40.015 Simulation Modeling and Analysis Nov 3, 2021

Homework Activity 4

Submission due on/before: Nov 16, 2021, 11:59pm

Instructions.

1. Please provide clearly written steps where necessary.
2. Please submit your solutions via eDimension before 11:59pm of the due date.
3. Please ensure that scanned copies (if any) are clearly readable.
4. Please submit the source code for questions where you have employed simulation.
5. Tasks 1 and 3 can be accomplished with content taught in Week 8 lectures
6. Task 2 can be accomplished with content taught in Week 9, Lecture 1

Description of the inventory model for the homework activity. Consider the following inventory managed to sell a single product. The demands for the given product are independent Poisson random variables with mean 100 units per day. If X_j is the stock level at the beginning of Day j and D_j is the demand on that day, then $\min\{D_j, X_j\}$ items are sold and the stock at the end of the day is $Y_j = \max\{0, X_j - D_j\}$. Each item is sold at the price \$2. Each item which remains stocked in the inventory during the night incurs a storage cost of \$0.1 per night.

The inventory orders are made according to the (s, S) policy which is described in terms of two positive numbers s, S as follows: If the inventory level Y_j falls below level s upon conclusion of sales on Day j, the inventory manager orders $S - Y_j$ items immediately; otherwise, no order is made. Basically, this ordering policy strives to replenish the inventory to be stocked at the level S whenever the inventory level falls below the level s [1]. When an order is made at the end of sales on any given day, with probability 0.9 it arrives before midnight and can be used for the next day. With probability 0.1, it never arrives and in which case, a new order will have to be made upon the conclusion of sales on the next day. For every order which successfully arrives, the inventory manger incurs a fixed ordering cost of \$10 and a marginal cost of \$1 per item.

For Tasks 1 - 3 below, suppose that the inventory is operated with the policy parameters $s = 80$ and $S = 500$. Also assume that at the beginning of Day 1, the inventory is stocked at the level S.

Task 1 (12 points) Plot an histogram of 1000 independent samples of the total profit earned over the first week by simulating the inventory profits over 7 days.

Task 2 (10 points) Obtain an estimate for the expected total profit earned over the first week. Our simulation estimate should be such that it does not differ from the true value by more than \$2 with 0.975 probability (97.5% confidence).

Task 3 (2 + 6 = 8 points) We have learnt the following in our first probability course: If X and Y are two random variables and c is a constant, then

$$\begin{aligned}\mathrm{Var}[X+Y] &= \mathrm{Var}[X] + \mathrm{Var}[Y] + 2\mathrm{Cov}(X,Y) \text{ and}\\ \mathrm{Var}[cX] &= c^2\mathrm{Var}[X].\end{aligned}$$

[1] Obviously, for this ordering policy to be meaningful, the parameter s has to be smaller than S.

Here we use $\mathrm{Var}[X]$ to denote the variance of the random variable X and $\mathrm{Cov}(X, Y)$ to denote the covariance between the random variables X and Y. Use the above observations to show the following:

a) If X and Y are independent random variables, show that $\mathrm{Var}[X + Y] = \mathrm{Var}[X] + \mathrm{Var}[Y]$.
b) Consider the sample mean $\bar{X}_n = (X_1 + \ldots + X_n)/n$, where $X_1, \ldots, X_n$ are independent samples of a random variable X. Show that

$$\mathrm{Var}[\bar{X}_n] = \frac{\mathrm{Var}[X]}{n}.$$