

Assignment-01

Q1 The asymptotic notation are used to tell the complexity of an algo when the input is very large.

Q2 Types of notation -

- 1) Big-oh (O)
- 2) Big-Omega (Ω)
- 3) Theta (Θ)
- 4) Small-oh (o)
- 5) Small-Omega (ω)

Q2 for ($i=1; i \leq n; i=i*2$)
- $O(\log n)$

Q3 $T(n) = 3T(n-1) \quad n > 0$

$$T(1) = 1$$

$$T(2) = 3T(1) = 3$$

$$T(3) = 3(T(2)) = 3^2 T(n-2)$$

$$T(4) = 3(T(3)) = 3^3 T(n-3)$$

$$3^0 + 3^1 + 3^2 + 3^3 \dots 3^n T(n-n)$$

$$= O(3^n)$$

Q4

$$T(n) = 2T(n-1) - 1$$

$$T(1) = 1$$

$$\rightarrow T(2) = 2T(1) - 1$$

$$= 2^2 T(n-2) - 2^1 - 2^0$$

$$= 2^3 T(n-3) - 2^2 - 2^1 - 2^0$$

$$= 2^n T(n-n) - 2^{n-1} - 2^{n-2} - \dots - 2^2 - 2^1 - 2^0$$

$$= 2^n \left(-\frac{1}{2}n + 1 \right)$$

$$\Rightarrow 2^n \left(\frac{1+2^{n+1}}{2^{n+1}} \times 2 \right)$$

$$\Rightarrow \frac{2^{n+1}}{3} + \frac{1}{3}$$

$$\frac{1}{3} (2^n \cdot 2 + 2^n)$$

$$2^n = k$$

$$n = \log_2 k$$

$$= 2^n - (2^n - 1)$$

$$= T(1)$$

Q5

int i = 1, s = 1;

while (s <= n)

i++,

s += i,

Print i

}

$$\Rightarrow T(n) = 1 + 2 + 3 + \dots + n$$

$$= \frac{n(n+1)}{2} \rightarrow k$$

$$S_i = S_{i-1} + i$$

value of i increase by one on each iteration.

The value of S at the i^{th} iteration is sum of first i^{th} terms.

If k is total no. of iterations.

So,

$$1+2+3+4+\dots+k = \frac{k(k+1)}{2} > n$$

$$k = \sqrt{n}$$

$$\boxed{T(\sqrt{n})}$$

Q6

```
void fun(int n)
{
    int i; count = 0;
    for (i = 1; i * i <= n; i++) // Executes (n^2) times
        count++;
}
```

$$\boxed{O(n^2)}$$

Q7

```
void fun(int n) {
    int i, j, k, count = 0;
    for (i = n/2; i <= n; i++) // (n/2)
        for (j = 1; j <= n; j *= 2) // (log n)
            for (k = 1; k <= 2; k = k * 2) // (log n)
                count++;
}
```

$$O\left(\frac{n}{2} \times \log n \times \log n\right)$$

$$O(n \log^2 n)$$

Q8

```

fun (int n) {
    if (n == 1) return;
    for (i = 1 to n)
        for (j = 1 to n)
            print (*);
    fun (n-3);
}
    
```

$$T(n) = n^2 + T(n-3)$$

Q9

```

void fun (int n) {
    for (i = 1 to n)
        for (j = i to n; j++)
            print ("*");
}
    
```

$$T(n) = n * (n + (n-1) + (n-2) + \dots + 1) = \frac{n(n+1)}{2}$$

$$\frac{n(n+1)}{2} \quad \cancel{O(n^2)} \quad O(n^2)$$