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DEPARTMENT OF MECHANICAL ENGINEERING

Control Theory: Assignment 2: Velocity control of the cart

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1 Velocity Controller

The DC motor ground model from Assignment 1 serves as the foundation for this study, aiming to design a velocity controller that ensures zero steady-state error for a constant velocity reference using frequency response methods.

1a Type of controller

The velocity controller must address two main objectives: eliminating steady-state error and ensuring system stability under varying conditions. To eliminate steady-state error, the controller requires an integrator, which increases the system's type and ensures infinite gain at DC. Without this, input disturbances could cause persistent errors, even if the system transfer function contains an integrator. Stability is equally critical, as the system must remain robust and avoid instability despite fluctuations.

Several controllers were evaluated but dismissed for not meeting these requirements. The PD and lead compensators were excluded because they lack an integrator, making them unsuitable for achieving zero steady-state error. A lead compensator with added feedforward could address this issue, but its complexity makes it less practical. The lag compensator, while providing low-frequency gain, does not deliver the infinite gain required to eliminate steady-state error.

The PI controller was selected as the best solution. It includes an integrator to ensure zero steady-state error, and its straightforward design makes it easy to implement.

1b Design process and choices of design parameters

The general formulation for a PI controller is:

$$D(s) = \frac{K}{s}(s + \frac{1}{T_i}) \tag{1}$$

The crossover frequency ω_c is determined as the frequency where the uncompensated open loop system has a phase.

$$\phi = -180° + PM + 15° \tag{2}$$

We choose a phase margin (PM) of 100°with a safety margin of 15°. ϕ is then equal to -105°. In Figure 1, the determination of the crossover frequency is shown. ω_c is equal to 30.3 rad/s. Next, we will determine T_i. This is done by taking a much lower value for $1/T_i$ than ω_c. This makes sure the phase lag of the PI controller around the crossover frequency doesn't heavily affect the PM. It is calculated using the following formula:

$$T_i\omega_c = tan(90° - 15°) \tag{3}$$

T_i is equal to 0.1228 s. The final parameter to determine is the gain K. This value is calculated to ensure that the compensated system achieves a gain of 1 at the crossover frequency. In order to find K, the transfer function of the controller is transformed to discrete time to match the identified model. This is done with the MATLAB commando c2d.

$$|G(j\omega_c) \cdot D(j\omega_c)| = 1 \tag{4}$$

This results in $K = 0.841$.

Each controller design parameter involves balancing specific trade-offs. The phase margin (PM),

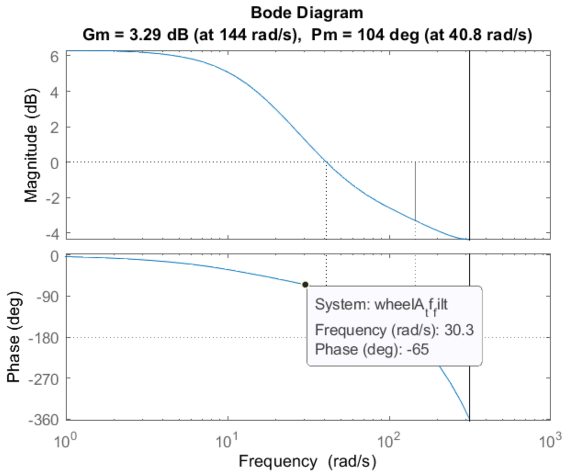


Figure 1: Crossover frequency

which indicates the system's stability, reflects the phase difference between the input and output signals. A higher phase margin enhances stability but may result in a slower transient response. Conversely, reducing the PM can improve the system's responsiveness but at the expense of stability, potentially causing overshoot and oscillations.

Similarly, adjusting the controller gain K comes with its own compromises. Increasing K can enhance both the transient response and steady-state accuracy of the system. However, this may also introduce instability, excessive overshoot, and oscillatory behavior. On the other hand, a lower K reduces instability and minimizes oscillations but often leads to slower response times and greater steady-state errors.

The Bode plot of the open loop is shown in Figure 2. The gain at crossover frequency is equal to 0 dB and the PM is also correct. We can conclude that the parameters of the controller were calculated correctly.

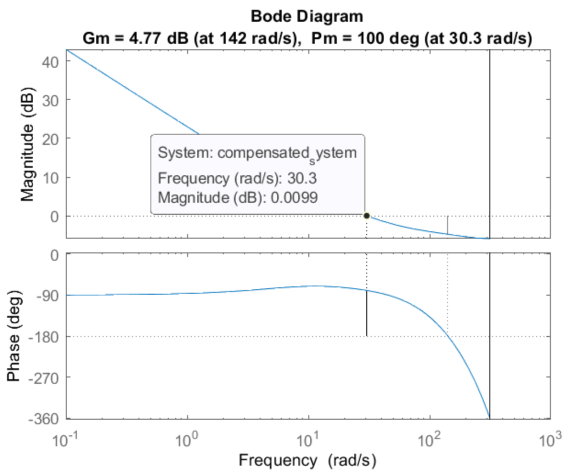


Figure 2: Open loop Bode diagram

1c Theoretical and practical limitation

The controller's crossover frequency determines the closed-loop bandwidth, as higher crossover frequencies theoretically enhance bandwidth. However, this comes with trade-offs: increasing the crossover frequency reduces the phase margin, potentially compromising stability. To ensure stability, a phase margin of 40° to 50° is theoretically recommended, as lower margins can lead to oscillations and overshoot, thereby constraining the achievable bandwidth. Conversely, excessively high phase margins expand the transient response, delaying the system's convergence to steady state.

Practical constraints further limit the closed-loop bandwidth. The microcontroller, operating with a 10 ms sampling period, imposes a maximum bandwidth of 50 Hz to prevent aliasing. Additionally, the motors require higher input voltages at elevated crossover frequencies, constrained by the practical voltage limit of 12V. These limitations restrict the achievable bandwidth despite theoretical possibilities.

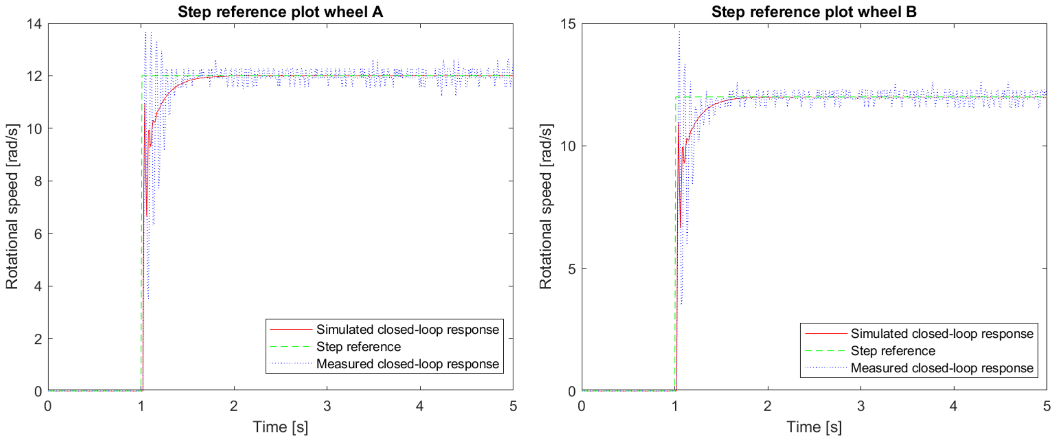
2 Experimental validation of designed controller

2a Controller implementation and step response testing

Figure 3 presents the step reference, the measured closed-loop response, and the simulated closed-loop response, tuned to a phase margin of 100°, identified experimentally as optimal. While theory suggests stability for phase margins between 40° and 50°, tests showed that a PM of 60° already led to instability at higher speeds. Increasing the PM to 90° improved stability but was insufficient for reliable high-speed performance. A PM of 100° ensured stability across all speeds, though further increases would slow the system's convergence to steady state due to a more pronounced transient response. Despite minor oscillations, the average rotational speed aligns closely with the reference, consistent with the PI controller's theoretical property of eliminating steady-state error.

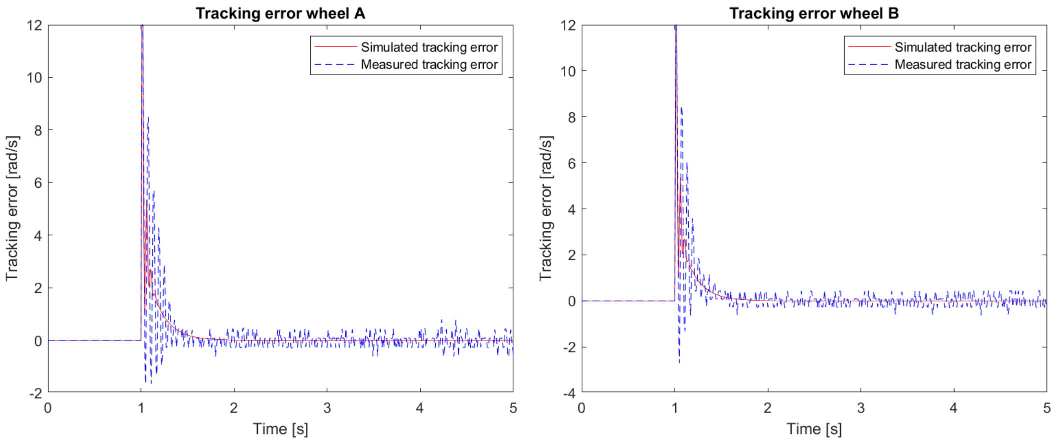
Figure 4 illustrates the measured and simulated tracking errors for the step reference. Both tracking errors converge to zero, confirming that the system achieves zero steady-state error, consistent with theoretical expectations.

Finally, Figure 5 depicts the measured control signal alongside the simulated control signal for the step reference. The simulated control shows higher peaks in the transient behavior and reaches the steady state more quickly than the measured signal, which can be attributed to the slower convergence of the measured rotational speed. These differences highlight the impact of system non-idealities on performance.



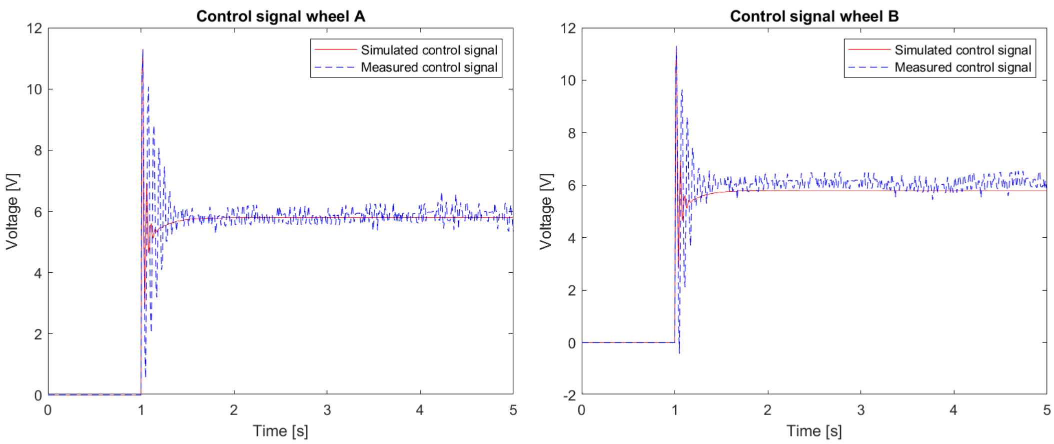
(a) Wheel A (b) Wheel B

Figure 3: Step reference, measured closed-loop response and simulated closed-loop response



(a) Wheel A (b) Wheel B

Figure 4: Simulated and measured tracking error



(a) Wheel A (b) Wheel B

Figure 5: Simulated and measured control signal

2b Disturbance response and steady-state performance validation

Figure 6 illustrates the point at which the disturbance f(k) is introduced into the control loop. The constant force is applied by having the cart move on an incline, where gravity serves as the constant force acting on the system.

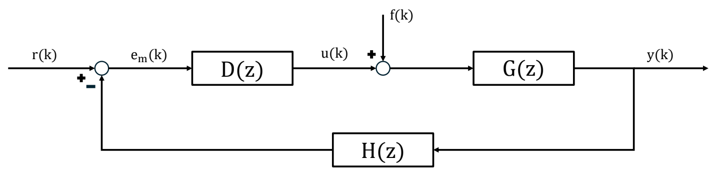
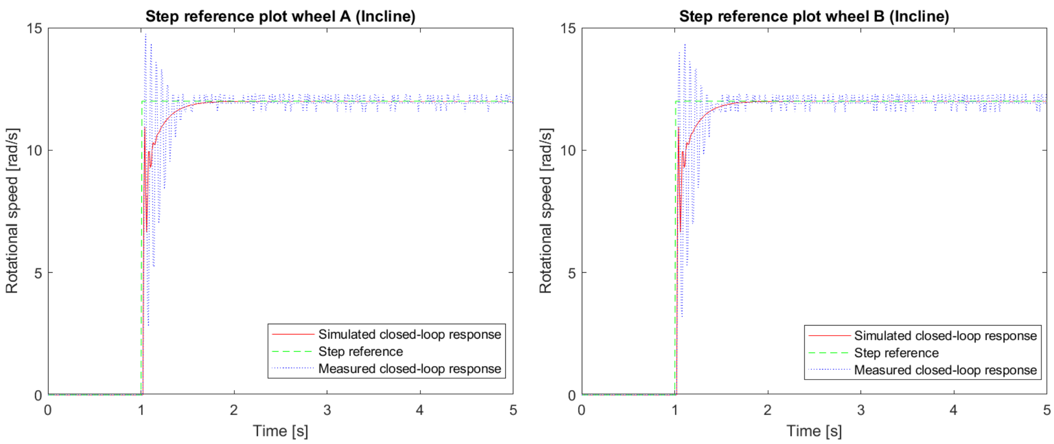


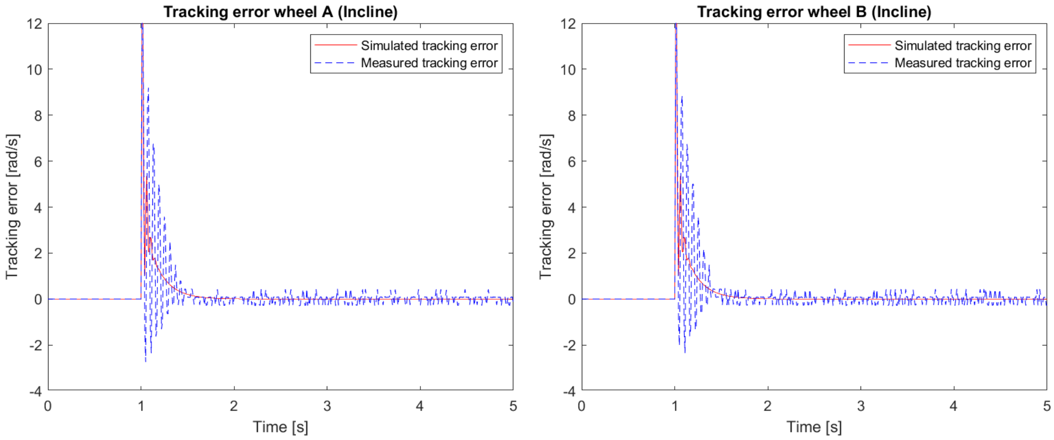
Figure 6: Block diagram with disturbance f(k)

Figures 7, 8, and 9 present the same plots as in Section 2a, now with the addition of a force disturbance signal. Comparing these graphs to those in Section 2a, it is evident that the controller continues to track the reference despite the disturbance. This performance is attributed to the type-one system, where the well-tuned PI controller effectively compensates for steady-state disturbances by adjusting the control signal. Additionally, in Figure 9, the measured motor voltage is higher than the simulated one, which is due to the additional gravitational force introduced by the incline.



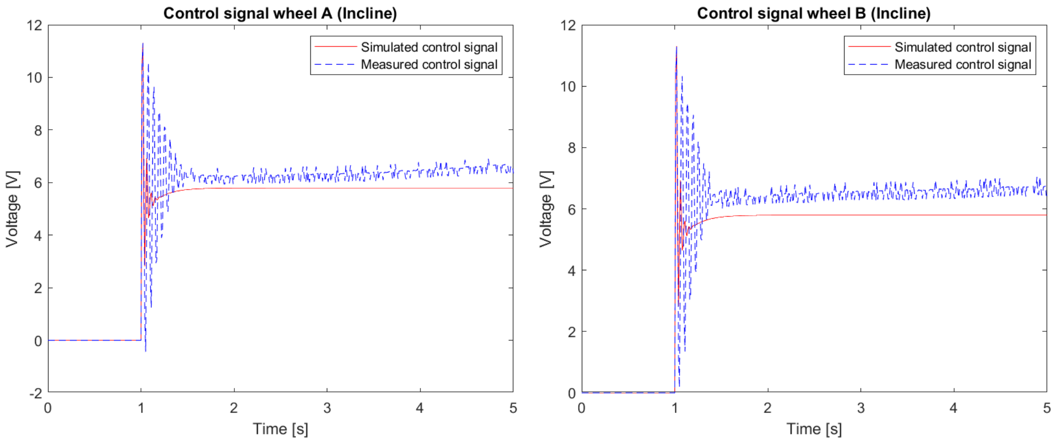
(a) Wheel A (b) Wheel B

Figure 7: Step reference, measured closed-loop response and simulated closed-loop response on inclined surface



(a) Wheel A (b) Wheel B

Figure 8: Simulated and measured tracking error on inclined surface



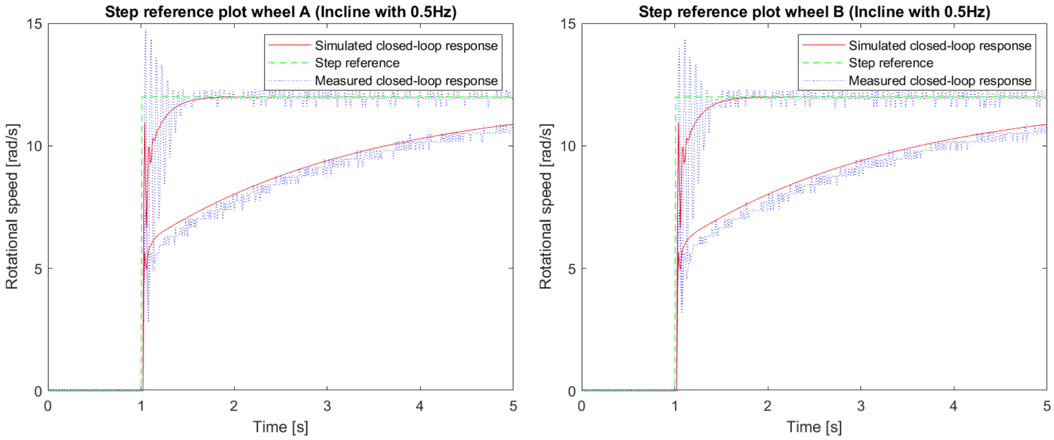
(a) Wheel A (b) Wheel B

Figure 9: Simulated and measured control signal on inclined surface

2c Controller redesign with 0.5 Hz cross-over frequency

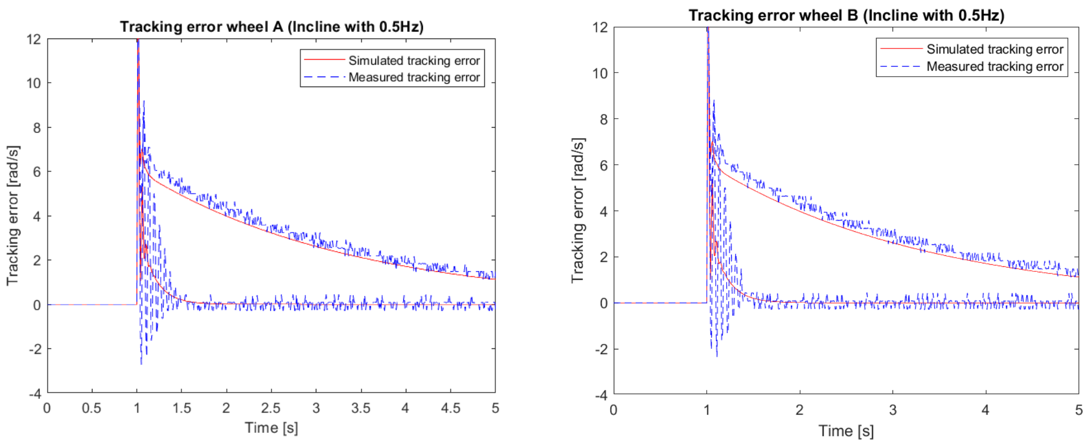
The results for this configuration are presented in Figures 10, 11 and 12, which include both the previously obtained data from 2b and the new results for $\omega_c = 0.5Hz = 3.14rad/s$. This crossover frequency corresponds to $K = 0.4741$.

The response for $\omega_c = 0.5\,\mathrm{Hz}$ exhibits slower dynamics, consistent with the higher stability margin and more damped behavior. This effect is particularly noticeable in the extended time required to reach steady-state. These findings are in accordance with the theoretical expectations, which predict that decreasing the crossover frequency increases system stability but also leads to a slower response.



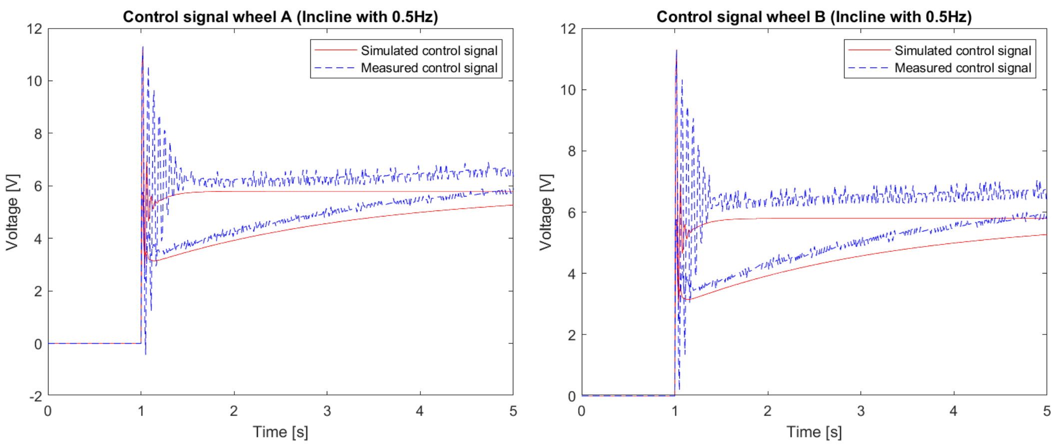
(a) Wheel A (b) Wheel B

Figure 10: Step reference, measured closed-loop response and simulated closed-loop response on inclined surface at 0.5 Hz



(a) Wheel A (b) Wheel B

Figure 11: Simulated and measured tracking error on inclined surface at 0.5 Hz



(a) Wheel A (b) Wheel B

Figure 12: Simulated and measured control signal on inclined surface at 0.5 Hz