B-KUL-H04X3A: Control Theory

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Assignment 3: State Feedback and State Estimation

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1 State estimator and state feedback controller design using pole placement

The goal of this assignment is to control the position of the cart along a straight line. The state is the cart position $x(t)$ [m] along the x-axis, with the wall at the origin $x = 0$ m. The cart is positioned in front of the wall, so $x < 0$. The infrared sensor measures the positive distance to the wall, i.e., it measures $-x$.

The system parameters are:

- Wheel radius: $r = 0.033$ m
- Sampling time: $T_s = 0.01$ s (100 Hz control loop)

The input $u(t) = \omega(t)$ [rad/s] is the common wheel angular velocity setpoint. The velocity control loop is assumed ideal, meaning the actual wheel velocity tracks the setpoint perfectly.

1.1 Discrete-time state equation

Continuous-time state equation: The continuous-time kinematic relation is:

$$\dot{x}(t) = v(t) = r \cdot \omega(t) = r \cdot u(t), \tag{1}$$

where $v(t)$ [m/s] is the linear cart velocity and $u(t) = \omega(t)$ [rad/s] is the wheel angular velocity input. In state-space form:

$$\dot{x}(t) = A_c x(t) + B_c u(t), \quad A_c = 0, \quad B_c = r = 0.033 \text{ m}. \tag{2}$$

Forward Euler discretization:

$$\frac{x[k+1] - x[k]}{T_s} = A_c x[k] + B_c u[k] = 0 \cdot x[k] + r \cdot u[k]. \tag{3}$$

Solving for $x[k+1]$:

$$x[k+1] = x[k] + T_s \cdot r \cdot u[k]. \tag{4}$$

The discrete-time state-space model is $x[k+1] = A_d x[k] + B_d u[k]$, with:

$$\boxed{A_d = 1, \qquad B_d = T_s \cdot r = 3.3 \times 10^{-4} \text{ m/(rad/s)}.} \tag{5}$$

1.2 Measurement equation

The measurement equation (ignoring noise) is:

$$y[k] = -x[k] \implies \boxed{C = -1, \qquad D = 0.} \tag{6}$$

1.3 Design of state feedback controller gain K

The control law is:

$$u[k] = K \cdot (x_{\text{ref}}[k] - x[k]), \tag{7}$$

Closed-loop pole:

$$x[k+1] = (1 - T_s r K) x[k] + T_s r K \cdot x_{\text{ref}}[k]. \tag{8}$$

The closed-loop system matrix and pole are:

$$\boxed{A_{\text{cl}} = z_{\text{cl}}(K) = 1 - T_s r K.} \tag{9}$$

Stability analysis: For stability, $|1 - T_s r K| < 1$, which yields:

$$\boxed{0 < K < \frac{2}{T_s r} \approx 6060 \text{ rad}/(\text{s} \cdot \text{m}).} \tag{10}$$

Yes, the system can become unstable if $K > 6060$ rad/(s·m).

Selected value:

$$\boxed{K = 40 \text{ rad}/(\text{s} \cdot \text{m}).} \tag{11}$$

1.4 Design of state estimator gain L

A discrete-time Luenberger observer is used:

$$\hat{x}[k+1] = \hat{x}[k] + T_s r \cdot u[k] + L \cdot (y[k] + \hat{x}[k]). \tag{12}$$

Estimator pole: The error dynamics $e[k+1] = (1 + L)e[k]$ have the pole:

$$\boxed{z_{\text{est}}(L) = 1 + L.} \tag{13}$$

Stability analysis: For stability, $|1 + L| < 1$, which yields:

$$\boxed{-2 < L < 0.} \tag{14}$$

Pole placement calculation: To make the estimator 4 times faster than the controller:

$$z_{\text{est}} = z_{\text{cl}}^4 = (0.9868)^4 \approx 0.948 \implies L = z_{\text{est}} - 1 \approx -0.052. \tag{15}$$

Selected value: $\boxed{L = -0.05.}$

2 Implementation and testing

2.1 Estimator only: wrong initial estimate

Figure 1 shows how the estimator converges. Larger $|L|$ yields faster convergence but more noise.

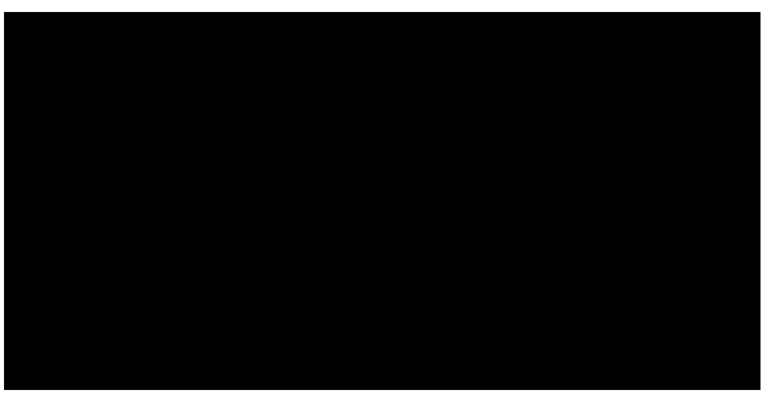

Figure 1: Measured vs. estimated distance for different L.

2.2 Controller only: proportional feedback

As K increases, the response speeds up but peak voltage increases.

Figure 2: Measured position step responses for different K.

2.3 Combined estimator and controller

Per specification, for a 10x slower estimator:

$$L_{\text{slow}} = -0.0013. \tag{16}$$

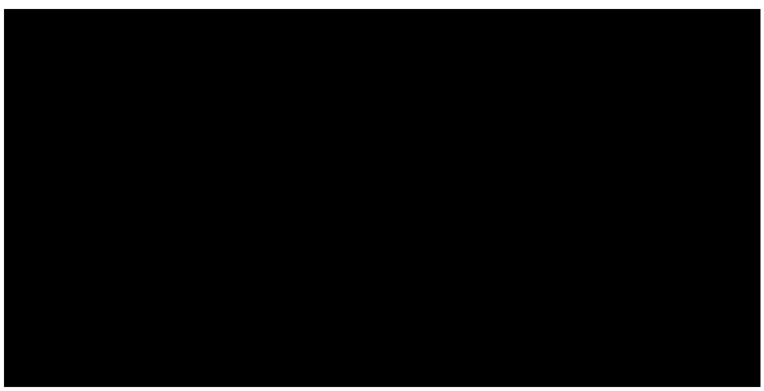

Figure 3: Combined system with wrong initial estimate and slow estimator.

Conclusion: A too-slow estimator severely reduces performance and introduces long transients, even if the system is technically stable. This is why estimators are typically designed to be faster than controllers.