CS 420\520 Winter 2019
HW 1

Written solutions must be typed using a word processor or text editor and submitted to Canvas. Submit a copy of all your code files and a README file that explains how to compile and run your code in a ZIP file to TEACH.

1. The ***square*** of a directed graph G = (V, E) is the graph $G^2 = (V, E^2)$, such that $(u, v) \in E^2$ if and only if G contains a path with two edges between u and v. Describe efficient algorithms for computing

(a) the adjacency-list for G^2 from the adjacency-list of *G*,
(b) the adjacency-matrix of G^2 from the adjacency matrix of G.

Analyze the running times of your algorithms.

2. A graph (V, E) is **bipartite** if the vertices V can be partitioned into two subsets L and R, such that every edge has one vertex in L and the other in R.

(a) Prove that every tree is a bipartite graph.
(b) Describe an efficient algorithm that determines whether a given undirected graph is bipartite.
(c) Analyze the running time of your algorithm in part (b)

3. In the **bottleneck-path problem**, you are given a graph G with edge weights, two vertices s and t and a particular weight W; your goal is to find a path from s to t in which every edge has at least weight W.

(a) Describe an efficient algorithm to solve this problem.
(b) What is the running time of your algorithm.

4. **Longest Path in a DAG (LP-DAG)**

(a) Describe an algorithm to find the longest path (measured in number of edges) in an unweighted DAG.
(b) What is the running time of the algorithm?

5. Implement the algorithm you described in (4) in C, C++ or Python, name the programs "lp-dag". Your program should read input from a file called "graph.txt" where the first line in the file is the number of vertices n (with n <= 100) in the graph and the second line is the number of edges. Assume the vertices are numbered 1,2, .., n. The following lines each contains an edge. The program should output to the terminal the length of the longest path and the path itself

Input: graph.txt:

```
5
6
1 4
1 3
2 3
2 4
3 4
5 2
```

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Output:

Length of longest path : 3
Path: 5 - 2- 3 - 4

Note: If there are more than one path of the longest length only one needs to be outputted.