CS 420
Section 001
Winter 2019
Mazen Alotaibi *(alotaima)* HW 4

Problem 1: Ford-Fulkerson.. In this problem, we will...

(a) Consider the network G shown in Fig. 1 a)...

(b) Find any augmenting path in G_f...

(c) Show the residual network...

(d) Is this the final maximum flow in this...

(e) Show the residual network for your...

(f) Show the cut that results by partitioning...

Solution:

(a) Please look at Graph 1 for the residual network G_f.

(b) Please look at Graph 1. The path $s- > d- > b- > a- > t$ has the maximum flow to push is 2.

(c) Please look at Graph 2 for the updated flow. Please look at Graph 4 for the updated residual network $G_{f'}$.

(d) There isn't any path from s to t. The maximum flow is 13.

(e) The residual graph is the same as (c) because the flow from (c) was already the maximum.

(f) Please look at Graph 5 for reference to my answer. The minimum cuts are $(\{s, b, c, d\}, \{a, t\})$ and $(\{s, a, b, c, d\}, \{t\})$, and the capacity of both of the cuts is equal to the sum of capacities from X region to Y region in both of the suggested minimum cuts, which they have a capacity of 13.

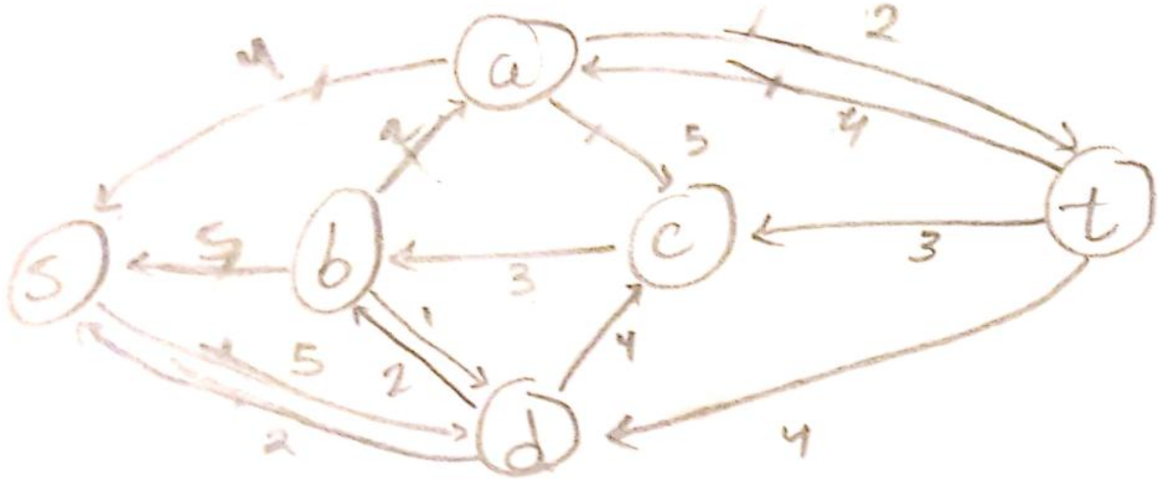
□

Problem 2: Disjoint Roads.. A number k of trucking companies, $c_1, \dots, c_k$...

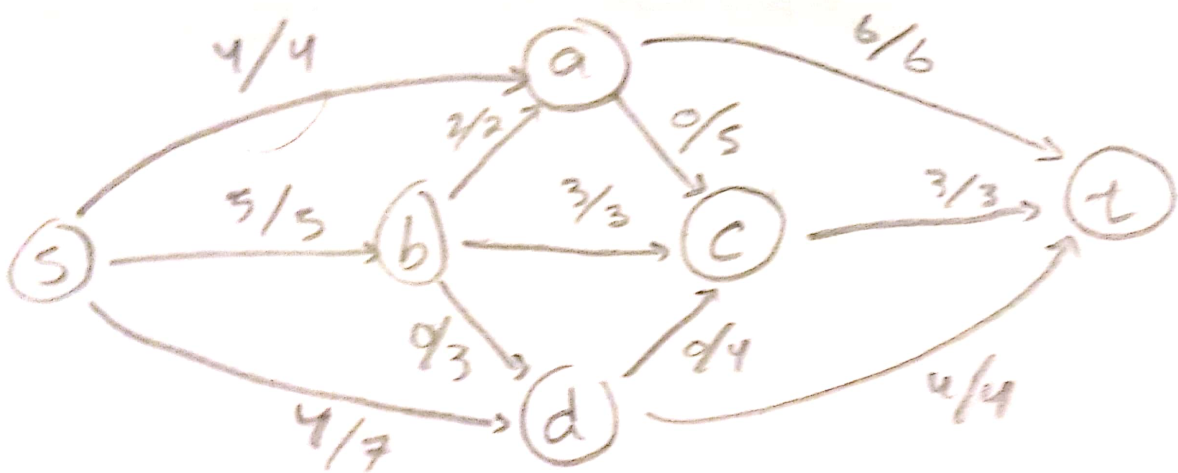
Solution: This problem is a max flow problem. By using Ford-Fulkerson algorithm, the result of the algorithm is a set of paths. If the value of the flow is less than k, the algorithm returns false, impossible. The time complexity is $O(Ek)$ as the max flow $|f| < k$. □

Problem 3: Food Truck Orders.. The Acme food truck produces a large variety of different...

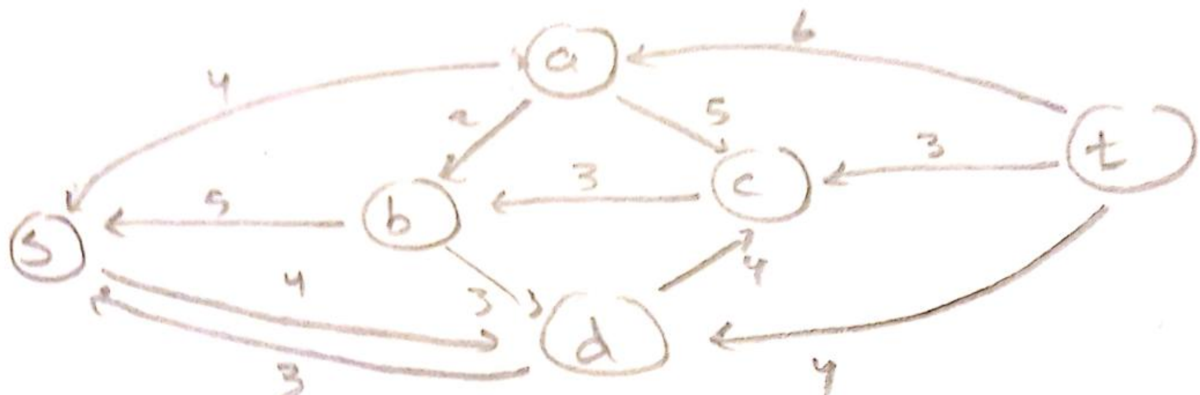
Solution: This problem is a max flow problem. By using Ford-Fulkerson algorithm, the result of the algorithm is a set of food assignments. The maximum number of happy customers means the minimum number of vouchers. The time complexity is $O(mn^2)$ as the number of edges is $E = mn$ and the max flow $|f| \leq n$. □



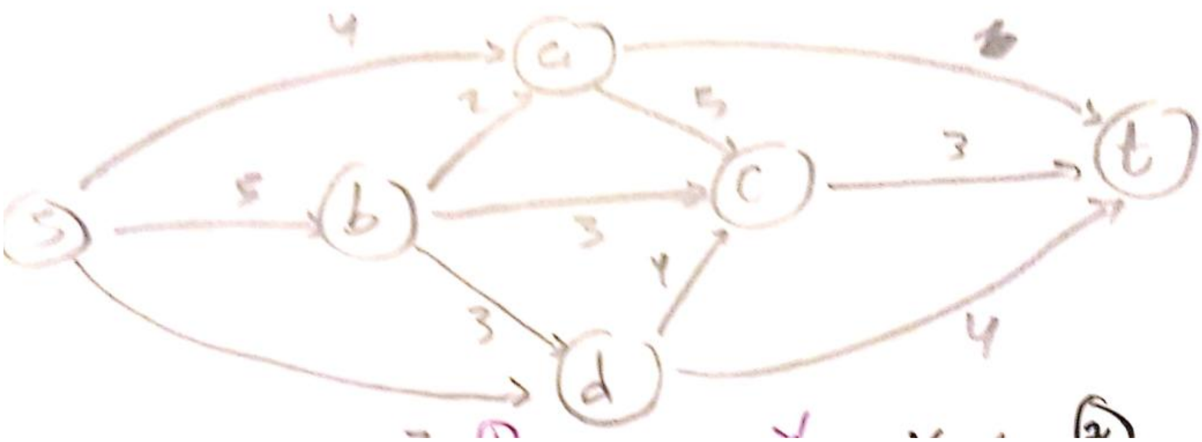
Graph 1: the residual network G_f



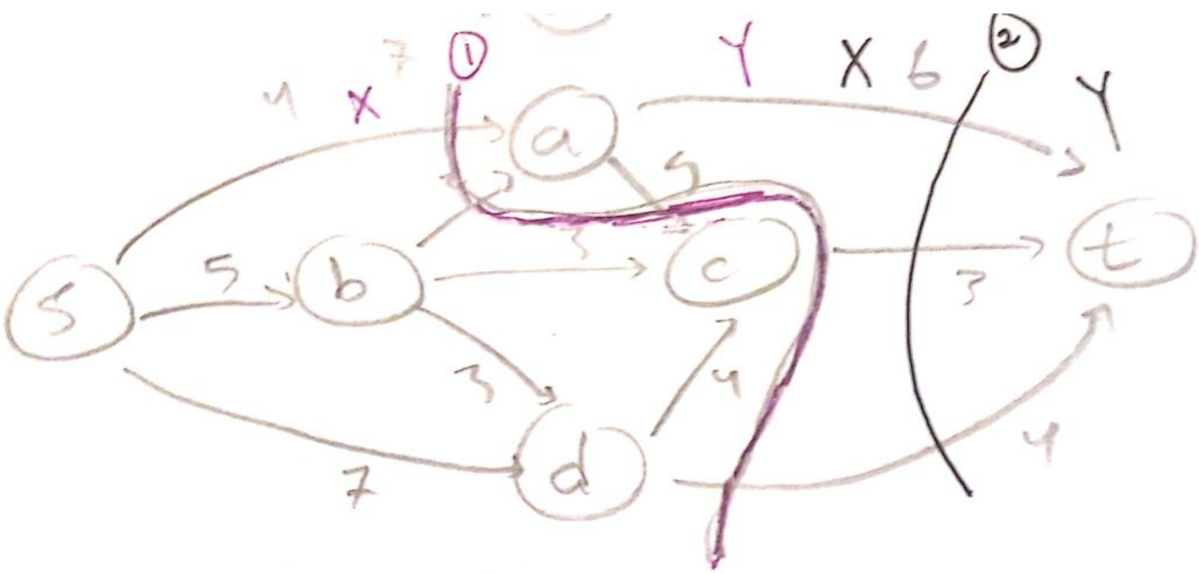
Graph 2: the updated flow



Graph 3: the updated residual network



Graph 4: the graph before applying the minimum cut. *Note: $s->d$ is 7 and $a->t$ is 6*



Graph 5: the minimum cut for two cuts. *Note: $b->a$ is 2 and $b->c$ is 3*
The purple line is for the $(\{s, b, c, d\}, \{a, t\})$ maximum cut.
The black line for the $(\{s, a, b, c, d\}, \{t\})$ maximum cut.