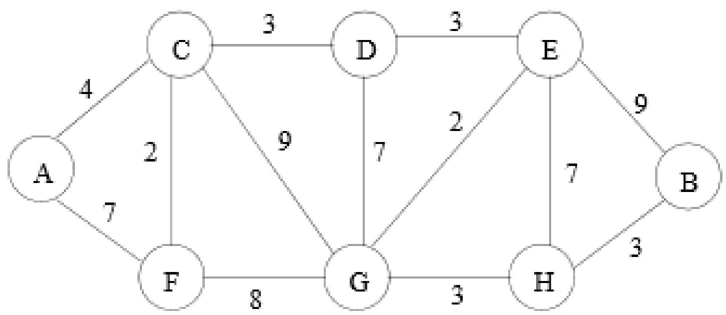
CS420/520 HW 3

1. A region contains a number of towns connected by roads. Each road is labeled by the average number of minutes required for a fire engine to travel to it. Each intersection is labeled with a circle. Suppose that you work for a city that has decided to place a fire station at location G. (While this problem is small, you want to devise a method to solve much larger problems).



(a) What algorithm would you recommend be used to find the fastest route from the fire station to each of the intersections? Demonstrate how it would work on the example above if the fire station is placed at G. Show the resulting routes.

(b) Suppose one "optimal" location (maybe instead of G) must be selected for the fire station such that it minimizes the distance to the farthest intersection. Devise an algorithm to solve this problem given an arbitrary road map. Analyze the time complexity of your algorithm when there are f possible locations for the fire station (which must be at one of the intersections) and r possible roads. In the above graph what is the "optimal" location to place one fire station?

(c) Now suppose you can build two fire stations. Where would you place them to minimize the farthest distance from an intersection to one of the fire stations? Devise an algorithm to solve this problem given an arbitrary road map. Analyze the time complexity of your algorithm when there are f possible locations for the fire station (which must be at one of the intersections) and r possible roads. In the above graph what is the "optimal" location to place two fire stations?

2. Consider an undirected graph $G=(V,E)$ with nonnegative edge weights $w(u,v)\geq 0$. Suppose that you have computed shortest paths to all vertices from vertex $s \in V$. Now suppose each edge weight is increased by 1: the new weights $w'(u,v) = w(u,v) + 1$. Does the shortest paths change? Give an example where they change or prove they cannot change.

3. Does the shortest path between two vertices on a graph always lie on a minimum spanning tree for the graph? Prove or give a counter example.

CS 520: CLRS Textbook Problem 25.1 (p 705) Transitive closure of a dynamic graph.