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$$\begin{aligned}
\sigma_{X|Z}^2 &= \int_{\mathcal{X}} (x - \mu_{X|Z})^2 f_{X|Z}(x|z) dx \\
&= \int_{\mathcal{X}} x^2 f_{X|Z}(x|z) dx - 2 \int_{\mathcal{Y}} \int_{\mathcal{X}} \mu_{X|Z} x f_{X|Y}(x|y) f_{Y|Z}(y|z) dx dy \\
&\quad + \mu_{X|Z}^2 \int_{\mathcal{X}} f_{X|Z}(x|z) dx \\
&= \int_{\mathcal{Y}} \int_{\mathcal{X}} x^2 f_{X|Y}(x|y) f_{Y|Z}(y|z) dx dy \\
&\quad - 2 \mu_{X|Y} \int_{\mathcal{Y}} \int_{\mathcal{X}} x f_{X|Y}(x|y) f_{Y|Z}(y|z) dx dy \\
&\quad + 2 \mu_{X|Y} \int_{\mathcal{Y}} \int_{\mathcal{X}} x f_{X|Y}(x|y) f_{Y|Z}(y|z) dx dy \\
&\quad + \mu_{X|Y}^2 \int_{\mathcal{Y}} \int_{\mathcal{X}} f_{X|Y}(x|y) f_{Y|Z}(y|z) dx dy \\
&\quad - \mu_{X|Y}^2 \int_{\mathcal{Y}} \int_{\mathcal{X}} f_{X|Y}(x|y) f_{Y|Z}(y|z) dx dy \\
&\quad + \int_{\mathcal{Y}} \mu_{X|Z}^2 f_{Y|Z}(y|z) dy - 2 \int_{\mathcal{Y}} \mu_{X|Z} \mu_{X|Y} f_{Y|Z}(y|z) dx dy \\
&= \int_{\mathcal{Y}} \sigma_{X|Y}^2 f_{Y|Z}(y|z) dy + \int_{\mathcal{Y}} (\mu_{X|Y} - \mu_{X|Z})^2 f_{Y|Z}(y|z) dy
\end{aligned} \tag{0.1}$$