Maria de los Angeles Prieto Ortega

CC. 1062278202

1. La distancia media entre dos señales periódicas $x_1(t) \in \mathbb{R}, \mathbb{C}$ y $x_2(t) \in \mathbb{R}, \mathbb{C}$; se puede expresar a partir de la potencia media de la diferencia entre ellas:

$$d^2(x_1, x_2) = \bar{P}_{x_1 - x_2} = \lim_{T \to \infty} \frac{1}{T} \int_T |x_1(t) - x_2(t)|^2 \, dt.$$

Sea $x_1(t)$ y $x_2(t)$ dos señales definidas como:

$$x_1(t) = Ae^{-jnw_0 t}$$

$$x_2(t) = Be^{jmw_0 t}$$

con $w_0 = \frac{2\pi}{T}$; $T, A, B \in \mathbb{R}^+$ y $n, m \in \mathbb{Z}$. Determine la distancia entre las dos señales. Compruebe sus resultados con Python.

$X_1(t) = Ae^{-jnW_0t}$, $X_2(t) = Be^{jmW_0t}$; $W_0 = \frac{2\pi}{T}$

$d^2(X_1, X_2) = \bar{P}_{X_1 - X_2} = \lim_{T \to \infty} \frac{1}{T} \int_T |X_1(t) - X_2(t)|^2 \partial t$

$$\begin{aligned}
\bar{P}_{X_1 - X_2} &= \frac{1}{T} \int_T |X_1(t) - X_2(t)|^2 \partial t \\
&= \frac{1}{T} \left(\int_T (X_1(t) - X_2(t))(X_1(t) - X_2(t))^* \right) \partial t \\
&= \frac{1}{T} \left(\int_T (X_1(t) - X_2(t))(X_1^*(t) - X_2^*(t)) \partial t \right) \\
&= \frac{1}{T} \left(\int_T X_1(t) X_1^*(t) \partial t - \int_T X_1(t) X_2^*(t) \partial t - \int_T X_2(t) X_1^*(t) \partial t + \int_T X_2(t) X_2^*(t) \partial t \right) \\
&= \underbrace{\frac{1}{T} \int |X_1(t)|^2 \partial t}_{\bar{P}_{X_1}} \underbrace{- \frac{2}{T} \int_T X_1(t) X_2^*(t) \partial t}_{C_X} + \underbrace{\frac{1}{T} \int_T |X_2(t)|^2 \partial t}_{\bar{P}_{X_2}}
\end{aligned}$$

$$\begin{aligned}
\bar{P}_{X_1} &= \frac{1}{T} \int_T X_1(t) X_1^*(t) \partial t = \frac{1}{T} \int_0^T Ae^{-jnW_0t} \left(Ae^{-jnW_0t}\right)^* = \frac{1}{T} \int_0^T Ae^{jnW_0t} Ae^{-jnW_0t} \partial t \\
&= \frac{A^2}{T} \int_0^T e^{\cancelto{0}{(n-n)}jW_0t} \partial t = \frac{A^2}{T} \int_0^T 1 \partial t = \frac{A^2}{T}(T - 0) = \frac{A^2}{\cancel{T}} \cdot \cancel{T} = A^2
\end{aligned}$$

$$\begin{aligned}
\bar{P}_{X_2} &= \frac{1}{T} \int_T X_2(t) X_2^*(t) \partial t = \frac{1}{T} \int_0^T Be^{jmW_0t} \left(Be^{jmW_0t}\right)^* = \frac{1}{T} \int_0^T Be^{jmW_0t} Be^{-jmW_0t} \partial t \\
&= \frac{B^2}{T} \int_0^T e^{\cancelto{0}{(m-m)}jW_0t} \partial t = \frac{B^2}{T} \int_0^T 1 \partial t = \frac{B^2}{T}(T - 0) = \frac{B^2}{\cancel{T}} \cdot \cancel{T} = B^2
\end{aligned}$$

El cruzado tiene dos casos:

① Si $n \neq -m \to$

$$\begin{aligned}
C_X &= -\frac{2}{T} \int_T X_1(t) X_2^*(t) \partial t = -\frac{2}{T} \int_0^T Ae^{-jnW_0t} \left(Be^{jmW_0t}\right)^* \partial t \\
&= -\frac{2}{T} \int_0^T Ae^{-jnW_0t} Be^{-jmW_0t} \partial t = -\frac{2AB}{T} \int_0^T e^{-(n+m)jW_0t} \partial t
\end{aligned}$$

$\to$ Si $n, m \in \mathbb{Z} \to n + m = r \in \mathbb{Z}$

$$\begin{aligned}
&= -\frac{2AB}{T} \int_0^T e^{-rjW_0t} \partial t = -\frac{j \cdot 2AB}{j \cdot -jrW_0T} e^{-jrW_0t} \Big|_0^T = -\frac{j2AB}{r\frac{2\pi}{\cancel{T}}\cancel{T}} \left(e^{-jr\frac{2\pi}{\cancel{T}}\cancel{T}} - e^{\cancelto{1}{-jr\frac{2\pi}{T}\cdot 0}} \right) \\
&= -\frac{jAB}{r\pi} \left(e^{-jr2\pi} - 1 \right) = -\frac{jAB}{\pi r} \left(\cancelto{1}{\cos(2\pi r)} - j \cancelto{0}{\operatorname{sen}(2\pi r)} - 1 \right) \\
&= \frac{jAB}{\pi r} (1 - 0 - 1) = \frac{jAB}{\pi r} \cdot 0 = 0
\end{aligned}$$

② Si $n = -m \to$

$$\begin{aligned}
C_X &= -\frac{2}{T} \int_T X_1(t) X_2^*(t) \partial t = -\frac{2}{T} \int_0^T Ae^{-jnW_0t} \left(Be^{jmW_0t}\right)^* \partial t = -\frac{2}{T} \int_0^T Ae^{-j(-m)W_0t} Be^{-jmW_0t} \partial t \\
&= -\frac{2}{T} \int_0^T Ae^{jmW_0t} Be^{-jmW_0t} \partial t = -\frac{2AB}{T} \int_0^T e^{\cancelto{0}{(m-m)}jW_0t} \partial t = -\frac{2AB}{T} \int_0^T 1 \partial t \\
&= -\frac{2AB}{T}(T - 0) = -\frac{2AB}{\cancel{T}} \cdot \cancel{T} = -2AB
\end{aligned}$$

$$\bar{P}_{X_1 - X_2} = \begin{cases} A^2 + B^2 & ; n \neq -m \\ A^2 + B^2 - 2AB & ; n = -m \end{cases}$$

$$d(X_1, X_2) = \begin{cases} \sqrt{A^2 + B^2} & ; n \neq -m \\ \sqrt{A^2 + B^2 - 2AB} & ; n = -m \end{cases}$$

2. Encuentre la señal en tiempo discreto al utilizar un conversor análogo digital con frecuencia de muestreo de $5kHz$ y 4 bits de capacidad de representación, aplicado a la señal continua: $x(t) = 3\cos(1000\pi t) + 5\sin(3000\pi t) + 10\cos(11000\pi t)$. Realizar la simulación del proceso de discretización (incluyendo al menos tres periodos de $x(t)$). En caso de que la discretización no sea apropiada, diseñe e implemente un conversor adecuado para la señal estudiada.

$X(t) = 3Cos(1000\pi t) + 5Sen(3000\pi t) + 10cos(11000\pi t)$

$F_s = 5KHz$, $\#$ de estados $= 2^{4Bits} = 16$

$t = nT_s \rightarrow F_s = \frac{1}{T_s}$; $F = \frac{1}{T} \rightarrow T = \frac{2\pi}{\omega}$

$A\cos(\Omega n)$
$A\cos(2\pi f n)$ $\rightarrow \Omega = 2\pi f \rightarrow f = \frac{T_s}{T} = \frac{F}{F_s}$

$X(t) = X_1(t) + X_2(t) + X_3(t) \rightarrow$

Para $X_1(t) = 3\cos(1000\pi t)$

$$W_1 = 1000\pi \rightarrow T_1 = \frac{2\pi}{W_1} = \frac{2\pi}{1000\pi} = \frac{1}{500} \rightarrow F_1 = \frac{1}{T} = \frac{1}{\frac{1}{500}} = 500\,Hz \rightarrow X_1[n] = A\cos\left(2\pi n \cdot \frac{F}{F_s}\right) = 3Cos\left(2\pi \cdot n \cdot \frac{500}{5000}\right) = 3Cos\left(\frac{n\pi}{5}\right)$$

Para $X_2(t) = 5Sen(3000\pi t)$

$$W_2 = 3000\pi \rightarrow T_2 = \frac{2\pi}{W_1} = \frac{2\pi}{3000\pi} = \frac{1}{1500} \rightarrow F_2 = \frac{1}{T} = \frac{1}{\frac{1}{1500}} = 1500\,Hz \rightarrow X_2[n] = A sen\left(2\pi n \cdot \frac{F}{F_s}\right) = 5Sen\left(2\pi \cdot n \cdot \frac{1500}{5000}\right) = 5Sen\left(\frac{3n\pi}{5}\right)$$

Para $X_3(t) = 10\cos(11000\pi t)$

$$W_3 = 11000\pi \rightarrow T_3 = \frac{2\pi}{W_1} = \frac{2\pi}{11000\pi} = \frac{1}{5500} \rightarrow F_3 = \frac{1}{T} = \frac{1}{\frac{1}{5500}} = 5500\,Hz \rightarrow X_3[n] = A\cos\left(2\pi n \cdot \frac{F}{F_s}\right) = 10Cos\left(2\pi \cdot n \cdot \frac{5500}{5000}\right) = 10Cos\left(\frac{11n\pi}{5}\right)$$

$$X[n] = 3Cos\left(\frac{n\pi}{5}\right) + 5Sen\left(\frac{3n\pi}{5}\right) + 10Cos\left(\frac{11n\pi}{5}\right)$$

$$X[n] = 3Cos\left(\frac{\pi}{5}n\right) + 5Sen\left(\frac{\pi}{5}n\right) + 10Cos\left(\frac{\pi}{5}n\right)$$

$$X[n] = 13Cos\left(\frac{\pi}{5}n\right) + 5Sen\left(\frac{\pi}{5}n\right)$$

$$\frac{W_1}{W_2} = \frac{1000\pi}{3000\pi} = \frac{1}{3} \in Q$$

$$\frac{W_1}{W_3} = \frac{1000\pi}{11000\pi} = \frac{1}{11} \in Q$$

$$\frac{W_2}{W_3} = \frac{3000\pi}{11000\pi} = \frac{3}{11} \in Q$$

↳ Como todas son racionales la señal es cuasiperiodica.

Si $F_s \geq 2F_{max} \rightarrow 2F_{max} = 2 \cdot 5500\,Hz = 11000\,Hz \rightarrow F_s = 5000\,Hz$.

↓ No cumple Nyquist

Vamos a comparar con las originales. $\rightarrow -\pi \leq \Omega < \pi$

$\Omega_1 = \frac{1}{5}\pi$ ✓ , $\Omega_2 = \frac{3}{5}\pi$ ✓ , $\Omega_3 = \frac{11}{5}\pi$ ✗ → Es una copia / aliasings

$$\Omega_{3original} = \frac{11}{5}\pi - 2\pi = \frac{1}{5}\pi$$

Suponemos una frecuencia de muestreo mucho más grande

$F_s = 4F_{max} \rightarrow F_s = 4(5500)\,Hz = 22\,KHz$ $\qquad T_s = \frac{1}{22000}$

Para $X_1(t) = 3\cos(1000\pi t)$

$$W_1 = 1000\pi \rightarrow T_1 = \frac{2\pi}{W_1} = \frac{2\pi}{1000\pi} = \frac{1}{500} \rightarrow F_1 = \frac{1}{T} = \frac{1}{\frac{1}{500}} = 500\,Hz \rightarrow X_1[n] = A\cos\left(2\pi n \cdot \frac{F}{F_s}\right) = 3Cos\left(2\pi \cdot n \cdot \frac{500}{22000}\right) = 3Cos\left(\frac{\pi}{22}n\right)$$

Para $X_2(t) = 5Sen(3000\pi t)$

$$W_2 = 3000\pi \rightarrow T_2 = \frac{2\pi}{W_1} = \frac{2\pi}{3000\pi} = \frac{1}{1500} \rightarrow F_2 = \frac{1}{T} = \frac{1}{\frac{1}{1500}} = 1500\,Hz \rightarrow X_2[n] = A sen\left(2\pi n \cdot \frac{F}{F_s}\right) = 5Sen\left(2\pi \cdot n \cdot \frac{1500}{22000}\right) = 5Sen\left(\frac{3\pi}{22}n\right)$$

Para $X_3(t) = 10\cos(11000\pi t)$

$$W_3 = 11000\pi \rightarrow T_3 = \frac{2\pi}{W_1} = \frac{2\pi}{11000\pi} = \frac{1}{5500} \rightarrow F_3 = \frac{1}{T} = \frac{1}{\frac{1}{5500}} = 5500\,Hz \rightarrow X_3[n] = A\cos\left(2\pi n \cdot \frac{F}{F_s}\right) = 10Cos\left(2\pi \cdot n \cdot \frac{5500}{22000}\right) = 10Cos\left(\frac{\pi}{2}n\right)$$

$$X[n] = 3Cos\left(\frac{\pi}{22}n\right) + 5Sen\left(\frac{3\pi}{22}n\right) + 10Cos\left(\frac{\pi}{2}n\right)$$

Vamos a comparar con las originales. $\rightarrow -\pi \leq \Omega < \pi$

$\Omega_1 = \frac{1}{22}\pi$ ✓ , $\Omega_2 = \frac{3}{22}\pi$ ✓ , $\Omega_3 = \frac{1}{22}\pi$ ✓ → las tres frecuencias digitales están en los originales.

$$X[n] = 3Cos\left(\frac{\pi}{22}n\right) + 5Sen\left(\frac{3\pi}{22}n\right) + 10Cos\left(\frac{\pi}{2}n\right)$$

Para saber el periodo a gráficar:

$$T = r\,\frac{2\pi}{\omega_1} = \ell\,\frac{2\pi}{\omega_2} = m\,\frac{2\pi}{\omega_3}$$

$$T = r\,\frac{2\cancel{\pi}}{1000\cancel{\pi}} = \ell\,\frac{2\cancel{\pi}}{3000\cancel{\pi}} = m\,\frac{2\cancel{\pi}}{11000\cancel{\pi}}$$

$$T = r\,\frac{1}{500} = \ell\,\frac{1}{1500} = m\,\frac{1}{5500}$$

$$5500T = r\,\frac{5500}{500} = \ell\,\frac{\cancel{5500}}{\cancel{1500}} = m\,\frac{\cancel{5500}}{\cancel{5500}}$$

$$5500T = r\,11 = \ell\,\frac{11}{3} = m$$

$$3\cdot 5500T = r\cdot 3\cdot 11 = \ell\cdot\cancel{3}\cdot\frac{11}{\cancel{3}} = m\cdot 3$$

$$16500T = 33r = 11\ell = 3m$$

$\text{mcm}\{33, 1, 11\} = 33$

$$16500\,T = 33$$

$$T = \frac{33}{16500}$$

$$T = \frac{1}{500}$$

3. Sea $x''(t)$ la segunda derivada de la señal $x(t)$, donde $t \in [t_i, t_f]$. Demuestre que los coeficientes de la serie exponencial de Fourier se pueden calcular según:

$$c_n = \frac{1}{(ti - tf)n^2 w_o^2} \int_{t_i}^{t_f} x''(t) e^{-jnw_o t} dt; \quad n \in \mathbb{Z}.$$

¿Cómo se pueden calcular los coeficientes a_n y b_n desde $x''(t)$ en la serie trigonométrica de Fourier?.

Con $X(t)$ para $t \in [t_i, t_f]$

Demostrar: $C_n = \frac{1}{(t_i - t_f) n^2 W_o^2} \int_T X''(t) e^{-jnW_o t} \partial t$ con $n \in \mathbb{Z}$

Tenemos: $X(t) = \sum_n C_n e^{jnW_o t}$

$$X'(t) = \frac{\partial}{\partial t} \sum_n C_n e^{jnW_o t} = \sum_n C_n \frac{\partial e^{jnW_o t}}{\partial t}$$

$$X''(t) = \frac{\partial X'(t)}{\partial t} = \frac{\partial}{\partial t} \sum_n C_n \frac{\partial e^{jnW_o t}}{\partial t} = \sum_n C_n \frac{\partial^2 e^{jnW_o t}}{\partial t^2}$$

$$\frac{\partial e^{jnW_o t}}{\partial t} = jnW_o e^{jnW_o t}; \quad \frac{\partial^2 e^{jnW_o t}}{\partial t^2} = \frac{\partial (jnW_o e^{jnW_o t})}{\partial t} = jnW_o \cdot jnW_o e^{jnW_o t} = -n^2 W_o^2 e^{jnW_o t}$$

$$X''(t) = \sum_n C_n \frac{\partial^2 e^{jnW_o t}}{\partial t^2} = \sum_n \underbrace{-C_n n^2 W_o^2}_{C\_n} e^{jnW_o t} = \sum_n C\_n e^{jnW_o t}; \text{ con } C\_n = -C_n n^2 W_o^2$$

Si $C_n = \frac{1}{T} \int_T X(t) e^{-jnW_o t} \partial t$ y $C\_n = \frac{1}{T} \int_T X''(t) e^{-jnW_o t} \partial t \longrightarrow$ Remplazamos el $C\_n \longrightarrow -C_n n^2 W_o^2 = \frac{1}{T} \int_T X''(t) e^{-jnW_o t} \partial t$

$$C_n = \frac{1}{-(t_f - t_i) n^2 W_o^2} \int_T X''(t) e^{-jnW_o t} \partial t$$

$$C_n = \frac{1}{(t_i - t_f) n^2 W_o^2} \int_T X''(t) e^{-jnW_o t} \partial t$$

$$C_n = \frac{1}{(t_i - t_f) n^2 W_o^2} \int_T X''(t) e^{-jnW_o t} \partial t = \frac{1}{(t_i - t_f) n^2 W_o^2} \int_T X''(t) (\cos(nW_o t) - j\,\text{sen}(nW_o t)) \partial t$$

$$= \frac{1}{(t_i - t_f) n^2 W_o^2} \int_T X''(t) \cos(nW_o t) \partial t - j \frac{1}{(t_i - t_f) n^2 W_o^2} \int_T X''(t)\,\text{sen}(nW_o t) \partial t$$

Sabiendo que: $a_n = \frac{2}{T} \int_T X(t) \cos(nW_o t) \partial t$; $b_n = \frac{2}{T} \int_T X(t)\,\text{sen}(nW_o t) \partial t$ y $a_n = 2\,\text{Re}\{C_n\}$; $b_n = -2\,\text{Im}\{C_n\}$

$$a_n = 2\,\text{Re}\{C_n\} = 2 \frac{1}{(t_i - t_f) n^2 W_o^2} \int_T X''(t) \cos(nW_o t) \partial t$$

$$b_n = -2\,\text{Im}\{C_n\} = -2 \cdot - \frac{1}{(t_i - t_f) n^2 W_o^2} \int_T X''(t)\,\text{sen}(nW_o t) \partial t = 2 \frac{1}{(t_i - t_f) n^2 W_o^2} \int_T X''(t)\,\text{sen}(nW_o t) \partial t$$

También se pueden calcular con: $X(t) = \sum_n a_n \cos(nW_o t) + b_n\,\text{Sen}(nW_o t)$

$$X(t) = \sum_n a_n \cos(nW_o t) + b_n\,\text{Sen}(nW_o t)$$

$$X'(t) = \sum_n -a_n\,\text{Sen}(nW_o t)\, nW_o + b_n \cos(nW_o t)\, nW_o$$

$$X''(t) = \sum_n -a_n \cos(nW_o t)\, nW_o \cdot nW_o - b_n\,\text{Sen}(nW_o t)\, nW_o \cdot nW_o = \sum_n \underbrace{-a_n n^2 W_o^2}_{a\_n} \cos(nW_o t) \underbrace{- b_n n^2 W_o^2}_{b\_n} \text{Sen}(nW_o t)$$

$$a\_n = -a_n n^2 W_o^2, \quad b\_n = -b_n n^2 W_o^2$$

$$a_n = \frac{2}{T} \int_T X(t) \cos(nW_o t) \partial t \longrightarrow a\_n = \frac{2}{T} \int_T X''(t) \cos(nW_o t) \partial t$$

$$-a_n n^2 W_o^2 = \frac{2}{T} \int_T X''(t) \cos(nW_o t) \partial t$$

$$a_n = \frac{2}{-(t_f - t_i) n^2 W_o^2} \int_T X''(t) \cos(nW_o t) \partial t$$

$$a_n = \frac{2}{(t_i - t_f) n^2 W_o^2} \int_T X''(t) \cos(nW_o t) \partial t$$

$$b_n = \frac{2}{T} \int_T X(t)\,\text{Sen}(nW_o t) \partial t \longrightarrow b\_n = \frac{2}{T} \int_T X''(t)\,\text{Sen}(nW_o t) \partial t$$

$$-b_n n^2 W_o^2 = \frac{2}{T} \int_T X''(t)\,\text{Sen}(nW_o t) \partial t$$

$$b_n = \frac{2}{-(t_f - t_i) n^2 W_o^2} \int_T X''(t)\,\text{Sen}(nW_o t) \partial t$$

$$b_n = \frac{2}{(t_i - t_f) n^2 W_o^2} \int_T X''(t)\,\text{Sen}(nW_o t) \partial t$$

4. Encuentre el espectro de Fourier, su parte real, imaginaria, magnitud, fase y el error relativo para $n \in \{0, \pm1, \pm2, \pm3, \pm4, \pm5\}$, a partir de $x''(t)$ para la señal $x(t)$ en la Figura 1. Compruebe el espectro obtenido con la estimación a partir de $x(t)$. Presente las simulaciones de Python respectivas.

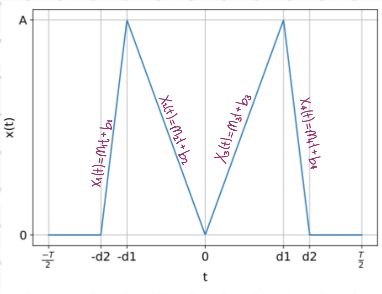


Figura 1: $x(t)$ para el ejercicio 1.4

$$X(t)=\begin{cases} 0 & -\frac{T}{2} \leq t < -d_2 \\ \frac{A}{d_2-d_1}(t+d_2) & -d_2 \leq t < -d_1 \\ \frac{-A}{d_1}t & -d_1 \leq t < 0 \\ \frac{A}{d_1}t & 0 \leq t < d_1 \\ -\frac{A}{d_2-d_1}(t+d_2) & d_1 \leq t < d_2 \\ 0 & d_2 \leq t < \frac{T}{2} \end{cases}$$

Para $X_1(t)=m_1t+b_1 \rightarrow m_1 = \frac{A-0}{-d_1-(-d_2)} = \frac{A}{d_2-d_1}$; En $t=-d_2 \rightarrow X(-d_2)=\frac{A}{d_2-d_1}\cdot -d_2 + b_1$

$0 = \frac{-d_2 A}{d_2-d_1} + b_1$

$b_1 = \frac{d_2}{d_2-d_1}A$

$X_1(t) = \frac{A}{d_2-d_1}t + \frac{d_2}{d_2-d_1}A$

$X_1(t) = \frac{A}{d_2-d_1}(t+d_2)$

Para $X_2(t)=m_2t+b_2 \rightarrow m_2 = \frac{0-A}{0-(-d_1)} = \frac{-A}{d_1}$; En $t=-d_1 \rightarrow X(-d_1)=\frac{-A}{d_1}\cdot -d_1 + b_2$

$A = A + b_2$

$b_2 = A - A = 0$

$X_2(t) = \frac{-A}{d_1}t$

Para $X_3(t)=m_3t+b_3 \rightarrow m_3 = \frac{A-0}{d_1-0} = \frac{A}{d_1}$; En $t=0 \rightarrow X(0)=\frac{A}{d_1}\cdot 0 + b_3$

$0 = 0 + b_3$

$b_3 = 0$

$X_3(t) = \frac{A}{d_1}t$

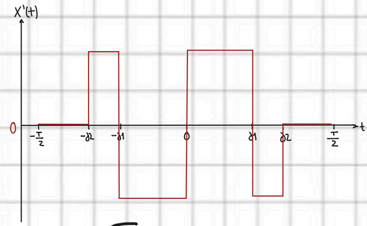
Para $X_4(t)=m_4t+b_4 \rightarrow m_4 = \frac{0-A}{d_2-d_1} = \frac{-A}{d_2-d_1}$; En $t=d_2 \rightarrow X(d_2)=\frac{-A}{d_2-d_1}d_2 + b_4$

$0 = \frac{-A}{d_2-d_1}\cdot d_2 + b_4$

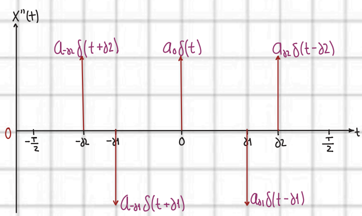
$b_4 = \frac{-Ad_2}{d_2-d_1}$

$X_4(t) = \frac{-A}{d_2-d_1}t + \frac{-Ad_2}{d_2-d_1}$

$X_4(t) = -\frac{A}{d_2-d_1}(t+d_2)$



$$X'(t)=\begin{cases} 0 & -\frac{T}{2} \leq t < -d_2 \\ \frac{A}{d_2-d_1} & -d_2 \leq t < -d_1 \\ \frac{-A}{d_1} & -d_1 \leq t < 0 \\ \frac{A}{d_1} & 0 \leq t < d_1 \\ -\frac{A}{d_2-d_1} & d_1 \leq t < d_2 \\ 0 & d_2 \leq t < \frac{T}{2} \end{cases}$$



$X''(t) = 0$ en cada tramo y en los puntos donde hay cambios tiende a $\pm\infty$ y estos cambios los podemos representar como $a_k\delta(t \pm t_k)$ donde a_k es la diferencia de la derivada por la derecha y la derivada por la izquierda y t_k donde ocurre el salto (cambio instantáneo)

$$a_{-\partial 2} = X'(-\partial 2^+) - X'(-\partial 2^-) = m_1 - 0 = \frac{A}{\partial 2-\partial 1}$$

$$a_{-\partial 1} = X'(-\partial 1^+) - X'(-\partial 1^-) = m_2 - m_1 = \frac{-A}{\partial 1} - \frac{A}{\partial 2-\partial 1} = -A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)$$

$$a_0 = X'(0^+) - X'(0^-) = m_3 - m_2 = \frac{A}{\partial 1} - \frac{-A}{\partial 1} = \frac{A}{\partial 1}(1+1) = \frac{2A}{\partial 1}$$

$$a_{\partial 1} = X'(\partial 1^+) - X'(\partial 1^-) = m_4 - m_3 = -\frac{A}{\partial 2-\partial 1} - \frac{A}{\partial 1} = -A\left(\frac{1}{\partial 2-\partial 1}+\frac{1}{\partial 1}\right)$$

$$a_{\partial 2} = X'(\partial 2^+) - X'(\partial 2^-) = 0 - m_4 = 0 - -\frac{A}{\partial 2-\partial 1} = \frac{A}{\partial 2-\partial 1}$$

$$X''(t) = a_{-\partial 2}\,\delta(t+\partial 2) + a_{-\partial 1}\,\delta(t+\partial 1) + a_0\,\delta(t) + a_{\partial 1}\,\delta(t-\partial 1) + a_{\partial 2}\,\delta(t-\partial 2)$$

$$X''(t) = \frac{A}{\partial 2-\partial 1}\,\delta(t+\partial 2) - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\delta(t+\partial 1) + \frac{2A}{\partial 1}\,\delta(t) - A\left(\frac{1}{\partial 2-\partial 1}+\frac{1}{\partial 1}\right)\delta(t-\partial 1) + \frac{A}{\partial 2-\partial 1}\,\delta(t-\partial 2)$$

$$X''(t) = \frac{A}{\partial 2-\partial 1}\big(\delta(t+\partial 2)+\delta(t-\partial 2)\big) - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\big(\delta(t+\partial 1)+\delta(t-\partial 1)\big) + \frac{2A}{\partial 1}\,\delta(t)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\int_T X''(t)\, e^{-jn\omega_0 t}\,\partial t \qquad \text{con } T = t_f - t_i = \frac{T}{2} - \left(-\frac{T}{2}\right) = T \quad \text{y} \quad \omega_0 = \frac{2\pi}{T}$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\int_{-T/2}^{T/2}\left(\frac{A}{\partial 2-\partial 1}\big(\delta(t+\partial 2)+\delta(t-\partial 2)\big) - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\big(\delta(t+\partial 1)+\delta(t-\partial 1)\big) + \frac{2A}{\partial 1}\,\delta(t)\right)e^{-jn\omega_0 t}\,\partial t$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\int_{-T/2}^{T/2}\frac{A}{2-\partial 1}\big(\delta(t+\partial 2)+\delta(t-\partial 2)\big)e^{-jn\omega_0 t}\,\partial t - \int_{-T/2}^{T/2}A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\big(\delta(t+\partial 1)+\delta(t-\partial 1)\big)e^{-jn\omega_0 t}\,\partial t + \int_{-T/2}^{T/2}\frac{2A}{\partial 1}\,\delta(t)\, e^{-jn\omega_0 t}\,\partial t\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\int_{-T/2}^{T/2}\frac{A}{\partial 2-\partial 1}\delta(t+\partial 2)e^{-jn\omega_0 t}\,\partial t + \int_{-T/2}^{T/2}\frac{A}{\partial 2-\partial 1}\delta(t-\partial 2)e^{-jn\omega_0 t}\,\partial t - \int_{-T/2}^{T/2}A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\delta(t+\partial 1)e^{-jn\omega_0 t}\,\partial t\right.$$

$$\left. - \int_{-T/2}^{T/2}A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\delta(t-\partial 1)e^{-jn\omega_0 t}\,\partial t + \int_{-T/2}^{T/2}\frac{2A}{\partial 1}\,\delta(t)\, e^{-jn\omega_0 t}\,\partial t\right) \qquad \text{Usando} \rightarrow \int_{-\infty}^{\infty}X(t)\,\delta(t\pm t_0)\,\partial t = X(\mp t_0)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\frac{A}{\partial 2-\partial 1}e^{-jn\omega_0\cdot -\partial 2} + \frac{A}{\partial 2-\partial 1}e^{-jn\omega_0\cdot\partial 2} - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)e^{-jn\omega_0\cdot -\partial 1} - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)e^{-jn\omega_0\cdot\partial 1} + \frac{2A}{\partial 1}\cdot\cancel{e^0}^{\,1}\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\frac{A}{\partial 2-\partial 1}\big(\cos(n\omega_0\partial 2)+\cancel{j\,\mathrm{sen}(n\omega_0\partial 2)} + \cos(n\omega_0\partial 2)-\cancel{j\,\mathrm{sen}(n\omega_0\partial 2)}\big)\right.$$

$$\left. - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\big(\cos(n\omega_0\partial 1)+\cancel{j\,\mathrm{sen}(n\omega_0\partial 1)} + \cos(n\omega_0\partial 1)-\cancel{j\,\mathrm{sen}(n\omega_0\partial 1)}\big) + \frac{2A}{\partial 1}\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2\omega_0^2}\left(\frac{A}{\partial 2-\partial 1}\cdot 2\cos(n\omega_0\partial 2) - A\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot 2\cos(n\omega_0\partial 1) + \frac{2A}{\partial 1}\right)$$

$$C_n = \frac{2A}{(t_i-t_f)n^2\omega_0^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1) + \frac{1}{\partial 1}\right)$$

$$C_n = \frac{2A}{\left(-\frac{T}{2}-\frac{T}{2}\right)n^2\frac{2^2\pi^2}{T^2}}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1) + \frac{1}{\partial 1}\right)$$

$$C_n = \frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1) + \frac{1}{\partial 1}\right)$$

Ahora confirmamos con x(t)

$$C_n=\frac{1}{T}\int_T x(t)e^{-jn\omega_0 t}dt \quad ; \quad T=t_f-t_i=\frac{T}{2}-\left(-\frac{T}{2}\right)=\frac{T}{2}(1+1)=T \quad ; \quad \omega_0=\frac{2\pi}{T}$$

$$C_n=\frac{1}{T}\left(\int_{-\frac{T}{2}}^{-d_2} 0\cdot e^{-jn\omega_0 t}dt+\int_{-d_2}^{-d_1}\frac{A}{d_2-d_1}(t+d_2)e^{-jn\omega_0 t}dt+\int_{-d_1}^{0}-\frac{A}{d_1}te^{-jn\omega_0 t}dt+\int_{0}^{d_1}\frac{A}{d_1}te^{-jn\omega_0 t}dt+\int_{d_1}^{d_2}-\frac{A}{d_2-d_1}(t+d_2)e^{-jn\omega_0 t}dt+\int_{d_2}^{\frac{T}{2}}0e^{-jn\omega_0 t}dt\right)$$

$$C_n=\frac{1}{T}\left(\frac{A}{d_2-d_1}\int_{-d_2}^{-d_1}(t+d_2)e^{-jn\omega_0 t}dt-\frac{A}{d_1}\int_{-d_1}^{0}te^{-jn\omega_0 t}dt+\frac{A}{d_1}\int_{0}^{d_1}te^{-jn\omega_0 t}dt-\frac{A}{d_2-d_1}\int_{d_1}^{d_2}(t+d_2)e^{-jn\omega_0 t}dt\right)$$

$$C_n=\frac{1}{T}\left(\frac{A}{d_2-d_1}\left(\underbrace{\int_{-d_2}^{-d_1}te^{-jn\omega_0 t}dt}_{1}+\underbrace{d_2\int_{-d_2}^{-d_1}e^{-jn\omega_0 t}dt}_{2}\right)-\frac{A}{d_1}\underbrace{\int_{-d_1}^{0}te^{-jn\omega_0 t}dt}_{3}+\frac{A}{d_1}\underbrace{\int_{0}^{d_1}te^{-jn\omega_0 t}dt}_{4}-\frac{A}{d_2-d_1}\left(\underbrace{\int_{d_1}^{d_2}te^{-jn\omega_0 t}dt}_{5}+\underbrace{d_2\int_{d_1}^{d_2}e^{-jn\omega_0 t}dt}_{6}\right)\right)$$

1 $\int_{-d_2}^{-d_1}te^{-jn\omega_0 t}dt \longrightarrow u=t \rightarrow du=dt$; $dv=e^{-jn\omega_0 t}\longrightarrow v=\int e^{-jn\omega_0 t}dt=\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\rightarrow uv-\int v\,du$

$$=t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{-d_2}^{-d_1}-\int_{-d_2}^{-d_1}\frac{e^{-jn\omega_0 t}}{-jn\omega_0}dt=t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{-d_2}^{-d_1}-\frac{e^{-jn\omega_0 t}}{n^2\omega_0^2}\Big|_{-d_2}^{-d_1}=\left(-d_1\frac{e^{jn\omega_0 d_1}}{-jn\omega_0}-(-d_2)\frac{e^{jn\omega_0 d_2}}{-jn\omega_0}\right)-\left(\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}-\frac{e^{jn\omega_0 d_2}}{n^2\omega_0^2}\right)$$

$$=d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0}-d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0}-\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}+\frac{e^{jn\omega_0 d_2}}{n^2\omega_0^2}=e^{jn\omega_0 d_1}\left(\frac{d_1}{jn\omega_0}-\frac{1}{n^2\omega_0^2}\right)+e^{jn\omega_0 d_2}\left(\frac{1}{n^2\omega_0^2}-\frac{d_2}{jn\omega_0}\right)$$

2 $$d_2\int_{-d_2}^{-d_1}e^{-jn\omega_0 t}dt=-d_2\frac{e^{-jn\omega_0 t}}{jn\omega_0}\Big|_{-d_2}^{-d_1}=-d_2\frac{e^{jn\omega_0 d_1}}{jn\omega_0}-(-d_2)\frac{e^{jn\omega_0 d_2}}{jn\omega_0}=d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0}-d_2\frac{e^{jn\omega_0 d_1}}{jn\omega_0}=\frac{d_2}{jn\omega_0}\left(e^{jn\omega_0 d_2}-e^{jn\omega_0 d_1}\right)$$

3 $$\int_{-d_1}^{0}te^{-jn\omega_0 t}dt=t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{-d_1}^{0}-\int_{-d_1}^{0}\frac{e^{-jn\omega_0 t}}{-jn\omega_0}dt=t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{-d_1}^{0}-\frac{e^{-jn\omega_0 t}}{n^2\omega_0^2}\Big|_{-d_1}^{0}=\left(0\cdot\frac{e^{-jn\omega_0 0}}{-jn\omega_0}-(-d_1)\frac{e^{jn\omega_0 d_1}}{-jn\omega_0}\right)-\left(\frac{e^{jn\omega_0\cdot 0}}{n^2\omega_0^2}-\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}\right)$$

$$=-d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0}-\left(\frac{1}{n^2\omega_0^2}-\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}\right)$$

4 $$\int_{0}^{d_1}te^{-jn\omega_0 t}dt=t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{0}^{d_1}-\int_{0}^{d_1}\frac{e^{-jn\omega_0 t}}{-jn\omega_0}dt=t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{0}^{d_1}-\frac{e^{-jn\omega_0 t}}{n^2\omega_0^2}\Big|_{0}^{d_1}=\left(d_1\frac{e^{-jn\omega_0 d_1}}{-jn\omega_0}-0\frac{e^{-jn\omega_0 0}}{-jn\omega_0}\right)-\left(\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}-\frac{e^{-jn\omega_0\cdot 0}}{n^2\omega_0^2}\right)$$

$$=-d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0}-\left(\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}-\frac{1}{n^2\omega_0^2}\right)$$

5 $$\int_{d_1}^{d_2}te^{-jn\omega_0 t}dt=t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{d_1}^{d_2}-\int_{d_1}^{d_2}\frac{e^{-jn\omega_0 t}}{-jn\omega_0}dt=t\cdot\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{d_1}^{d_2}-\frac{e^{-jn\omega_0 t}}{n^2\omega_0^2}\Big|_{d_1}^{d_2}=\left(d_2\frac{e^{-jn\omega_0 d_2}}{-jn\omega_0}-d_1\frac{e^{-jn\omega_0 d_1}}{-jn\omega_0}\right)-\left(\frac{e^{-jn\omega_0 d_2}}{n^2\omega_0^2}-\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}\right)$$

$$=\left(d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0}-d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}\right)-\left(\frac{e^{-jn\omega_0 d_2}}{n^2\omega_0^2}-\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}\right)$$

6 $$d_2\int_{d_1}^{d_2}e^{-jn\omega_0 t}dt=d_2\frac{e^{-jn\omega_0 t}}{-jn\omega_0}\Big|_{d_1}^{d_2}=d_2\frac{e^{-jn\omega_0 d_2}}{-jn\omega_0}-d_2\frac{e^{-jn\omega_0 d_1}}{-jn\omega_0}=d_2\frac{e^{-jn\omega_0 d_1}}{jn\omega_0}-d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}$$

$$C_n=\frac{1}{T}\left(\frac{A}{d_2-d_1}\left(\left(d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0}-d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0}\right)-\left(\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}-\frac{e^{jn\omega_0 d_2}}{n^2\omega_0^2}\right)+d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0}-d_2\frac{e^{jn\omega_0 d_1}}{jn\omega_0}\right)-\frac{A}{d_1}\left(-d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0}-\left(\frac{1}{n^2\omega_0^2}-\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}\right)\right)\right.$$

$$\left.+\frac{A}{d_1}\left(-d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0}-\left(\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}-\frac{1}{n^2\omega_0^2}\right)\right)-\frac{A}{d_2-d_1}\left(\left(d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0}-d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}\right)-\left(\frac{e^{-jn\omega_0 d_2}}{n^2\omega_0^2}-\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}\right)+d_2\frac{e^{-jn\omega_0 d_1}}{jn\omega_0}-d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}\right)\right)$$

$$=\frac{1}{T}\left(\frac{A}{d_2-d_1}d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0}-\frac{A}{d_2-d_1}d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0}-\frac{A}{d_2-d_1}\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}+\frac{A}{d_2-d_1}\frac{e^{jn\omega_0 d_2}}{n^2\omega_0^2}+\frac{A}{d_2-d_1}d_2\frac{e^{jn\omega_0 d_2}}{jn\omega_0}-\frac{A}{d_2-d_1}d_2\frac{e^{jn\omega_0 d_1}}{jn\omega_0}\right.$$

$$+\frac{A}{d_1}d_1\frac{e^{jn\omega_0 d_1}}{jn\omega_0}+\frac{A}{d_1}\frac{1}{n^2\omega_0^2}-\frac{A}{d_1}\frac{e^{jn\omega_0 d_1}}{n^2\omega_0^2}-\frac{A}{d_1}d_1\frac{e^{-jn\omega_0 d_1}}{jn\omega_0}-\frac{A}{d_1}\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}+\frac{A}{d_1}\frac{1}{n^2\omega_0^2}$$

$$\left.-\frac{A\,d_1}{d_2-d_1}\frac{e^{-jn\omega_0 d_1}}{jn\omega_0}+\frac{A\,d_2}{d_2-d_1}\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}+\frac{A}{d_2-d_1}\frac{e^{-jn\omega_0 d_2}}{n^2\omega_0^2}-\frac{A}{d_2-d_1}\frac{e^{-jn\omega_0 d_1}}{n^2\omega_0^2}-\frac{A}{d_2-d_1}d_2\frac{e^{-jn\omega_0 d_1}}{jn\omega_0}+\frac{A}{d_2-d_1}d_2\frac{e^{-jn\omega_0 d_2}}{jn\omega_0}\right)$$

$$= \frac{1}{T}\Bigg(\frac{\delta_1 A e^{jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} - \frac{\delta_2 A e^{jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} - \frac{A e^{jn\omega_0\delta_1}}{n^2\omega_0^2(\delta_2-\delta_1)} + \frac{A e^{jn\omega_0\delta_2}}{n^2\omega_0^2(\delta_2-\delta_1)} + \frac{\delta_2 A e^{jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} - \frac{\delta_2 A e^{jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)}$$

$$+ \frac{A e^{jn\omega_0\delta_1}}{jn\omega_0} + \frac{A}{\delta_1 n^2\omega_0^2} - \frac{A e^{jn\omega_0\delta_1}}{\delta_1 n^2\omega_0^2} - \frac{A e^{-jn\omega_0\delta_1}}{jn\omega_0} - \frac{A e^{-jn\omega_0\delta_1}}{\delta_1 n^2\omega_0^2} + \frac{A}{\delta_1 n^2\omega_0^2}$$

$$- \frac{\delta_1 A e^{-jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} + \frac{\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} + \frac{A e^{-jn\omega_0\delta_2}}{n^2\omega_0^2(\delta_2-\delta_1)} - \frac{A e^{-jn\omega_0\delta_1}}{n^2\omega_0^2(\delta_2-\delta_1)} - \frac{\delta_2 A e^{-jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} + \frac{\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} \Bigg)$$

Acomodamos los terminos por aparte.

$$\frac{\delta_1 A e^{jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} - \frac{\delta_1 A e^{-jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} \longrightarrow \frac{\delta_1 A}{jn\omega_0(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} - e^{-jn\omega_0\delta_1}\right)$$

$$-\frac{\delta_2 A e^{jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} + \frac{\delta_2 A e^{jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} + \frac{\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} + \frac{\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} \longrightarrow \frac{2\,\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)}$$

$$-\frac{\delta_2 A e^{jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} - \frac{\delta_2 A e^{-jn\omega_0\delta_1}}{jn\omega_0(\delta_2-\delta_1)} \longrightarrow \frac{-\delta_2 A}{jn\omega_0(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right)$$

$$-\frac{A e^{jn\omega_0\delta_1}}{n^2\omega_0^2(\delta_2-\delta_1)} - \frac{A e^{-jn\omega_0\delta_1}}{n^2\omega_0^2(\delta_2-\delta_1)} \longrightarrow \frac{-A}{n^2\omega_0^2(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right)$$

$$+\frac{A}{\delta_1 n^2\omega_0^2} + \frac{A}{\delta_1 n^2\omega_0^2} \longrightarrow \frac{2A}{\delta_1 n^2\omega_0^2}$$

$$+\frac{A e^{jn\omega_0\delta_2}}{n^2\omega_0^2(\delta_2-\delta_1)} + \frac{A e^{-jn\omega_0\delta_2}}{n^2\omega_0^2(\delta_2-\delta_1)} \longrightarrow \frac{A}{n^2\omega_0^2(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_2} + e^{-jn\omega_0\delta_2}\right)$$

$$+\frac{A e^{jn\omega_0\delta_1}}{jn\omega_0} - \frac{A e^{-jn\omega_0\delta_1}}{jn\omega_0} \longrightarrow \frac{A\left(e^{jn\omega_0\delta_1} - e^{-jn\omega_0\delta_1}\right)}{jn\omega_0}$$

$$-\frac{A e^{jn\omega_0\delta_1}}{\delta_1 n^2\omega_0^2} - \frac{A e^{-jn\omega_0\delta_1}}{\delta_1 n^2\omega_0^2} \longrightarrow \frac{-A\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right)}{\delta_1 n^2\omega_0^2}$$

$$\frac{\delta_1 A}{jn\omega_0(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} - e^{-jn\omega_0\delta_1}\right) = \frac{\delta_1 A\left(\cancel{\cos(n\omega_0\delta_1)} + j\,\text{sen}(n\omega_0\delta_1) - \cancel{\cos(n\omega_0\delta_1)} + j\,\text{sen}(n\omega_0\delta_1)\right)}{jn\omega_0(\delta_2-\delta_1)} = \frac{\delta_1 A}{jn\omega_0(\delta_2-\delta_1)}\,2j\,\text{sen}(n\omega_0\delta_1)$$

$$\frac{2\,\delta_2 A e^{-jn\omega_0\delta_2}}{jn\omega_0(\delta_2-\delta_1)} = \frac{2\,\delta_2 A \cos(n\omega_0\delta_2)}{jn\omega_0(\delta_2-\delta_1)} - \frac{2\,\delta_2 A j\,\text{sen}(n\omega_0\delta_2)}{jn\omega_0(\delta_2-\delta_1)}$$

$$\frac{-\delta_2 A}{jn\omega_0(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right) = \frac{-\delta_2 A\left(\cos(n\omega_0\delta_1) + \cancel{j\,\text{sen}(n\omega_0\delta_1)} + \cos(n\omega_0\delta_1) - \cancel{j\,\text{sen}(n\omega_0\delta_1)}\right)}{jn\omega_0(\delta_2-\delta_1)} = \frac{-\delta_2 A\cdot 2\cos(n\omega_0\delta_1)}{jn\omega_0(\delta_2-\delta_1)}$$

$$\frac{-A}{n^2\omega_0^2(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right) = \frac{-A\left(\cos(n\omega_0\delta_1) + \cancel{j\,\text{sen}(n\omega_0\delta_1)} + \cos(n\omega_0\delta_1) - \cancel{j\,\text{sen}(n\omega_0\delta_1)}\right)}{n^2\omega_0^2(\delta_2-\delta_1)} = \frac{-A\cdot 2\cos(n\omega_0\delta_1)}{n^2\omega_0^2(\delta_2-\delta_1)}$$

$$\frac{2A}{\delta_1 n^2\omega_0^2}$$

$$\frac{A}{n^2\omega_0^2(\delta_2-\delta_1)}\left(e^{jn\omega_0\delta_2} + e^{-jn\omega_0\delta_2}\right) = \frac{A\left(\cos(n\omega_0\delta_2) + \cancel{j\,\text{sen}(n\omega_0\delta_2)} + \cos(n\omega_0\delta_2) - \cancel{j\,\text{sen}(n\omega_0\delta_2)}\right)}{n^2\omega_0^2(\delta_2-\delta_1)} = \frac{A\cdot 2\cos(n\omega_0\delta_2)}{n^2\omega_0^2(\delta_2-\delta_1)}$$

$$\frac{A\left(e^{jn\omega_0\delta_1} - e^{-jn\omega_0\delta_1}\right)}{jn\omega_0} = \frac{A\left(\cancel{\cos(n\omega_0\delta_1)} + j\,\text{sen}(n\omega_0\delta_1) - \cancel{\cos(n\omega_0\delta_1)} + j\,\text{sen}(n\omega_0\delta_1)\right)}{jn\omega_0} = \frac{A\cdot 2j\,\text{sen}(n\omega_0\delta_1)}{jn\omega_0}$$

$$\frac{-A\left(e^{jn\omega_0\delta_1} + e^{-jn\omega_0\delta_1}\right)}{\delta_1 n^2\omega_0^2} = \frac{-A\left(\cos(n\omega_0\delta_1) + \cancel{j\,\text{sen}(n\omega_0\delta_1)} + \cos(n\omega_0\delta_1) - \cancel{j\,\text{sen}(n\omega_0\delta_1)}\right)}{\delta_1 n^2\omega_0^2} = \frac{-A\cdot 2\cos(n\omega_0\delta_1)}{\delta_1 n^2\omega_0^2}$$

$$C_n = \frac{1}{T}\Bigg(\frac{\delta_1 A}{jn\omega_0(\delta_2-\delta_1)}\,2j\,\text{sen}(n\omega_0\delta_1) + \frac{2\,\delta_2 A\cos(n\omega_0\delta_2)}{jn\omega_0(\delta_2-\delta_1)} - \frac{2\,\delta_2 A j\,\text{sen}(n\omega_0\delta_2)}{jn\omega_0(\delta_2-\delta_1)} - \frac{\delta_2 A\cdot 2\cos(n\omega_0\delta_1)}{jn\omega_0(\delta_2-\delta_1)} + \frac{2A}{\delta_1 n^2\omega_0^2} + \frac{A\cdot 2\cos(n\omega_0\delta_2)}{n^2\omega_0^2(\delta_2-\delta_1)}$$

$$+ \frac{A\cdot 2j\,\text{sen}(n\omega_0\delta_1)}{jn\omega_0} - \frac{A\cdot 2\cos(n\omega_0\delta_1)}{\delta_1 n^2\omega_0^2}\Bigg)$$

$$= \frac{1}{T}\Bigg(\frac{2\delta_1 A}{n\omega_0(\delta_2-\delta_1)}\,\text{sen}(n\omega_0\delta_1) + \frac{2A\delta_2\cdot\cos(n\omega_0\delta_2)}{jn\omega_0(\delta_2-\delta_1)} - \frac{2\delta_2 A}{n\omega_0(\delta_2-\delta_1)}\,\text{sen}(n\omega_0\delta_2) - \frac{\delta_2 A 2}{jn\omega_0(\delta_2-\delta_1)}\cos(n\omega_0\delta_1) + \frac{2A}{\delta_1 n^2\omega_0^2}$$

$$+ \frac{A\cdot 2\cos(n\omega_0\delta_2)}{n^2\omega_0^2(\delta_2-\delta_1)} + \frac{2A}{n\omega_0}\,\text{sen}(n\omega_0\delta_1) - \frac{2A}{\delta_1 n^2\omega_0^2}\cos(n\omega_0\delta_1)\Bigg)$$

$$= \frac{1}{T}\Bigg(\frac{2A\cdot\text{sen}(n\omega_0\delta_1)}{n\omega_0}\left(\frac{\delta_1}{\delta_2-\delta_1} + 1\right) + \frac{2A\cos(n\omega_0\delta_2)}{n\omega_0(\delta_2-\delta_1)}\left(\frac{\delta_2}{j} + \frac{1}{n\omega_0}\right)$$

$$- \frac{2A\cos(n\omega_0\delta_1)}{n\omega_0}\left(\frac{1}{\delta_1 n\omega_0} + \frac{\delta_2}{\delta_2-\delta_1}\right) - \frac{2\delta_2 A}{n\omega_0(\delta_2-\delta_1)}\,\text{sen}(n\omega_0\delta_2) + \frac{2A}{\delta_1 n^2\omega_0^2}\Bigg)$$

$$\mathrm{Re}\{C_n\} = \frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right) \quad y \quad \mathrm{Im}\{C_n\}=0 \longrightarrow \text{Simetría par}$$

$$|C_n| = \sqrt{\mathrm{Re}^2\{C_n\}+\mathrm{Im}^2\{C_n\}} = \sqrt{\left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^2}$$

$$\theta_{C_n} = \tan^{-1}\left\{\frac{\mathrm{Im}\{C_n\}}{\mathrm{Re}\{C_n\}}\right\} = \tan^{-1}\left(\frac{0}{\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)}\right) = 0$$

$$e_r[\%] = \left(1-\sum |C_n|^2\frac{P_n}{P_x}\right)\cdot 100[\%] \quad ; \quad P_n = \frac{1}{T}E_n = \frac{1}{T}\int_T |e^{jn\omega_0 t}|^2\partial t = \frac{1}{T}\cdot T = 1$$

Como la señal es par solo vamos a integrar de 0 a $\partial 2$ $(x(-t)=x(t))$

$$P_x = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|x(t)|^2\partial t = \frac{2}{T}\int_0^{\frac{T}{2}}|x(t)|^2\partial t \longrightarrow \text{Pero de } \partial 2 \text{ a } \tfrac{T}{2} \text{ es } 0 \longrightarrow P_x = \frac{2}{T}\int_0^{\partial 2}|x(t)|^2\partial t = \frac{2}{T}\left(\int_0^{\partial 1}\left(\frac{A}{\partial 1}t\right)^2\partial t + \int_{\partial 1}^{\partial 2}\left(-\frac{A}{\partial 2-\partial 1}(t+\partial 2)\right)^2\partial t\right)$$

$$P_x = \frac{2}{T}\left(\int_0^{\partial 1}\left(\frac{A}{\partial 1}t\right)^2\partial t + \int_{\partial 1}^{\partial 2}\left(-\frac{A}{\partial 2-\partial 1}(t+\partial 2)\right)^2\partial t\right) = \frac{2}{T}\left(\frac{A^2}{\partial 1^2}\int_0^{\partial 1}t^2\partial t - \frac{A^2}{(\partial 2-\partial 1)^2}\int_{\partial 1}^{\partial 2}(t+\partial 2)^2\partial t\right)$$

$$\int_0^{\partial 1}t^2\partial t = \frac{t^3}{3}\Big|_0^{\partial 1} = \frac{\partial 1^3}{3} \quad ; \quad \int_{\partial 1}^{\partial 2}(t+\partial 2)^2\partial t = \int_{\partial 1+\partial 2}^{2\partial 2}u^2\partial u = \frac{u^3}{3}\Big|_{\partial 1+\partial 2}^{2\partial 2} = \frac{(2\partial 2)^3}{3} - \frac{(\partial 1+\partial 2)^3}{3} = \frac{1}{3}(8\partial 2^3-(\partial 1+\partial 2^3))$$

$u = t+\partial 2$
$\partial u = \partial t$

Si $t=\partial 1 \to u = \partial 1+\partial 2$
Si $t=\partial 2 \to u = 2\partial 2$
$u \in [\partial 1+\partial 2, 2\partial 2]$

$$P_x = \frac{2}{T}\left(\frac{A^2}{\partial 1^2}\frac{\partial 1^3}{3} - \frac{A^2}{(\partial 2-\partial 1)^2}\left(\frac{1}{3}(8\partial 2^3-(\partial 1+\partial 2)^3)\right)\right)$$

$$P_x = \frac{2}{T}\left(\frac{A^2}{\partial 1^2}\frac{\partial 1^3}{3} - \frac{A^2}{3(\partial 2-\partial 1)^2}(8\partial 2^3-(\partial 1+\partial 2)^3)\right)$$

$$P_x = \frac{2}{T}\left(\frac{A^2}{\cancel{\partial 1^2}}\frac{\cancel{\partial 1^3}}{3} - \frac{A^2}{3(\partial 2-\partial 1)^2}(8\partial 2^3-(\partial 1+\partial 2)^3)\right)$$

$$P_x = \frac{2A^2\partial 1}{3T} - \frac{2A^2}{3T(\partial 2-\partial 1)^2}(8\partial 2^3-(\partial 1+\partial 2)^3)$$

$$|C_n|^2 = \cancel{\sqrt{\left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^2}}^{\,2}$$

$$= \left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^2$$

$$e_r(\%) = \left(1-\sum_n \frac{\left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2) - \left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^2}{\frac{2A^2\partial 1}{3T} - \frac{2A^2}{3T(\partial 2-\partial 1)^2}(8\partial 2^3-(\partial 1+\partial 2)^3)}\right)\cdot 100(\%)$$