits} = 16$

$t = nT_s \longrightarrow F_s = \frac{1}{T_s}$; $F = \frac{1}{T} \longrightarrow T = \frac{2\pi}{\omega}$

$A\cos(\Omega n)$
$A\cos(2\pi f n)$ $\longrightarrow \Omega = 2\pi f \longrightarrow f = \frac{T_s}{T} = \frac{F}{F_s}$

$X(t) = X_1(t) + X_2(t) + X_3(t) \rightarrow$

Para $X_1(t) = 3\cos(1000\pi t)$

$$W_1 = 1000\pi \longrightarrow T_1 = \frac{2\pi}{W_1} = \frac{2\pi}{1000\pi} = \frac{1}{500} \longrightarrow F_1 = \frac{1}{T} = \frac{1}{\frac{1}{500}} = 500\,Hz \longrightarrow X_1[n] = A\cos\left(2\pi n\frac{F}{F_s}\right) = 3\cos\left(2\pi\cdot n\cdot\frac{500}{5000}\right) = 3\cos\left(\frac{n\pi}{5}\right)$$

Para $X_2(t) = 5\,\text{Sen}(3000\pi t)$

$$W_2 = 3000\pi \longrightarrow T_2 = \frac{2\pi}{W_1} = \frac{2\pi}{3000\pi} = \frac{1}{1500} \longrightarrow F_2 = \frac{1}{T} = \frac{1}{\frac{1}{1500}} = 1500\,Hz \longrightarrow X_2[n] = A\,\text{sen}\left(2\pi n\frac{F}{F_s}\right) = 5\,\text{Sen}\left(2\pi\cdot n\cdot\frac{1500}{5000}\right) = 5\,\text{Sen}\left(\frac{3n\pi}{5}\right)$$

Para $X_3(t) = 10\cos(11000\pi t)$

$$W_3 = 11000\pi \longrightarrow T_3 = \frac{2\pi}{W_1} = \frac{2\pi}{11000\pi} = \frac{1}{5500} \longrightarrow F_3 = \frac{1}{T} = \frac{1}{\frac{1}{5500}} = 5500\,Hz \longrightarrow X_3[n] = A\cos\left(2\pi n\frac{F}{F_s}\right) = 10\cos\left(2\pi\cdot n\cdot\frac{5500}{5000}\right) = 10\cos\left(\frac{11n\pi}{5}\right)$$

$X[n] = 3\cos\left(\frac{n\pi}{5}\right) + 5\,\text{Sen}\left(\frac{3n\pi}{5}\right) + 10\cos\left(\frac{11n\pi}{5}\right)$

$X[n] = 3\cos\left(\frac{\pi}{5}n\right) + 5\,\text{Sen}\left(\frac{\pi}{5}n\right) + 10\cos\left(\frac{\pi}{5}n\right)$

$X[n] = 13\cos\left(\frac{\pi}{5}n\right) + 5\,\text{Sen}\left(\frac{\pi}{5}n\right)$

$\frac{W_1}{W_2} = \frac{1000\pi}{3000\pi} = \frac{1}{3} \in Q$

$\frac{W_1}{W_3} = \frac{1000\pi}{11000\pi} = \frac{1}{11} \in Q$

$\frac{W_2}{W_3} = \frac{3000\pi}{11000\pi} = \frac{3}{11} \in Q$

↳ Como todas son racionales la señal es cuasiperiodica.

Si $F_s \geq 2F_{max} \rightarrow 2F_{max} = 2\cdot 5500\,Hz = 11000\,Hz \rightarrow F_s = 5000\,Hz$.

No cumple Nyquist

Vamos a comparar con las originales. $\longrightarrow -\pi \leq \Omega < \pi$

$\Omega_1 = \frac{1}{5}\pi$ ✓ , $\Omega_2 = \frac{3}{5}\pi$ ✓ , $\Omega_3 = \frac{11}{5}\pi$ x $\longrightarrow$ Es una copia / aliasings

$\Omega_{3\,original} = \frac{11}{5}\pi - 2\pi = \frac{1}{5}\pi$

Suponemos una frecuencia de muestreo mucho más grande

$F_s = 4F_{max} \longrightarrow F_s = 4(5500)\,Hz = 22\,KHz$ $\qquad T_s = \frac{1}{22000}$

Para $X_1(t) = 3\cos(1000\pi t)$

$$W_1 = 1000\pi \longrightarrow T_1 = \frac{2\pi}{W_1} = \frac{2\pi}{1000\pi} = \frac{1}{500} \longrightarrow F_1 = \frac{1}{T} = \frac{1}{\frac{1}{500}} = 500\,Hz \longrightarrow X_1[n] = A\cos\left(2\pi n\frac{F}{F_s}\right) = 3\cos\left(2\pi\cdot n\cdot\frac{500}{22000}\right) = 3\cos\left(\frac{\pi}{22}n\right)$$

Para $X_2(t) = 5\,\text{Sen}(3000\pi t)$

$$W_2 = 3000\pi \longrightarrow T_2 = \frac{2\pi}{W_1} = \frac{2\pi}{3000\pi} = \frac{1}{1500} \longrightarrow F_2 = \frac{1}{T} = \frac{1}{\frac{1}{1500}} = 1500\,Hz \longrightarrow X_2[n] = A\,\text{sen}\left(2\pi n\frac{F}{F_s}\right) = 5\,\text{Sen}\left(2\pi\cdot n\cdot\frac{1500}{22000}\right) = 5\,\text{Sen}\left(\frac{3\pi}{22}n\right)$$

Para $X_3(t) = 10\cos(11000\pi t)$

$$W_3 = 11000\pi \longrightarrow T_3 = \frac{2\pi}{W_1} = \frac{2\pi}{11000\pi} = \frac{1}{5500} \longrightarrow F_3 = \frac{1}{T} = \frac{1}{\frac{1}{5500}} = 5500\,Hz \longrightarrow X_3[n] = A\cos\left(2\pi n\frac{F}{F_s}\right) = 10\cos\left(2\pi\cdot n\cdot\frac{5500}{22000}\right) = 10\cos\left(\frac{\pi}{2}n\right)$$

$X[n] = 3\cos\left(\frac{\pi}{22}n\right) + 5\,\text{Sen}\left(\frac{3\pi}{22}n\right) + 10\cos\left(\frac{\pi}{2}n\right)$

Vamos a comparar con las originales. $\longrightarrow -\pi \leq \Omega < \pi$

$\Omega_1 = \frac{1}{22}\pi$ ✓ , $\Omega_2 = \frac{3}{22}\pi$ ✓ , $\Omega_3 = \frac{1}{22}\pi$ ✓ $\longrightarrow$ las tres frecuencias digitales están en los originales.

$X[n] = 3\cos\left(\frac{\pi}{22}n\right) + 5\,\text{Sen}\left(\frac{3\pi}{22}n\right) + 10\cos\left(\frac{\pi}{2}n\right)$

Para saber el periodo a gráficar:

$$T = r\,\frac{2\pi}{w_1} = \ell\,\frac{2\pi}{w_2} = m\,\frac{2\pi}{w_3}$$

$$T = r\,\frac{2\cancel{\pi}}{1000\cancel{\pi}} = \ell\,\frac{2\cancel{\pi}}{3000\cancel{\pi}} = m\,\frac{2\cancel{\pi}}{11000\cancel{\pi}}$$

$$T = r\,\frac{1}{500} = \ell\,\frac{1}{1500} = m\,\frac{1}{5500}$$

$$5500T = r\,\frac{5500}{500} = \ell\,\frac{5500}{1500} = m\,\frac{\cancel{5500}}{\cancel{5500}}$$

$$5500T = r\,11 = \ell\,\frac{11}{3} = m$$

$$3\cdot 5500T = r\cdot 3\cdot 11 = \ell\cdot\cancel{3}\cdot\frac{11}{\cancel{3}} = m\cdot 3$$

$$16500T = 33r = 11\ell = 3m$$

$\text{mcm}\{33, 1, 11\} = 33$

$$16500\,T = 33$$

$$T = \frac{33}{16500}$$

$$T = \frac{1}{500}$$

3. Sea $x''(t)$ la segunda derivada de la señal $x(t)$, donde $t \in [t_i, t_f]$. Demuestre que los coeficientes de la serie exponencial de Fourier se pueden calcular según:

$$c_n = \frac{1}{(ti - tf)n^2 w_o^2}\int_{t_i}^{t_f} x''(t) e^{-jnw_o t} dt; \quad n \in \mathbb{Z}.$$

¿Cómo se pueden calcular los coeficientes a_n y b_n desde $x''(t)$ en la serie trigonométrica de Fourier?.

Con $X(t)$ para $t \in [t_i, t_f]$

Demostrar: $C_n = \frac{1}{(t_i - t_f)n^2\omega_0^2}\int_T X''(t) e^{-jn\omega_0 t}\partial t$ con $n \in \mathbb{Z}$

Tenemos: $X(t) = \sum_n C_n e^{jn\omega_0 t}$

$$X'(t) = \frac{\partial}{\partial t}\sum_n C_n e^{jn\omega_0 t} = \sum_n C_n \frac{\partial e^{jn\omega_0 t}}{\partial t}$$

$$X''(t) = \frac{\partial X'(t)}{\partial t} = \frac{\partial}{\partial t}\sum_n C_n \frac{\partial e^{jn\omega_0 t}}{\partial t} = \sum_n C_n \frac{\partial^2 e^{jn\omega_0 t}}{\partial t^2}$$

$$\frac{\partial e^{jn\omega_0 t}}{\partial t} = jn\omega_0 e^{jn\omega_0 t}; \quad \frac{\partial^2 e^{jn\omega_0 t}}{\partial t^2} = \frac{\partial (jn\omega_0 e^{jn\omega_0 t})}{\partial t} = jn\omega_0 \cdot jn\omega_0 e^{jn\omega_0 t} = -n^2\omega_0^2 e^{jn\omega_0 t}$$

$$X''(t) = \sum_n C_n \frac{\partial^2 e^{jn\omega_0 t}}{\partial t^2} = \sum_n \underbrace{-C_n n^2 \omega_0^2}_{C\_n} e^{jn\omega_0 t} = \sum_n C\_n e^{jn\omega_0 t}; \text{ con } C\_n = -C_n n^2 \omega_0^2$$

Si $C_n = \frac{1}{T}\int_T X(t) e^{-jn\omega_0 t}\partial t$ y $C\_n = \frac{1}{T}\int_T X''(t) e^{-jn\omega_0 t}\partial t \longrightarrow$ Remplazamos el $C\_n \longrightarrow -C_n n^2\omega_0^2 = \frac{1}{T}\int_T X''(t) e^{-jn\omega_0 t}\partial t$

$$C_n = \frac{1}{-(t_f - t_i)n^2\omega_0^2}\int_T X''(t) e^{-jn\omega_0 t}\partial t$$

$$C_n = \frac{1}{(t_i - t_f)n^2\omega_0^2}\int_T X''(t) e^{-jn\omega_0 t}\partial t$$

$$C_n = \frac{1}{(t_i - t_f)n^2\omega_0^2}\int_T X''(t) e^{-jn\omega_0 t}\partial t = \frac{1}{(t_i - t_f)n^2\omega_0^2}\int_T X''(t)(\cos(n\omega_0 t) - j\,\text{sen}(n\omega_0 t))\partial t$$

$$= \frac{1}{(t_i - t_f)n^2\omega_0^2}\int_T X''(t)\cos(n\omega_0 t)\partial t - j\frac{1}{(t_i - t_f)n^2\omega_0^2}\int_T X''(t)\,\text{sen}(n\omega_0 t)\partial t$$

Sabiendo que: $a_n = \frac{2}{T}\int_T X(t)\cos(n\omega_0 t)\partial t$; $b_n = \frac{2}{T}\int_T X(t)\,\text{sen}(n\omega_0 t)\partial t$ y $a_n = 2\,\text{Re}\{c_n\}$; $b_n = -2\,\text{Im}\{c_n\}$

$$a_n = 2\,\text{Re}\{c_n\} = 2\frac{1}{(t_i - t_f)n^2\omega_0^2}\int_T X''(t)\cos(n\omega_0 t)\partial t$$

$$b_n = -2\,\text{Im}\{c_n\} = -2\cdot -\frac{1}{(t_i - t_f)n^2\omega_0^2}\int_T X''(t)\,\text{sen}(n\omega_0 t)\partial t = 2\frac{1}{(t_i - t_f)n^2\omega_0^2}\int_T X''(t)\,\text{sen}(n\omega_0 t)\partial t$$

También se pueden calcular con: $X(t) = \sum_n a_n \cos(n\omega_0 t) + b_n\,\text{sen}(n\omega_0 t)$

$$X(t) = \sum_n a_n\cos(n\omega_0 t) + b_n\,\text{sen}(n\omega_0 t)$$

$$X'(t) = \sum_n -a_n\,\text{sen}(n\omega_0 t)n\omega_0 + b_n\cos(n\omega_0 t)n\omega_0$$

$$X''(t) = \sum_n -a_n\cos(n\omega_0 t)n\omega_0 \cdot n\omega_0 - b_n\,\text{sen}(n\omega_0 t)n\omega_0 \cdot n\omega_0 = \sum_n \underbrace{-a_n n^2\omega_0^2}_{a\_n}\cos(n\omega_0 t) \underbrace{- b_n n^2\omega_0^2}_{b\_n}\,\text{sen}(n\omega_0 t)$$

$$a\_n = -a_n n^2\omega_0^2, \quad b\_n = -b_n n^2\omega_0^2$$

$$a_n = \frac{2}{T}\int_T X(t)\cos(n\omega_0 t)\partial t \longrightarrow a\_n = \frac{2}{T}\int_T X''(t)\cos(n\omega_0 t)\partial t$$

$$-a_n n^2\omega_0^2 = \frac{2}{T}\int_T X''(t)\cos(n\omega_0 t)\partial t$$

$$a_n = \frac{2}{-(t_f - t_i)n^2\omega_0^2}\int_T X''(t)\cos(n\omega_0 t)\partial t$$

$$a_n = \frac{2}{(t_i - t_f)n^2\omega_0^2}\int_T X''(t)\cos(n\omega_0 t)\partial t$$

$$b_n = \frac{2}{T}\int_T X(t)\,\text{sen}(n\omega_0 t)\partial t \longrightarrow b\_n = \frac{2}{T}\int_T X''(t)\,\text{sen}(n\omega_0 t)\partial t$$

$$-b_n n^2\omega_0^2 = \frac{2}{T}\int_T X''(t)\,\text{sen}(n\omega_0 t)\partial t$$

$$b_n = \frac{2}{-(t_f - t_i)n^2\omega_0^2}\int_T X''(t)\,\text{sen}(n\omega_0 t)\partial t$$

$$b_n = \frac{2}{(t_i - t_f)n^2\omega_0^2}\int_T X''(t)\,\text{sen}(n\omega_0 t)\partial t$$

4. Encuentre el espectro de Fourier, su parte real, imaginaria, magnitud, fase y el error relativo para $n \in \{0, \pm1, \pm2, \pm3, \pm4, \pm5\}$, a partir de $x''(t)$ para la señal $x(t)$ en la Figura 1. Compruebe el espectro obtenido con la estimación a partir de $x(t)$. Presente las simulaciones de Python respectivas.

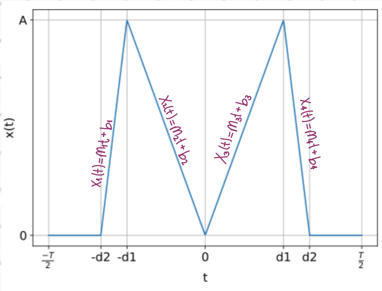


Figura 1: $x(t)$ para el ejercicio 1.4

$$X(t)=\begin{cases} 0 & -\frac{T}{2}\leq t < -d_2 \\ \frac{A}{d_2-d_1}(t+d_2) & -d_2 \leq t < -d_1 \\ \frac{-A}{d_1}t & -d_1 \leq t < 0 \\ \frac{A}{d_1}t & 0 \leq t < d_1 \\ \frac{A}{d_2-d_1}(d_2-t) & d_1 \leq t < d_2 \\ 0 & d_2 \leq t < \frac{T}{2} \end{cases}$$

Para $X_1(t)=m_1t+b_1 \rightarrow m_1=\frac{A-0}{-d_1-(-d_2)}=\frac{A}{d_2-d_1}$; En $t=-d_2 \rightarrow X(-d_2)=\frac{A}{d_2-d_1}\cdot -d_2+b_1$

$$0=\frac{-d_2A}{d_2-d_1}+b_1$$

$$b_1=\frac{d_2}{d_2-d_1}A$$

$$X_1(t)=\frac{A}{d_2-d_1}t+\frac{d_2}{d_2-d_1}A$$

$$X_1(t)=\frac{A}{d_2-d_1}(t+d_2)$$

Para $X_2(t)=m_2t+b_2 \rightarrow m_2=\frac{0-A}{0-(-d_1)}=\frac{-A}{d_1}$; En $t=-d_1 \rightarrow X(-d_1)=\frac{-A}{d_1}\cdot -d_1+b_2$

$$A=A+b_2$$

$$b_2=A-A=0$$

$$X_2(t)=\frac{-A}{d_1}t$$

Para $X_3(t)=m_3t+b_3 \rightarrow m_3=\frac{A-0}{d_1-0}=\frac{A}{d_1}$; En $t=0 \rightarrow X(0)=\frac{A}{d_1}\cdot 0+b_3$

$$0=0+b_3$$

$$b_3=0$$

$$X_3(t)=\frac{A}{d_1}t$$

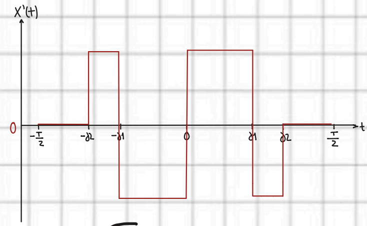
Para $X_4(t)=m_4t+b_4 \rightarrow m_4=\frac{0-A}{d_2-d_1}=\frac{-A}{d_2-d_1}$; En $t=d_2 \rightarrow X(d_2)=\frac{-A}{d_2-d_1}d_2+b_4$

$$0=\frac{-A}{d_2-d_1}\cdot d_2+b_4$$

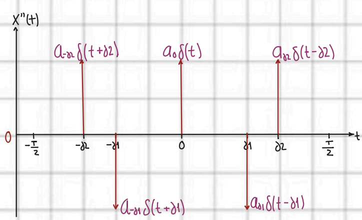
$$b_4=\frac{Ad_2}{d_2-d_1}$$

$$X_4(t)=\frac{-A}{d_2-d_1}t+\frac{Ad_2}{d_2-d_1}$$

$$X_4(t)=\frac{A}{d_2-d_1}(d_2-t)$$



$$X'(t)=\begin{cases} 0 & -\frac{T}{2}\leq t < -d_2 \\ \frac{A}{d_2-d_1} & -d_2 \leq t < -d_1 \\ \frac{-A}{d_1} & -d_1 \leq t < 0 \\ \frac{A}{d_1} & 0 \leq t < d_1 \\ -\frac{A}{d_2-d_1} & d_1 \leq t < d_2 \\ 0 & d_2 \leq t < \frac{T}{2} \end{cases}$$



$X''(t)=0$ en cada tramo y en los puntos donde hay cambios tiende a $\pm\infty$ y estos cambios los podemos representar como $a_k\delta(t\pm t_k)$ donde a_k es la diferencia de la derivada por la derecha y la derivada por la izquierda y t_k donde ocurre el salto (cambio instantáneo)

$$a_{-\partial 2} = x'(-\partial 2^{+}) - x'(-\partial 2^{-}) = m_1 - 0 = \frac{A}{\partial 2-\partial 1}$$

$$a_{-\partial 1} = x'(-\partial 1^{+}) - x'(-\partial 1^{-}) = m_2 - m_1 = \frac{-A}{\partial 1} - \frac{A}{\partial 2-\partial 1} = -A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)$$

$$a_{0} = x'(0^{+}) - x'(0^{-}) = m_3 - m_2 = \frac{A}{\partial 1} - \frac{-A}{\partial 1} = \frac{A}{\partial 1}(1+1) = \frac{2A}{\partial 1}$$

$$a_{\partial 1} = x'(\partial 1^{+}) - x'(\partial 1^{-}) = m_4 - m_3 = -\frac{A}{\partial 2-\partial 1} - \frac{A}{\partial 1} = -A\left(\frac{1}{\partial 2-\partial 1}+\frac{1}{\partial 1}\right)$$

$$a_{\partial 2} = x'(\partial 2^{+}) - x'(\partial 2^{-}) = 0 - m_4 = 0 - -\frac{A}{\partial 2-\partial 1} = \frac{A}{\partial 2-\partial 1}$$

$$x''(t) = a_{-\partial 2}\,\delta(t+\partial 2) + a_{-\partial 1}\,\delta(t+\partial 1) + a_0\,\delta(t) + a_{\partial 1}\,\delta(t-\partial 1) + a_{\partial 2}\,\delta(t-\partial 2)$$

$$x''(t) = \frac{A}{\partial 2-\partial 1}\,\delta(t+\partial 2) - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\delta(t+\partial 1) + \frac{2A}{\partial 1}\,\delta(t) - A\left(\frac{1}{\partial 2-\partial 1}+\frac{1}{\partial 1}\right)\delta(t-\partial 1) + \frac{A}{\partial 2-\partial 1}\,\delta(t-\partial 2)$$

$$x''(t) = \frac{A}{\partial 2-\partial 1}\big(\delta(t+\partial 2)+\delta(t-\partial 2)\big) - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\big(\delta(t+\partial 1)+\delta(t-\partial 1)\big) + \frac{2A}{\partial 1}\,\delta(t)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\int_T x''(t)\,e^{-jn\omega_0 t}\,\partial t \qquad \text{con } T = t_f - t_i = \frac{T}{2} - \left(-\frac{T}{2}\right) = T \quad y \quad \omega_0 = \frac{2\pi}{T}$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\int_{-T/2}^{T/2}\left(\frac{A}{\partial 2-\partial 1}\big(\delta(t+\partial 2)+\delta(t-\partial 2)\big) - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\big(\delta(t+\partial 1)+\delta(t-\partial 1)\big) + \frac{2A}{\partial 1}\,\delta(t)\right)e^{-jn\omega_0 t}\,\partial t$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\int_{-T/2}^{T/2}\frac{A}{\partial 2-\partial 1}\big(\delta(t+\partial 2)+\delta(t-\partial 2)\big)e^{-jn\omega_0 t}\,\partial t - \int_{-T/2}^{T/2}A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\big(\delta(t+\partial 1)+\delta(t-\partial 1)\big)e^{-jn\omega_0 t}\,\partial t + \int_{-T/2}^{T/2}\frac{2A}{\partial 1}\,\delta(t)\,e^{-jn\omega_0 t}\,\partial t\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\int_{-T/2}^{T/2}\frac{A}{\partial 2-\partial 1}\delta(t+\partial 2)e^{-jn\omega_0 t}\,\partial t + \int_{-T/2}^{T/2}\frac{A}{\partial 2-\partial 1}\delta(t-\partial 2)e^{-jn\omega_0 t}\,\partial t - \int_{-T/2}^{T/2}A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\delta(t+\partial 1)e^{-jn\omega_0 t}\,\partial t\right.$$

$$\left. - \int_{-T/2}^{T/2}A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\delta(t-\partial 1)e^{-jn\omega_0 t}\,\partial t + \int_{-T/2}^{T/2}\frac{2A}{\partial 1}\,\delta(t)\,e^{-jn\omega_0 t}\,\partial t\right) \quad \text{Usando} \rightarrow \int_{-\infty}^{\infty}x(t)\,\delta(t\pm t_0)\,\partial t = x(\mp t_0)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\frac{A}{\partial 2-\partial 1}e^{-jn\omega_0\cdot -\partial 2} + \frac{A}{\partial 2-\partial 1}e^{-jn\omega_0\cdot\partial 2} - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)e^{-jn\omega_0\cdot -\partial 1} - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)e^{-jn\omega_0\cdot\partial 1} + \frac{2A}{\partial 1}\cdot\cancelto{1}{e^{0}}\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\frac{A}{\partial 2-\partial 1}\big(\cos(n\omega_0\partial 2)\cancel{+j\,sen(n\omega_0\partial 2)} + \cos(n\omega_0\partial 2)\cancel{-j\,sen(n\omega_0\partial 2)}\big)\right.$$

$$\left. - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\big(\cos(n\omega_0\partial 1)\cancel{+j\,sen(n\omega_0\partial 1)} + \cos(n\omega_0\partial 1)\cancel{-j\,sen(n\omega_0\partial 1)}\big) + \frac{2A}{\partial 1}\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\frac{A}{\partial 2-\partial 1}\cdot 2\cos(n\omega_0\partial 2) - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot 2\cos(n\omega_0\partial 1) + \frac{2A}{\partial 1}\right)$$

$$C_n = \frac{2A}{(t_i-t_f)n^2\omega_0^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1) + \frac{1}{\partial 1}\right)$$

$$C_n = \frac{2A}{\left(-\frac{T}{2}-\frac{T}{2}\right)n^2\frac{2^2\pi^2}{T^2}}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1) + \frac{1}{\partial 1}\right)$$

$$C_n = \frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1) + \frac{1}{\partial 1}\right)$$

Ahora confirmamos con $x(t)$ → Tenemos en cuenta que la señal es par y que el espectro solo va depender de a_n

$$a_n=\frac{2}{T}\int_T x(t)\cos(n\omega_0 t)\,dt=\frac{2}{T}\left(\int_{-\frac{T}{2}}^{-d_2}\cancelto{0}{0\cdot\cos(n\omega_0 t)\,dt}+\int_{-d_2}^{-d_1}\frac{A}{d_2-d_1}(t+d_2)\cos(n\omega_0 t)\,dt+\int_{-d_1}^{0}-\frac{A}{d_1}t\cos(n\omega_0 t)\,dt+\int_{0}^{d_1}\frac{A}{d_1}t\cos(n\omega_0 t)\,dt+\int_{d_1}^{d_2}\frac{A}{d_2-d_1}(d_2-t)\cos(n\omega_0 t)\,dt\right.$$
$$\left.+\int_{d_2}^{\frac{T}{2}}\cancelto{0}{0\cdot\cos(n\omega_0 t)\,dt}\right)$$

$$=\frac{2}{T}\left(\frac{A}{d_2-d_1}\left(\int_{-d_2}^{-d_1}t\cos(n\omega_0 t)\,dt+\int_{-d_2}^{-d_1}d_2\cos(n\omega_0 t)\,dt\right)-\frac{A}{d_1}\int_{-d_1}^{0}t\cos(n\omega_0 t)\,dt+\frac{A}{d_1}\int_{0}^{d_1}t\cos(n\omega_0 t)\,dt+\frac{A}{d_2-d_1}\left(\int_{d_1}^{d_2}d_2\cos(n\omega_0 t)\,dt-\int_{d_1}^{d_2}t\cos(n\omega_0 t)\,dt\right)\right)$$

$$\int_{-d_2}^{-d_1}t\cos(n\omega_0 t)\,dt\rightarrow\begin{matrix}u=t\\du=dt\end{matrix}\quad\begin{matrix}dv=\cos(n\omega_0 t)\\v=\frac{\text{sen}(n\omega_0 t)}{n\omega_0}\end{matrix}\longrightarrow uv-\int v\,du$$

Nota: $\text{sen}(-x)=-\text{sen}(x)$ y $\cos(-x)=\cos(x)$

$$t\frac{\text{sen}(n\omega_0 t)}{n\omega_0}\Big|_{-d_2}^{-d_1}-\int_{-d_2}^{-d_1}\frac{\text{sen}(n\omega_0 t)}{n\omega_0}\,dt=t\frac{\text{sen}(n\omega_0 t)}{n\omega_0}\Big|_{-d_2}^{-d_1}-\frac{-\cos(n\omega_0 t)}{n^2\omega_0^2}\Big|_{-d_2}^{-d_1}=\left(-d_1\frac{\text{sen}(-n\omega_0 d_1)}{n\omega_0}+d_2\frac{\text{sen}(-n\omega_0 d_2)}{n\omega_0}\right)-\left(\frac{-\cos(-n\omega_0 d_1)}{n^2\omega_0^2}-\frac{-\cos(-n\omega_0 d_2)}{n^2\omega_0^2}\right)$$
$$=d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}-d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}+\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}-\frac{\cos(n\omega_0 d_2)}{n^2\omega_0^2}$$

$$\int_{-d_2}^{-d_1}d_2\cos(n\omega_0 t)\,dt=\frac{d_2}{n\omega_0}\text{sen}(n\omega_0 t)\Big|_{-d_2}^{-d_1}=\frac{d_2}{n\omega_0}\left(\text{sen}(-n\omega_0 d_1)-\text{sen}(-n\omega_0 d_2)\right)=-d_2\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}$$

$$\int_{-d_1}^{0}t\cos(n\omega_0 t)\,dt=t\frac{\text{sen}(n\omega_0 t)}{n\omega_0}\Big|_{-d_1}^{0}-\frac{-\cos(n\omega_0 t)}{n^2\omega_0^2}\Big|_{-d_1}^{0}=\left(\cancelto{0}{0\cdot\frac{\text{sen}(0)}{n\omega_0}}+d_1\frac{\text{sen}(-n\omega_0 d_1)}{n\omega_0}\right)-\left(\cancelto{1}{\frac{-\cos(0)}{n^2\omega_0^2}}-\frac{-\cos(-n\omega_0 d_1)}{n^2\omega_0^2}\right)=-d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{1}{n^2\omega_0^2}-\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}$$

$$\int_{0}^{d_1}t\cos(n\omega_0 t)\,dt=t\frac{\text{sen}(n\omega_0 t)}{n\omega_0}\Big|_{0}^{d_1}-\frac{-\cos(n\omega_0 t)}{n^2\omega_0^2}\Big|_{0}^{d_1}=\left(d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}-\cancelto{0}{0\cdot\frac{\text{sen}(0)}{n\omega_0}}\right)-\left(\frac{-\cos(n\omega_0 d_1)}{n^2\omega_0^2}-\cancelto{1}{\frac{-\cos(0)}{n^2\omega_0^2}}\right)=d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}-\frac{1}{n^2\omega_0^2}$$

$$\int_{d_1}^{d_2}d_2\cos(n\omega_0 t)\,dt=\frac{d_2}{n\omega_0}\text{sen}(n\omega_0 t)\Big|_{d_1}^{d_2}=d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}-d_2\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}$$

$$\int_{d_1}^{d_2}t\cos(n\omega_0 t)\,dt=t\frac{\text{sen}(n\omega_0 t)}{n\omega_0}\Big|_{d_1}^{d_2}-\frac{-\cos(n\omega_0 t)}{n^2\omega_0^2}\Big|_{d_1}^{d_2}=\left(d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}-d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}\right)-\left(\frac{-\cos(n\omega_0 d_2)}{n^2\omega_0^2}-\frac{-\cos(n\omega_0 d_1)}{n^2\omega_0^2}\right)=d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}-d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{\cos(n\omega_0 d_2)}{n^2\omega_0^2}-\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}$$

$$a_n=\frac{2}{T}\left(\frac{A}{d_2-d_1}\left(d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}-d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}+\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}-\frac{\cos(n\omega_0 d_2)}{n^2\omega_0^2}-d_2\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}\right)-\frac{A}{d_1}\left(-d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{1}{n^2\omega_0^2}-\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}\right)\right.$$
$$\left.+\frac{A}{d_1}\left(d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}-\frac{1}{n^2\omega_0^2}\right)+\frac{A}{d_2-d_1}\left(d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}-d_2\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}-\left(d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}-d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{\cos(n\omega_0 d_2)}{n^2\omega_0^2}-\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}\right)\right)\right)$$

$$=\frac{2}{T}\left(\frac{A}{d_2-d_1}d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}-\frac{A}{d_2-d_1}d_2\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}-\cancel{\frac{A}{d_2-d_1}d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}}+\cancel{\frac{A}{d_2-d_1}d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}}+\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}+\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}\right.$$
$$-\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_2)}{n^2\omega_0^2}-\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_2)}{n^2\omega_0^2}-\frac{A}{d_2-d_1}d_2\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{A}{d_2-d_1}d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\cancel{\frac{A}{d_2-d_1}d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}}-\cancel{\frac{A}{d_2-d_1}d_2\frac{\text{sen}(n\omega_0 d_2)}{n\omega_0}}$$
$$\left.+\frac{A}{\cancel{d_1}}\cancel{d_1}\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}-\frac{A}{d_1}\frac{1}{n^2\omega_0^2}+\frac{A}{d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}+\frac{A}{\cancel{d_1}}\cancel{d_1}\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{A}{d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}-\frac{A}{d_1}\frac{1}{n^2\omega_0^2}\right)$$

$$=\frac{2}{T}\left(\frac{A}{d_2-d_1}d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}-\frac{A}{d_2-d_1}d_2\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}+\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}-\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_2)}{n^2\omega_0^2}-\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_2)}{n^2\omega_0^2}\right.$$
$$\left.-\frac{A}{d_2-d_1}d_2\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{A}{d_2-d_1}d_1\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{A}{\cancel{d_1}}\cancel{d_1}\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}-\frac{A}{d_1}\frac{1}{n^2\omega_0^2}+\frac{A}{d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}+\frac{A}{\cancel{d_1}}\cancel{d_1}\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}+\frac{A}{d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}-\frac{A}{d_1}\frac{1}{n^2\omega_0^2}\right)$$

$$=\frac{2}{T}\left(\frac{-A\,\text{sen}(d_1)\cancel{(d_2-d_1)}}{\cancel{(d_2-d_1)}\,n\omega_0}+2\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}-2\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_2)}{n^2\omega_0^2}-\frac{A\,\text{sen}(n\omega_0 d_1)\cancel{(d_2-d_1)}}{\cancel{(d_2-d_1)}\,n\omega_0}+2A\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}-\frac{A}{d_1}\frac{2}{n^2\omega_0^2}+2\frac{A}{d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}\right)$$

$$=\frac{2}{T}\left(2\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}-2\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_2)}{n^2\omega_0^2}-\cancel{2\frac{A\,\text{sen}(n\omega_0 d_1)}{n\omega_0}}+\cancel{2A\frac{\text{sen}(n\omega_0 d_1)}{n\omega_0}}-\frac{A}{d_1}\frac{2}{n^2\omega_0^2}+2\frac{A}{d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}\right)$$

$$=\frac{2}{T}\left(2\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}-2\frac{A}{d_2-d_1}\frac{\cos(n\omega_0 d_2)}{n^2\omega_0^2}-\frac{A}{d_1}\frac{2}{n^2\omega_0^2}+2\frac{A}{d_1}\frac{\cos(n\omega_0 d_1)}{n^2\omega_0^2}\right)$$

$$= \frac{-4A}{Tn^2\omega_0^2}\left(\frac{1}{\partial 2-\partial 1}\cos(n\omega_0\partial 2) - \cos(n\omega_0\partial 1)\left(\frac{1}{\partial 2-\partial 1}+\frac{1}{\partial 1}\right)+\frac{1}{\partial 1}\right)$$

$$= \frac{-4A}{Tn^2\frac{2\pi^2}{T^2}}\left(\frac{1}{\partial 2-\partial 1}\cos(n\omega_0\partial 2) - \cos(n\omega_0\partial 1)\left(\frac{1}{\partial 2-\partial 1}+\frac{1}{\partial 1}\right)+\frac{1}{\partial 1}\right)$$

$$a_n = \frac{-TA}{n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cos(n\omega_0\partial 2) - \cos(n\omega_0\partial 1)\left(\frac{1}{\partial 2-\partial 1}+\frac{1}{\partial 1}\right)+\frac{1}{\partial 1}\right)$$

Si $a_n = 2Re\{C_n\} \longrightarrow Re\{C_n\} = a_n \cdot \frac{1}{2}$

$$Re\{C_n\} = \frac{1}{2}a_n = \frac{1}{2}\cdot\frac{-TA}{n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cos(n\omega_0\partial 2) - \cos(n\omega_0\partial 1)\left(\frac{1}{\partial 2-\partial 1}+\frac{1}{\partial 1}\right)+\frac{1}{\partial 1}\right) = \frac{-TA}{2n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cos(n\omega_0\partial 2) - \cos(n\omega_0\partial 1)\left(\frac{1}{\partial 2-\partial 1}+\frac{1}{\partial 1}\right)+\frac{1}{\partial 1}\right)$$

$$Re\{C_n\} = \frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right) \quad y \quad Im\{C_n\} = 0 \longrightarrow \text{Simetría par}$$

$$|C_n| = \sqrt{Re^2\{C_n\} + Im^2\{C_n\}} = \sqrt{\left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^2}$$

$$\theta_{C_n} = \tan^{-1}\left\{\frac{Im\{C_n\}}{Re\{C_n\}}\right\} = \tan^{-1}\left(\frac{0}{\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)}\right) = 0$$

$$e_r[\%] = \left(1 - \sum |C_n|^2\frac{P_n}{P_x}\right)\cdot 100[\%] \quad ; \quad P_n = \frac{1}{T}E_n = \frac{1}{T}\int_T |e^{jn\omega_0 t}|^2\partial t = \frac{1}{T}\cdot T = 1$$

Como la señal es par solo vamos a integrar de 0 a $\partial 2$ $(x(-t) = x(t))$

$$P_x = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|x(t)|^2\partial t = \frac{2}{T}\int_0^{\frac{T}{2}}|x(t)|^2\partial t \rightarrow \text{Pero de } \partial 2 \text{ a } \tfrac{T}{2} \text{ es } 0 \longrightarrow P_x = \frac{2}{T}\int_0^{\partial 2}|x(t)|^2\partial t = \frac{2}{T}\left(\int_0^{\partial 1}\left(\frac{A}{\partial 1}t\right)^2\partial t + \int_{\partial 1}^{\partial 2}\left(\frac{A}{\partial 2-\partial 1}(\partial 2 - t)\right)^2\partial t\right)$$

$$P_x = \frac{2}{T}\left(\int_0^{\partial 1}\left(\frac{A}{\partial 1}t\right)^2\partial t + \int_{\partial 1}^{\partial 2}\left(\frac{A}{\partial 2-\partial 1}(t+\partial 2)\right)^2\partial t\right) = \frac{2}{T}\left(\frac{A^2}{\partial 1^2}\int_0^{\partial 1}t^2\partial t + \frac{A^2}{(\partial 2-\partial 1)^2}\int_{\partial 1}^{\partial 2}(\partial 2 - t)^2\partial t\right)$$

$$\int_0^{\partial 1}t^2\partial t = \frac{t^3}{3}\Big|_0^{\partial 1} = \frac{\partial 1^3}{3} \quad ; \quad \int_{\partial 1}^{\partial 2}(\partial 2 - t)^2\partial t = \int_0^{\partial 2-\partial 1}u^2\partial u = \frac{u^3}{3}\Big|_0^{\partial 2-\partial 1} = \frac{(\partial 2-\partial 1)^3}{3}$$

$u = \partial 2 - t$
$\partial u = \partial t$

Si $t = \partial 1 \rightarrow u = \partial 2 - \partial 1$
Si $t = \partial 2 \rightarrow u = \partial 2 - \partial 2 = 0$
$u \in [0, \partial 2 - \partial 1]$

$$P_x = \frac{2}{T}\left(\frac{A^2}{\partial 1^2}\frac{\partial 1^3}{3} + \frac{A^2}{(\partial 2-\partial 1)^2}\frac{(\partial 2-\partial 1)^3}{3}\right)$$

$$P_x = \frac{2}{T}\left(\frac{A^2}{\partial 1^2}\frac{\partial 1^3}{3} + \frac{A^2(\partial 2-\partial 1)}{3}\right)$$

$$P_x = \frac{2}{T}\left(\cancel{\frac{A^2\partial 1}{3}} + \frac{A^2\partial 2}{3} - \frac{A^2\partial 1}{3}\right)$$

$$P_x = \frac{2A^2\partial 2}{T3}$$

$$|C_n|^2 = \sqrt{\left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^2}^{\,2}$$

$$= \left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^2$$

$$e_r(\%) = \left(1 - \sum_n \frac{\left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^2}{\frac{2A^2\partial 2}{T3}}\right)\cdot 100(\%)$$