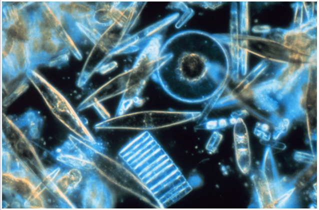
Extinction, Fixation, and Invasion in an Ecological Niche

MattheW Badali

Internal defense
performed as a requirement for
the degree of Doctor of Philosophy
4 July 2019

Motivation: Biodiversity

Paradox of the Plankton
a problem of biodiversity



corp2365, NOAA Corps Collection

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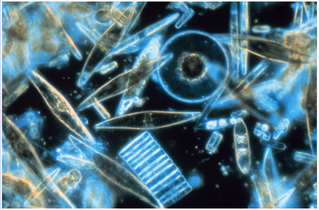

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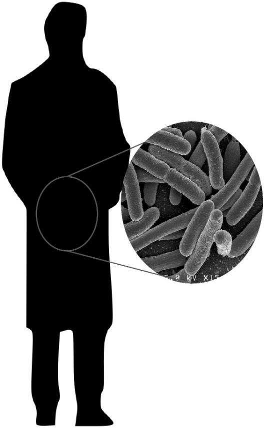
Gut Microbiome
important to health



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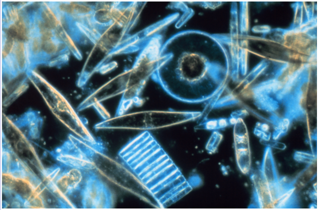

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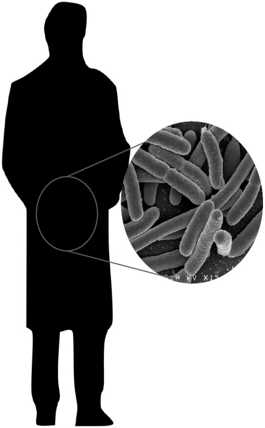

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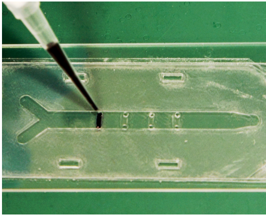
Small Populations
microfluidics



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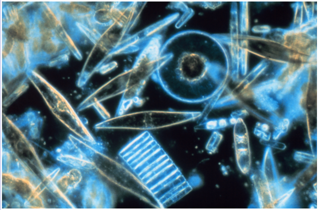

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Conservation Efforts

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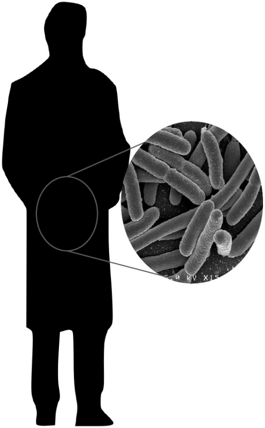

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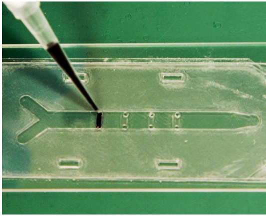

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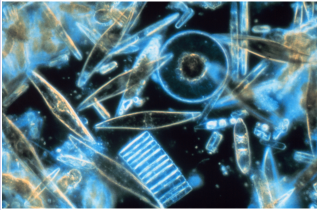

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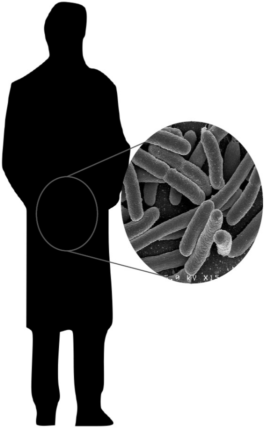

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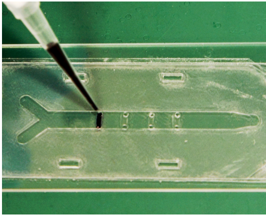

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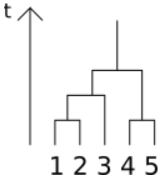
Coalescent Trees



Niche Apportionment Explains Abundance Curves

Competitive Exclusion Principle

- “two species cannot coexist if they share a single [ecological] niche”
- niche/resource apportionment explains abundance distributions (biodiversity)
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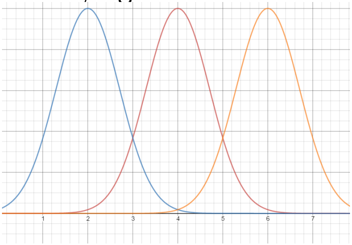
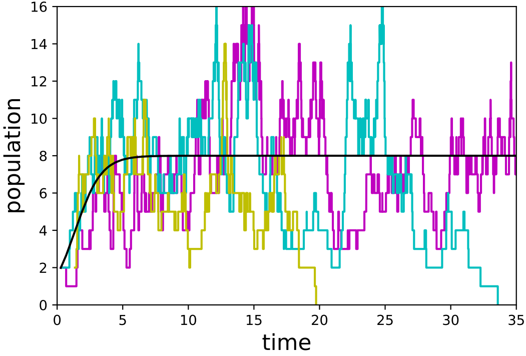
- species: group with the same birth and death rates
- niche: survivable values of those factors which affect the birth and death rates



Stochasticity Leads To Extinction

logistic equation $\dot{x} = r\,x\,\left(1 - \frac{x}{K}\right)$



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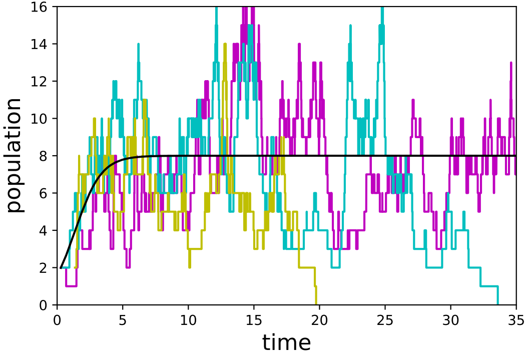

- demographic stochasticity = fluctuations, noise
- probability of population n: P_n
- mean time to extinction: $\tau \sim e^K$

Neutral Theory Explains Abundance Curves Better

- better prediction of abundance curves (Hubbell), also allele frequencies (Kimura), fixation (Moran)
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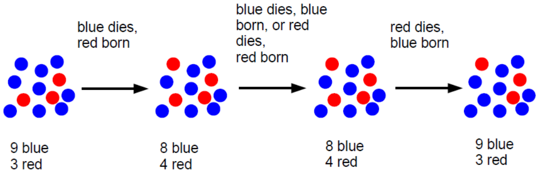

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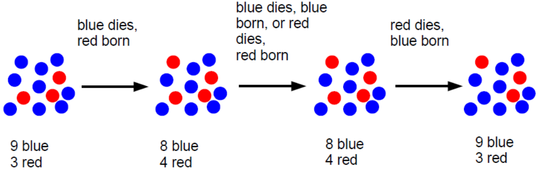

- $\tau \sim K$
- biodiversity comes from a balance of species exiting (extinction, fixation) and species entering (invasion, immigration) the system

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Coexistence and Extinction of Competing Species

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Background

Fixation

Invasion

Discussion

Extra Slides

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- Invasion is fast unless two species occupy the same niche

Fixation versus Coexistence of Two Species

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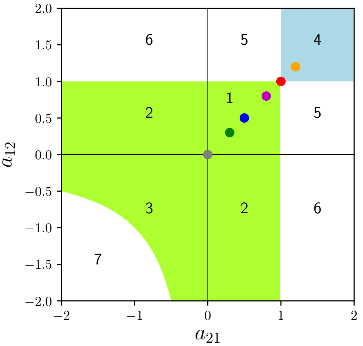
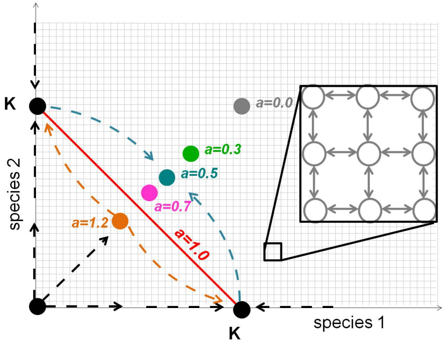
- WKB ansatz: $P_n \propto \exp\left\{K\sum_i \frac{1}{K^i}S_i(n)\right\}$ with
$S_0(n) = \int_0^n dx \ln\left(b_x/d_x\right)$ along extinction trajectory

Coupled Logistic Equations Include Competitive Exclusion

$$\dot{x}_1 = r_1 x_1 \left(1 - \frac{x_1 + a_{12} x_2}{K_1}\right) \text{ and } \dot{x}_2 = r_2 x_2 \left(1 - \frac{a_{21} x_1 + x_2}{K_2}\right)$$





$O = (0,0),\ A = (0, K_2),\ B = (K_1, 0),\ C = \left(\frac{K_1 - a_{12} K_2}{1 - a_{12} a_{21}}, \frac{K_2 - a_{21} K_1}{1 - a_{12} a_{21}}\right)$

2,6 = parasitism/predation/antagonism, 3,7 = mutualism,

4,5 = competitive exclusion, 1 = (weak) competition

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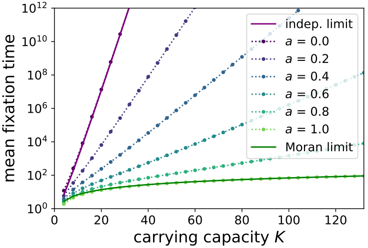

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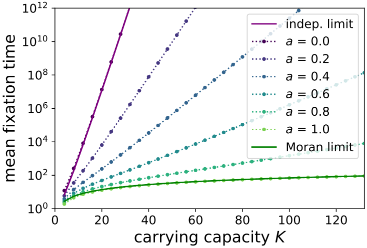

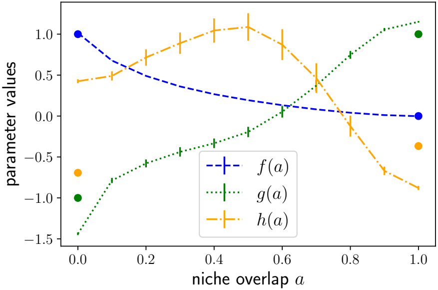

Effective coexistence except with complete niche overlap!

The Speed of Invasion of a Second Species

Invasion - Definition And Expectations

Invasion is going from one organism to half the population.

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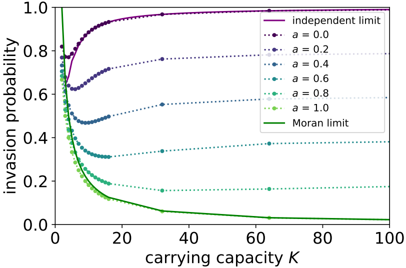
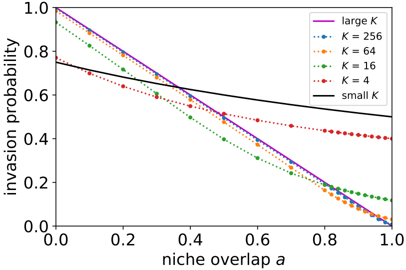
- invasion with a fixed point should be fast (logarithmic)
- invasion on the line should be slower (linear)

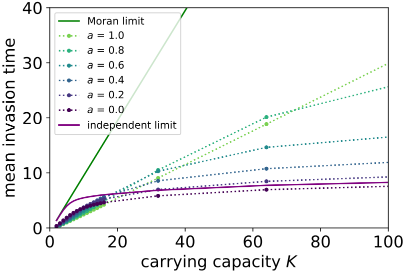
Invasion Is Less Probable As Niche Overlap Increases

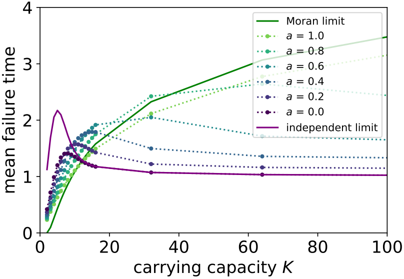




Probability of a successful invasion. Invasion probability lessens as niche overlap increases. The trend is more stark for large K.

Invasion Times Are Fast





Mean time of a successful or failed invasion attempt.

Left: Mean time conditioned on eventual invasion success.

Right: Mean time conditioned on failed attempt.

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- similarly, greater niche overlap leads to longer invasion times, and less likelihood of success of an attempt;
- incomplete niche overlap is a niche theory with carrying capacities modified by niche overlaps;

Utility Of My Results

- human health (gut microbiome)
- planet health (conservation)
- minimal working models
- coalescent theory
- plasmids
- mitochondria

Potential Future Research

- predator-prey model (centre fixed point)
- rock-paper-scissors model (limit cycle)
- other 3D models (chaos)
- SIR model (epidemics)
- evolving parameters (ecology and evolutionary biology)

Thank You