Supplementary

Proof for lemma 1:

Proof. **Rewritability** By construction, each rule added to $\Gamma_i^{\ddagger}$ explicitly passes the constraints defined by the FUS fragment, verified via the procedure IsFUS. Thus, rewritability trivially holds.

Recoverability Each rule $\gamma \in \Gamma_i^{\ddagger}$ is guaranteed to have confidence $f_\gamma = 1$. Hence, it does not derive any triples beyond those present in the original knowledge graph $\mathcal{G}$, thus: $\Phi_{\mathcal{G} \cup \Gamma_i^{\ddagger}} = \mathcal{G}$. Given the subset relation $\mathcal{G}_i^{\ddagger} \subseteq \mathcal{G}$, it follows that: $\Phi_{\mathcal{G}_i^{\ddagger} \cup \Gamma_i^{\ddagger}} \subseteq \Phi_{\mathcal{G} \cup \Gamma_i^{\ddagger}} = \mathcal{G}$. Conversely, by the explicit construction of the compression process, for each triple δ_j removed from $\mathcal{G}$, we verify explicitly: $(\mathcal{G}_i^{\ddagger} \cup \Gamma_i^{\ddagger}) \models \delta_j$. Let Δ_j be the set of all removed triples. Thus: $\Delta_j \subseteq \Phi_{\mathcal{G}_i^{\ddagger} \cup \Gamma_i^{\ddagger}}$. Given $\mathcal{G} = \mathcal{G}_i^{\ddagger} \cup \Delta_j$, it follows that: $\mathcal{G} \subseteq \Phi_{\mathcal{G}_i^{\ddagger} \cup \Gamma_i^{\ddagger}}$. Therefore, combining both directions, we obtain: $\Phi_{\mathcal{G}_i^{\ddagger} \cup \Gamma_i^{\ddagger}} = \mathcal{G}$.

Minimality: Let $\mathcal{G}_{i-1}^{\ddagger}$ be the remaining triple set before adding γ_i, and let $\Gamma_{i-1}^{\ddagger} = \{\gamma_1, \ldots, \gamma_{i-1}\}$ and $\Gamma_i^{\ddagger} = \Gamma_{i-1}^{\ddagger} \cup \{\gamma_i\}$. We first show that, if a triple $\delta \in \mathcal{G}_{i-1}^{\ddagger}$ is newly redundant at step i, i.e.,

$$(\mathcal{G}_{i-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_{i-1}^{\ddagger} \not\models \delta \quad \text{and} \quad (\mathcal{G}_{i-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_i^{\ddagger} \models \delta,$$

then $\mathrm{pred}(\delta) = H_{\gamma_i}$.

Let $\mathcal{G}_0 = \mathcal{G}_{i-1}^{\ddagger} \setminus \{\delta\}$ and iterate forward with $\Gamma_i^{\ddagger}$:

$$\mathcal{G}_t = \mathcal{G}_{t-1} \cup \left\{ \sigma(H_\gamma) \,\middle|\, \sigma(B_\gamma) \subseteq \mathcal{G}_{t-1}, \gamma \in \Gamma_i^{\ddagger} \right\}.$$

Since $(\mathcal{G}_{i-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_i^{\ddagger} \models \delta$, there exists a least stage $\tau \geq 1$ such that $\delta \in \mathcal{G}_\tau \setminus \mathcal{G}_{\tau-1}$. Hence there exist $\gamma \in \Gamma_i^{\ddagger}$ and a ground substitution σ with

$$\sigma(B_\gamma) \subseteq \mathcal{G}_{\tau-1} \quad \text{and} \quad \sigma(H_\gamma) = \delta.$$

If $\gamma \in \Gamma_{i-1}^{\ddagger}$, we have $\mathcal{G}_{\tau-1} \subseteq \Phi_{(\mathcal{G}_{i-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_{i-1}^{\ddagger}}$, and thus $(\mathcal{G}_{i-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_{i-1}^{\ddagger} \models \delta$, contradicting the premise that δ is not redundant before step i. Therefore $\gamma = \gamma_i$, and hence $\mathrm{pred}(\delta) = H_{\gamma_i}$.

Suppose, for contradiction to minimality, that there exists a subset $\mathcal{G}' \subsetneq \mathcal{G}_i^{\ddagger}$ such that $\Phi_{\mathcal{G}' \cup \Gamma_i^{\ddagger}} = \mathcal{G}$. Then there exists $\delta \in \mathcal{G}_i^{\ddagger} \setminus \mathcal{G}'$ with $\mathcal{G}' \cup \Gamma_i^{\ddagger} \models \delta$. Consider the forward iteration from $\mathcal{G}_0 = \mathcal{G}'$ with set of rules $\Gamma_i^{\ddagger}$ as above. Let $\tau \geq 1$ be the least stage such that $\delta \in \mathcal{G}_\tau \setminus \mathcal{G}_{\tau-1}$, and pick (γ, σ) with $\sigma(B_\gamma) \subseteq \mathcal{G}_{\tau-1}$ and $\sigma(H_\gamma) = \delta$. Since $\mathcal{G}' \subseteq \mathcal{G}_{i-1}^{\ddagger} \setminus \{\delta\} \subseteq \mathcal{G}_{i-1}^{\ddagger}$, it follows for all $t \geq 0$ that

$$\mathcal{G}_t \subseteq \Phi_{(\mathcal{G}_{i-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_i^{\ddagger}}.$$

If $\gamma \in \Gamma_{i-1}^{\ddagger}$, then as above $(\mathcal{G}_{i-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_{i-1}^{\ddagger} \models \delta$, so δ would have been removed at some step $\leq i-1$, contradicting $\delta \in \mathcal{G}_i^{\ddagger}$. Hence $\gamma = \gamma_i$, and therefore $\mathrm{pred}(\delta) = H_{\gamma_i}$. Moreover, the inclusion above implies

$$(\mathcal{G}_{i-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_i^{\ddagger} \models \delta,$$

so δ is exactly among those removed at step i by IdentifyRedundant-*Triples*$(\mathcal{G}_{i-1}^{\ddagger}, \Gamma_i^{\ddagger}, \gamma_i) := \{\delta' \in \mathcal{G}_{i-1}^{\ddagger} \mid \mathrm{pred}(\delta') = H_{\gamma_i},\ (\mathcal{G}_{i-1}^{\ddagger} \setminus \{\delta'\}) \cup \Gamma_i^{\ddagger} \models \delta'\}$. Thus $\delta \notin \mathcal{G}_i^{\ddagger}$, a contradiction. Therefore $\Phi_{\mathcal{G}' \cup \Gamma_i^{\ddagger}} \subsetneq \mathcal{G}$ for every proper $\mathcal{G}' \subsetneq \mathcal{G}_i^{\ddagger}$, i.e., minimality holds.

Non-redundancy: Let subsumption be as in the definition: γ' subsumes γ if there exists a substitution σ such that $\sigma(H_{\gamma'}) = H_\gamma$ and $\sigma(B_{\gamma'}) \subseteq B_\gamma$. Assume, for contradiction, that there exist distinct $\gamma, \gamma' \in \Gamma_i^{\ddagger}$ with γ' subsuming γ. By the algorithmic policy, any candidate that is subsumed by another candidate with the same support is eliminated before evaluation. Hence, if $\mathrm{supp}(\gamma') = \mathrm{supp}(\gamma)$, then γ is removed a priori and never enters $\Gamma_i^{\ddagger}$, which is a contradiction. If $\mathrm{supp}(\gamma') \neq \mathrm{supp}(\gamma)$, then necessarily $\mathrm{supp}(\gamma') > \mathrm{supp}(\gamma)$, so γ' is processed earlier. Consider the state $(\mathcal{G}_{j-1}^{\ddagger}, \Gamma_{j-1}^{\ddagger})$ at the point when γ is evaluated, with $\gamma' \in \Gamma_{j-1}^{\ddagger}$ if it has been accepted. For any triple δ with $\mathrm{pred}(\delta) = H_\gamma$, if

$$(\mathcal{G}_{j-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_{j-1}^{\ddagger} \cup \{\gamma\} \models \delta,$$

then there exists a ground substitution τ such that

$$\tau(B_\gamma) \subseteq \Phi_{(\mathcal{G}_{j-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_{j-1}^{\ddagger}}$$

and $\tau(H_\gamma) = \delta$. Since γ' subsumes γ, there exists a substitution σ with $\sigma(H_{\gamma'}) = H_\gamma$ and $\sigma(B_{\gamma'}) \subseteq B_\gamma$. Then $\tau(\sigma(B_{\gamma'})) = \tau(\sigma(B_{\gamma'})) \subseteq \tau(\sigma(B_\gamma)) \subseteq \Phi_{(\mathcal{G}_{j-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_{j-1}^{\ddagger}}$, $\quad \tau(\sigma(H_{\gamma'})) = \tau(\sigma(H_{\gamma'})) = \tau(H_\gamma) = \delta$, and therefore $(\mathcal{G}_{j-1}^{\ddagger} \setminus \{\delta\}) \cup \Gamma_{j-1}^{\ddagger} \models \delta$. Thus every triple that would be made redundant by adding γ is already entailed by the previously processed set of rules (in particular by γ'), so the set of newly redundant triples for γ is empty and γ is rejected. In both cases, γ cannot be in $\Gamma_i^{\ddagger}$, contradicting the assumption. Thus $\Gamma_i^{\ddagger}$ contains no distinct subsumed pair, i.e., $\forall \gamma \in \Gamma_i^{\ddagger}, \nexists \gamma' \in \Gamma_i^{\ddagger}$ with $\gamma' \neq \gamma$ and γ' subsumes γ.

□