Time-Constrained Bipartite Vehicle Routing Problem (TCBVRP)

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Problem description

Our task in the TCBVRP[1] is to supply a given number of demand nodes by minimizing the unit of time by the number of vehicles used. A vehicle tour starts at the depot node and after an alternating ordering of getting first to a supply and then to a demand node the tour finishes at the depot again. The time capacity of every crossed edge is added up and has a given constraint of time limit per vehicle that must not be exceeded. Aim is to minimize the sum of time periods of all used vehicles in the graph.
For this problem we developed the following integer programming formulations, which we solved with CPLEX.

Single Commodity Flow (SCF)

Variables

$$\begin{aligned} n &: \text{Total number of Nodes} \\ m &: \text{Total number of Nodes} \\ T &: \text{Timelimit for one vehicle} \\ N &: \text{Set of all nodes} \\ S &: \text{Set of supply nodes} \\ D &: \text{Set of demand nodes} \\ V &: \text{Set of vehicles} \end{aligned}$$

Declaration of the decision variables

$$x_{ij}^k : \begin{cases} 1 & \text{if edge from i to j is crossed by vehicle k} \\ 0 & \text{otherwise} \end{cases}$$

Objective Function

Minimize the total time of all used vehicles

$$min. \quad \sum_{k=1}^{m} \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij}^k * d_{ij} \tag{1}$$

[1]Programming Exercise / ILOG CPLEX Tutorial, Benjamin Biesinger, Bin Hu, Gunther R. Raidl, Algorithms and Data Structures Group Institute of Computer Graphics and Algorithms, Vienna University of Technology, VU Algorithmics, WS 2014/15

Constraints

Für jedes Fahrzeug muss das timelimit T eingehalten werden:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}^{k} * d_{ij} <= T \qquad \forall k \in V \tag{2}$$

Jeder Demand muss von einem Supply aus angefahren werden

$$\sum_{i \in S}\sum_{k \in V} x_{ij}^{k} = 1 \qquad \forall j \in D \tag{3}$$

Supplies dürfen maximal einmal angefahren werden - von Depot aus oder Demand

$$\sum_{i \in D}\sum_{k \in V} x_{ij}^{k} <= 1 \qquad \forall j \in S \tag{4}$$

Zumindest ein Fahrzeug muss das Depot verlassen

$$\sum_{k \in V}\sum_{j \in S} x_{0j}^{k} => 0 \tag{5}$$

Die Anzahl der Fahrzeuge die in einen Node reinfahren muss gleich sein der Summe die rausfahren

$$\sum_{j=1}^{n} x_{ij}^{k} = \sum_{j=1}^{n} x_{ji}^{k} \qquad \forall i \in N \quad \forall k \in V \tag{6}$$

Sub-tour Elimination Constraint

$$\sum_{j, j \neq 1} f_{1j} = n - 1 \tag{7}$$

$$\sum_{i, i \neq j} f_{ij} - \sum_{l, l \neq j} f_{lj} <= 1 \qquad \forall j \in N \setminus 1 \tag{8}$$

$$0 \leq f_{ij} \leq (n-1) * x_{ij} \qquad \forall i, j \in N, i \neq j \tag{9}$$

Multi Commodity Flow (MCF)

Miller-Tucker-Zemlin subtour elimination constraints (MTZ)

Results and Discussion

Table 1: Solutions with the Single Commodity Flow Constraints

Nodes [n]	Time [sec]	Vehicles [m]	Objective Function Values	# of Branch & Bound Nodes	Runtime [sec]
10	240	2	347		
10	480	2	308		
10	240	2	275		
10	480	2	252		
20	240	3	533		
20	480	2	517		
20	240	4	594		
20	480	2	566		
30	240	4	763		
30	480	2	717		
30	240	4	896		
30	480	2	852		
60	360	6	1699		
60	480	4	1671		
90	480	8	2031		
120	480	8	2663		
180	720	10	5782		

Table 2: Solutions with the Miller-Tucker-Zemlin subtour elimination constraints

Nodes [n]	Time [sec]	Vehicles [m]	Objective Function Values	# of Branch & Bound Nodes	Runtime [sec]
10	240	2	347		
10	480	2	308		
10	240	2	275		
10	480	2	252		
20	240	3	533		
20	480	2	517		
20	240	4	594		
20	480	2	566		
30	240	4	763		
30	480	2	717		
30	240	4	896		
30	480	2	852		
60	360	6	1699		
60	480	4	1671		
90	480	8	2031		
120	480	8	2663		
180	720	10	5782		