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1. La distancia media entre dos señales periódicas $x_1(t) \in \mathbb{R},\mathbb{C}$ y $x_2(t) \in \mathbb{R},\mathbb{C}$; se puede expresar a partir de la potencia media de la diferencia entre ellas:

$$d^2(x_1,x_2) = \bar{P}_{x_1-x_2} = \lim_{T\to\infty} \frac{1}{T}\int_T |x_1(t)-x_2(t)|^2\,dt.$$

Sea $x_1(t)$ y $x_2(t)$ dos señales definidas como:

$$x_1(t) = Ae^{-jnw_0t}$$

$$x_2(t) = Be^{jmw_0t}$$

con $w_0 = \frac{2\pi}{T}$; $T, A, B \in \mathbb{R}^+$ y $n, m \in \mathbb{Z}$. Determine la distancia entre las dos señales. Compruebe sus resultados con Python.

$X_1(t) = Ae^{-jn\omega_0 t}, \quad X_2(t) = Be^{jm\omega_0 t} \quad ; \quad \omega_0 = \frac{2\pi}{T}$

$d^2(X_1,X_2) = \bar{P}_{X_1-X_2} = \lim_{T\to\infty} \frac{1}{T}\int_T |X_1(t)-X_2(t)|^2 \partial t$

$$\begin{aligned}
\bar{P}_{X_1-X_2} &= \frac{1}{T}\int_T |X_1(t)-X_2(t)|^2 \partial t \\
&= \frac{1}{T}\left(\int_T (X_1(t)-X_2(t))(X_1(t)-X_2(t))^*\right)\partial t \\
&= \frac{1}{T}\left(\int_T (X_1(t)-X_2(t))(X_1^*(t)-X_2^*(t))\partial t\right) \\
&= \frac{1}{T}\left(\int_T X_1(t)X_1^*(t)\partial t - \int_T X_1(t)X_2^*(t)\partial t \right.\\
&\quad \left. - \int_T X_2(t)X_1^*(t)\partial t + \int_T X_2(t)X_2^*(t)\partial t\right) \\
&= \underbrace{\frac{1}{T}\int_T |X_1(t)|^2\partial t}_{\bar{P}_{X_1}} \underbrace{- \frac{2}{T}\int_T X_1(t)X_2^*(t)\partial t}_{C_X} + \underbrace{\frac{1}{T}\int_T |X_2(t)|^2\partial t}_{\bar{P}_{X_2}}
\end{aligned}$$

$$\begin{aligned}
\bar{P}_{X_1} &= \frac{1}{T}\int_T X_1(t)X_1^*(t)\partial t = \frac{1}{T}\int_0^T Ae^{-jn\omega_0 t}\left(Ae^{-jn\omega_0 t}\right)^* = \frac{1}{T}\int_0^T Ae^{jn\omega_0 t}Ae^{-jn\omega_0 t}\partial t \\
&= \frac{A^2}{T}\int_0^T e^{\overbrace{(n-n)}^{0}j\omega_0 t}\partial t = \frac{A^2}{T}\int_0^T 1\,\partial t = \frac{A^2}{T}(T-0) = \frac{A^2}{\cancel{T}}\cdot\cancel{T} = A^2
\end{aligned}$$

$$\begin{aligned}
\bar{P}_{X_2} &= \frac{1}{T}\int_T X_2(t)X_2^*(t)\partial t = \frac{1}{T}\int_0^T Be^{jm\omega_0 t}\left(Be^{jm\omega_0 t}\right)^* = \frac{1}{T}\int_0^T Be^{jm\omega_0 t}Be^{-jm\omega_0 t}\partial t \\
&= \frac{B^2}{T}\int_0^T e^{\overbrace{(m-m)}^{0}j\omega_0 t}\partial t = \frac{B^2}{T}\int_0^T 1\,\partial t = \frac{B^2}{T}(T-0) = \frac{B^2}{\cancel{T}}\cdot\cancel{T} = B^2
\end{aligned}$$

El cruzado tiene dos casos:

① Si $n \neq -m \rightarrow$

$$\begin{aligned}
C_X &= -\frac{2}{T}\int_T X_1(t)X_2^*(t)\partial t = -\frac{2}{T}\int_0^T Ae^{-jn\omega_0 t}\left(Be^{jm\omega_0 t}\right)^*\partial t \\
&= -\frac{2}{T}\int_0^T Ae^{-jn\omega_0 t}Be^{-jm\omega_0 t}\partial t = -\frac{2AB}{T}\int_0^T e^{-(n+m)j\omega_0 t}\partial t \rightarrow \text{Si } n,m\in\mathbb{Z} \rightarrow n+m = r \in \mathbb{Z} \\
&= -\frac{2AB}{T}\int_0^T e^{-rj\omega_0 t}\partial t = -\frac{j\cdot 2AB}{j\cdot -jr\omega_0 T}e^{-jr\omega_0 t}\Big|_0^T = -\frac{j2AB}{r\frac{2\pi}{\cancel{T}}\cancel{T}}\left(e^{-jr\frac{2\pi}{\cancel{T}}\cancel{T}} - e^{\overbrace{-jr\frac{2\pi}{T}\cdot 0}^{0}}\right) \\
&= -\frac{jAB}{r\pi}\left(e^{-jr2\pi}-1\right) = -\frac{jAB}{\pi r}\left(\overbrace{\cos(2\pi r)}^{1} - j\,\overbrace{\mathrm{sen}(2\pi r)}^{0} - 1\right) \\
&= \frac{jAB}{\pi r}(1-0-1) = \frac{jAB}{\pi r}\cdot 0 = 0
\end{aligned}$$

② Si $n = -m \rightarrow$

$$\begin{aligned}
C_X &= -\frac{2}{T}\int_T X_1(t)X_2^*(t)\partial t = -\frac{2}{T}\int_0^T Ae^{-jn\omega_0 t}\left(Be^{jm\omega_0 t}\right)^*\partial t = -\frac{2}{T}\int_0^T Ae^{-j(-m)\omega_0 t}Be^{-jm\omega_0 t}\partial t \\
&= -\frac{2}{T}\int_0^T Ae^{jm\omega_0 t}Be^{-jm\omega_0 t}\partial t = -\frac{2AB}{T}\int_0^T e^{\overbrace{(m-m)}^{0}j\omega_0 t}\partial t = -\frac{2AB}{T}\int_0^T 1\,\partial t \\
&= -\frac{2AB}{T}(T-0) = -\frac{2AB}{\cancel{T}}\cdot\cancel{T} = -2AB
\end{aligned}$$

$$\bar{P}_{X_1-X_2} = \begin{cases} A^2+B^2 & ; n \neq -m \\ A^2+B^2-2AB & ; n = -m \end{cases}$$

2. Encuentre la señal en tiempo discreto al utilizar un conversor análogo digital con frecuencia de muestreo de $5kHz$ y 4 bits de capacidad de representación, aplicado a la señal continua: $x(t) = 3\cos(1000\pi t) + 5\sin(3000\pi t) + 10\cos(11000\pi t)$. Realizar la simulación del proceso de discretización (incluyendo al menos tres periodos de $x(t)$). En caso de que la discretización no sea apropiada, diseñe e implemente un conversor adecuado para la señal estudiada.

$X(t) = 3\cos(1000\pi t) + 5\,\text{Sen}(3000\pi t) + 10\cos(11000\pi t)$

$F_S = 5\,KHz$, $\#$ de estados $= 2^{4\,Bits} = 16$

$t = nT_s \longrightarrow F_s = \frac{1}{T_s}$; $F = \frac{1}{T} \longrightarrow T = \frac{2\pi}{\omega}$

$A\cos(\Omega n)$
$A\cos(2\pi f n)$ $\longrightarrow \Omega = 2\pi f \longrightarrow f = \frac{T_s}{T} = \frac{F}{F_s}$

$X(t) = X_1(t) + X_2(t) + X_3(t) \rightarrow$

Para $X_1(t) = 3\cos(1000\pi t)$

$W_1 = 1000\pi \longrightarrow T_1 = \frac{2\pi}{W_1} = \frac{2\pi}{1000\pi} = \frac{1}{500} \longrightarrow F_1 = \frac{1}{T} = \frac{1}{\frac{1}{500}} = 500\,Hz \longrightarrow X_1[n] = A\cos\left(2\pi n \cdot \frac{F}{F_s}\right) = 3\cos\left(2\pi \cdot n \cdot \frac{500}{5000}\right) = 3\cos\left(\frac{n\pi}{5}\right)$

Para $X_2(t) = 5\,\text{Sen}(3000\pi t)$

$W_2 = 3000\pi \longrightarrow T_2 = \frac{2\pi}{W_1} = \frac{2\pi}{3000\pi} = \frac{1}{1500} \longrightarrow F_2 = \frac{1}{T} = \frac{1}{\frac{1}{1500}} = 1500\,Hz \longrightarrow X_2[n] = A\,\text{sen}\left(2\pi n \cdot \frac{F}{F_s}\right) = 5\,\text{Sen}\left(2\pi \cdot n \cdot \frac{1500}{5000}\right) = 5\,\text{Sen}\left(\frac{3n\pi}{5}\right)$

Para $X_3(t) = 10\cos(11000\pi t)$

$W_3 = 11000\pi \longrightarrow T_3 = \frac{2\pi}{W_1} = \frac{2\pi}{11000\pi} = \frac{1}{5500} \longrightarrow F_3 = \frac{1}{T} = \frac{1}{\frac{1}{5500}} = 5500\,Hz \longrightarrow X_3[n] = A\cos\left(2\pi n \cdot \frac{F}{F_s}\right) = 10\cos\left(2\pi \cdot n \cdot \frac{5500}{5000}\right) = 10\cos\left(\frac{11n\pi}{5}\right)$

$X[n] = 3\cos\left(\frac{n\pi}{5}\right) + 5\,\text{Sen}\left(\frac{3n\pi}{5}\right) + 10\cos\left(\frac{11n\pi}{5}\right)$

$X[n] = 3\cos\left(\frac{\pi}{5}n\right) + 5\,\text{Sen}\left(\frac{\pi}{5}n\right) + 10\cos\left(\frac{\pi}{5}n\right)$

$X[n] = 13\cos\left(\frac{\pi}{5}n\right) + 5\,\text{Sen}\left(\frac{\pi}{5}n\right)$

$\frac{W_1}{W_2} = \frac{1000\pi}{3000\pi} = \frac{1}{3} \in Q$

$\frac{W_1}{W_3} = \frac{1000\pi}{11000\pi} = \frac{1}{11} \in Q$

$\frac{W_2}{W_3} = \frac{3000\pi}{11000\pi} = \frac{3}{11} \in Q$

$\hookrightarrow$ Como todas son racionales la señal es cuasiperiodica.

Si $F_s \geq 2F_{max} \rightarrow 2F_{max} = 2 \cdot 5500\,Hz = 11000\,Hz \rightarrow F_s = 5000\,Hz.$

$\downarrow$
No cumple Nyquist

Vamos a comparar con las originales. $\longrightarrow -\pi \leq \Omega < \pi$

$\Omega_1 = \frac{1}{5}\pi$ ✓ , $\Omega_2 = \frac{3}{5}\pi$ ✓ , $\Omega_3 = \frac{11}{5}\pi$ ✗ $\longrightarrow$ Es una copia / aliasings

$\Omega_{3\,original} = \frac{11}{5}\pi - 2\pi = \frac{1}{5}\pi$

Suponemos una frecuencia de muestreo mucho más grande

$F_s = 4F_{max} \longrightarrow F_s = 4(5500)\,Hz = 22\,KHz$ $\quad T_s = \frac{1}{22000}$

Para $X_1(t) = 3\cos(1000\pi t)$

$W_1 = 1000\pi \longrightarrow T_1 = \frac{2\pi}{W_1} = \frac{2\pi}{1000\pi} = \frac{1}{500} \longrightarrow F_1 = \frac{1}{T} = \frac{1}{\frac{1}{500}} = 500\,Hz \longrightarrow X_1[n] = A\cos\left(2\pi n \cdot \frac{F}{F_s}\right) = 3\cos\left(2\pi \cdot n \cdot \frac{500}{22000}\right) = 3\cos\left(\frac{\pi}{22}n\right)$

Para $X_2(t) = 5\,\text{Sen}(3000\pi t)$

$W_2 = 3000\pi \longrightarrow T_2 = \frac{2\pi}{W_1} = \frac{2\pi}{3000\pi} = \frac{1}{1500} \longrightarrow F_2 = \frac{1}{T} = \frac{1}{\frac{1}{1500}} = 1500\,Hz \longrightarrow X_2[n] = A\,\text{sen}\left(2\pi n \cdot \frac{F}{F_s}\right) = 5\,\text{Sen}\left(2\pi \cdot n \cdot \frac{1500}{22000}\right) = 5\,\text{Sen}\left(\frac{3\pi}{22}n\right)$

Para $X_3(t) = 10\cos(11000\pi t)$

$W_3 = 11000\pi \longrightarrow T_3 = \frac{2\pi}{W_1} = \frac{2\pi}{11000\pi} = \frac{1}{5500} \longrightarrow F_3 = \frac{1}{T} = \frac{1}{\frac{1}{5500}} = 5500\,Hz \longrightarrow X_3[n] = A\cos\left(2\pi n \cdot \frac{F}{F_s}\right) = 10\cos\left(2\pi \cdot n \cdot \frac{5500}{22000}\right) = 10\cos\left(\frac{\pi}{2}n\right)$

$X[n] = 3\cos\left(\frac{\pi}{22}n\right) + 5\,\text{Sen}\left(\frac{3\pi}{22}n\right) + 10\cos\left(\frac{\pi}{2}n\right)$

Vamos a comparar con las originales. $\longrightarrow -\pi \leq \Omega < \pi$

$\Omega_1 = \frac{1}{22}\pi$ ✓ , $\Omega_2 = \frac{3}{22}\pi$ ✓ , $\Omega_3 = \frac{1}{22}\pi$ ✓ $\longrightarrow$ las tres frecuencias digitales están en los originales.

$X[n] = 3\cos\left(\frac{\pi}{22}n\right) + 5\,\text{Sen}\left(\frac{3\pi}{22}n\right) + 10\cos\left(\frac{\pi}{2}n\right)$

3. Sea $x''(t)$ la segunda derivada de la señal $x(t)$, donde $t \in [t_i, t_f]$. Demuestre que los coeficientes de la serie exponencial de Fourier se pueden calcular según:

$$c_n = \frac{1}{(ti - tf)n^2 w_o^2}\int_{t_i}^{t_f} x''(t)e^{-jnw_o t}dt; \qquad n \in \mathbb{Z}.$$

¿Cómo se pueden calcular los coeficientes a_n y b_n desde $x''(t)$ en la serie trigonométrica de Fourier?.

Con $X(t)$ para $t \in [t_i, t_f]$

Demostrar: $C_n = \frac{1}{(t_i - t_f)n^2 W_o^2}\int_T X''(t)e^{-jnW_o t}\partial t$ con $n \in \mathbb{Z}$

Tenemos: $X(t) = \sum_n C_n e^{jnW_o t}$

$$X'(t) = \frac{\partial}{\partial t}\sum_n C_n e^{jnW_o t} = \sum_n C_n \frac{\partial e^{jnW_o t}}{\partial t}$$

$$X''(t) = \frac{\partial X'(t)}{\partial t} = \frac{\partial}{\partial t}\sum_n C_n \frac{\partial e^{jnW_o t}}{\partial t} = \sum_n C_n \frac{\partial^2 e^{jnW_o t}}{\partial t^2}$$

$$\frac{\partial e^{jnW_o t}}{\partial t} = jnW_o e^{jnW_o t}; \quad \frac{\partial^2 e^{jnW_o t}}{\partial t^2} = \frac{\partial (jnW_o e^{jnW_o t})}{\partial t} = jnW_o \cdot jnW_o e^{jnW_o t} = -n^2W_o^2 e^{jnW_o t}$$

$$X''(t) = \sum_n C_n \frac{\partial^2 e^{jnW_o t}}{\partial t^2} = \sum_n \underbrace{-C_n n^2 W_o^2}_{C\_n} e^{jnW_o t} = \sum_n C\_n e^{jnW_o t}; \text{ con } C\_n = -C_n n^2 W_o^2$$

Si $C_n = \frac{1}{T}\int_T X(t)e^{-jnW_o t}\partial t$ y $C\_n = \frac{1}{T}\int_T X''(t)e^{-jnW_o t}\partial t \longrightarrow$ Remplazamos el $C\_n \longrightarrow -C_n n^2 W_o^2 = \frac{1}{T}\int_T X''(t)e^{-jnW_o t}\partial t$

$$C_n = \frac{1}{-(t_f - t_i)n^2 W_o^2}\int_T X''(t)e^{-jnW_o t}\partial t$$

$$C_n = \frac{1}{(t_i - t_f)n^2 W_o^2}\int_T X''(t)e^{-jnW_o t}\partial t$$

$$C_n = \frac{1}{(t_i - t_f)n^2 W_o^2}\int_T X''(t)e^{-jnW_o t}\partial t = \frac{1}{(t_i - t_f)n^2 W_o^2}\int_T X''(t)(\cos(nW_o t) - j\,\mathrm{sen}(nW_o t))\partial t$$

$$= \frac{1}{(t_i - t_f)n^2 W_o^2}\int_T X''(t)\cos(nW_o t)\partial t - \frac{1}{j(t_i - t_f)n^2 W_o^2}\int_T X''(t)\,\mathrm{sen}(nW_o t)\partial t$$

Sabiendo que: $a_n = \frac{2}{T}\int_T X(t)\cos(nW_o t)\partial t$; $b_n = \frac{2}{T}\int_T X(t)\,\mathrm{sen}(nW_o t)\partial t$ y $a_n = 2\,\mathrm{Re}\{c_n\}$; $b_n = -2\,\mathrm{Im}\{c_n\}$

$$a_n = 2\,\mathrm{Re}\{c_n\} = 2\frac{1}{(t_i - t_f)n^2 W_o^2}\int_T X''(t)\cos(nW_o t)\partial t$$

$$b_n = -2\,\mathrm{Im}\{c_n\} = -2\cdot -\frac{1}{(t_i - t_f)n^2 W_o^2}\int_T X''(t)\,\mathrm{sen}(nW_o t)\partial t = 2\frac{1}{(t_i - t_f)n^2 W_o^2}\int_T X''(t)\,\mathrm{sen}(nW_o t)\partial t$$

También se pueden calcular con: $X(t) = \sum_n a_n\cos(nW_o t) + b_n\,\mathrm{Sen}(nW_o t)$

$$X(t) = \sum_n a_n\cos(nW_o t) + b_n\,\mathrm{Sen}(nW_o t)$$

$$X'(t) = \sum_n -a_n\,\mathrm{Sen}(nW_o t)nW_o + b_n\cos(nW_o t)nW_o$$

$$X''(t) = \sum_n -a_n\cos(nW_o t)nW_o \cdot nW_o - b_n\,\mathrm{Sen}(nW_o t)nW_o \cdot nW_o = \sum_n \underbrace{-a_n n^2 W_o^2}_{a\_n}\cos(nW_o t) \underbrace{- b_n n^2 W_o^2}_{b\_n}\,\mathrm{Sen}(nW_o t)$$

$$a\_n = -a_n n^2 W_o^2, \quad b\_n = -b_n n^2 W_o^2$$

$$a_n = \frac{2}{T}\int_T X(t)\cos(nW_o t)\partial t \longrightarrow a\_n = \frac{2}{T}\int_T X''(t)\cos(nW_o t)\partial t$$

$$-a_n n^2 W_o^2 = \frac{2}{T}\int_T X''(t)\cos(nW_o t)\partial t$$

$$a_n = \frac{2}{-(t_f - t_i)n^2 W_o^2}\int_T X''(t)\cos(nW_o t)\partial t$$

$$a_n = \frac{2}{(t_i - t_f)n^2 W_o^2}\int_T X''(t)\cos(nW_o t)\partial t$$

$$b_n = \frac{2}{T}\int_T X(t)\,\mathrm{Sen}(nW_o t)\partial t \longrightarrow b\_n = \frac{2}{T}\int_T X''(t)\,\mathrm{Sen}(nW_o t)\partial t$$

$$-b_n n^2 W_o^2 = \frac{2}{T}\int_T X''(t)\,\mathrm{Sen}(nW_o t)\partial t$$

$$b_n = \frac{2}{-(t_f - t_i)n^2 W_o^2}\int_T X''(t)\,\mathrm{Sen}(nW_o t)\partial t$$

$$b_n = \frac{2}{(t_i - t_f)n^2 W_o^2}\int_T X''(t)\,\mathrm{Sen}(nW_o t)\partial t$$

4. Encuentre el espectro de Fourier, su parte real, imaginaria, magnitud, fase y el error relativo para $n\in\{0,\pm1,\pm2,\pm3,\pm4,\pm5\}$, a partir de $x''(t)$ para la señal $x(t)$ en la Figura 1. Compruebe el espectro obtenido con la estimación a partir de $x(t)$. Presente las simulaciones de Python respectivas.

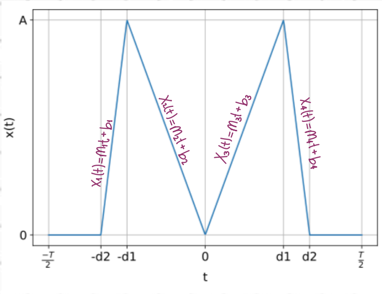


Figura 1: $x(t)$ para el ejercicio 1.4

$$X(t)=\begin{cases} 0 & -\frac{T}{2}\le t<-\partial_2 \\ \frac{A}{\partial_2-\partial_1}(t+\partial_2) & -\partial_2\le t<-\partial_1 \\ \frac{-A}{\partial_1}t & -\partial_1\le t<0 \\ \frac{A}{\partial_1}t & 0\le t<\partial_1 \\ -\frac{A}{\partial_2-\partial_1}(t+\partial_2) & \partial_1\le t<\partial_2 \\ 0 & \partial_2\le t<\frac{T}{2} \end{cases}$$

Para $X_1(t)=m_1t+b_1 \rightarrow m_1=\frac{A-0}{-\partial_1-(-\partial_2)}=\frac{A}{\partial_2-\partial_1}$; En $t=-\partial_2 \rightarrow X(-\partial_2)=\frac{A}{\partial_2-\partial_1}\cdot-\partial_2+b_1$

$$0=\frac{-\partial_2 A}{\partial_2-\partial_1}+b_1$$

$$b_1=\frac{\partial_2}{\partial_2-\partial_1}A$$

$$X_1(t)=\frac{A}{\partial_2-\partial_1}t+\frac{\partial_2}{\partial_2-\partial_1}A$$

$$X_1(t)=\frac{A}{\partial_2-\partial_1}(t+\partial_2)$$

Para $X_2(t)=m_2t+b_2 \rightarrow m_2=\frac{0-A}{0-(-\partial_1)}=\frac{-A}{\partial_1}$; En $t=-\partial_1 \rightarrow X(-\partial_1)=\frac{-A}{\cancel{\partial_1}}\cdot\cancel{-\partial_1}+b_2$

$$X_2(t)=\frac{-A}{\partial_1}t$$

$$A=A+b_2$$

$$b_2=A-A=0$$

Para $X_3(t)=m_3t+b_3 \rightarrow m_3=\frac{A-0}{\partial_1-0}=\frac{A}{\partial_1}$; En $t=0 \rightarrow X(0)=\frac{A}{\partial_1}\cdot 0+b_3$

$$X_3(t)=\frac{A}{\partial_1}t$$

$$0=0+b_3$$

$$b_3=0$$

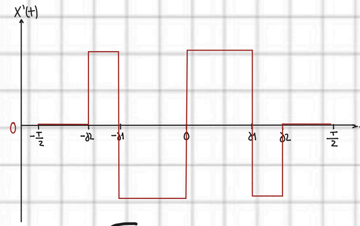
Para $X_4(t)=m_4t+b_4 \rightarrow m_4=\frac{0-A}{\partial_2-\partial_1}=\frac{-A}{\partial_2-\partial_1}$; En $t=\partial_2 \rightarrow X(\partial_2)=\frac{-A}{\partial_2-\partial_1}\partial_2+b_4$

$$0=\frac{-A}{\partial_2-\partial_1}\cdot\partial_2+b_4$$

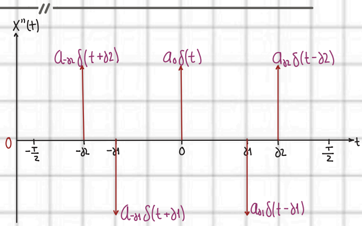
$$b_4=\frac{-A\partial_2}{\partial_2-\partial_1}$$

$$X_4(t)=\frac{-A}{\partial_2-\partial_1}t+\frac{-A\partial_2}{\partial_2-\partial_1}$$

$$X_4(t)=-\frac{A}{\partial_2-\partial_1}(t+\partial_2)$$



$$X'(t)=\begin{cases} 0 & -\frac{T}{2}\le t<-\partial_2 \\ \frac{A}{\partial_2-\partial_1} & -\partial_2\le t<-\partial_1 \\ \frac{-A}{\partial_1} & -\partial_1\le t<0 \\ \frac{A}{\partial_1} & 0\le t<\partial_1 \\ -\frac{A}{\partial_2-\partial_1} & \partial_1\le t<\partial_2 \\ 0 & \partial_2\le t<\frac{T}{2} \end{cases}$$



$X''(t)=0$ en cada tramo y en los puntos donde hay cambios tiende a $\pm\infty$ y estos cambios los podemos representar como $a_k\delta(t\pm t_k)$ donde a_k es la diferencia de la derivada por la derecha y la derivada por la izquierda y t_k donde ocurre el salto (cambio instantáneo)

$$a_{-d_2} = X'(-d_2^+) - X'(-d_2^-) = m_1 - 0 = \frac{A}{d_2-d_1}$$

$$a_{-d_1} = X'(-d_1^+) - X'(-d_1^-) = m_2 - m_1 = \frac{-A}{d_1} - \frac{A}{d_2-d_1} = -A\left(\frac{1}{d_1} + \frac{1}{d_2-d_1}\right)$$

$$a_0 = X'(0^+) - X'(0^-) = m_3 - m_2 = \frac{A}{d_1} - \frac{-A}{d_1} = \frac{A}{d_1}(1+1) = \frac{2A}{d_1}$$

$$a_{d_1} = X'(d_1^+) - X'(d_1^-) = m_4 - m_3 = -\frac{A}{d_2-d_1} - \frac{A}{d_1} = -A\left(\frac{1}{d_2-d_1} + \frac{1}{d_1}\right)$$

$$a_{d_2} = X'(d_2^+) - X'(d_2^-) = 0 - m_4 = 0 - -\frac{A}{d_2-d_1} = \frac{A}{d_2-d_1}$$

$$X''(t) = a_{-d_2}\,\delta(t+d_2) + a_{-d_1}\,\delta(t+d_1) + a_0\,\delta(t) + a_{d_1}\,\delta(t-d_1) + a_{d_2}\,\delta(t-d_2)$$

$$X''(t) = \frac{A}{d_2-d_1}\,\delta(t+d_2) - A\left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)\delta(t+d_1) + \frac{2A}{d_1}\,\delta(t) - A\left(\frac{1}{d_2-d_1}+\frac{1}{d_1}\right)\delta(t-d_1) + \frac{A}{d_2-d_1}\,\delta(t-d_2)$$

$$X''(t) = \frac{A}{d_2-d_1}\big(\delta(t+d_2)+\delta(t-d_2)\big) - A\left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)\big(\delta(t+d_1)+\delta(t-d_1)\big) + \frac{2A}{d_1}\,\delta(t)$$

$$C_n = \frac{1}{(t_i-t_f)n^2 w_0^2}\int_T X''(t)\, e^{-jnw_0 t}\, dt \qquad \text{con } T = t_f - t_i = \frac{T}{2} - \left(-\frac{T}{2}\right) = T \quad \text{y} \quad w_0 = \frac{2\pi}{T}$$

$$C_n = \frac{1}{(t_i-t_f)n^2 w_0^2}\int_{-T/2}^{T/2}\left(\frac{A}{d_2-d_1}\big(\delta(t+d_2)+\delta(t-d_2)\big) - A\left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)\big(\delta(t+d_1)+\delta(t-d_1)\big) + \frac{2A}{d_1}\,\delta(t)\right)e^{-jnw_0 t}\, dt$$

$$C_n = \frac{1}{(t_i-t_f)n^2 w_0^2}\left(\int_{-T/2}^{T/2}\frac{A}{2-d_1}\big(\delta(t+d_2)+\delta(t-d_2)\big)e^{-jnw_0 t}\,dt - \int_{-T/2}^{T/2} A\left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)\big(\delta(t+d_1)+\delta(t-d_1)\big)e^{-jnw_0 t}\,dt + \int_{-T/}^{T/2}\frac{2A}{d_1}\,\delta(t)\, e^{-jnw_0 t}\,dt\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2 w_0^2}\left(\int_{-T/2}^{T/2}\frac{A}{d_2-d_1}\delta(t+d_2)e^{-jnw_0 t}\,dt + \int_{-T/2}^{T/2}\frac{A}{d_2-d_1}\delta(t-d_2)e^{-jnw_0 t}\,dt - \int_{-T/2}^{T/2} A\left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)\delta(t+d_1)e^{-jnw_0 t}\,dt\right.$$

$$\left. - \int_{-T/2}^{T/2} A\left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)\delta(t-d_1)e^{-jnw_0 t}\,dt + \int_{-T/2}^{T/2}\frac{2A}{d_1}\,\delta(t)\, e^{-jnw_0 t}\,dt\right) \quad \text{Usando} \rightarrow \int_{-\infty}^{\infty} X(t)\,\delta(t \pm t_0)\,dt = X(\mp t_0)$$

$$C_n = \frac{1}{(t_i-t_f)n^2 w_0^2}\left(\frac{A}{d_2-d_1}e^{-jnw_0\cdot -d_2} + \frac{A}{d_2-d_1}e^{-jnw_0\cdot d_2} - A\left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)e^{-jnw_0\cdot -d_1} - A\left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)e^{-jnw_0\cdot d_1} + \frac{2A}{d_1}\cdot \cancelto{1}{e^{0}}\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2 w_0^2}\left(\frac{A}{d_2-d_1}\big(\cos(nw_0d_2) + \cancel{j\,\text{sen}(nw_0d_2)} + \cos(nw_0d_2) - \cancel{j\,\text{sen}(nw_0d_2)}\big)\right.$$

$$\left. - A\left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)\big(\cos(nw_0d_1) + \cancel{j\,\text{sen}(nw_0d_1)} + \cos(nw_0d_1) - \cancel{j\,\text{sen}(nw_0d_1)}\big) + \frac{2A}{d_1}\right)$$

$$C_n = \frac{1}{(t_i-t_f)n^2 w_0^2}\left(\frac{A}{d_2-d_1}\cdot 2\cos(nw_0d_2) - A\left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)\cdot 2\cos(nw_0d_1) + \frac{2A}{d_1}\right)$$

$$C_n = \frac{2A}{(t_i-t_f)n^2 w_0^2}\left(\frac{1}{d_2-d_1}\cdot\cos(nw_0d_2) - \left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)\cdot\cos(nw_0d_1) + \frac{1}{d_1}\right)$$

$$C_n = \frac{2A}{\left(-\frac{T}{2}-\frac{T}{2}\right)n^2\,\frac{2^2\pi^2}{T^2}}\left(\frac{1}{d_2-d_1}\cdot\cos(nw_0d_2) - \left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)\cdot\cos(nw_0d_1) + \frac{1}{d_1}\right)$$

$$C_n = \frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{d_2-d_1}\cdot\cos(nw_0d_2) - \left(\frac{1}{d_1}+\frac{1}{d_2-d_1}\right)\cdot\cos(nw_0d_1) + \frac{1}{d_1}\right)$$

$$Re\{C_n\}=\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2)-\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right) \quad y \quad Im\{C_n\}=0 \longrightarrow \text{Simetría par}$$

$$|C_n|=\sqrt{Re^2\{C_n\}+Im^2\{C_n\}}=\sqrt{\left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2)-\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^2}$$

$$\theta_{C_n}=\tan^{-1}\left\{\frac{Im\{C_n\}}{Re\{C_n\}}\right\}=\tan^{-1}\left(\frac{0}{\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2)-\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)}\right)=0$$

$$e_r[\%]=\left(1-\sum|C_n|^2\frac{P_n}{P_x}\right)\cdot 100[\%] \quad ; \quad P_n=\frac{1}{T}E_n=\frac{1}{T}\int_T|e^{jn\omega_0 t}|^2\partial t=\frac{1}{T}\cdot T=1$$

Como la señal es par solo vamos a integrar de 0 a $\partial 2$ $(x(-t)=x(t))$

$$P_x=\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}}|x(t)|^2\partial t=\frac{2}{T}\int_0^{\frac{T}{2}}|x(t)|^2\partial t \longrightarrow \text{Pero de } \partial 2 \text{ a } \tfrac{T}{2} \text{ es } 0 \longrightarrow P_x=\frac{2}{T}\int_0^{\partial 2}|x(t)|^2\partial t=\frac{2}{T}\left(\int_0^{\partial 1}\left(\frac{A}{\partial 1}t\right)^2\partial t+\int_{\partial 1}^{\partial 2}\left(-\frac{A}{\partial 2-\partial 1}(t+\partial 2)\right)^2\partial t\right)$$

$$P_x=\frac{2}{T}\left(\int_0^{\partial 1}\left(\frac{A}{\partial 1}t\right)^2\partial t+\int_{\partial 1}^{\partial 2}\left(-\frac{A}{\partial 2-\partial 1}(t+\partial 2)\right)^2\partial t\right)=\frac{2}{T}\left(\frac{A^2}{\partial 1^2}\int_0^{\partial 1}t^2\partial t-\frac{A^2}{(\partial 2-\partial 1)^2}\int_{\partial 1}^{\partial 2}(t+\partial 2)^2\partial t\right)=\frac{2}{T}\left(\frac{A^2}{\partial 1^2}\left.\frac{t^3}{3}\right|_0^{\partial 1}-\frac{A^2}{(\partial 2-\partial 1)^2}\int_{\partial 1}^{\partial 2}(t^2+2t\partial 2+\partial 2^2)\partial t\right)$$

$$=\frac{2}{T}\left(\frac{A^2}{\partial 1^2}\left(\frac{\partial 1^3}{3}-0\right)-\frac{A^2}{(\partial 2-\partial 1)^2}\left.\left(\frac{t^3}{3}+\frac{2t^2\partial 2}{2}+\partial 2^2 t\right)\right|_{\partial 1}^{\partial 2}\right)=\frac{2}{T}\left(\frac{A^2\partial 1^3}{3\partial 1^2}-\frac{A^2}{(\partial 2-\partial 1)^2}\left(\frac{\partial 2^3}{3}+\frac{2\partial 2^2\partial 2}{2}+\partial 2^2\cdot\partial 2-\frac{\partial 1^3}{3}-\frac{2\partial 1^2\partial 2}{2}+\partial 2^2\partial 1\right)\right)$$

$$=\frac{2}{T}\left(\frac{A^2\partial 1}{3}-\frac{A^2}{(\partial 2-\partial 1)^2}\left(\frac{\partial 2^3}{3}+\partial 2^3+\partial 2^3-\frac{\partial 1^3}{3}-\partial 1^2\partial 2-\partial 2^2\partial 1\right)\right)$$

$$|C_n|^2=\cancel{\sqrt{\left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2)-\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^{\cancel{2}}}}^{2}$$

$$=\left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2)-\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^2$$

$$e_r(\%)=\left(1-\sum_n\frac{\left(\frac{-AT}{2\cdot n^2\pi^2}\left(\frac{1}{\partial 2-\partial 1}\cdot\cos(n\omega_0\partial 2)-\left(\frac{1}{\partial 1}+\frac{1}{\partial 2-\partial 1}\right)\cdot\cos(n\omega_0\partial 1)+\frac{1}{\partial 1}\right)\right)^2}{\frac{2}{T}\left(\frac{A^2\partial 1}{3}-\frac{A^2}{(\partial 2-\partial 1)^2}\left(\frac{\partial 2^3}{3}+\partial 2^3+\partial 2^3-\frac{\partial 1^3}{3}-\partial 1^2\partial 2-\partial 2^2\partial 1\right)\right)}\right)\cdot 100(\%)$$