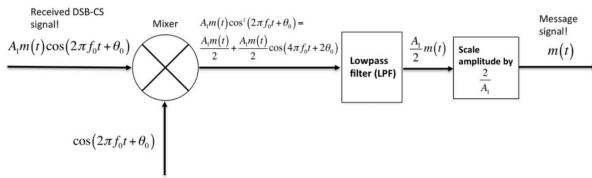


1. Sea el demodulador en amplitud presentado en la siguiente Figura:



Asumiendo $\theta_0 = 0$, determine el espectro de Fourier (teórico) en cada una de las etapas del sistema. Luego, con base en la simulación de modulación en amplitud del Taller 2 y utilizando cinco segundos de una canción de Youtube como mensaje, grafique cada una de las etapas principales del proceso de modulación y demodulación en el tiempo y la frecuencia (reproduzca el segmento de la canción en cada etapa).

Nota: Para la etapa de filtrado pasa bajas, realice su implementación a partir de la transformada rápida de Fourier.

Demodulador coherente de AM DSB-SC/DSB-SC

Señal mensaje base: $m(t) \rightarrow M(\omega)$

Señal recibida: $s_{rec}(t) = A_1 m(t) \cos(2\pi f_c t + \theta_0)$

A_1 : Ganancia, f_c : frecuencia de la portadora.

Segunda señal que entra al mixer: $\cos(2\pi f_c t + \theta_0)$

$$\omega_0 = 2\pi f_c = \frac{2\pi}{T}$$

Después del mixer:

$$y(t) = A_1 m(t) \cos(2\pi f_c t + \theta_0) \cdot \cos(2\pi f_c t + \theta_0)$$

$$y(t) = A_1 m(t) \cos^2(2\pi f_c t + \theta_0) ; \omega_0 = 2\pi f_c$$

$$\text{Si } \cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

$$y(t) = \frac{A_1}{2} m(t) (1 + \cos(2(2\pi f_c t + \theta_0)))$$

$$y(t) = \frac{A_1}{2} m(t) + \frac{A_1}{2} m(t) \cos(4\pi f_c t + 2\theta_0)$$

Filtro pasa-bajas (LPF):

Entrada: $y(t)$

$$\text{Salida: } y_f(t) = \frac{A_1}{2} m(t) \rightarrow y_f(\omega) = \frac{A_1}{2} M(\omega)$$

Escalador de amplitud:

$$\text{Entrada: } y_f(t) = \frac{A_1}{2} m(t)$$

$$\text{Ganancia: } G = \frac{2}{A_1}$$

$$\text{Salida: } \hat{m}(t) = \frac{A_1}{2} m(t) \cdot \frac{2}{A_1} = m(t)$$

$$X(\omega) = F\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Espectro de señal recibida al mixer:

$$s_{rec}(t) = A_1 m(t) \cos(2\pi f_c t + \theta_0) \quad \text{Si } \cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2} \text{ y } \omega_0 = 2\pi f_c$$

$$= A_1 m(t) \cos(\omega_0 t)$$

$$= A_1 m(t) \left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2} \right)$$

$$= \frac{1}{2} A_1 m(t) (e^{j\omega_0 t} + e^{-j\omega_0 t})$$

$$\text{Si } f(t) e^{j\omega_0 t} \leftrightarrow F(\omega - \omega_0)$$

$$S_{rec}(\omega) = F\left\{ \frac{1}{2} A_1 m(t) (e^{j\omega_0 t} + e^{-j\omega_0 t}) \right\} = \frac{1}{2} A_1 F\{m(t) e^{j\omega_0 t} + m(t) e^{-j\omega_0 t}\} = \frac{1}{2} A_1 (M(\omega - \omega_0) + M(\omega + \omega_0))$$

Espectro de señal de salida del mixer:

$$y(t) = \frac{A_1}{2} m(t) + \frac{A_1}{2} m(t) \cos(4\pi f_c t + 2\theta_0) \quad \text{Si } \omega_0 = 2\pi f_c$$

$$y(t) = \frac{A_1}{2} m(t) + \frac{A_1}{2} m(t) \cos(2\omega_0 t)$$

$$y(\omega) = F\left\{ \frac{A_1}{2} m(t) + \frac{A_1}{2} m(t) \cos(2\omega_0 t) \right\} = F\left\{ \frac{A_1}{2} m(t) \right\} + F\left\{ \frac{A_1}{2} m(t) \cos(2\omega_0 t) \right\} = \frac{A_1}{2} M(\omega) + F\left\{ \frac{A_1}{2} m(t) \cos(2\omega_0 t) \right\} \quad \text{Si } \cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$= \frac{A_1}{2} M(\omega) + F\left\{ \frac{A_1}{2} m(t) \frac{e^{j2\omega_0 t} + e^{-j2\omega_0 t}}{2} \right\} = \frac{A_1}{2} M(\omega) + \frac{A_1}{4} F\{m(t) e^{j2\omega_0 t} + m(t) e^{-j2\omega_0 t}\} \quad \text{Si } f(t) e^{j\omega_0 t} \leftrightarrow F(\omega - \omega_0)$$

$$= \frac{A_1}{2} M(\omega) + \frac{A_1}{4} (M(\omega - 2\omega_0) + M(\omega + 2\omega_0))$$

Espectro de señal de salida del mixer: $H_f(\omega) = \begin{cases} 1, & |\omega| \leq \omega_c \\ 0, & |\omega| > \omega_c \end{cases}$

$$H_f(\omega) = \frac{y_f(\omega)}{y(\omega)}$$

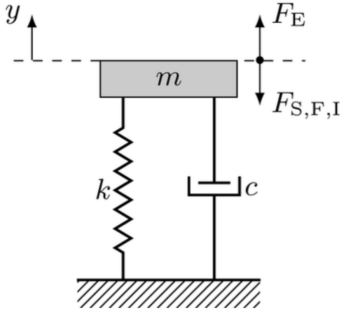
$$y_f(t) = \frac{A_1}{2} m(t) \rightarrow y_f(\omega) = \frac{A_1}{2} M(\omega)$$

Espectro de señal del escalador:

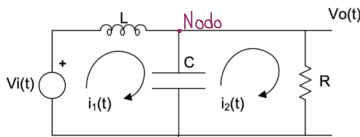
$$\hat{m}(t) = \frac{A_1}{2} m(t) \cdot \frac{2}{A_1} = m(t)$$

$$\hat{m}(\omega) = \mathcal{F}\{m(t)\} = M(\omega)$$

2. Encuentre la función de transferencia que caracteriza el sistema masa, resorte, amortiguador, presentado en la siguiente Figura (asuma condiciones iniciales cero):



Posteriormente, encuentre el sistema equivalente del modelo masa, resorte, amortiguador, a partir del siguiente circuito eléctrico:



Finalmente, proponga unos valores de m , k y c y sus equivalentes R , L y C , para simular un sistema subamortiguado, sobreamortiguado y de amortiguamiento crítico (determine el factor de amortiguamiento, la frecuencia natural amortiguada, la frecuencia natural no amortiguada, el tiempo pico, tiempo de levantamiento y el tiempo de establecimiento en cada caso). Para cada caso, grafique el diagrama de polos y ceros, el diagrama de Bode, la respuesta impulso, respuesta escalón y respuesta rampa. Repita el proceso para modo lazo cerrado.

$$V_L(t) = L \frac{d i_L(t)}{dt} = V_i(t) - V_o(t) \rightarrow \frac{d i_L(t)}{dt} = \frac{V_i(t) - V_o(t)}{L}$$

Si le aplicamos derivada a: $i_L(t) = C \frac{d V_o(t)}{dt} + \frac{1}{R} V_o(t)$

$$\frac{d i_L(t)}{dt} = C \frac{d^2 V_o(t)}{dt^2} + \frac{1}{R} \frac{d V_o(t)}{dt}$$

$$\frac{V_i(t) - V_o(t)}{L} = C \frac{d^2 V_o(t)}{dt^2} + \frac{1}{R} \frac{d V_o(t)}{dt} \rightarrow V_i(t) - V_o(t) = CL \frac{d^2 V_o(t)}{dt^2} + \frac{L}{R} \frac{d V_o(t)}{dt} \rightarrow V_i(t) = CL \frac{d^2 V_o(t)}{dt^2} + \frac{L}{R} \frac{d V_o(t)}{dt} + V_o(t) \rightarrow \text{E.D.O del sistema}$$

$$V_i(s) = CLs^2 V_o(s) + \frac{L}{R} s V_o(s) + V_o(s)$$

$$V_i(s) = V_o(s) (CLs^2 + \frac{L}{R} s + 1)$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{1}{(CLs^2 + \frac{L}{R} s + 1)} \cdot \frac{R}{R} = \frac{R}{(RCLs^2 + Ls + R)} \rightarrow \text{función de transferencia}$$

Si comparamos los sistemas (1) y (2) $m=L$, $C=\frac{1}{R}$, $K=\frac{1}{C}$, $F_E(t)=\frac{1}{C} V_i(t)$, $y(t)=V_o(t)$

masa (bloque): m , resorte: K , amortiguador: C

fuerza externa aplicada sobre la masa: $F_E(t)$ (hacia arriba)

fuerza del resorte: F_S

$F_S = KY(t)$: (opuesta al desplazamiento)

$F_d = CY'(t)$: (opuesta a la velocidad) $F = m \cdot a$

tomaremos $y(t)$ como el desplazamiento

$$m y''(t) = F_E(t) - F_d(t) - F_S(t)$$

$$m y''(t) + C y'(t) + K y(t) = F_E(t) \quad (1) \rightarrow \text{E.D.O del sistema}$$

$$\mathcal{L}\left\{\frac{d^2 X(t)}{dt^2}\right\} = s^2 X(s)$$

$$F_E(s) = \mathcal{L}\{m y''(t) + C y'(t) + K y(t)\}$$

$$F_E(s) = m s^2 Y(s) + C s Y(s) + K Y(s)$$

$$F_E(s) = Y(s) (m s^2 + C s + K)$$

$$\frac{1}{(m s^2 + C s + K)} = \frac{y(s)}{F_E(s)}$$

$$H(s) = \frac{y(s)}{F_E(s)} = \frac{1}{(m s^2 + C s + K)}$$

$$H(s) = \frac{1}{m s^2 + C s + K} \rightarrow \text{función de transferencia}$$

Calculamos la función de transferencia del circuito:

En paralelo: $i_L(t) = i_C(t) + i_R(t) \rightarrow i_L(t) = C \frac{d V_o(t)}{dt} + \frac{1}{R} V_o(t)$ Si $V_L(t) = V_R(t) = V_o(t)$

$$i_L(t) = C \frac{d V_o(t)}{dt} + \frac{1}{R} V_o(t)$$

$$\frac{1}{C} V_i(t) = L \frac{d^2 V_o(t)}{dt^2} + \frac{L}{C R} \frac{d V_o(t)}{dt} + \frac{1}{C} V_o(t) \quad (2) \rightarrow \text{E.D.O del sistema}$$

$$H(s) = \frac{1}{m} \cdot \frac{1}{s^2 + \frac{c}{m}s + \frac{k}{m}}$$

Para llevarlo a su forma estandar de segundo orden:

$$\text{Si la frecuencia natural no amortiguada es: } \omega_n^2 = \frac{k}{m} \leftrightarrow \omega_n = \sqrt{\frac{k}{m}}$$

$2\zeta\omega_n = \frac{c}{m}$: ζ es factor de amortiguamiento adimensional

$$\zeta = \frac{c}{2m\omega_n} = \frac{c}{2m\omega_n} = \frac{c}{2\sqrt{km}} \quad \text{Si no hay amortiguamiento } c=0$$

$$H(s) = \frac{1}{m} \cdot \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Para los polos:

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

$$\text{Con } \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \leftrightarrow p_{1,2} = \frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2} = \frac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2} = \frac{-2\zeta\omega_n \pm \sqrt{4(\zeta^2\omega_n^2 - \omega_n^2)}}{2}$$

$$= \frac{-2\zeta\omega_n \pm 2\sqrt{\zeta^2\omega_n^2 - \omega_n^2}}{2} = -\zeta\omega_n \pm \sqrt{\zeta^2\omega_n^2 - \omega_n^2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\zeta\omega_n \pm \omega_n\sqrt{1 - \zeta^2}$$

Para subamortiguado: $0 < \zeta < 1 \rightarrow \zeta^2 - 1 < 0: \sqrt{\zeta^2 - 1} = j\sqrt{1 - \zeta^2}$

$$p_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2} \rightarrow \text{La frecuencia amortiguada es la velocidad angular: } \omega_d = \omega_n\sqrt{1 - \zeta^2}$$

$$\text{Tiempo de establecimiento } (\pm 5\%): T_s = \frac{3}{\zeta\omega_n}$$

$$\text{Tiempo de pico: } T_p = \frac{\pi}{\omega_n\sqrt{1 - \zeta^2}}$$

Para amortiguamiento critico: $\zeta = 1 \quad p_{1,2} = -\omega_n$

Para sobreamortiguado: $\zeta > 1 \rightarrow \zeta^2 - 1 > 0: p_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{1 - \zeta^2}$

Ahora $H(s)$ con lazo cerrado:

$$H(s) = \frac{1}{m} \cdot \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \text{Para lazo cerrado hay una realimentación:}$$

$$G_{cl}(s) = \frac{\frac{1}{m} \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}}{1 + \frac{1}{m} \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}} = \frac{\frac{1}{m} \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2}}{\frac{m(s^2 + 2\zeta\omega_n s + \omega_n^2) + 1}{m(s^2 + 2\zeta\omega_n s + \omega_n^2)}} = \frac{1}{m(s^2 + 2\zeta\omega_n s + \omega_n^2)} \cdot \frac{m(s^2 + 2\zeta\omega_n s + \omega_n^2)}{m(s^2 + 2\zeta\omega_n s + \omega_n^2) + 1}$$

$$= \frac{1}{m(s^2 + 2\zeta\omega_n s + \omega_n^2) + 1} = \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2 + \frac{1}{m}}$$

Respuesta al impulso (Subamortiguado): Si $F_E(s) = \delta(t) \rightarrow F_E(s) = 1 \quad H(s) = \frac{y(s)}{F_E(s)} = \frac{y(s)}{1} = y(s)$

$$H(s) = \frac{1}{m} \cdot \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \rightarrow (s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2) = (s + \zeta\omega_n)^2 + \omega_d^2$$

$$H(s) = \frac{1}{m} \cdot \frac{1}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

$$\text{Si } \frac{1}{(s-a)^2 + b^2} \leftrightarrow e^{at} \frac{\sin(bt)}{b}$$

$$H(s) = \mathcal{L}\{h(t)\}$$

$$\mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\{\mathcal{L}\{h(t)\}\}$$

$$\mathcal{L}^{-1}\{H(s)\} = h(t)$$

$$h(t) = \frac{1}{m\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t)$$

Respuesta al escalón (Subamortiguado): Si $F_E(t) = u(t) \rightarrow F_E(s) = \frac{1}{s}$ $H(s) = \frac{Y(s)}{F_E(s)} \rightarrow Y(s) = F_E(s)H(s)$

$$Y(s) = \frac{1}{m} \cdot \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{1}{s} = \frac{A}{s} + \frac{Bs+C}{s^2 + 2\zeta\omega_n s + \omega_n^2} \rightarrow Bs+C = B((s+\zeta\omega_n) - \zeta\omega_n) + C = B(s+\zeta\omega_n) + (C-B\zeta\omega_n)$$

$$s^2 + 2\zeta\omega_n s + \omega_n^2 \rightarrow (s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2) = (s+\zeta\omega_n)^2 + \omega_d^2$$

$$\frac{1}{m} = A(s^2 + 2\zeta\omega_n s + \omega_n^2) + (Bs+C)s$$

$$\frac{1}{m} = As^2 + A2\zeta\omega_n s + A\omega_n^2 + Bs^2 + Cs$$

$$\frac{1}{m} = (A+B)s^2 + (A2\zeta\omega_n + C)s + A\omega_n^2$$

Si comparamos coeficientes de ambos lados:

$$A = \frac{1}{m\omega_n^2}, \text{ De } \omega_n^2 = \frac{K}{m} \rightarrow \frac{1}{K} = \frac{1}{m\omega_n^2}$$

$$A+B=0, A2\zeta\omega_n + C=0, A\omega_n^2 = \frac{1}{m}$$

$$A = \frac{1}{K}, B = -A = -\frac{1}{m\omega_n^2}, C = -A2\zeta\omega_n = -2\zeta\omega_n \frac{1}{m\omega_n^2} = -\frac{2\zeta}{m\omega_n}$$

$$\frac{Bs+C}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{Bs+C}{(s+\zeta\omega_n)^2 + \omega_d^2} = B \frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2} + \frac{(C-B\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2}$$

$$Y(s) = \frac{1}{m} \cdot \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{1}{s} = A \frac{1}{s} + B \frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2} + \frac{(C-B\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2}$$

$$Y(s) = A \frac{1}{s} + B \frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2} + \frac{(C-B\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2}$$

Si usamos: $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$

$$\mathcal{L}^{-1}\left\{\frac{s+a}{(s+a)^2 + b^2}\right\} = e^{-at} \cos(bt)$$

$$\mathcal{L}^{-1}\left\{\frac{b}{(s+a)^2 + b^2}\right\} = e^{-at} \sin(bt)$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+a)^2 + b^2}\right\} = \frac{1}{b} e^{-at} \sin(bt)$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{A \frac{1}{s}\right\} + \mathcal{L}^{-1}\left\{B \frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2}\right\} + \mathcal{L}^{-1}\left\{\frac{(C-B\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2}\right\}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = A \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} + B \mathcal{L}^{-1}\left\{\frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2}\right\} + (C-B\zeta\omega_n) \mathcal{L}^{-1}\left\{\frac{1}{(s+\zeta\omega_n)^2 + \omega_d^2}\right\}$$

$$y(t) = A + B e^{-\zeta\omega_n t} \cos(\omega_d t) + \frac{(C-B\zeta\omega_n)}{\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t)$$

$$y(t) = A + e^{-\zeta\omega_n t} (B \cos(\omega_d t) + \frac{C-B\zeta\omega_n}{\omega_d} \sin(\omega_d t))$$

$$y(t) = A + e^{-\zeta\omega_n t} (B \cos(\omega_d t) + \frac{C-B\zeta\omega_n}{\omega_d} \sin(\omega_d t))$$

$$y(t) = \frac{1}{m\omega_n^2} + e^{-\zeta\omega_n t} \left(-\frac{1}{m\omega_n^2} \cos(\omega_d t) + \left(-\frac{2\zeta}{m\omega_n} - \left(-\frac{1}{m\omega_n^2} \zeta\omega_n \right) \frac{1}{\omega_d} \sin(\omega_d t) \right) \right)$$

$$y(t) = \frac{1}{m\omega_n^2} + e^{-\zeta\omega_n t} \left(-\frac{1}{m\omega_n^2} \cos(\omega_d t) + \left(-\frac{2\zeta}{m\omega_n} - \left(-\frac{1}{m\omega_n^2} \zeta\omega_n \right) \frac{1}{\omega_d} \sin(\omega_d t) \right) \right)$$

$$y(t) = \frac{1}{m\omega_n^2} + e^{-\zeta\omega_n t} \left(-\frac{1}{m\omega_n^2} \cos(\omega_d t) + \left(-\frac{2\zeta}{m\omega_n} + \frac{\zeta}{m\omega_n} \right) \frac{1}{\omega_d} \sin(\omega_d t) \right)$$

$$y(t) = \frac{1}{m\omega_n^2} + e^{-\zeta\omega_n t} \left(-\frac{1}{m\omega_n^2} \cos(\omega_d t) - \frac{\zeta}{m\omega_n \omega_d} \sin(\omega_d t) \right)$$

$$y(t) = \frac{1}{m\omega_n^2} \left(1 - e^{-\zeta\omega_n t} (\cos(\omega_d t) + \frac{\zeta\omega_n}{\omega_d} \sin(\omega_d t)) \right)$$

$$\text{Si } K = m\omega_n^2$$

$$y(t) = \frac{1}{K} \left(1 - e^{-\zeta\omega_n t} (\cos(\omega_d t) + \frac{\zeta\omega_n}{\omega_d} \sin(\omega_d t)) \right)$$

Respuesta al escalón (amortiguamiento critico): $\zeta=1 \rightarrow \omega_d=0$

$$y(s) = \frac{1}{m} \cdot \frac{1}{s(s+\omega_n)^2} = \frac{A}{s} + \frac{B}{s+\omega_n} + \frac{C}{(s+\omega_n)^2}$$

$$\frac{1}{m} = A(s+\omega_n)^2 + B s(s+\omega_n) + C s$$

$$A = \frac{1}{m\omega_n^2} \cdot B = -\frac{1}{m\omega_n^2} \cdot C = -\frac{1}{m\omega_n}$$

$$y(t) = \mathcal{L}^{-1}\{y(s)\} = \mathcal{L}^{-1}\left\{\frac{A}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{B}{s+\omega_n}\right\} + \mathcal{L}^{-1}\left\{\frac{C}{(s+\omega_n)^2}\right\}$$

$$y(t) = A + B e^{-\omega_n t} + C t e^{-\omega_n t}$$

$$y(t) = \frac{1}{m\omega_n^2} - \frac{1}{m\omega_n^2} e^{-\omega_n t} - \frac{1}{m\omega_n} t e^{-\omega_n t}$$

$$y(t) = \frac{1}{m\omega_n^2} (1 - e^{-\omega_n t} - \omega_n t e^{-\omega_n t})$$

$$\text{Si } K = m\omega_n^2$$

$$y(t) = \frac{1}{K} (1 - e^{-\omega_n t} (1 + \omega_n t))$$

Respuesta al escalón (Sobreamortiguado): $p_{1,2} = -\zeta\omega_n \pm \omega_n \sqrt{1-\zeta^2}$

$$y(s) = \frac{1}{m} \cdot \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{1}{s}$$

$$\frac{1}{m} \cdot \frac{1}{s(s-s_1)(s-s_2)} = \frac{A}{s} + \frac{B}{s-s_1} + \frac{C}{s-s_2}$$

$$\frac{1}{m} = A(s-s_1)(s-s_2) + B s(s-s_2) + C s(s-s_1)$$

$$\text{Para } s=0 \rightarrow A = \frac{1}{m s_1 s_2} = \frac{1}{m\omega_n^2}$$

$$\text{Para } s=s_1 \rightarrow B = \frac{1}{m s_1 (s_1 - s_2)}$$

$$\text{Para } s=s_1 \rightarrow C = \frac{1}{m s_2 (s_2 - s_1)}$$

$$y(s) = \frac{A}{s} + \frac{B}{s-s_1} + \frac{C}{s-s_2}$$

$$y(t) = \mathcal{L}^{-1}\{y(s)\} = \mathcal{L}^{-1}\left\{\frac{A}{s}\right\} + \mathcal{L}^{-1}\left\{\frac{B}{s-s_1}\right\} + \mathcal{L}^{-1}\left\{\frac{C}{s-s_2}\right\}$$

$$y(t) = A + B e^{s_1 t} + C e^{s_2 t}$$

$$y(t) = \frac{1}{m\omega_n^2} + \frac{1}{m s_1 (s_1 - s_2)} e^{(s_1 \omega_n + \omega_n (1-\zeta^2))t} + \frac{1}{m s_2 (s_2 - s_1)} e^{(s_2 \omega_n - \omega_n (1-\zeta^2))t}$$

Respuesta a la rampa:

$$y(s) = \frac{1}{m} \cdot \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad \text{Si } F_E(t) = r(t) = t u(t) \rightarrow F_E(s) = \rho(s) = \frac{1}{s^2}$$

$$H(s) = \frac{y(s)}{F_E(s)} \rightarrow y(s) = F_E(s) H(s) = \frac{1}{m} \cdot \frac{1}{s^2 + 2\zeta\omega_n s + \omega_n^2} \cdot \frac{1}{s^2}$$

$$y(s) = \frac{1}{m} \cdot \frac{1}{s^2 (s^2 + 2\zeta\omega_n s + \omega_n^2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\frac{1}{m} = A s (s^2 + 2\zeta\omega_n s + \omega_n^2) + B (s^2 + 2\zeta\omega_n s + \omega_n^2) + (Cs+D) s^2$$

$$A s (s^2 + 2\zeta\omega_n s + \omega_n^2) = A s^3 + 2A\zeta\omega_n s^2 + A\omega_n^2 s$$

$$B (s^2 + 2\zeta\omega_n s + \omega_n^2) = B s^2 + 2B\zeta\omega_n s + B\omega_n^2$$

$$(Cs+D) s^2 = C s^3 + D s^2$$

$$\frac{1}{m} = (A+C) s^3 + (2A\zeta\omega_n + B+D) s^2 + (A\omega_n^2 + 2B\zeta\omega_n) s + B\omega_n^2$$

Si comparamos terminos de cada lado

$$\begin{aligned} A+C &= 0, & 2A\zeta\omega_n + B+D &= 0, & A\omega_n^2 + 2B\zeta\omega_n &= 0, & B\omega_n^2 &= \frac{1}{m} \\ C &= -A, & D &= -A2\zeta\omega_n - B, & A\omega_n^2 &= -2B\zeta\omega_n, & B &= \frac{1}{m\omega_n^2} \\ C &= \frac{2\zeta}{m\omega_n^3}, & D &= \frac{2\zeta}{m\omega_n^3} 2\zeta\omega_n - \frac{1}{m\omega_n^2}, & A\omega_n^2 &= -\frac{1}{m\omega_n^2} 2\zeta\omega_n, & & \\ D &= \frac{4\zeta^2}{m\omega_n^3} - \frac{1}{m\omega_n^2}, & A &= -\frac{2\zeta}{m\omega_n^3}, & & & \\ D &= \frac{4\zeta^2 - 1}{m\omega_n^3} \end{aligned}$$

Para (Subamortiguado):

$$y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{s^2 + 2\zeta\omega_n s + \omega_n^2} \rightarrow Cs+D = C((s+\zeta\omega_n) - \zeta\omega_n) + D = C(s+\zeta\omega_n) + (D-C\zeta\omega_n)$$

$$\rightarrow s^2 + 2\zeta\omega_n s + \omega_n^2 = (s+\zeta\omega_n)^2 + \omega_d^2$$

$$y(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{C(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2} + \frac{(D-C\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2}$$

$$y(t) = A \mathcal{L}^{-1} \left\{ \frac{1}{s} \right\} + B \mathcal{L}^{-1} \left\{ \frac{1}{s^2} \right\} + C \mathcal{L}^{-1} \left\{ \frac{(s+\zeta\omega_n)}{(s+\zeta\omega_n)^2 + \omega_d^2} \right\} + (D-C\zeta\omega_n) \mathcal{L}^{-1} \left\{ \frac{1}{(s+\zeta\omega_n)^2 + \omega_d^2} \right\}$$

$$y(t) = A + Bt + C e^{-\zeta\omega_n t} \cos(\omega_d t) + (D-C\zeta\omega_n) \frac{1}{\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t)$$

$$y(t) = -\frac{2\zeta}{m\omega_n^3} + \frac{t}{m\omega_n^3} + \frac{2\zeta}{m\omega_n^3} e^{-\zeta\omega_n t} \cos(\omega_d t) + \left(\frac{4\zeta^2 - 1}{m\omega_n^3} - \frac{2\zeta\omega_n}{m\omega_n^3} \right) \frac{1}{\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t)$$

$$y(t) = -\frac{2\zeta}{m\omega_n^3} + \frac{t}{m\omega_n^3} + \frac{2\zeta}{m\omega_n^3} e^{-\zeta\omega_n t} \cos(\omega_d t) + \left(\frac{4\zeta^2 - 1}{m\omega_n^3} - \frac{2\zeta^2}{m\omega_n^3} \right) \frac{1}{\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t)$$

$$y(t) = -\frac{2\zeta}{m\omega_n^3} + \frac{t}{m\omega_n^3} + \frac{2\zeta}{m\omega_n^3} e^{-\zeta\omega_n t} \cos(\omega_d t) + \frac{2\zeta^2 - 1}{m\omega_n^3} \frac{1}{\omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t)$$

$$y(t) = -\frac{2\zeta}{m\omega_n^3} + \frac{t}{m\omega_n^3} + \frac{2\zeta}{m\omega_n^3} e^{-\zeta\omega_n t} \cos(\omega_d t) + \frac{2\zeta^2 - 1}{m\omega_n^3 \omega_d} e^{-\zeta\omega_n t} \sin(\omega_d t)$$

Nota: En los polos del sistema la parte real $\rightarrow -\zeta\omega_n$ determina la tasa de decaimiento exponencial
la parte imaginaria determina la oscilación angular ω_d

\rightarrow natural
frecuencia amortiguada:
 $\omega_d = \omega_n \sqrt{1 - \zeta^2}$